SMC-ABC

- Round 1 : SMC-ABC is initialised as rejection ABC with a loose threshold ϵ_1
    - $\theta_i^{(1)} \sim p(\theta \mid d(S(x), S(x^o)) \leq \epsilon_1), \quad i = 1, \ldots, N$
    - Set weight $w_i^{(1)} = N^{-1}$
    - Calculate sample variances (for each dimension $j = 1, \ldots, M$)

$$\hat{\mu} = \sum_{i=1}^{N} \frac{\theta_i}{N}, \quad \hat{\sigma}_j^2 = \sum_{i=1}^{N} \frac{1}{N}(\theta_{i_j} - \hat{\mu}_{i_j})^2$$

    - Set proposal density $q(\theta^{(2)} \mid \theta^{(1)}) = \text{Normal}(\theta^{(1)},\ 2 \cdot \text{diag}(\hat{\sigma}_1^2, \ldots, \hat{\sigma}_M^2))$

SMC-ABC

- Rounds $t = 2, \ldots, T$: Set a tighter threshold $\epsilon_t < \epsilon_{t-1}$
  - (1) Select $\theta_i^{(t-1)}$ with probability $\propto w_i^{(t-1)}$
  - (2) Draw $\theta^* \sim$ Normal$(\theta_i^{(t-1)}$, diag$(\hat{\sigma}_1^2, \ldots, \hat{\sigma}_M^2))$
  - (3) Simulate $x^* \sim p(x \mid \theta^*)$
  - Repeat (1)-(3) until $d(S(x^*), S(x^o)) < \epsilon_t$ and set $\theta_i^{(t)} = \theta^*$
    - Set weights $w_i^{(t)} \propto p(\theta_i^{(t)}) \cdot \left[\sum_{k=1}^{N} w_k^{(t-1)} \sum_{j=1}^{M} \phi \left(\frac{\theta_{i_j}^{(t)} - \theta_{k_j}^{(t)}}{\hat{\sigma}_j^{(t-1)}} \right) \right]^{-1}$
    - Calculate weighted sample variances $\hat{\sigma}_j^2$ (for each dimension $j = 1, \ldots, M$)
    - Set proposal density $q(\theta^{(t+1)} \mid \theta^{(t)}) =$ Normal$(\theta^{(t)}, 2 \cdot$ diag$(\hat{\sigma}_1^2, \ldots, \hat{\sigma}_M^2))$