Quantitative International Economics

Heitor S. Pellegrina

University of Notre Dame

Weeks 9 and 10 - Dynamic GE Models - Part 1
Spring 2025

Outline

- Dynamic Models
- We will cover a series of tools to bring dynamic considerations
  - Path dependence
  - Forward looking agents

Why should we bring dynamics?

- Climate change and natural resources
- Migration and sectoral relocation
- Consumption vs. investment decisions

Class of Models

- Spatial Development
  - Desmet, Nagy and Rossi-Hansberg (2018)
- Migration
  - Caliendo, Parro and Dvorkin (2019), Artuc, Chaudhuri, and Mclaren (2010), Allen and Donaldson (2024)
  - Pellegrina and Sotelo (2024)

Motivating Examples

Growth and Global Migration

- What would be the gains from relaxing migration restrictions on welfare and growth? (Desmet, Nagy and Rossi-Hansberg 2018)

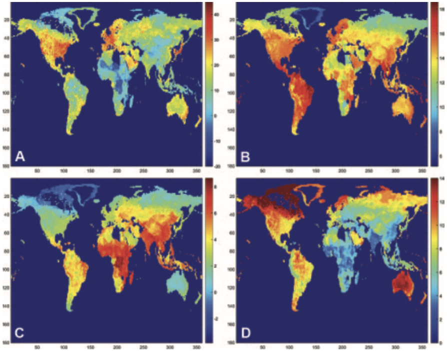


Fig. 1.—Benchmark calibration: results from the inversion and migration costs. *A*, Fundamental productivities: $\tau_1(r)$. *B*, Fundamental amenities: $\bar{a}(r)$. *C*, Amenities over utility: $\bar{a}(r)/u_0(r)$. *D*, Migration costs: $m_2(r)$.

Growth and Global Migration

- What would be the gains from relaxing migration restrictions on welfare and growth? (Desmet, Nagy and Rossi-Hansberg 2018)

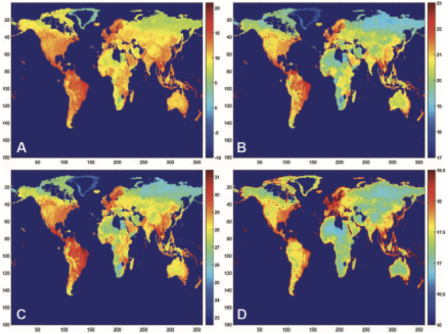
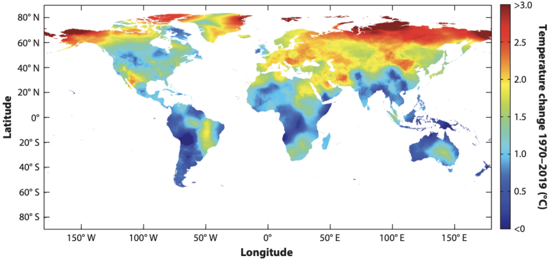


FIG. 7.—Equilibrium with free migration (period 600). *A*, Population density for $\vartheta = 0$. *B*, Productivity for $\vartheta = 0$: $[\tau_t(r)\bar{L}_t(r)^{\alpha}]^{1/\theta}$. *C*, Utility for $\vartheta = 0$: $u_t(r)$. *D*, Real income per capita for $\vartheta = 0$: $y_t(r)$.

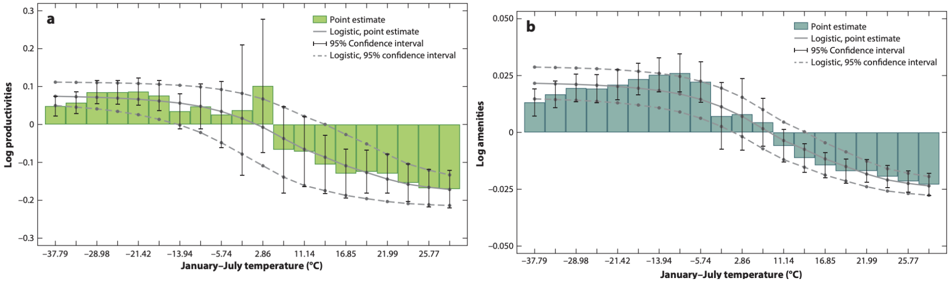
Growth, Migration and Global Warming

- Using the same class of models, Cruz and Rossi-Hansberg (2024) study the impact of global warming



Growth, Migration and Global Warming

- Using the same class of models, Cruz and Rossi-Hansberg (2024) study the impact of global warming



Growth, Migration and Global Warming

- Using this class of models, Cruz and Rossi-Hansberg (2024) study the impact of global warming

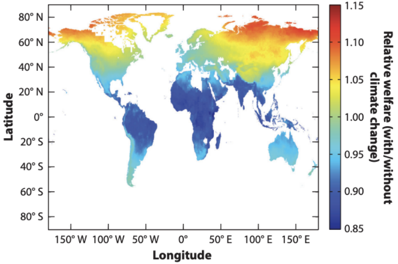


Figure 4

Ratio of welfare with global warming to welfare without global warming in baseline scenario. Figure adapted with permission from Cruz & Rossi-Hansberg (2024).

- Conte et al. (2021) works on a model with multiple sectors

Anticipatory Effects

- The applications are based on models with no anticipatory effects
  - The model is dynamic $\Rightarrow$ a shock today affects the path of the economy
  - However, while agents react to past shocks, they do not react to future ones
  - In other words, there are no anticipatory effects. Computationally, this assumption is extremely useful, as we will discuss later.

China Shock, Employment and Welfare

- Large literature following Autor, Dorn and Hanson (2016)
  - Reduced-form impact of China import penetration on employment
  - Comparison between metropolitan areas in the US
  - Identification based on relative effects $\Rightarrow$ No perfectly isolated control group
- Caliendo, Dvorkin and Parro (2018)

China Shock, Employment and Welfare

- Effects on Employment

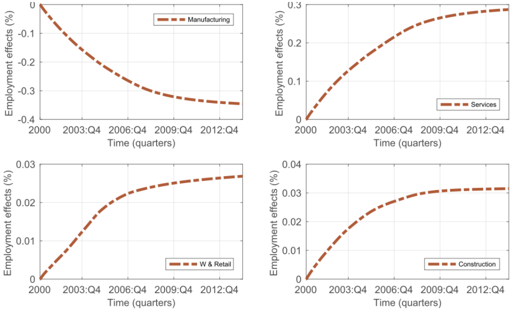


FIGURE 1.—The effect of the China shock on employment shares. Note: The figure presents the effects of the China shock measured as the change in employment shares by sector (manufacturing, services, wholesale and retail, and construction) over total employment between the economy with all fundamentals changing as in the data and the economy with all fundamentals changing except for the estimated sectoral changes in productivities in China (the economy without the China shock).

China Shock, Employment and Welfare

- Effects on Employment

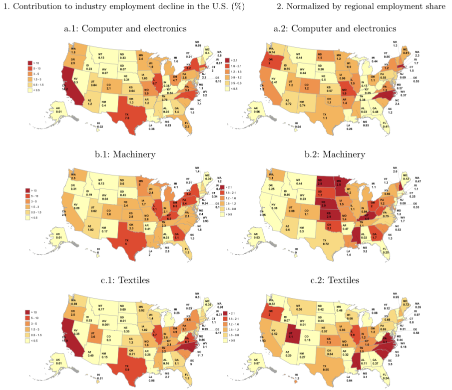


FIGURE 7.—Regional employment declines in manufacturing industries. Note: This figure presents the reduction in local employment in manufacturing industries. Column 1 presents the contribution of each state to the U.S. aggregate reduction in industry employment due to the China shock. Column 2 presents the contribution of each state to the U.S. aggregate reduction in industry employment normalized by the employment size of each state relative to U.S. aggregate employment. Panels a present the results for the computer and electronics industry. Panels b present the results for the machinery industry. Panels c present the results for the textiles industry.

China Shock, Employment and Welfare

- Effects on Employment

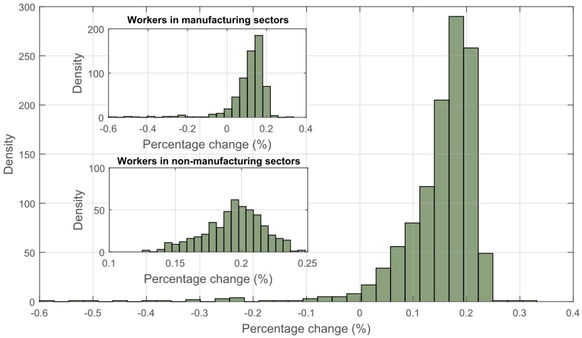


FIGURE 10.—Welfare effects of the China shock across U.S. labor markets. Note: The figure presents the change in welfare across all labor markets (central figure), for workers in manufacturing sectors (top-left panel), and for workers in non-manufacturing sectors (bottom-left panel) as a consequence of the China shock. The largest and smallest 1 percentile are excluded in each figure. The percentage change in welfare is measured in terms of consumption equivalent variation.

Infrastructure Investments and Coastal Flooding

- Balboni (2025) studies the impact of coastal flooding in Vietnam

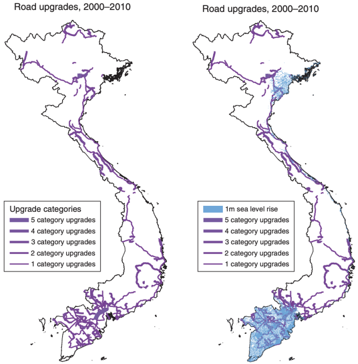


Figure 1. Road Investments in Vietnam, 2000–2010

Note: Data sources and construction are described in Section II and Supplemental Appendix II.

Infrastructure Investments and Coastal Flooding

- Balboni (2025) studies the impact of coastal flooding in Vietnam

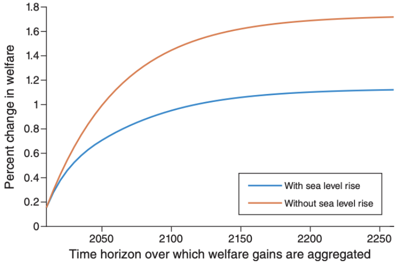


FIGURE 6. DYNAMIC AGGREGATE WELFARE GAINS FROM REALIZED ROAD INVESTMENTS WITH AND WITHOUT SEA LEVEL RISE

Notes: The figure compares aggregate welfare gains for the realized road upgrades with (blue line) and without (orange line) a 1 m rise in the sea level realized gradually over 100 years, as the time horizon over which welfare gains are aggregated increases. The percentage change in welfare is measured in terms of consumption equivalent variation, relative to a scenario in which no roads had been upgraded.

Desmet, Nagy and Rossi-Hansberg (2018)

Environment

- A continuum of locations $r \in S$
  - Location r has density $H(r) > 0$ where $\int_S H(r)\, dr = 1$
- There are C countries
  - Countries constitute a partition of S: $(S_1, ..., S_C)$
- Endowment
  - $\bar{L}$ population for the world economy
- Initial population distribution $\bar{L}_0(r)$

Preferences

- Utility of an agent i who resides in r in period t and lived in $\bar{r}_- = (r_0, ..., r_{t-1})$

$$u_t^i(\bar{r}_-, r) = a_t(r) \left[\int_0^1 c_t^\omega(r)^\rho \, d\omega \right]^{1/\rho} \varepsilon_t^i(r) \prod_{s=1}^{t} m(r_{s-1}, r_s)^{-1}$$

  - $1/(1-\rho)$ elasticity of subtitution
  - $a_t(r)$ is the amenity term
  - $c_t^\omega(r)$ is the consumption of good ω at location r
  - $m(r_{s-1}, r_s)$ represents the permanent flow-utility cost of moving from r_{s-1} in $s-1$ to r_s in s
  - $\varepsilon_t^i(r)$ is a taste shock Fréchet distributed with dispersion Ω

Preferences

- Assume

$$a_t(r) = \bar{a}(r)\bar{L}_t(r)^{-\lambda}$$

  - $\bar{a}(r)$ is the natural amenity

- Agents earn income from work and from land rents, which are distributed equally among local workers. Total income is therefore $w_t(r) + R_t(r)/\bar{L}_t(r)$

- Every period agents consume their income (hand-to-mouth)

$$\begin{aligned} u_t^i(\bar{r}_-, r) &= \frac{a_t(r)}{\prod_{s=1}^t m(r_{s-1}, r_s)} \frac{w_t(r) + R_t(r)/\bar{L}_t(r)}{P_t(r)} \varepsilon_t^i(r) \\ &= \frac{a_t(r)}{\prod_{s=1}^t m(r_{s-1}, r_s)} y_t(r)\varepsilon_t^i(r) \end{aligned}$$

  - $y_t(r)$ is the real income of the region, where $P_t(r)$ is the usual price index
  - Migration costs are given in units of utility

Migration Costs

- Migration costs are assumed to be multiplicative

$$m(s,r) = m_1(s)\, m_2(r)$$

  - In other words, any worker from any origin s' moving to r share a destination cost $m_2(r)$

- Migration costs of a region with itself is

$$m(r,r) = 1$$

  - That implies $m_1(r) = \dfrac{1}{m_2(r)}$

- Now, think of a worker who migrated from s to r, and then back from r to s. She has to pay

$$(m_1(s)\, m_2(r)) \times (m_1(r)\, m_2(s))$$

Migration Costs

- With this formulation, workers only pay the migration costs *while they live* in the region
  - This assumption means that workers will not consider the history of their migration choices
  - It also means that they do not consider the future consequences of their migration decisions
- The value of moving to 1 for an individual is

$$\begin{aligned}
V\left(r_0, \bar{\varepsilon}_1^i\right) &= \max_{r_1}\left[\frac{a_1\left(r_1\right)}{m\left(r_0, r_1\right)} y_1\left(r_1\right) \varepsilon_1^i\left(r_1\right) + \beta E\left(\frac{V\left(r_1, \bar{\varepsilon}_2^i\right)}{m\left(r_0, r_1\right)}\right)\right] \\
&= \frac{1}{m_1\left(r_0\right)} \max_{r_1}\left[\frac{a_1\left(r_1\right)}{m_2\left(r_1\right)} y_1\left(r_1\right) \varepsilon_1^i\left(r_1\right) + \beta E\left(\frac{V\left(r_1, \bar{\varepsilon}_2^i\right)}{m_2\left(r_1\right)}\right)\right] \\
&= \frac{1}{m_1\left(r_0\right)}\left\{\max_{r_1}\left[\frac{a_1\left(r_1\right)}{m_2\left(r_1\right)} y_1\left(r_1\right) \varepsilon_1^i\left(r_1\right)\right] + \beta E\left(\frac{V\left(r_1, \bar{\varepsilon}_2^i\right)}{m_2\left(r_1\right)}\right)\right\}
\end{aligned}$$

Migration

- Let us write the period utility of an agent from r_0 who lives in r_t is ($\tilde{x} = \ln x$)

$$\tilde{u}_t^i\left(r_0, r_t\right) = \tilde{u}_t\left(r_t\right) - \tilde{m}_1\left(r_0\right) - \tilde{m}_2\left(r_t\right) + \tilde{\varepsilon}_t^i\left(r_t\right)$$

- Share of population from origin s in $t-1$ who move to r is

$$Pr\left(\tilde{u}_t\left(s,r\right) \geq \tilde{u}_t\left(s,v\right), \forall v \in S\right) = \frac{\exp\left(\left[\tilde{u}_t\left(r\right) - \tilde{m}_2\left(r\right)\right]/\Omega\right)}{\int_S \exp\left(\left[\tilde{u}_t\left(v\right) - \tilde{m}_2\left(v\right)\right]/\Omega\right) dv}$$

which can be rewritten as

$$\begin{aligned} Pr\left(\tilde{u}_t\left(s,r\right) \geq \tilde{u}_t\left(s,v\right), \forall v \in S\right) &= \frac{\exp\left(\left[\log u_t\left(r\right)/m_2\left(r\right)\right]/\Omega\right)}{\int_S \exp\left(\left[\log u_t\left(v\right)/m_2\left(v\right)\right]/\Omega\right) dv} \\ &= \frac{\left(u_t\left(r\right)/m_2\left(r\right)\right)^{\frac{1}{\Omega}}}{\int_S \left(u_t\left(v\right)/m_2\left(v\right)\right)^{\frac{1}{\Omega}} dv} \end{aligned}$$

Technology

- Firms produce a good $\omega \in [0,1]$ using labor and land

$$q_t^{\omega}(r) = \phi_t^{\omega}(r)^{\gamma_1} z_t^{\omega}(r) L_t^{\omega}(r)^{\mu}$$

  - $L_t^{\omega}(r)$ workers per unit of land
  - $\phi_t^{\omega}(r)$ is an endogenous innovation term that costs $\nu\phi_t^{\omega}(r)^{\xi}$
  - $z_t^{\omega}(r)$ is an exogenous shifter, distributed Fréchet with dispersion θ and level

$$T_t(r) = \tau_t(r) \bar{L}_t(r)^{\alpha}$$

$$\tau_t(r) = \phi_{t-1}(r)^{\theta\gamma_1} \left[\int_S \eta\tau_{t-1}(s)\, ds\right]^{1-\gamma_2} \tau_{t-1}(r)^{\gamma_2}$$

    - $\gamma_1, \gamma_2 \in (0,1)$ helps with convergence
    - $\gamma_2 = 1 \Rightarrow$ technology is independent of the rest of economy $\Rightarrow$ economy in one location
    - $\gamma_1 = \gamma_2 = 0 \Rightarrow$ only aggregate matters $\Rightarrow$ no incentives to innovate and economy stagnates

Technology

- Firms maximize

$$\pi_t^{\omega}(r) = \max_{L_t^{\omega}(r),\phi_t^{\omega}(r)} p_t^{\omega}(r,r)\,\phi_t^{\omega}(r)^{\gamma_1} z_t^{\omega}(r)\,L_t^{\omega}(r)^{\mu} - w_t(r)\,L_t^{\omega}(r) - w_t(r)\,\nu\phi_t^{\omega}(r)^{\xi} - R_t(r)$$

- Rents per unit of land are set to extract all profits

$$R_t(r) = \pi_t^{\omega}(r)$$

- It can be shown that total employment of workers (production + innovation) is

$$\bar{L}_t^{\omega}(r) = L_t^{\omega}(r) + \nu\phi_t^{\omega}(t)^{\xi} = \frac{L_t^{\omega}(r)}{\mu}\left[\mu + \frac{\gamma_1}{\xi}\right]$$

- It can also be shown that

$$R_t(r) = \left[\frac{\xi(1-\mu)}{\gamma_1} - 1\right] w_t(r)\,\nu\phi_t^{\omega}(r)^{\xi}$$

Technology

- Local competition implies that firms will bid for land up to the point at which they obtain zero profits (after covering for their innovation costs)
- Innovation decisions affect future productivity
- However, investment decision does not consider future profits, only current profits, which are forced to zero
- Solution to the dynamic problem equals a sequence of static equilibrium

Technology

- Decisions to innovate does not depend on idiosyncratic productivity $z_t^{\omega}(r)$
- That allows the price of a firm to be written as (similarly to EK)

$$p_t^{\omega}(r,r) = \left[\frac{1}{\mu}\right]^{\mu}\left[\frac{\nu\xi}{\gamma_1}\right]^{1-\mu}\left[\frac{\gamma_1 R_t(r)}{w_t(r)\,\nu\,(\xi(1-\mu)-\gamma_1)}\right]^{(1-\mu)-(\gamma_1/\xi)}\frac{w_t(r)}{z_t^{\omega}(r)}$$

- That allows us to further write this as

$$p_t^{\omega}(r,r) = \frac{mc_t(r)}{z_t^{\omega}(r)}$$

$$mc_t(r) = \left[\frac{1}{\mu}\right]^{\mu}\left[\frac{\nu\xi}{\gamma_1}\right]^{1-\mu}\left[\frac{\gamma_1 R_t(r)}{w_t(r)\,\nu\,(\xi(1-\mu)-\gamma_1)}\right]^{(1-\mu)-(\gamma_1/\xi)} w_t(r)$$

Trade

- Based on the Eaton and Kortum structure, we now have

$$\pi_t(s,r) = \frac{T_t(r)\left[mc_t(r)\,\varsigma(r,s)\right]^{-\theta}}{\int_S T_t(s)\left[mc_t(u)\,\varsigma(u,s)\right]^{-\theta} du}$$

- Total revenue in r is

$$w_t(r)\,H(r)\left[L_t(r) + \nu\phi_t(r)^{\xi}\right] + H(r)\,R_t(r) = \frac{1}{\mu} w_t(r)\,H(r)\,L_t(r)$$

- So the trade balance can be written as

$$w_t(r)\,H(r)\,\bar{L}_t(r) = \int_S \pi_t(s,r)\,w_t(s)\,H(s)\,\bar{L}_t(s)\,ds$$

- We substituted $L_t(r)$ by $\bar{L}_t(r)$ because the proportion of total workers to productive workers is the same across locations

Static Equilibrium

- Income of workers

$$w_t(r)\,H(r)\,\bar{L}_t(r) = \int_S \pi_t(s,r)\,w_t(s)\,H(s)\,\bar{L}_t(s)\,ds \tag{1}$$

- Trade flows

$$\pi_t(s,r) = \frac{T_t(r)\left[mc_t(r)\,\varsigma(r,s)\right]^{-\theta}}{\int_S T_t(s)\left[mc_t(u)\,\varsigma(u,s)\right]^{-\theta} du} \tag{2}$$

- Marginal cost

$$mc_t(r) = \left[\frac{1}{\mu}\right]^{\mu}\left[\frac{\nu\xi}{\gamma_1}\right]^{1-\mu}\left[\frac{\gamma_1 R_t(r)}{w_t(r)\,\nu\left(\xi\left(1-\mu\right)-\gamma_1\right)}\right]^{(1-\mu)-(\gamma_1/\xi)} w_t(r) \tag{3}$$

- Land rent

$$R_t(r) = \left[\frac{\xi\left(1-\mu\right)-\gamma_1}{\mu\xi+\gamma_1}\right] w_t(r)\,\bar{L}_t(r) \tag{4}$$

Definition: Given technology parameters $\{T_t(r), \theta, \xi, \gamma_1, \mu, \nu\}$, trade costs $\{\varsigma(r,s)\}$, and worker location $\{\bar{L}_t(r)\}$, an equilibrium is a vector of prices $\{w_t(r)\}$ that satisfies equations (1)-(4).

Dynamic equilibrium

- Given initial conditions $\{\tau_0, \bar{a}, \bar{L}_0, H\}$, trade and migration costs $\{\varsigma(r,s), m(r,s)\}$ the sequence of workers location $\{\bar{L}_t(t)\}$, productivities $\{T_t(r)\}$ and wages $\{w_t(r)\}$ satisfy for every period t:
  - Technology evolution

$$\tau_t(r) = \phi_{t-1}(r)^{\theta\gamma_1} \left[\int_S \eta\tau_{t-1}(s)\, ds\right]^{1-\gamma_2} \tau_{t-1}(r)^{\gamma_2}$$

  - Worker location

$$H(r)\bar{L}_t(r) = \frac{(u_t(r)/m_2(r))^{\frac{1}{\Omega}}}{\int_S (u_t(v)/m_2(v))^{\frac{1}{\Omega}}\, dv}\bar{L} \tag{5}$$

$$u_t(r) = \bar{a}(r)\bar{L}_t(r)^{-\lambda} \frac{\xi}{\mu\xi + \gamma_1} \frac{w_t(r)}{P_t(r)} \tag{6}$$

$$P_t(r) = \bar{\Gamma}\left[\int_S \int_S T_t(s)\left[mc_t(u)\varsigma(u,s)\right]^{-\theta} du\right]^{-\frac{1}{\theta}} \tag{7}$$

  - Static equilibrium

Existence of Equilibrium

- There exists an equilibrium (and its unique) if

$$\frac{\alpha}{\theta} + \frac{\gamma_1}{\xi} \leq \lambda + 1 - \mu + \Omega$$

  – Static agglomeration $\frac{\alpha}{\theta}$ (as in AA)
    - α agglomeration from population density
    - θ dispersion of idiossyncratic shocks

  – Returns to innovation $\frac{\gamma_1}{\xi}$
    - γ_1 importance of past innovation
    - ξ convexity of innovation costs

  – Congestion forces
    - λ static congestion
    - Ω dispersion in preferences
    - $1 - \mu$ importance of fixed factor

Balanced Growth Path

- The growth rate is given by

$$\frac{\tau_{t+1}(r)}{\tau_t(r)} = \phi_t(r)^{\theta\gamma_1}\left[\int_S \eta \frac{\tau_t(s)}{\tau_t(r)} ds\right]^{1-\gamma_2}$$

- A BGP is a path in which $\frac{\tau_{t+1}(r)}{\tau_t(r)}$ is constant over time and also that $\frac{\tau_t(s)}{\tau_t(r)}$ is constant over time

- Dividing both side by the corresponding one for s and rearranging gives

$$\frac{\tau_t(s)}{\tau_t(r)} = \left[\frac{\phi_t(s)}{\phi_t(r)}\right]^{\frac{\theta\gamma_1}{1-\gamma_2}} = \left[\frac{\bar{L}(s)}{\bar{L}(r)}\right]^{\frac{\theta\gamma_1}{[1-\gamma_2]\xi}}$$

Balanced Growth Path

- A unique solution exists for the BGP if

$$\frac{\alpha}{\theta} + \frac{\gamma_1}{\xi} + \frac{\gamma_1}{[1 - \gamma_2]\, \xi} \leq \lambda + 1 - \mu + \Omega$$

  - which is stronger than the previous condition

Extensions

- Conte et al. (JEG, 2021)
  - Multiple sectors (agriculture and non-agriculture) + Impact of climate change on TFP + Energy use impact climate change
- Conte et al. (2025)
  - Implications for carbon taxes
- Cruz and Rossi-Hanbserg (RESTUD, 2024)
  - One sector + Impact of climate change on TFP + Natality rates

Taking Stock

- We developed a model with forward looking agents (in principle)
- The model, however, does not incorporate <u>anticipatory effects</u>
  - We learned which assumptions convert the model into an essentially sequence of static equilibrium, in which TFPs in t depend on TFPs in $t-1$
  - Such models are computationally fast and tractable, which is really important for implementation
- Recent papers incorporate anticipatory effects
  - Bilal and Rossi-Hansberg (“Anticipating Climate Change across the US”)
  - Allen and Donaldson $\Rightarrow$ Next class