# Quantitative International Economics

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Weeks 1 and 2 - The Basic Structure of Quantitative Trade Models - Part 2
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# Outline

- EK model with 1 sector and no intermediate inputs
  - Solving the model in levels (DONE)
  - Policy counterfactuals (DONE)
  - Solving the model in changes (hat-algebra) (DONE)
  - Inverting the model
  - Estimating trade costs
  - Estimating trade elasticities

# Model Inversion

# From Previous Lecture

- So far, we have learned how to solve the EK model **given parameters**
  - What do we need to simulate the model in levels?
    - Productivity shifters $T_i$
    - Trade costs $d_{ni}$
    - Population $N_i$ (observable)
    - Trade elasticity $\theta$
  - What do we need to simulate the model in differences?
    - Baseline trade shares $\pi_{ni}^0$, wages $w_n^0$ and population $N_n^0$
    - Trade elasticity $\theta$
- This set of lectures
  - How to recover $T_i$, $d_{ni}$, and $\theta$
- I will not cover all possible methods, but will give references

# Calibrating $T_i$ with Model Inversion

- **GOAL**: Calibrate $T_n$ given $d_{ni}$ and data on $X_n$ and $N_n$
  - $d_{ni}$ could come from a first step in which we estimate trade costs
  - Set $T_n$ so that gross output in the model matches the data

- Allen and Arkolakis (2014) prove that $T_n$ is identified "up-to-scale", which means that we need a normalization of $T_n$. To see this point, notice that identification of $T_n$ comes from the following equation

$$X_n = \sum_i \frac{T_n \left(w_n d_{ni}\right)^{-\theta}}{\sum_s T_s \left(w_s d_{is}\right)^{-\theta}} X_i$$

  - If $T_n$ is too low, then county $n$'s gross output will be too low relative to the data
  - $X_n$ is homogeneous of degree zero in $T_n$ (what does that mean?)
  - *Relative* gross output identifies *relative* $T_n$

# Algorithm

- Step 1: Guess $T_n^g$ (pick a normalization)
- Step 2: Solve the model given $T_n^g$
- Step 3: Obtain model-implied gross output $X_n^{model}$ (pick a normalization)
- Step 4: Check gross output
  - If $\max_n\{|X_n^{data} - X_n^{model}|\} < \epsilon$, then stop algorithm
  - If $\max_n\{|X_n^{data} - X_n^{model}|\} > \epsilon$, then update $T_n^g$ and go back to step 2
    - Increase $T_n^g$ if $X_n^{data} - X_n^{model} > 0$
    - Decrease $T_n^g$ if $X_n^{data} - X_n^{model} < 0$
    - Remember to re-normalize $T_n^g$ before starting the algorithm again

# Calibration of $T_n$

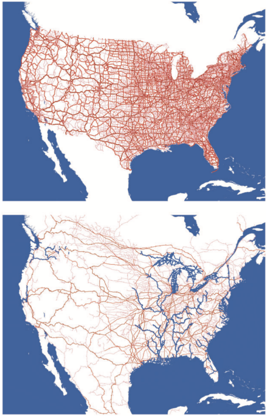
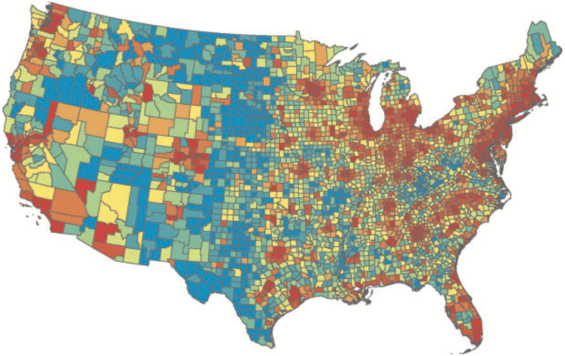


FIGURE IX
U.S. Transportation Networks

- Allen and Arkolakis (2014) $\rightarrow$ First estimate trade cost $d_{ni}$ using data on transportation infrastructure and trade flows between CFS and between MSAs

# Calibration of $T_n$



Composite productivity

- Allen and Arkolakis (2014) $\rightarrow$ Then calibrate $T_n$

# Estimation of Trade Costs

# Estimation of Trade Costs

- A (very) large literature on the estimation of trade costs
- We will discuss two approaches, each one of them is based on

  1. Price data
  2. Trade data

- The goal here is not to be exhaustive. I'll provide what I believe are the main intuitions and approaches used in the literature

# Estimation of Trade Costs based on Prices

- Fundamental challenge
  - If we look at the price gap between two regions. Does it tell us something about the trade costs?

# Estimation of Trade Costs based on Prices

- Define $d_{in}$ as the trade cost to take a good from $i$ to $n$
  - For example, if the good costs 2 at $i$ and $d_{in}$ is 1.5, then price at $n$ is $3$
- If a product is not exported from $i$ to $n$, then it must mean that producers at $i$ found it not profitable to sell it to $n$, therefore
$$p_n < p_i d_{in}$$
  Similarly, we get
$$p_i < p_n d_{ni}$$
- That puts a bound on price differentials
$$\frac{1}{d_{ni}} < \frac{p_n}{p_i} < d_{in}$$
- Price ratios are bounded above and below by trade costs. If we only look at price ratios, we will understate trade costs!

# Estimation of Trade Costs based on Prices

- Problem 1
  - We do not observe whether regions are actually trading
- Problem 2
  - Even if they are, we might not know if the product is exactly the same
- Problem 3
  - Even if products are exactly the same, price ratios could also reflect regional differences in markups

# Estimation of Trade Costs based on Prices

- Solution 1: EK (2002) approach

$$\widehat{d}_{in} = \max_{\omega \in \Omega} \frac{p_n(\omega)}{p_i(\omega)}$$

  – where $\omega$ is a good from country $n$ and country $i$

- Solution 2: Donaldson (2018) approach

  – Pick an homogeneous variety of salt $\Rightarrow$ price differentials do not reflect quality and perfect competition is reasonable assumption for salt
  – Data shows origin and destination $\Rightarrow$ location of production is unique!
  – Make the following parametric assumption on $d_{in}^{v}$ and take logs

$$p_n^v = p_i^v \underbrace{exp\left(\beta \mathsf{dist}_{in}\right) exp\left(\epsilon_{in}^v\right)}_{d_{in}^v}$$

$$\log \frac{p_n^v}{p_i^v} = \beta \mathsf{dist}_{in} + \epsilon_{in}^v$$

  – Simulate model using fitted value $\widehat{d}_{ni}$ as trade cost, for example

$$\widehat{d}_{ni} = \widehat{\beta} \mathsf{dist}_{in}$$

# Estimation of Trade Costs based on Prices

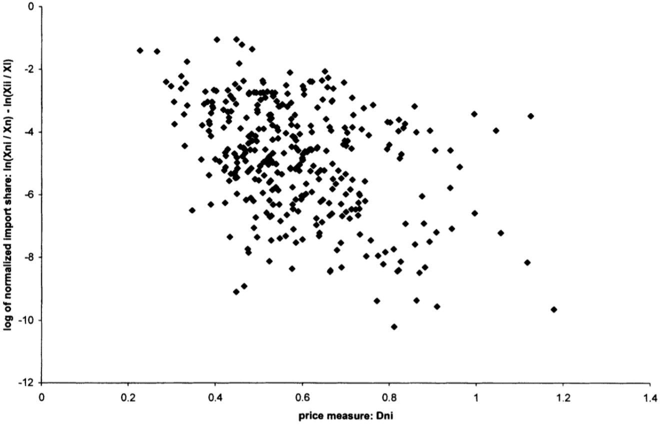



FIGURE 2.—Trade and prices.

- Eaton and Kortum (2002) $\Rightarrow$ Price gaps are negatively correlated with normalized import shares

# Estimation of Trade Costs based on Prices

- Previous approaches are based on perfectly competitive markets
- The following papers address the issue of market power when estimating trade costs
  - See Atkin and Donaldson (2015)
  - See Dominguez-Iino (2024)
- A key insight here is that pass-throughs contain information about market power that can be used as additional information to recover trade costs

# Estimation of Trade Costs based on Trade Data

- Trade data contains information about trade costs. To see why, imagine a world with two countries and the following data structure.

$$\begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix}$$

where $X_{in}$ are the sales of country $i$ to country $n$. We also define $X_1 = X_{11} + X_{12}$ and $X_2 = X_{22} + X_{21}$.

- The corresponding equations in the EK model are

$$\begin{bmatrix} \frac{T_1 (w_1 d_{11})^{-\theta}}{T_1 (w_1 d_{11})^{-\theta} + T_2 (w_2 d_{21})^{-\theta}} X_1 & \frac{T_1 (w_1 d_{12})^{-\theta}}{T_1 (w_1 d_{12})^{-\theta} + T_2 (w_2 d_{22})^{-\theta}} X_2 \\ \frac{T_2 (w_2 d_{21})^{-\theta}}{T_2 (w_2 d_{21})^{-\theta} + T_1 (w_1 d_{11})^{-\theta}} X_1 & \frac{T_2 (w_2 d_{22})^{-\theta}}{T_2 (w_2 d_{22})^{-\theta} + T_1 (w_1 d_{12})^{-\theta}} X_2 \end{bmatrix}$$

# Estimation of Trade Costs based on Trade Data

- Say we already know $\theta$, then we have 6 parameters $T_1, T_2, d_{11}, d_{12}, d_{21}, d_{22}$, to rationalize 6 observations $X_{11}, X_{12}, X_{21}, X_{22}, X_1$ and $X_2$.
  - However, we already have 2 restrictions $\Rightarrow X_1 = X_{11} + X_{12}$ and $X_2 = X_{22} + X_{21}$
  - Recall that we can only recover *relative* gross output (we have to pick a numeraire for the economy)
  - We are therefore down to 6 observations and 3 restrictions, with 6 parameters.
  - Normalize $d_{11} = d_{22} = 1$, now we have 4 parameters.
  - Normalize $T_1 = 1$, now we have 3 parameters and 3 degrees of freedom.
- We can perfectly rationalize the data if we pick $T_2$, $d_{12}$ and $d_{21}$. This is for the 2 by 2 case. Here, we are perfectly identified and these 3 parameters fully saturate the data. For $N > 2$ countries, however, we will be under-identified, so that there is more than one combination of TFPs and trade costs that can perfectly rationalize the data. How should we proceed with $N$ countries?

# Estimation of Trade Costs based on Trade Data

- Estimate
$$\log X_{ni} = \alpha_n + \gamma_i + \underbrace{\epsilon_{ni}}_{\log\left(\tilde{d}_{ni}^{-\theta}\right)}$$
recover residuals from the regression and obtain
$$\tilde{d}_{ni} = \exp\left(\widehat{\epsilon_{ni}}\right)^{-\frac{1}{\theta}}$$
- Ensure that $\widehat{d}_{ii} = 1$ by constructing
$$\widehat{d}_{ni} = \frac{\tilde{d}_{ni}}{\tilde{d}_{ii}}$$
- Use $\widehat{d}_{ni}$ as your estimates of trade costs to simulate the model

# Estimation of Trade Costs based on Trade Data

- Why don't we need price data here? Let's take the Armington trade expression

$$X_{in} = \frac{(p_i d_{in})^{1-\sigma}}{P_n^{1-\sigma}} E_n$$

  – $p_i$ is the price at origin country $i$, $P_n$ the price index, $X_{in}$ sales of country $i$ to $n$ and $E_n$ total expenditure

- Take logs

$$\log X_{in} = \underbrace{\log p_i^{1-\sigma}}_{\text{origin FE}} + \underbrace{\log \frac{E_n}{P_n^{1-\sigma}}}_{\text{destination DE}} + \underbrace{\log d_{in}^{1-\sigma}}_{\text{trade cost}}$$

- Price is fully absorbed by the origin FE
  – After taking logs, it is additive to trade costs!
  – If I increase price by 1 percent, **every country** will decrease their imports by $1-\sigma$ percent

# Estimation of Trade Costs based on Trade Data

- Head and Ries approach

$$
\begin{aligned}
\frac{X_{21}}{X_{22}} \times \frac{X_{12}}{X_{11}} &= \frac{T_2 \left(w_2 d_{21}\right)^{-\theta}}{T_2 \left(w_2 d_{22}\right)^{-\theta}} \times \frac{T_1 \left(w_1 d_{12}\right)^{-\theta}}{T_1 \left(w_1 d_{11}\right)^{-\theta}} \\
&= \left(d_{21}\right)^{-\theta} \times \left(d_{12}\right)^{-\theta}
\end{aligned}
$$

- Assume trade costs are symmetric, then

$$
\widehat{d}_{12} = \left(\frac{X_{21}}{X_{22}} \times \frac{X_{12}}{X_{11}}\right)^{-\frac{1}{2\theta}}
$$

- Useful statistic for counterfactuals
  - Farrokhi and Pellegrina (2023) recover the symmetric component between 1980 and 2010
  - Jacks, Meissner and Novy (2008) study trade costs changes between 1870 and 2000

# Estimation of Trade Costs based on Trade Data

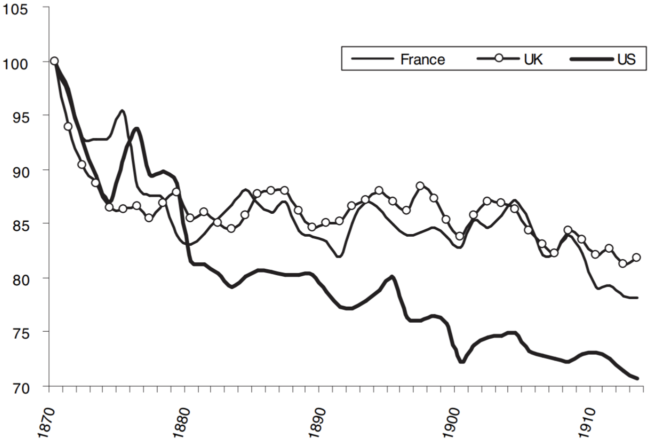



FIGURE 1A. TRADE COST INDICES, 1870–1913 (1870 = 100)

- Jacks, Meissner and Novy (2008)

# Estimation of Trade Costs based on Trade Data


FIGURE 1B. TRADE COST INDICES, 1921–1939 (1921 = 100)

- Jacks, Meissner and Novy (2008)

# Estimation of Trade Costs based on Trade Data

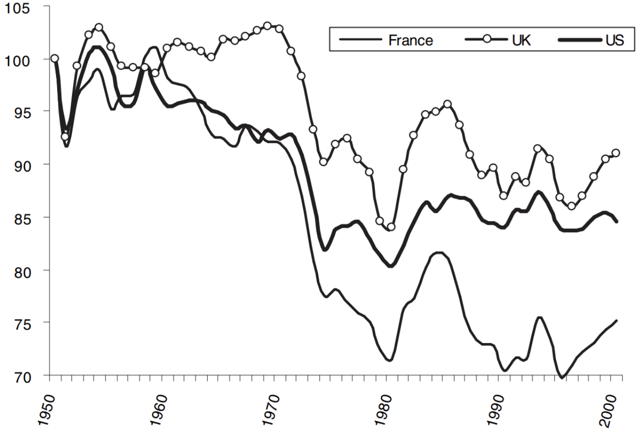



FIGURE 1C. TRADE COST INDICES, 1950–2000 (1950 = 100).

- Jacks, Meissner and Novy (2008)

# Estimation of Trade Elasticity

# Estimation of Trade Elasticity

- Approach 1: Once with estimates of trade costs, we can estimate

$$\log X_{ni} = \alpha_n + \gamma_i - \theta \log \widehat{d_{ni}} + \epsilon_{ni}$$

via OLS and use $\widehat{\theta}^{OLS}$ as the trade elasticity.

  – Donaldson (2018), Pellegrina (2022), EK (2002)

- Approach 2: Double ratio estimator. Define

$$d_{in} = \tau_{in}\kappa_{in}^{sym}\mu_n\gamma_i\epsilon_{in}$$

  – where $\kappa_{in}^{sym}$ is a symmetric component of trade cost, $\mu_n$ is a destination specific cost, $\gamma_i$ is an origin specific cost, and $\tau_{in}$ is 1 plus the tariff
  – Take the triple ratio

$$\log \frac{X_{ni}}{X_{in}}\frac{X_{ih}}{X_{hi}}\frac{X_{hn}}{X_{nh}} = -\theta \log \frac{\tau_{ni}}{\tau_{in}}\frac{\tau_{ih}}{\tau_{hi}}\frac{\tau_{hn}}{\tau_{nh}} + \tilde{e}$$

  – It removes $\kappa_{in}^{sym}$, $\mu_n$ and $\gamma_i$. Here, the tariffs $\tau_{ni}$ are observable, so we can simply estimate the equation above by OLS

# Estimation of Trade Elasticity

TABLE 1
*Dispersion-of-productivity estimates*

| | Full sample | | | 99% sample | | | 97.5% sample | | |
|---|---|---|---|---|---|---|---|---|---|
| Sector | $\theta^j$ | s.e. | N | $\theta^j$ | s.e. | N | $\theta^j$ | s.e. | N |
| Agriculture | 8.11 | (1.86) | 496 | 9.11 | (2.01) | 430 | 16.88 | (2.36) | 364 |
| Mining | 15.72 | (2.76) | 296 | 13.53 | (3.67) | 178 | 17.39 | (4.06) | 152 |
| Manufacturing | | | | | | | | | |
| Food | 2.55 | (0.61) | 495 | 2.62 | (0.61) | 429 | 2.46 | (0.70) | 352 |
| Textile | 5.56 | (1.14) | 437 | 8.10 | (1.28) | 314 | 1.74 | (1.73) | 186 |
| Wood | 10.83 | (2.53) | 315 | 11.50 | (2.87) | 191 | 11.22 | (3.11) | 148 |
| Paper | 9.07 | (1.69) | 507 | 16.52 | (2.65) | 352 | 2.57 | (2.88) | 220 |
| Petroleum | 51.08 | (18.05) | 91 | 64.85 | (15.61) | 86 | 61.25 | (15.90) | 80 |
| Chemicals | 4.75 | (1.77) | 430 | 3.13 | (1.78) | 341 | 2.94 | (2.34) | 220 |
| Plastic | 1.66 | (1.41) | 376 | 1.67 | (2.23) | 272 | 0.60 | (2.11) | 180 |
| Minerals | 2.76 | (1.44) | 342 | 2.41 | (1.60) | 263 | 2.99 | (1.88) | 186 |
| Basic metals | 7.99 | (2.53) | 388 | 3.28 | (2.51) | 288 | −0.05 | (2.82) | 235 |
| Metal products | 4.30 | (2.15) | 404 | 6.99 | (2.12) | 314 | 0.52 | (3.02) | 186 |
| Machinery n.e.c. | 1.52 | (1.81) | 397 | 1.45 | (2.80) | 290 | −2.82 | (4.33) | 186 |
| Office | 12.79 | (2.14) | 306 | 12.95 | (4.53) | 126 | 11.47 | (5.14) | 62 |
| Electrical | 10.60 | (1.38) | 343 | 12.91 | (1.64) | 269 | 3.37 | (2.63) | 177 |
| Communication | 7.07 | (1.72) | 312 | 3.95 | (1.77) | 143 | 4.82 | (1.83) | 93 |
| Medical | 8.98 | (1.25) | 383 | 8.71 | (1.56) | 237 | 1.97 | (1.36) | 94 |
| Auto | 1.01 | (0.80) | 237 | 1.84 | (0.92) | 126 | −3.06 | (0.86) | 59 |
| Other Transport | 0.37 | (1.08) | 245 | 0.39 | (1.08) | 226 | 0.53 | (1.15) | 167 |
| Other | 5.00 | (0.92) | 412 | 3.98 | (1.08) | 227 | 3.06 | (0.83) | 135 |
| Test equal parameters | | | F(17, 7294) = 7.52 | | | | Prob > F = 0.00 | | |
| Aggregate elasticity | 4.55 | (0.35) | 7212 | 4.49 | (0.39) | 5102 | 3.29 | (0.47) | 3482 |

- Caliendo and Parro (2015)

# Estimation of Trade Elasticity

- Donaldson (2018)
- Estimate trade cost

$$\ln p^o_{dt} = \beta^o_{ot} + \underbrace{\beta^o_{od} + \delta \ln LCRED\left(R_t, \alpha\right)_{odt} + \epsilon^o_{odt}}_{T^o_{odt}}$$

  - $p^o_{dt}$ is price in destination $d$ of type $o$ in period $t$
  - $LCRED\left(R_t, \alpha\right)_{odt}$ is the lowest cost route given railroads in period $t$

- Estimate trade elasticity using

$$\ln X^k_{odt} = \beta^k_{ot} + \beta^k_{dt} + \beta^k_{od} - \theta_k \hat{\delta} \ln LCRED\left(R_t, \alpha\right)_{odt} + \epsilon^k_{odt}$$

# Estimation of Trade Elasticity

TABLE 2—RAILROADS AND TRADE COSTS: STEP 1

| Dependent variable: log salt price at destination | (1) | (2) |
|---|---|---|
| log effective distance to source, along lowest-cost route | 0.088 | |
| (at historical freight rates) | (0.028) | |
| log effective distance to source, along lowest-cost route | | 0.169 |
| (at estimated mode costs) | | [0.062, 0.296] |
| Estimated mode costs per unit distance: | | 1 |
| Railroad (normalized to 1) | | N/A |
| Road | | 2.375 |
| | | [1.750, 10.000] |
| River | | 2.250 |
| | | [1.500, 6.250] |
| Coast | | 6.188 |
| | | [5.875, 10.000] |
| Observations | 7,345 | 7,345 |
| $R^2$ | 0.946 | 0.946 |

- Donaldson (2018)

# Estimation of Trade Elasticity

TABLE 3—RAILROADS AND TRADE FLOWS: STEP 2

| Dependent variable: log value of exports | (1) | (2) |
|---|---|---|
| log effective distance beween origin and destination along lowest-cost route | −1.603 | −1.701 |
| | (0.533) | (1.141) |
| (log effective distance beween origin and destination along lowest-cost route) × (weight per unit value of commodity in 1890) | | −0.946 |
| | | (3.634) |
| (log effective distance beween origin and destination along lowest-cost route) × (high-value railroad freight class of commodity in 1859) | | 1.286 |
| | | (1.243) |
| Observations | 142,541 | 142,541 |
| $R^2$ | 0.901 | 0.901 |

- Donaldson (2018)

# Estimation of Trade Elasticity

- What if trade costs or tariffs are measured with error?
  - If we understate their variance, we would overstate the trade elasticity
- Literature finds a large range of values
  - Eaton and Kortum (2002) finds 4.17 to 9.6 (single sector)
  - Simonovska and Waugh (2014) $\rightarrow \approx$ 4.17 (single sector)
  - Caliendo and Parro (2015) $\rightarrow$0.37 to 51.08 (multiple sector)
- Many papers pick values from the literature

# Conclusion

# Recent Developments on these Tools

- Eaton and Kortum (2002) microfoundations
  - EK (2002) formulation can be microfounded with a model in which firms draw ideas and there is a probability that the ideas are better than previous ones – see the excellent JEP from EK (2012). A few papers have explored these microfoundations to study innovation and diffusion.
  - Lind and Ramondo (2022), Buera and Ozberfield (2019)
- Non-parametric approach
  - Large scale CGE models contain thousands of parameters and regions to simulate counterfatuals. EK has 1, but it comes at a cost. Can we simulate counterfactuals with fewer parametric assumptions?
  - Adao, Costinot and Donaldson (2016) $\Rightarrow$ It turns out that the hat-algebra approach does not depend on parametric assumptions such as CES
- Can we test trade models?
  - Because we saturate the data, it is challenging to test the model.
  - Adao, Costinot and Donaldson (2024) $\Rightarrow$ Develop new tests for trade models.

# Next class

- Welfare gains from trade
- Multi-sectoral models and input-output structures
- Sectoral comparative advantage