HPC sheet 8

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Ex 1: NVIDIA Tesla V100 SXM2

- memory size: 16|32 Gb (HBM2)
- 1/SM L1 Cache: 128 KB (per SM)
- 1 L2 Cache: 6MB (6144 KB)
- memory bandwidth: 900 GB/sec
- max. number of threads per block: 1024
- TFlops: 7,8 (double prec.), 15,7 (single), 120 (tensor)
- 80 SM (streaming multiprocessors), 640 TC (8 per SM)

Ex 2: Tensor cores

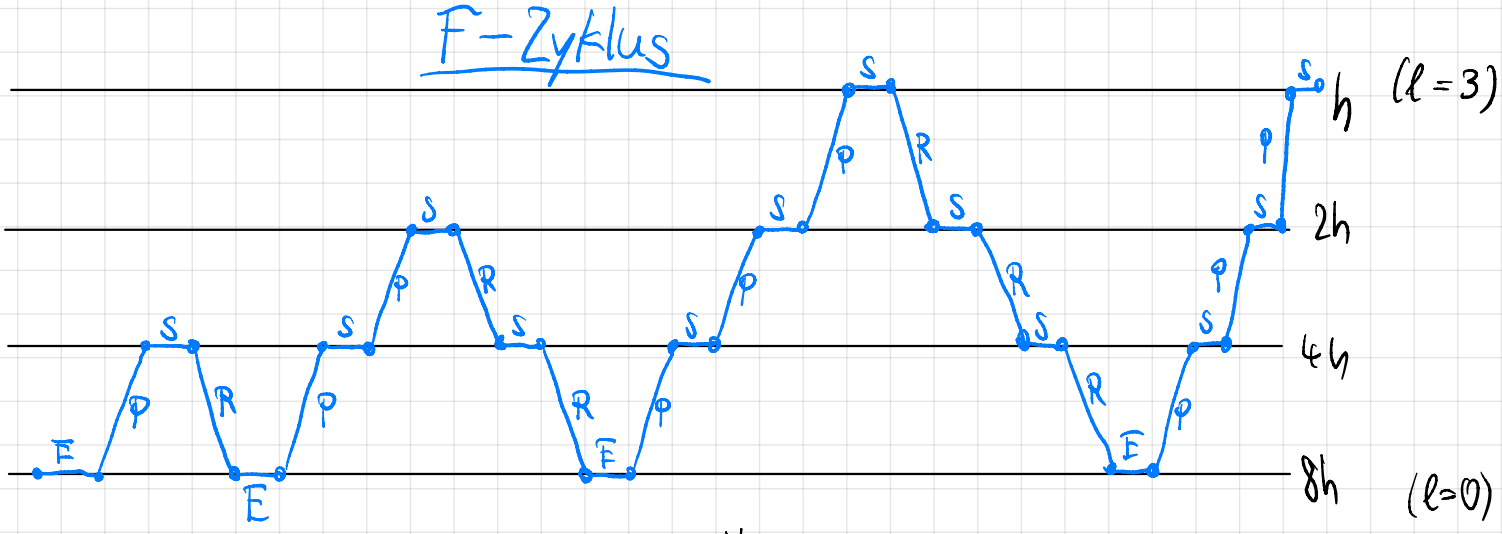
- introduced in Volta architecture of NVIDIA (Tesla V100)
- developed for machine learning
- each tensor core can multiply two 4×4 (16bit) matrices into one 4×4 (16/32bit) matrix (and add a matrix - fused multiplication addition).
  - therefore called mixed precision multiply/add operations

Ex. 3:

In a V-cycle we start at the finest grid, where we do smoothing and then keep reducing (and smoothing) the grid size until we reach a size at which we solve the problem directly. After that we prolongate until we get to the finest grid.

Instead in a F-cycle we start at the coarsest grid, where we use a direct solver to get a solution. This solution is prolongated to the next finer grid, where we start a v-cycle. We iteratively start a new V-cycle from a finer grid until we reach the finest.

Sketch:

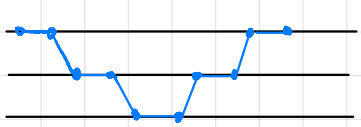


Flops per iteration of 1D - multigrid:

- N grid size (finest level)
- 3 levels ($\ell=0$: $\frac{N-3}{4}$, $\ell=1$: $\frac{N-1}{2}$, $\ell=2$: N)
- 5 times smoothing (jacobi method)
- exact solving with LU decomposition and forward/backward substitution

	Flops in dense implementation
$\ell=2$ smoothing:	$5 \cdot 2(N^2-N)$
$\ell=1$ smoothing:	$5 \cdot 2\left(\left(\frac{N-1}{2}\right)^2 - \frac{N-1}{2}\right)$
restriction $\ell=2 \to \ell=1$: $R_{2h}^{h} \in \mathbb{R}^{\frac{N-1}{2} \times N}$:	$2\left(\frac{N-1}{2} \cdot N - \frac{N-1}{2}\right)$
restriction $\ell=1 \to \ell=0$:	$2\left(\frac{N-3}{4} \cdot \frac{N-1}{2} - \frac{N-3}{4}\right)$
lu decomposition:	$\approx \frac{2}{3}\left(\frac{N-3}{4}\right)^3$
forward/backward subst:	$2\left(\frac{N-3}{4}\right)^2$
prolongation $\ell=0 \to 1$:	$2\left(\frac{N-1}{2} \cdot \frac{N-3}{4} - \frac{N-1}{2}\right)$
prolongation $\ell=1 \to 2$:	$2\left(N \cdot \frac{N-1}{2} - N\right)$

a) V-cycle:



$\ell=2$, N

$\ell=1$, $\frac{N-1}{2}$

$\ell=0$, $\frac{\frac{N-1}{2}-1}{2} = \frac{N-3}{4}$

2 times smoothening $\ell=2$

2 times smoothening $\ell=1$

1 time exact solving

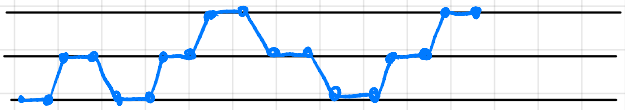
1 restriction $\ell=2\to1$

1 restriction $\ell=1\to0$

1 prolongation $\ell=0\to1$

1 prolongation $\ell=1\to2$

b) F-cycle:



3 times exact solving

3 times prolongation $\ell=0\to1$

2 times restriction $\ell=1\to0$

4 times smoothening $\ell=1$

2 times smoothening $\ell=2$

2 times prolongation $\ell=1\to2$

1 times restriction $\ell=2\to1$