Quantitative International Economics

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Weeks 1 and 2 - The Basic Structure of Quantitative Trade Models - Part 1
Spring 2025

Outline

- Logistics
- A bit of history
- EK model with 1 sector and no intermediate inputs
  - Solving the model in levels
  - Policy counterfactuals
  - Solving the model in changes (hat-algebra)
  - Inverting the model
  - Estimating trade costs

Course Structure

Logistics

- Instructor: Heitor S. Pellegrina
- My office location: Jenkins Hall Office 3042
- Office hour: Thursday 9-11 am
- Location: Main Campus - 3005 until spring break, Haggar Hall 212 after then
- Lectures: TT and TH at 2-3:15 pm
- 12 weeks

Organization

- Every other week we will do
  - A model
  - A computational section
- There will be 3 problem sets.
- Presentation of a project by the end of the course. The presentation will consist of:
  - Motivation
  - Data
  - Proof of concept **calibration**

Grading

- Problem sets: 50 percent
- Presentation: 10 percent
- Final project: 40 percent

Method

- We will start with the basic quantitative trade model
  - In each section, we will add a new piece
    - For example, mobility, non-homothetic preferences, dynamics
  - We will study how to incorporate each new piece quantitatively
- I will do my best to cover the simplest (bare-bones) version of each model. When you write your own papers, you will need to adjust these basic frameworks to the context that you are studying.
- **GOAL**: By the end of the course, you should be able to understand papers using quantitative trade models. How to write them. How to code them. Hopefully, the final project will be the beginning of one of your dissertation chapters.

A (Very) Brief History of International Economics

A Brief History of International Economics

- International Economics is one of the oldest fields in Economics
  - Question: Why do countries trade?
    - Comparative Advantage (David Ricardo, 1817)
    - Factor Endowment (Heckscher-Ohlin, 1933)
    - Increasing Returns to Scale (Krugman, 1980)
- Chipman "A Survey of the Theory of International Trade" (1965)
- Krugman's Nobel Lecture (a must!)

A Brief History of International Economics

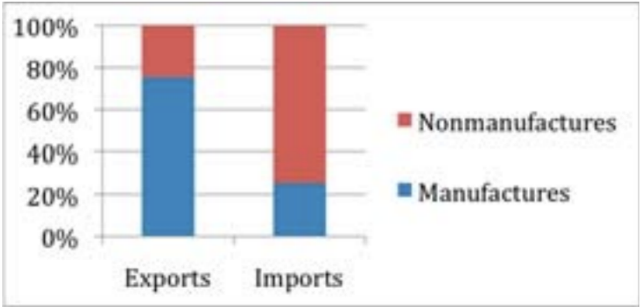

Figure 1. Composition of British trade circa 1910. Source: Baldwin and Martin, 1999.

A Brief History of International Economics

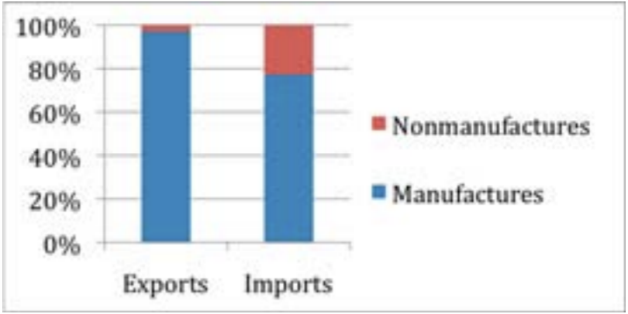

Figure 2. Composition of British trade in the 1990s. Source: Baldwin and Martin, 1999.

A Brief History of International Economics

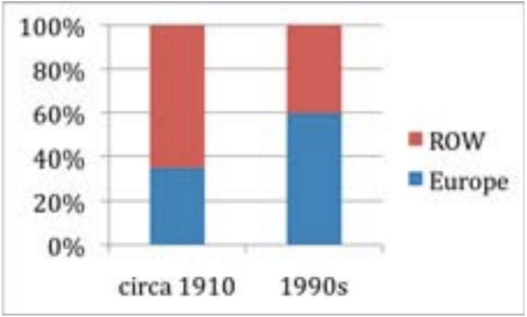

Figure 3. Destination of British exports. Source: Baldwin and Martin, 1999.

Modern Quantitative Trade Models (Foundation)

- Anderson (1979) $\Rightarrow$ Armington Model
  - At the time, growing empirical literature on gravity models, without theory
- Eaton and Kortum (ECTA 2002)
  - Brought Ricardo back to quantitative trade models (see EK JEL 2012)
- Melitz (ECTA 2003)
  - Theory paper, but framework became the basis of quantitative trade models with firms
  - Firm level data in the 1990s revolutionized the field

Recent Developments in Quantitative Trade Models

- Rich internal geography (trade between regions)
- Development Economics
  - Non-homothetic preferences and structural change
  - Agriculture and land use
- Spatial and Urban Economics
  - Factor mobility
- Climate Change
  - Natural resources, CO_2 emissions and optimal policy
- Economic Growth
  - Factor accumulation

The Eaton and Kortum Model

Motivation

- Four Empirical Facts
  - Trade diminishes with distance
  - Prices vary across locations, particularly between locations that are farther apart
  - Factor rewards are different across locations
  - Countries' relative productivities vary substantially
- This paper: A Ricardian Framework that captures all of them

Environment

- Countries are denoted by i or n and goods are denoted by j
  - Productivity of country i in good j is $z_i(j)$
  - There is a continuum of goods $j \in [0, 1]$
- Factors are fully mobile *within* countries + CRS
  - Cost or producing good j in country i is $c_i / z_i(j)$
  - c_i is the cost of a bundle of inputs (here, we will assume only labor, then $c_i = w_i$)
- Geographic barriers are iceberg (Samuelson's formulation)
  - For every unit of good j sold in destination country n, d_{ni} units of that good must be produced in i.
  - $d_{ni} > 1$ if $n \neq i$ and $d_{ni} = 1$ if $n = i$
  - Cost of delivering a unit of good j produced in country i to country n is

$$p_{ni}(j) = \left(\frac{c_i}{z_i(j)} \right) d_{ni}$$

Demand

- Consumers purchase amounts $Q(j)$ to maximize

$$U = \left[\int_0^1 Q(j)^{(\sigma-1)/\sigma} \, dj \right]^{\sigma/(\sigma-1)}$$

subject to budget constraints. The elasticity of substitution satisfies $\sigma > 0$.

- Consumers buy from the origin that can offer the lowest price for good j

$$p_n(j) = \min \{ p_{ni}(j) \, ; i = 1, ..., N \}$$

- Countries have the option of buying from many different sources
  - What is the distribution of *realized* prices in this economy?
  - EK take a probabilistic approach to solve this question

Technology

- Each $z_i\left(j\right)$ is drawn from a random variable Z_i
  - $F_i\left(z\right) = Pr\left(Z_i \leq z\right)$
- Assume that $z_i\left(j\right)$ comes from Fréchet (type II extreme value distribution)

$$F_i\left(z\right) = e^{-T_i z^{-\theta}}$$

  - T_i governs the level of productivity draws (**country-specific**)
  - θ governs the dispersion of these draws (**common across countries**)
    - Lower θ means larger dispersion
    - Here, θ will govern CA forces
- What is the probability that i will buy a good from n, π_{ni}?

Prices

- Country i presents to country n with a distribution of **potential** prices

$$G_{ni}(p) = Pr(P_{ni} \leq p) = 1 - F_i(c_i d_{ni}/p) = 1 - e^{-T_i (c_i d_{ni}/p)^{-\theta}} = 1 - e^{-T_i (c_i d_{ni})^{-\theta} p^{\theta}}$$

- What is the distribution of **realized** prices in destination country n? Let's go by steps

$$\begin{aligned} G_{ni}(p) &\Rightarrow \text{probability that country } i \text{ offers a price below } p \\ 1 - G_{ni}(p) &\Rightarrow \text{probability that country } i \text{ offers a price above } p \\ \prod_{i=1}^{N} (1 - G_{ni}(p)) &\Rightarrow \text{probability that } \textbf{all countries} \text{ offer price above } p \\ 1 - \prod_{i=1}^{N} (1 - G_{ni}(p)) &\Rightarrow \text{probability that } \textbf{at least 1 country} \text{ offers a price below } p \end{aligned}$$

- The probability that the **realized** price is below p in destination n is

$$G_n(p) = 1 - \prod_{i=1}^{N} (1 - G_{ni}(p))$$

Prices

- Combining

$$G_{ni}(p) = 1 - e^{-T_i (c_i d_{ni})^{-\theta} p^{\theta}}$$

and

$$G_n(p) = 1 - \prod_{i=1}^{N} (1 - G_{ni}(p))$$

we get

$$G_n(p) = 1 - e^{-\Phi_n p^{\theta}} \text{ where } \Phi_n = \sum_{i=1}^{N} T_i (c_i d_{ni})^{-\theta}$$

- Φ_n incorporates (often called the "multilateral resistance term")
  - Trade costs: d_{ni}
  - States of technology: T_i
  - Input costs: c_i

- What happens when $d_{ni} \to \infty$ for $n \neq i$?

- Note the similarity with the initial distribution $F_i(z) = e^{-T_i z^{-\theta}}$
  - Frechét preserves its shape under minimization transformations

Prices

- What is the probability that i is the lowest cost provider for n?

$$\pi_{ni} = Pr\left[P_{ni}(j) \leq \min\left\{P_{ns}(j); s \neq i\right\}\right]$$
$$= \int_0^\infty \prod_{s \neq i} \left[1 - G_{ns}(p)\right] dG_{ni}(p)$$
$$= \int_0^\infty \prod_{s \neq i} \left[1 - G_{ns}(p)\right] g_{ni}(p)\, dp$$

- The probability of price p times the probability that every other price is above that p

$$\pi_{ni} = \int_0^\infty \prod_{s \neq i} \left[e^{-T_s(c_s d_{ns})^{-\theta} p^\theta}\right] \left[e^{-T_i(c_i d_{ni})^{-\theta} p^\theta} T_i (c_i d_{ni})^{-\theta}\, \theta p^{\theta-1}\right] dp$$
$$= T_i (c_i d_{ni})^{-\theta} \int_0^\infty \left[e^{-\sum_i T_i(c_i d_{ni})^{-\theta} p^\theta}\right] \theta p^{\theta-1} dp$$
$$= \frac{T_i (c_i d_{ni})^{-\theta}}{\Phi_n} \int_0^\infty \underbrace{\left[e^{-\Phi_n p^\theta}\right] \theta \Phi_n p^{\theta-1}}_{g_n(p)}\, dp$$
$$= \frac{T_i (c_i d_{ni})^{-\theta}}{\Phi_n}$$

Prices

- π_{ni} is the probability that country n buys a good j from i
  - Because there is a continuum of goods (an infinite number), π_{ni} is also the fraction of goods
- What is the share of **expenditure?** We need to know prices as well. Quantities are not enough.
  - Average price of **realized** transactions between origin i and destination n is the same! Why?
    - Imagine a country A selling to country B experiences an increase in T_A
      $\Rightarrow$ Avg. price of goods sold by country A to country B falls
    - Country B will drop goods from other countries to buy from A, particularly those with higher prices
      $\Rightarrow$ Avg. price of goods sold by other countries to country B also falls!
    - Under Fréchet, the relative avg. price of goods sold to B *from any origin falls.* The degree to which they fall is such that after the increase in T_a the average price of any origin is still equal to the average price of goods from A.
  - This property carries over to other applications of the Fréchet, including land and labor allocation
- **LLN + Price distribution is independent of origin $\Rightarrow$ Expenditure shares = Fraction of goods**

Welfare

- Welfare is then given by

$$U_n = \frac{w_n}{P_n}$$

where P_n is the price index of the economy

$$P_n = \left[\Gamma\left(\frac{\theta + 1 - \sigma}{\theta}\right)\right]^{\frac{1}{1-\sigma}} \left(\sum_i T_i \left(w_i d_{in}\right)^{-\theta}\right)^{-\frac{1}{\theta}}$$

Trade Flows and Gravity

- Trade flows (in values) from country n to country i

$$X_{ni} = \pi_{ni} X_n = \frac{T_i \left(c_i d_{ni}\right)^{-\theta}}{\sum_{s=1}^{N} T_s \left(c_s d_{ns}\right)^{-\theta}} X_n$$

  - where X_n is total expenditure

- Take logs to get

$$\log\left(X_{ni}\right) = \underbrace{\log\left(T_i c_i^{-\theta}\right)}_{\text{Origin Component}} + \underbrace{\log\left(X_n\right) - \log\left(\sum_{s=1}^{N} T_s \left(c_s d_{ns}\right)^{-\theta}\right)}_{\text{Destination Component}} + \log\left(d_{ni}^{-\theta}\right)$$

  - What is the elasticity of trade flows with respect to trade costs?
  - What does happen when θ is larger, why? What is the intuition?
  - What about shocks to c_i? What about the importance of T_i versus $c_i d_{ni}$?
  - Where is σ?

Equilibrium

- Total expenditure equals total payments to factors of production (N_n is total labor)

$$X_n = w_n N_n \tag{1}$$

- Labor market clears

$$w_n N_n = \sum_{i=1}^{N} X_{in} \tag{2}$$

- Consumers choices are optimal

$$X_{ni} = \frac{T_i \left(w_i d_{in}\right)^{-\theta}}{\Phi_n} X_n \tag{3}$$

  – where

$$\Phi_n = \sum_s T_s \left(w_s d_{ns}\right)^{-\theta} \tag{4}$$

Definition: Given technology $\{\{T_n\}_n, \theta\}$, trade costs $\{d_{ni}\}_{ni}$, and population $\{N_n\}_n$, an equilibrium is a vector of wages $\{w_n\}_n$ that satisfies equations (1)-(4).

Algorithm

- Step 1: Guess a vector of wages w_n^g (pick a normalization)
- Step 2: Construct X_n
- Step 3: Construct X_{ni}
- Step 4: Obtain the demand for workers $N_n^{dem} = \sum_{i=1}^{N} X_{in}/w_n^g$
- Step 5: Check labor market clearing
  - If $\max_n\{|N_n^{dem} - N_n|\} < \epsilon$, then stop algorithm
  - If $\max_n\{|N_n^{dem} - N_n|\} > \epsilon$, then update w_n^g and go back to step 2
    - Increase w_n^g if $N_n^{dem} - N_n > 0$
    - Decrease w_n^g if $N_n^{dem} - N_n < 0$

Policy Counterfactual

- **Goal**: Evaluate the welfare impact of a productivity increase of 10 percent in a given country n
- Solve the model for a baseline vector of productivity T_n^{bline}
- Solve the model for a counterfactual vector of productivity $T_n^{cfactual} = 1.1 \times T_n^{bline}$
- Compare $U_n = \frac{X_n}{P_n}$ in the baseline and in the counterfactual

The Hat-Algebra Method

What parameters do we need to simulate the model?

- The entire matrix of trade d_{ni} plus productivity shifter T_i
  - $N \times N - N$ trade cost parameters + N productivity parameters
  - If we have K sectors, we multiple the above number by K
  - Calibrating all of these parameters can be a daunting task
- Let's revisit the policy counterfactual in which we increase productivity level by 10 percent
  - It turns out that we can solve the model in changes without calibrating d_{ni} and T_i!
  - This is the so-called "Hat-Algebra"
  - As I will discuss later, there are some tradeoffs

Solving the model in differences

- Using superscripts 0 and 1 for baseline and counterfactual respectively
- Equation (1) gives

$$X_n^0 = w_n^0 N_n \text{ and } X_n^1 = w_n^1 N_n$$

  which becomes

$$\widehat{X}_n = \widehat{w}_n$$

- Equation (2) can be written as

$$\frac{w_n^1 N_n^1}{w_0 N_n^0} \times w_n^0 N_n^0 = \sum_{i=1}^{N} \frac{X_{in}^1}{X_{in}^0} \times \frac{X_{in}^0}{X_n^0} \times X_n^0$$

  which becomes

$$w_n^0 N_n^0 \widehat{w} = \sum_{i=1}^{N} \pi_{in}^0 X_n^0 \widehat{X_{in}}$$

Solving the model in differences

- Equation (3) gives

$$X_{ni}^0 = \frac{T_i^0 \left(w_i^0 d_{ni}^0\right)^{-\theta}}{\Phi_n^0} X_n^0 \text{ and } X_{ni}^1 = \frac{T_i^1 \left(w_i^1 d_{ni}^1\right)^{-\theta}}{\Phi_n^0} X_n^1$$

which becomes

$$\widehat{X}_{ni} = \widehat{T}_i \left(\widehat{w}_i \widehat{d}_{ni}\right)^{-\theta} \widehat{\Phi}_n^{-1} \widehat{X}_n$$

- Equation (4) can be written as

$$\Phi_n^1 = \sum_i \frac{T_i^1 \left(w_i^1 d_{ni}^1\right)^{-\theta}}{T_i^0 \left(w_i^0 d_{ni}^0\right)^{-\theta}} \times \frac{T_i^0 \left(w_i^0 d_{ni}^0\right)^{-\theta}}{\Phi_n^0} \times \Phi_n^0$$

which becomes

$$\widehat{\Phi}_n = \sum_i \pi_{ni}^0 \widehat{T}_i \left(\widehat{w}_i \widehat{d_{ni}}\right)^{-\theta}$$

Equilibrium in Changes

- Changes in total expenditure equals changes in total payments to factors of production

$$\widehat{X}_n = \widehat{w}_n \tag{5}$$

- Labor market clears

$$w_n^0 N_n^0 \widehat{w}_n = \sum_{i=1}^{N} \pi_{in}^0 X_n^0 \widehat{X_{in}} \tag{6}$$

- Consumers choices are optimal

$$\widehat{X}_{ni} = \widehat{T}_i \left(\widehat{w}_i \widehat{d}_{ni}\right)^{-\theta} \widehat{\Phi}_n^{-1} \widehat{X}_n \tag{7}$$

  – where

$$\widehat{\Phi}_n = \sum_i \pi_{ni}^0 \widehat{T}_i \left(\widehat{w_i d_{ni}}\right)^{-\theta} \tag{8}$$

Definition: Given baseline trade shares $\{\pi_{ni}^0\}_{ni}$ and trade elasticity θ, an equilibrium in changes is a vector of changes in wages $\{\widehat{w}_n\}_n$ that satisfies equations (5)-(8)

Algorithm

- Step 1: Guess a vector of changes in wages $\widehat{w}_n^g$
- Step 2: Construct $\widehat{X}_n$
- Step 3: Construct $\widehat{X}_{ni}$
- Step 4: Obtain the implied change in wages from demand $\widehat{w}_n^{implied} = \left(\sum_{i=1}^{N} \pi_{in}^0 X_n^0 \widehat{X_{in}}\right) / w_n^0 N_n^0$
- Step 5:
  - If $\max_n\{|\widehat{w}_n^{implied} - \widehat{w}_n^g|\} < \epsilon$, then stop algorithm
  - If $\max_n\{|\widehat{w}_n^{implied} - \widehat{w}_n^g|\} > \epsilon$, then update $\widehat{w}_n^g$ and go back to step 2
    - Increase $\widehat{w}_n^g$ if $|\widehat{w}_n^{implied} - \widehat{w}_n^g| > 0$
    - Decrease $\widehat{w}_n^g$ if $|\widehat{w}_n^{implied} - \widehat{w}_n^g| < 0$

Observations about Solving in Levels and in Changes

- Sometimes you are interested in understanding the levels of T_i
  - We will see a case in one of the applications
- Sometimes you do not have data on trade flows π_{ni}^0
  - This is common in economic geography settings, where you do not observe π_{ni}^0 between every geographic unit.
  - In that case, we can parametrize trade cost. For example using

$$d_{ni} = d^0 \left(dist_{ni}\right)^{-\nu}$$

    and calibrate or estimate d^0 and ν. Then recover T_n by inverting the model (more later on this!)
  - In other words, you will need only N pieces of data to calibrate and simulate the model if you parametrize the trade costs. With hat-algebra, you need instead $N \times N$ pieces of data. (Of course, the difference comes from the assumptions we imposed on the trade cost matrix. So less data requirement comes at a cost.)

Taking Stock

Taking Stock

- So far, we have learned how to solve the EK model **given parameters**
  - What do we need to simulate the model in levels?
    - Productivity shifters T_i
    - Trade costs d_{ni}
    - Population N_n (observable)
    - Trade elasticity θ
  - What do we need to simulate the model in differences?
    - Baseline trade shares π_{ni}^0, wages w_n^0 and population N_n^0
    - Trade elasticity θ
- Next set of slides
  - How can we recover T_i, d_{ni}, and θ