Mobile Chair Lift

MENG 4350: Machine Design

Spring 2019

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Executive Summary

Since 1950 the average life expectancy has increased by ten years and continues to rise with the aid of better technology and medical advancement. Often, the increased longevity brings a host of physical failure. Many seniors experience muscle atrophy and loss of coordination. This group was tasked with designing and manufacturing a device with the ability to aid a fallen person to stand when they otherwise lacked the muscle and coordination to achieve a standing position unassisted. The request came from a senior veteran who has been experiencing falling and hence a loss of his independence due to relying on emergency personnel for assistance. This group researched current assisted lifting devices available in hospitals and senior care centers and gained insight although many of those devices required the person to be located in a bed or wheelchair to start or required several people to use. Taking some ideas from the current lifting devices in use, this group designed a lift that could be easily positioned next to the fallen individual and be sat in like a chair. This “chair” would then lift the person to a height where they could regain their footing and eventually stand without requiring emergency services.

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Objectives

The main objective is to design and build a product in the given timeframe, under budget, and most importantly meeting the expectations of the customer. This project is designed to utilize and showcase this group's knowledge of machine design by properly designing and analyzing a safe and reliable product that can be used easily and efficiently to suit the customer's needs.

1. Introduction

Mechanical Engineering Design is used to solve a problem to satisfy a need relating to anything of physical properties. A design of something, whether it be a machine, tool, structure, or other object, is very reliant on the designer, therefore, it needs to be intelligently and carefully created [1].

1.1. Design Process

Mechanical Engineering Design is a complex method that requires diligent adherence to the design process in order to create a safe and effective product. In an effort to simplify, the process can be broken down into several stages as seen in Figure 1 [1]. The design process starts with the identification of a need, this can be in the form of something as simple as solving an everyday problem that has no solution or noticing that a product that has been designed already is not meeting the need that it was designed for. The second step in the design process focuses on the requirements that must be addressed to solve the need identified in the first step. The third and most important step in the design process is synthesis. This step requires multiple variations of a solution to the problem that was identified in the first step.

When a concept has been approved or parts from different variations have been implemented into a different concept, that concept will then move to the analysis and optimization phase. In this segment it is necessary to perform calculations based on the design and determine the optimal conditions of each of the components that constitution your concept. This step allows for optimization if the results of the completed calculations do not meet the standards of the need identified in step one. At this point in the process, adequate evaluation of the design concept is necessary in order to determine the suitability of each component and the total product. If the design or components are found to be inadequate the design team would return to the synthesis step and recalculate another potential solution. The arrival in the design process at the evaluation step indicates that a design has successfully navigated the design process and a suitable prototype is completed. Evaluation and testing of the prototype commence at this juncture. The testing of the prototype proves the concept and design solves the need that was identified in step one. The evaluation step shows whether the prototype is reliable, competitive with similar products, economical to manufacture, and easy to use. The final segment of the design process is the presentation of the concept and final design to the consumer. The presentation of inception, conceptual research, design, creation, calculations, and testing prove the product's value for the application to the consumer, investor, or professor.

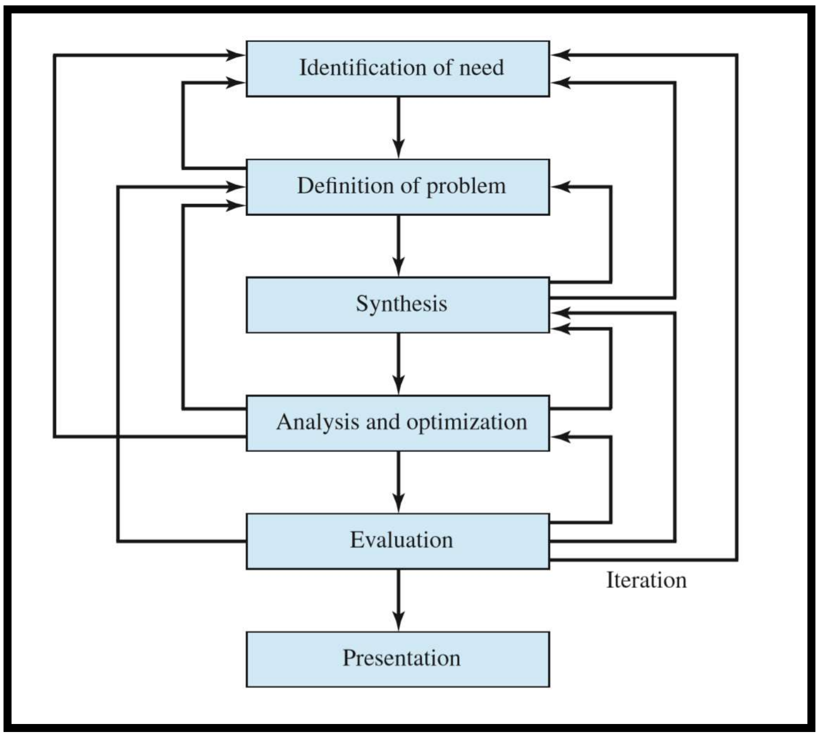


Figure 1 - Design Process Flowchart

1.2. General Design Requirements

Designing engineers work to create solutions to problems or issues while satisfying and meeting current policies, and codes. Several sub-factors considered when designing are corrosion, friction, marketability, noise levels, as well as economic, safety and reliability standards. A correct mechanical design needs to meet or exceed all of the aforementioned standards and expectations.

1.3. Problem Definition

1.3.1. Scope

The problem this group was tasked with solving was the creation of an assisted lifting device for a person who has fallen that requires minimal aid from others. The problem was that a relatively independent elderly veteran would occasionally lose his balance and fall at home and was unable to rise on his own power. The man requested of this group, a solution to his problem that would allow him to keep his pride and independence with minimal aid from his wife instead of emergency services.

1.3.2. Technical Review

This group's client is an elderly veteran who in the past has fallen and was unable to rise without calling emergency services. The elderly man's wife was unable to help him rise on her own and the man has lost some of his independence and pride during these incidents. The client requested a mechanical design that would allow his elderly wife to assist him in rising after a fall thus removing the need for emergency services and further embarrassment. The client is around five foot six inches tall and around three hundred pounds. The device will aim to lift him to a normal chair height sitting position from which he can easily stand. The client set a budget of one thousand dollars for the project that this group intended to remain below during the creation of the prototype and also attempted to get donations in appreciation of his service.

1.3.3. Design Requirements

Limiting factors of the design were reliability, maneuverability, weight, size, and ease of use. Considering the machine is essentially a medical aid device, the goal is to design and analyze the product to be as safe and reliable as possible. The device needs to be portable enough to be used in and around the customers house. To meet the requirement of indoor use, it would need a foot print that will allow it to be maneuvered through a standard doorway while still being sturdy enough to have a lifting capacity that would be suitable for the client. To meet the requirement of outdoor use, it would have to be corrosion resistant, be battery powered, and able to move along uneven surfaces. To meet the criteria of maneuverability, OSHA requires a maximum pushing force of 50lbs [2]. Since our assisted lifting device is not intended to carry the client while pushing it, this maximum pushing force is intended for the lifting device while unoccupied.

1.3.4. Constraints

Constraints experienced while designing this project were budget, tool access and fabrication. The budget set by the customer was one thousand dollars and this group was determined complete the project closer to seven hundred and fifty dollars with the help of donations towards veteran assistance. Access to tools was found not to be a concern once the design was settled on and material chosen. The fabrication of the prototype was done through welding which was done by a group member who is a professional welder. Further access to a CNC laser was available through the workplace of a group member. The CNC laser was to be used to cut components necessary to complete the build. Many of the components were found within the local area and only minimal parts were outsourced. The ordering of these outsourced parts was done early in an attempt to gain part access early on for testing and analysis.

2. Concept Design

2.1. Preliminary Design Concepts

Post discussions with the client regarding his needs and specifications, this group met and brainstormed design concepts. Over the course of five group meetings, the group assessed each design and extracted good concepts from each to utilize in the final product. Through these meetings and discussions, the group produced four valid potential designs. Each design was drawn using Fusion 360 and then a Pugh matrix was used to determine the pros and cons of each design which the group used to choose the final concept.

2.1.1. Dolly Fork-Lift Concept

Figure 2 - Dolly Fork Lift Style Concept shows the first design concept which was a dolly and fork lift style combination for assisted lifting. This design was to be constructed using lightweight and easily accessed metal for structure base and the forks would be a strong coated material that would make the lifting process as comfortable as possible for the client. The forks would have been placed under the client's arms and would function as a strong man lifting him up from behind. The pros of this design were the cost, weight and manufacturability as shown in Table 1 - Design Pugh Matrix. The cons of this design were ease of use, portability and compactness. It was also surmised by the group that it may be painful for the client to be lifted in this manner.

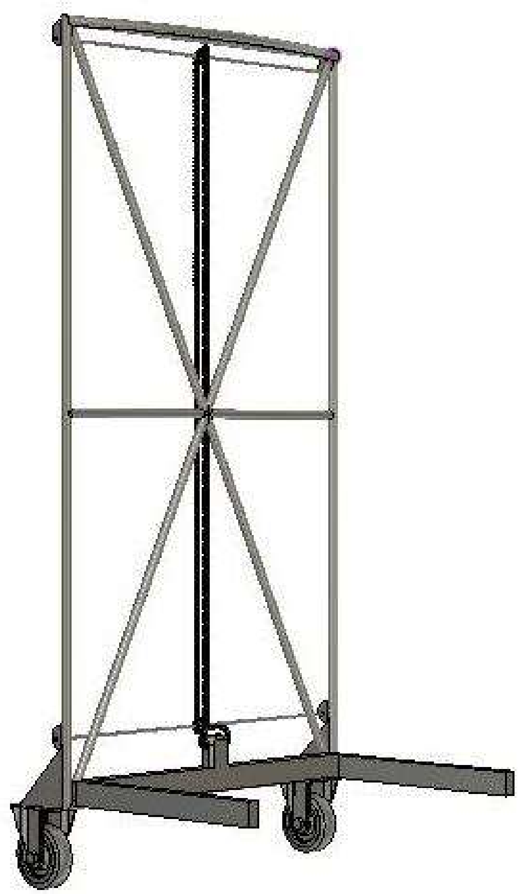


Figure 2 - Dolly Fork Lift Style Concept

2.1.2. Platform Stretcher Concept

The second design concept as shown in Figure 3 - Platform/Ambulance Stretcher was a platform stretcher similar to those used in ambulatory services. This device would allow the client to roll onto it and in a horizontal position be lifted. The pros of this design were ease of use and compactness according to Table 1 - Design Pugh Matrix, and the cons were cost, weight, manufacturability, and portability. It was further discussed by the group that there could be a concern with pinch points and functionality with this design concept.

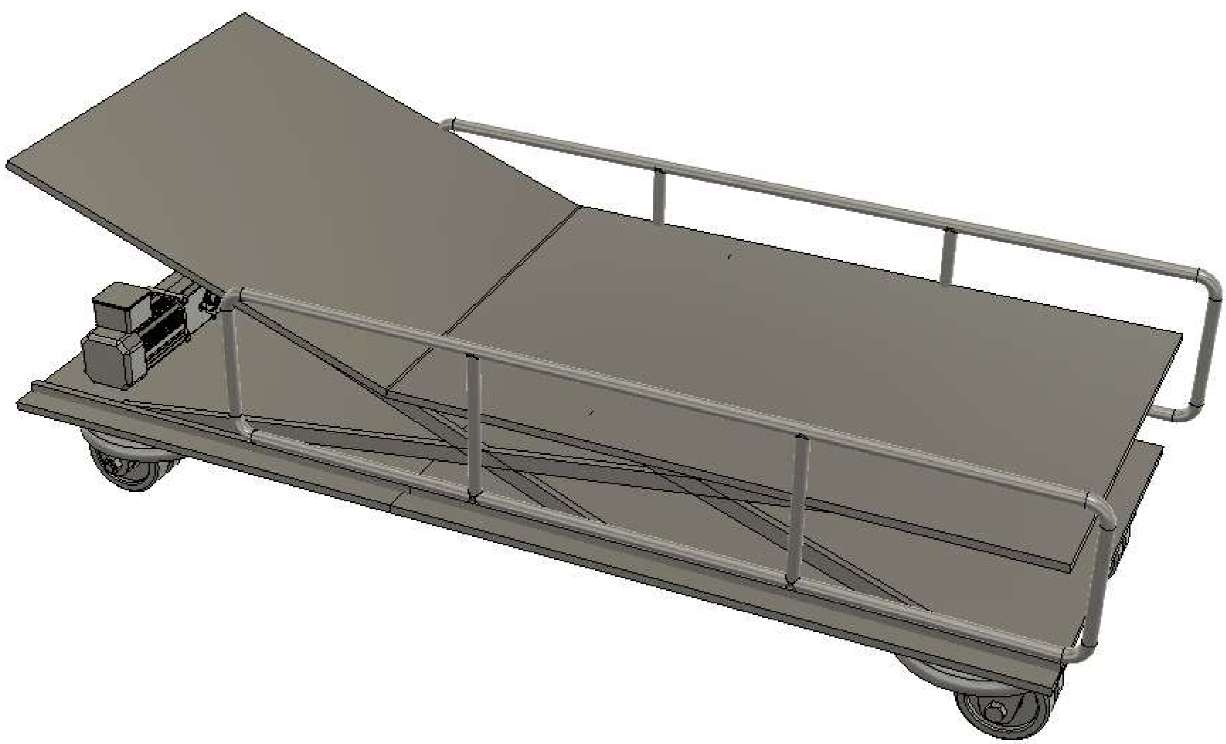


Figure 3 - Platform/Ambulance Stretcher

2.1.3. Chair – Dolly Concept

Figure 4 - Chair/Dolly design shows a third concept. This design is a basic elevating platform on a dolly that can be moved around similar to a furniture dolly. This device would be wheeled to the customer when in need, and after they get positioned on the seat, it would raise them vertically to the height required for them to get back on their feet. The pros to this design were the cost of manufacture and the weight of the final build. The drawbacks were the stability and safety while in use.

Figure 4 - Chair/Dolly design

2.1.4. Walker/Lift Chair

The walker/chair concept was selected as the optimum preliminary design, but it had to be optimized to reduce weight. The group used this as a base for the final design, and incorporated aspects of some of the other concepts to improve the product and meet the design requirements.

Figure 5 - Walker Style Chair

2.2. Selection of Design Concept

The Pugh matrix is a tool that assists in the selection of a concept or product by rating the various concepts by their strengths and weaknesses.

2.2.1. Criteria and scaling

The importance of each criteria is rated from one to six, with one being least important and six as the most important. Each design is then evaluated according to the set criteria using a binary scale from -1 to 1, where -1 indicates that the concept fails to meet that particular criteria requirement, 0 meaning the concept met the criteria and 1 if the criteria was met and surpassed.

2.2.2. Design Pugh Matrix

For the design Pugh matrix, the group used a scaling system to rate the criteria of the design. The most important advantage was ease of use. This advantage was decided as the most important due to the client and his wife being elderly. The second most important concept was the weight of the device also due to the client being elderly, and in order to meet OSHA standards. The cost was chosen as third in line of importance because of the parameters set by the client himself. Portability was fourth in importance as the couple is older and the device will be used in a variety of locations including outdoors. Manufacturability was second to last in order of importance due to the group members having experience and access to necessary equipment. Compactness was listed last in order of importance as it was not a criterion set by the client as well as considerations of time and budget constraints. Table 1 - Design Pugh Matrix shows that several of the design concepts met the criterial well, whereas the platform stretcher design had difficulty meeting the advantages listed on the matrix.

Table 1 - Design Pugh Matrix

		Design							
Advantage	**Importance**	**Platform/Ambulance Stretcher**		**Chair/Dolly**		**Walker Style Chair**		**Fork/Dolly**	
Cost	4	-1	-4	1	4	0	0	1	4
Weight	5	-1	-5	1	5	1	5	1	5
Manufacturability	2	-1	-2	0	0	1	2	0	0
Ease of Use	6	0	0	0	0	1	6	-1	-6
Portability	3	-1	-3	0	0	1	3	-1	-3
Compactness	1	0	0	0	0	1	1	-1	-1
Total		**-14**		**9**		**17**		**-1**	

This group determined that since the client was a private individual and not a company, it was important that the cost of the build remain as minimal as possible which caused the group to choose more feasible material. The calculations performed on the material showed that the framing metal was larger than required for stability. The smaller sized material was not chosen for use however, because the uncommon size was more expensive and harder to obtain than the larger pipe as seen in Table 2 - Pipe Cost of the device if need be.

Table 2 - Pipe Cost

Supplier	Build material	Size	$/ft
Silver Star (Ama)	ALM SCH 40	1"	2.39
Klockner metals	STEEL SCH 40 ASTM A53 PIPE GRADE B	1/4"	1.8
		3/8"	2.4
		1/2"	0.77
		3/4"	1.02
		1"	1.51
Scrap Proc	STEEL SCH 40 ASTM A53 PIPE GRADE B	1/4"	1.67
		3/8"	2.16
		1/2"	0.76
		3/4"	1.09
		1"	1.51

2.2.3. Drive Mechanism

Many drive options were available to power the chair lift, so a Pugh matrix was used to select the optimal choice for the final product. The rating criteria for this matrix was the same as previous Pugh matrix used, and the winch was selected as the most efficient drive for the design.

Table 3 - Lifting Drive Pugh Matrix

		Lifting Drive					
Advantage	**Importance**	**Linear Actuator**		**Hydraulic Cylinder**		**Winch**	
Cost	3	-1	-3	-1	-3	1	3
Weight	1	1	1	-1	-1	0	0
Compactness	2	1	2	-1	-2	1	2
Lifting Capacity	4	0	0	1	4	1	4
Total		**0**		**-2**		**9**	

2.2.4. Battery Selection

There were two main batteries we could choose from; Lead Acid and Lithium Ion. Lead acid is used in most vehicles for starting and are known for their continuous cycles where lithium-ion batteries are most commonly found in electronics and are known for their large current output. Our group kept going back and forth between the two and finally chose the Lithium-Ion battery due to its weight, as seen in **Error! Reference source not found.**.

Table 4-Battery Pugh Matrix

		Design			
Advantage	**Importance**	**Lead Acid**		**Lithium-Ion Battery**	
Cost	2	0	0	-1	-2
Weight	3	-1	-3	1	3
Manufacturability	1	1	1	0	0
Total		**-2**		**1**	

3. Design Description

3.1. Component Design:

After our design had been selected, we immediately began locating critical points of stress and areas of high stress concentration. Considering that the reliability of the lift needs to be as close to 100% as other conditions will allow, our safety factor guarding against yielding should not fall below 2.5. Since the budget is allowing, our group concluded that for that requirement to be met, we would also take a conservative approach in finding the safety factor. All deductions of safety factor are calculated then, using Maximum Sheer Stress Theory from the equation below from Shigley's Mechanical Engineering Design book as equation 5-1 [1].

$$n := \frac{S_y}{\sigma_1 - \sigma_3} \qquad [1]$$

3.1.1. Frame Shaft for Pulley

To start off, we first analyzed segments of our pipe frame. We know, as decided in Section 2.2.2., the pipe is going to be made of ASTM A53 Steel Grade B pipe. This gives us the yield strength of 35000 psi.[3]

$$S_y := 35000 \ \frac{lb}{in^2}$$

The upper shaft supporting the pulley is going to be one of the most load bearing in this design. Figure 31 We first analyzed the maximum shear and stress of this support as can be seen in Figure 32 and Figure 33 in Appendix B. From the diagram we can see that the max moment and shear values are

$$M_{max} := 422.67 \ lb \cdot in$$

$$V_{max} := 400 \ lb$$

To find the most appropriate pipe size for this support, we compiled a list of marketed pipe sizes to work with, along with their corresponding actual dimensions.

$$d := \begin{bmatrix} 0.125\ in \\ 0.25\ in \\ 0.375\ in \\ 0.5\ in \\ 0.75\ in \\ 1\ in \end{bmatrix} \quad d_o := \begin{bmatrix} 0.41\ in \\ 0.54\ in \\ 0.675\ in \\ 0.840\ in \\ 1.050\ in \\ 1.315\ in \end{bmatrix} \quad d_i := \begin{bmatrix} 0.269\ in \\ 0.364\ in \\ 0.493\ in \\ 0.622\ in \\ 0.824\ in \\ 1.049\ in \end{bmatrix}$$

Our vector, d, represents the list of marketed pipe sizes, and d_o and d_i represent actual pipe outer and inner diameters. From these we can calculate the pipe's cross-sectional properties.

First, our area is given by the following equation:

$$A := \frac{1}{4}\pi \cdot \left(d_o^{\ 2} - d_i^{\ 2}\right) \qquad [2]$$

Then our second moment of area about the neutral axis is calculated.

$$I := \frac{1}{64} \cdot \pi \cdot \left(d_o^{\ 4} - d_i^{\ 4}\right) \qquad [3]$$

Then our second polar moment of area about the centroid is given as:

$$J := \frac{1}{32} \cdot \pi \cdot \left(d_o^{\ 4} - d_i^{\ 4}\right) \qquad [4]$$

To simplify the number of equations required, we compiled a matrix of critical point diameters based on market pipe sizes.

$$cpr := \text{augment}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} in, \left(\frac{d_i}{2}\right), \left(\frac{d_o}{2}\right)\right) = \begin{bmatrix} 0 & 0.135 & 0.205 \\ 0 & 0.182 & 0.27 \\ 0 & 0.247 & 0.338 \\ 0 & 0.311 & 0.42 \\ 0 & 0.412 & 0.525 \\ 0 & 0.525 & 0.658 \end{bmatrix} in$$

From the resulting matrix, cpr (critical point radius), each row represents the six different sold pipe sizes and the three columns represent the radius to each critical point.

In the case of the pipe, the three critical points are the neutral axis (Critical Point 0), inner diameter (Critical Point 1), and outer diameter (Critical Point 2) as seen in Figure 6 - Pipe Cross-Section.

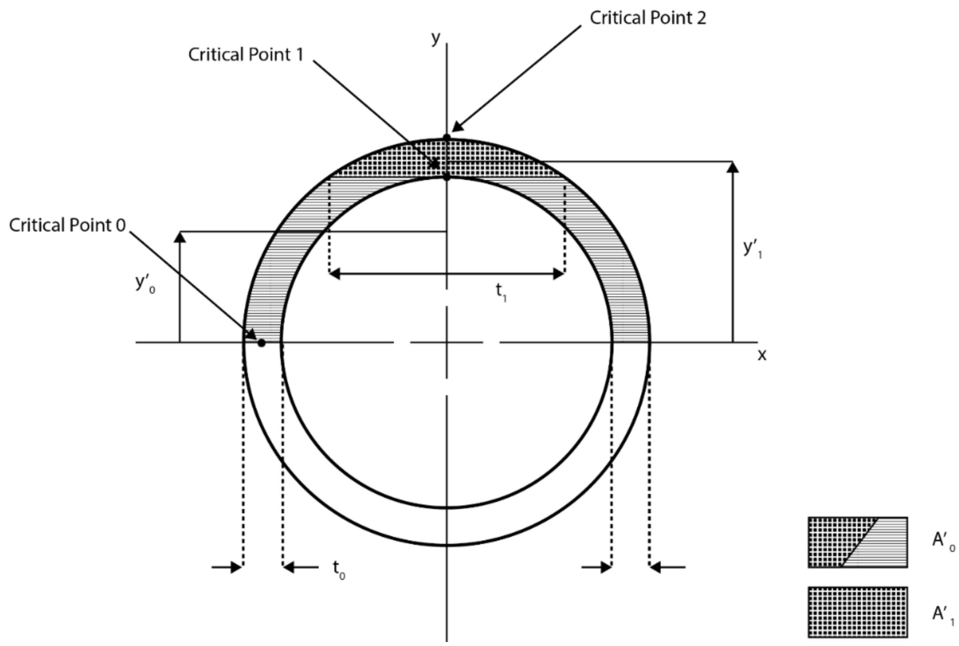


Figure 6 - Pipe Cross-Section

With this matrix, we can then find the component stresses at each critical point.

3.1.1.1. Critical Point 0: Neutral Axis

This critical point is the neutral axis. Although there is no axial stress on this axis (hence the name), the shear stress is maximum along this axis. Therefore, to calculate stress, we only need the thickness of the shear axis, which is given as

$$t_0 := d_o - d_i \qquad [5]$$

And the total cross section area above neutral axis created by the two concentric circles

$$A'_0 := \frac{A}{2} \qquad [6]$$

3.1.1.2. Critical Point 1: Inner Wall

This critical point lies on the inner wall of the pipe. It experiences both axial and shear stress.

Its thickness is given by

$$t_1 := 2 \cdot \sqrt{d_o^{\ 2} - d_i^{\ 2}} \qquad [7]$$

And the area of minor segment of circle created by the chord on the critical point and parallel to the x-axis is given by taking the arc created between the integral of the function of a circle given by the Pythagorean theorem, shown below and in Figure 6 - Pipe Cross-Section.

$$A'_{1_i} := \int_{-\sqrt{cpr_{i,2}^{\ 2} - cpr_{i,1}^{\ 2}}}^{\sqrt{cpr_{i,2}^{\ 2} - cpr_{i,1}^{\ 2}}} \sqrt{cpr_{i,2}^{\ 2} - x^2} - cpr_{i,1} \, dx \qquad [8]$$

Then, the centroid of that area is given as the following equation.

$$y'_{1_i} := \frac{\displaystyle\int_{-\sqrt{cpr_{i,2}^{\ 2} - cpr_{i,1}^{\ 2}}}^{\sqrt{cpr_{i,2}^{\ 2} - cpr_{i,1}^{\ 2}}} \frac{1}{2} \left(\sqrt{cpr_{i,2}^{\ 2} - x^2}^{\,2} - cpr_{i,1}^{\ 2} \right) dx}{A'_{1_i}} \qquad [9]$$

3.1.1.3. Critical Point 2: Outer Wall

The outer wall of the pipe has a parallel thickness of zero, therefore, it experiences no shear stress. Also, the area outside of it is zero as well.

$$t_{2_{5,0}} := 0 \text{ in} \qquad [10]$$

$$A'_{2_{5,0}} := 0 \text{ in}^2 \qquad [11]$$

Since the outside area of this point is zero, the distance to the centroid of it is same as the distance to the critical point.

$$y'_{2_i} := cpr_{i,2} \qquad [12]$$

3.1.1.4. Stress Analysis

Once all critical point properties have been calculated for each pipe size, we proceeded to find the stress at each point and size. First, we had to compile all values of each size into a matrix for each property.

$$A' := \text{augment}\left(A'_0, A'_1, A'_2\right)$$

$$y' := \text{augment}\left(y'_0, y'_1, y'_2\right)$$

$$t := \text{augment}\left(t_0, t_1, t_2\right)$$

$$I := \text{augment}\left(I, I, I\right)$$

Now we could continue making equations for the matrices. The axial stress per critical point, from eq. 3-24 of [1], is given as

$$\sigma_{cp} := \frac{\overrightarrow{M_{max} \cdot cpr}}{I} \qquad [13]$$

And the shear stress, from eq. 3-32 of [1], is given as follows.

$$Q := \overrightarrow{y' \cdot A'} \qquad [14]$$

$$\tau_{cp} := \frac{\overrightarrow{\left(V_{max} \cdot Q\right)}}{\overrightarrow{I \cdot t}} \qquad [15]$$

With these next equations, we then can find the principal stress using the following equation, from eq. 3-13 of [1].

$$\sigma_A := \overrightarrow{\frac{\sigma_{cp}}{2} + \overrightarrow{\sqrt{\overrightarrow{\left(\frac{\sigma_{cp}}{2}\right)^2} + \overrightarrow{\tau_{cp}^{\,2}}}}} \qquad [16]$$

$$\sigma_B := \overrightarrow{\frac{\sigma_{cp}}{2} - \overrightarrow{\sqrt{\overrightarrow{\left(\frac{\sigma_{cp}}{2}\right)^2} + \overrightarrow{\tau_{cp}^{\,2}}}}} \qquad [17]$$

Following the Maximum Shear Stress theory, σ_A and σ_B are used to find σ_1 and σ_3. With these values, we can then use the safety factor equation (equation 5-19 of [1]), along with the yield strength of our material, to graph the factor of safety for each critical point at each pipe diameter. This resulted in Figure 7 - Safety Factor of different pipe sizes and their Critical Points for the pulley support.

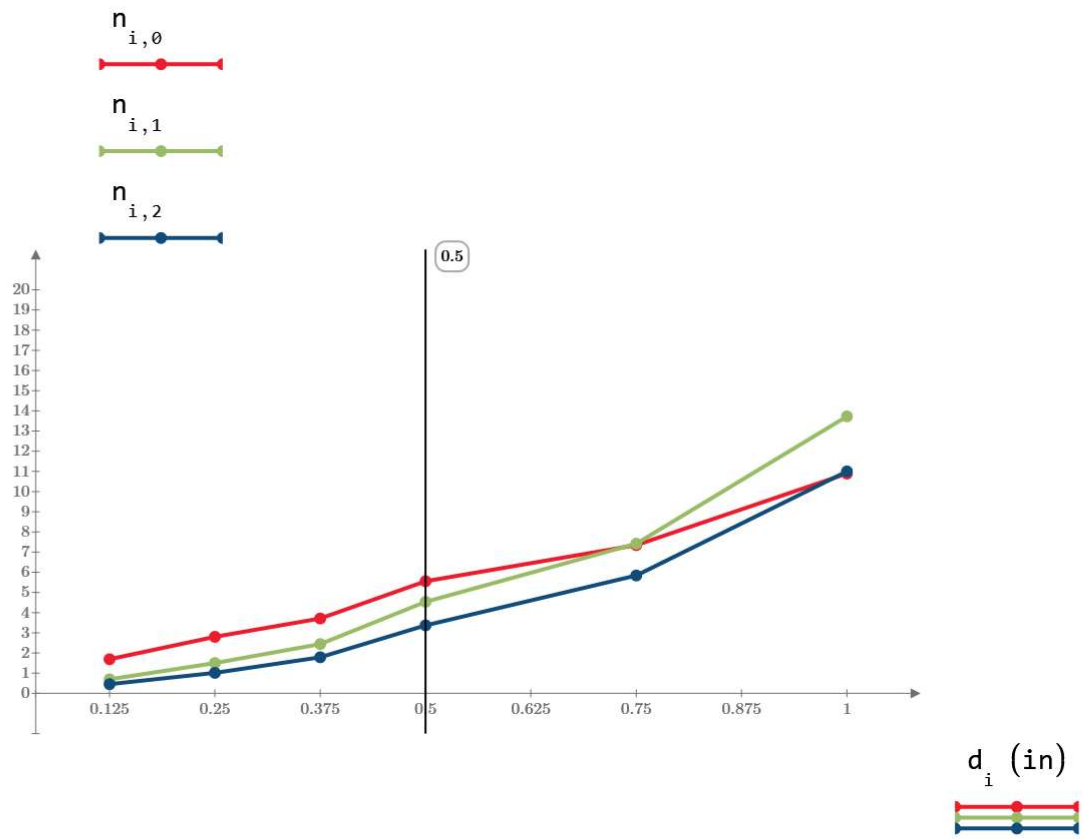


Figure 7 - Safety Factor of different pipe sizes and their Critical Points for the pulley support

From this graph, we can see Critical Point 2 ($n_{i,2}$), the outer edge, experiences the most stress and has the lowest safety factor. Singling out this critical point, we then determined which pipe size would keep our factor of safety at a good level, observed in Figure 8.

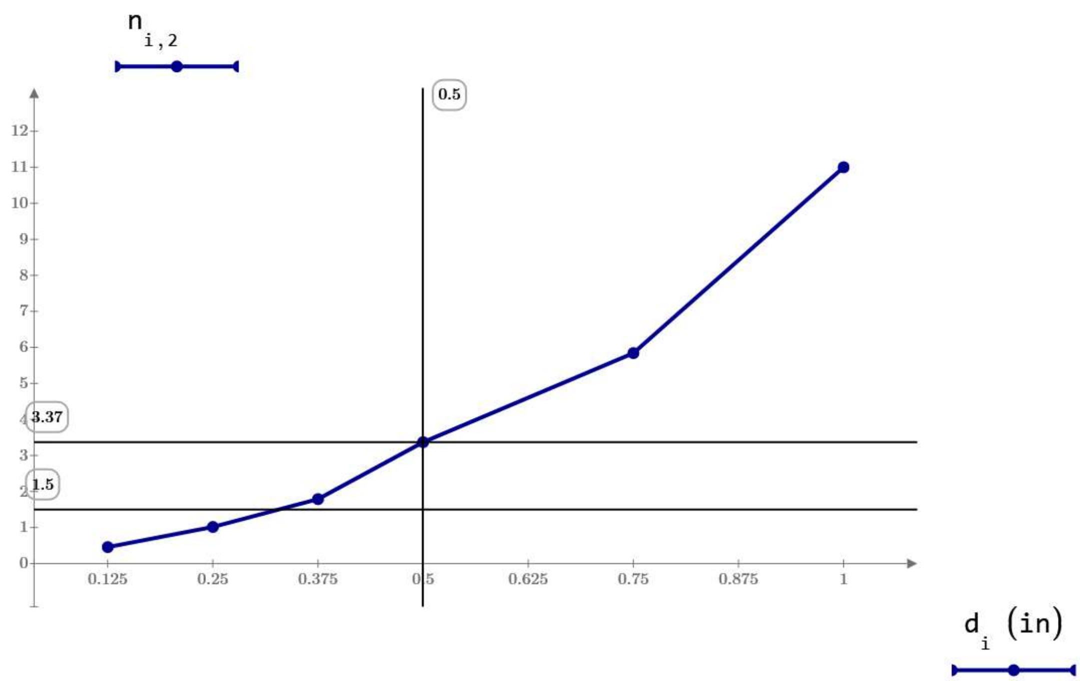


Figure 8 - Safety Factor at most critical point of pulley support pipe

From this graph we see that a $3 / 8$ in. pipe provides a safety factor of just over 1.5 while a $1 / 2$ in. pipe offers a safety factor of 3.37, which is well beyond safe and meets our established safety factor of 2.5.

3.1.2. Pulley Axle

Our pulley axle went through some changes in order to get it as safe as possible. At first, our group opted to put the pulley on the frame bar that went across the top of the frame (Figure 9). After analysis, the stress created on the pipe was much larger than it could bear. After that, we knew we were going to have to reinforce the axle for the pulley. Instead of changing the whole top pipe, we opted to create a new axle made of a solid steel that was mounted just below the top pipe. Furthermore, we added a gusset around the whole axle and pulley to prevent it from bending, shown in Figure 9.

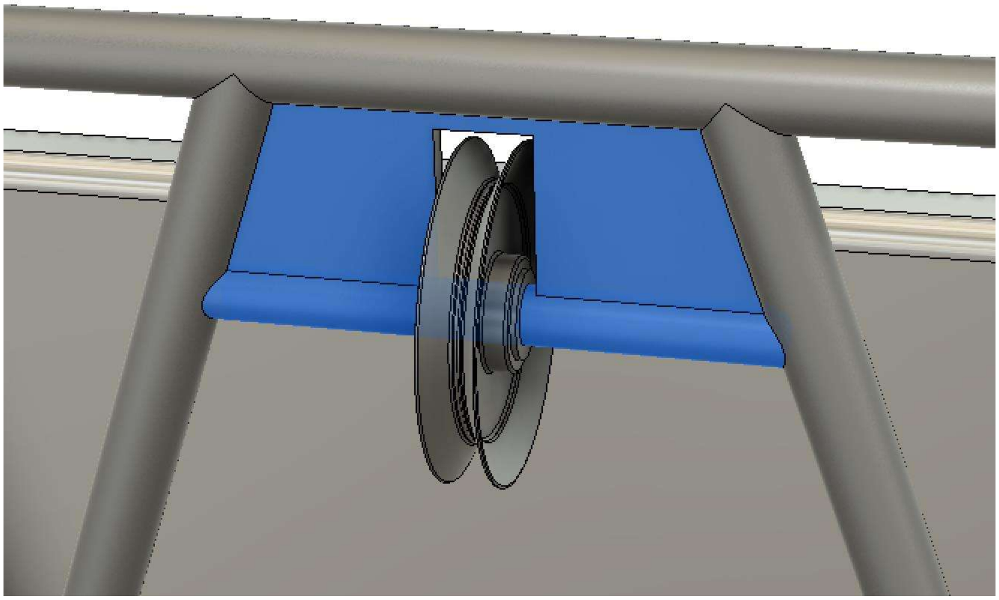


Figure 9 - Pulley Axle

The pulley axle, even after its design changes, was estimated to be the second most critical point. This component is a solid pipe that is going to be the axle for our pulley. Using the same material as our frame pipe, our yield strength is going to be the same. To find max shear and moment, we inserted the loading diagram into SkyCiv to produce elegant graphs and accurate results. See Figure 36, Figure 37, and Figure 38 in Appendix B. From SkyCiv, our shear and moment resulted in the following.

$$M_{max} := 292.7 \text{ lb} \cdot \text{in}$$

$$V_{max} := 400 \text{ lb}$$

Solid pipes are sold in $1 / 8$ in. increments, giving us the diameters:

$$d := 0 \text{ in}, 0.125 \text{ in} .. 1 \text{ in}$$

The cross-sectional properties required, in order, are area, second moment of area about neutral axis, second polar moment of area about the centroid, shear section area, distance from neutral axis to shear section area, and first moment of area.

$$A(d) := \frac{1}{4} \pi \cdot d^2 \qquad [18]$$

$$I(d) := \frac{1}{64} \cdot \pi \cdot d^4 \qquad [19]$$

$$J(d) := \frac{1}{32} \cdot \pi \cdot d^4 \qquad [20]$$

$$A'(d) := \frac{A(d)}{2} \qquad [21]$$

$$y'_{bar}(d) := \frac{d}{4} \qquad [22]$$

$$Q(d) := y'_{bar}(d) \cdot A'(d) \qquad [23]$$

Once these cross-sectional properties were determined, we could then find the internal component stresses of the axle using the following equations used previously for axial and shear stress, respectively.

$$\sigma_{max}(d) := \frac{M_{max} \cdot c(d)}{I(d)} \qquad [24]$$

$$\tau_{max}(d) := \frac{\left(V_{max} \cdot Q(d)\right)}{I(d) \cdot d} \qquad [25]$$

Once our component stresses for the pipe were found, we converted them to principal stresses via the previously referenced equations below.

$$\sigma_A(d) := \frac{\sigma_{max}(d)}{2} + \sqrt{\left(\frac{\sigma_{max}(d)}{2}\right)^2 + \tau_{max}(d)^2} \qquad [26]$$

$$\sigma_B(d) := \frac{\sigma_{max}(d)}{2} - \sqrt{\left(\frac{\sigma_{max}(d)}{2}\right)^2 + \tau_{max}(d)^2} \qquad [27]$$

Using the conservative approach of calculating safety factor, Maximum Shear Stress (MSS), section 5-4 of [1], these principal stresses can be manipulated to find maximum shear stress, $\sigma_1 - \sigma_3$, through the manipulation shown in Figure 10 below.

$$\sigma_1(d) := \begin{array}{l} \text{if } \sigma_A(d) \geq \sigma_B(d) \geq 0 \\ \quad \text{return } \sigma_A(d) \\ \text{else if } \sigma_A(d) \geq 0 \geq \sigma_B(d) \\ \quad \text{return } \sigma_A(d) \\ \text{else if } 0 \geq \sigma_A(d) \geq \sigma_B(d) \\ \quad \text{return } 0 \end{array} \qquad \sigma_3(d) := \begin{array}{l} \text{if } \sigma_A(d) \geq \sigma_B(d) \geq 0 \\ \quad \text{return } 0 \\ \text{else if } \sigma_A(d) \geq 0 \geq \sigma_B(d) \\ \quad \text{return } \sigma_B(d) \\ \text{else if } 0 \geq \sigma_A(d) \geq \sigma_B(d) \\ \quad \text{return } \sigma_B(d) \end{array}$$

Figure 10 - Principal stress to max shear stress conversion

These are then put into the safety factor equation for Maximum shear stress and results in Figure 11.

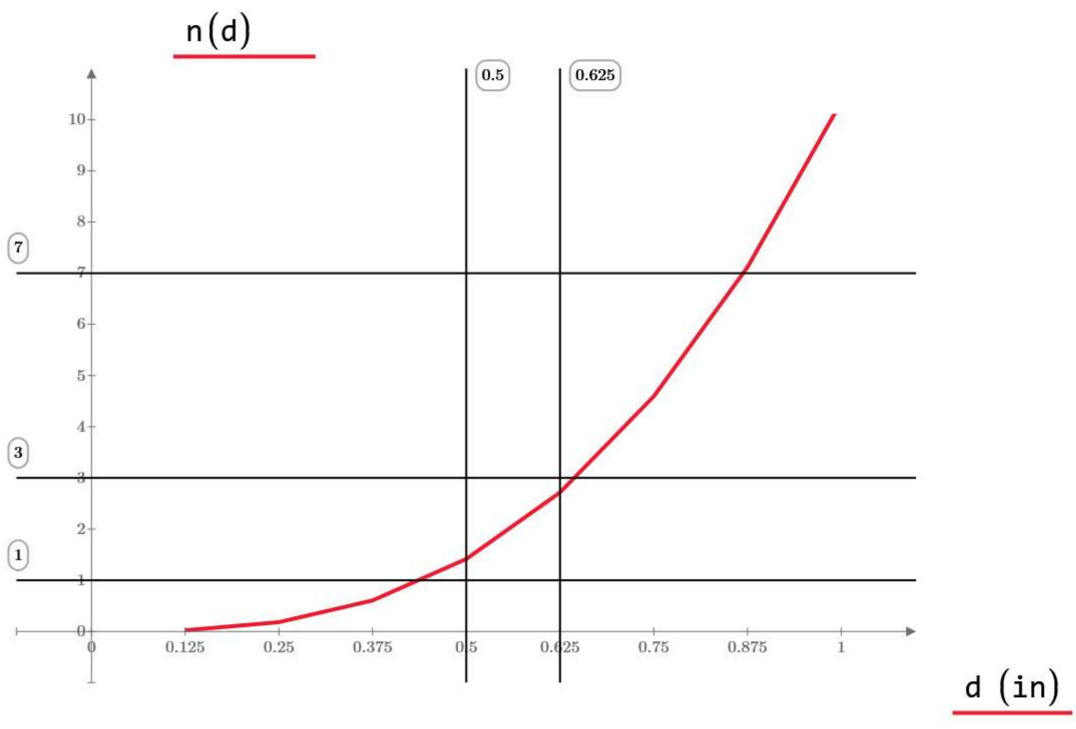


Figure 11 - Factor of safety for pulley axle

From this chart we concluded that a half-inch rod would not give us the desired safety factor, so we opted for the five-eighths inch rod. This gave us a safety factor about 2.8 which is very safe. Since the axle will not be rotating, there is not fatigue on it.

3.1.3. Seat

The design of the seat was one of our most though out processes. We wanted it to be thin and light while being structurally rigid. Our group opted for a sheet metal seat with a frame that lines the underneath. The seat analysis consists of two parts, the frame and the plate sides.

We knew that the plate itself would not be able to withstand the weight with simple supports on the edges, so a frame is inserted underneath. The frame is important in providing support to the base plate preventing bending. The seat edge is the main support for keeping the seat's shape and from collapsing. The main concern in the designing the seat frame was to keep the seat from bending and warping. A simple design was utilized that consisted of a bar that runs along the front, lower back and upper back then one bar that connects them together in the middle. Analysis is performed on the lower middle beam and front edge.

3.1.3.1. Front Seat Frame Support

Our group decided the front of the seat had to be no more than one inch of the ground in the lowest position, to provide ease of use. We went through many materials and styles. After considering all options, the 1-inch square frame proved to be the most reliable and accessible type. Similar processes for beam analysis as the previous beams except using a square cross section for the beam. Seen in the figure below.

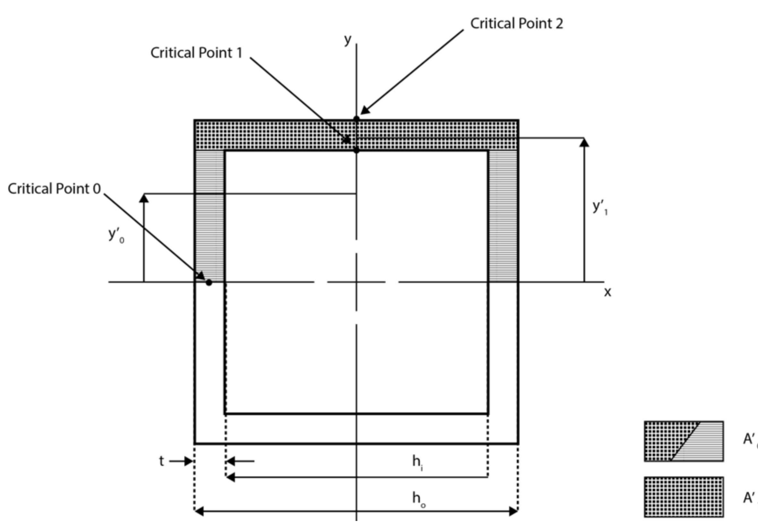


Figure 12 - Hollow Square Cross Section

After analyzing maximum internal stresses, the square tube gave us a minimum safety factor of 3.13. See Appendix B Front Seat Frame Analysis at Critical Points.

$$n_1 = 3.132$$

3.1.3.2. Center Seat Frame

The center seat frame was analyzed similarly to the front seat frame. To be consistent with our design, we used the same material also. After analysis, the safety factor at the most critical point came out to be 1.78. See Appendix B Front Center Frame Analysis at Critical Points.

3.1.3.3. Seat sheet metal design

The process in finding a safe shape for our seat was a more complicated problem. We know that we wanted our entire seat to be made of one piece of sheet metal. The sheet metal was then to be cut out using an advanced laser cutting machine from a sponsored company. We also knew the arms of the seat were going to carry most of the load applied. To design the arm of the seat, we did a reverse stress analysis of the cross-section at and point along the chair (x-axis) with a safety factor of 2.5.

$$\sigma := \frac{S_y}{n} = \left(1.6 \cdot 10^4\right) \frac{lb}{in^2}$$

$$h(x) := \sqrt{\left(\frac{M(x) \cdot 3}{t \cdot \sigma}\right)}$$

This resulted in an equation that gave us the height of the cross-section at a given distance from the edge. When graphed, it represents the shape and dimensions of the desired seat cutout (Figure 13).

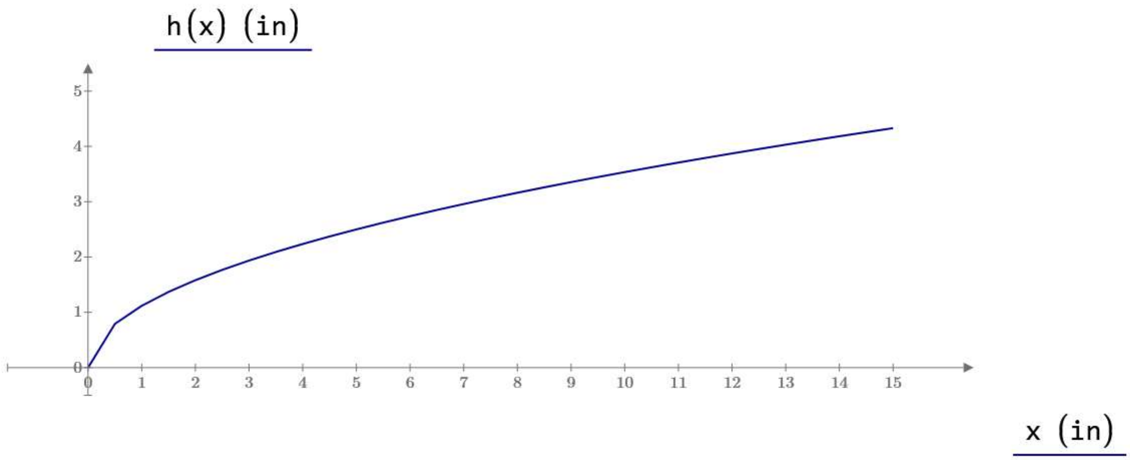


Figure 13 - Height of seat arm at distance from edge

We then layered and aligned the chart into Autodesk to create a template for the cutout and began sketching the seat cutout. The seat cutout was drawn with an offset to provide additional safety, represented in Figure 14. Highlighted in yellow is the needed height per length of the seat edge and in white is the actual cut height of the seat edge.

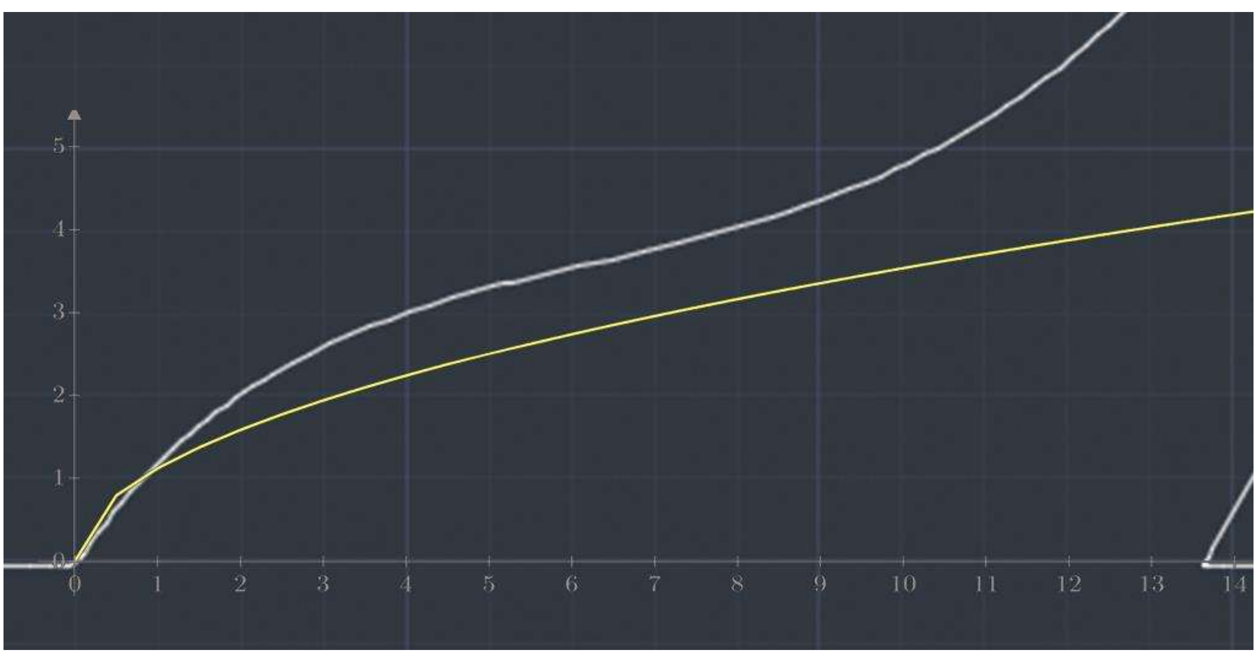


Figure 14 - Autodesk sketch of seat cutout with calculated seat height graph

3.1.4. Winch Bolt

The winch is mounted to the frame via 3 bolts. The winch came with 3 mounting holes, so we decided to use those that were already there. Because the budged allowed, and the price difference was insignificant, we opted for Grade 8 bolts. The analysis of the bolt is a little different because of its torque. To find the axial stress due to torque, we used the following equation

$$F := \frac{T}{K \cdot t} \qquad [28]$$

With this, we then combined it with the shear stress created from the perpendicular force of the winch on the mounting plate to create the maximum shear stress. This gave us a safety factor of 8.4.

3.1.5. Electronics

The machine is going to be powered by a Schumacher 800-Amp rechargeable 12v Jump-Start lithium-ion battery pack running a 2000lb utility winch. To determine the proper duty-cycle of the power supply to reliably handle the work load of the winch was a critical calculation. The battery is rated for 1000 cycle life, thus being adequate for our product. The battery is capable of giving a charge of 12 Volts (V) at 37 Watt-hours (Wh). Using the equation below, we can convert the power into Amp-hours (Ah).

$$P = I * V \qquad [29]$$

Resulting in a total of 3.08 Amp-hours. At fully loaded the winch draws 29 amps from the source. Since our machine is not going to load the winch at 2000 lbs. we calculated that the winch draws 10 amps when loaded with 400 lbs.

At that rate, we calculated how long the battery will run the winch by simply dividing the Amp-hours supplied by the Amps used.

$$t(hours) = \frac{Ah}{A} \qquad [30]$$

That results in a fully loaded run time of 18.48 minutes.

The winch has a speed of 1.7 ft/min at full load and 4.5 ft/min with no load. To find how long one cycle the machine takes to execute, following equation can be used.

$$t = \frac{2.5\,ft}{1.7\,ft/min} + \frac{2.5\,ft}{4.5\,ft/min} \qquad [31]$$

This gives one cycle an average time of 2.25 minutes at with a load of 400lb.
With that noted we can safely say that the machine can run 8 cycles before needing a charge. In comparison to a real-life scenario, that time is more than plenty. We had set our initial requirement that the machine needed to cycle at least 2 times before needing charged.

Limits have to be set to the winch in order for it to not over-extend the seat too far up or drop it too low. Limit switches, labeled TOP and BOTTOM, are placed at both top and bottom along the sliders to break the circuit when the chair is slid up or down to the desired maximum. The switches are simple interrupt switches that when activated, they disengage the user's ability to move either further up or further down, respectively. The switches are hooked to the controller switches as shown in Figure 15.

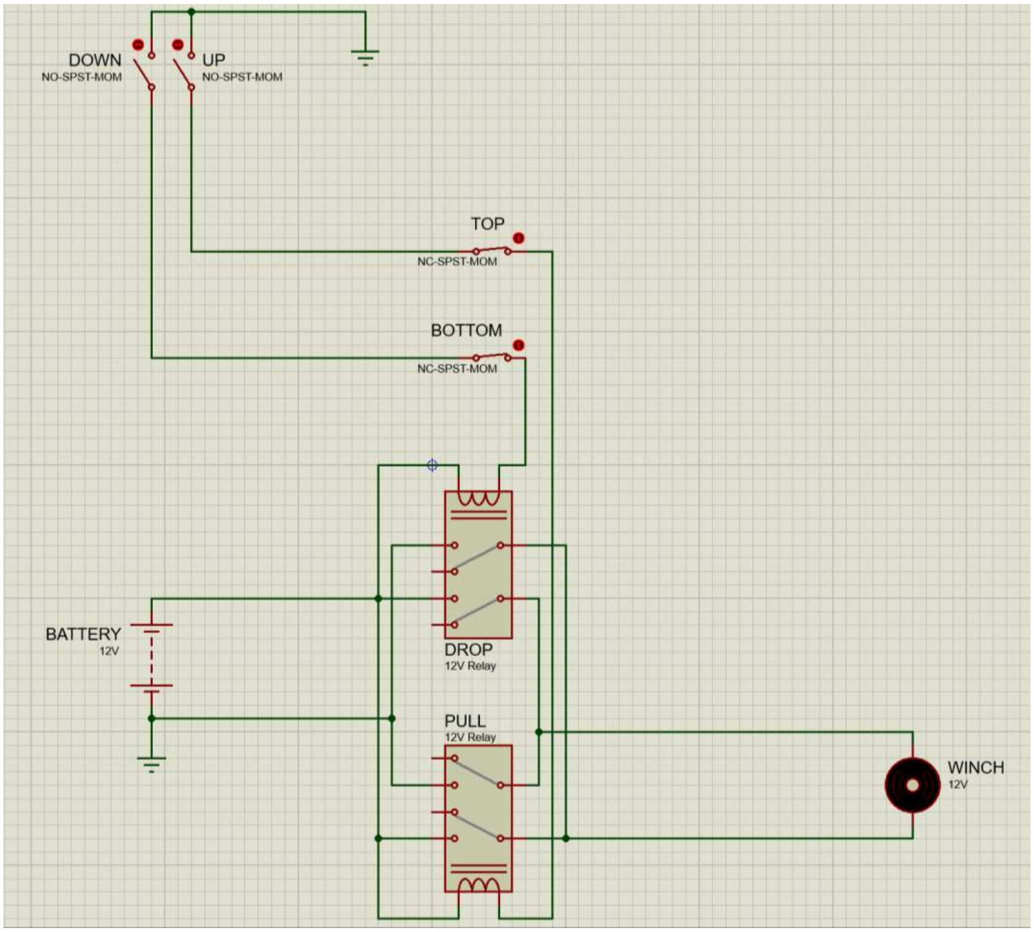


Figure 15 - Wiring Diagram

3.2. Dimensioning and Assembly

The frame is cut from one size pipe and some pipe is needed to be bent into shape. All edges that are welded were rounded off to create a saddle joint for the welds. Then the pipes are held in place and welded together. The frame itself stands 3 ft tall.

Figure 16 - Frame Cad Drawing

The casters are 3 inches and provide swivel functionality in the front of the seat and locking functionality in the back. Together they raise the system off the ground 4 inches. Each wheel is rated to hold a 300lb capacity and 1200lb total with all 4 together. The caster's heads are trimmed down and welded to the bottom of the frame vertically.

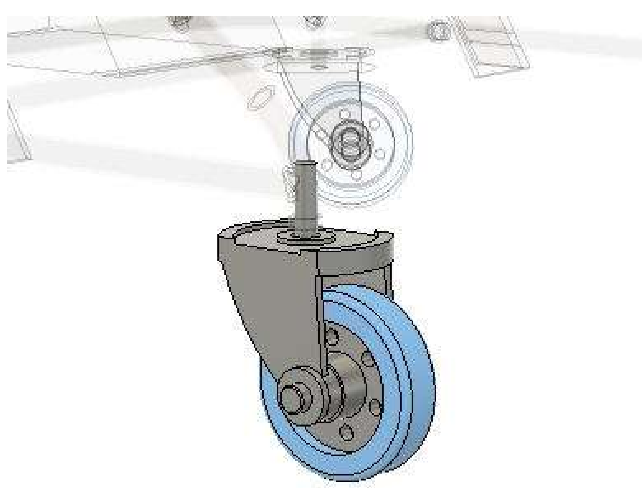


Figure 17 - Caster Cad Drawing

The pulley is a welded 3-inch pulley with a 5/8” diameter bore to fit the 6-inch cross bar supporting it.

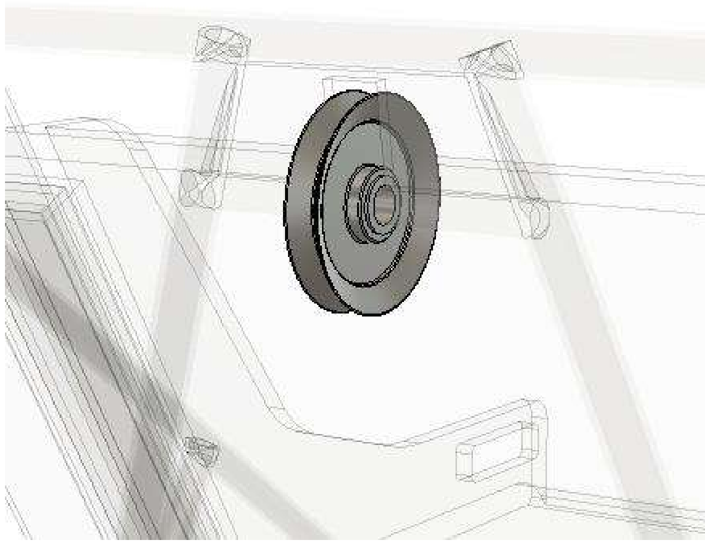


Figure 18 - Pulley Cad Drawing

A gusset is cut in a shape to fit around the pulley and within the surrounding structure to create a stable frame. The pulley is first placed on the bar and a couple guide washers on each side. Then the gusset is welded to that bar then fitted into the corner and welded.

The rail system has two basic components. The rail or L-channel and the sliders or C-channel with Teflon. The L-channel is # inches long and welded to the frame at an exact angle of 30° from the vertical. To ensure each side is aligned exactly with each other several guide bars are placed across, measured, then clamped to hold each in place while it is being welded together. Very fine sanding and polishing is done to the rail where the seat is going to be sliding to minimize friction along the metal. The C-channel is 10 inches long has a Teflon core that is cut from a solid block and so all sides of the rail are kept away from grinding. The Teflon fits perfectly into the C-channel and is held in place by glue and brackets.

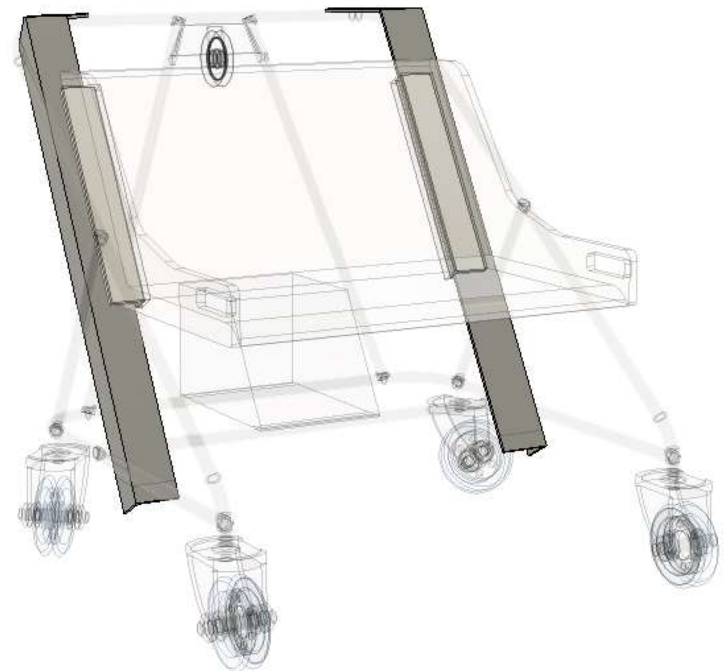


Figure 19 - Rail and Slider System Cad Drawing

The seat is dimensioned and cut from a template using AutoCAD, analysis from Section 3.1.3.3., and a Laser CNC from one piece of sheet metal. The seat has a width of 24 inched, a sitting length of 13 inches and a back-support height of13 inches.

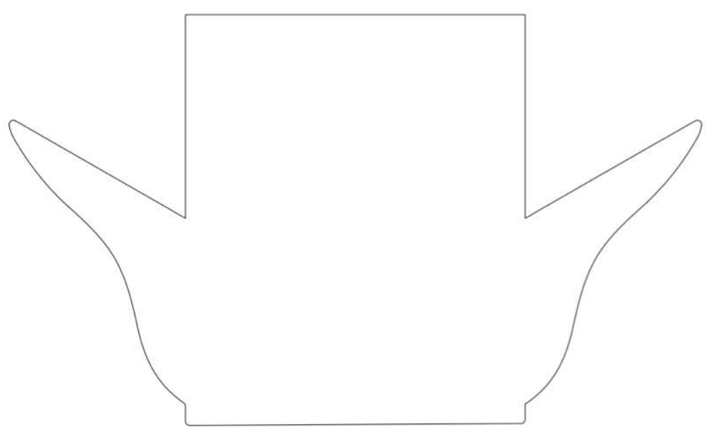


Figure 20 - Sheet Metal Shape for Seat

After the sheet is cut a press is then used to fold the seat into shape. First the two sides are folder 90° upward then the back is had bent at 30° to line up with the sides. Then all the edges are welded together. After shape of the seat is set, the frame is built and welded underneath the seat. The material we selected for the seat frame is ASTM A-36 steel, with a yield strength of 36kpsi [4].

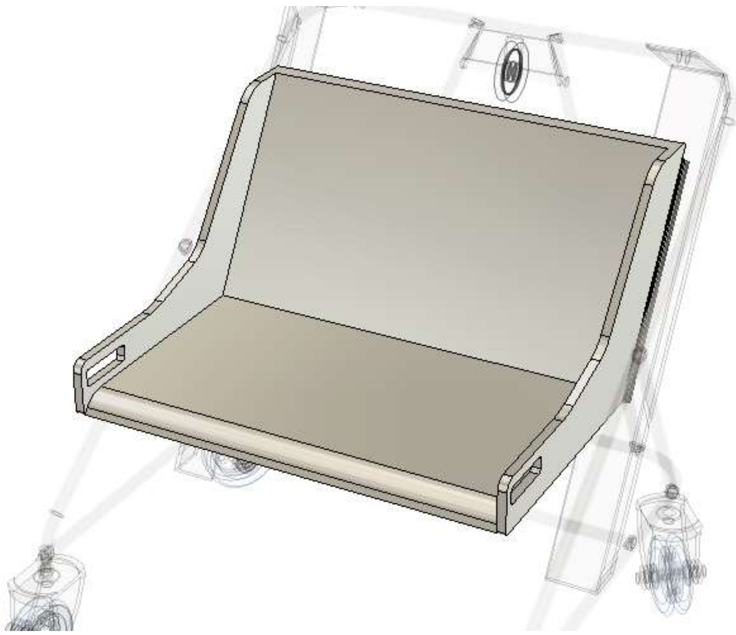


Figure 21 - Seat Cad Drawing

The seat and seat frame are lined up and welded to the C-channel sliders ensuring the base plate is level when on the rails.

The winch is bolted to a plate that is then welded to the bottom of the vertical A-frame pipes. The cable is sent through the pulley and attached to the back of the seat using a pin. The battery and switches are hooked up and the device is tested.

Figure 22 - Final Design

3.3. Working Principles of Design

This device was designed so that the customer could leave it plugged in and charging until they may need it. When the chair needed to be utilized, it would simply be disconnected from the charging cable and moved to the client. The chair is easily maneuvered next to the person in need, who would then just ease up on the seat and be lifted high enough to get back on their feet. The seat is operated by means of a simple two button remote control. After the seat was raised, and the customer was back on their feet, the chair would simply be moved back to the storage location where the battery-pack could be placed on charge, ready for the next use.

Figure 23 - Exploded Component Render

3.4. Use of Advanced tools for Optimization

The use of Software in design was necessary to obtain the best results possible in designing our structure. Starting from the concepts we put to test in simulation how each would perform and what constraints each have right off the bat. Fusion 360 allowed us to eliminate some of the concept models from the beginning. Then taking our desired model we needed to break down each critical component to find stresses and factors of safety. First, our shear and moment needed to be located and graphically represented. SkyCiv offered very presentable and accurate shear and bending moment diagrams and numbers. That was the extent of SkyCiv utility. Next, Mathcad presented us with the cleanest and neatest solution to show our work and calculations. Mathcad is what we used to optimize as much as we could on our individual components. The seat took a different approach and required Mathcad's dimensions to be transferred over to AutoCAD for a template to be cut out of sheet metal. Finally, after making neat all the calculations and optimizations we made another test in Fusion 360 to see what it would look like all together. And these are the results.

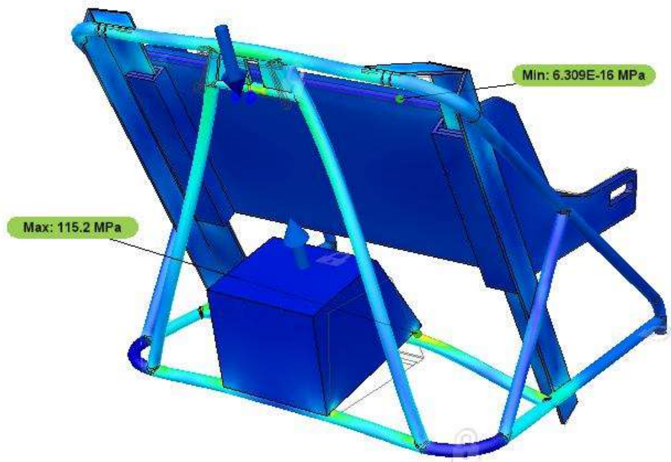


Figure 24 - Fusion 360 Stress Distribution

4. Construction and Evaluation

4.1. Prototype Building

4.1.1. Bill of quantities

Table 5 - Bill of Quantities

Part	**Price**
Winch	$60.00
Battery	$70.00
Channel (rectangular tubing)	$18
Angle	$18.97
Teflon	$62.00
Pully	$11.15
Frame	$32.04
Misc.	$2.70
Chair & Support	$33
Seat	$10.00
Seat Belt	$8.50
Wheels/Casters	$23.99
Paint	$20.00
Arduino/Phone	$20.00
Total	**$390.35**

4.1.2. Manufacturing Process:

Manufacturing of the assisted lifting device began soon after completion of the analysis and design. The bill of quantities was listed, and all the parts and components were purchased. The initial plan was to pay an outside company to bend, and depending on cost, assemble the frame structure. After the group learned that there would be ready-access to a pipe-bended and welder, the decision was made to cut cost by doing the labor using group provided skill sets.

After a short trial-and-error experimental session with the pipe-bender, the team confidently proceeded to assemble the frame and attach the wheels. Seeing the height of what the final product would be, it was decided that a handle bar would be added.

The seat portion of the device was cut and bent courtesy of Hand Industrial. Once the group received the finished seat, the slide-rail system was aligned on the seat as was the frame. The rest of the components were married into the system and into one semi-completed model.

The winch was next to be fixed to the frame, and adjustment of the cable-pulley system was simple. A detailed overlook of the safety of the device was completed after the build during which the group ensured the device had no sharp edges as to cut the user as well as to address potential pinch points thus limiting chances of injury.

4.1.3. Challenges

There were several challenges faced during the conception and completion of this project in both the design phase as well as in the manufacturing phase. The design challenge was determining the best design in terms of ease and safety. The group decided to create a frame with as few separate pieces as possible to allow for an easier and safer building process. The concern during the manufacturing phase was in regard to the correct type of welding process to use when constructing the pieces together for time reasons. The group chose MIG welding in the end because the equipment was easily acceptable timely. Due to this type of welding it was considered to have a professional company build the intricate welds to cut down on time, this however was abandoned due to cost of professional labor.

One of the important challenges that took careful consideration in finding a solution, was how was the chair going to be lifted. The group brainstormed many different options from a simple scissor lift design with a screw system that consisted of many pinch points, to a heavy hydraulic system, as well as a winch cable system. After talking to the customer about his wishes and concerns he suggested that we use the winch system with a pully allowing for the lightest and smoothest lifting system of the available options.

4.1.4. Optimization

Once the general concept for the design was chosen, the group worked towards optimizing the frame. The group looked at assisted walking and lifting devices used in the medical field as inspiration and adapted those frame styles to fit our design specifications. Initially the group designed the device to have an A-frame type support system but started looking at other options for accessibility when the professor pointed out potential issues. After more trial and error, the group devised a design that allowed for an easier side loading option with more clearance options as needed for maneuverability.

Many potential optimizations were presented for lifting systems. The initial idea proposed for lifting was a simple device of linear actuators, however this was decided against due to high cost of actuators. A rail system consisting of a trolley and I-beam was also considered, though it increased the weight of the device so drastically that it was ruled out. Merging these ideas together, the group designed a rail system similar to the trolley and linear actuators. The concept of the trolley with the smooth action of the actuators was incorporated into the rail system using an Ultra High Molecular Weight (UHMW) Teflon to have smooth motion. The group applied it to a C-channel type trolley that attached to the chair portion of the device. In the case of the actual chair it was discussed within the group as to how thick of material would be required to build the chair allowing for safety as well as comfort for the client. After doing the analysis for the design of the chair it was decided the group would design and build a frame that the chair would be welded to in order to simplify the amount of calculations as well as the weight of the chair. The handles on the sides of the chair were also removed to prevent stress concentration. The group added the use of casters to the design because of the potential for the device to be used on carpet, different indoor flooring and outside surfaces. The handle was also added to the back of the device to allow the client's wife a more ergonomic position while maneuvering the device. The group also implemented a simple operation control for the lifting mechanism allowing for two options when controlling the movement of the chair. Due to the client being elderly

4.2. Testing and Results

4.2.1. Design requirement

The overall design needed to fall within parameters of weight, budget, lifting capacity, ease of use and footprint size. Some of these requirements were given by OSHA and others were from client preference or need. Due to the client and his wife being elderly and having health conditions, it was necessary to make the device as lightweight and easy to use as possible. The total weight of the assisted lifting device ended up being just over one hundred pounds. Another parameter we had to work within on the design was the overall footprint because the device was going to be used indoors. The doorways in the client's house were measured, and with standard 36" doors, we concluded that our device would be compact enough to be used by the customer without difficulty. Considering that the client weighed about 300lbs, the group decided to ensure that the device had to be capable of lifting up to four hundred pounds to ensure safety and reliability. The other limitation was that of budget, which was set at one thousand dollars, this group was able to gain some donations due to the client being a veteran and much of the manufacturing work was done by the group members themselves which resulted in a final cost of $390.35.

4.2.2. Testing Methodology

For the testing of our machine, for our initial condition we designed the device to have a capability of lifting a max of 400 pounds. To test the design in this aspect we put a total of 400 pounds on the device and operated it repeatedly to ensure that the device was going to be handle the operation that it was designed for. To ensure that the footprint of the device was correct we took the device and maneuvered it through as many doors that we could find that measured in at thirty-six inches or less. To see if the device was designed with ease of operations in mind, we contacted a few people that had never seen the device and that had no idea on what the device was designed for and asked them to operate it without telling them what to expect. The weight of the device was found using a standard bathroom scale which gives an error of approximately plus or minus five pounds.

4.2.3. Results and discussion

During the testing we found that the device was more than capable of handling the operation of lifting a person from a sitting position to a height that would allow them to regain a standing position. The testing to ensure the footprint was the correct size was clarified that the size of the device was more than capable of going through a standard door frame of 36” with ease which the group felt was sufficient for eliminating pinch points and allowing for easier mobility. As for testing to ensure that the device was easy to use it was made clear by the time that it took for random people to operate the device with no help from the group. The group felt that one hundred pounds was a good final weight for the device when compared to a standard shopping cart with four gallons of milk weighing roughly the same amount.

4.3. Assessment

After building and testing our design, it was made apparent that some of the weaknesses in our design came from the type of material that was chosen to build the frame based on the budget and availability of the material. While the weight was acceptable, it could have been even further reduced through the use of a lighter, if more expensive material. The potential for making this device collapsible would have added to the design by creating an easier storage option for the client and other potential future clients. There were also further ideas for other applications in the medical field once the design took form whether for the morbidly obese or severely handicapped, etc. The group also felt that there could be further optimization for placement of the casters for better footing in outdoor settings.

5. Conclusion and Recommendation

5.1. Summary of Results

The client asked for a solution to a simple and specific problem. This group was inclined to help the aging veteran in his quest to keep his independence after suffering some falls at home. This group's solution was an assisted lifting device specified to meet the needs of a fallen individual lacking the muscle or coordination to stand on their own power. The device is wheeled to their side, powered down to ground level at which point the user can slide themselves onto the chair-style seat. Once seated the chair is powered up to a normal seated height from which point the person can stand on their own without the need for emergency services. This device is lightweight, comfortable, easy to use, safe to lift up to four hundred pounds, maneuverable both inside and outside the home, and the prototype came in well under the given budget. The cons of this device are limited to the post design discoveries for the potential to have made it lighter and collapsible which may have been an added advantage to the elderly clients ultimately using this device.

5.2. Recommendations

Depending on the budget that is available to the next group of designers, a recommendation that this group has is to use a lighter material such as aluminum. A system designed out of aluminum would allow easier portability and a more creative frame system. Due to the cost of aluminum and the budget that was available, it was not thought to be an option. Further recommendations for improvements to this device would be to make it easily collapsible for storage and or travel. Furthermore, this group recognizes the potential for aiding the morbidly obese with an assisted lifting device like this and could be done so with minimal alterations including enlarged footprint and lifting capacity.

Figure 25 - Rendered Final Design

Acknowledgments

In gratitude to those who contributed to the success of this project:

- Hand Industrial for donating materials, labor, and assisting in manufacturing of parts.
- Multiple Systems for providing materials.
- Panhandle harvesting for allowing shop equipment.
- Tim Podzemny for use of facility and equipment for building process.

References

[1] Budynas, R. G., Nisbett, J. K., and Shigley, J. E., 2002, *Shigley's Mechanical Engineering Design*, McGraw-Hill Education.

[2] "Ergonomics ETool: Solutions for Electrical Contractors - Materials Handling: Pushing, Pulling and Carrying."

[3] "Differences between ASTM A53 B and ASTM A106 B Steel Pipe."

[4] "ASTM A36 Steel, Bar."

Appendix A

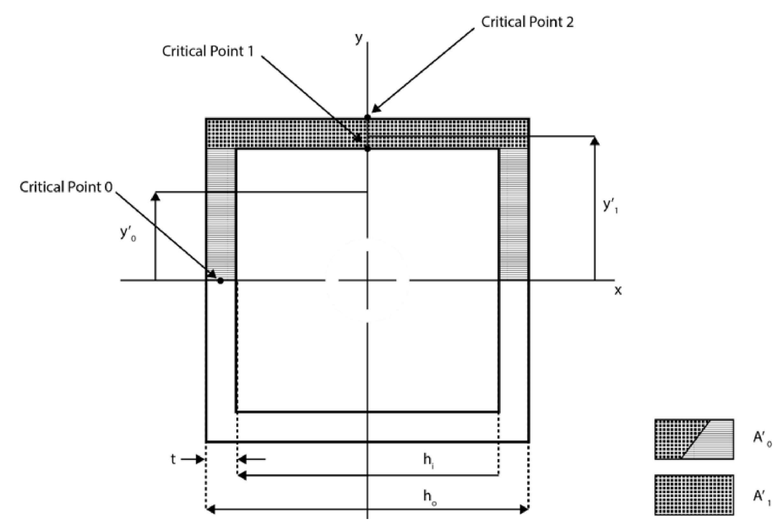


Figure 28 - Hollow Square Cross-Section

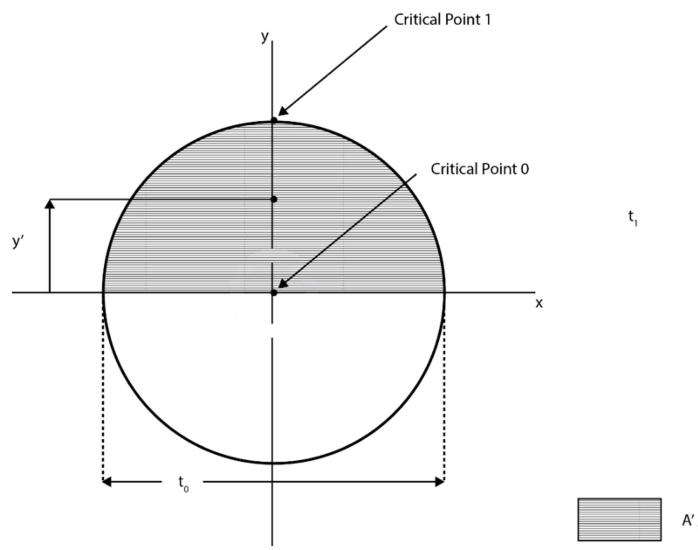


Figure 29 - Solid Circle Cross-Section

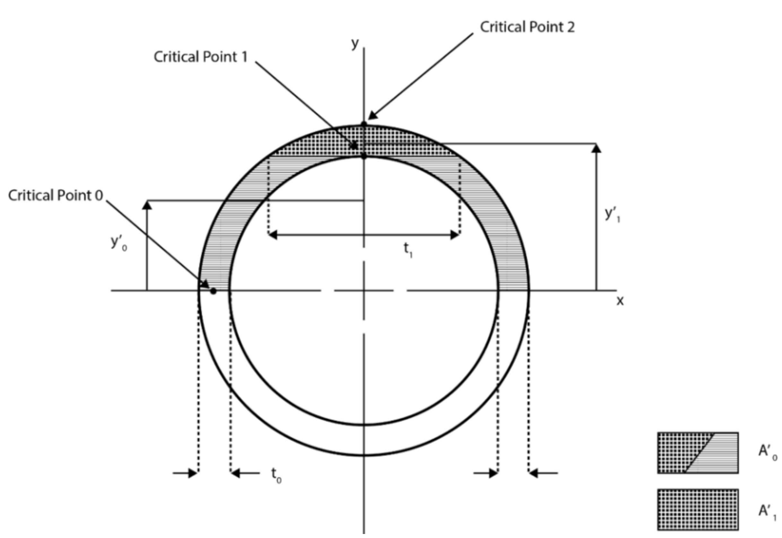


Figure 27 - Hollow Circle Cross-Section

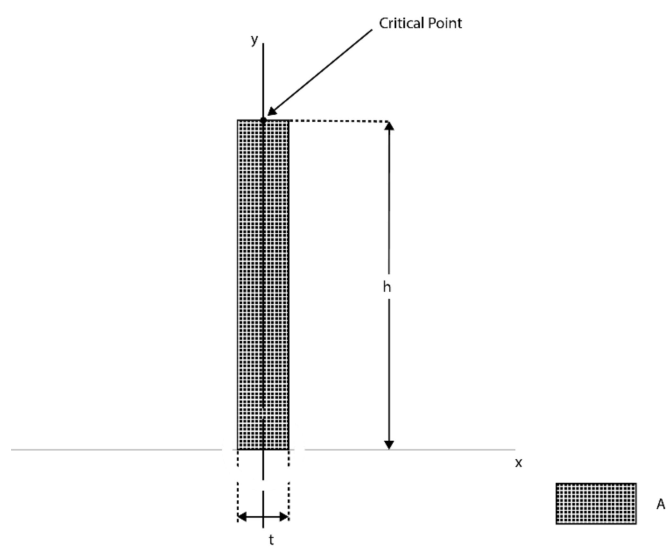


Figure 26 - Vertical Flat Plate Cross-Section

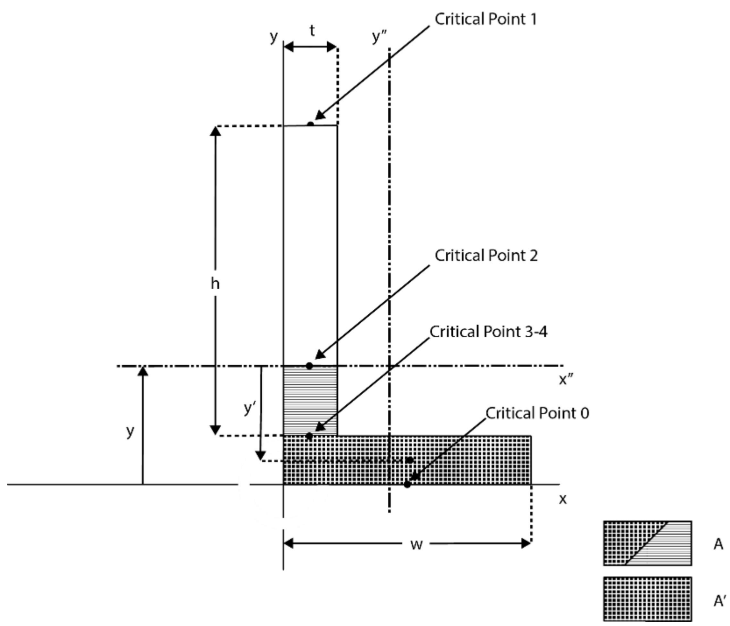


Figure 30 - L Beam Cross-Section

Appendix B

HANDLE BAR SUPPORT SHEAR AND BENDING MOMENT ANALYSIS

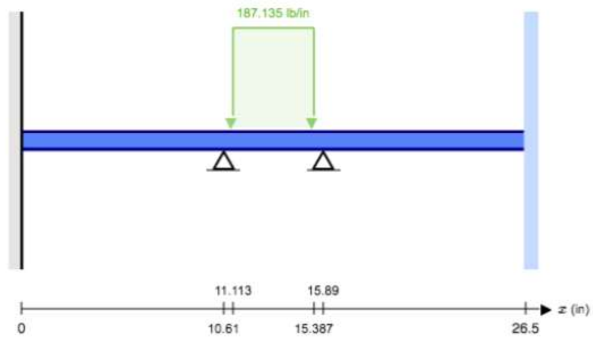


Figure 31 - Handle Bar Free Body Diagram

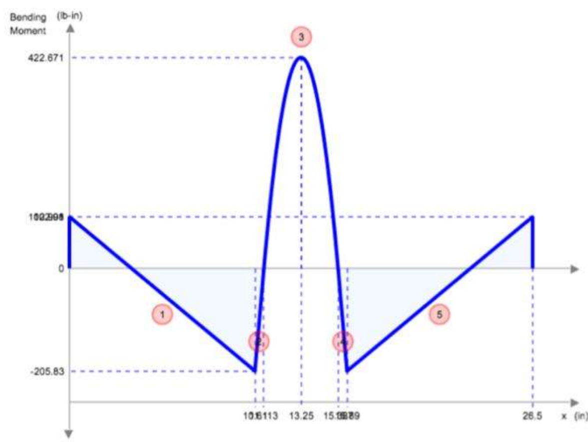


Figure 32 - Handle Bar Shear Diagram

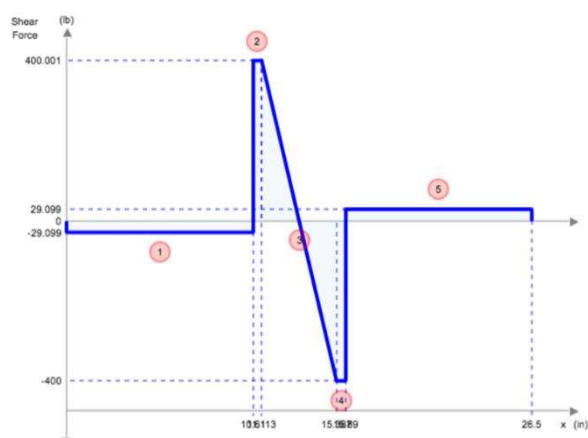


Figure 33 - Handle Bar Moment Diagram

HANDLE BAR SUPPORT ANALYSIS AT CRITICAL POINTS

Static Load

Material: ASTM A53 Steel Grade B Solid Pipe

$$S_y := 35000 \ \frac{lb}{in^2}$$

From SkyCiv: Figure 32 and Figure 33

$$M_{max} := 422.67 \ lb \cdot in$$

$$V_{max} := 400 \ lb$$

SCH 40 Pipe Dimensions

$$d := \begin{bmatrix} 0.125 \ in \\ 0.25 \ in \\ 0.375 \ in \\ 0.5 \ in \\ 0.75 \ in \\ 1 \ in \end{bmatrix} \quad d_o := \begin{bmatrix} 0.41 \ in \\ 0.54 \ in \\ 0.675 \ in \\ 0.840 \ in \\ 1.050 \ in \\ 1.315 \ in \end{bmatrix} \quad d_i := \begin{bmatrix} 0.269 \ in \\ 0.364 \ in \\ 0.493 \ in \\ 0.622 \ in \\ 0.824 \ in \\ 1.049 \ in \end{bmatrix}$$

Analysis:

Cross Sectional Properties: **Error!**

$$A := \frac{1}{4} \pi \cdot \left(d_o^{\,2} - d_i^{\,2}\right) = \begin{bmatrix} 0.075 \\ 0.125 \\ 0.167 \\ 0.25 \\ 0.333 \\ 0.494 \end{bmatrix} in^2$$

$$I := \frac{1}{64} \cdot \pi \cdot \left(d_o^{\,4} - d_i^{\,4}\right) = \begin{bmatrix} 0.001 \\ 0.003 \\ 0.007 \\ 0.017 \\ 0.037 \\ 0.087 \end{bmatrix} in^4$$

$$J := \frac{1}{32} \cdot \pi \cdot \left(d_o^{\,4} - d_i^{\,4}\right) = \begin{bmatrix} 0.002 \\ 0.007 \\ 0.015 \\ 0.034 \\ 0.074 \\ 0.175 \end{bmatrix} in^4$$

Critical Point Radii at different pipe sizes

Rows: Pipe Sizes Columns: Critical Point Radii

$$cpr := \text{augment}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \text{in}, \left(\frac{d_i}{2}\right), \left(\frac{d_o}{2}\right)\right) = \begin{bmatrix} 0 & 0.135 & 0.205 \\ 0 & 0.182 & 0.27 \\ 0 & 0.247 & 0.338 \\ 0 & 0.311 & 0.42 \\ 0 & 0.412 & 0.525 \\ 0 & 0.525 & 0.658 \end{bmatrix} \text{in}$$

$cp := 0..2$ Number of Critical Points

$i := 0..5$ Pipe Size

$j := 0..2$ Critical Point

Dimensions of Critical Point matrix

Critical Point 0: Neutral Axis

$$A'_0 := \frac{A}{2}$$

Total cross section area above neutral axis created by concentric circles

$$y'_0 := \frac{4\left(\left(\frac{d_o}{2}\right)^3 - \left(\frac{d_i}{2}\right)^3\right)}{3 \cdot \pi \cdot \left(\left(\frac{d_o}{2}\right)^2 - \left(\frac{d_i}{2}\right)^2\right)}$$

Centroid of Semi-Circular Arch

$$t_0 := d_o - d_i$$

Thickness of pipe along neutral axis

Critical Point 1: Inner Wall on y-axis

$$A'_{1_i} := \int_{-\sqrt{cpr_{i,2}{}^2 - cpr_{i,1}{}^2}}^{\sqrt{cpr_{i,2}{}^2 - cpr_{i,1}{}^2}} \sqrt{cpr_{i,2}{}^2 - x^2} - cpr_{i,1} \, dx$$

Area of minor segment of circle created by the chord on the critical point and parallel to the x-axis.

$$y'_{1_i} := \frac{\displaystyle\int_{-\sqrt{cpr_{i,2}{}^2 - cpr_{i,1}{}^2}}^{\sqrt{cpr_{i,2}{}^2 - cpr_{i,1}{}^2}} \frac{1}{2}\left(\sqrt{cpr_{i,2}{}^2 - x^2}^{\,2} - cpr_{i,1}{}^2\right) dx}{A'_{1_i}}$$

Centroid of A'(1)

$$t_1 := 2 \cdot \sqrt{d_o{}^2 - d_i{}^2}$$

Critical Point 2: Outer wall on y-axis

$$A'_{2_{5,0}} := 0 \text{ in}^2$$

The critical point lies on outer edge of cross-section, therefore, A'=0

$$y'_{2_i} := cpr_{i,2}$$

y'= outer radius

$$t_{2_{5,0}} := 0 \text{ in}$$

$$t_2 := t_2 + 0.00001 \text{ in}$$

A small number was added to the cross-sectional thickness to avoid divide-by-zero in the shear stress equation

Critical Point Properties per Pipe Size

$$A' := \text{augment}\left(A'_0, A'_1, A'_2\right)$$
$$y' := \text{augment}\left(y'_0, y'_1, y'_2\right)$$
$$t := \text{augment}\left(t_0, t_1, t_2\right)$$

$$I := \text{augment}(I, I, I)$$

Stress Analysis

$$Q := \overrightarrow{y' \cdot A'}$$

$$\sigma_{cp} := \frac{\overrightarrow{M_{max} \cdot cpr}}{I}$$

$$\tau_{cp} := \frac{\overrightarrow{\left(V_{max} \cdot Q\right)}}{\overrightarrow{I \cdot t}}$$

$$\sigma_A := \overrightarrow{\frac{\sigma_{cp}}{2} + \overrightarrow{\sqrt{\overrightarrow{\left(\frac{\sigma_{cp}}{2}\right)^2} + \overrightarrow{\tau_{cp}^{\,2}}}}}$$ Normal Stress Due to Bending (3-26a)

$$\sigma_B := \overrightarrow{\frac{\sigma_{cp}}{2} - \overrightarrow{\sqrt{\overrightarrow{\left(\frac{\sigma_{cp}}{2}\right)^2} + \overrightarrow{\tau_{cp}^{\,2}}}}}$$ Moment at outer diameter

(3-31)

Maximum Shear Stress Theory (MSS):

$$\sigma_{1_{i,j}} := \begin{array}{|l} \text{if } \sigma_{A_{i,j}} \ge \sigma_{B_{i,j}} \ge 0 \\ \quad \text{return } \sigma_{A_{i,j}} \\ \text{else if } \sigma_{A_{i,j}} \ge 0 \ge \sigma_{B_{i,j}} \\ \quad \text{return } \sigma_{A_{i,j}} \\ \text{else if } 0 \ge \sigma_{A_{i,j}} \ge \sigma_{B_{i,j}} \\ \quad \text{return } 0 \end{array}$$

$$\sigma_{3_{i,j}} := \begin{array}{|l} \text{if } \sigma_{A_{i,j}} \ge \sigma_{B_{i,j}} \ge 0 \\ \quad \text{return } 0 \\ \text{else if } \sigma_{A_{i,j}} \ge 0 \ge \sigma_{B_{i,j}} \\ \quad \text{return } \sigma_{B_{i,j}} \\ \text{else if } 0 \ge \sigma_{A_{i,j}} \ge \sigma_{B_{i,j}} \\ \quad \text{return } \sigma_{B_{i,j}} \end{array}$$ (5-1)

Factor of Safety:

$$n := \frac{S_y}{\sigma_1 - \sigma_3}$$ (5-3)

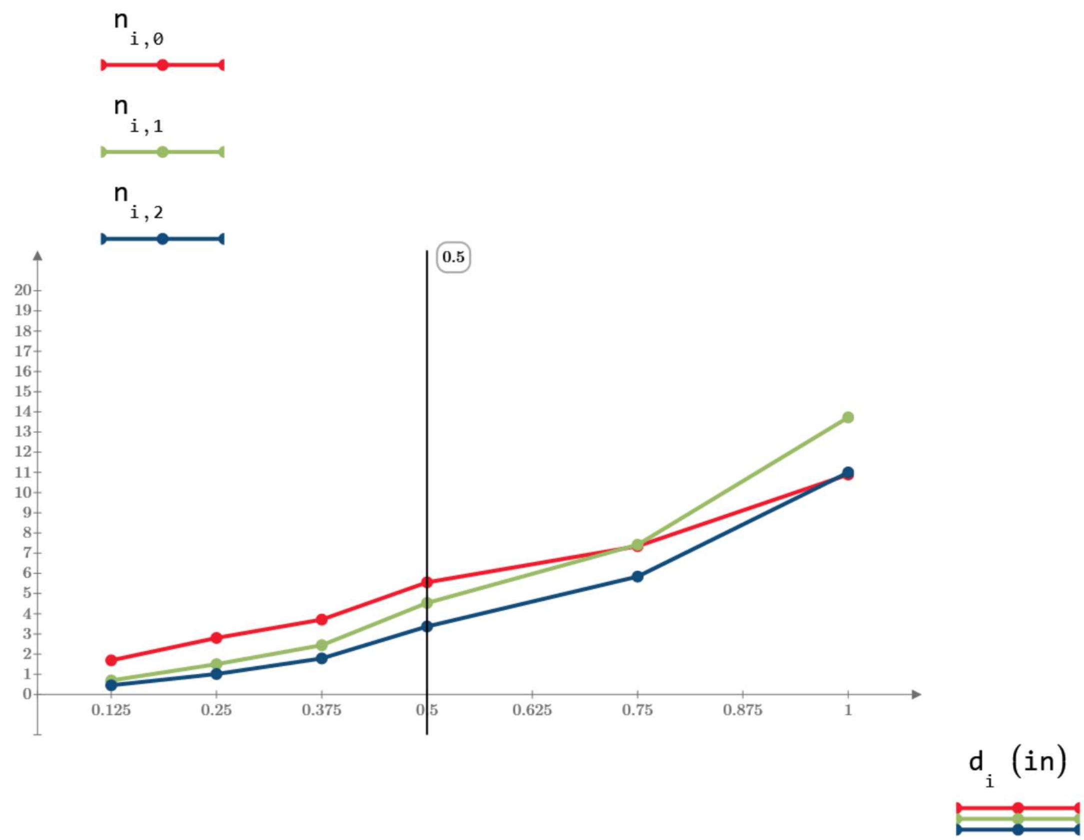
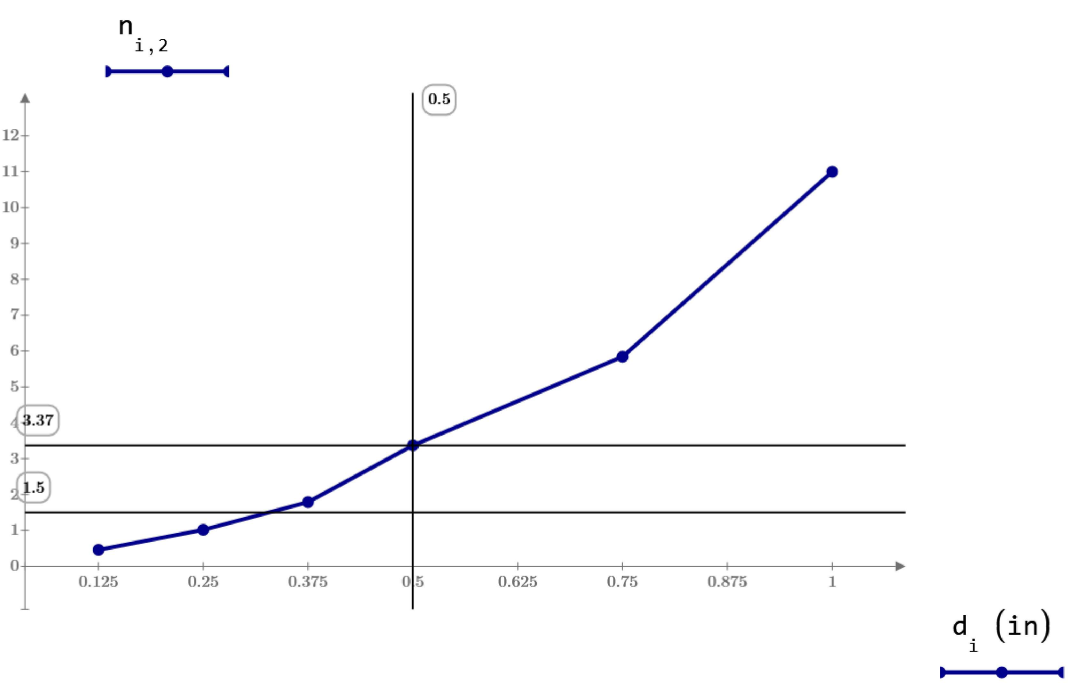


Figure 35 - Handle Bar Safety factors for different critical locations and different size pipes



PULLEY BEAM SHEAR AND BENDING MOMENT ANALYSIS

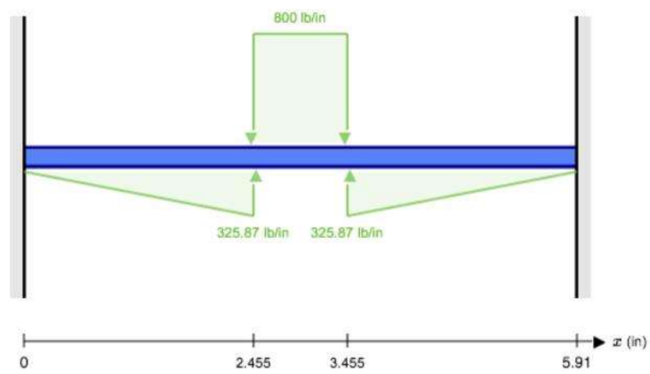


Figure 36 - Pulley Beam Free Body Diagram

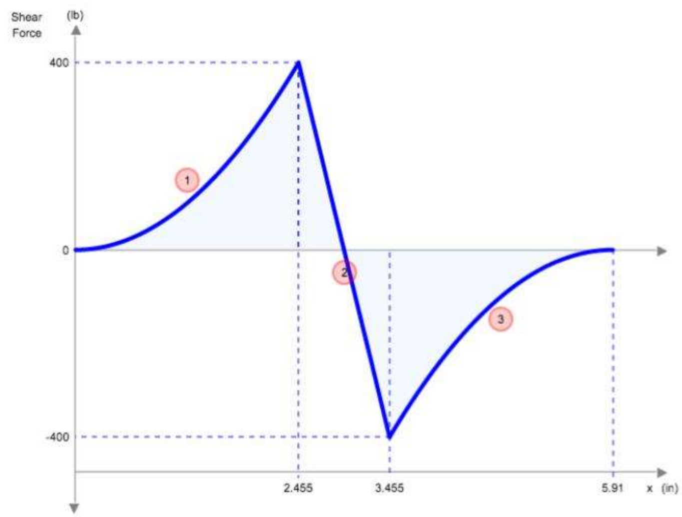


Figure 37 - Pulley Beam Shear Diagram

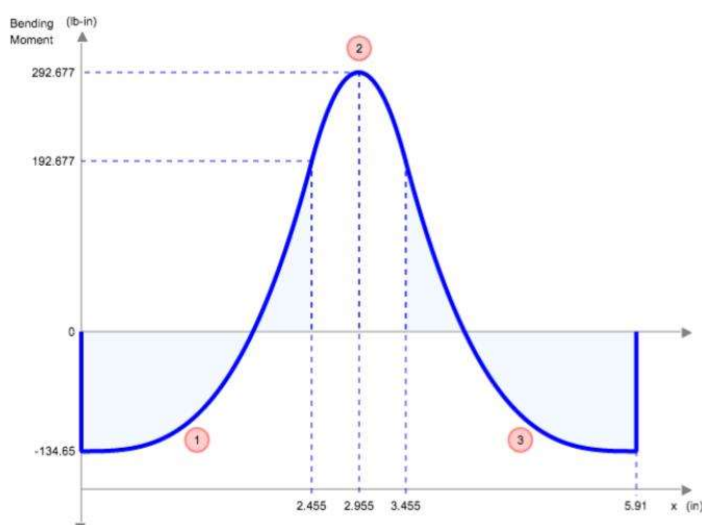


Figure 38 - Pulley Beam Moment Diagram

PULLEY BEAM ANALYSIS AT CRITICAL POINTS

Static Load

Properties:

Material: ASTM A53 Steel Grade B Solid Pipe

$$S_y := 35000 \frac{lb}{in^2}$$

Maximum Reaction Forces from SkyCiv: Figure 37 and Figure 38

$$M_{max} := 292.7\ lb \cdot in$$

$$V_{max} := 400\ lb$$

Analysis:

Cross-Sectional Properties: Figure 29

$$d := 0.0001\ in, 0.125\ in..1\ in$$

Pipes are measured in 1/8" increments

$$A(d) := \frac{1}{4}\pi \cdot d^2$$

Cross-Sectional Properties (Table A-18)

$$I(d) := \frac{1}{64} \cdot \pi \cdot d^4$$

(Table A-18)

$$J(d) := \frac{1}{32} \cdot \pi \cdot d^4$$

(Table A-18)

Critical Point 1: Outer Edge

$$c(d) := \frac{d}{2}$$

Neutral Axis where Shear is Max

$$\sigma_{max}(d) := \frac{M_{max} \cdot c(d)}{I(d)}$$

Normal Stress Due to Bending Moment at outer diameter

$$\tau_{max}(d) := 0 \ \frac{lb}{in^2}$$

Shear is zero at y=radius (3-26a)

$$\sigma_A(d) := \frac{\sigma_{max}(d)}{2} + \sqrt{\left(\frac{\sigma_{max}(d)}{2}\right)^2 + \tau_{max}(d)^2}$$

$$\sigma_B(d) := \frac{\sigma_{max}(d)}{2} - \sqrt{\left(\frac{\sigma_{max}(d)}{2}\right)^2 + \tau_{max}(d)^2}$$

Maximum Shear Stress Theory (MSS):

$$\sigma_1(d) := \begin{array}{|l} \text{if } \sigma_A(d) \geq \sigma_B(d) \geq 0 \\ \quad \text{return } \sigma_A(d) \\ \text{else if } \sigma_A(d) \geq 0 \geq \sigma_B(d) \\ \quad \text{return } \sigma_A(d) \\ \text{else if } 0 \geq \sigma_A(d) \geq \sigma_B(d) \\ \quad \text{return } 0 \end{array}$$

$$\sigma_3(d) := \begin{array}{|l} \text{if } \sigma_A(d) \geq \sigma_B(d) \geq 0 \\ \quad \text{return } 0 \\ \text{else if } \sigma_A(d) \geq 0 \geq \sigma_B(d) \\ \quad \text{return } \sigma_B(d) \\ \text{else if } 0 \geq \sigma_A(d) \geq \sigma_B(d) \\ \quad \text{return } \sigma_B(d) \end{array}$$

Factor of Safety: (5-3)

$$n_1(d) := \frac{S_y}{\sigma_1(d) - \sigma_3(d)}$$

$$n_1(d) = \begin{bmatrix} 1.174 \cdot 10^{-11} \\ 0.023 \\ 0.183 \\ 0.618 \\ 1.465 \\ 2.861 \\ 4.943 \\ 7.848 \\ 11.715 \end{bmatrix}$$

Critical Point 0: Neutral Axis

$$A'(d) := \frac{A(d)}{2}$$

Distance from edge to Neutral Axis

$$y'_{bar}(d) := \frac{4\ \frac{d}{2}}{3\ \pi}$$

Distance to centroid of A'(d)

$$Q(d) := y'_{bar}(d) \cdot A'(d)$$

$$\sigma_{max}(d) := 0\ \frac{lb}{in^2}$$

Normal Stress is equal to zero along neutral axis

$$\tau_{max}(d) := \frac{\left(V_{max} \cdot Q(d)\right)}{I(d) \cdot d}$$

Shear at neutral axis

$$\sigma_A(d) := \frac{\sigma_{max}(d)}{2} + \sqrt{\left(\frac{\sigma_{max}(d)}{2}\right)^2 + \tau_{max}(d)^2}$$

$$\sigma_B(d) := \frac{\sigma_{max}(d)}{2} - \sqrt{\left(\frac{\sigma_{max}(d)}{2}\right)^2 + \tau_{max}(d)^2}$$

Maximum Shear Stress Theory (MSS):

$$\sigma_1(d) := \begin{array}{l} \text{if } \sigma_A(d) \ge \sigma_B(d) \ge 0 \\ \quad \text{return } \sigma_A(d) \\ \text{else if } \sigma_A(d) \ge 0 \ge \sigma_B(d) \\ \quad \text{return } \sigma_A(d) \\ \text{else if } 0 \ge \sigma_A(d) \ge \sigma_B(d) \\ \quad \text{return } 0 \end{array}$$

$$\sigma_3(d) := \begin{array}{l} \text{if } \sigma_A(d) \ge \sigma_B(d) \ge 0 \\ \quad \text{return } 0 \\ \text{else if } \sigma_A(d) \ge 0 \ge \sigma_B(d) \\ \quad \text{return } \sigma_B(d) \\ \text{else if } 0 \ge \sigma_A(d) \ge \sigma_B(d) \\ \quad \text{return } \sigma_B(d) \end{array}$$

Factor of Safety: (5-3)

$$n_0(d) := \frac{S_y}{\sigma_1(d) - \sigma_3(d)}$$

We concluded that a 5/8" rod of ASTM A53 steel is needed to safely support the weight of 400 lbs.

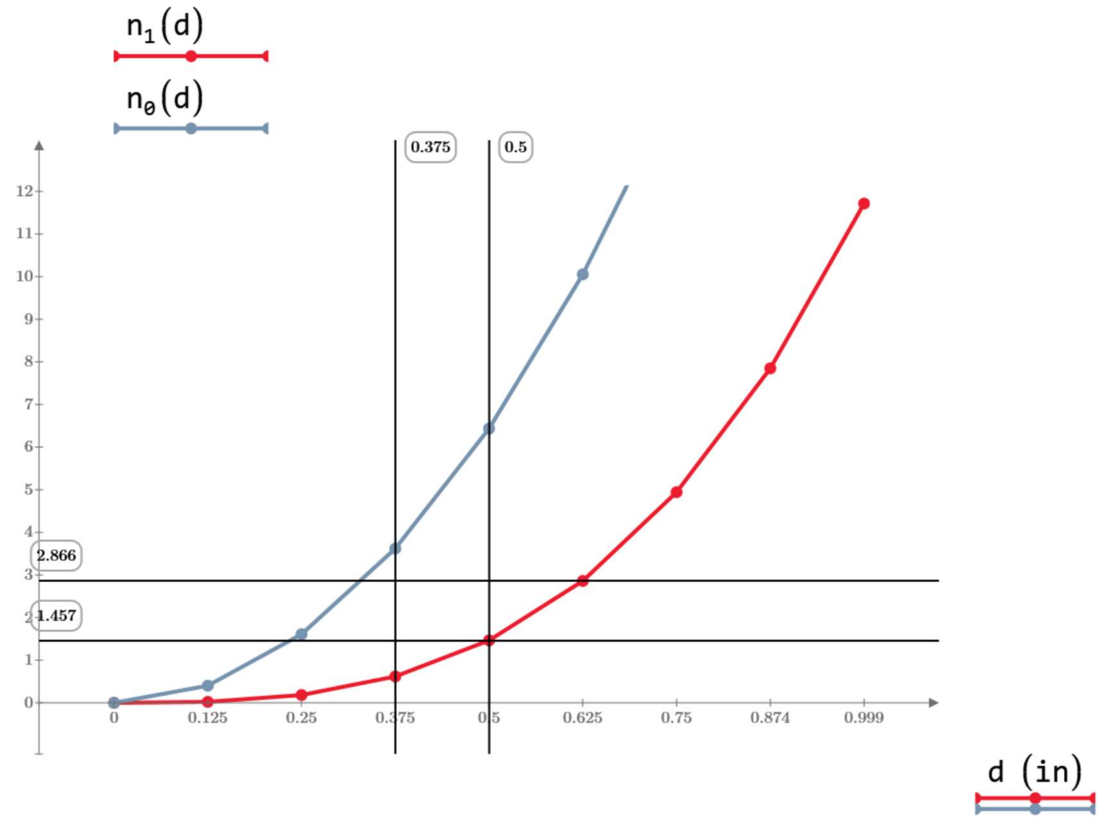


Figure 39- Safety Factor Graph for Pully Shaft

SEAT EDGE HEIGHT DETERMINATION ANALYSIS

Given Cross Sectional Properties: Figure 26

Width $t := 0.060\ \boldsymbol{in}$

Length $l := 14\ \boldsymbol{in}$

Strength $S_y := 40000\ \dfrac{\boldsymbol{lb}}{\boldsymbol{in}^2}$

Force $V := 400\ \boldsymbol{lb}$

Safety Factor

$n := 2.5$

Assuming: Seat base offers no distribution of force, each of the two supports takes half of the

$x := 0\ \boldsymbol{in}, 0.5\ \boldsymbol{in} .. 15\ \boldsymbol{in}$

Find: Slope of Support

Solve:

$$\sigma := \frac{S_y}{n} = \left(1.6 \cdot 10^4\right) \frac{\boldsymbol{lb}}{\boldsymbol{in}^2}$$

$$M(x) := V \cdot x$$

$$h(x) := \sqrt{\left(\frac{M(x) \cdot 3}{t \cdot \sigma}\right)}$$

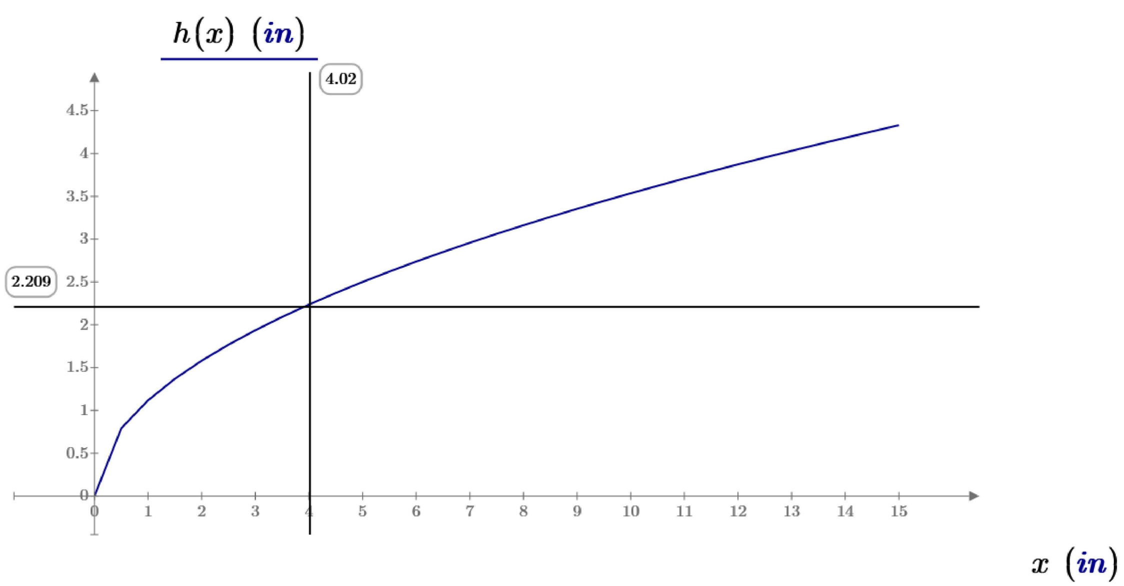


Figure 40 - Height Distribution for Seat Edge

CENTER SEAT FRAME SHEAR AND BENDING MOMENT ANALYSIS

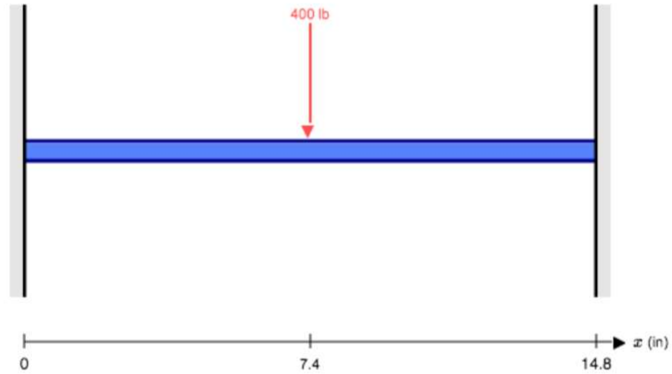


Figure 41 - Center Seat Free Body Diagram

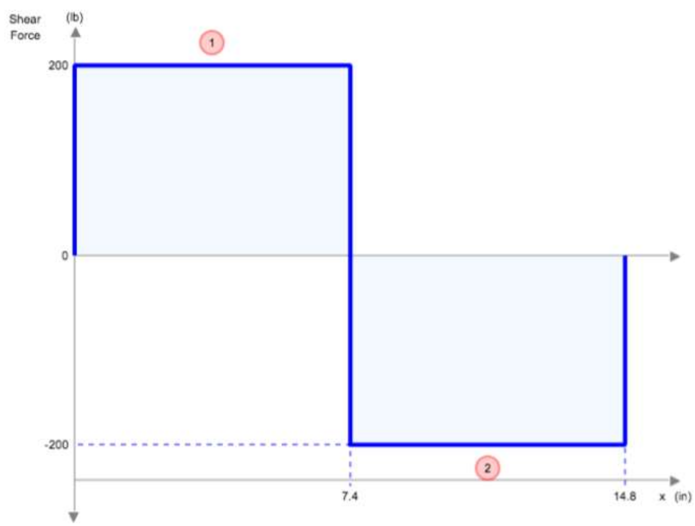


Figure 42 - Center Seat Shear Diagram

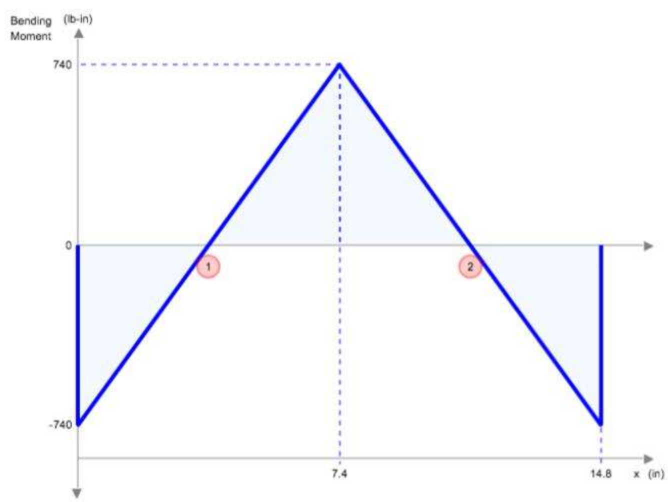


Figure 43 - Center Seat Moment Diagram

CENTER SEAT FRAME ANALYSIS AT CRITICAL POINTS

Static Load

Properties:

Material: ASTM A36 Hot Rolled Steel Hollow Square Pipe - 16 Gauge

$$S_y := 36000 \ \frac{lb}{in^2}$$

Maximum Reaction Forces from SkyCiv: Figure 42 and Figure 43

$$M_{max} := 740 \ lb \cdot in$$

$$V_{max} := 200 \ lb$$

Analysis:

Cross-Sectional Properties: Figure 28

$$h_o := 1 \ in \qquad h_i := 0.94 \ in$$

$$t := 0.06 \ in$$

1" square pipe dimensions

$$I := \frac{\left(h_o\right)^4}{12} - \frac{\left(h_i\right)^4}{12} = 0.018 \ in^4$$

Cross-Sectional Properties

Critical Point 2: Outer Edge

$$c := \frac{h_o}{2} = 0.5\ in$$

$$\sigma_{max} := \frac{M_{max} \cdot c}{I} = \left(2.025 \cdot 10^{4}\right) \frac{lb}{in^2}$$

$$\tau_{max} := 0\ \frac{lb}{in^2}$$

$$\sigma_A := \frac{\sigma_{max}}{2} + \sqrt{\left(\frac{\sigma_{max}}{2}\right)^2 + \tau_{max}^{\ 2}} = \left(2.025 \cdot 10^{4}\right) \frac{lb}{in^2}$$

$$\sigma_B := \frac{\sigma_{max}}{2} - \sqrt{\left(\frac{\sigma_{max}}{2}\right)^2 + \tau_{max}^{\ 2}} = 0\ \frac{lb}{in^2}$$

Maximum Shear Stress Theory (MSS):

$$\sigma_1 := \begin{array}{l} \text{if } \sigma_A \ge \sigma_B \ge 0 \\ \quad \text{return } \sigma_A \\ \text{else if } \sigma_A \ge 0 \ge \sigma_B \\ \quad \text{return } \sigma_A \\ \text{else if } 0 \ge \sigma_A \ge \sigma_B \\ \quad \text{return } 0 \end{array}$$

$$\sigma_3 := \begin{array}{l} \text{if } \sigma_A \ge \sigma_B \ge 0 \\ \quad \text{return } 0 \\ \text{else if } \sigma_A \ge 0 \ge \sigma_B \\ \quad \text{return } \sigma_B \\ \text{else if } 0 \ge \sigma_A \ge \sigma_B \\ \quad \text{return } \sigma_B \end{array}$$

Factor of Safety: (5-3)

$$n_1 := \frac{S_y}{\sigma_1 - \sigma_3}$$

$$n_1 = 1.778$$

Critical Point 0: Neutral Axis

$$A' := \frac{\left(\left(h_o\right)^2 - \left(h_i\right)^2\right)}{2} = 0.058\ \mathbf{in} \cdot \mathbf{in}$$

$$y'_{bar} := \frac{\frac{h_o}{4} \frac{\left(h_o\right)^2}{2} - \frac{h_i}{4} \frac{\left(h_i\right)^2}{2}}{A'} = 0.364\ \mathbf{in}$$

$$Q := y'_{bar} \cdot A' = 0.021\ \mathbf{in}^3$$

$$\sigma_{max} := 0\ \frac{\mathbf{lb}}{\mathbf{in}^2}$$

$$\tau_{max} := \frac{\left(V_{max} \cdot Q\right)}{I \cdot \left(h_o - h_i\right)} = \left(3.864 \cdot 10^3\right)\ \frac{\mathbf{lb}}{\mathbf{in}^2}$$

$$\sigma_A := \frac{\sigma_{max}}{2} + \sqrt{\left(\frac{\sigma_{max}}{2}\right)^2 + \tau_{max}^{\ 2}} = \left(3.864 \cdot 10^3\right)\ \frac{\mathbf{lb}}{\mathbf{in}^2}$$

$$\sigma_B := \frac{\sigma_{max}}{2} - \sqrt{\left(\frac{\sigma_{max}}{2}\right)^2 + \tau_{max}^{\ 2}} = -3.864 \cdot 10^3\ \frac{\mathbf{lb}}{\mathbf{in}^2}$$

Maximum Shear Stress Theory (MSS):

$$\sigma_1 := \begin{array}{l} \text{if } \sigma_A \ge \sigma_B \ge 0 \\ \quad \text{return } \sigma_A \\ \text{else if } \sigma_A \ge 0 \ge \sigma_B \\ \quad \text{return } \sigma_A \\ \text{else if } 0 \ge \sigma_A \ge \sigma_B \\ \quad \text{return } 0 \end{array}$$

$$\sigma_3 := \begin{array}{l} \text{if } \sigma_A \ge \sigma_B \ge 0 \\ \quad \text{return } 0 \\ \text{else if } \sigma_A \ge 0 \ge \sigma_B \\ \quad \text{return } \sigma_B \\ \text{else if } 0 \ge \sigma_A \ge \sigma_B \\ \quad \text{return } \sigma_B \end{array}$$

Factor of Safety: (5-3)

$$n_2 := \frac{S_y}{\sigma_1 - \sigma_3}$$

$$n_2 = 4.659$$

Critical Point 1: Inner Wall

$$c_1 := \frac{h_i}{2} = 0.47\ \boldsymbol{in}$$

$$A' := h_o \cdot t = 0.06\ \boldsymbol{in}^2$$

$$y'_{bar} := \frac{h_i}{2} + \frac{t}{2} = 0.5\ \boldsymbol{in}$$

$$Q := y'_{bar} \cdot A' = 0.03\ \boldsymbol{in}^3$$

$$\sigma_{max} := \frac{M_{max} \cdot c_1}{I} = \left(1.904 \cdot 10^4\right) \frac{\boldsymbol{lb}}{\boldsymbol{in}^2}$$

$$\tau_{max} := \frac{\left(V_{max} \cdot Q\right)}{I \cdot \left(h_o\right)} = 328.391\ \frac{\boldsymbol{lb}}{\boldsymbol{in}^2}$$

$$\sigma_A := \frac{\sigma_{max}}{2} + \sqrt{\left(\frac{\sigma_{max}}{2}\right)^2 + \tau_{max}^{\ 2}} = \left(1.904 \cdot 10^4\right) \frac{\boldsymbol{lb}}{\boldsymbol{in}^2}$$

$$\sigma_B := \frac{\sigma_{max}}{2} - \sqrt{\left(\frac{\sigma_{max}}{2}\right)^2 + \tau_{max}^{\ 2}} = -5.663\ \frac{\boldsymbol{lb}}{\boldsymbol{in}^2}$$

Maximum Shear Stress Theory (MSS):

$$\sigma_1 := \begin{Vmatrix} \text{if } \sigma_A \ge \sigma_B \ge 0 \\ \quad \text{return } \sigma_A \\ \text{else if } \sigma_A \ge 0 \ge \sigma_B \\ \quad \text{return } \sigma_A \\ \text{else if } 0 \ge \sigma_A \ge \sigma_B \\ \quad \text{return } 0 \end{Vmatrix}$$

$$\sigma_3 := \begin{Vmatrix} \text{if } \sigma_A \ge \sigma_B \ge 0 \\ \quad \text{return } 0 \\ \text{else if } \sigma_A \ge 0 \ge \sigma_B \\ \quad \text{return } \sigma_B \\ \text{else if } 0 \ge \sigma_A \ge \sigma_B \\ \quad \text{return } \sigma_B \end{Vmatrix}$$

Factor of Safety: (5-3)

$$n_3 := \frac{S_y}{\sigma_1 - \sigma_3}$$

$$n_3 = 1.89$$

FRONT SEAT FRAME SHEAR AND BENDING MOMENT ANALYSIS

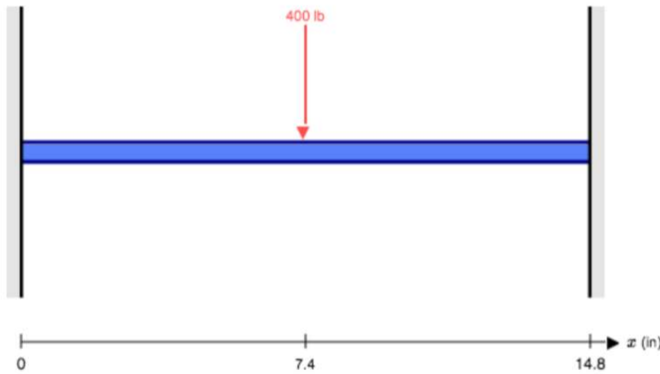


Figure 44 - Front Seat Free Body Diagram

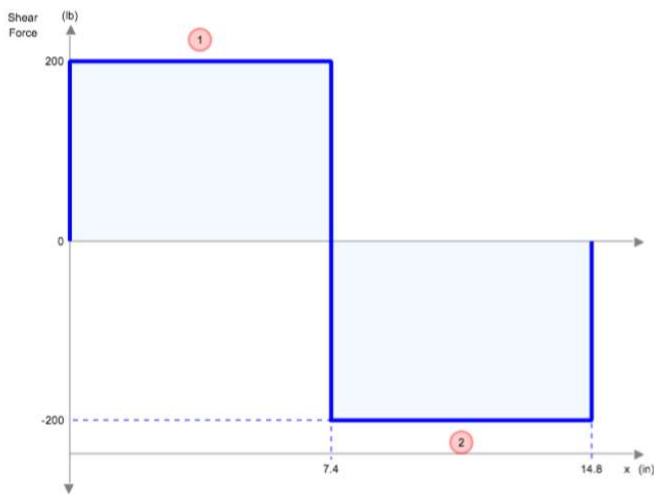


Figure 45 - Front Seat Shear Diagram

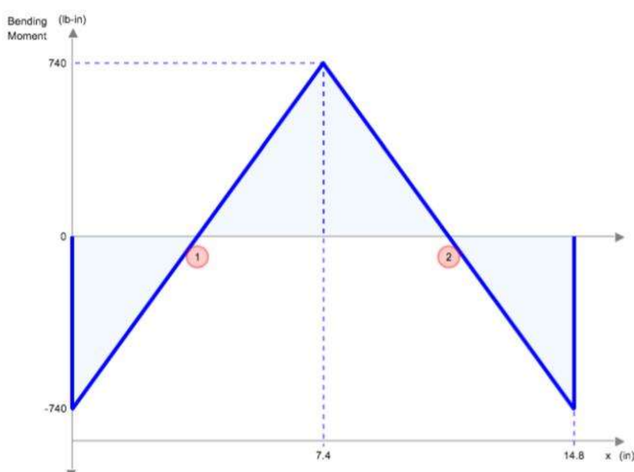


Figure 46 -Front Seat Moment Diagram

FRONT SEAT FRAME ANALYSIS AT CRITICAL POINTS

Static Load

Properties:

Material: ASTM A36 Hot Rolled Steel Hollow Square Pipe - 16 Gauge

$$S_y := 36000 \ \frac{lb}{in^2}$$

Maximum Reaction Forces from SkyCiv: Figure 45 and Figure 46

$$M_{max} := 420 \ lb \cdot in$$

$$V_{max} := 100 \ lb$$

Analysis:

Cross-Sectional Properties Figure 28

$h_o := 1 \ in \qquad h_i := 0.94 \ in$ 1" square pipe dimensions

$t := 0.06 \ in$

Cross-Sectional Properties

$$I := \frac{(h_o)^4}{12} - \frac{(h_i)^4}{12} = 0.018 \ in^4$$

Critical Point 2: Outer Edge

$$c := \frac{h_o}{2} = 0.5\ in$$

$$\sigma_{max} := \frac{M_{max} \cdot c}{I} = \left(1.149 \cdot 10^4\right) \frac{lb}{in^2}$$

$$\tau_{max} := 0\ \frac{lb}{in^2}$$

$$\sigma_A := \frac{\sigma_{max}}{2} + \sqrt{\left(\frac{\sigma_{max}}{2}\right)^2 + {\tau_{max}}^2} = \left(1.149 \cdot 10^4\right) \frac{lb}{in^2}$$

$$\sigma_B := \frac{\sigma_{max}}{2} - \sqrt{\left(\frac{\sigma_{max}}{2}\right)^2 + {\tau_{max}}^2} = 0\ \frac{lb}{in^2}$$

Maximum Shear Stress Theory (MSS):

$$\sigma_1 := \begin{array}{l} \text{if } \sigma_A \ge \sigma_B \ge 0 \\ \quad \text{return } \sigma_A \\ \text{else if } \sigma_A \ge 0 \ge \sigma_B \\ \quad \text{return } \sigma_A \\ \text{else if } 0 \ge \sigma_A \ge \sigma_B \\ \quad \text{return } 0 \end{array} \qquad \sigma_3 := \begin{array}{l} \text{if } \sigma_A \ge \sigma_B \ge 0 \\ \quad \text{return } 0 \\ \text{else if } \sigma_A \ge 0 \ge \sigma_B \\ \quad \text{return } \sigma_B \\ \text{else if } 0 \ge \sigma_A \ge \sigma_B \\ \quad \text{return } \sigma_B \end{array}$$

Factor of Safety: (5-3)

$$n_1 := \frac{S_y}{\sigma_1 - \sigma_3}$$

$$n_1 = 3.132$$

Critical Point 0: Neutral Axis

$$A' := \frac{\left(\left(h_o\right)^2 - \left(h_i\right)^2\right)}{2} = 0.058\ \boldsymbol{in \cdot in}$$

$$y'_{bar} := \frac{\frac{h_o}{4} \cdot \frac{\left(h_o\right)^2}{2} - \frac{h_i}{4} \cdot \frac{\left(h_i\right)^2}{2}}{A'} = 0.364\ \boldsymbol{in}$$

$$Q := y'_{bar} \cdot A' = 0.021\ \boldsymbol{in}^3$$

$$\sigma_{max} := 0\ \frac{\boldsymbol{lb}}{\boldsymbol{in}^2}$$

$$\tau_{max} := \frac{\left(V_{max} \cdot Q\right)}{I \cdot \left(h_o - h_i\right)} = \left(1.932 \cdot 10^3\right)\ \frac{\boldsymbol{lb}}{\boldsymbol{in}^2}$$

$$\sigma_A := \frac{\sigma_{max}}{2} + \sqrt{\left(\frac{\sigma_{max}}{2}\right)^2 + \tau_{max}^{\ 2}} = \left(1.932 \cdot 10^3\right)\ \frac{\boldsymbol{lb}}{\boldsymbol{in}^2}$$

$$\sigma_B := \frac{\sigma_{max}}{2} - \sqrt{\left(\frac{\sigma_{max}}{2}\right)^2 + \tau_{max}^{\ 2}} = -1.932 \cdot 10^3\ \frac{\boldsymbol{lb}}{\boldsymbol{in}^2}$$

Maximum Shear Stress Theory (MSS):

$$\sigma_1 := \begin{array}{l} \text{if } \sigma_A \ge \sigma_B \ge 0 \\ \quad \text{return } \sigma_A \\ \text{else if } \sigma_A \ge 0 \ge \sigma_B \\ \quad \text{return } \sigma_A \\ \text{else if } 0 \ge \sigma_A \ge \sigma_B \\ \quad \text{return } 0 \end{array}$$

$$\sigma_3 := \begin{array}{l} \text{if } \sigma_A \ge \sigma_B \ge 0 \\ \quad \text{return } 0 \\ \text{else if } \sigma_A \ge 0 \ge \sigma_B \\ \quad \text{return } \sigma_B \\ \text{else if } 0 \ge \sigma_A \ge \sigma_B \\ \quad \text{return } \sigma_B \end{array}$$

Factor of Safety: (5-3)

$$n_2 := \frac{S_y}{\sigma_1 - \sigma_3}$$

$$n_2 = 9.318$$

Critical Point 1: Inner Wall

$$c_1 := \frac{h_i}{2} = 0.47\ \boldsymbol{in}$$

$$A' := h_o \cdot t = 0.06\ \boldsymbol{in}^2$$

$$y'_{bar} := \frac{h_i}{2} + \frac{t}{2} = 0.5\ \boldsymbol{in}$$

$$Q := y'_{bar} \cdot A' = 0.03\ \boldsymbol{in}^3$$

$$\sigma_{max} := \frac{M_{max} \cdot c_1}{I} = \left(1.08 \cdot 10^4\right)\ \frac{\boldsymbol{lb}}{\boldsymbol{in}^2}$$

$$\tau_{max} := \frac{\left(V_{max} \cdot Q\right)}{I \cdot \left(h_o\right)} = 164.195\ \frac{\boldsymbol{lb}}{\boldsymbol{in}^2}$$

$$\sigma_A := \frac{\sigma_{max}}{2} + \sqrt{\left(\frac{\sigma_{max}}{2}\right)^2 + \tau_{max}^{\ 2}} = \left(1.081 \cdot 10^4\right)\ \frac{\boldsymbol{lb}}{\boldsymbol{in}^2}$$

$$\sigma_B := \frac{\sigma_{max}}{2} - \sqrt{\left(\frac{\sigma_{max}}{2}\right)^2 + \tau_{max}^{\ 2}} = -2.495\ \frac{\boldsymbol{lb}}{\boldsymbol{in}^2}$$

Maximum Shear Stress Theory (MSS):

$$\sigma_1 := \begin{Vmatrix} \text{if } \sigma_A \ge \sigma_B \ge 0 \\ \quad \Vert\ \text{return } \sigma_A \\ \text{else if } \sigma_A \ge 0 \ge \sigma_B \\ \quad \Vert\ \text{return } \sigma_A \\ \text{else if } 0 \ge \sigma_A \ge \sigma_B \\ \quad \Vert\ \text{return } 0 \end{Vmatrix}$$

$$\sigma_3 := \begin{Vmatrix} \text{if } \sigma_A \ge \sigma_B \ge 0 \\ \quad \Vert\ \text{return } 0 \\ \text{else if } \sigma_A \ge 0 \ge \sigma_B \\ \quad \Vert\ \text{return } \sigma_B \\ \text{else if } 0 \ge \sigma_A \ge \sigma_B \\ \quad \Vert\ \text{return } \sigma_B \end{Vmatrix}$$

Factor of Safety: (5-3)

$$n_3 := \frac{S_y}{\sigma_1 - \sigma_3}$$

$$n_3 = 3.331$$

WINCH BOLTS ANALYSIS AT CRITICAL POINTS

Static Load

Properties:

Material: Carbon Alloy Steel Bolts - SAE Grade 8

$$S_y := 130000 \ \frac{lb}{in^2}$$

Maximum Reaction Forces

$$V_{max} := 400 \ lb \qquad M_{max} := 0 \ lb \cdot in \qquad T := 100 \ in \cdot lb$$

Analysis:

Cross-Sectional Properties: Figure 29

Bolt Diameter

$$t := \frac{3}{8} \ in$$

$$r := \frac{t}{2}$$

$$I := \frac{1}{4} \cdot \pi \cdot (r)^4$$

$$y' := \frac{4\ r}{3 \cdot \pi}$$

$$A' := \frac{\pi \cdot r^2}{2}$$

$$Q := y' \cdot A'$$

$$\tau_{max} := \frac{V_{max} \cdot Q}{I \cdot t}$$

$$K := 0.2$$

$$F := \frac{T}{K \cdot t} \qquad (8\text{-}27)$$

$$A := \pi \cdot r^2$$

$$\sigma_{max} := \frac{F}{A}$$

$$\sigma_A := \frac{\sigma_{max}}{2} + \sqrt{\left(\frac{\sigma_{max}}{2}\right)^2 + \tau_{max}^{\ 2}} = \left(1.377 \cdot 10^4\right) \ \frac{lb}{in^2}$$

$$\sigma_B := \frac{\sigma_{max}}{2} - \sqrt{\left(\frac{\sigma_{max}}{2}\right)^2 + \tau_{max}^{\ 2}} = -1.694 \cdot 10^3 \ \frac{lb}{in^2}$$

Maximum Shear Stress Theory (MSS):

$$\sigma_1 := \begin{array}{l} \text{if } \sigma_A \ge \sigma_B \ge 0 \\ \quad \text{return } \sigma_A \\ \text{else if } \sigma_A \ge 0 \ge \sigma_B \\ \quad \text{return } \sigma_A \\ \text{else if } 0 \ge \sigma_A \ge \sigma_B \\ \quad \text{return } 0 \end{array}$$

$$\sigma_3 := \begin{array}{l} \text{if } \sigma_A \ge \sigma_B \ge 0 \\ \quad \text{return } 0 \\ \text{else if } \sigma_A \ge 0 \ge \sigma_B \\ \quad \text{return } \sigma_B \\ \text{else if } 0 \ge \sigma_A \ge \sigma_B \\ \quad \text{return } \sigma_B \end{array}$$

Factor of Safety:

$$n_1 := \frac{S_y}{\sigma_1 - \sigma_3}$$

$$n_1 = 8.409$$

L-BEAM RAIL SHEAR AND BENDING MOMENT ANALYSIS

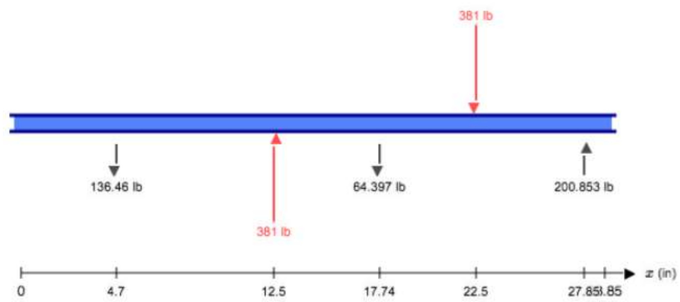


Figure 47 - Rail Free Body Diagram

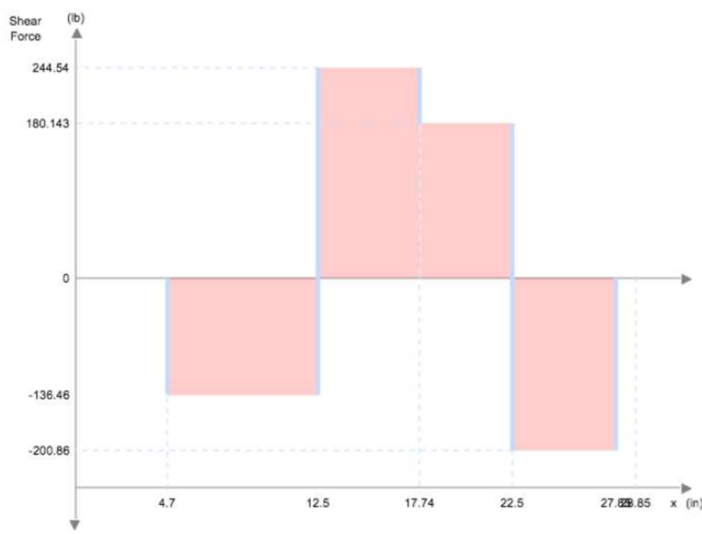


Figure 48 - Rail Shear Diagram

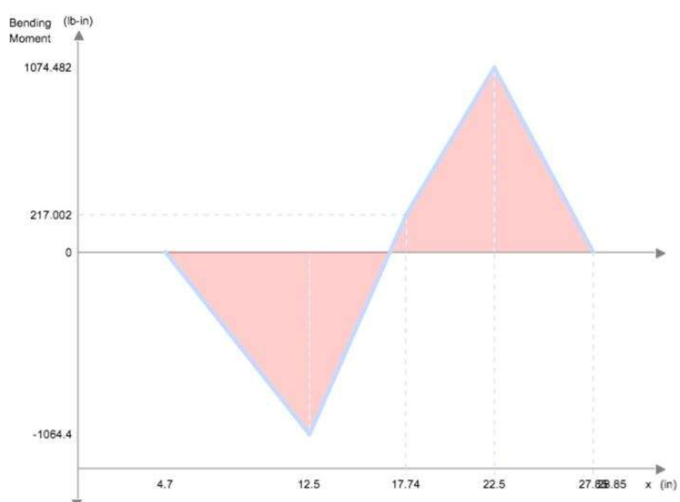


Figure 49 - Moment Diagram

L-BEAM RAIL ANALYSIS AT CRITICAL POINTS

Static Load

Properties:

Material: ASTM A36 Hot Rolled Steel L-Beam: Figure 30

$$S_y := 36000\ \frac{lb}{in^2}$$

Maximum Reaction Forces from SkyCiv: Figure 48 and Figure 49

$V_{max} := 244\ lb$ $\qquad$ $M_{max} := 1074\ lb \cdot in$

Analysis:

Cross-Sectional Properties: Figure 30

$t := 0.0001\ in, 0.05\ in..0.4\ in$ $\qquad$ $t_{actual} := 0.187\ in$

$w := 2.6\ in$ — Outside width

$h := 1.75\ in$ — Inside Height

$A(t) := w \cdot t + h \cdot t$ — Combined Area

$$A_w(t) := w \cdot t$$

$$A_h(t) := h \cdot t$$

$$y_w(t) := \frac{t}{2}$$

$$y_h(t) := t + \frac{h}{2}$$

$$y(t) := \frac{\left(y_w(t) \cdot A_w(t)\right) + \left(y_h(t) \cdot A_h(t)\right)}{A(t)}$$

Area moment of inertia axis

$$I_w(t) := \frac{1}{12}\ w \cdot t^3$$

$$I_h(t) := \frac{1}{12} \cdot t \cdot h^3$$

$$I(t) := \left(I_w(t) + A_w(t) \cdot \left(y(t) - y_w(t)\right)^2\right) + \left(I_h(t) + A_h(t) \cdot \left(y_h(t) - y(t)\right)^2\right)$$

Parallel Axis theorem

Critical Point 0: Bottom Thick Edge

$$\sigma_{max}(t) := \frac{M_{max} \cdot y(t)}{I(t)}$$

$$\tau_{max}(t) := 0\ \frac{lb}{in^2}$$

$$\tau_{max\_0}(t) := \sqrt{\left(\frac{\sigma_{max}(t)}{2}\right)^2 + \tau_{max}(t)^2}$$

$$n_0(t) := \frac{S_y}{2 \cdot \tau_{max\_0}(t)}$$

Critical Point 1: Top Edge

$$c(t) := (h + t) - y(t)$$

$$\sigma_{max}(t) := \frac{M_{max} \cdot c(t)}{I(t)}$$

$$\tau_{max}(t) := 0\ \frac{lb}{in^2}$$

$$\tau_{max\_1}(t) := \sqrt{\left(\frac{\sigma_{max}(t)}{2}\right)^2 + \tau_{max}(t)^2}$$

$$n_1(t) := \frac{S_y}{2 \cdot \tau_{max\_1}(t)}$$

Critical Point 2: Neutral Axis

$A'(t) := t \cdot (h - y(t))$

Area from top edge to neutral axis

$y'(t) := \frac{h - y(t)}{2}$

Distance from neutral axis to A' centroid

$Q(t) := y'(t) \cdot A'(t)$

$\sigma_{max}(t) := 0 \frac{lb}{in^2}$

$\tau_{max}(t) := \frac{V_{max} \cdot Q(t)}{I(t) \cdot t}$

$\tau_{max\_2}(t) := \sqrt{\left(\frac{\sigma_{max}(t)}{2}\right)^2 + \tau_{max}(t)^2}$

$n_2(t) := \frac{S_y}{2 \cdot \tau_{max\_2}(t)}$

Critical Point 3: Upper Contact Point

$A'(t) := A_w(t)$

$y'(t) := y(t) - \frac{t}{2}$

$Q(t) := y'(t) \cdot A'(t)$

$c(t) := y(t) - t$

$\sigma_{max}(t) := \frac{M_{max} \cdot c(t)}{I(t)}$

$\tau_{max}(t) := \frac{V_{max} \cdot Q(t)}{I(t) \cdot t}$

$\tau_{max\_3}(t) := \sqrt{\left(\frac{\sigma_{max}(t)}{2}\right)^2 + \tau_{max}(t)^2}$

$n_3(t) := \frac{S_y}{2 \cdot \tau_{max\_3}(t)}$

Critical Point 4: Upper Contact Point

$$A'(t) := A_w(t)$$

$$y'(t) := y(t) - \frac{t}{2}$$

$$Q(t) := y'(t) \cdot A'(t)$$

$$c(t) := y(t) - t$$

$$\sigma_{max}(t) := \frac{M_{max} \cdot c(t)}{I(t)}$$

$$\tau_{max}(t) := \frac{V_{max} \cdot Q(t)}{I(t) \cdot w}$$

$$\tau_{max\_4}(t) := \sqrt{\left(\frac{\sigma_{max}(t)}{2}\right)^2 + \tau_{max}(t)^2}$$

$$n_4(t) := \frac{S_y}{2 \cdot \tau_{max\_4}(t)}$$

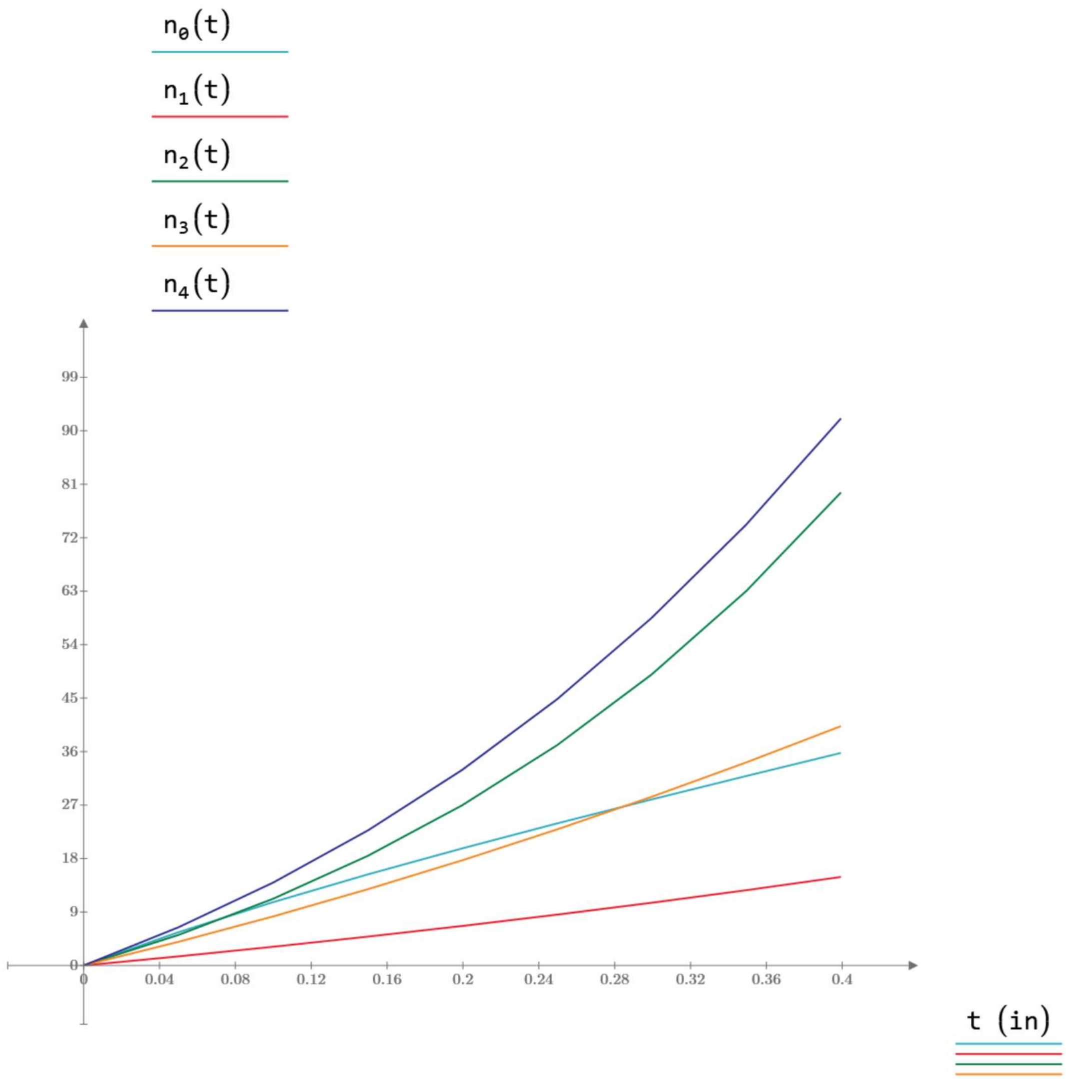


Figure 50 - Factor of Safety of all critical points of L-Bean for different thickness

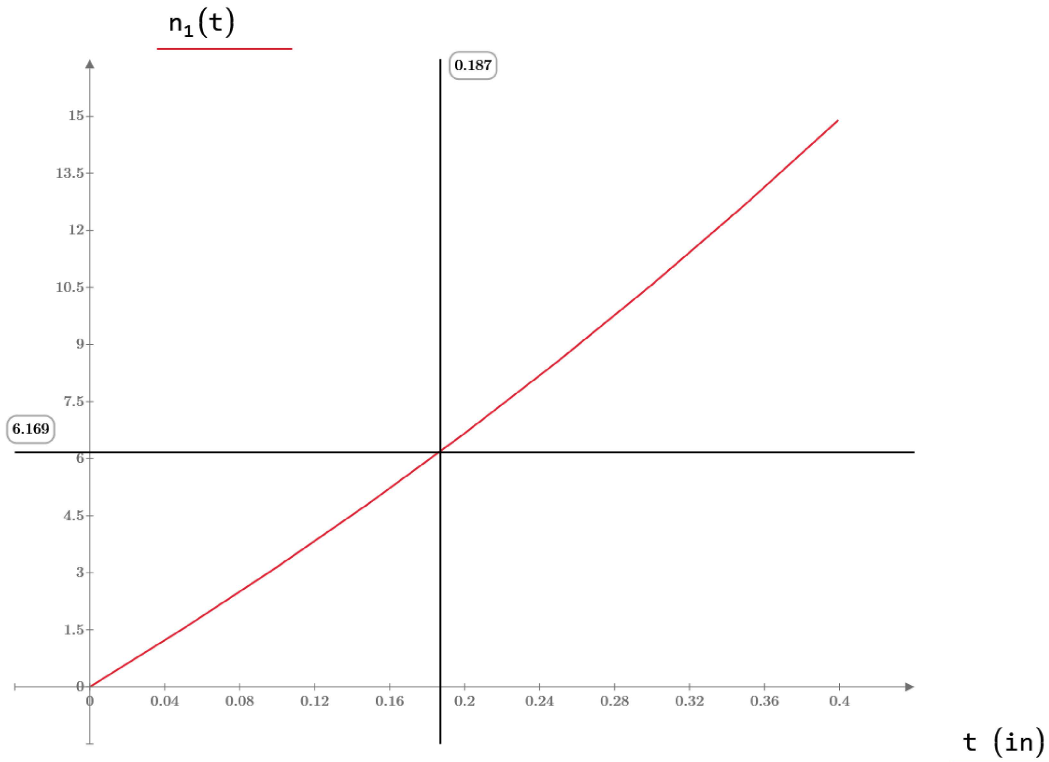


Figure 51 - Factor of Safety for most critical point at L-Beam at different sizes