CHAP05 - Solutions for Ragsdale 2015

Decision Models in Business Analytics (HEC Montréal)

Chapter 5
Network Modeling

1. If supplies are represented by positive numbers and demands are represented numbers, the balance-of-flow rule would be stated as follows:

For Minimum Cost Network Flow Problems Where:	Apply This Balance-of-Flow Rule At Each Node:
Total Supply > Total Demand	Outflow - Inflow ≤ Supply or Demand
Total Supply < Total Demand	Outflow - Inflow ≥ Supply or Demand
Total Supply = Total Demand	Outflow - Inflow = Supply or Demand

2. Multiply both sides of the constraint by -1 and reverse the sign of the inequality.

3. See file: Prb5_3.xlsm

a. Total cost = $3,398

	Node	Net Flow	Supply/Demand
1	Newspaper	-80.0	-80
2	Mixed Paper	-50.0	-50
3	White Office	-30.0	-30
4	Cardboard	-40.0	-40
5	Process 1	0.0	0
6	Process 2	0.0	0
7	Newsprint	60.0	60
8	Packaging	40.0	40
9	Print Stock	50.0	50

b. Total cost = $3,129

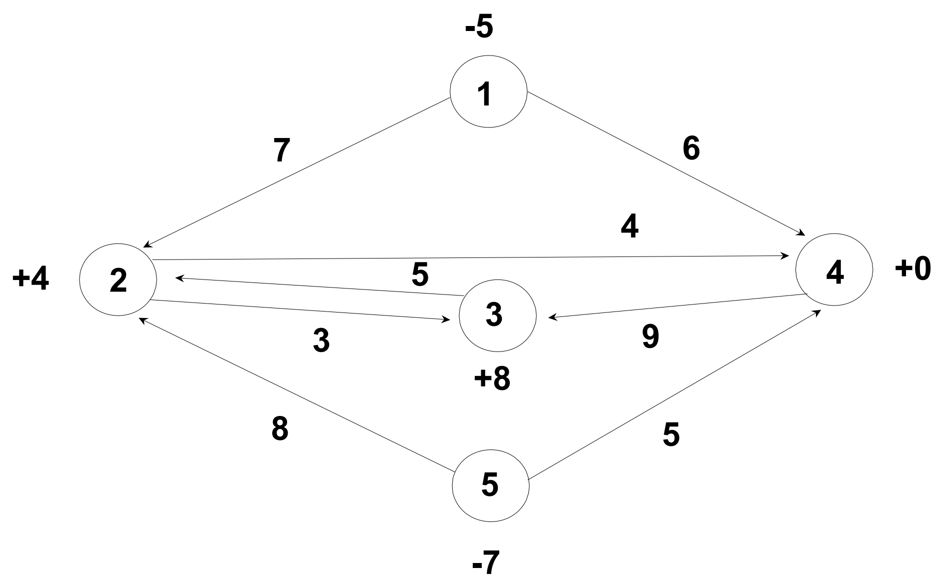
	Node	Net Flow	Supply/Demand
1	Newspaper	-80.0	-80
2	Mixed Paper	-50.0	-50
3	White Office	-30.0	-30
4	Cardboard	-25.4	-40
5	Process 1	0.0	0
6	Process 2	0.0	0
7	Newsprint	60.0	60
8	Packaging	40.0	40
9	Print Stock	50.0	50

c. In part a, if we assume supply is inadequate the demand (when, in fact, it is adequate) we require all the supply to be used – even if it is not all needed. This results in higher than necessary costs. In part b, assuming supply is adequate to meet demand (when, in fact, it is) resulted in a smaller total cost as this solution does not require all the supply to be used.

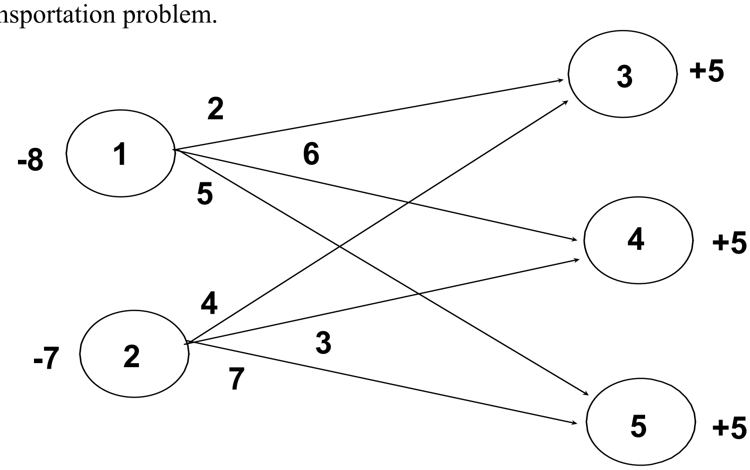
d. The shortage on print stock pulp can be reduced to 4 units (without sacrificing newspaper or packing paper pulp) at an additional cost of $307.

4. Because there is a 10% loss of flow on all arcs going to node 4, a total of 702/0.9 = 780 units must flow into node 4. Thus, we can simply increase the demand at node 4 to 780 and assume no loss of flow occurs on arcs leading into this node. Similarly, only 608/1.05 = 579.05 units must flow into node 5. Thus, we can simply decrease the demand at node 5 to 579.05 and assume no gain of flow occurs on the arcs leading into this node.

5.



6. This is a transportation problem.



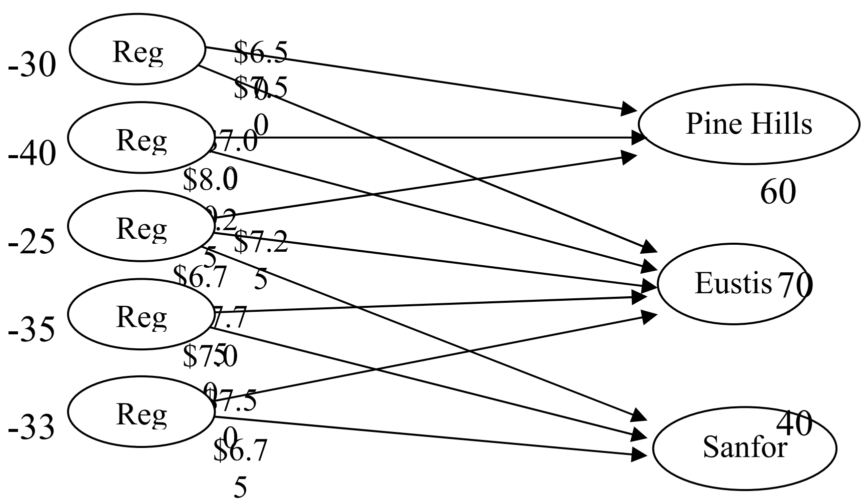
7. The cost on each arc increases by \$2,000. The optimal plan under both leasing options is to replace the equipment at the beginning of years 3 and 5. However, leasing option 2 provides the lowest total cost (\$122,965 + \$62,000) and is therefore the preferred alternative. See file: Prb5_7.xlsm

8. a. One solution is:

	1	2	3	4	5	6	7
1		1					
2	1	1	1		1	1	1
3	1		1		1	1	1
4	1	1	1			1	1
5	1	1	1		1		1
6							
7	1						
8				S			1

b. 310 feet.

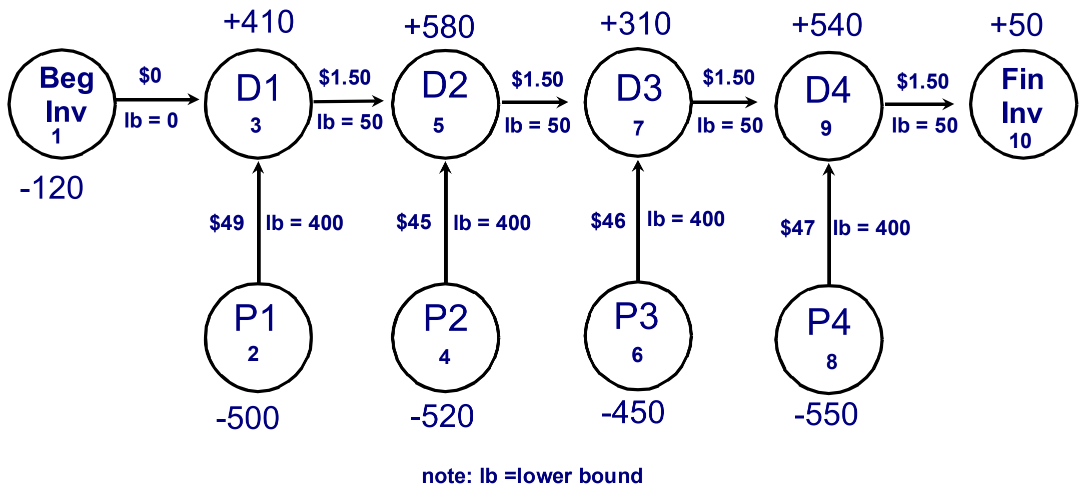
9. a.



b. See file Prb5_9.xlsm

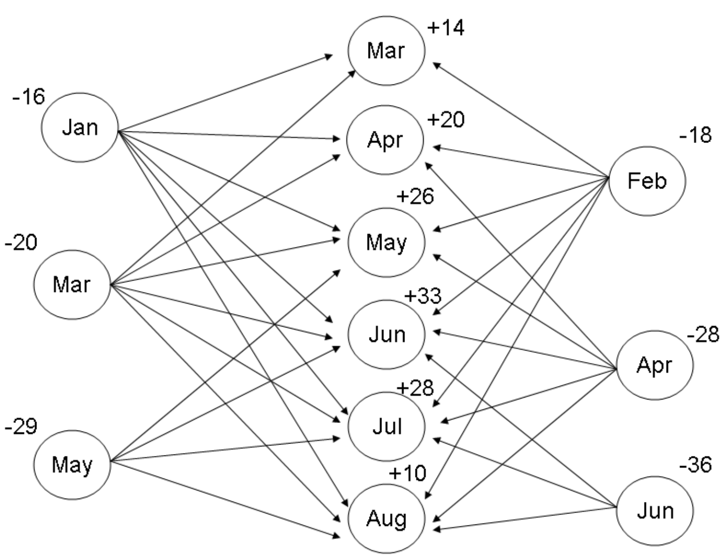
c. 20,000 from Region 1 to Pine Hills, 10,000 from Region 1 to Eustis, 40,000 from Region 2 to Pine Hills, 25,000 from Region 3 to Eustis, 35,000 from Region 4 to Sanford, 25,000 from Region 5 to Eustis, 5,000 from Region 5 to Sanford. Total cost $1,132,500.

10. a.



b. See file Prb5_10.xlsm
c. Produce 420 in month 1, 520 in month 2, 400 in month 3, 450 in month 4, carry 110 in inventory from month 1 to 2, 50 from month 2 to 3, 140 from month 3 to 4, and 50 at the end of month 4. Total Cost = $83,565.
d. Not much, only $45.

11. a. Production costs (per 1000) in January, February, March, April, May and June are $7,100, $7,700, $7,600, $7,800, $7,900, and $7,400, respectively.



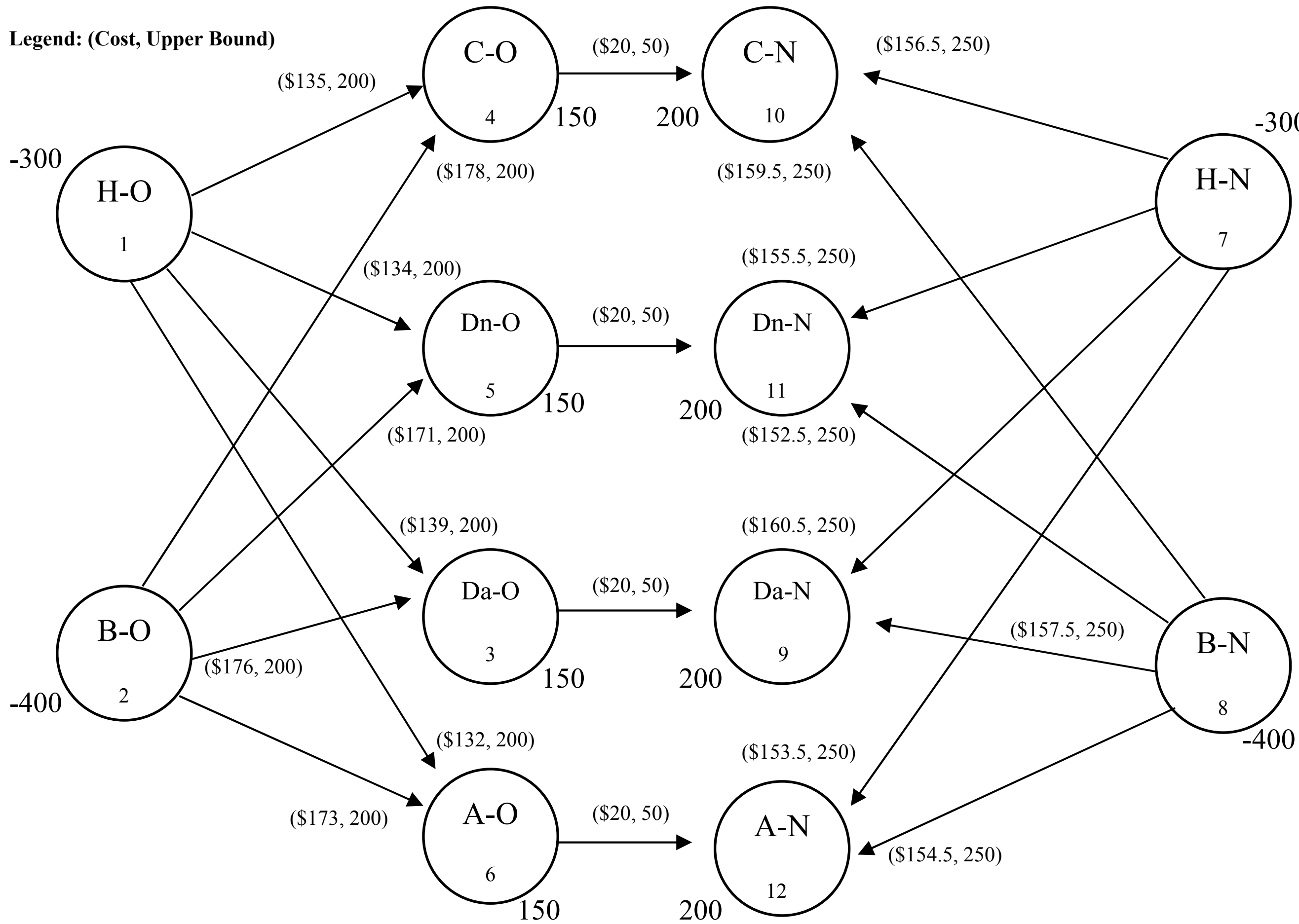
Costs for each arc are shown in the spreadsheet.

b. See file: Prb5_11.xlsm

c.

Ship	From			To	Unit Cost
0	1	Jan	13	Mar	$7,265
0	1	Jan	14	Apr	$7,320
0	1	Jan	15	May	$7,375
0	1	Jan	16	Jun	$7,430
6	1	Jan	17	July	$7,485
10	1	Jan	18	Aug	$7,540
0	2	Feb	13	Mar	$7,810
0	2	Feb	14	Apr	$7,865
0	2	Feb	15	May	$7,920
0	2	Feb	16	Jun	$7,975
13	2	Feb	17	July	$8,030
0	2	Feb	18	Aug	$8,085
14	3	Mar	13	Mar	$7,600
0	3	Mar	14	Apr	$7,720
0	3	Mar	15	May	$7,775
0	3	Mar	16	Jun	$7,830
6	3	Mar	17	July	$7,885
0	3	Mar	18	Aug	$7,940
20	4	Apr	14	Apr	$7,800
0	4	Apr	15	May	$7,935
0	4	Apr	16	Jun	$7,990
0	4	Apr	17	July	$8,045
0	4	Apr	18	Aug	$8,100
26	5	May	15	May	$7,900
0	5	May	16	Jun	$8,050
0	5	May	17	July	$8,105
0	5	May	18	Aug	$8,160
33	6	Jun	16	Jun	$7,400
3	6	Jun	17	July	$7,555
0	6	Jun	18	Aug	$7,610
			Total Cost		$1,006,675

12. a.



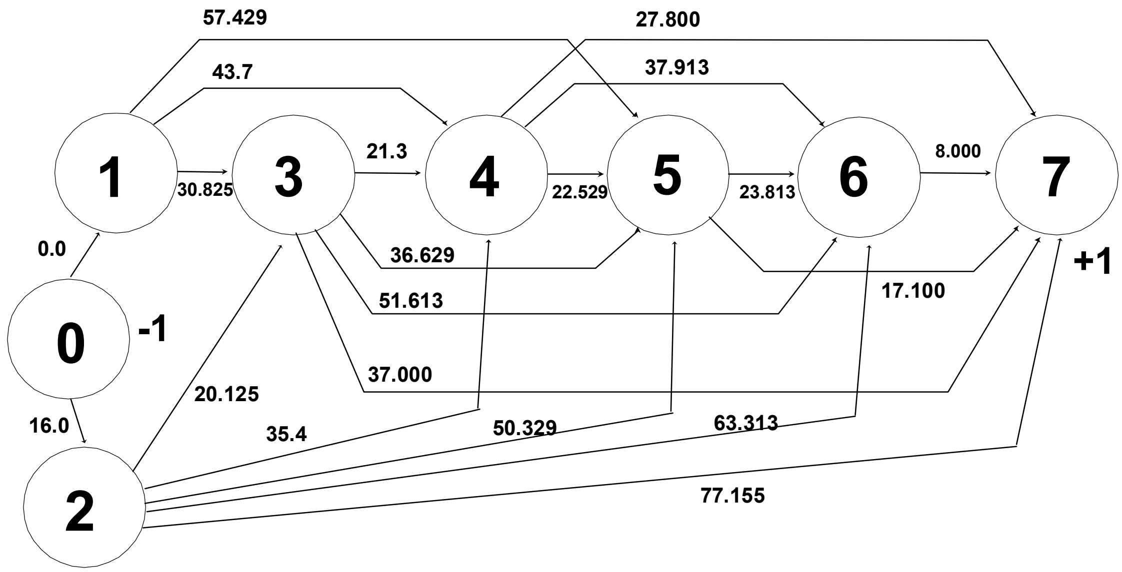
b. See file: Prb5_12.xlsm

c.

Flow	From		To		Cost
0	1	Huntington-O	3	Dallas-O	$139.00
200	1	Huntington-O	4	Chicago-O	$135.00
0	1	Huntington-O	5	Denver-O	$134.00
100	1	Huntington-O	6	Atlanta-O	$132.00
200	2	Bakersfield-O	3	Dallas-O	$176.00
0	2	Bakersfield-O	4	Chicago-O	$178.00
150	2	Bakersfield-O	5	Denver-O	$171.00
50	2	Bakersfield-O	6	Atlanta-O	$173.00
0	7	Huntington-N	9	Dallas-N	$160.50
150	7	Huntington-N	10	Chicago-N	$156.50
0	7	Huntington-N	11	Denver-N	$155.50
150	7	Huntington-N	12	Atlanta-N	$153.50
150	8	Bakersfield-N	9	Dallas-N	$157.50
0	8	Bakersfield-N	10	Chicago-N	$159.50
200	8	Bakersfield-N	11	Denver-N	$152.50
50	8	Bakersfield-N	12	Atlanta-N	$154.50
50	3	Dallas-O	9	Dallas-N	$20.00
50	4	Chicago-O	10	Chicago-N	$20.00

0	5	Denver-O	11	Denver-N	$20.00
0	6	Atlanta-O	12	Atlanta-N	$20.00
				Total Cost	$220,050.0 0

13. a.

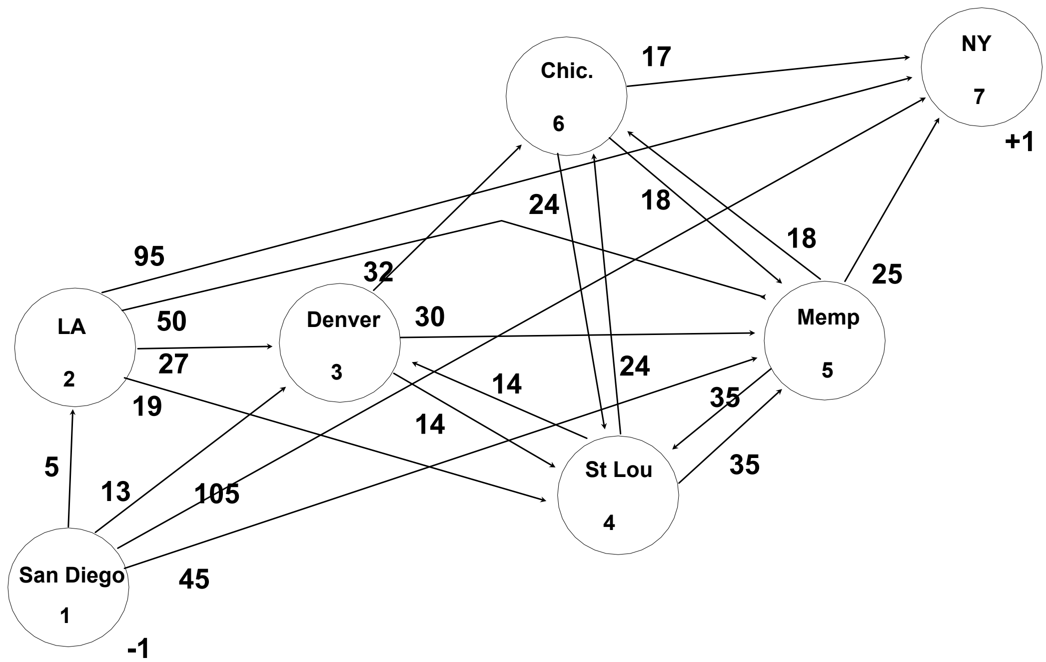


Notes: 0-1 = keep current equipment to use during the coming year
0-2 = trade-in current equipment immediately and use new equipment during the coming year

b. See file: Prb5_13.xlsm
The solution is: $X_{01}=X_{13}= X_{37}=1$ with a minimum total cost of $67,825.

c. The problem could be made more realistic by considering the tax savings associated with depreciation on the equipment. Also, the current problem does not consider the time value of money (i.e., we might attempt to minimize the net present value of the cash flows). Both of these considerations could be accommodated easily by altering the objective function coefficients.

14. a.

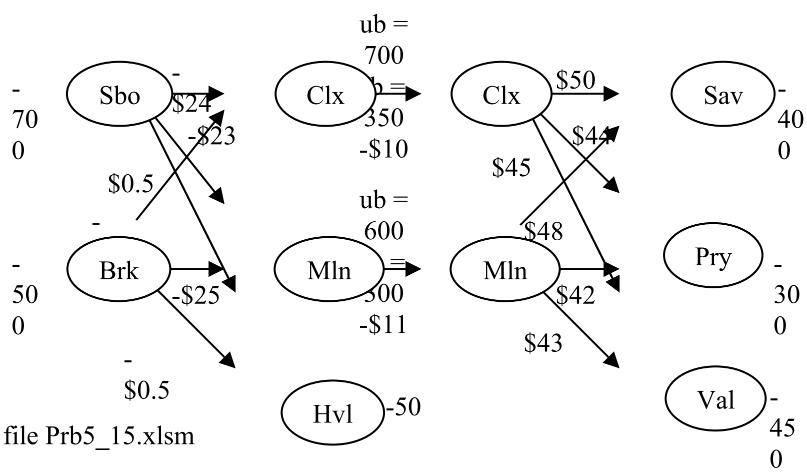


b. MIN $5X_{12} + 13X_{13} + 45X_{15} + 105X_{17} + 27X_{23} + 19X_{24} + 50X_{25} + 95X_{27} +$
$14X_{34} + 30X_{35} + 32X_{36} + 14X_{43} + 35X_{45} + 24X_{46} + 35X_{54} + 18X_{56} + 25X_{57} +$
$24X_{64} + 18X_{65} + 17X_{67}$

ST $-X_{12} - X_{13} - X_{15} - X_{17} = -1$
$+X_{12} - X_{23} - X_{24} - X_{25} - X_{27} = 0$
$+X_{13} + X_{23} + X_{43} - X_{34} - X_{35} - X_{36} = 0$
$+X_{24} + X_{34} + X_{54} + X_{64} - X_{43} - X_{45} - X_{46} = 0$
$+X_{15} + X_{25} + X_{35} + X_{45} + X_{65} - X_{54} - X_{56} - X_{57} = 0$
$+X_{36} + X_{46} + X_{56} - X_{64} - X_{65} - X_{67} = 0$
$+X_{17} + X_{27} + X_{57} + X_{67} = +1$
$X_{ij} \geq 0$

c. See file: Prb5_14.xlsm
The solution is: $X_{13}=X_{36}=X_{67}=1$ with a minimum total cost of \$62.

15. a.



b. See file Prb5_15.xlsm

c. Ship 250 from Statesboro to Claxton, 450 from Statesboro from Millen, 450 from Brooklet to Claxton, 50 from Brooklet to Hinesville, 250 from Claxton to Perry, 450 Claxton to Valdosta, 400 Millen to Savannah, 50 from Millen to Perry. Total Profit = $12,750.

16. a. See file: Prb5_16.xlsm

b.

Ship	From		To		Unit Cost
0	1	Pittsburgh	3	Charleston	$3
1300	1	Pittsburgh	4	Roanoke	$4
1100	2	Staunton	3	Charleston	$3
0	2	Staunton	4	Roanoke	$4
0	3	Charleston	4	Roanoke	$7
800	3	Charleston	5	Richmond	$9
0	3	Charleston	6	Norfolk	$6
0	4	Roanoke	3	Charleston	$7
1100	4	Roanoke	6	Norfolk	$4
0	4	Roanoke	7	Suffolk	$8
0	5	Richmond	6	Norfolk	$3
400	6	Norfolk	7	Suffolk	$2
		Total Transportation Cost			$20,900

c.

Ship	From		To		Unit Cost
1000	1	Pittsburgh	3	Charleston	$3
300	1	Pittsburgh	4	Roanoke	$4
200	2	Staunton	3	Charleston	$3
900	2	Staunton	4	Roanoke	$4
0	3	Charleston	4	Roanoke	$7
800	3	Charleston	5	Richmond	$9
100	3	Charleston	6	Norfolk	$6
0	4	Roanoke	3	Charleston	$7
1000	4	Roanoke	6	Norfolk	$4
0	4	Roanoke	7	Suffolk	$8
0	5	Richmond	6	Norfolk	$3
400	6	Norfolk	7	Suffolk	$2
		Total Transportation Cost			$21,000

17. a. Supply nodes: 1, 2
Demand node: 6
Transshipment nodes: 3, 4 & 5

b. See file: Prb5_17.xlsm
The solution is: X_{13}=20, X_{24}=10, X_{25}=30, X_{36}=40, with a minimum total cost of $2,700.

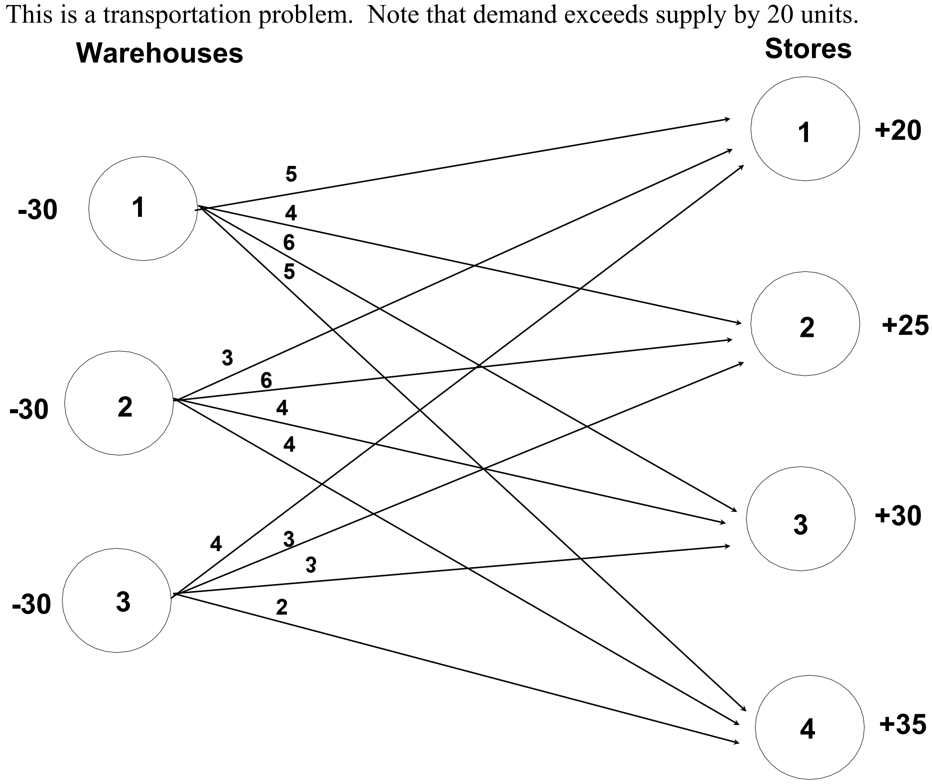
18. MAX X_{71}

ST
$+X_{71} - X_{12} - X_{13} - X_{14} = 0$
$+X_{12} - X_{23} - X_{25} = 0$
$+X_{13} + X_{23} + X_{43} - X_{35} - X_{36} - X_{37} = 0$
$+X_{14} - X_{43} - X_{46} = 0$

$+X_{25} + X_{35} - X_{57} = 0$
$+X_{36} + X_{46} - X_{67} = 0$
$+X_{37} + X_{57} + X_{67} = 0$
$0 \le X_{12} \le 8$
$0 \le X_{13} \le 9$
$0 \le X_{14} \le 7$
$0 \le X_{23} \le 7$
$0 \le X_{25} \le 10$
$0 \le X_{35} \le 8$
$0 \le X_{36} \le 7$
$0 \le X_{37} \le 9$
$0 \le X_{43} \le 6$
$0 \le X_{46} \le 9$
$0 \le X_{57} \le 9$
$0 \le X_{67} \le 11$

See file: Prb5_18.xlsm
The optimal solution is: $X_{12} = 8$, $X_{13} = 9$, $X_{14} = 7$, $X_{25} = 8$, $X_{37} = 9$, $X_{46} = 7$, $X_{57} = 8$, $X_{67} = 7$.
Maximal flow = 24 tons of sewage per hour.

19. a. This is a transportation problem. Note that demand exceeds supply by 20 units.



b. MIN $5X_{11} + 4X_{12} + 6X_{13} + 5X_{14}$
$+3X_{21} + 6X_{22} + 4X_{23} + 4X_{24}$
$+4X_{31} + 3X_{32} + 3X_{33} + 2X_{34}$

ST $-X_{11} - X_{12} - X_{13} - X_{14} = -30$
$-X_{21} - X_{22} - X_{23} - X_{24} = -30$
$-X_{31} - X_{32} - X_{33} - X_{34} = -30$
$+X_{11} + X_{21} + X_{31} + X_{D1} \le +20$
$+X_{12} + X_{22} + X_{32} + X_{D2} \le +25$

$+X_{13} + X_{23} + X_{33} + X_{D3} \le +30$
$+X_{14} + X_{24} + X_{34} + X_{D4} \le +35$
$X_{ij} \ge 0$

c. See file: Prb5_19.xlsm
The optimal solution is: $X_{12} = 25$, $X_{14} = 5$, $X_{21} = 20$, $X_{23} = 10$, $X_{34} = 30$, $X_{D3} = 20$.
Minimum total cost = \$285.
Note that store 3 receives 20 units less than demanded.

d. Assign arbitrarily large costs (such as \$999) to the arcs representing these flows.
The optimal solution is then: $X_{13} = 30$, $X_{21} = 20$, $X_{24} = 10$, $X_{32} = 5$, $X_{34} = 25$.
Minimum total cost = \$345.
Note that store 2 receives 20 units less than demanded.

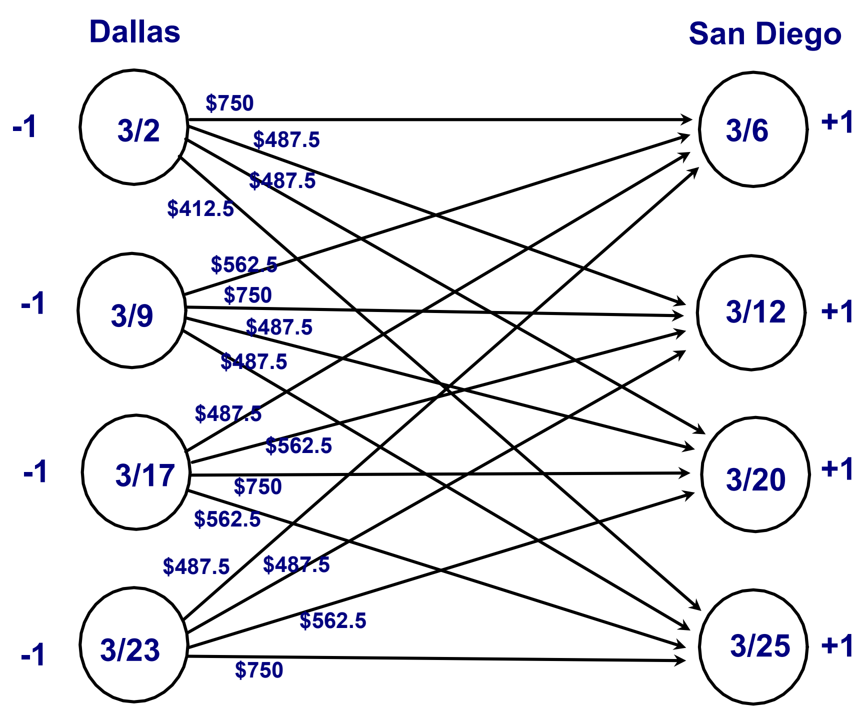
20. The LP model is:

MIN $12 X_{12} + 8 X_{13} + 15 X_{14} + 9 X_{23} + 16 X_{25} + 6 X_{34} + 7 X_{35} + 12 X_{54}$

ST
$-X_{12} - X_{13} - X_{14} = -15$
$-X_{23} - X_{25} = -15$
$+X_{13} + X_{23} - X_{34} - X_{35} = 0$
$+X_{14} + X_{34} + X_{54} = 20$
$+X_{25} + X_{35} - X_{54} = 10$
$0 \le X_{ij} \le 10$

The solution is: $X_{13} = 5$, $X_{14} = 10$, $X_{23} = 5$, $X_{25} = 10$, $X_{34} = 10$
Minimum total cost = \$455
See file: Prb5_20.xlsm

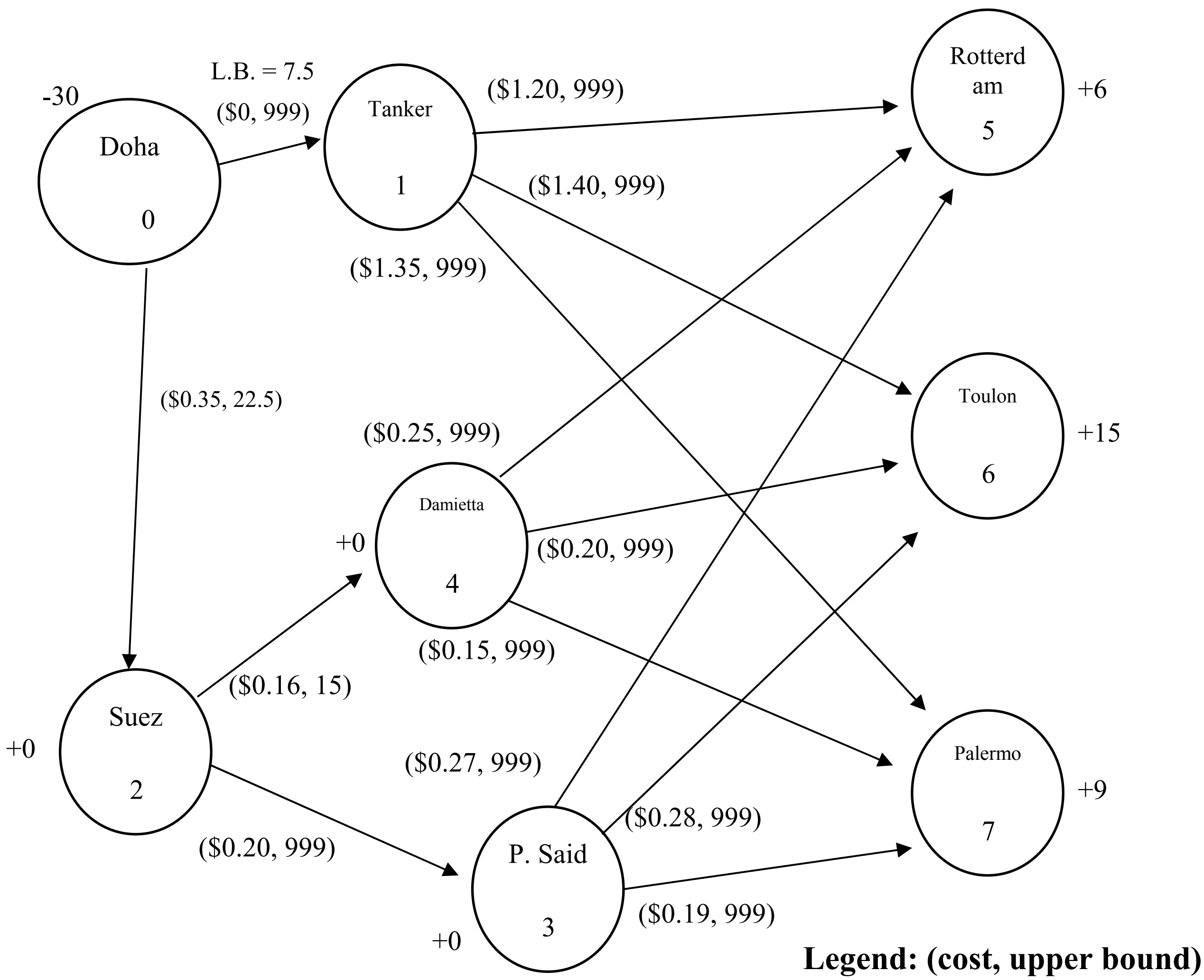
21. a.



Note: each arc represents a possible round trip ticket departing from nodes with the earliest dates (e.g., we could buy a round trip ticket departing San Diego on March 6, returning to San Diego March 23).

b. See file: Prb5_21.xlsm
c. The optimal solution is to buy the following 4 tickets:
Leave Dallas March 2, returning March 25
Leave Dallas March 9, returning March 20
Leave San Diego March 6, returning March 17
Leave San Diego March 12, returning March 23
Total cost = $1,875. This saves $1,125 off the full-fare price of $3,000.

22. a.

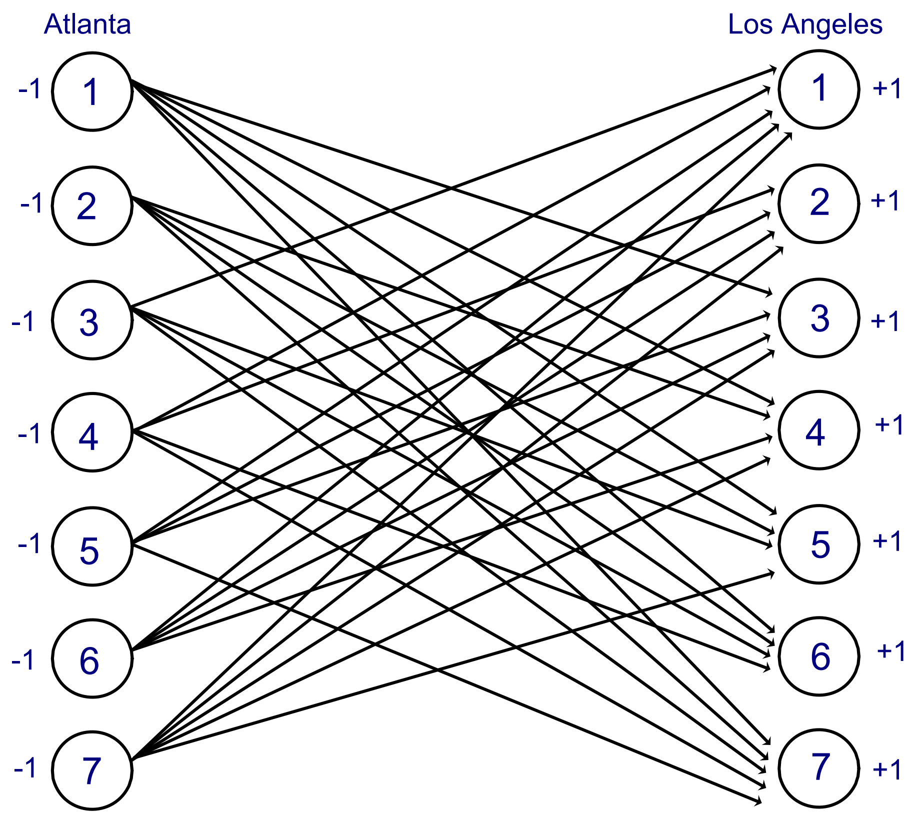


b. See file: Prb5_22.xlsm
c. Total Cost = $25.43 million

Flow	L.B.	U.B.	From		To		Unit Cost ($1,000,000s)
22.5	0	22.5	0	Doha	2	Suez	$0.35
7.5	7.5	999	0	Doha	1	Tanker	$0.00
6	0	999	1	Tanker	5	Rotterdam	$1.20
1.5	0	999	1	Tanker	6	Toulon	$1.40

0	0	999	1	Tanker	7	Palermo	$1.35
7.5	0	999	2	Suez	3	Port Said	$0.20
15	0	15	2	Suez	4	Damietta	$0.16
0	0	999	3	Port Said	5	Rotterdam	$0.27
0	0	999	3	Port Said	6	Toulon	$0.28
7.5	0	999	3	Port Said	7	Palermo	$0.19
0	0	999	4	Damietta	5	Rotterdam	$0.25
13.5	0	999	4	Damietta	6	Toulon	$0.20
1.5	0	999	4	Damietta	7	Palermo	$0.15

23. a.



Note: each arc represents a possible round trip flight assignment departing from nodes with the earliest times. See spreadsheet for arc costs.

b. See file: Prb5_23.xlsm

c.

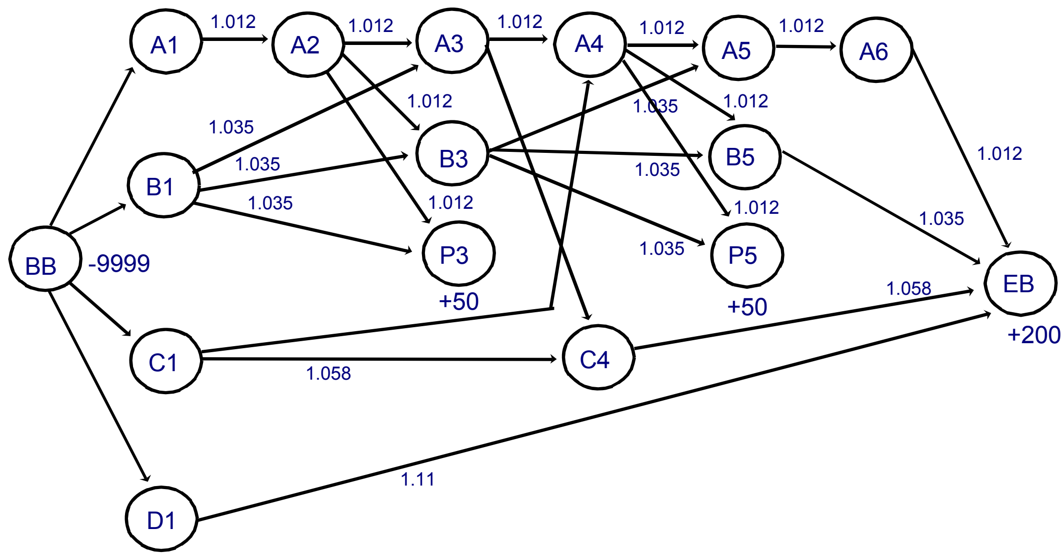
Leave	Return flight leaves at
Atlanta at 6 am	9 am
Atlanta at 8 am	5 pm
Los Angeles at 5 am	10 am
Los Angeles at 6 am	Noon
Atlanta at 4 pm	7 pm
Los Angeles at noon	6 pm
Los Angeles at 2 pm	7 pm

Total layover hours = 15, longest layover time = 7 hours.

d. There is no solution with a smaller longest layover time.

24. a. See file: Prb5_24.xlsm
    b. 1-> 3-> 6-> 8-> 10-> 12, Total distance = 1863
    c. 1-> 4-> 7-> 6-> 8-> 9-> 12, Total distance = 3280
    d. For the longest route, the solution is now unbounded.

25. a.



Minimize the total flow from BB (beginning balance) to A1, B1, C1 and D1.

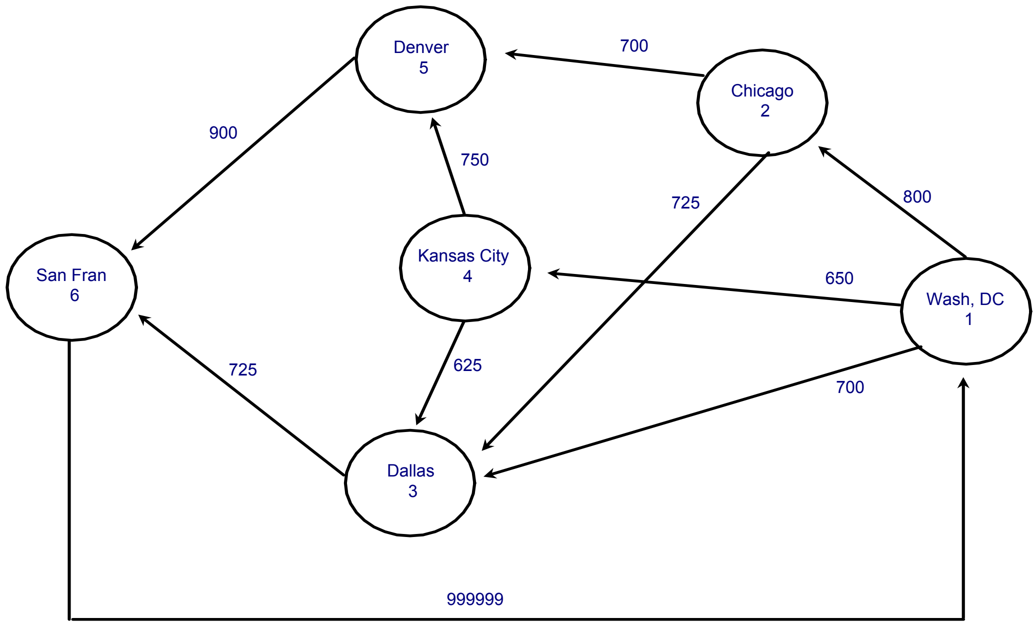
b. See file: Prb5_25.xlsm

c.

Flow	From		To	
$94.985	1	BB	3	B1
$178.673	1	BB	4	C1
$46.676	3	B1	8	B3
$48.309	3	B1	9	P3
$178.673	4	C1	11	C4
$48.309	8	B3	14	P5
$189.036	11	C4	16	EB

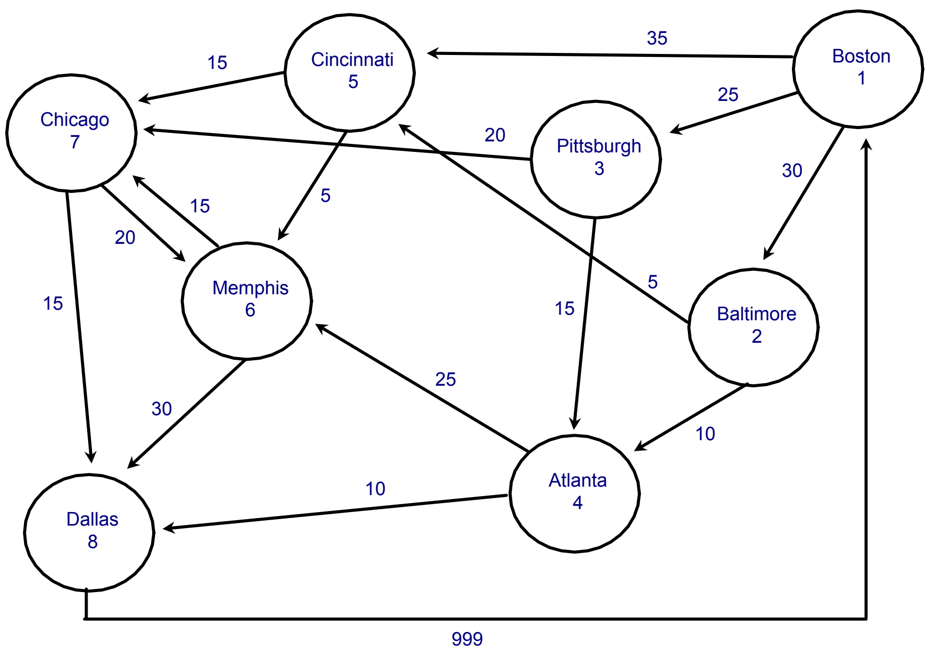
Total cash required = $273,658

26. a.



b. See file: Prb5_26.xlsm
c. The system can handle 1,625,000 calls

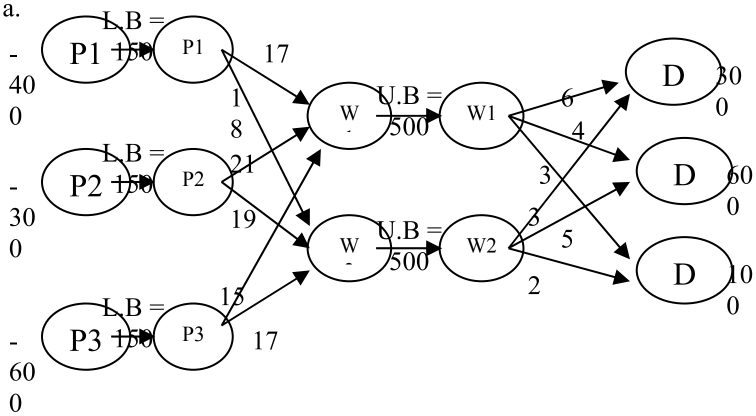
27. a.



b. See file: Prb5_27.xlsm
c. The maximum flow is 55 tons.

28. a. See file Prb5_28.xlsm
b. 7,000,000 packets per minute

29. a.



b. See file Prb5_29.xlsm

c.

Ship	From		To	
250	1	Plant 1	4	Plant 1 out
150	2	Plant 2	5	Plant 2 out
600	3	Plant 3	6	Plant 3 out
0	4	Plant 1 out	7	Whse 1 in
250	4	Plant 1 out	8	Whse 2 in
0	5	Plant 2 out	7	Whse 1 in
150	5	Plant 2 out	8	Whse 2 in
500	6	Plant 3 out	7	Whse 1 in
100	6	Plant 3 out	8	Whse 2 in
500	7	Whse 1 in	9	Whse 1 out
500	8	Whse 2 in	10	Whse 2 out
0	9	Whse 1 out	11	Dist. 1
500	9	Whse 1 out	12	Dist. 2
0	9	Whse 1 out	13	Dist. 3
300	10	Whse 2 out	11	Dist. 1
100	10	Whse 2 out	12	Dist. 2
100	10	Whse 2 out	13	Dist. 3

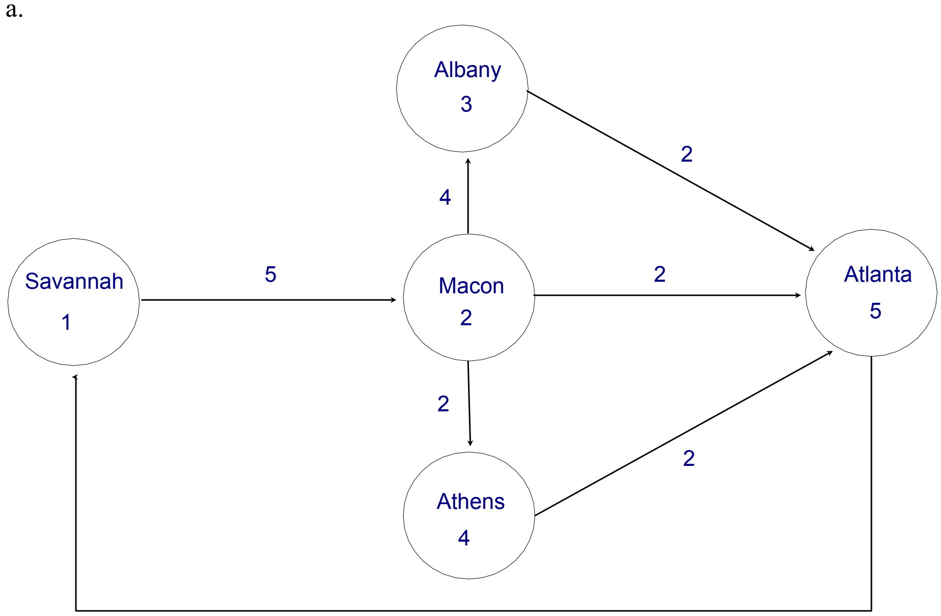
Total cost: $20,150.

30. MAX $X_{71} + X_{81}$

ST
$+X_{71} + X_{81} - X_{12} - X_{13} - X_{15} = 0$
$+X_{12} - X_{24} - X_{25} = 0$
$+X_{13} - X_{35} - X_{36} = 0$
$+X_{24} + X_{54} - X_{45} - X_{47} = 0$
$+X_{15} + X_{25} + X_{35} + X_{45} + X_{65} - X_{54} - X_{56} - X_{57} - X_{58} = 0$
$+X_{36} + X_{56} - X_{65} - X_{68} = 0$
$+X_{47} + X_{57} - X_{71} = 0$
$+X_{58} + X_{68} - X_{81} = 0$
$0 \le X_{12} \le 100$
$0 \le X_{13} \le 100$
$0 \le X_{15} \le 200$
$0 \le X_{24} \le 100$
$0 \le X_{25} \le 150$
$0 \le X_{35} \le 150$
$0 \le X_{36} \le 150$
$0 \le X_{45} \le 100$
$0 \le X_{47} \le 150$
$0 \le X_{54} \le 100$
$0 \le X_{56} \le 100$
$0 \le X_{57} \le 100$
$0 \le X_{58} \le 100$
$0 \le X_{65} \le 100$
$0 \le X_{68} \le 150$
$0 \le X_{71} \le \infty$
$0 \le X_{81} \le \infty$

b. The optimal solution is: $X_{15} = 200$, $X_{24} = 100$, $X_{25} = 50$, $X_{36} = 150$, $X_{47} = 150$, $X_{54} = 50$, $X_{57} = 100$, $X_{58} = 100$, $X_{68} = 150$, $X_{81} = 200$, $X_{72} = 100$, $X_{82} = 50$, $X_{73} = 150$.
Maximum flow = 500 bags per minute.
See file: Prb5_30.xlsm

31. a.



b. MAX X_{51}

ST
$+X_{51} - X_{12} = 0$
$+X_{12} - X_{23} - X_{24} - X_{25} = 0$
$+X_{23} - X_{35} = 0$
$+X_{24} - X_{45} = 0$
$+X_{25} + X_{35} + X_{45} - X_{51} = 0$
$0 \le X_{12} \le 5$
$0 \le X_{23} \le 4$
$0 \le X_{24} \le 2$
$0 \le X_{25} \le 2$
$0 \le X_{35} \le 2$
$0 \le X_{45} \le 2$
$0 \le X_{51} \le \infty$

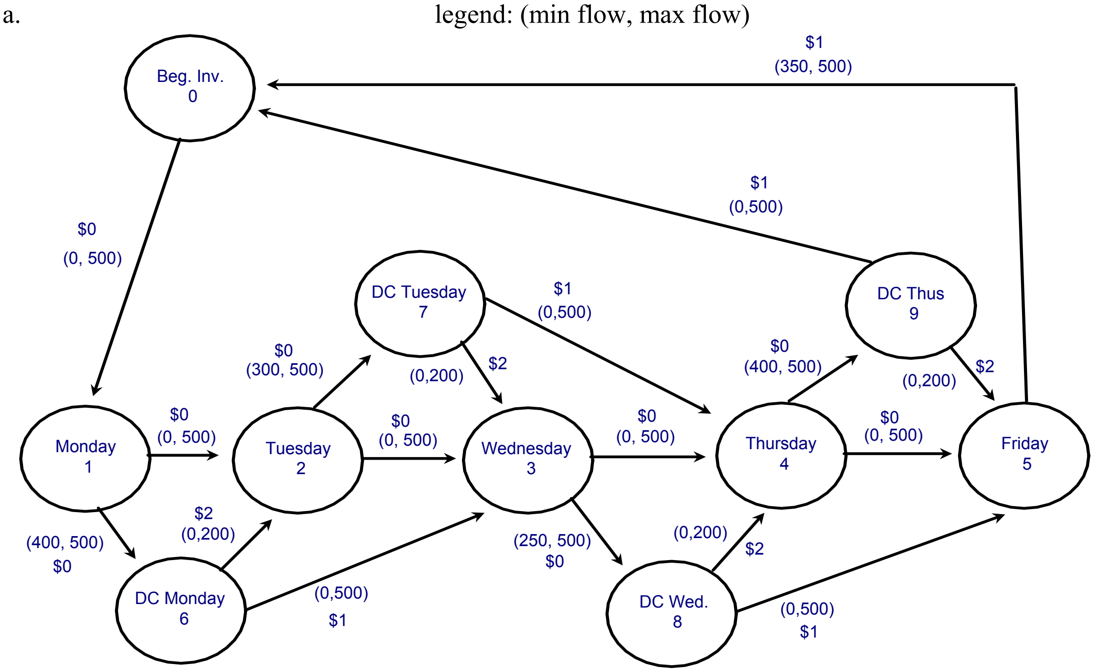
c. The optimal solution is: $X_{12} = 5$, $X_{23} = 2$, $X_{24} = 2$, $X_{25} = 1$, $X_{35} = 2$, $X_{45} = 2$, $X_{51} = 5$
Maximum flow = 5 sets of connecting flight plans.
See file: Prb5_31.xlsm

32. The optimal solution is: $X_{14}=X_{49}=X_{9,12}= 1$, Minimum cost = $8 million
See file: Prb5_32.xlsm

33.

Iteration	Node Added	Cost
1	1	$0
2	5	$85
3	4	$20
4	3	$25
5	7	$30
6	6	$20
7	2	$30
8	8	$35
9	9	$25
	Total Cost	$270

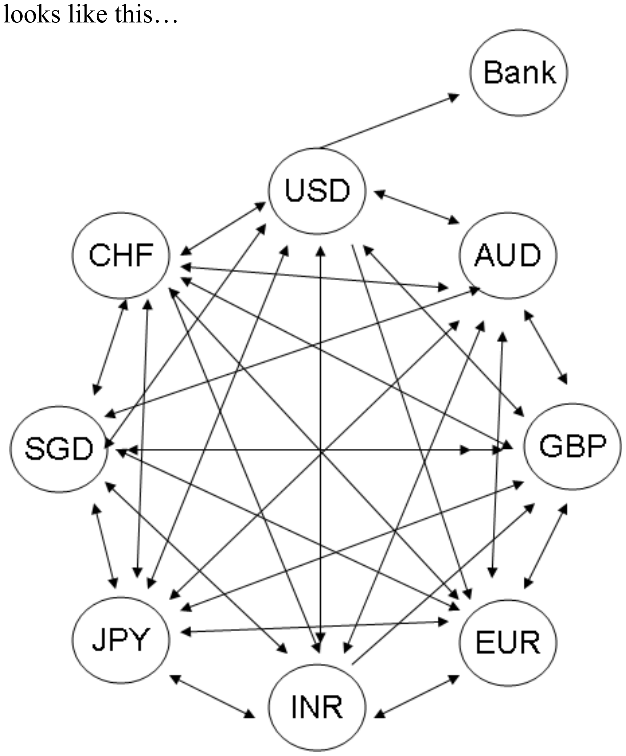
34. a. legend: (min flow, max flow)



b. See file: Prb5_34.xlsm, Total cost = $2,350

Case 5-1: Hamilton & Jacobs

1. The basic problem looks like this…



   This is a generalized network flow model. The exchange rates and transaction costs comprise the coefficients that appear on each arc (and are calculated in the spreadsheet). The key is to realize that you want to maximize the amount that makes it into the bank in US currency.

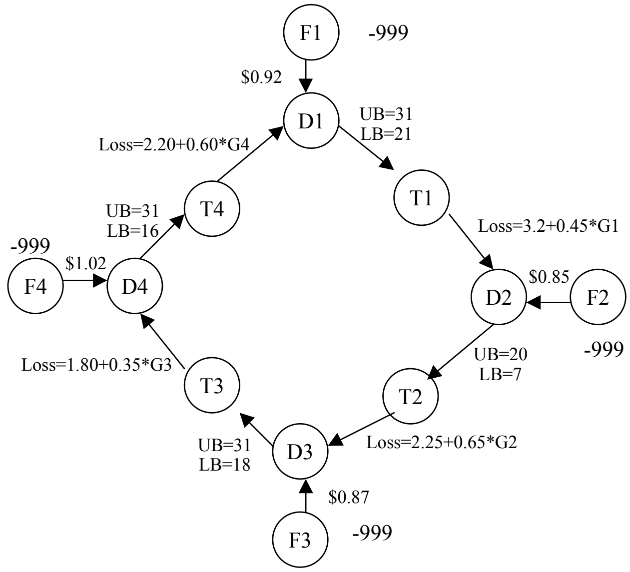
2. See file: Case5_1.xlsm
3. See file: Case5_1.xlsm. Total US dollars = $ 42,234,228
4. Total US dollars = $ 42,211,654
5. See file: Case5_1.xlsm. Total US dollars = $ 38,367,337

Case 5-2: Old Dominion Energy

See file: Case5_2.xlsm
1. Katy to Leidy = 70,000 cf; Katy to Juliet = 70,000 cf
2. Leidy = 35,000 cf, Joliet = 35,000 cf, Profit = $213.5
3. No.
4. Increase the capacity from Katy to Carthage would be the easiest way to meet more of the demand at Joliet (since the pipe from Carthage to Joliet is not at full capacity). Additionally, increasing the capacity from Carthage to Lebanon would allow Bruce to satisfy more of the demand at Leidy (since the pipe from Lebanon to Leidy is not at full capacity).

Case 5-3: US Express

1.

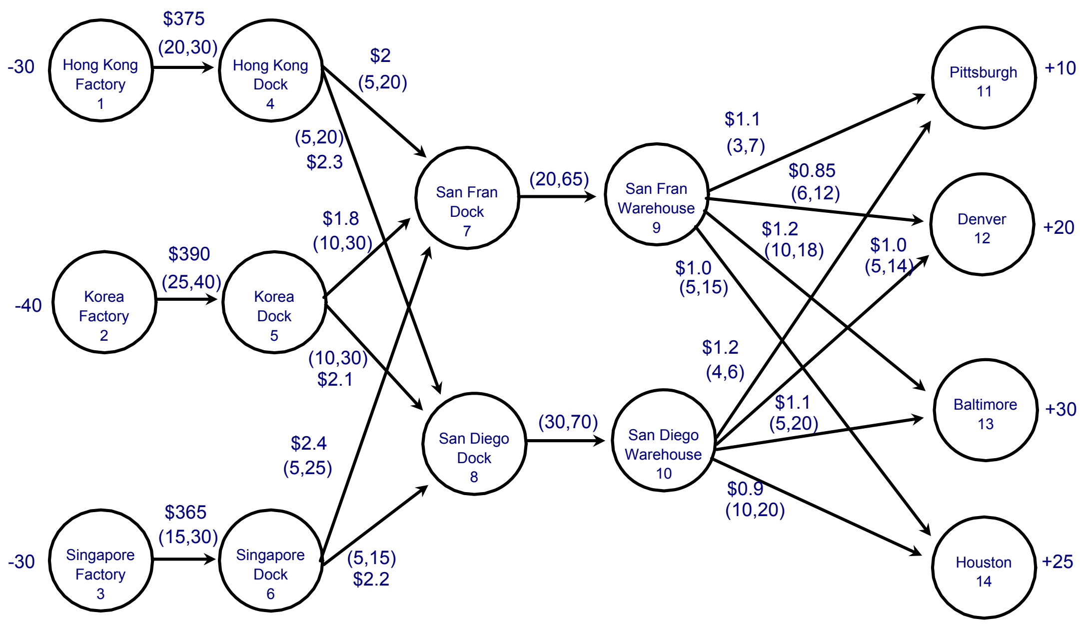


Di = departure point i
Fi = fuel depot at departure point i
Ti = take off from departure point i
Gi = fuel on board when taking off from departure point I

2. See file Case5_3.xlsm
3. Buy 16,800 at departure point 1, 17,328 from departure point 3, 6,100 from departure point 4.
   Total cost = $36,753.

Case 5-4: The Major Electric Corporation

1.



2. See file: Case5_4.xlsm

3. See file: Case5_4.xlsm, Total cost = $32,218,300