Quantitative International Economics

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Weeks 3 - Welfare Gains from Trade
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Outline

- Welfare gains from trade
  - EK
  - Armington with multiple sectors
  - ACR
  - Applications
  - First Order Impact

Welfare Gains in EK

Welfare Gains in EK

- We will start with a one sector model, basic EK
- Slowly we will add other elements to show how the formulas for the welfare gains change
- We will use this as an opportunity also to present the Armington model

Equilibrium Equations

- Real wage can be written as

$$u_i = \frac{w_i}{P_i}$$

  where

$$P_i = \Phi_i^{-\frac{1}{\theta}} \text{ and } \pi_{ii} = \frac{T_i \left(w_i\right)^{-\theta}}{\Phi_i}$$

- We can therefore write

$$\pi_{ii} = T_i \left(\frac{w_i}{P_i}\right)^{-\theta} = T_i \left(u_i\right)^{-\theta}$$

- Which after some simple algebra gives

$$\widehat{u}_i = \left(\frac{\widehat{\pi}_{ii}}{\widehat{T}_i}\right)^{-\frac{1}{\theta}}$$

Welfare Impact of Changes in Fundamentals

- Changes in welfare for any counterfactual change in T_i and d_{in} can be written as

$$\widehat{u}_i = \left(\frac{\widehat{\pi}_{ii}}{\widehat{T}_i}\right)^{-\frac{1}{\theta}}$$

  - Trade of a country with itself is a sufficient statistic for the impact of any change in trade cost and productivities in other countries
  - One single variable captures the role of all the general equilibrium effects associated with trade

- What is the gain from trade? Trade gains as a fraction of current utility.

$$\begin{aligned} GFT_i &= \frac{u_i - u_i^{autarky}}{u_i} \\ &= 1 - (\pi_{ii})^{-\frac{1}{\theta}} \end{aligned}$$

  - π_{ii} is data. What happens when $\pi_{ii} = 1$?
  - What happens with the dispersion of efficiencies when θ is large?
  - What happens to the gains from trade when θ is large?

Welfare Gains from Trade

Table 4.1 Welfare Gains from Trade

	G_j Expressed in Percentages Computed Using:						
	One Sector (12)	Multiple Sectors, No Intermediates (23)		Multiple Sectors, with Intermediates (29)			
		Perfect Competition	Monopolistic Competition	Perfect Competition (Data Alphas)	Perfect Competition	Monop. Comp. (Krugman)	Monop. Comp. (Melitz)
Country	1	2	3	4	5	6	7
AUS	2.3%	8.6%	3.7%	15.8%	15.7%	6.9%	6.8%
AUT	5.7%	30.3%	30.5%	49.5%	49.0%	57.6%	64.3%
BEL	7.5%	32.7%	32.4%	54.6%	54.2%	63.0%	70.9%
BRA	1.5%	3.7%	4.3%	6.3%	6.4%	9.7%	12.7%
CAN	3.8%	17.4%	15.3%	30.2%	29.5%	33.0%	39.8%
CHN	2.6%	4.0%	4.0%	11.5%	11.2%	28.0%	77.9%
CZE	6.0%	16.8%	21.2%	34.0%	37.2%	65.1%	86.7%
DEU	4.5%	12.7%	17.6%	21.3%	22.5%	41.4%	52.9%
DNK	5.8%	30.2%	24.8%	41.4%	45.0%	42.0%	44.8%
ESP	3.1%	9.0%	9.5%	18.3%	17.5%	24.4%	30.5%
FIN	4.4%	11.1%	10.5%	20.2%	20.3%	24.2%	28.0%
FRA	3.0%	9.4%	11.1%	17.2%	16.8%	25.8%	32.1%
GBR	3.2%	12.9%	11.7%	21.6%	22.4%	22.2%	23.5%
GRC	4.2%	16.3%	4.7%	23.7%	24.7%	6.8%	6.1%
HUN	8.1%	29.8%	31.3%	53.5%	55.3%	75.7%	91.0%

- Are the GFT small? Large? For whom?

Welfare Changes with Intermediate Inputs

- With intermediate inputs we have

$$u_i = \frac{w_i}{P_i}$$

where instead

$$P_i = \Phi_i^{-\frac{1}{\theta}} \text{ and } \pi_{ii} = \frac{T_i \left(w_i^{\beta} P_i^{1-\beta}\right)^{-\theta}}{\Phi_i}$$

we can manipulate to get

$$\pi_{ii} = \frac{T_i \left(u_i^{\beta} P_i\right)^{-\theta}}{\Phi_i}$$

- After some manipulations we get

$$\widehat{u}_i = \left(\frac{\widehat{\pi}_{ii}}{\widehat{T}_i}\right)^{-\frac{1}{\beta\theta}}$$

  – Now we have β in the formula. Does it increase or decrease the gains from trade?

Welfare Gains with Intermediate Inputs

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	One Sector (12)	Multiple Sectors, No Intermediates (23)		Multiple Sectors, with Intermediates (29)			
		Perfect Competition	Monopolistic Competition	Perfect Competition (Data Alphas)	Perfect Competition	Monop. Comp. (Krugman)	Monop. Comp. (Melitz)
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AUS	2.3%	8.6%	3.7%	15.8%	15.7%	6.9%	6.8%
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BEL	7.5%	32.7%	32.4%	54.6%	54.2%	63.0%	70.9%
BRA	1.5%	3.7%	4.3%	6.3%	6.4%	9.7%	12.7%
CAN	3.8%	17.4%	15.3%	30.2%	29.5%	33.0%	39.8%
CHN	2.6%	4.0%	4.0%	11.5%	11.2%	28.0%	77.9%
CZE	6.0%	16.8%	21.2%	34.0%	37.2%	65.1%	86.7%
DEU	4.5%	12.7%	17.6%	21.3%	22.5%	41.4%	52.9%
DNK	5.8%	30.2%	24.8%	41.4%	45.0%	42.0%	44.8%
ESP	3.1%	9.0%	9.5%	18.3%	17.5%	24.4%	30.5%
FIN	4.4%	11.1%	10.5%	20.2%	20.3%	24.2%	28.0%
FRA	3.0%	9.4%	11.1%	17.2%	16.8%	25.8%	32.1%
GBR	3.2%	12.9%	11.7%	21.6%	22.4%	22.2%	23.5%
GRC	4.2%	16.3%	4.7%	23.7%	24.7%	6.8%	6.1%
HUN	8.1%	29.8%	31.3%	53.5%	55.3%	75.7%	91.0%

Armington Model with Multiple Sectors

Armington

- Multiple countries indexed by i and n
- Each country is endowed with labor N_i
- Multiple sectors k
  - Each country produces a unique variety of good for sector k
- CRS technologies with productivity $a_{i,k}$
- Perfectly competitive markets

Demand

- Cobb-Douglas over composite goods $C_{n,k}$ (upper tier)

$$U_n = \sum_{k=1}^{K} \frac{1}{\beta_{n,k}} C_{n,k}^{\beta_{n,k}}$$

  – $\beta_{n,k}$ share of consumption of goods from sector k

- CES over varieties of goods $c_{ni,k}$ (lower tier)

$$C_{n,k} = \left(\sum_{i=1}^{N} b_{in,k}^{\frac{1}{\sigma_k}} c_{in,k}^{\frac{\sigma_k - 1}{\sigma_k}} \right)^{\frac{\sigma_k}{\sigma_k - 1}}$$

  – $b_{in,k}$ demand shifters
  – σ_k elasticity of substitution between origins (trade elasticity)

Demand

- Value of trade flows of industry k from i to n

$$X_{in,k} = \pi_{in,k} X_{n,k} = \underbrace{\frac{b_{in,k} p_{in,k}^{1-\sigma_k}}{P_{n,k}^{1-\sigma_k}}}_{\equiv \pi_{in,k}} \underbrace{\beta_{n,k} X_n}_{\equiv X_{n,k}}$$

  where the CES price index $P_{n,k}$ is

$$P_{n,k} = \left(\sum_{i=1}^{N} b_{in,k} p_{in,k}^{1-\sigma_k} \right)^{\frac{1}{1-\sigma_k}}$$

  where $p_{in,k}$ is the price of taking a good from i to n

- Final consumer price index P_n is

$$P_n = \prod_{k=1}^{K} P_{n,k}^{\beta_{n,k}}$$

Supply

- Supply

$$Q_{i,k} = a_{i,k} N_{i,k}$$

  – $a_{i,k}$ and $N_{i,k}$ are labor productivity and employment

- Prices

$$p_{in,k} = d_{in,k} p_{ii,k} = d_{in,k} \frac{w_i}{a_{i,k}}$$

  – w_i is the wage, $d_{in,k}$ is an iceberg trade cost

General Equilibrium

- Total expenditure equals total payments to factors of production

$$X_n = w_n N_n \tag{1}$$

- Labor market clearing

$$w_n N_n = \sum_{i=1}^{N} \sum_{k=1}^{K} X_{ni,k} \tag{2}$$

- Trade Flows

$$X_{in,k} = \frac{b_{in,k} p_{in,k}^{1-\sigma_k}}{P_{n,k}^{1-\sigma_k}} \beta_{n,k} X_n \tag{3}$$

- Firms maximize profits

$$p_{in,k} = d_{in,k} \frac{w_i}{a_{i,k}} \tag{4}$$

- Price Indexes

$$P_n = \prod_{k=1}^{K} P_{n,k}^{\beta_{n,k}} \quad \text{and} \quad P_{n,k} = \left(\sum_{i=1}^{N} b_{in,k} p_{in,k}^{1-\sigma_k} \right)^{\frac{1}{1-\sigma_k}} \tag{5}$$

Definition: Given technology $\{a_{n,k}\}_{n,k}$, preferences $\left\{ \left\{ \{b_{in,k}\}_i , \beta_{n,k} \right\}_n , \sigma_k \right\}_k$, trade costs $\{d_{ni}\}_{ni}$, and population $\{N_n\}_n$, an equilibrium is a vector of wages $\{w_n\}_n$ that satisfies equations (1)-(5).

Welfare Changes

- Trade share equation

$$\pi_{in,k} = \frac{b_{in,k}\left(w_i/a_{i,k}\right)^{1-\sigma_k}}{P_{n,k}^{1-\sigma_k}}$$

- Consider a change in productivity $\{\widehat{a}_{i,k}\}$ or trade cost $\left\{\widehat{d}_{in,k}\right\}_{i\neq n}$

$$\widehat{\pi}_{in,k} = \frac{\left(\widehat{w}_i/\widehat{a}_{i,k}\right)^{1-\sigma_k}}{\widehat{P}_{n,k}^{1-\sigma_k}} \Rightarrow \left(\frac{\widehat{w}_i}{\widehat{P}_{i,k}}\right) = \widehat{a}_{i,k}\widehat{\pi}_{ii,k}^{\frac{1}{1-\sigma_k}}$$

- Change in welfare

$$\widehat{u}_i = \frac{\widehat{w}_i}{\widehat{P}_i} = \prod_k \left(\frac{\widehat{w}_i}{\widehat{P}_{i,k}}\right)^{\beta_{i,k}} = \prod_k \widehat{a}_{i,k}^{\beta_{i,k}}\left(\widehat{\pi}_{ii,k}\right)^{\frac{\beta_{i,k}}{1-\sigma_k}}$$

  – Again! All the impact of tfps and trade costs that occurs via trade are encapsulated by the $\widehat{\pi}_{ii,k}$ term

Welfare Gains from Trade

- Consider now a move to autarky, as before. Then we have

$$G_i = \frac{u_i - u_i^{autarky}}{u_i} = 1 - \prod_k \left(\pi_{ii,k}\right)^{\frac{\beta_{i,k}}{1-\sigma_k}}$$

  - Notice that $\pi_{ii,k}$ is, in principle, observable
  - β_k is also readily available from the data
  - We still need to estimate σ_k

Comparison

- The EK model gave us

$$G_i = 1 - (\pi_{ii})^{-\frac{1}{\theta}}$$

- The EK model with intermediate inputs gave us

$$G_i = 1 - (\pi_{ii})^{-\frac{1}{\beta\theta}}$$

- The multi-sectoral version Armington gave us

$$G_i = 1 - \prod_k (\pi_{ii,k})^{\frac{\beta_{i,k}}{1-\sigma_k}}$$

- Is that a coincidence? If we adopt different micro-economic foundation for trade, do we get a different formula? Under what conditions do we get a gains from trade formula that has this particular shape? If so, why?
  - It turns out that many different models will give you that equation, despite differences in market structure, preferences and technology, which would suggest dissimilarities a priori

ACR (2012)

Basic Idea

- A large class of models will lead to a formula for the gains from trade as

$$\widehat{W} = \widehat{\lambda}^{\frac{1}{\epsilon}}$$

  – $\widehat{\lambda}$ is the change in the share of domestic expenditure and ϵ is the trade elasticity

- Trade models that satisfy the following primitive assumptions

  1. Dixit-Stiglitz preferences
  2. One factor of production
  3. Linear cost of production
  4. Perfect or monopolistic competition

- As well as the following macro-level restrictions

  1. Trade is balanced
  2. Aggregate profits are a constant share of aggregate revenues
  3. Import demand system is CES

Building Intuition with Armington 1 sector

- Countries indexed by i and j and utility is

$$U_j = \left[\sum_{j=1}^{n} q_{ij}^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)} \text{, giving a price index of } P_j = \left[\sum_{i=1}^{n} (w_i \tau_{ij})^{1-\sigma}\right]^{\frac{1}{1-\sigma}}.$$

- Total imports is

$$X_{ij} = \left(\frac{w_i \tau_{ij}}{P_j}\right)^{1-\sigma} Y_j \quad \text{, where} \quad Y_j = \sum_{i=1}^{n} X_{ij},$$

and $1 - \sigma < 0$ is the partial elasticity of relative imports w.r.t. variable trade cost

$$\frac{\partial \ln (X_{ij}/X_{jj})}{\partial \ln \tau_{ij}}$$

- Lastly, trade balances

$$Y_j = w_j L_j$$

Building Intuition with Armington 1 sector

- Consider a shock that affects labor endowments L_i and trade costs τ_{ij} , but leaves endowments in j and τ_{jj} fixed. What is the change in real income $W_j \equiv Y_j / P_j$

$$d \ln W_j = d \ln Y_j - d \ln P_j$$

- Let's work with the first term

$$d \ln Y_j = d \ln w_j + d \ln L_j$$

  – We can pick w_j as numeraire, so that $d \ln w_j = 0$. Also, since there is no change in L_j, $d \ln L_j = 0$.

- Now, let us turn to $d \ln P_j$

$$\begin{aligned}
d \ln P_j &= \frac{1}{1-\sigma} \frac{1}{\sum_{i=1}^{n} \left(w_i \tau_{ij}\right)^{1-\sigma}} \sum_{i=1}^{n} (1-\sigma) \left(exp \left(\ln \left(w_i \tau_{ij}\right)^{1-\sigma} \right) \right) \left(d \ln w_i + d \ln \tau_{ij}\right) \\
&= \sum_{i=1}^{n} \frac{\left(w_i \tau_{ij}\right)^{1-\sigma}}{\sum_{i=1}^{n} \left(w_i \tau_{ij}\right)^{1-\sigma}} \left(d \ln w_i + d \ln \tau_{ij}\right) \\
&= \sum_{i=1}^{n} \lambda_{ij} \left(d \ln w_i + d \ln \tau_{ij}\right)
\end{aligned}$$

Building Intuition with Armington 1 sector

- Notice that changes in price index contain a “direct” and a “GE effect”

$$d \ln W_j = -d \ln P_j = -\sum_{i=1}^{n} \lambda_{ij} \left(\underbrace{d \ln w_i}_{\text{GE impact}} + \underbrace{d \ln \tau_{ij}}_{\text{direct impact}} \right)$$

- Changes in the price index are a function of the change in wage and trade cost (tariffs) weighted by how important that country is in the expenditure share of destination j

  – When we are building intuition about how big should the effect of a shock be on welfare, we can multiply the shock by the expenditure share (for example, impact of a tariff change on consumers). Notice that w_i is endogenous (depends on GE) and τ_{ij} is the direct impact of changes in trade costs

- ACR refers to $-\sum_{i=1}^{n} \lambda_{ij} \left(d \ln w_i + d \ln \tau_{ij} \right)$ as the *changes in terms of trade*. How expensive a bundle of goods is relative to its own cost of producing goods w_j (recall that w_j and τ_{jj} are fixed).

Building Intuition with Armington 1 sector

- Let's look at $\lambda_{ij}/\lambda_{jj} = \left(w_i\tau_{ij}\right)^{1-\sigma} / \left(w_j\tau_{jj}\right)^{1-\sigma}$, which gives

$$d\ln\lambda_{ij} - d\ln\lambda_{jj} = (1-\sigma)\left(d\ln w_i + d\ln\tau_{ij}\right)$$

- Changes in terms of trade can be inferred from changes in trade shares! Let's substitute

$$d\ln W_j = -d\ln P_j = -\sum_{i=1}^{n}\lambda_{ij}\left(d\ln w_i + d\ln\tau_{ij}\right) = -\frac{1}{(1-\sigma)}\sum_{i=1}^{n}\lambda_{ij}\left(d\ln\lambda_{ij} - d\ln\lambda_{jj}\right)$$

- Notice that, because $\sum_{i=1}^{n}\lambda_{ij} = 1$, then the expression above simplifies to

$$d\ln W_j = \frac{1}{(1-\sigma)}d\ln\lambda_{jj}$$

- Because of the macro-economic restrictions, aggregating across the change in trade shares across all importers gives the equation above

Applications

Some Applications

- Now that we understand better the idea of the ACR (2012), let us turn to some applications
  - Donaldson (railroads of the raj)
  - Costinot, Donaldson and Komunjer (2012)

Donaldson (2018)

- Evaluate the impact of railroads with a DID

$$\ln\left(\frac{r_{ot}}{\tilde{P}_{ot}}\right) = \beta_o + \beta_t + \gamma RAIL_{ot} + \epsilon_{ot}$$

  – where $r_{ot}/\tilde{P}_{ot}$ is the real agricultural income per acre

- Question 1: What was the impact of railroads on real agricultural income per acre

- Question 2: How much of these gains can be attributed to trade?

Donaldson (2018)

Table 4—Railroads and Real Income Levels: Step 3

Dependent variable: log real agricultural income	(1)	(2)	(3)	(4)
Railroad in district	0.164	0.158	0.160	0.167
	(0.049)	(0.048)	(0.050)	(0.050)
Unbuilt railroad in district, abandoned after proposal stage		0.057		
		(0.058)		
Unbuilt railroad in district, abandoned after reconnaissance stage		0.013		
		(0.099)		
Unbuilt railroad in district, abandoned after survey stage		−0.069		
		(0.038)		
(Unbuilt railroad in district, included in Lawrence Plan 1869–1873)			0.067	
× (post-1871 indicator)			(0.104)	
(Unbuilt railroad in district, included in Lawrence Plan 1874–1878)			−0.019	
× (post-1874 indicator)			(0.092)	
(Unbuilt railroad in district, included in Lawrence Plan 1879–1883)			0.095	
× (post-1879 indicator)			(0.084)	
(Unbuilt railroad in district, included in Lawrence Plan 1884–1888)			−0.072	
× (post-1884 indicator)			(0.075)	
(Unbuilt railroad in district, included in Lawrence Plan 1889–1893)			0.047	
× (post-1889 indicator)			(0.049)	
(Unbuilt railroad in district, included in Lawrence Plan 1894–1898)			−0.088	
× (post-1894 indicator)			(0.086)	
(Unbuilt railroad in district, included in Kennedy plan, high-priority)				−0.0001
× (year-1848)				(0.002)
(Unbuilt railroad in district, included in Kennedy plan, low-priority)				0.001
× (year-1848)				(0.003)
Observations	7,086	7,086	7,086	7,086
R^2	0.848	0.848	0.848	0.848

Donaldson (2018)

TABLE 5—A SUFFICIENT STATISTIC FOR RAILROAD IMPACT: STEP 4

log real ag. income, corrected for rainfall:	(1)	(2)
Railroad in district	0.258	0.124
	(0.050)	(0.050)
"Trade share," as computed in model		−1.587
		(0.177)
Observations	7,086	7,086
R^2	0.835	0.844

Costinot, Donaldson and Komunjer (2012)

- What are the gains from trade driven by Ricardian comparative advantage?
- Ricardian models predict that countries should produce and export relatively more in industries in which they are relatively more productive
- This paper uses the EK (2002) framework to address these points

Costinot, Donaldson and Komunjer (2012)

- **A1.** Technology

$$F_i^k(z) = exp\left[-\left(z/z_i^k\right)^{-\theta}\right]$$

  - z_i^k fundamental productivity, determines relative productivities between sectors
  - θ captures within industry heterogeneity

- **A2.** Trade frictions are iceberg
- **A3.** Perfect competition and CRS
- **A4.** Preferences for sectors are CD and within sector are CES
- **A5.** Trade is balanced

Costinot, Donaldson and Komunjer (2012)

- Lemma 1. Country i and i' exporting to the same destination j

$$\ln\left(\frac{x_{ij}^{k}}{x_{ij}^{k'}}\frac{x_{i'j}^{k'}}{x_{i'j}^{k}}\right) = \theta\ln\left(\frac{z_{i}^{k}}{z_{i}^{k'}}\frac{z_{i'}^{k'}}{z_{i'}^{k}}\right) - \theta\ln\left(\frac{d_{ij}^{k}}{d_{ij}^{k'}}\frac{d_{i'j}^{k'}}{d_{i'j}^{k}}\right)$$

- If trade costs are

$$d_{ij}^{k} = d_{ij}d_{j}^{k}$$

- Then $\frac{d_{ij}^{k}}{d_{ij}^{k'}}\frac{d_{i'j}^{k'}}{d_{i'j}^{k}} = 1$. In that case, then for any importer j and any pair of exporters i ($i \neq j$), the ranking of relative fundamental productivities determines the ranking of relative exports

$$\frac{z_{i}^{1}}{z_{i'}^{1}} \leq \ldots \leq \frac{z_{i}^{K}}{z_{i'}^{K}} \iff \frac{x_{ij}^{1}}{x_{i'j}^{1}} \leq \ldots \leq \frac{x_{ij}^{K}}{x_{i'j}^{K}}$$

Costinot, Donaldson and Komunjer (2012)

- To take the model to data, they notice that z_i^k in the model is not the productivity measure by statistical agencies. What these agencies provide is $\tilde{z}_i^k \equiv E\left[z_i^k(\omega)\,|\Omega_i^k\right]$, which is the productivity conditional on the subset of varieties that are actually produced. They therefore give the following alternative expression

$$\ln\left(\frac{\tilde{x}_{ij}^k}{\tilde{x}_{ij}^{k'}}\frac{\tilde{x}_{i'j}^{k'}}{\tilde{x}_{i'j}^k}\right)=\theta\ln\left(\frac{\tilde{z}_i^k}{\tilde{z}_i^{k'}}\frac{\tilde{z}_{i'}^{k'}}{\tilde{z}_{i'}^k}\right)-\theta\ln\left(\frac{d_{ij}^k}{d_{ij}^{k'}}\frac{d_{i'j}^{k'}}{d_{i'j}^k}\right)$$

where $\tilde{x}_{ij}^k = x_{ij}^k/\pi_{ii}^k$, which gives

$$\frac{z_i^k}{z_{i'}^k}=\left(\frac{\tilde{z}_i^k}{\tilde{z}_{i'}^k}\right)\left(\frac{\pi_{ii}^k}{\pi_{i'i'}^k}\right)^{\frac{1}{\theta}}$$

Costinot, Donaldson and Komunjer (2012)

- **Counterfactual**. If there were no fundamental relative productivity differences across industries, what would be the consequence for aggregate trade flows and welfare?
  - Reference country i_0. Fundamental productivity is the same there $\left(z_{i_0}^k\right)' \equiv z_{i_0}^k$ for all k
  - For others $i \neq i_0$. Productivity changes from z_i^k to $\left(z_i^k\right)'$ such that $\left(z_i^k\right)' \equiv Z_i \times z_{i_0}^k$. Z_i is chosen so that relative wages w_i / w_{i_0} is the same in $\left(w_i / w_{i_0}\right)'$
- Then predicted changes in bilateral trade flows

$$\widehat{x}_{ij}^k = \frac{\left(z_i^k / Z_i\right)^{-\theta}}{\sum_{i'} \pi_{i'j}^k \left(z_{i'}^k / Z_{i'}\right)^{-\theta}}$$

and

$$\widehat{W}_{i_0} = \prod_{k=1}^{K} \left[\sum_{i=1}^{I} \pi_{i i_0}^k \left(\frac{z_i^k}{z_{i_0}^k Z_i} \right)^{-\theta} \right]^{\alpha_{i_0}^k / \theta}$$

Constinot, Donaldson and Komunjer (2015)

- Revealed Comparative Advantage measure

$$RCA_i^k = \frac{X_{i,world}^k}{\sum_{k'=1}^K X_{i,world}^{k'}} \bigg/ \frac{\sum_{i'=1}^I X_{i',world}^k}{\sum_{i'=1}^I \sum_{k'=1}^K X_{i',world}^{k'}}$$

- Very famous measure from Balassa (1965). This measure, commonly used, is not theoretically consistent. If we run a gravity equation of

$$\ln x_{ij}^k = \delta_{ij} + \delta_j^k + \theta \ln z_i^k + \epsilon_{ij}^k$$

- Then look at $\delta_{US}^k - \delta_i^k$, which gives the ranking of $\ln z_i^k - \ln z_{US}^k$. Balassa's measure would not, however, provide this ranking. Their measure is also a pairwise comparison between two goods, which is at the core of Ricardian's CA. The RCA_i^k, however, does not provide that relative benchmark.
  - I find these statistics to be useful when I'm doing a first inspection at the data.

Costinot, Donaldson and Komunjer (2015)

TABLE 7
Counterfactual results—baseline

	Outcome variable of interest			
	% change in in total exports	Change in index of interindustry trade	% change in welfare	% change in welfare relative to the total gains from trade
Reference country	(1)	(2)	(3)	(4)
Australia	18·52	24·57	−2·90	−39·11
Belgium and Luxembourg	−1·76	4·12	0·71	2·64
Czech Republic	3·91	5·62	−0·12	−1·26
Denmark	0·60	−2·64	−0·40	−2·18
Spain	3·68	−3·89	−0·46	−7·08
Finland	−5·62	3·44	0·14	1·65
France	0·80	−0·49	−0·20	−3·09
Germany	−2·10	−8·46	0·14	2·22
Greece	26·35	−11·23	−4·37	−40·47
Hungary	1·70	−5·28	−0·25	−1·62
Ireland	−5·48	−4·31	0·20	0·74
Italy	−4·76	−9·85	0·14	2·78
Japan	−6·12	−24·75	0·35	24·48
Korea	2·68	−10·15	−0·44	−9·60
Netherlands	1·95	−0·94	−0·64	−2·81
Poland	12·33	−22·35	−1·68	−23·09
Portugal	8·44	−13·62	−0·92	−9·12
Slovakia	2·33	14·11	0·82	4·64
Sweden	−2·98	3·03	0·34	3·30
U.K.	3·45	−4·04	−0·26	−2·94
U.S.	3·82	−3·83	−0·42	−11·71
World average	2·94	−5·72	−0·49	−5·32

Other Applications

- Levchenko and Zhang (JME 2015)
  - Evolution of CA over 5 decades. Convergence of relative sectoral productivity had a modest impact on welfare.
- Ossa (JIE 2015)
  - Abstract. "I show that accounting for cross-industry variation in trade elasticities greatly magnifies the estimated gains from trade. The main idea is as simple as it is general: while imports in the average industry do not matter too much, imports in some industries are critical to the functioning of the economy, so that a complete shutdown of international trade is very costly overall."
  - One sector model. GFT of 16.5% in average. Multi-sectoral models GFT raises to 55.9% in average.
- Egger, Erhardt and Nigai (WP 2024)
  - What happens when productivities take the form of distribution across firms?
  - Impressive estimation of firm-level productivity distribution across multiple countries

Conclusion

Conclusion

- Overall GFT in one sector version of the EK model is not especially high, but with multiple sectors conclusions change
- Analysis of counterfactuals hinge heavily on functional form (CES demand in particular)
  - Adao, Costinot, and Donaldson (2017) study counterfactual predictions in non-parametric versions
    - Generalize hat-algebra to non-parametric demand function systems
    - Use mixed CES (avoid Independence of Irrelevant Alternatives, IIA)
- Some papers first show that their object of study is identified without (or with weaker) parametric assumptions, before imposing the necessary parametric assumptions which allow for the quantification of the model

Conclusion

- We finish here our analysis of the basic foundation of quantitative trade models
  - EK 2002
    - Solution in levels
    - Solution in changes
    - Estimation of parameters
  - Gains from trade
    - Single sector
    - Multiple sectors (Armington)
  - We covered a few applications
- Next week
  - Firms