Rejection ABC uses samples from the prior

- Unless we have plenty of prior information about the parameters, sampling from prior is hardly effective
- Sequentially formulate importance sampling distributions with more probability mass in interesting regions
  - Sequential Monte Carlo ABC

SMC-ABC

- Round 1 : SMC-ABC is initialised as rejection ABC with a loose threshold ϵ_1
  - $\theta_i^{(1)} \sim p(\theta \mid d(S(x), S(x^o)) \leq \epsilon_1), \quad i = 1, \ldots, N$
  - Set weight $w_i^{(1)} = N^{-1}$
  - Calculate sample variances (for each dimension $j = 1, \ldots, M$)

$$\hat{\mu} = \sum_{i=1}^{N} \frac{\theta_i}{N}, \quad \hat{\sigma}_j^2 = \sum_{i=1}^{N} \frac{1}{N} (\theta_{i_j} - \hat{\mu}_{i_j})^2$$

  - Set proposal density $q(\theta^{(2)} \mid \theta^{(1)}) = \text{Normal}(\theta^{(1)}, 2 \cdot \text{diag}(\hat{\sigma}_1^2, \ldots, \hat{\sigma}_M^2))$