CSCI 4/5576: Project Proposal
The Random Logistic Map

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1 Introduction

The purpose of this project is to characterize the Random Logistic Map. In particular, we will be studying the stability of the map, which includes locating fixed points and generating bifurcation diagrams. As the map has an element of randomness to it, many simulations (a large N) would be required for statistical analysis. This project is a subset of a larger work in progress and requires optimization.

2 Description and Background

2.1 Deterministic Logistic Map

The Logistic map is a popularly studied topic in nonlinear dynamics. The Logistic function is described as follows:

$$\hat{f}(x) = rx(1-x) \tag{1}$$

where r is a parameter that takes on values in [0,4]. To get the Logistic Map, one simply discretizes the function by replacing $\hat{f}(x)$ with x_{n+1} and x with x_n:

$$x_{n+1} = rx_n(1-x_n)$$

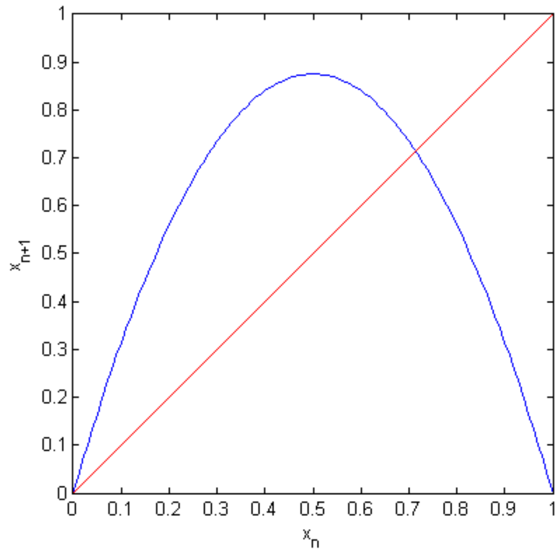

Figure 1: The deterministic Logistic Map.

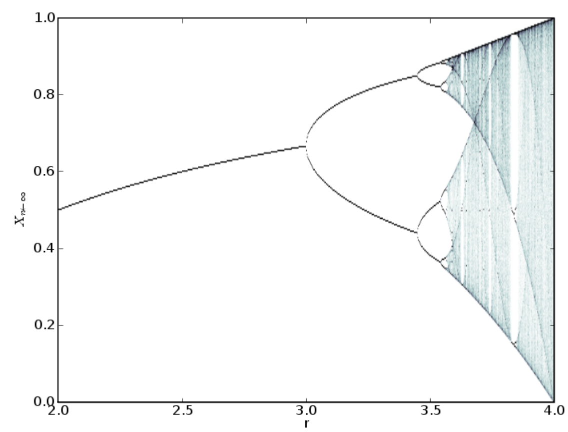

Figure 2: The bifurcation diagram for the deterministic logistic map.

Fixed points are located where the line $y = x$ intersects with the Logistic Map. As the parameter r is increased or decreased over the interval [0,4], the map undergoes period doubling. Around $r = 3.6$, the map begins to display chaotic behavior.

2.2 Random Logistic Map

The Random Logistic function is a special case of the Logistic function, which is not as popularly studied. In the random case, r becomes a value dependent on x. Essentially, the Random Logistic Map introduces the notion of spatial randomness to the deterministic function (1). The Random Logistic function and map are as follows:

$$f(x) = R(x)x(1-x)$$
$$x_{n+1} = R(x_n)x_n(1-x_n) \tag{2}$$

$R(x)$ consists of a Fourier series whose coefficients are independent (but not identical) random variables drawn from a uniform distribution.

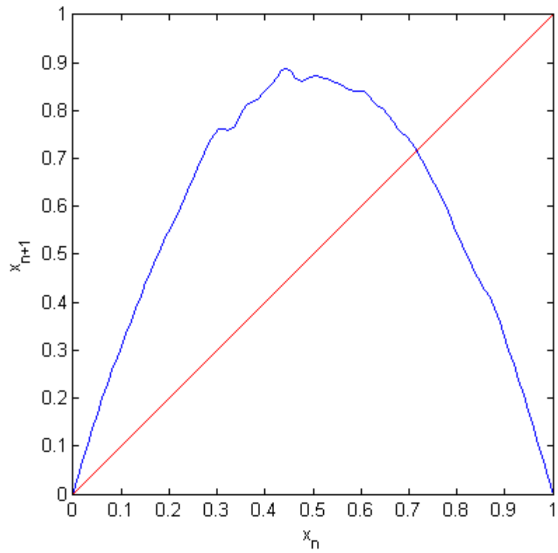

Figure 3: One instance of a random logistic map.

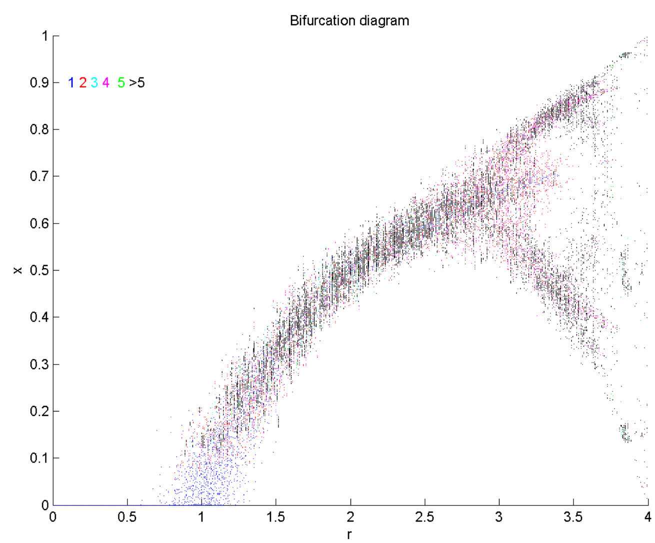

Figure 4: The colors of the bifurcation diagram coincide with the period order in the legend (top left corner).

An initial serial implementation has been developed for this project and mildly tested in MATLAB. Our project would consist of converting the existing code to C++ and implementing parallelization through OpenMP. Other possible improvements include IO optimization and memory optimization (possible improvements through HDF5).

2.3 Application to engineering

This project has potential applications to civil engineering in terms of treating contaminated groundwater [1]. Essentially, the distribution of rocks underground can be viewed as a random field distributed spatially, through which treatment solution must mix with contaminated groundwater. Understanding the problem of spatial randomness in one dimension can be generalized to two dimensions, with the end goal of successfully simulating chaotic advection and reaction in two dimensions.

3 Analysis

The analysis of our simulation will encompass the following:

1. Compare the MFLOP/s (using profiling from psrun) for a variable number of simulation sizes.
2. Measure the speedup after implementing OpenMP.
3. Analyze the Karp-Flatt Metric to see how the code performs in parallel.
4. Compare the performance of our simulation in terms of GFLOP/s to the bite:flop ratio in the Roofline plot

References

[1] Roseanna M. Neupauer, James D. Meiss, and David C. Mays. Chaotic advection and reaction during engineered injection and extraction in heterogeneous porous media. *Water Resources Research*, Volume 50, 2014.