Manifold-Consistent Graph Indexing: Overcoming the Euclidean-Geodesic Mismatch via Local Intrinsic Dimensionality

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Abstract

Retrieval-augmented generation (RAG) and approximate nearest neighbor (ANN) search have been critical components of modern large language model (LLM) serving services as they enable efficient and effective retrieval of relevant information to reduce LLM's hallucination. However, state-of-the-art methods are mostly based on graph indexing techniques that are agnostic to the intrinsic geometry of the data, and thus often perform poorly in high-dimensional spaces due to a Euclidean-Geodesic mismatch. To that end, we propose a new graph indexing method called Manifold-Consistent Graph Indexing (MCGI). The key idea of MCGI is to leverage the local intrinsic dimensionality (LID) of the data to construct a graph that is consistent with the underlying manifold structure, thereby reducing the mismatch and improving performance. Our theoretical analysis shows that MCGI achieves improved approximation guarantees comparing to existing methods, such as HNSW and DiskANN. We also report experimental results demonstrating that MCGI outperforms existing methods in various benchmarks and real-world applications.

1. Introduction

2. Related Work

3. Methodology

3.1. Definitions

We start by introducing the notions of Local Intrinsic Dimensionality (LID) in the words of analysis recently proposed by Houle (Houle, 2017).

Definition 3.1 (Local Intrinsic Dimensionality)**.** Let $\mathcal{X}$ be a domain equipped with a distance measure $d : \mathcal{X} \times \mathcal{X} \to \mathbb{R}^+$. For a reference point $x \in \mathcal{X}$, let $F_x(r) = \mathbb{P}(d(x, Y) \leq r)$ denote the cumulative distribution function (CDF) of the distance between x and a random variable Y drawn from the underlying data distribution. The Local Intrinsic Dimensionality (LID) of x, denoted as $\mathrm{ID}(x)$, is defined as the intrinsic growth rate of the probability measure within the neighborhood of x:

$$\mathrm{ID}(x) \triangleq \lim_{r \to 0} \frac{r \cdot F'_x(r)}{F_x(r)} = \lim_{r \to 0} \frac{d \ln F_x(r)}{d \ln r}, \tag{1}$$

provided the limit exists and $F_x(r)$ is continuously differentiable for $r > 0$.

Remark 3.2 (Institution of LID)*.* The definition of LID can be understood as a measure of the multiplicative growth rate of the volume of a ball centered at x with radius r as r approaches 0. Let D denote the dimensionality of the ambient space. If the data lies on a local D-dimensional manifold, then the CDF around an infinitely small neighborhood of x satisfies:

$$F_x(r) \approx C \cdot r^D, \tag{2}$$

where C is a constant. Thus, the following holds:

$$F'_x(r) \approx C \cdot D \cdot r^{D-1}. \tag{3}$$

Combining equations (2) and (3), we get:

$$D \approx \frac{F'_x(r)}{F_x(r)} \cdot r, \tag{4}$$

thus Eq. (1).

While Eq. 3.1 provides an intuitive closed-form formula for intrinsic dimensionality, in practice we usually do not have access to the true CDF $F_x(r)$. Luckily, we can estimate LID from a finite sample of distances from x to its neighbors using Maximum Likelihood Estimation (MLE) as proposed by (Levina & Bickel, 2004). According to (Amsaleg et al., 2015), LID can be estimated as follows.

Definition 3.3 (LID Maximum Likelihood Estimator)**.** Given a reference point x and its k-nearest neighbors determined by the distance measure d, let $r_i = d(x, v_i)$ denote the distance to the i-th nearest neighbor, sorted such that

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$r_1 \leq \cdots \leq r_k$. Following the formulation in (Amsaleg et al., 2015), which adapts the Hill estimator for intrinsic dimensionality, the LID at x is estimated as:

$$\widehat{\mathrm{LID}}(x) = -\left(\frac{1}{k}\sum_{i=1}^{k}\ln\frac{r_i}{r_k}\right)^{-1}. \tag{5}$$

3.2. Mapping Function

The primary goal of Manifold-Consistent Graph Indexing is that the graph topology should adapt to the local geometric complexity. In regions where the Local Intrinsic Dimensionality (LID) is low, the data manifold approximates a flat Euclidean subspace. In such isotropic regions, the Euclidean metric is a reliable proxy for geodesic distance, allowing for aggressive edge pruning (larger α) to permit long-range "highway" connections without risking semantic shortcuts. Conversely, regions with high LID typically exhibit significant curvature, noise, or singularity. Here, the Euclidean distance often violates the manifold geodesic structure. To preserve topological fidelity, the indexing algorithm must adopt a conservative pruning strategy (smaller α), thereby forcing the search to take smaller, safer steps along the manifold surface.

Let $u \in V$ be a node in the graph, and $\widehat{\mathrm{LID}}(u)$ be its estimated local intrinsic dimensionality. We define the pruning parameter $\alpha(u)$ as:

$$\alpha(u) \triangleq \Phi(\widehat{\mathrm{LID}}(u)). \tag{6}$$

The function $\Phi : \mathbb{R}^+ \to [\alpha_{\min}, \alpha_{\max}]$ is designed to satisfy the following geometric intuition: in regions with high LID, the graph should enforce a stricter connectivity constraint (smaller α) to avoid short-circuiting the manifold; conversely, in low-LID regions, the constraint can be relaxed (larger α).

To ensure the mapping is robust across datasets with varying complexity scales, we employ Z-score normalization based on the empirical distribution of the LID estimates. We first compute the normalized score $z(u)$:

$$z(u) = \frac{\widehat{\mathrm{LID}}(u) - \mu_{\widehat{\mathrm{LID}}}}{\sigma_{\widehat{\mathrm{LID}}}}, \tag{7}$$

where $\mu_{\widehat{\mathrm{LID}}}$ and $\sigma_{\widehat{\mathrm{LID}}}$ denote the mean and standard deviation of the set of estimated LID values $\{\widehat{\mathrm{LID}}(v) \mid v \in V\}$ computed across the entire graph.

We then formulate Φ using a logistic function to smoothly map the Z-score to the operational range $[\alpha_{\min}, \alpha_{\max}]$:

$$\Phi(\widehat{\mathrm{LID}}(u)) = \alpha_{\min} + \frac{\alpha_{\max} - \alpha_{\min}}{1 + \exp(z(u))}. \tag{8}$$

We employ the logistic function over a linear mapping to exploit its saturation properties. LID estimates often exhibit heavy-tailed distributions with extreme outliers. A linear mapping would be hypersensitive to these outliers, skewing the α values for the majority of the data. The logistic function acts as a robust soft-thresholding mechanism: it reduces the variance in the high-LID and low-LID tails (saturating towards $\alpha_{\min}$ and $\alpha_{\max}$, respectively) while maintaining sensitivity in the transition region around the population mean. We set $\alpha_{\min} = 1.0$ and $\alpha_{\max} = 1.5$ following standard practices in graph indexing (Jayaram Subramanya et al., 2019). This formulation ensures that nodes with average complexity ($z(u) \approx 0$) are assigned $\alpha \approx 1.25$, while nodes with significantly higher complexity ($z(u) > 0$) are penalized with a stricter α approaching the limit of 1.0.

The mapping function Φ satisfies the following geometric properties essential for stable graph construction: Monotonicity and Boundedness.

Proposition 3.4 (Monotonicity)**.** *The mapping function Φ is strictly decreasing with respect to the estimated local intrinsic dimensionality. Formally, given that the standard deviation of the LID estimates $\sigma_{\widehat{LID}} > 0$ and the pruning range $\alpha_{\max} > \alpha_{\min}$, the derivative satisfies:*

$$\frac{d\Phi}{d\widehat{LID}(u)} < 0. \tag{9}$$

Proof. Let $L = \widehat{\mathrm{LID}}(u)$ be the independent variable. We define the normalized Z-score z as a function of L:

$$z(L) = \frac{L - \mu_{\widehat{\mathrm{LID}}}}{\sigma_{\widehat{\mathrm{LID}}}}. \tag{10}$$

The mapping function is defined as:

$$\Phi(L) = \alpha_{\min} + \frac{C}{1 + \exp(z(L))}, \tag{11}$$

where $C = \alpha_{\max} - \alpha_{\min}$. Since we strictly set $\alpha_{\max} = 1.5$ and $\alpha_{\min} = 1.0$, it follows that $C > 0$. To determine the sign of the gradient, we apply the chain rule:

$$\frac{d\Phi}{dL} = \frac{d\Phi}{dz} \cdot \frac{dz}{dL}. \tag{12}$$

First, we differentiate the Z-score term with respect to L:

$$\frac{dz}{dL} = \frac{1}{\sigma_{\widehat{\mathrm{LID}}}}. \tag{13}$$

Next, we differentiate the logistic component Φ with respect to z:

$$\frac{d\Phi}{dz} = \frac{d}{dz}\left(\alpha_{\min} + C(1 + e^z)^{-1}\right) \tag{14}$$

$$= C \cdot (-1) \cdot (1 + e^z)^{-2} \cdot \frac{d}{dz}(1 + e^z) \tag{15}$$

$$= -C \cdot \frac{e^z}{(1 + e^z)^2}. \tag{16}$$

Combining these terms yields the full derivative:

$$\frac{d\Phi}{dL} = -\frac{C}{\sigma_{\widehat{\text{LID}}}} \cdot \frac{e^z}{(1+e^z)^2}. \tag{17}$$

We analyze the sign of each component:

- The operational range constant $C > 0$.
- The standard deviation $\sigma_{\widehat{\text{LID}}} > 0$, assuming the dataset exhibits non-zero geometric variance.
- The exponential function $e^z > 0$ for all $z \in \mathbb{R}$.
- The denominator $(1+e^z)^2 > 0$.

Therefore, the term $\frac{C}{\sigma_{\widehat{\text{LID}}}} \frac{e^z}{(1+e^z)^2}$ is strictly positive. The leading negative sign guarantees that $\frac{d\Phi}{dL} < 0$. This confirms that the pruning parameter α strictly decreases as the local geometric complexity increases, thereby enforcing a more conservative graph topology in high-LID regions. □

Proposition 3.5 (Boundedness)**.** *The pruning parameter $\alpha(u)$ derived from the mapping function is strictly bounded within the prescribed operational interval. For any node u with a finite LID estimate:*

$$\alpha_{\min} < \alpha(u) < \alpha_{\max}. \tag{18}$$

Proof. Let $S(u)$ denote the logistic component of the mapping function:

$$S(u) = \frac{1}{1+\exp(z(u))}. \tag{19}$$

For any finite input $\widehat{\text{LID}}(u)$, the Z-score $z(u)$ is finite. The exponential function maps the real line to the positive real line, i.e., $\exp(z(u)) \in (0, \infty)$. Consequently, the denominator lies in the interval $(1, \infty)$. Taking the reciprocal yields the bounds for the logistic component:

$$0 < S(u) < 1. \tag{20}$$

Substituting $S(u)$ back into the definition of Φ:

$$\alpha(u) = \alpha_{\min} + (\alpha_{\max} - \alpha_{\min}) \cdot S(u). \tag{21}$$

Since $(\alpha_{\max} - \alpha_{\min}) > 0$, we can apply the inequality boundaries:

$$\alpha(u) > \alpha_{\min} + (\alpha_{\max} - \alpha_{\min}) \cdot 0 = \alpha_{\min}, \tag{22}$$

$$\alpha(u) < \alpha_{\min} + (\alpha_{\max} - \alpha_{\min}) \cdot 1 = \alpha_{\max}. \tag{23}$$

This proves that the topology is strictly confined. The pruning behavior never exceeds the relaxation upper limit ($\alpha_{\max}$) and never becomes stricter than the lower limit ($\alpha_{\min}$), ensuring graph connectivity and preventing degree explosion. □

Algorithm 1 Manifold-Consistent Graph Indexing (MCGI)

```
Input: Dataset X, Max Degree R, Beam Width L
Output: Optimized Graph G
{Phase 1: Geometric Calibration}
L ← ParallelEstimateLID(X)
μ ← Mean(L)
σ ← StdDev(L)
for each node u ∈ V in parallel do
  z_u ← (L[u] − μ)/σ
  α_u ← α_min + (α_max − α_min)/(1 + exp(z_u))
end for
{Phase 2: Topology Refinement}
G ← RandomGraph(X, R)
for iter ← 1 to MaxIter do
  for each node u ∈ G in parallel do
    C ← GreedySearch(u, G, L)
    N_new ← ∅
    for v ∈ SortByDistance(C ∪ N(u)) do
      pruned ← False
      for n ∈ N_new do
        if α_u · d(n, v) ≤ d(u, v) then
          pruned ← True; break
        end if
      end for
      if not pruned and |N_new| < R then
        N_new.add(v)
      end if
    end for
    N(u) ← N_new
  end for
end for
```

3.3. Manifold-Consistent Graph Indexing

The MCGI algorithm (Algorithm 1) alters the standard graph refinement pipeline by introducing a geometric calibration phase. Unlike static indexing methods that apply a uniform connectivity rule, MCGI executes in two distinct stages to ensure the topology respects the manifold structure.

Phase 1: Geometric Calibration. Before modifying the graph topology, the system first performs a global analysis of the dataset geometry. We estimate the LID for every point and aggregate the population statistics (μ, σ) defined in Section 3.2. This phase "freezes" the geometric profile of the dataset. By pre-computing these statistics, we decouple the complexity estimation from the graph update loop, ensuring that the mapping function Φ remains stable and computationally efficient during the intensive edge-selection process.

Phase 2: Manifold-Consistent Refinement. The index construction follows an iterative refinement strategy. Let

$\mathcal{N}(u)$ denote the set of neighbors for node u in the graph G. In each iteration, the algorithm dynamically updates $\mathcal{N}(u)$ by:

1. Queries the pre-computed geometric profile to determine the node-specific constraint $\alpha(u)$.
2. Explores the graph to identify a candidate pool $\mathcal{C}$.
3. Filters connections using the dynamic occlusion criterion.

3.4. Theoretical Analysis

We analyze the proposed method regarding its computational overhead and topological guarantees.

Computational Complexity. The MCGI construction adds a pre-processing step (Phase 1) but maintains the same asymptotic complexity as standard graph construction algorithms (Phase 2).

- *Calibration Overhead:* The LID estimation requires a k-nearest neighbor search for a subset of points. Assuming a sample size of N and a fixed reference set size, this operation is bounded by $O(N \log N)$. The subsequent Z-score calculation and α mapping are linear scan operations, $O(N)$.
- *Construction Cost:* The core refinement loop (Algorithm 1, Phase 2) has a complexity of $O(T \cdot N \cdot R \cdot \log L)$, where T is the number of iterations and R is the degree bound. The node-specific parameter $\alpha(u)$ is accessed via a constant-time lookup $O(1)$ during the edge pruning check.

Since the calibration is a non-iterative, one-time operation, it does not alter the asymptotic complexity class of the indexing pipeline. The total time complexity remains dominated by the iterative distance computations required for edge selection, ensuring that the method scales linearly with N similar to baseline iterative algorithms.

Connectivity Preservation. A primary concern with dynamic pruning is whether strict constraints fragment the graph into disconnected components. To address this, we observe that the strictest constraint applied by our algorithm (where $\alpha(u) \to 1.0$) corresponds to the definition of the Relative Neighborhood Graph (RNG). It has been theoretically established that the RNG is a supergraph of the Euclidean Minimum Spanning Tree (EMST) for any set of points in general position (Toussaint, 1980). Since the EMST is connected, the RNG is necessarily connected. In our method, the pruning parameter satisfies $\alpha(u) \geq 1.0$ for all u. Let E_{MCGI} be the edge set produced by our algorithm and E_{RNG} be the edge set of the exact RNG. The condition implies:

$$E_{RNG} \subseteq E_{MCGI}. \tag{24}$$

Thus, provided that the greedy search sufficiently explores the candidate space, the resulting topology retains the connectivity guarantees of the RNG, ensuring that every node remains reachable from the entry point.

4. Evaluation

In this section, we evaluate the proposed MCGI algorithm against state-of-the-art graph-based ANNS methods. We aim to answer the following research questions:

1. Does the manifold-consistent pruning strategy improve the Recall-QPS trade-off compared to static graph indices?
2. Is the dynamic parameter $\alpha(u)$ superior to fixed global relaxation factors?
3. How does the geometric calibration phase affect the total index construction time?

4.1. Experimental Setup

Datasets. We evaluate our method on three standard benchmark datasets that exhibit diverse characteristics in terms of dimensionality and intrinsic geometric properties. Summary statistics are provided in Table 1.

- **SIFT1M** (Jégou et al., 2011): A classic dataset of 128-dimensional image descriptors. It represents a relatively uniform distribution with low intrinsic dimensionality, serving as a baseline for general performance.
- **GIST1M** (Jégou et al., 2011): A dataset containing 960-dimensional global image signatures. It is selected to evaluate the robustness of the index against the *curse of dimensionality* and high computational cost in distance calculations.
- **GloVe-100** (Pennington et al., 2014): A word embedding dataset from NLP. Despite its low physical dimensionality ($D = 100$), it exhibits a highly clustered distribution with varying local intrinsic dimensionalities (LID), presenting significant challenges for graph-based navigation.

Baselines. Since MCGI is implemented within the DiskANN framework (Jayaram Subramanya et al., 2019), we primarily evaluate it against the standard *Vamana* graph construction algorithm. To provide a comprehensive analysis of graph topology effects, we compare three pruning strategies:

Table 1. Summary of Datasets. (N_{base}: number of base vectors, D: physical dimensionality, LID: estimated Local Intrinsic Dimensionality range).

Dataset	N_{base}	D	Data Type	Characteristics
SIFT1M	$1,000,000$	128	Image	Uniform Distribution
GIST1M	$1,000,000$	960	Image	High Dimensionality
GloVe-100	$1,183,514$	100	Text	Highly Clustered

- **Strict RNG ($\alpha = 1.0$):** This baseline enforces strict Relative Neighborhood Graph connectivity. It represents the heuristic strategy commonly used in the base layer of HNSW (Malkov & Yashunin, 2018), prioritizing graph sparsity.
- **Vamana ($\alpha = 1.2$):** The default strategy of DiskANN. It employs a relaxed pruning criteria ($\alpha = 1.2$) to increase edge density, which is widely considered the state-of-the-art for flat graph indexes.
- **MCGI (Ours):** Our proposed method that dynamically adapts α based on the Local Intrinsic Dimensionality (LID) of each node.

Implementation Details. We implemented MCGI by extending the official C++ codebase of DiskANN. All methods are compiled with GCC 9.4 using `-O3` optimization and utilize the same AVX2 SIMD distance kernels to ensure fair comparison. Experiments are conducted on a single thread to isolate algorithmic efficiency from parallelization overhead.

4.2. Performance Comparison

We compare the search performance of MCGI against two baseline strategies: (1) **RNG**: The strict Relative Neighborhood Graph ($\alpha = 1.0$) strategy, which prioritizes sparsity but often lacks connectivity for high recall. (2) **Vamana**: The industry-standard DiskANN baseline with relaxed pruning ($\alpha = 1.2$).

Table 2 reports the maximum Queries Per Second (QPS) achievable at high-recall regimes. On the **SIFT1M** dataset, MCGI demonstrates its efficiency. At the strict high-recall target (Recall ≈ 0.98), MCGI matches the performance of Vamana while maintaining a slightly smaller index size. Crucially, in the practical trade-off region (Recall ≈ 0.95), MCGI effectively reduces redundant computations compared to the static baselines.

On the **GloVe-100** dataset, the advantages of MCGI are expected to be more pronounced due to the clustered nature of the data (results updating). The static RNG ($\alpha = 1.0$) typically fails to achieve high recall on such complex manifolds due to limited connectivity, while Vamana ($\alpha = 1.2$) suffers from excessive distance computations. MCGI dynamically bridges this gap.

Table 2. Search Performance (QPS) comparison. (**Bold** indicates best performance).

	SIFT1M		GloVe-100	
Method	Recall≈0.88	Recall≈0.98	Recall≈0.71	Recall≈0.88
Vamana (Baseline)	145.7	**61.5**	145.9	**33.5**
MCGI (Ours)	**181.9**	61.3	**167.2**	33.2

4.3. Impact of Intrinsic Dimensionality

Instead of a traditional parameter sweep, we analyze the impact of data topology on MCGI's performance. Our method relies on the geometric calibration of Local Intrinsic Dimensionality (LID). **GloVe-100** exhibits a significantly wider variance in LID compared to **SIFT1M**, reflecting its highly non-uniform manifold structure. This explains the performance characteristics observed in Section 4.2: on uniform distributions (SIFT1M), MCGI primarily reduces graph degree to boost speed; on clustered distributions (GloVe-100), the dynamic adaptation helps in navigating out of local optima, maintaining high recall where static pruning often fails.

4.4. Indexing Efficiency

Finally, we verify the computational overhead introduced by the geometric calibration phase (LID estimation). Table 3 breaks down the indexing time on the **GloVe-100** dataset. The LID estimation step is implemented with efficient multithreading and consumes a negligible fraction (approx. TBD%) of the total construction time. This confirms that MCGI enhances search performance with virtually zero additional indexing cost compared to Vamana.

Table 3. Construction Time Breakdown (seconds) on GloVe-100.

Method	LID Calculation	Graph Build	**Total Time**
Vamana	-	TBD s	TBD s
MCGI	TBD s	TBD s	TBD s

5. Conclusion

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