

APS April Meeting 2024

Interpolating the 't Hooft Model Between IFD and LFD in the Coulomb Gauge

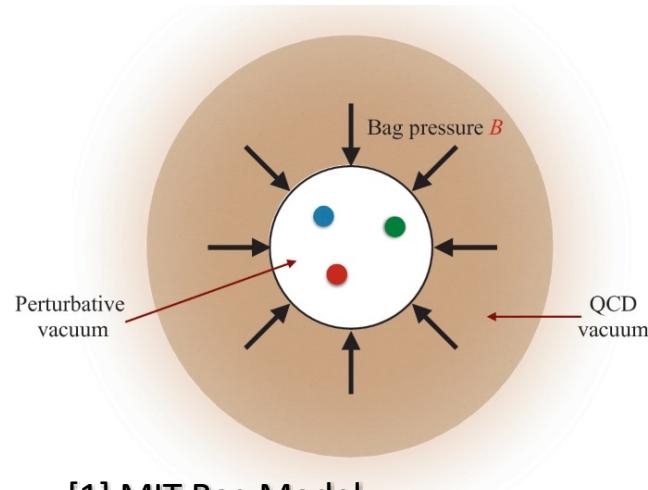
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Funded by Provost Professional Experience Program

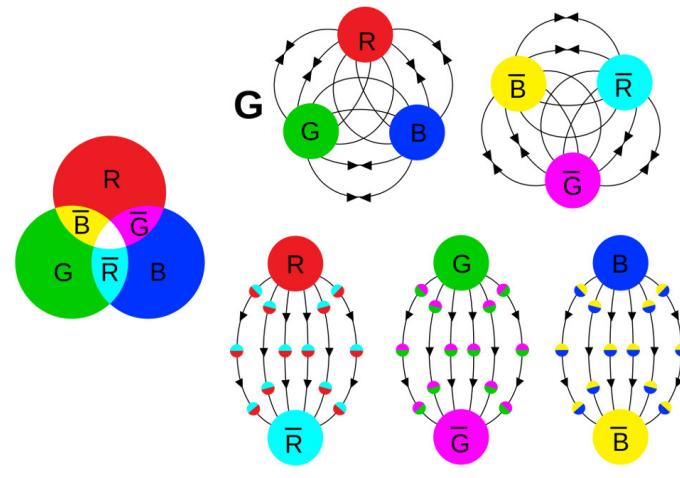
Outline:

- Short QCD introduction
- Introduction to Light Front Dynamics
- Interpolation between instant and front forms
- Motivation behind Interpolation
- 't Hooft's toy meson model
- Why use the Coulomb gauge?
- Mass gap
- Mass Spectra
- An interesting application

Quark Model in QCD



[1] MIT Bag Model

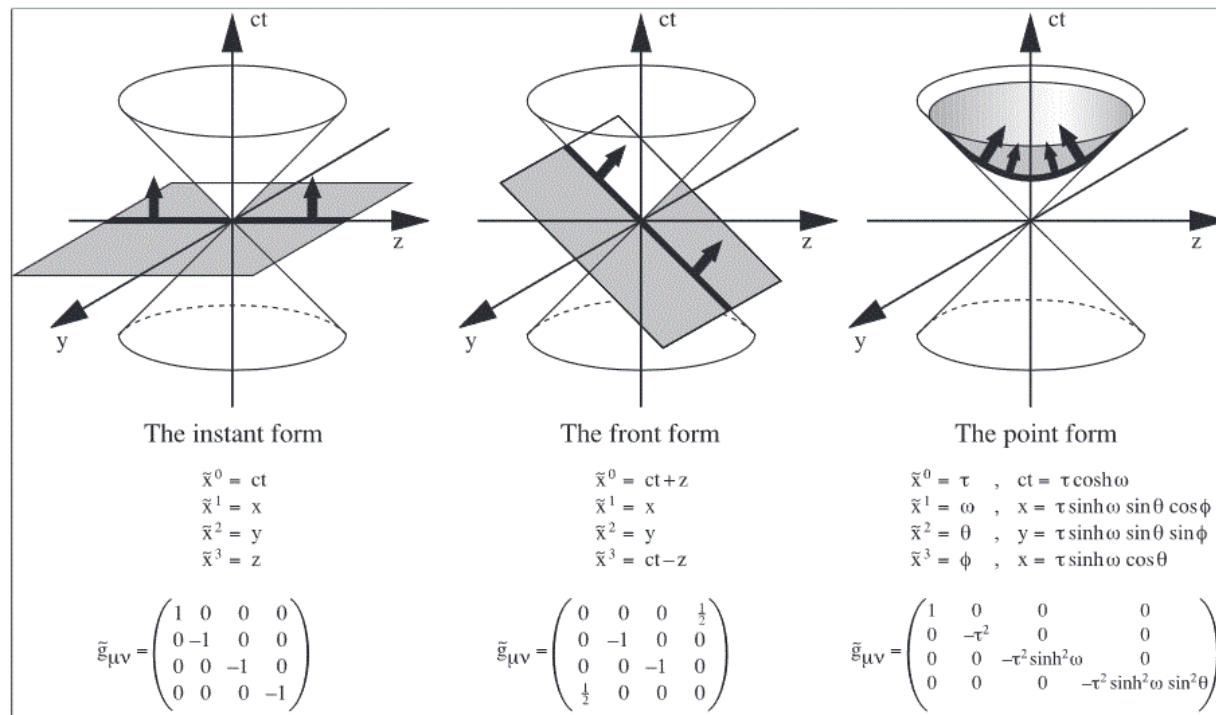


[2] QCD Illustration

Baryons
Mesons

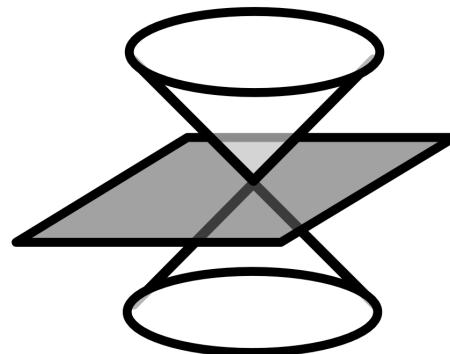
- Baryons are an N_c valence quark bound state
- Mesons are valence quark-antiquark bound states
- Running coupling constant unlike QED

Light Front Dynamics

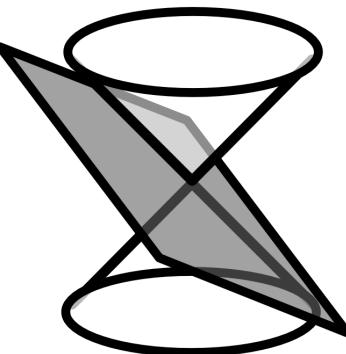
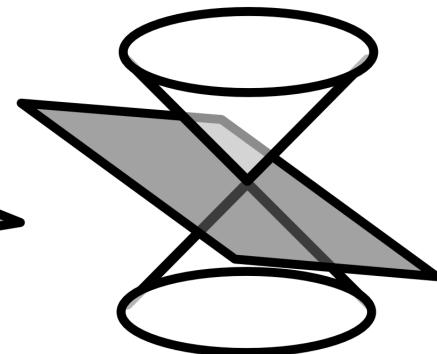
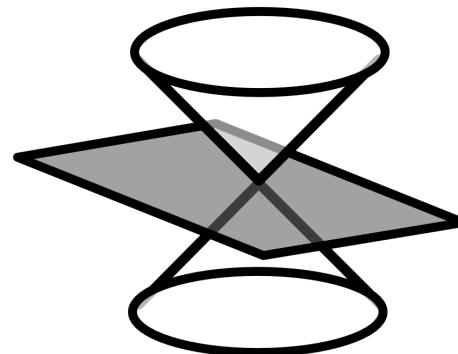


[3] Illustration of Dirac's proposed coordinates

Interpolation between Instant and Front Form

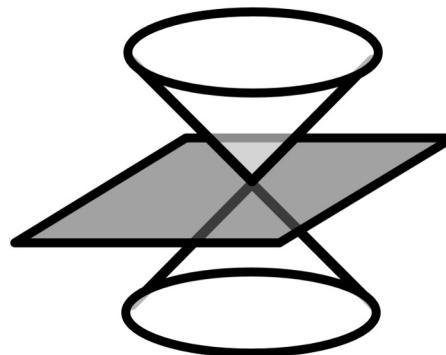


IFD

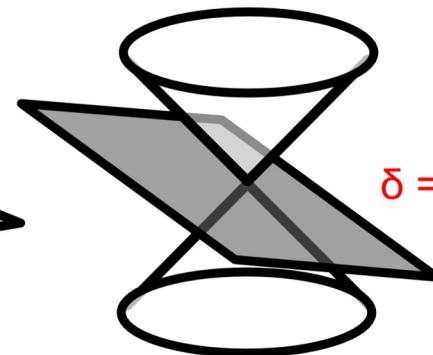
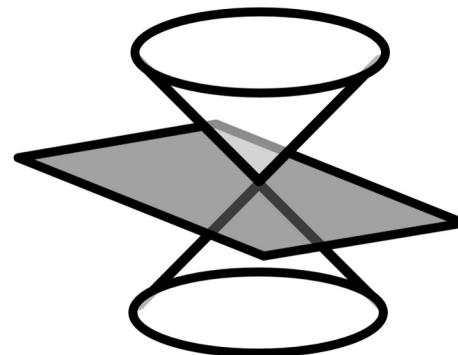


LFD

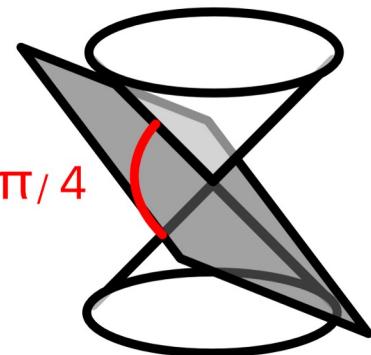
Interpolation between Instant and Front Form



IFD



$$\delta = \pi/4$$



LFD

‘t Hooft’s Meson Model

$$\mathcal{L} = -\frac{1}{4} F^{\hat{\mu}\hat{\nu}} F_{\hat{\mu}\hat{\nu}} + \bar{\psi} (i\gamma^{\hat{\mu}} D_{\hat{\mu}} - m) \psi$$

- 1 space dimension, 1 time dimension
- Large N_c limit
- Predicts color confinement
- Coupling constant has mass dimension

$$D_{\hat{\mu}} = \partial_{\hat{\mu}} - ig A_{\hat{\mu}}^a t_a$$

$$F_{\hat{\mu}\hat{\nu}}^a = \partial_{\hat{\mu}} A_{\hat{\nu}}^a - \partial_{\hat{\nu}} A_{\hat{\mu}}^a + g f^{abc} A_{\hat{\mu}}^b A_{\hat{\nu}}^c$$

Motivation

$$\begin{pmatrix} x^+ \\ x^\perp \end{pmatrix} = \begin{pmatrix} \cos \delta & \sin \delta \\ \sin \delta & -\cos \delta \end{pmatrix} \begin{pmatrix} x^0 \\ x^1 \end{pmatrix}$$

$$g^{\hat{\mu}\hat{\nu}} = g_{\hat{\mu}\hat{\nu}} = \begin{bmatrix} \cos(2\delta) & \sin(2\delta) \\ \sin(2\delta) & -\cos(2\delta) \end{bmatrix}$$

- To establish a formal link between IFD and LFD
- Not the same as Lorentz transformation (equal time)
- Simplify calculations ordinarily too difficult to solve in IFD
- Explore an alternative quasi-PDF implementable in Lattice QCD

Why Coulomb Gauge?

*In IFD 1+1

$$\partial_\mu A^\mu = 0 \implies \begin{cases} A^0 = 0 & := \text{Coulomb Gauge} \\ \text{OR} \\ A^1 = 0 & := \text{Axial Gauge} \end{cases}$$

Lorentz Condition

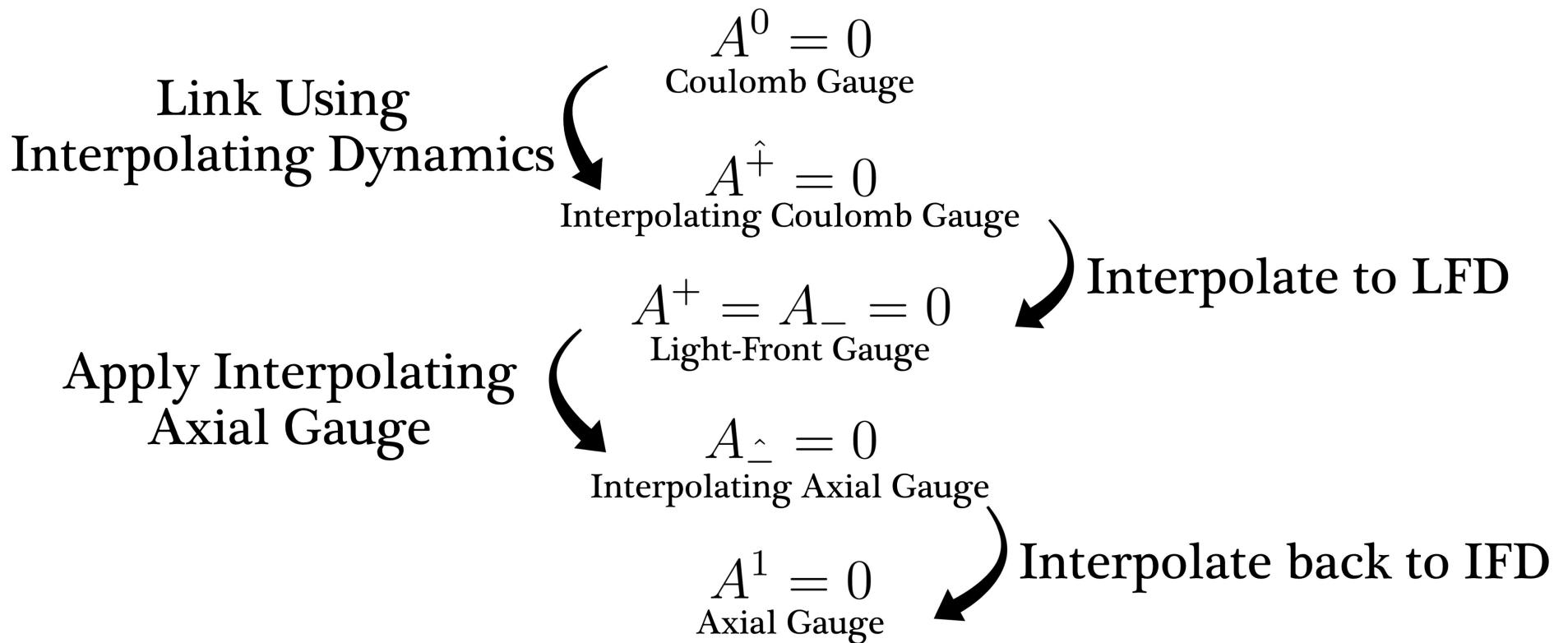
- Axial gauge calculations have already been done [2]
- Consistent with QED
- All degrees of freedom are physical
- Proposed resolution to the QCD (1+1) Conjugate Field Problem

$$\partial_\perp A_\perp = 0$$

Conjugate Field

$$\Pi^0 = \frac{\partial \mathcal{L}}{\partial (\partial_0 A^0)} \quad G = \partial_\mu E^\mu (t, y) - e \psi^\dagger \psi$$
$$G|phys> = 0$$

- Pi-0 field as an operator can generate and annihilate particles
- In the Coulomb gauge, the Pi-0 field does not exist
- The “color electric field” still appears as a constraint of the equation of motion in the Coulomb gauge (analogous to Gauss’s law in QED).



Comparison of Axial and Coulomb Gauge

Lagrangian

Axial Gauge:

$$\mathcal{L} = \frac{1}{2} \left(\partial_{\hat{+}} A_{\hat{+}}^a \right)^2 + \bar{\psi} \left[i\gamma^{\hat{+}} D_{\hat{+}} + i\gamma^{\hat{-}} \partial_{\hat{-}} - m \right] \psi$$

Coulomb Gauge:

$$\mathcal{L} = \frac{1}{2} \left(\partial_{\hat{+}} A_{\hat{-}}^a \right)^2 + \bar{\psi} \left[i\gamma^{\hat{-}} D_{\hat{-}} + i\gamma^{\hat{+}} \partial_{\hat{+}} - m \right] \psi$$

Comparison of Axial and Coulomb Gauge

Axial Gauge:

- The field equation is dependent on the interpolating space dimension.
- The source of the potential is explained by an effective charge density, which acts like a background field.

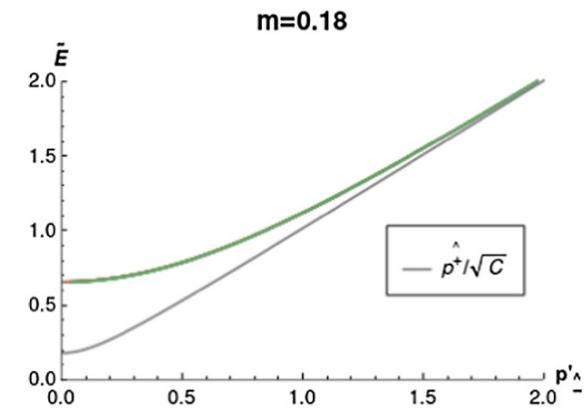
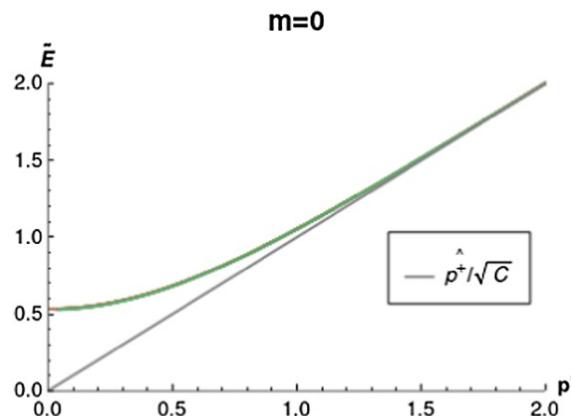
Coulomb Gauge:

- The field equation is dependent on the interpolating time dimension.
- The source of the potential is explained by an effective current density, which acts like a globally changing field through time.

$$\partial_{\hat{\mu}} \partial^{\hat{\mu}} A^{\hat{\nu}} = J^{\hat{\nu}}$$

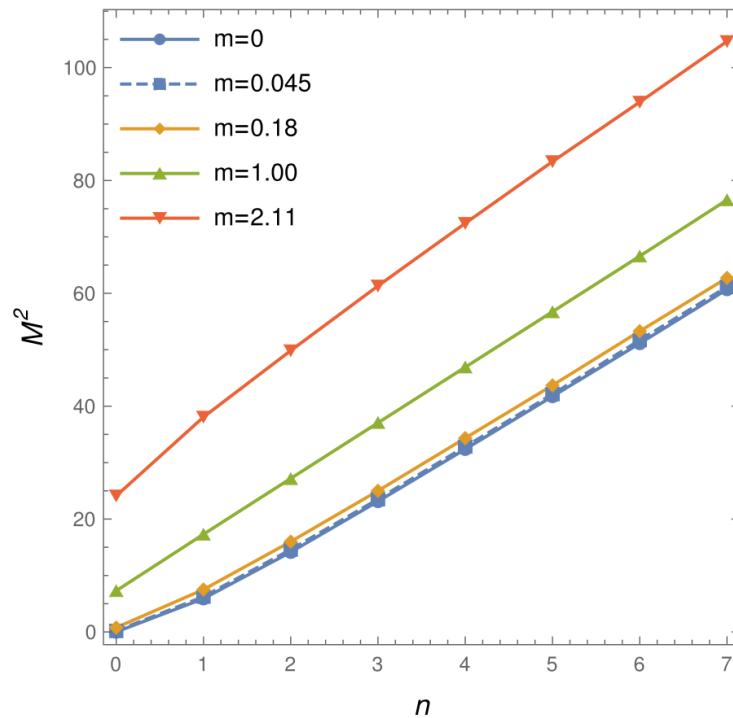
Expected Mass Gap Solution

$$\tilde{E} (p'_\perp)^2 = p'^2_\perp + M (p'_\perp)^2$$



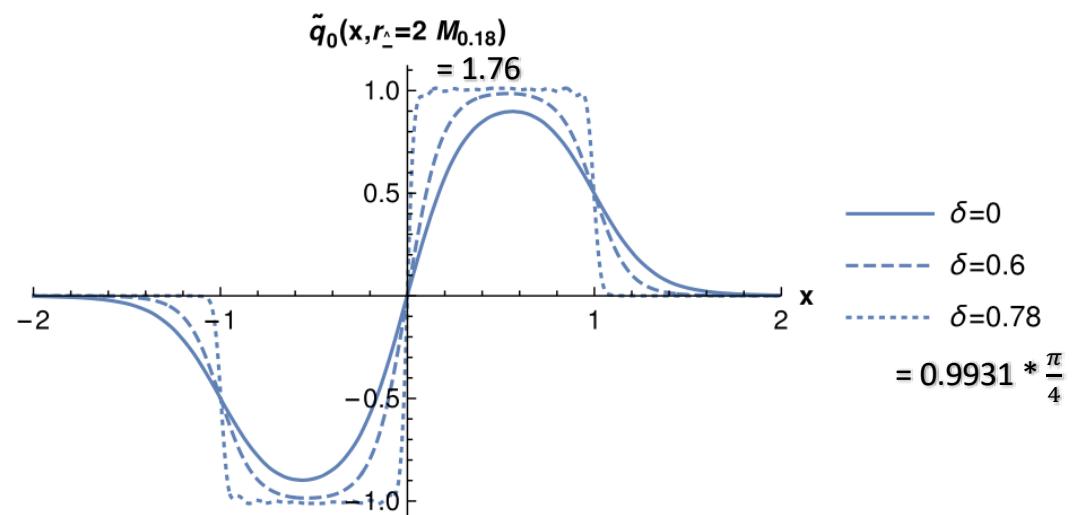
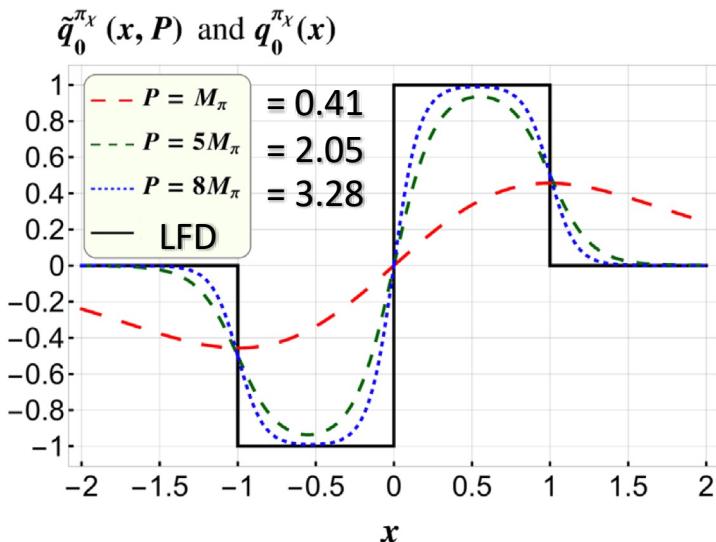
Note: This is independent of the interpolation angle

Bound State Mass Spectra and Regge Behavior



- Mass increases (roughly) linearly with principal quantum number

Application: Alternative Quasi-PDFs



Jia, Y., Liang, S., Xiong, X., & Yu, R. (2018). [ARXIV.1804.04644](https://arxiv.org/abs/1804.04644)

- Dependent on Interpolation angle and momentum
- Does not suffer from the large momentum boost

Conclusion and Future Work

Conclusion:

- The 't Hooft model is interpolated between the instant form and the front form in the axial gauge, looking to explore the Coulomb gauge
- Interpolating dynamics is used to help resolve conjugate field operator issue in the axial gauge.
- The mass gap equation is solved and plotted
- The mass spectra shows Regge behavior, which we see in nature
- Bound state wave functions are used to calculate alternative quasi-PDFs

Future work:

- (3+1) and $N_c = 3$ needs to be explored
- Testing the alternative quasi-PDF on the lattice
- Exploring the time-like region using LFD

Bibliography

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2. Zwanziger, D. (1998). Coulomb-Gauge in QCD: Renormalization and confinement. *Progress of Theoretical Physics Supplement*, 131, 233-242. <https://doi.org/10.1143/ptps.131.233>
3. Hoyer, P. (2021). Journey to the Bound States. arXiv. <https://doi.org/10.48550/ARXIV.2101.06721>
4. Jia, Y., Liang, S., Xiong, X., & Yu, R. (2018). Partonic quasidistributions in two-dimensional QCD. arXiv. <https://doi.org/10.48550/ARXIV.1804.04644>

Images:

- [1] Hoyer, P. (2021). Journey to the Bound States. arXiv. <https://doi.org/10.48550/ARXIV.2101.06721>
 - [2] <https://webifid.ific.uv.es/web/en/content/lattice-qcd-numerical-approach-strong-force>
 - [3] <https://ncatlab.org/nlab/show/light-cone+quantization>
- [Else] Ma, B., & Ji, C.-R. (2021). Interpolating 't Hooft model between instant and front forms. In Physical Review D (Vol. 104, Issue 3). American Physical Society (APS). <https://doi.org/10.1103/physrevd.104.036004>