Quantitative International Economics

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Weeks 4 - Firms
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Outline

- Firms
  - Krugman, 1980
  - Facts about firm heterogeneity
  - Melitz 2003 and Chaney 2008
  - Multinational Firms

Krugman 1980

Krugman 1980

- Firms will be **homogeneous** (we will add firm heterogeneity in Melitz and Chaney)
- Krugman introduces Increasing Returns to Scale (IRS)
- What does it deliver?
  - **Result 1**: All else equal, larger countries have higher welfare
  - **Result 2**: All else equal, larger countries have better terms of trade
  - **Result 3**: Home market effect (originally from Linder, 1961)
- I will use a different notation relative to the paper to make it closer to the notation that we have been using in previous lectures

Environment

- Many countries indexed by i and n
- 1 sector – We will add 2 sectors later
- Each country hosts potentially many symmetric firms producing differentiated goods
  - Each firm is indexed by ω
- Labor endowment N_n
- Iceberg trade cost d_{in}

Demand

- Demand for potential varieties is

$$U_n = \left[\sum_i \int_{\omega \in \Omega_i} q_{in}(\omega)^{\frac{\sigma-1}{\sigma}} \, d\omega \right]^{\frac{\sigma}{\sigma-1}}$$

  - σ is the elasticity of substitution between varieties coming from different firms
  - Ω_i is the range of goods produced in country i
  - $q_{in}(\omega)$ are the goods from firm ω from country i being sold to country n

Demand

- One can show that

$$\frac{p_{in}(\omega)\, q_{in}(\omega)}{X_n} = \frac{p_{in}(\omega)^{1-\sigma}}{P_n^{1-\sigma}}$$

  - where X_n is expenditure and the price index is

$$P_n = \left[\sum_i \int_0^{\Omega_i} p_{in}(\omega)^{1-\sigma}\, d\omega\right]^{\frac{1}{1-\sigma}}$$

- The quantity produced by the firm is

$$q_{in}(\omega) = \frac{p_{in}(\omega)^{-\sigma}}{P_n^{1-\sigma}} X_n$$

Supply

- Labor requirement

$$l_i(\omega) = f + \frac{q_i(\omega)}{\beta}$$

  - $f \Rightarrow$ fixed cost in terms of labor
  - $\beta \Rightarrow$ productivity of variable labor
  - Average cost falls with quantities

- Maximization problem is

$$\max_{p_{in}(\omega)} \sum_n \left(p_{in}(\omega)\, q_{in}(\omega) - \frac{w_i d_{in} q_{in}(\omega)}{\beta} \right) - f w_i$$

Supply

- Substitute the demand equation into the maximization problem

$$\max_{p_{in}(\omega)} \sum_n p_{in}(\omega) \frac{p_{in}(\omega)^{-\sigma}}{P_n^{1-\sigma}} X_n - f w_i - w_i \frac{1}{\beta} \frac{d_{in} p_{in}(\omega)^{-\sigma}}{P_n^{1-\sigma}} X_n$$

which gives

$$p_{in}(\omega) = \underbrace{\frac{\sigma}{\sigma - 1}}_{\text{Markup}} \times \underbrace{\frac{w_i}{\beta}}_{\text{Mg Cost}} \times \underbrace{d_{in}}_{\text{Trade Cost}}$$

- Two comments
  - Constant markups (recall the macro-economic conditions from ACR)
  - Therefore, price does not depend on the demand, but it will influence the set of varieties produced

Supply

- Total Sales

$$r_i(\omega) = \sum_n p_{in}(\omega)\, q_{in}(\omega) = \sum_n \left(\frac{\sigma}{\sigma - 1}\frac{1}{\beta} w_i d_{in}\right)^{1-\sigma} \frac{X_n}{P_n^{1-\sigma}} = \beta^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} (w_i)^{1-\sigma} \sum_n d_{in}^{1-\sigma} \frac{X_n}{P_n^{1-\sigma}}$$

- Payments to variable labor

$$w_i l_i^V(\omega) = \sum_n \frac{w_i d_{in} q_{in}(\omega)}{\beta} = \sum_n \frac{1}{\beta} w_i d_{in} \left(\frac{\sigma}{\sigma - 1}\frac{1}{\beta} w_i d_{in}\right)^{-\sigma} \frac{X_n}{P_n^{1-\sigma}} = \beta^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} (w_i)^{1-\sigma} \sum_n d_{in}^{1-\sigma} \frac{X_n}{P_n^{1-\sigma}}$$

- Share of payments to **variable labor**

$$\frac{w_i l_i^V(\omega)}{r_i(\omega)} = \frac{\sigma - 1}{\sigma}$$

- Payments to **gross profits**

$$\pi_i^G(\omega) = \frac{r_i(\omega)}{\sigma}$$

- Lastly, **net profits**

$$\pi_i^N(\omega) = \pi_i^G(\omega) - f w_i$$

Supply

- Free entry condition implies zero net profits

$$\frac{1}{\sigma}\sum_{n} p_{in}(\omega)\, q_{in}(\omega) = f w_i$$

which, because of symmetry, implies

$$\frac{1}{\sigma}\sum_{n} p_{in} q_{in} = f w_i$$

- Let $\overline{\Omega}_i = |\Omega_i|$ be the mass of firms in country i. We get

$$\overline{\Omega}_i \sum_{n} p_{in} q_{in} = w_i N_i$$

- The mass of firms is therefore

$$\overline{\Omega}_i = \frac{N_i}{\sigma f}$$

  - Larger countries will produce a larger number of varieties. Notice that this measure is fixed, it does not depend on trade costs nor demand for products coming from i.

Gravity

- Coming back to gravity

$$X_{in} = \frac{\overline{\Omega}_i \beta^{\sigma-1} \left(\frac{\sigma}{\sigma-1} w_i d_{in} \right)^{1-\sigma}}{P_n^{1-\sigma}} X_n$$

where the price index is now

$$P_n = \left[\sum_i \overline{\Omega}_i \beta^{\sigma-1} \left(\frac{\sigma}{\sigma-1} w_i d_{in} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

- Taking logs

$$\ln X_{in} = \underbrace{\ln \left(\overline{\Omega}_i \beta^{\sigma-1} \left(\frac{\sigma}{\sigma-1} w_i \right)^{1-\sigma} \right)}_{\text{Origin FE}} + \underbrace{\ln \left(\frac{X_n}{P_n^{1-\sigma}} \right)}_{\text{Destination FE}} + \ln d_{in}^{1-\sigma}$$

  – Notice that we retained the gravity structure in this model

Equilibrium

- Expenditure

$$X_n = w_n N_n \tag{1}$$

- Labor market clearing

$$w_n N_n = \sum_{i=1}^{N} X_{in} \tag{2}$$

- Sales

$$X_{in} = \frac{\overline{\Omega}_i \beta^{\sigma-1} \left(\frac{\sigma}{\sigma-1} w_i d_{in} \right)^{1-\sigma}}{P_n^{1-\sigma}} X_n \tag{3}$$

  - where

$$P_n^{1-\sigma} = \sum_i \overline{\Omega}_i \beta^{\sigma-1} \left(\frac{\sigma}{\sigma-1} w_i d_{in} \right)^{1-\sigma} \tag{4}$$

- Total payments of factors of production

$$\overline{\Omega}_n = \frac{N_n}{\sigma f} \tag{5}$$

Definition: Given technology $\{\beta, f\}$, trade costs $\{d_{ni}\}_{ni}$, preferences $\{\sigma\}$, and population $\{N_n\}_n$, an equilibrium is a vector of wages $\{w_n\}_n$ that satisfies equations (1)-(5).

Welfare: EK vs Krugman

- In EK, we get

$$u_i = \underbrace{\Gamma^{\frac{\theta}{1-\sigma}}}_{\text{cte}} \times \underbrace{T_i^{\frac{1}{\theta}}}_{\text{Own Efficiency}} \times \underbrace{(\pi_{ii})^{-\frac{1}{\theta}}}_{\text{Trade}}$$

  – Does not depend directly on country size N_i

- In Krugman, we get

$$u_i = \underbrace{\left(\frac{\sigma-1}{\sigma}\beta\right)}_{\text{cte}} \times \underbrace{\overline{\Omega}_i^{\frac{1}{\sigma-1}}}_{\text{Own Efficiency}} \times \underbrace{(\pi_{ii})^{\frac{1}{1-\sigma}}}_{\text{Trade}}$$

  – **Result 1**: Welfare rises with country size N_i via $\overline{\Omega}_i$, with an elasticity $\dfrac{1}{\sigma-1}$

Two countries example

- To better understand the gains from trade, let us focus on two countries now
  - Common strategy to sharpen intuition

- Define $p_i = \dfrac{\sigma}{\sigma - 1}\frac{w_i}{\beta}$ and assume symmetric trade costs $d_{in} = d_{ni} = d$

- Trade can be written as

$$X_{in} = \frac{N_i\left(w_i d\right)^{1-\sigma}}{N_i\left(w_i d\right)^{1-\sigma} + N_n\left(w_n\right)^{1-\sigma}} w_n N_n$$

- And write

$$\tilde{\pi}_{in} = \frac{\pi_{in}}{\pi_{nn}} = \frac{N_i\left(w_i d\right)^{1-\sigma}}{N_n\left(w_n\right)^{1-\sigma}}$$

Two countries example

- Setting $w_i = 1$ as the numeraire, the balance of payments can be written as

$$B_{in} = \frac{N_i \left(w_i d\right)^{1-\sigma}}{N_i \left(w_i d\right)^{1-\sigma} + N_n \left(w_n\right)^{1-\sigma}} w_n N_n - \frac{N_n \left(w_n d\right)^{1-\sigma}}{N_n \left(w_n d\right)^{1-\sigma} + N_i \left(w_i\right)^{1-\sigma}} w_i N_i$$

$$= d^{1-\sigma} w_n N_n \left(\underbrace{\frac{1}{d^{1-\sigma} + \frac{N_n}{N_i} w_n^{1-\sigma}} - \frac{w_n^{-\sigma}}{1 + \frac{N_n}{N_i} w_n^{1-\sigma} d^{1-\sigma}}}_{\tilde{B}} \right)$$

- If $N_n = N_i$, then

$$\tilde{B} = \frac{1}{d^{1-\sigma} + w_n^{1-\sigma}} - \frac{w_n^{-\sigma}}{1 + w_n^{1-\sigma} d^{1-\sigma}}$$

and we need $w_n = 1$ to satisfy the balance of payments

Two countries example

- Balance of payments

$$\tilde{B} = \frac{1}{d^{1-\sigma} + \frac{N_n}{N_i} w_n^{1-\sigma}} - \frac{w_n^{-\sigma}}{1 + \frac{N_n}{N_i} w_n^{1-\sigma} d^{1-\sigma}} \tag{6}$$

- The right hand side of equation (6) falls with N_n/N_i and rises with w_n. Therefore, if we increase N_n/N_i, we have to rise the wage of w_n.
  - **Result 2**: Welfare rises with country size N_i because it improves the terms of trade (w_n relative to w_i)

- Intuition from Krugman:
  - "In a world with economies of scale, we would expect workers to be better off in larger economies, because of the larger size of the local market. In this model, however, there is a secondary benefit in the form of better terms of trade with workers in the rest of the world."
  - "If production costs were the same in both countries, it would always be more profitable to produce near the larger market, thus minimizing transportation costs. To keep labor employed in both countries, this advantage must be offset by a wage differential."

Home Market Effects

- With increasing returns to scale, production tends to be concentrated in one place, to realize the scale economies
  - Production concentrates near larger markets
  - Countries will tend to export those kind of products for which they have relatively large domestic demand
  - Import protection can work as export promotion!
- This is not the case with DRS $\Rightarrow$ Strong domestic demand for a good will tend to make a country an importer rather than an exporter of that good
- Next $\Rightarrow$ 2 countries 2 goods example

Two-Industry Economy

- Economy has 2 types of goods with preferences for two different types of goods as follows

$$U_n = \prod_{k \in \{A,B\}} \left[\sum_i \int_{\omega \in \Omega_i} q_{in}^k(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\mu_n^k \frac{\sigma}{\sigma-1}}$$

- On the supply side, we have the same production function

$$l_i^k(\omega) = f + \frac{q_i^k(\omega)}{\beta}$$

- Each country has the same endowment of individuals L
- Two symmetric countries, but with opposite preferences $\mu_n^k = 1 - \mu_n^{k'}$
  - One country prefers A over B. The other prefers B over A with the exact strength.
- Wages must be the same $w_i = w_n = w$
- Trade cost are symmetric $d_{in} = d_{ni} = d$

Two-Industry Economy

- Because firms are symmetric

$$\frac{1}{\sigma}\sum_n p_{in}^k q_{in}^k = f w_i$$

- Similarly to before, payments to workers employed in sector k

$$\overline{\Omega}_i^k \sum_n p_{in}^k q_{in}^k = w_i N_i^k$$

which gives

$$\overline{\Omega}_i^k = \frac{N_i^k}{\sigma f}$$

- Here, N_i^k is endogenous

Two-Industry Economy

- Because of symmetry, we get

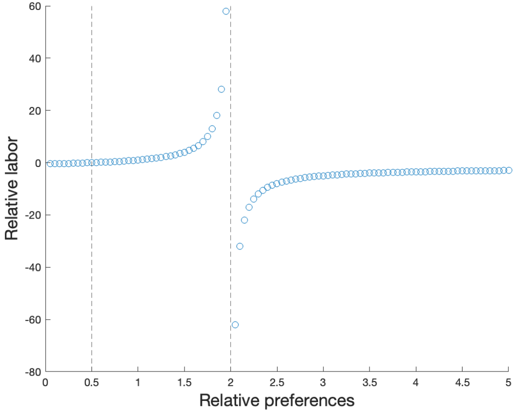
$$\frac{X_i^A}{X_i^B}=\frac{\dfrac{\overline{\Omega}_i^A\left(\frac{\sigma}{\sigma-1}\frac{w}{\beta}\right)^{1-\sigma}}{\overline{\Omega}_i^A\left(\frac{\sigma}{\sigma-1}\frac{w}{\beta}\right)^{1-\sigma}+\overline{\Omega}_n^A\left(\frac{\sigma}{\sigma-1}\frac{w}{\beta}d\right)^{1-\sigma}}\mu_i^A+\dfrac{\overline{\Omega}_i^A\left(\frac{\sigma}{\sigma-1}\frac{w}{\beta}d\right)^{1-\sigma}}{\overline{\Omega}_i^A\left(\frac{\sigma}{\sigma-1}\frac{w}{\beta}d\right)^{1-\sigma}+\overline{\Omega}_n^A\left(\frac{\sigma}{\sigma-1}\frac{w}{\beta}\right)^{1-\sigma}}\mu_n^A}{\dfrac{\overline{\Omega}_i^B\left(\frac{\sigma}{\sigma-1}\frac{w}{\beta}\right)^{1-\sigma}}{\overline{\Omega}_i^B\left(\frac{\sigma}{\sigma-1}\frac{w}{\beta}\right)^{1-\sigma}+\overline{\Omega}_i^B\left(\frac{\sigma}{\sigma-1}\frac{d}{\beta}\right)^{1-\sigma}}\mu_i^B+\dfrac{\overline{\Omega}_i^B\left(\frac{\sigma}{\sigma-1}\frac{w}{\beta}d\right)^{1-\sigma}}{\overline{\Omega}_i^B\left(\frac{\sigma}{\sigma-1}\frac{w}{\beta}d\right)^{1-\sigma}+\overline{\Omega}_i^B\left(\frac{\sigma}{\sigma-1}\frac{w}{\beta}\right)^{1-\sigma}}\mu_n^B}$$

- After some tedious algebra, you get

$$\frac{\mu_{i,A}}{\mu_{i,B}}=\frac{(d)^{1-\sigma}+N_{i,A}/N_{i,B}}{N_{i,A}/N_{i,B}\,(d)^{1-\sigma}+1}\Rightarrow\frac{N_{i,A}}{N_{i,B}}=\frac{\frac{\mu_{i,A}}{\mu_{i,B}}-d^{1-\sigma}}{1-\frac{\mu_{i,A}}{\mu_{i,B}}d^{1-\sigma}}$$

  - If $\frac{\mu_{i,A}}{\mu_{i,B}}$ is too small or too large, the ratio is negative (not equilibrium). Equation does not represent equilibrium. We have full specialization!

Two-Industry Economy



- Within vertical lines, we have impartial specialization
- Increasing preferences for $\mu_{i,A}$ leads to an increase in $N_{i,A}$ relative to $N_{i,B}$. Here, countries become net exporters of the goods that they demand!

Empirical Evidence

- "The more we die the more we sell" (Costinot, Donaldson, Kyle, and Williams, QJE 2019)
  - Demographic composition of the country predicts the diseases of its inhabitants
  - Countries tend to be net sellers of the drugs they demand the most
- They run

$$\ln x_{ij}^n = \delta_{ij} + \delta^n + \beta_M \ln \theta_j^n + \beta_X \ln \theta_i^n + \epsilon_{ij}^n$$

  - origin i destination j disease n
  - β_M: elasticity of trade flows with respect to demand shocks in the importing country
  - β_X: elasticity of trade flows with respect to demand shocks in the exporting country
- Weak home market effect $\beta_X > 0$
- Strong home market effect $\beta_X > \beta_M$

Empirical Evidence

TABLE III

TEST OF THE HOME-MARKET EFFECT (BASELINE)

	Log (bilateral sales)		
	(1)	(2)	(3)
Log (PDB, destination)	0.520		0.545
	(0.097)		(0.107)
Log (PDB, origin)		0.947	0.928
		(0.174)	(0.123)
p-value for $H_0 : \tilde{\beta}_X \leqslant 0$		.000***	.000***
p-value for $H_0 : \tilde{\beta}_X \leqslant \tilde{\beta}_M$			.018**
Origin $\times$ disease FE	$\surd$		
Destination $\times$ disease FE		$\surd$	
Disease FE			$\surd$
Adjusted R^2	0.630	0.563	0.540
Observations	18,756	18,905	19,150

- PDB $\rightarrow$ predicted diseases burden

Empirical Facts about Firms

Empirical Facts about Firms

- Krugman assumes firms are homogeneous in his model
- During the 1990s, there was a revolution in terms of data availability
- The data that emerged showed massive firm heterogeneity (which could be now used to discipline models)
  - "Firms in International Trade", Andrew Bernard, J. Bradford Jensen, Stephen J. Redding, Peter K. Schott

Empirical Facts about Firms

- Engaging in trade is rare. In 2000, 4 percent of firms were exporters.
- Exporting firms are
  - Larger
  - More productive
  - more skill- and capital-intensive
  - Pay higher wages
- When trade costs fall, larger firms tend to survive, while smaller firms tend to fail

Empirical Facts about Firms

Table 3
Exporter Premia in U.S. Manufacturing, 2002

	Exporter premia		
	(1)	*(2)*	*(3)*
Log employment	1.19	0.97	
Log shipments	1.48	1.08	0.08
Log value-added per worker	0.26	0.11	0.10
Log TFP	0.02	0.03	0.05
Log wage	0.17	0.06	0.06
Log capital per worker	0.32	0.12	0.04
Log skill per worker	0.19	0.11	0.19
Additional covariates	None	Industry fixed effects	Industry fixed effects, log employment

Sources: Data are for 2002 and are from the U.S. Census of Manufactures.
Notes: All results are from bivariate ordinary least squares regressions of the firm characteristic in the first column on a dummy variable indicating firm's export status. Regressions in column 2 include industry fixed effects. Regressions in column 3 include industry fixed effects and log firm employment as controls. Total factor productivity (TFP) is computed as in Caves, Christensen, and Diewert (1982). "Capital per worker" refers to capital stock per worker. "Skill per worker" is nonproduction workers per total employment. All results are significant at the 1 percent level.

Empirical Facts about Firms

Table 5

The Intensive and Extensive Margins of Exporters, 1997

	Exporter premia	
	(1)	*(2)*
Log number of products	0.23	0.27
Log mean shipments/# products	1.25	0.73
Additional covariates	None	Industry fixed effects

Sources: Data are for 1997 and are from the U.S. Census of Manufactures.
Notes: All results are from bivariate ordinary least squares regressions of the firm characteristic in the first column on a dummy variable indicating firm export status. Regressions in column two include four-digit SIC industry fixed effects. The first dependent variable is the log of the number of five-digit SIC products produced by the firm in 1997. The second dependent variable is the log of total firm shipments divided by the number of products.

Melitz 2003

Melitz 2003

- Structure is very similar to Krugman's model
  - Preferences for varieties
  - Each firm produces one type of variety

- Firms now differ in their labor productivity
  - Instead of β being the same for all firms ω, it will come from a distribution
  - We will index a firm by β instead of ω to simplify notation

- Iceberg trade costs

- To understand the mechanics of these models, we will first cover a simpler version of the model with no selection into exporting

Supply - as before

- Total sales

$$r_i(\beta) = \sum_n p_{in}(\beta)\, q_{in}(\beta) = \beta^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}(w_i)^{1-\sigma}\sum_n d_{in}^{1-\sigma}\frac{X_n}{P_n^{1-\sigma}}$$

- Payments to variable labor

$$w_i l_i^V(\beta) = \beta^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma}(w_i)^{1-\sigma}\sum_n d_{in}^{1-\sigma}\frac{X_n}{P_n^{1-\sigma}}$$

- Share of payments to **variable labor**

$$\frac{w_i l_i^V(\beta)}{r_i(\beta)} = \frac{\sigma-1}{\sigma}$$

- Payments to **gross profits**

$$\pi_i^G(\beta) = \frac{r_i(\beta)}{\sigma}$$

- Payments to **net profits**

$$\pi_i^N(\beta) = \pi_i^G(\beta) - w_i f$$

Aggregation

- Before, because of symmetry, obtaining aggregate outcomes from firm-level behavior was trivial. We need to do some extra work here. Let us start by defining the average β

$$\tilde{\beta} = \left(\int_0^\infty \beta^{\sigma-1} \mu(\beta) \, d\beta \right)^{\frac{1}{\sigma-1}}$$

- Use it to derive aggregate sales

$$\begin{aligned}
R_i &= \bar{\Omega}_i \int_0^\infty r_i(\beta) \mu(\beta) \, d\beta \\
&= \bar{\Omega}_i \int_0^\infty \beta^{\sigma-1} \left(\frac{\sigma}{\sigma-1} \right)^{1-\sigma} (w_i)^{1-\sigma} \sum_n d_{in}^{1-\sigma} \frac{X_n}{P_n^{1-\sigma}} \mu(\beta) \, d\beta \\
&= \bar{\Omega}_i \tilde{\beta}_i^{\sigma-1} \left(\frac{\sigma}{\sigma-1} \right)^{1-\sigma} (w_i)^{1-\sigma} \sum_n d_{in}^{1-\sigma} \frac{X_n}{P_n^{1-\sigma}} \\
&= \bar{\Omega}_i r\left(\tilde{\beta}_i\right)
\end{aligned}$$

  - $\bar{\Omega}_i$ is the mass of operating firms

Aggregation

- Similarly, we get the price index as

$$P_n^{1-\sigma} = \sum_i \bar{\Omega}_i \int_0^\infty p_{in}(\beta)^{1-\sigma} \mu(\beta)\, d\beta$$

$$P_n^{1-\sigma} = \sum_i \bar{\Omega}_i \int_0^\infty \left(\frac{\sigma}{\sigma-1} d_{in} \frac{w_i}{\beta} \right)^{1-\sigma} \mu(\beta)\, d\beta$$

$$P_n^{1-\sigma} = \sum_i \bar{\Omega}_i \tilde{\beta}_i^{\sigma-1} \left(\frac{\sigma}{\sigma-1} d_{in} w_i \right)^{1-\sigma}$$

- And similar to Krugman

$$\bar{\Omega}_i r\left(\tilde{\beta}_i\right) = w_i N_i$$

$$\bar{\Omega}_i \sigma \left(\pi_i^N \left(\tilde{\beta}_i\right) + w_i f \right) = w_i N_i$$

$$\bar{\Omega}_i = \frac{w_i N_i}{\sigma \left(\pi_i^N \left(\tilde{\beta}_i\right) + w_i f \right)}$$

- We need additional equations to $\tilde{\beta}$ and $\mu(\beta)$ which are endogenous

Entry and Exit

- Timing
  - Firms pay fixed cost f^E to enter
  - Firms observe productivity β, drawn from a distribution $g(.)$
  - Firms who enter decide between two options
    - Leave if $\beta < \beta_i^*$
    - Stay if $\beta \geq \beta_i^*$
    - β_i^* is the cutoff above which firms make positive profits. β_i^* is given by $\pi_i\left(\beta_i^*\right) = 0$
- The endogenous density of firms $\mu(\beta)$ is

$$\mu_i(\beta) = \begin{cases} \dfrac{g(\beta)}{1 - G\left(\beta_i^*\right)} & \text{if } \beta \geq \beta_i^* \\ 0 & \text{otherwise} \end{cases}$$

Entry and Exit

- Ex-Post
  - Let's use $\pi_i^N\left(\beta_i^*\right)=0$ to find the cutoff value of β_i^*, we use

$$\pi_i^N\left(\beta_i^*\right)=\left(\beta_i^*\right)^{\sigma-1}\sigma^{-\sigma}\left(\sigma-1\right)^{\sigma-1}w_i^{1-\sigma}\sum_n d_{in}^{1-\sigma}\frac{X_n}{P_n^{1-\sigma}}-w_if$$

  - Average productivity of surviving firms is a function of β_i^*

$$\tilde{\beta}_i^{\sigma-1}=\frac{1}{1-G\left(\beta_i^*\right)}\int_{\beta_i^*}^{\infty}\beta^{\sigma-1}g\left(\beta\right)d\beta$$

  - The net profit of the average firm can be written as

$$\pi_i^N\left(\tilde{\beta}_i\right)=w_if\left(\left(\tilde{\beta}_i/\beta_i^*\right)^{\sigma-1}-1\right)$$

- Ex-ante
  - Free entry drives profits to zero

$$E\left(\pi^N\left(\beta\right)\right)-w_if^E=0$$

    using the previous results we get

$$\left(1-G\left(\beta_i^*\right)\right)\pi_i^N\left(\tilde{\beta}_i\right)-w_if^E=0\iff\pi_i^N\left(\tilde{\beta}_i\right)=\frac{w_if^E}{\left(1-G\left(\beta_i^*\right)\right)}$$

Equilibrium

- Market clearing

$$X_n = w_n N_n \tag{7}$$

- Labor market clearing

$$w_n N_n = \sum_{i=1}^{N} X_{in} \tag{8}$$

- Sales

$$X_{in} = \sum_n \left(\bar{\Omega}_i \beta^{\sigma-1} \left(\frac{\sigma}{\sigma-1} w_i d_{in} \right)^{1-\sigma} / P_n^{1-\sigma} \right) X_n \tag{9}$$

  – where

$$P_n^{1-\sigma} = \sum_i \bar{\Omega}_i \tilde{\beta}_i \left(\frac{\sigma}{\sigma-1} w_i d_{in} \right)^{1-\sigma} \tag{10}$$

- Total payments of factors of production

$$\bar{\Omega}_n = \frac{w_n N_n}{\sigma \left(\pi_n^N \left(\tilde{\beta}_n \right) + w_n f \right)}$$

- Entry and exit equations

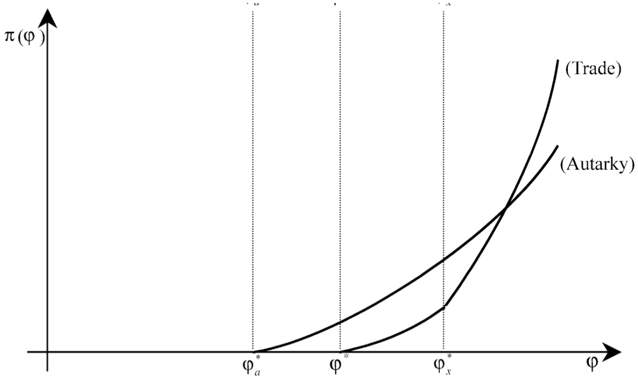
$$\tilde{\beta}_i^{\sigma-1} = \frac{1}{1 - G\left(\beta_i^*\right)} \int_{\beta_i^*}^{\infty} \beta^{\sigma-1} g\left(\beta\right) d\beta \qquad \pi_i^N \left(\tilde{\beta}_i\right) = w_i f \left(\left(\tilde{\beta}_i / \beta_i^* \right)^{\sigma-1} - 1 \right) \qquad \pi_i^N \left(\tilde{\beta}_i\right) = \frac{w_i f^E}{1 - G\left(\beta_i^*\right)} \tag{11}$$

Definition: Given technology $\{f^E, f, g(\beta)\}$, trade costs $\{d_{ni}\}_{ni}$, preferences $\{\sigma\}$, and population $\{N_n\}_n$, an equilibrium is a vector of wages $\{w_n\}_n$ that satisfies equations (7)-(11).

Selection into Exporting

- In Melitz, we have an extra fixed cost for exporting
  - We then have two cutoff values defining
  - (1) firms that operate only in the domestic market
  - (2) firms that operate in both domestic and foreign markets

Selection into Exporting



- When a country opens to trade, what happens to
  - Mass of firms in operation?
  - Average productivity?
  - Which firms export?

Chaney 2008

Chaney 2008

- We will make two adjustments to our previous setting

1. Firms from i have to pay a fixed cost of f_{in} in labor to penetrate market n
2. Firms will draw their productivity distribution from a Pareto

$$G_i(\beta) = 1 - T_i^{\theta}\beta^{-\theta}$$
$$g_i(\beta) = \theta T_i^{\theta}\beta^{-\theta-1}$$

   - θ controls the dispersion of productivities
   - T_i controls the level of productivities

Price Index

- The average firm productivity becomes origin-destination specific

$$\tilde{\beta}_{in}^{\sigma-1} = \frac{1}{1 - G\left(\beta_{in}^{*}\right)} \int_{\beta_{in}^{*}}^{\infty} \beta^{\sigma-1} g\left(\beta\right) d\beta$$

- Price index becomes

$$P_n^{1-\sigma} = \sum_i \bar{\Omega}_{in} \tilde{\beta}_{in}^{\sigma-1} \left(\frac{\sigma}{\sigma-1} d_{in} w_i \right)^{1-\sigma}$$

- Here, $\bar{\Omega}_{in}$ is the mass of firms from i operating in n

$$\bar{\Omega}_{in} = \bar{\Omega}_i \left(1 - G\left(\beta_{in}^{*}\right)\right)$$

Price Index

- Let's use the Pareto to derive

$$
\begin{aligned}
\tilde{\beta}_{in}^{\sigma-1} &= \frac{1}{1-G\left(\beta_{in}^{*}\right)}\left(\int_{\beta_{in}^{*}}^{\infty} \beta^{\sigma-1} \theta T_i^{\theta} \beta^{-\theta-1} d\beta\right) \\
&= \frac{1}{T_i^{\theta} \beta_{in}^{*\,-\theta}}\left(\theta T_i^{\theta} \int_{\beta_{in}^{*}}^{\infty} \beta^{\sigma-\theta-2} d\beta\right) \\
&= \frac{1}{T_i^{\theta} \beta_{in}^{*\,-\theta}}\left(T_i^{\theta} \frac{\theta}{\theta-\sigma+1}\left(\beta_{in}^{*}\right)^{\sigma-\theta-1}\right) \\
&= \frac{\theta}{\theta-\sigma+1} \beta_{in}^{*\,\sigma-1}
\end{aligned}
$$

and also

$$
\begin{aligned}
\bar{\Omega}_{in} &= \bar{\Omega}_i\left(1-G\left(\beta_{in}^{*}\right)\right) \\
&= \bar{\Omega}_i T_i^{\theta} \beta_{in}^{*\,-\theta}
\end{aligned}
$$

- Price index becomes

$$
P_n^{1-\sigma} = \sum_i \bar{\Omega}_i T_i^{\theta} \frac{\theta}{\theta-\sigma+1} \beta_{in}^{*\,(\sigma-1)-\theta}\left(\frac{\sigma}{\sigma-1} d_{in} w_i\right)^{1-\sigma}
$$

Supply

- **Revenues** from i to j

$$r_{in}(\beta) = p_{in}(\beta)\, q_{in}(\beta) = \beta^{\sigma-1}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}(w_i d_{in})^{1-\sigma}\frac{X_n}{P_n^{1-\sigma}}$$

- **Net profits**

$$\pi_{in}^{N}(\beta) = \beta^{\sigma-1}\sigma^{-\sigma}(\sigma-1)^{1-\sigma}(w_i d_{in})^{1-\sigma}\frac{X_n}{P_n^{1-\sigma}} - w_i f_{in}$$

- **Cutoff**, given by $\pi_{in}^{N}(\beta_{in}^{*}) = 0$, is

$$\beta_{in}^{*\sigma-1} = \frac{w_i f_{in}}{\sigma^{-\sigma}(\sigma-1)^{1-\sigma}(w_i d_{in})^{1-\sigma}\frac{X_n}{P_n^{1-\sigma}}}$$

Gravity

- Sales from i to n

$$X_{in} = \bar{\Omega}_i T_i^\theta \frac{\theta}{\theta - \sigma + 1} \beta_{in}^{*\,(\sigma-1)-\theta} \left(\frac{\sigma}{\sigma - 1} d_{in} w_i \right)^{1-\sigma} \frac{X_n}{P_n^{1-\sigma}}$$

- Substitute β_{in}^*

$$X_{in} = \bar{\Omega}_i T_i^\theta \frac{\theta}{\theta - \sigma + 1} \left(\frac{w_i f_{ij}}{\sigma^{-\sigma} (\sigma - 1)^{1-\sigma} (w_i d_{in})^{1-\sigma} \frac{X_n}{P_n^{1-\sigma}}} \right)^{\frac{(\sigma-1)-\theta}{\sigma-1}} \left(\frac{\sigma}{\sigma - 1} d_{in} w_i \right)^{1-\sigma} \frac{X_n}{P_n^{1-\sigma}}$$

which gives

$$X_{in} = \underbrace{\xi_i}_{\text{Origin FE}} \times \underbrace{\Delta_j}_{\text{Destination FE}} \times d_{in}^{-\theta} \times f_{in}^{-\left(\frac{\theta}{\sigma-1} - 1\right)}$$

  – Impact of trade cost on trade flows only depend on θ!
  – The impact of fixed costs is inversely related to elasticity of substitution

Krugman vs Chaney

- Krugman
  - Lower elast. of substitution means that consumers are willing to buy foreign goods even at a higher cost
  - Lower elast. of substitution $\Rightarrow$ Lower impact of trade barriers on trade flows
  - All firms are symmetric and exporters $\Rightarrow$ No extensive margin
- Chaney
  - We now have the extensive margin
    - Trade costs impact how much firms export (*extensive margin*)
    - Trade costs impact which firms export (*intensive margin*)
  - Higher elasticity of substitution has opposite effects on intensive and extensive margin
    - Higher elast. of substitution $\Rightarrow$ Higher impact of trade barriers from intensive margin
    - Higher elast. of substitution $\Rightarrow$ Lower impact of trade barriers from extensive margin
  - Why?
    - With higher elasticity of subtitution, less productive firms capture a smaller share of the market, so their entry and exit decisions have a smaller impact on total trade flows

Krugman vs Chaney

- Which effects dominate?
  - With Pareto distribution of firms, the extensive margin dominates
- Chaney shows that

$$-\frac{d\ln X_{in}}{d\ln d_{in}} = \underbrace{(\sigma-1)}_{\text{Intensive Margin}} + \underbrace{(\theta-(\sigma-1))}_{\text{Extensive Margin}} = \theta$$

Conclusion

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- This week, we close the fundamentals of trade
- Next week, we will cover economic geography
  - Allow for labor mobility