Quantitative International Economics

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Weeks 5 and 6 - Economic Geography
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Outline

- Economic Geography - An introduction
- Krugman (1991)
- Allen and Arkolakis (2014)
- Ahlfeldt, Redding, Sturm, and Wolf (2015)

Introduction

Some Facts

- From Allen and Donaldson (2022)
  - Economic activity is extremely concentrated across space
  - 1/6th of value-added in the US is produced in 3 cities that occupy less than 1/160th of its land area
  - One was a Dutch trading post, one a pueblo for 22 adult and 22 children settlers designated by a Spanish governor to honor the angels, and one a river mouth known to Algonquin residents for its wild garlic (or, chicago-ua)
  - Ex-ante, (perhaps) not clear these would be the “natural” candidates for largest cities in US

Economic Geography

- Key feature: Workers are now mobile
  - IRS + trade costs + labor mobility $\Rightarrow$ Multiple equilibria comes to the forefront

- Because of IRS, production will take place in a few number of sites
  - Preferred sites are those with larger demand (to save on transportation)
  - Where do we have a larger demand?
    - Demand for manufacturing goods comes in part from manufacturing itself
    - "Circular causation" $\Rightarrow$ manufacturing production tends to concentrate where there is a large market, but a large market will be large where manufactures production is concentrated

- Similar reasoning for labor mobility
  - It is more desirable to live in places with low price of manufacturing goods, incentivizes workers to move to regions with large population

Economic Geography

- Researchers often put the forces driving the location of workers into two categories
  - **First nature**: Natural characteristics, such as trading spots, proximity to the ocean, proximity to rivers
  - **Second nature**: Those conditions created by human intervention
- Do we live in a world of multiple equilibria? Can temporary economic shocks generate permanent effects?
  - Hard to test
    - We need a large and temporary shock
    - We also need a sufficiently large period of time to see relocation of economic activity
  - A small shock may not shift allocation of factors enough to put the economy into a new path
- Before doing some models, let us briefly cover 3 key papers

Paper 1: Davis and Weinstein (2002)

- Application: Japan and the WWII
  - How do regions grow after being hit by bombs?

Paper 1: Davis and Weinstein (2002)

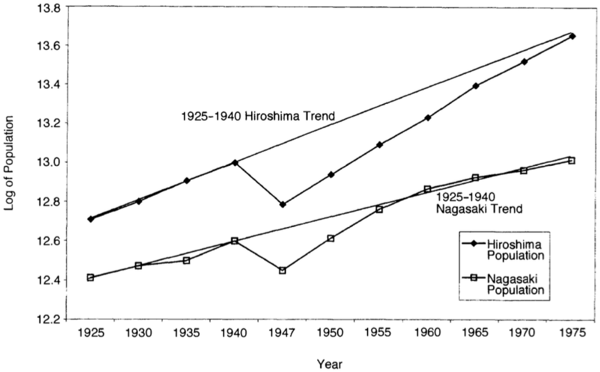


FIGURE 2. POPULATION GROWTH

Paper 1: Davis and Weinstein (2002)

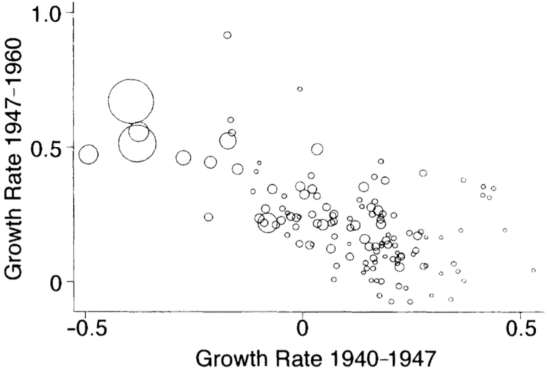


FIGURE 1. EFFECTS OF BOMBING ON CITIES WITH MORE THAN 30,000 INHABITANTS

Paper 2: Bleakley and Lin (2012)

- A natural advantage that is no longer needed
- With no agglomeration economies, once we remove the advantage, we would go back to previous equilibrium
- But if agglomeration forces are large enough and we have multiple equilibria, then the region may retain its advantage
  - They look at portage sites: locations where there is a relocation of goods from one mode of transportation to another

Paper 2: Bleakley and Lin (2012)

FIGURE A.1
The Density Near Fall-Line/River Intersections
This map shows the contemporary distribution of economic activity across the southeastern United States measured by the 2003 nighttime lights layer. For information on sources, see notes for Figures II and IV.

Paper 2: Bleakley and Lin (2012)

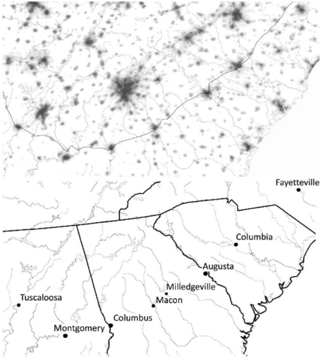


FIGURE II
Fall-Line Cities from Alabama to North Carolina

The map in the upper panel shows the contemporary distribution of economic activity across the southeastern United States, measured by the 2003 nighttime lights layer from NationalAtlas.gov. The nighttime lights are used to present a nearly continuous measure of present-day economic activity at a high spatial frequency. The fall line (solid) is digitized from *Physical Divisions of the United States*, produced by the U.S. Geological Survey. Major rivers (dashed gray) are from NationalAtlas.gov, based on data produced by the United States Geological Survey. Contemporary fall-line cities are labeled in the lower panel.

Paper 2: Bleakley and Lin (2012)

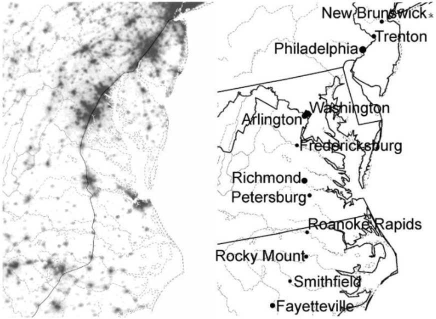


FIGURE IV

Fall-Line Cities from North Carolina to New Jersey

The map in the left panel shows the contemporary distribution of economic activity across the southeastern United States measured by the 2003 nighttime lights layer from NationalAtlas.gov. The nighttime lights are used to present a nearly continuous measure of present-day economic activity at a high spatial frequency. The fall line (solid) is digitized from *Physical Divisions of the United States*, produced by the U.S. Geological Survey. Major rivers (dashed gray) are from NationalAtlas.gov, based on data produced by the U.S. Geological Survey. Contemporary fall-line cities are labeled in the right panel.

Paper 3: Kline and Moretti (2014)

- What is the impact of a large and temporary shock?
  - Can it lead the economy to a new equilibrium?
  - Does it induce structural change?

Paper 3: Kline and Moretti (2014)

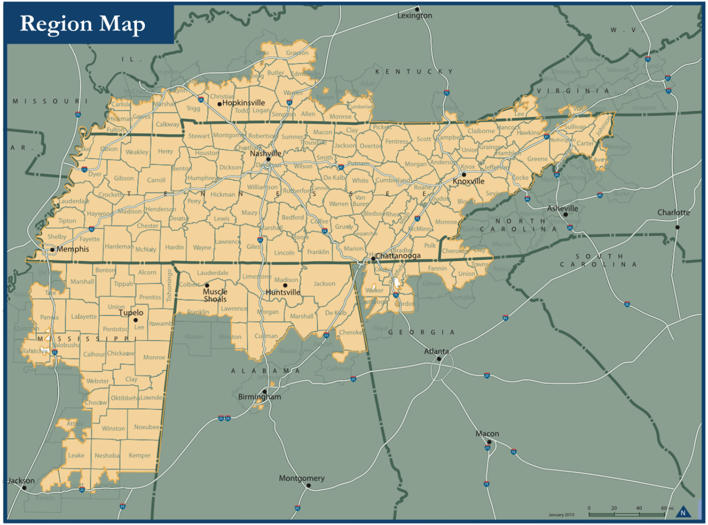


FIGURE I

The TVA Service Area (as of 2010)

Paper 3: Kline and Moretti (2014)

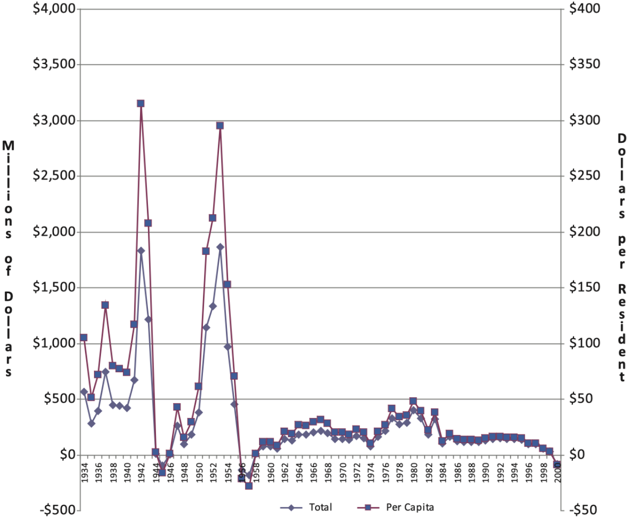


FIGURE II

Federal Transfers to TVA by Year (2000 Dollars)

Paper 3: Kline and Moretti (2014)

TABLE III

DECADALIZED IMPACT OF TVA ON GROWTH RATE OF OUTCOMES (1940–2000)

Outcome	(1) Point estimate (unadjusted)	(2) Clustered std. err.	(3) Point estimate (controls)	(4) Clustered std. err.	(5) Spatial HAC	(6) N
Panel A: TVA region versus rest of U.S.						
Population	0.004	(0.021)	0.007	(0.020)	(0.018)	1,907
Average manufacturing wage	0.027***	(0.006)	0.005	(0.004)	(0.005)	1,172
Agricultural employment	−0.130***	(0.026)	−0.056**	(0.024)	(0.027)	1,907
Manufacturing employment	0.076***	(0.013)	0.059***	(0.015)	(0.023)	1,907
Value of farm production	−0.028	(0.028)	0.002	(0.032)	(0.026)	1,903
Median family income (1950–2000 only)	0.072***	(0.014)	0.021	(0.013)	(0.011)	1,905
Average agricultural land value	0.066***	(0.013)	−0.002	(0.012)	(0.016)	1,906
Median housing value	0.040**	(0.017)	0.005	(0.015)	(0.015)	1,906
Panel B: TVA region versus U.S. South						
Population	−0.007	(0.018)	0.014		(0.019)	942
Average manufacturing wage	0.003	(0.006)	0.001		(0.005)	610
Agricultural employment	−0.097***	(0.030)	−0.051*		(0.027)	942
Manufacturing employment	0.079***	(0.023)	0.063***		(0.024)	942
Value of farm production	−0.005	(0.025)	−0.006		(0.026)	939
Median family income (1950–2000 only)	0.041***	(0.012)	0.024**		(0.011)	942
Average agricultural land value	0.031*	(0.018)	−0.003		(0.017)	942
Median housing value	0.019	(0.017)	0.007		(0.016)	942

Krugman 1991

Krugman 1991

- Krugman 1980 + factor mobility
  - IRS
  - love for variety
- Question: How can we endogenously generate a “core-periphery” structure, in which the core is the city and absorbs most of the workers and periphery is the hinterland?
  - How does the answer to that question depend on the parameters of the model?

Krugman 1991

- Let's first work out the intuition

Krugman 1991

- 2 equal regions indexed by i or n
- 2 sectors $\Rightarrow$ Agriculture and Manufacturing
  - Peasant population produces agriculture and can not move
  - Workers produce manufacturing goods and can move
- IRS in manufacturing production and CES in agriculture
- Iceberg trade cost τ
  - OBS Different from before, we are defining τ as the fraction of goods that arrive at the destination. This is the inverse of what we had before. Higher τ means lower trade costs.

Preferences

- Upper tier

$$U = C_M^{\mu} C_A^{1-\mu}$$

- Lower tier for manufacturing

$$C_M = \left[\sum_{i=1}^{N} c_i^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}$$

- In agriculture, goods are homogeneous

Technology and Worker Allocation

- Peasant population
- 1 peasant produces 1 unit of an agricultural good (so price is also 1)
- Workers employed in agriculture in each region is

$$\frac{(1-\mu)}{2}$$

- Manufacturing workers in each region is L_1 and L_2 (endogenous)

$$L_1 + L_2 = \mu$$

Technology

- Production of an individual manufactured good

$$L_{M_i} = f + \frac{x_i}{\beta}$$

- In region 1, the price of firms there will be

$$p_i = \left(\frac{\sigma}{\sigma - 1}\right)\frac{w_i}{\beta}$$

- Free entry leads to

$$\left(p_i - \frac{w_i}{\beta}\right) q_i = \alpha w_i$$

- The two equations above imply

$$q_i = \frac{f\left(\sigma - 1\right)}{\beta}$$

  – Output per firm is the same in each region. So the number of firms and goods will be proportional to the number of people

$$\frac{n_i}{n_n} = \frac{L_i}{L_n}$$

Short-Run Equilibrium

- Let's take the allocation of workers in each region as given
- Given that allocation, let us think about how workers will move between regions
- Sale of region i to region j

$$X_{in} = \frac{(p_i/\tau_{in})}{P_n^{1-\sigma}}^{1-\sigma} X_n$$

- Let c_{in} be the consumption of goods from origin i in destination n

$$\frac{c_{ii}}{c_{ni}} = \left(\frac{p_i}{p_n/\tau_{ni}}\right)^{-\sigma}$$

- Let z_{ii} be the ratio of region i expenditure on goods from i versus goods from n

$$z_{ii} = \frac{n_i p_i c_{ii}}{n_n p_n c_{ni}/\tau_{ni}} = \frac{L_i}{L_n}\left(\frac{w_i}{w_n/\tau_{ni}}\right)^{-(\sigma-1)}$$

Short-Run Equilibrium

- The ratio of sales

$$z_{ii} = \frac{L_i}{L_n} \left(\frac{w_i}{w_n/\tau_{ni}} \right)^{-(\sigma-1)}$$

- The labor market clearing condition is

$$w_i L_i = \mu \sum_{i'} \left(\frac{z_{ii'}}{1 + z_{ii'}} \right) X_{i'}$$

- Expenditure

$$Y_i = \frac{1-\mu}{2} + w_i L_i$$

- For given labor allocation L_i and L_n, we can solve for wages w_i.

- By inspection, if $L_1 = L_2$ then $w_1 = w_2$.
  - If labor is shifted to region 1, relative wage w_1/w_2 can move either way
  - **Mechanism 1**: Home market effect (driven by the demand from manufacturing itself)
  - **Mechanism 2**: Less competition for the local peasant market (driven by the demand from agriculture)

Long-run Equilibrium

- We now solve for L_1 and L_2
- The price index of manufacturing goods is

$$P_n = \left[\sum_i \frac{L_i}{\mu} \left(w_i / \tau_{in} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

and real wages in region 1 is

$$u_i = \frac{w_i}{P_i^{\mu} P_{A_i}^{1-\mu}} = \frac{w_i}{P_i^{\mu}}$$

- How does u_1/u_2 vary with L_1/L_2?
  - We know that when $L_1/L_2 = 1$, then $u_1/u_2 = 1$. But is this a stable equilibrium?
  - If u_1/u_2 falls with L_1/L_2, then workers out-migrate from the larger region
  - If u_1/u_2 raises with L_1/L_2, then workers in-migrate from the larger region
- In what follows, Krugman defines f as L_1/μ. Let us now look at the role of σ, τ and μ

Long-run equilibrium

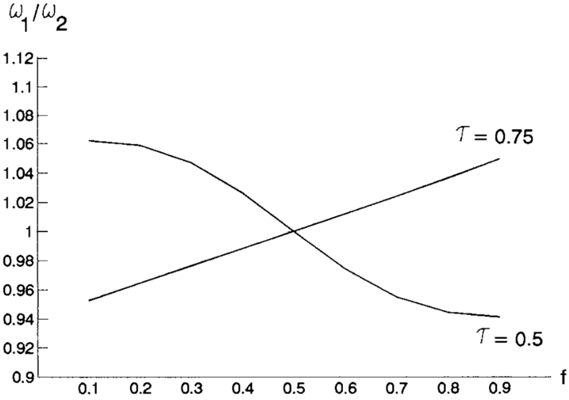


FIG. 1

Necessary conditions

- Is an equilibrium in which every worker lives in 1 stable? Assume that region 1 has all manufacturing workers, then

$$\frac{Y_2}{Y_1} = \frac{(1-\mu)/2}{(1-\mu)/2+\mu} = \frac{1-\mu}{1+\mu}$$

- Let n be the total number of manufacturing firms, then the value of sales is

$$V_1 = \left(\frac{\mu}{n}\right)(Y_1+Y_2)$$

- Does any individual firm have incentives to deviate from this equilibrium and move to 2? They have to convince workers to move

$$\frac{u_2}{u_1} = \frac{w_2/(P_1/\tau)^{\mu}}{w_1/(P_1)^{\mu}} \Rightarrow 1 = \frac{w_2/(P_1/\tau)^{\mu}}{w_1/(P_1)^{\mu}} \Rightarrow \frac{w_2}{w_1} = \left(\frac{1}{\tau}\right)^{\mu}$$

  - Firm is just an infinitesimal entity in a continuum, so on its own action doesn't affect the price index

- The total value of a defecting firm is

$$V_2 = \left(\frac{\mu}{n}\right)\left[\left(\frac{w_2/\tau}{w_1}\right)^{1-\sigma} Y_1 + \left(\frac{w_2}{w_1/\tau}\right)^{1-\sigma} Y_2\right]$$

- Putting these together gives

$$\frac{V_2}{V_1} = \frac{1}{2}\tau^{\mu(\sigma-1)}\left[(1+\mu)\,\tau^{\sigma-1} + (1-\mu)\,\tau^{1-\sigma}\right]$$

Necessary conditions

- $V_2/V_1 > 1$ is not enough, we still need to take the fixed costs into account.
  - We need $V_2/V_1 > w_2/w_1 = \tau^{-\mu}$
- That gives

$$\nu = \frac{1}{2}\tau^{\mu\sigma}\left[(1+\mu)\,\tau^{\sigma-1} + (1-\mu)\,\tau^{1-\sigma}\right]$$

- Let us analyze ν in the vicinity of 1

$$\frac{\partial \nu}{\partial \mu} = \nu\sigma\,(\ln\tau) + \frac{1}{2}\tau^{\sigma\mu}\left[\tau^{\sigma-1} - \tau^{1-\sigma}\right] < 0$$

  - The larger the share of income spent in manufactured goods, the lower the relative sales of the defecting firm
  - Why? (1) workers demand a larger wage premium ("*forward linkage*") (2) the larger the home market effect ("*backward linkage*")

Necessary conditions

- Now we look at transportation costs

$$\frac{\partial \nu}{\partial \tau} = \frac{\mu \sigma \nu}{\tau} + \frac{\tau^{\mu\sigma}(\sigma - 1)\left[(1+\mu)\tau^{\sigma-1} - (1-\mu)\tau^{1-\sigma}\right]}{2\tau}$$

- For τ close to 1, the second term goes to zero and the derivative is always positive

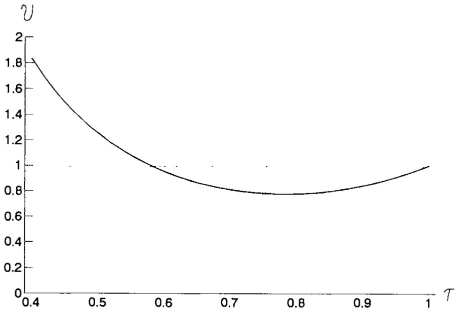


FIG. 2

Necessary conditions

- Higher elasticity of substitution works against regional convergence
- With higher σ, higher $1/\tau$ and higher μ are necessary to ensure stability

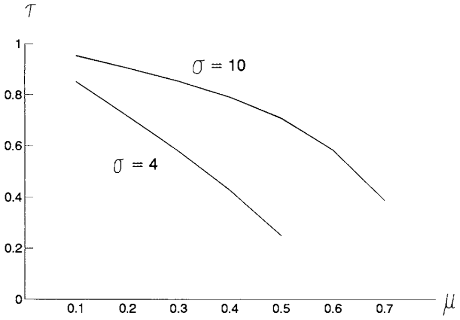


FIG. 3

Allen and Arkolakis 2014

Allen and Arkolakis 2014

- Trade and worker location model on a continuous space
- I will present a model with a discrete number of regions
- I should emphasize that their proofs are based on the continuum

Environment

- Many regions i and n
- Armington $\Rightarrow$ Every region produces a differentiated product
- 1 sector
- Workers can freely move between regions
- The economy is endowed with a mass of workers of $\bar{L}$
- Iceberg trade costs τ_{in} between i and n

Preferences

- Utility of workers in location n is

$$U_n = B_i \left(\sum_i q_{in}^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)}$$

  – where B_i is an amenity term

- Consumption share

$$\pi_{in} = \frac{(p_i \tau_{in})^{1-\sigma}}{P_n^{1-\sigma}}$$

  and

$$P_n^{1-\sigma} = \sum_i (p_i \tau_{in})^{1-\sigma}$$

- Utility from living in location n

$$U_i = B_i \frac{w_i}{P_i}$$

Technology

- One worker produces A_i units of a good (CRS)
- Perfect competition ensures that price of goods from i in destination n are

$$p_{in} = \frac{w_i \tau_{in}}{A_i}$$

  - A_i is the productivity of region i

Agglomeration and Congestion

- Amenity is given by

$$B_i = \bar{B}_i L_i^{\beta}$$

  - $\bar{B}_i$ is an exogenous utility derived from living in i
  - L_i is the population living in i
  - β captures the extent to which amenities are determined by pop density

- Productivity is given by

$$A_i = \bar{A}_i L_i^{\alpha}$$

  - $\bar{A}_i$ is a productivity shifter
  - L_i is the population living in i
  - α captures the extent to which productivity depends on pop density

- β and α are strength of productivity and amenity spillover

Agglomeration and Congestion

- Their formulation is isomorphic to many models
  - $\alpha = \frac{1}{\sigma - 1} \Rightarrow$ monopolistic competition model (Krugman, 1991)
  - $\alpha = \frac{1}{\sigma - 1}$ and $\beta = -\frac{1-\delta}{\delta} \Rightarrow 1 - \delta$ is the share of immobile factor, negative β captures the inelastic supply of land (Helpman 1991 and Redding 2012)
  - $\alpha = \delta - 1 \Rightarrow$ land as a factor of production where δ is share of labor in the production function (notice that $\alpha < 0$)

Mobility

- Welfare must be equalized if there exists U such that

$$U > 0$$

  and

$$U_i \leq U \text{ with equalit if } L_i > 0$$

- For any location with a positive population, we must have $U > 0$

Gravity

- Trade flows satisfy

$$X_{ij} = \frac{(d_{in} w_i / A_i)^{1-\sigma}}{P_n^{1-\sigma}} w_n L_n$$

where

$$P_n^{1-\sigma} = \sum_i (d_{in} w_i / A_i)^{1-\sigma}$$

Equilibrium

- Welfare is given by

$$U_i = \frac{w_i}{P_i} B_i \tag{1}$$

- Welfare equalizes

$$U_i \leq U \text{ with equality if } L_i > 0 \tag{2}$$

- Local labor market clears

$$w_i L_i = \sum_n X_{in} \tag{3}$$

- Sales are given by

$$X_{in} = \frac{(d_{in} w_i / A_i)^{1-\sigma}}{P_n^{1-\sigma}} w_n L_n \quad \text{where} \quad P_n^{1-\sigma} = \sum_i (d_{in} w_i / A_i)^{1-\sigma} \tag{4}$$

- Aggregate labor market clears

$$L = \sum_i L_i \tag{5}$$

- Productivity and amenity shifters are given by

$$A_i = \bar{A}_i L_i^{\alpha} \text{ and } B_i = \bar{B}_i L_i^{\beta} \tag{6}$$

Definition: Given technology $\{\bar{A}_i\}$, amenities $\{\bar{B}_i\}$ trade costs $\{d_{ni}\}_{ni}$, preferences $\{\sigma\}$, and total $\{L\}$, an equilibrium is a vector of wages $\{w_n\}_n$, labor allocation $\{L_n\}_n$ and an utility $\{U\}$ that satisfies equations (1)-(6).

An Algorithm

- Guess a vector of wages $\{w_n\}_n$ and a vector of labor allocation $\{L_n\}_n$
  - Construct A_i and B_i
  - Construct P_n
  - Construct X_{in}
  - Construct demand for workers
  - Construct utility of workers
- XXX

Existence and Uniqueness

- To study the properties of this system of equations, AA use reduce the equilibrium equations into two, which do not contain the price index. Specifically, they substitute equation (1) into (3) to get

$$w_i^{\sigma} L_i = \sum_n U_n^{1-\sigma} d_{in}^{1-\sigma} A_i^{\sigma-1} B_n^{\sigma-1} w_n^{\sigma} L_n$$

  and again equation (1) into the price index (4) to get

$$w_i^{1-\sigma} = \sum_n U_i^{1-\sigma} d_{ni}^{1-\sigma} A_n^{\sigma-1} B_i^{\sigma-1} w_n^{1-\sigma}$$

- We have a vector of wages $\{L_n\}_n$, a vector of wages $\{w_n\}_n$, and a welfare equalization $U_i = U$. These two equations are linear operators whose eigenfunctions are $w_i^{\sigma} L_i$ and $w_i^{1-\sigma}$ whose eigenvalues are $U_i^{1-\sigma}$.

- When there are no spillovers $\alpha = \beta = 0$, for a regular geography (i.e., for $d_{ni} < \infty$ for any ni pair and $A_i > 0$ for any i), then we have a unique equilibrium and it can be computed with a simple iterative procedure.

Existence and Uniqueness

- Let us substitute the values of A_i and B_i. That gives

$$L_i^{1-\alpha(\sigma-1)} w_i^{\sigma} = U^{1-\sigma} \sum_n d_{in}^{1-\sigma} \bar{A}_i^{\sigma-1} \bar{B}_i^{\sigma-1} L_n^{1+\beta(\sigma-1)} w_n^{\sigma} \tag{7}$$

  and

$$w_i^{1-\sigma} L_i^{\beta(1-\sigma)} = U^{1-\sigma} \sum_n d_{ni}^{1-\sigma} \bar{A}_n^{\sigma-1} \bar{B}_n^{\sigma-1} w_n^{1-\sigma} L_n^{\alpha(\sigma-1)} \tag{8}$$

- By assuming that trade costs are symmetric, the following equation must hold

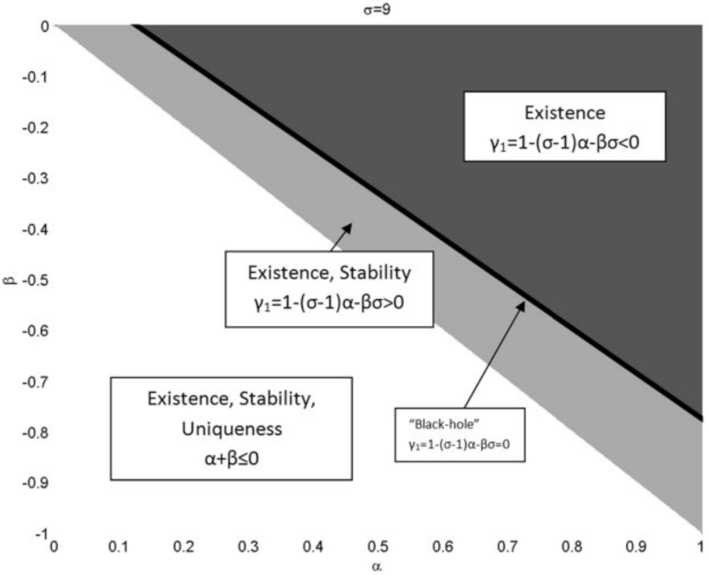
$$L_i A_i^{1-\sigma} w_i^{\sigma} = \phi w_i^{1-\sigma} B_i^{1-\sigma}$$

- Using that equation, they reduce the two equations above, (7) and (8), to

$$L_i^{\tilde{\sigma}\gamma_1} = \bar{B}_i^{(1-\tilde{\sigma})(\sigma-1)} \bar{A}_i^{\tilde{\sigma}(\sigma-1)} U^{1-\sigma} \sum_n d_{ni}^{1-\sigma} \bar{A}_n^{(1-\tilde{\sigma})(\sigma-1)} \bar{B}_n^{\tilde{\sigma}(\sigma-1)} \left(L_n^{\tilde{\sigma}\gamma_1}\right)^{\frac{\gamma_2}{\gamma_1}}$$

$$\gamma_1 \equiv 1 - \alpha(\sigma-1) - \beta\sigma \qquad \gamma_2 \equiv 1 + \alpha\sigma + (\sigma-1)\beta \qquad \tilde{\sigma} = \frac{\sigma-1}{2\sigma-1}$$

Existence and Uniqueness



Existence and Uniqueness

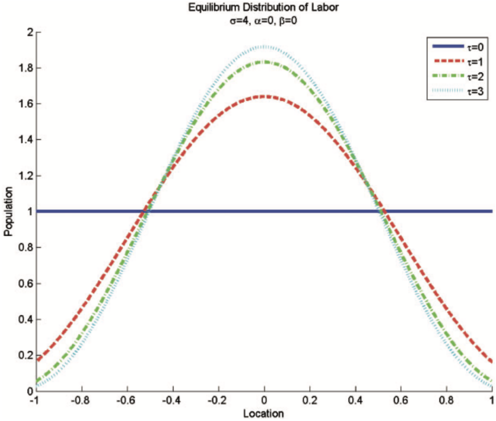


FIGURE III

Economic Activity on a Line: Trade Costs

This figure shows how the equilibrium distribution of population along a line is affected by changes in the trade cost. When trade is costless, the population is equal along the entire line. As trade becomes more costly, the population becomes increasingly concentrated in the center of the line where the consumption bundle is cheapest.

Existence and Uniqueness

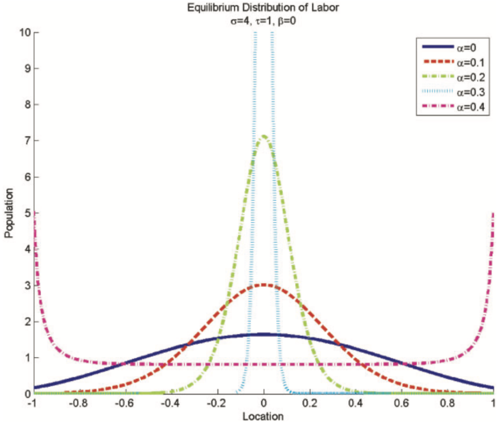


FIGURE VI

Economic Activity on a Line: Productivity Spillovers

This figure shows how the equilibrium distribution of population along a line is affected by varying degrees of productivity spillovers. As the productivity spillovers increase, the population becomes increasingly concentrated in the center of the line. A nondegenerate equilibrium can be maintained as long as $\gamma_1 = 1 - \alpha(\sigma - 1) - \sigma\beta > 0$.

Equilibrium economic activity

- We can further derive the following two equations (assuming symmetric trade costs)

$$\gamma_1 \ln w_i = C_w - \beta (\sigma - 1) \ln \bar{A}_i - (1 - \alpha (\sigma - 1)) \ln \bar{B}_i + (1 + (\sigma - 1)(\beta - \alpha)) \ln P_i$$

$$\gamma_1 \ln L_i = C_L - (\sigma - 1) \ln \bar{A}_i + \sigma \ln \bar{B}_i + (1 - 2\sigma) \ln P_i$$

- P_i is a summary statistic for the role of geography. They combine the equations above to derive

$$\frac{\gamma_1}{\sigma - 1} \ln Y_i = \frac{C_w + C_L}{\sigma - 1} + (1 - \beta) \ln \bar{A}_i + (1 + \alpha) \ln \bar{B}_i - (2 + \alpha - \beta) \ln P_i$$

- At 20% of the variation can be attributed to geographic location

Inversion

- XXX

Inversion

- Notice that we need to know α and β to recover $\bar{A}_i$ and $\bar{B}_i$
- We can always rationalize the data by choosing shifters A_i and B_i so that the model matches the gross output and the location of workers
  - Even without any knowledge of agglomeration and congestion forces
- A large part of the literature focuses on the estimation of these terms

Parameters

- Before, we learned how to recover
  - Productivity shifters $\bar{A}_i$
  - Trade costs τ_{in}
  - Trade elasticity θ

- We are now adding
  - Amenity shifters $\bar{B}_i$
  - Congestion and agglomeration forces α and β

- In AA, workers are freely mobile. Later, in migration models, we will also need
  - Migration costs + Migration elasticity

- Allen and Donaldson (2024) extend many of the existence and uniqueness results to the case in which we have migration costs and dynamics

Ahlfeldt, Redding, Sturm, and Wolf 2015

Berlin wall paper

- XXX

Berlin wall

- XXX

Berlin wall

- XXX

Taking stock

Taking stock

- XXX