A novel Bayesian linear regression model for analysing event-related potentials

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1 Introduction

This technical report along with provided MATLAB code are intended as a companion report for the article *Single-subject analysis of N400 event-related potential component with four different methods* by Kallionpää et al.

2 Model

A Bayesian functional regression model for event related potential (ERP) analysis is presented. For subject s the voltage-amplitude of the jth trial at a single channel i at time t is formulated as a linear model

$$Y_j^{(s,i)}(t) = X(t)\theta^{(s,i)} + \epsilon_j^{(s,i)}(t), \quad j = 1, \ldots, N_{\text{trial}},\ i = 1, \ldots, N_{\text{channel}} \tag{1}$$

$$X(t)\theta^{(s,i)} = \sum_{k=0}^{M} \phi_k(t)\theta_k^{(s,i)}, \tag{2}$$

where $X(t)$ is defined by a set of basis functions and $\epsilon_j^{(s,i)}(t)$ is the background EEG noise. The coefficient vector to be solved $\theta^{(s,i)}$ defines the EEG signal curve.

There are a number of reasonable basis sets $\phi_k(t)$ that can be used to model the ERP-signal, but in the current analyses we employ Fourier basis

$$
\begin{aligned}
\phi_0(t) &= 1 \\
\phi_1(t) &= \sin(\omega t) \\
\phi_2(t) &= \cos(\omega t) \\
\phi_3(t) &= \sin(2\omega t) \\
\phi_4(t) &= \cos(2\omega t) \\
&\vdots \\
\phi_{M-1}(t) &= \sin\left(\frac{M\omega t}{2}\right) \\
\phi_M(t) &= \cos\left(\frac{M\omega t}{2}\right),
\end{aligned}
$$

where M is an even natural number $M \geq 2$ and ω defines the periodicity of the signal.

The signal is measured within a time window at sample points $\{t_1, \ldots, t_N\}$ and there the model can be written in matrix form as

$$
Y_j^{(s,i)} = X\theta^{(s,i)} + \epsilon_j^{(s,i)}, \quad j = 1, \ldots, N_{\text{trial}}, \tag{3}
$$

where

$$
\begin{aligned}
Y_j^{(s,i)} &= \begin{bmatrix} Y_j^{(s,i)}(t_1) \\ \vdots \\ Y_j^{(s,i)}(t_N) \end{bmatrix}, \\
X &= \begin{bmatrix} 1 & \sin(\omega t_1) & \cos(\omega t_1) & \cdots & \sin\left(\frac{M\omega t_1}{2}\right) & \cos\left(\frac{M\omega t_1}{2}\right) \\ 1 & \sin(\omega t_2) & \cos(\omega t_2) & \cdots & \sin\left(\frac{M\omega t_2}{2}\right) & \cos\left(\frac{M\omega t_2}{2}\right) \\ \vdots & & & & & \\ 1 & \sin(\omega t_N) & \cos(\omega t_N) & \cdots & \sin\left(\frac{M\omega t_N}{2}\right) & \cos\left(\frac{M\omega t_N}{2}\right) \end{bmatrix}, \\
\epsilon_j^{(s,i)} &= \begin{bmatrix} \epsilon_j^{(s,i)}(t_1) \\ \vdots \\ \epsilon_j^{(s,i)}(t_N) \end{bmatrix}.
\end{aligned}
$$

The background EEG-signal $\epsilon_j^{(s,i)}(t)$ is modelled as a stationary discrete-time Gaussian process which is within the time window

$$
\epsilon_j^{(s,i)} \sim \mathsf{N}\left(0, R^{(s,i)}\right)
$$

where the covariance matrix

$$R^{(s,i)} = \mathsf{E}\left(\epsilon_j^{(s,i)} \epsilon_j^{(s,i)^T}\right)$$

is estimated using the autocorrelation matrix of the background EEG recorded pre- and post-stimulus. The mean operator is denoted as $\mathsf{E}(\cdot)$

Furthermore, the measurement from all trials can be stacked into one linear model

$$\begin{aligned} \begin{bmatrix} Y_1^{(s,i)} \\ \vdots \\ Y_{N_{\text{trial}}}^{(s,i)} \end{bmatrix} &= \underbrace{\mathbf{1}_{N_{\text{trial}}} \otimes X}_{=\mathcal{X}} \theta^{(s,i)} + \begin{bmatrix} \epsilon_1^{(s,i)} \\ \vdots \\ \epsilon_{N_{\text{trial}}}^{(s,i)} \end{bmatrix} \\ \Rightarrow \quad Y^{(s,i)} &= \mathcal{X}\theta^{(s,i)} + \epsilon^{(s,i)}, \end{aligned} \tag{4}$$

where $\mathbf{1}_{N_{\text{trial}}}$ is a vector of ones with length of N_{trial}, and $\otimes$ is the Kronecker product. If the background EEG is assumed independent from trial to another, then

$$\epsilon^{(s,i)} \sim \mathsf{N}\left(0, \mathcal{R}^{(s,i)}\right)$$

where

$$\mathcal{R}^{(s,i)} = I_{N_{\text{trial}}} \otimes R^{(s,i)}$$

and $I_{N_{\text{trial}}}$ is an identity matrix of size $N_{\text{trial}} \times N_{\text{trial}}$.

2.1 N400 effect model

As explained in Kallionpää et al, a possible way to investigate investigate N400 effect in the voltage amplitude is to expose subjects to two types of stimuli. The auditory presented stimuli consists of sentences that end to congruent or incongruent word, and the voltage amplitude after incongruent last word is expected to be more negative than after congruent word, peaking around 400 ms after the stimulus. The model for voltage amplitude response (1) is extended to take into account the different response given different test type, i.e. different stimulus after the time point t_0. The voltage amplitude under the last words of incongruent sentences is modelled as

$$Y_j^{(s,i)}(t) = X(t)\theta^{(s,i)} + \mathcal{H}_{t_0}(t)X(t)\xi^{(s,i)} + \epsilon_j^{(s,i)}(t), \tag{5}$$

where

$$\mathcal{H}_{t_0}(t) = \begin{cases} 0, & t < t_0 \\ 1, & t \geq t_0, \end{cases}$$

and $\xi^{(i)}$ is the additional component in the signal under incongruent sentences. By introducing an indicator variable $k \in \{0, 1\}$, the equations (1) and (5) are stacked as

$$Y_{j,k}^{(s,i)}(t) = \begin{bmatrix} X(t) & k\mathcal{H}_{t_0}(t)X(t) \end{bmatrix} \underbrace{\begin{bmatrix} \theta^{(s,i)} \\ \xi^{(s,i)} \end{bmatrix}}_{\alpha^{(s,i)}} + \epsilon_{j,k}^{(s,i)}(t), \tag{6}$$

where $k = 0$ under congruent stimuli and $k = 1$ under incongruent stimuli. Let

$$\mathcal{H}_{t_0} = \begin{bmatrix} \mathcal{H}_{t_0}(t_1) & 0 & \dots & 0 \\ 0 & \mathcal{H}_{t_0}(t_2) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathcal{H}_{t_0}(t_N) \end{bmatrix}.$$

Then the discrete time series process can be expressed as a vector observation model

$$Y_{j,k}^{(s,i)} = \underbrace{\begin{bmatrix} X & k\mathcal{H}_{t_0}X \end{bmatrix}}_{=X_k} \alpha^{(i,j)} + \epsilon_{j,k}^{(s,i)}. \tag{7}$$

The model can be expressed in matrix form analogously to (8)

$$\begin{bmatrix} Y_{1,0}^{(s,i)} \\ \vdots \\ Y_{N_{\text{trial}},0}^{(s,i)} \\ Y_{1,1}^{(s,i)} \\ \vdots \\ Y_{N_{\text{trial}},1}^{(s,i)} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{1}_{N_{\text{trial}},0} \otimes X_0 \\ \mathbf{1}_{N_{\text{trial}},1} \otimes X_1 \end{bmatrix}}_{=\mathcal{X}} \alpha^{(s,i)} + \begin{bmatrix} \epsilon_{1,0}^{(s,i)} \\ \vdots \\ \epsilon_{N_{\text{trial}},0}^{(s,i)} \\ \epsilon_{1,1}^{(s,i)} \\ \vdots \\ \epsilon_{N_{\text{trial}},1}^{(s,i)} \end{bmatrix}$$

$$\Rightarrow \quad Y^{(s,i)} = \mathcal{X}\alpha^{(s,i)} + \epsilon^{(s,i)}, \tag{8}$$

3 Estimation

Bayesian approach to estimation is used to evaluate the N400 effect. In this case, the posterior distribution evaluated from the prior distribution and likelihood function contains all the information of the effect obtained from the models and the data. To estimate subject-level and group-level effect from the repeated channel-level data, a simplified hierarchical structure is used. Given Gaussian likelihood and prior, the Gaussian posterior distribution can be evaluated in closed form.

3.1 Channel-level estimation

Bayesian linear regression is used to estimate the posterior distribution of the parameter vector α at each level of modelling. Let the prior of the parameter α be a Gaussian distribution

$$p(\alpha^{(s,i)}) = \mathsf{N}\left(\bar{\alpha}_0^{(s,i)}, \Sigma_0^{(s,i)}\right),$$

where $\bar{\alpha}_0^{(s,i)}$ and $\Sigma_0^{(s,i)}$ are the prior mean and covariance, respectively.

Given the modelling assumptions, the posterior distribution of the channel-level data is evaluated as

$$p(\alpha^{(s,i)} \mid Y^{(s,i)}) \propto p(Y^{(s,i)} \mid \alpha^{(s,i)}) p(\alpha^{(s,i)}) \propto \mathsf{N}\left(\bar{\alpha}^{(s,i)}, \Sigma^{(s,i)}\right), \tag{9}$$

$$\bar{\alpha}^{(s,i)} = \left[\Sigma_0^{(s,i)^{-1}} + \mathcal{X}^T \mathcal{R}^{(s,i)^{-1}} \mathcal{X}\right]^{-1} \left[\mathcal{X}^T \mathcal{R}^{(s,i)^{-1}} Y^{(s,i)} + \Sigma_0^{(s,i)^{-1}} \bar{\alpha}_0^{(s,i)}\right] \tag{10}$$

$$\Sigma^{(s,i)} = \left[\Sigma_0^{(s,i)^{-1}} + \mathcal{X}^T \mathcal{R}^{(s,i)^{-1}} \mathcal{X}\right]^{-1}. \tag{11}$$

3.2 Subject-level estimation

On the subject-level, the channel-level estimates are used as independent observations of the subject-level average response. The measurement model is

$$\underbrace{\begin{bmatrix} \bar{\alpha}^{(s,1)} \\ \vdots \\ \bar{\alpha}^{(s,N_{\text{channel}})} \end{bmatrix}}_{=Y^{(s)}} = \underbrace{\left(\mathbf{1}_{N_{\text{Channel}}} \otimes I_N\right)}_{=\mathcal{H}^{(s)}} \alpha^{(s)} + \underbrace{\begin{bmatrix} e^{(s,1)} \\ \vdots \\ e^{(s,N_{\text{channel}})} \end{bmatrix}}_{=e^{(s)}} \tag{12}$$

The error part is modelled as

$$p(e^{(s)}) = \mathsf{N}\left(0, \mathcal{R}^{(s)}\right), \tag{13}$$

where

$$\mathcal{R}^{(s)} = \begin{bmatrix} \Sigma^{(s,1)} & 0 & \cdots & 0 \\ 0 & \Sigma^{(s,2)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Sigma^{(s,N_{\text{channel}})} \end{bmatrix} \tag{14}$$

The average response prior distribution is modelled as

$$p(\alpha^{(s)}) = \mathsf{N}\left(\bar{\alpha}_0^{(s)}, \Sigma_0^{(s)}\right). \tag{15}$$

Under these assumptions, the posterior distribution is

$$p(\alpha^{(s)} \mid Y^{(s)}) = \mathsf{N}\left(\bar{\alpha}^{(s)}, \Sigma^{(s)}\right) \tag{16}$$

$$\bar{\alpha}^{(s)} = \left[\Sigma_0^{(s)^{-1}} + \mathcal{H}^{(s)^T}\mathcal{R}^{(s)^{-1}}\mathcal{H}^{(s)}\right]^{-1}\left[\mathcal{H}^{(s)^T}\mathcal{R}^{(s)^{-1}}Y^{(s)} + \Sigma_0^{(s)^{-1}}\bar{\alpha}_0^{(s)}\right] \tag{17}$$

$$\Sigma^{(s)} = \left[\Sigma_0^{(s)^{-1}} + \mathcal{H}^{(s)^T}\mathcal{R}^{(s)^{-1}}\mathcal{H}^{(s)}\right]^{-1} \tag{18}$$

3.3 Group-level estimation

On the group-level, the subject-level estimates are used as conditionally independent observations of the group-level average response. The observation model is now

$$\underbrace{\begin{bmatrix} \bar{\alpha}^{(1)} \\ \vdots \\ \bar{\alpha}^{(N_{\text{subject}})} \end{bmatrix}}_{=Y} = \underbrace{\left(\mathbf{1}_{N_{\text{subject}}} \otimes I_N\right)}_{=\mathcal{H}} \alpha + \underbrace{\begin{bmatrix} e^{(1)} \\ \vdots \\ e^{(s)} \end{bmatrix}}_{=e} \tag{19}$$

The error part is modelled as

$$p(e) = \mathsf{N}\left(0, \mathcal{R}\right), \tag{20}$$

where

$$\mathcal{R} = \begin{bmatrix} \Sigma^{(1)} & 0 & \cdots & 0 \\ 0 & \Sigma^{(s)} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Sigma^{(N_{\text{subject}})} \end{bmatrix} \tag{21}$$

The average response prior distribution is modelled as

$$p(\alpha) = \mathsf{N}\left(\bar{\alpha}_0, \Sigma_0\right). \tag{22}$$

Under these assumptions, the posterior distribution is

$$p(\alpha \mid Y) = \mathsf{N}\left(\bar{\alpha}, \Sigma\right) \tag{23}$$

$$\bar{\alpha} = \left[{\Sigma_0}^{-1} + \mathcal{H}^T\mathcal{R}^{-1}\mathcal{H}\right]^{-1}\left[\mathcal{H}^T\mathcal{R}^{-1}Y + {\Sigma_0}^{-1}\bar{\alpha}_0\right] \tag{24}$$

$$\Sigma = \left[{\Sigma_0}^{-1} + \mathcal{H}^T\mathcal{R}^{-1}\mathcal{H}\right]^{-1} \tag{25}$$

3.4 The detection of N400 effect

The detection of N400 is determined by investigating the average fitted signal within time window $300 - 600$ms after the stimulus. If the voltage amplitude under incongruent stimuli is on average more negative than that of congruent stimuli within the time window, then N400 effect is detected. The effect size of the ERP effect within this time window can be defined as

$$\hat{\xi}^{(s,i)} = \frac{1}{K}\mathbf{1}_N^T(\mathcal{H}_{300} - \mathcal{H}_{600})X\xi^{(s,i)}, \tag{26}$$

where K is the number of samples within the time window under investigation. Similarly for subject-level and group-level it is defined that

$$\hat{\xi}^{(s)} = \frac{1}{K}\mathbf{1}_N^T(\mathcal{H}_{300} - \mathcal{H}_{600})X\xi^{(s)}, \tag{27}$$

$$\hat{\xi} = \frac{1}{K}\mathbf{1}_N^T(\mathcal{H}_{300} - \mathcal{H}_{600})X\xi, \tag{28}$$

where $\hat{\xi}^{(s)}$ and $\hat{\xi}$ are the N400 effect sizes of subject-level and group-level, respectively, and are evaluated using the subvectors $\xi^{(s)}$ and ξ of $\alpha^{(s)}$ and α. Given the Gaussian assumptions, the posterior distributions of random variables $\hat{\xi}^{(s,i)}, \hat{\xi}^{(s)}$ and $\hat{\xi}$ also have Gaussian distributions, and the probabilities that the effects are negative, i.e. conforming the assumptions of N400 effect, can be computed as

$$\mathsf{P}(\hat{\xi}^{(s,i)} < 0 \mid Y^{(s,i)}) = \int_{-\infty}^{0} p(\hat{\xi}^{(s,i)} \mid Y^{(s,i)})\mathsf{d}\hat{\xi}^{(s,i)} \tag{29}$$

$$\mathsf{P}(\hat{\xi}^{(s)} < 0 \mid Y^{(s)}) = \int_{-\infty}^{0} p(\hat{\xi}^{(s)} \mid Y^{(s)})\mathsf{d}\hat{\xi}^{(s)} \tag{30}$$

$$\mathsf{P}(\hat{\xi} < 0 \mid Y) = \int_{-\infty}^{0} p(\hat{\xi} \mid Y)\mathsf{d}\hat{\xi} \tag{31}$$

4 Example

To illustrate the performance of the presented estimation method, there is a example dataset attached with the report. In the files `cong.mat` and `incong.mat`, there are recorded and pre-processed EEG-data from 10 trials for 5 randomly selected subjects and for 10 randomly selected channels. The dataset includes as variables the 800ms long stimulus-locked trial sets (`Y0` and `Y1`) recorded with 250Hz and the similarly preprocessed epochs of stimulus-free background EEG recorded before and after experimental epochs (`epsilon_pre` and `epsilon_post`). The dimensions of the variables are channels × sample points × epochs × participants

$(10 \times 201 \times 10 \times 10)$. The file `bayesfuncreg` processes the data sets and plots the results as a mean of the posteriors at channel-, subject- and group-level. The posterior means of the subject-level and and the group-level ERP effects are plotted in Figure 1.

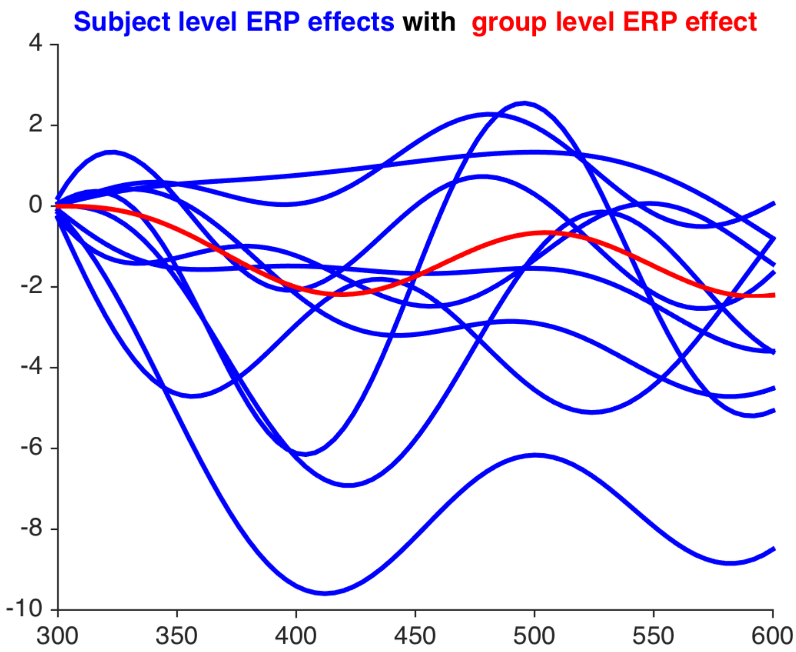

Figure 1: Subject-level posterior mean estimates of ERP effect within time window 300–600ms with group-level posterior estimate.