Goal Programming and Multiple Objective Optimization (MOLP)

Lecture 11

Chapter 3: Optimization

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Chapter 7

Goal Programming and Multiple Objective Optimization

Spreadsheet Modeling & Decision Analysis:
A Practical Introduction to Management Science, 3e
by Cliff Ragsdale

Introduction

- Most of the optimization problems considered to this point have had a single objective.
- Often, more than one objective can be identified for a given problem
  - Maximize Return or Minimize Risk
  - Maximize Profit or Minimize Pollution
- These objectives often conflict with one another.
- This chapter describes how to deal with such problems.

Goal Programming (GP)

- Most LP problems have ***hard constraints*** that cannot be violated...
  - There are 1,566 labor hours available.
  - There is $850,00 available for projects.

- In some cases, hard constraints are too restrictive...
  - You have a maximum price in mind when buying a car (this is your “goal” or target price).
  - If you can’t buy the car for this price you’ll likely find a way to spend more.

- We use ***soft constraints*** to represent such goals or targets we’d like to achieve.

A Goal Programming Example: Myrtle Beach Hotel Expansion

- Davis McKeown wants to expand the convention center at his hotel in Myrtle Beach, South Carolina.
- The types of conference rooms being considered are:

	Size (sq ft)	Unit Cost
Small	400	$18,000
Medium	750	$33,000
Large	1,050	$45,150

- Davis would like to add 5 small, 10 medium and 15 large conference rooms.
- He would also like the total expansion to be 25,000 square feet and to limit the cost to $1,000,000.

Defining the Decision Variables

X_1 = number of small rooms to add

X_2 = number of medium rooms to add

X_3 = number of large rooms to add

Defining the Goals

- Goal 1: The expansion should include *approximately* 5 small conference rooms.
- Goal 2: The expansion should include *approximately* 10 medium conference rooms.
- Goal 3: The expansion should include *approximately* 15 large conference rooms.
- Goal 4: The expansion should consist of *approximately* 25,000 square feet.
- Goal 5: The expansion should cost *approximately* $1,000,000.

Defining the Goal Constraints

- Small Rooms

$$X_1 + d_1^- - d_1^+ = 5$$

- Medium Rooms

- Large Rooms

where

d_i^+ and d_i^- are deviational variables, represents the amount by which each goal's target value is overachieved and underachieved respectively

		d^+	d^-
x_1	3	0	2
x_2	13	3	0
x_3	15	0	0

Defining the Goal Constraints (cont'd)

- Total Expansion

- Total Cost (in $1,000s)

where

GP Objective Functions

- There are numerous objective functions we could formulate for a GP problem.

Minimize the sum of the deviations:

MIN

- Problem: The deviations measure different things, so what does this objective represent?

GP Objective Functions *(cont'd)*

- **Minimize the sum of *percentage* deviations**

  MIN

  where t_i represents the target value of goal i

- Problem: Suppose the first goal is underachieved by 1 small room and the fifth goal is overachieved by $20,000.
  - We underachieve goal 1 by 1/5=20%
  - We overachieve goal 5 by 20,000/1,000,000= 2%
  - This implies being $200,000 over budget is just as undesirable as having one too few small rooms.
  - Is this true? Only the decision maker can say for sure.

GP Objective Functions *(cont'd)*

- Weights can be used in the previous objectives to allow the decision maker indicate
  - desirable vs. undesirable deviations
  - the relative importance of various goals

Minimize the weighted sum of deviations

MIN

- Minimize the weighted sum of % deviations

MIN

- w_i^+ and w_i^- represents numeric values assigned to weight the various deviational variables,
- Highly undesirable deviations is assigned a large weight making it undesirable for that variable to assume a value larger than 0
- Desirable deviation is assigned weight of 0 or lower than 0

Defining the Objective

- Assume
  - It is undesirable to underachieve any of the first three room goals
  - It is undesirable to overachieve or underachieve the 25,000 sq. ft. expansion goal
  - It is undesirable to overachieve the \$1,000,000 total cost goal

$$\text{MIN}: \frac{w_1^-}{5}d_1^- + \frac{w_2^-}{5}d_2^- + \frac{w_3^-}{5}d_3^- + \frac{w_4^-}{25{,}000}d_4^- + \frac{w_4^+}{25{,}000}d_4^+ + \frac{w_5^+}{1{,}000{,}000}d_5^+$$

Initially, we will assume all the above weights equal 1.

Implementing the Model

	A	B	C	D	E	F
1						
2		**Davis McKeown Hotel Expansion**				
3						
4	**Problem Data**	**Small**	**Medium**	**Large**		
5	Square Footage	**400**	**750**	**1,050**		
6	Building Cost	**$18,000**	**$33,000**	**$45,150**		
7						
8	**Goal Constraints**	**Small**	**Medium**	**Large**	**Sq. Ft.**	**Cost**
9	Actual Amount	5	10	15	25,250	$1,097,250
10	+ Under	0	0	0	0	$0
11	- Over	0	0	0	250	$97,250
12	= Goal	5	10	15	25,000	$1,000,000
13	Target Value	**5**	**10**	**15**	**25,000**	**$1,000,000**
14						
15	**Percentage Deviation**					
16	Under	0.00%	0.00%	0.00%	0.00%	0.00%
17	Over	0.00%	0.00%	0.00%	1.00%	9.73%
18						
19	**Weights**					
20	Under	**1**	**1**	**1**	**1**	**0**
21	Over	**0**	**0**	**0**	**1**	**1**
22						
23	**Objective**	11%				

Minimize: B23
By changing: B9:D9,B10:F11
Subject to: B12:F12=B13:F13
B9:D9>=0 & integer
B10:F11>=0

Implementing the Model

	A	B	C	D	E	F
1						
2		**Davis McKeown Hotel Expansion**				
3						
4	**Problem Data**	**Small**	**Medium**	**Large**		
5	Square Footage	**400**	**750**	**1,050**		
6	Building Cost	**$18,000**	**$33,000**	**$45,150**		
7						
8	**Goal Constraints**	**Small**	**Medium**	**Large**	**Sq. Ft.**	**Cost**
9	Actual Amount	5	10	13	23,150	$1,006,950
10	+ Under	0	0	2	1,850	$0
11	- Over	0	0	0	0	$6,950
12	= Goal	5	10	15	25,000	$1,000,000
13	Target Value	**5**	**10**	**15**	**25,000**	**$1,000,000**
14						
15	**Percentage Deviation**					
16	Under	0.00%	0.00%	13.33%	7.40%	0.00%
17	Over	0.00%	0.00%	0.00%	0.00%	0.70%
18						
19	**Weights**					
20	Under	**1**	**1**	**1**	**1**	**0**
21	Over	**0**	**0**	**0**	**1**	**10**
22						
23	**Objective**	28%				

Minimize: B23
By changing: B9:D9,B10:F11
Subject to: B12:F12=B13:F13
B9:D9>=0 & integer
B10:F11>=0

Implementing the Model

	A	B	C	D	E	F
1						
2		Davis McKeown Hotel Expansion				
3						
4	**Problem Data**	**Small**	**Medium**	**Large**		
5	Square Footage	400	750	1,050		
6	Building Cost	$18,000	$33,000	$45,150		
7						
8	**Goal Constraints**	**Small**	**Medium**	**Large**	**Sq. Ft.**	**Cost**
9	Actual Amount	5	7	15	23,000	$998,250
10	+ Under	0	3	0	2,000	$1,750
11	- Over	0	0	0	0	$0
12	= Goal	5	10	15	25,000	$1,000,000
13	Target Value	5	10	15	25,000	$1,000,000
14						
15	**Percentage Deviation**					
16	Under	0.00%	30.00%	0.00%	8.00%	0.18%
17	Over	0.00%	0.00%	0.00%	0.00%	0.00%
18						
19	**Weights**					
20	Under	1	1	10	1	0
21	Over	0	0	0	1	10
22						
23	**Objective**	38%				

Minimize: B23
By changing: B9:D9,B10:F11
Subject to: B12:F12=B13:F13
B9:D9>=0 & integer
B10:F11>=0

Comments About GP

- GP involves making trade-offs among the goals until the most satisfying solution is found

- GP objective function values should not be compared because the weights are changed in each iteration. Compare the solutions!

- An arbitrarily large weight will effectively change a soft constraint to a hard constraint.

- Hard constraints can be place on deviational variables.

The MiniMax Objective

Can be used to minimize the maximum deviation from any goal.

MIN: Q

$$d_1^- \leq Q$$

$$d_2^- \leq Q$$

etc...

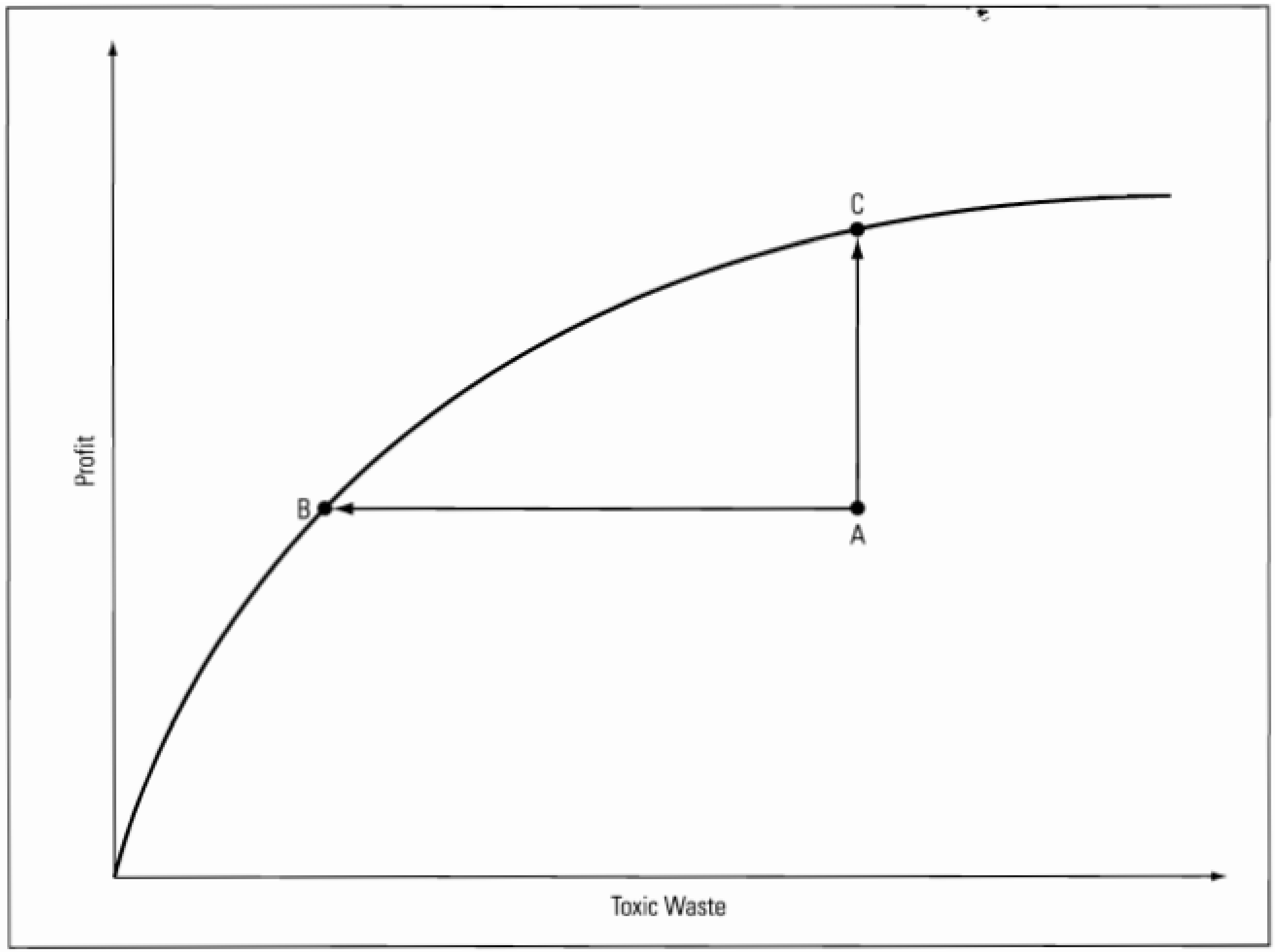
Summary of Goal Programming

1. Identify the decision variables in the problem.
2. Identify any hard constraints in the problem and formulate them in the usual way.
3. State the goals of the problem along with their target values.
4. Create constraints using the decision variables that would achieve the goals exactly.
5. Transform the above constraints into goal constraints by including deviational variables.
6. Determine which deviational variables represent undesirable deviations from the goals.
7. Formulate an objective that penalizes the undesirable deviations.
8. Identify appropriate weights for the objective.
9. Solve the problem.
10. Inspect the solution to the problem. If the solution is unacceptable, return to step 8 and revise the weights as needed.

Multiple Objective Linear Programming (MOLP)

Multiple Objective Linear Programming (MOLP)

- MOLP problem is an LP problem with more than one objective function.
- MOLP problems can be viewed as special types of GP problems where we must also determine target values for each goal or objective.
- Analyzing these problems effectively also requires that we use the MiniMax objective described earlier.


Profit
C
B
A
Toxic Waste

An MOLP Example: The Blackstone Mining Company

- Blackstone Mining operates two coal mines in Southwest Virginia.
- Monthly production by a shift workers at each mine is summarized as follows:

Type of Coal	Wythe Mine	Giles Mine
High-grade	12 tons	4 tons
Medium-grade	4 tons	4 tons
Low-grade	10 tons	20 tons
Cost per month	$40,000	$32,000
Gallons of toxic water produced	800	1,250
Life-threatening accidents	0.20	0.45

- Blackstone needs to produce 48 more tons of high-grade, 28 more tons of medium-grade, and 100 more tons of low-grade coal.

Defining the Decision Variables

X_1 = number of months to schedule an extra shift at the Wythe county mine

X_2 = number of months to schedule an extra shift at the Giles county mine

Defining the Objective

There are three objectives:

Min: $\$40\ X_1 + \$32\ X_2$ } Production costs

Min: $800\ X_1 + 1250\ X_2$ } Toxic water

Min: $0.20\ X_1 + 0.45\ X_2$ } Accidents

Defining the Constraints

- High-grade coal required

  $12 X_1 + 4 X_2 >= 48$

- Medium-grade coal required

  $4 X_1 + 4 X_2 >= 28$

- Low-grade coal required

  $10 X_1 + 20 X_2 >= 100$

- Non-negativity conditions

  $X_1, X_2 >= 0$

Handling Multiple Objectives

- If the objectives had target values we could treat them like the following goals:

Goal 1: The total cost of productions cost should be approximately t_1.
Goal 2: The amount of toxic water produce should be approximately t_2.
Goal 3: The number of life-threatening accidents should be approximately t_3.

- We can solve 3 separate LP problems, independently optimizing each objective, to find values for t_1, t_2 and t_3.

Implementing the Model

See file Fig7-7.xls

	Wythe	Giles		
Months to operate	2.50	4.50		
Objectives			Totals	
Cost per month	$40	$32	$244.0	
Toxins per month	800	1,250	7,625.0	
Accidents per month	0.20	0.45	2.53	
Constraints			Available	Required
HG coal produced	12	4	48	48
MG coal produced	4	4	28	28
LG coal produced	10	20	115	100

Minimize Cost

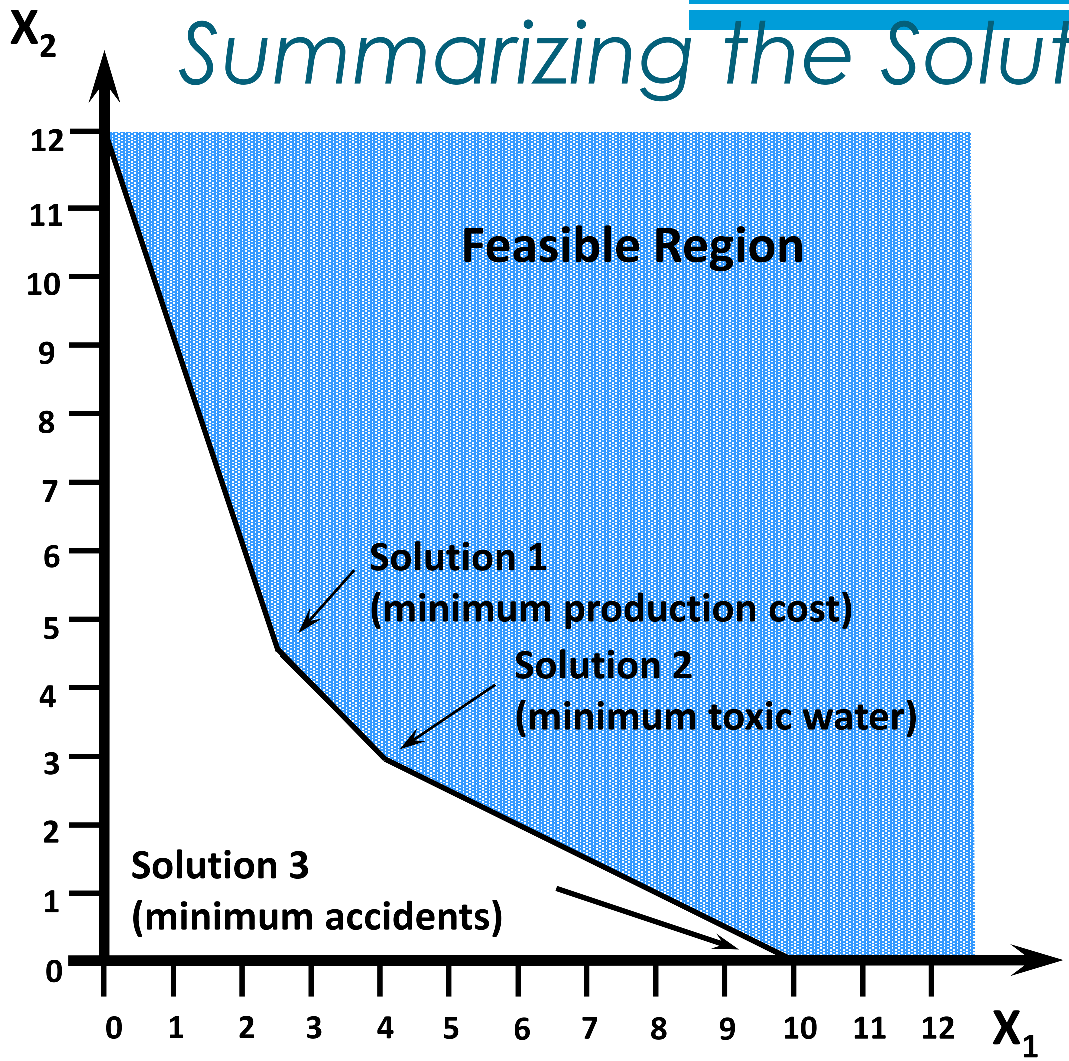
	Wythe	Giles		
Months to operate	4.00	3.00		
Objectives			Totals	
Cost per month	$40	$32	$256.0	
Toxins per month	800	1,250	6,950.0	
Accidents per month	0.20	0.45	2.15	
Constraints			Available	Required
HG coal produced	12	4	60	48
MG coal produced	4	4	28	28
LG coal produced	10	20	100	100

Minimize Toxins

	Wythe	Giles		
Months to operate	10.00	0.00		
Objectives			Totals	
Cost per month	$40	$32	$400.0	
Toxins per month	800	1,250	8,000.0	
Accidents per month	0.20	0.45	2.00	
Constraints			Available	Required
HG coal produced	12	4	120	48
MG coal produced	4	4	40	28
LG coal produced	10	20	100	100

Minimize accident

Summarizing the Solutions



Solution	X_1	X_2	Cost	Toxic Water	Accidents
1	2.5	4.5	$244	7,625	2.53
2	4.0	3.0	$256	6,950	2.15
3	10.0	0.0	$400	8,000	2.00

Defining The Goals

Goal 1: The total cost of productions cost should be approximately $244.

Goal 2: The gallons of toxic water produce should be approximately 6,950.

Goal 3: The number of life-threatening accidents should be approximately 2.0.

Defining an Objective

- Formulate GP objective to allow decision maker to explore possible solutions
- We can minimize the sum of % deviations as follows:

$$\text{MIN}: w_1\left(\frac{(40X_1+32X_2)-244}{244}\right)+w_2\left(\frac{(800X_1+1250X_2)-6950}{6950}\right)+w_3\left(\frac{(0.20X_1+0.45X_2)-2}{2}\right)$$

- It can be shown that this is just a linear combination of the decision variables.

- This objective will only generate solutions at the corner points of the feasible region (no matter what weights are used).

Defining a Better Objective

MIN: Q

Subject to the additional constraints:

$$w_1\left(\frac{(40X_1+32X_2)-244}{244}\right)\le Q$$

$$w_2\left(\frac{(800X_1+1250X_2)-6950}{6950}\right)\le Q$$

$$w_3\left(\frac{(0.20X_1+0.45X_2)-2}{2}\right)\le Q$$

- This objective will allow the decision maker to explore non-corner point solutions of the feasible region.

	A	B	C	D	E	F	G	H
1	-1							
2		Blackstone Mining Co.						
3								
4		Wythe	Giles					
5	Months to operate							
6					Target			Weighted %
7	Goals			Total	Value	% Deviation	Weight	Deviation
8	Cost per month	$40	$32	$0.0	$244.0	-100.00%	1	-100.00%
9	Toxins per month	800	1,250	0.0	6950.0	-100.00%	1	-100.00%
10	Accidents per month	0.20	0.45	0.0000	2.00	-100.00%	5	-500.00%
11								
12	Constraints			Available	Required			
13	HG coal produced	12	4	0.00	48			
14	MG coal produced	4	4	0.00	28			
15	LG coal produced	10	20	0.00	100			
16								
17	Objective							
18	MiniMax Variable							

Minimize: B18
By changing: B5:C5 & B18
Subject to: D13:D15.>=E13:E15
B5:C5>=0
H8:H10<=B18

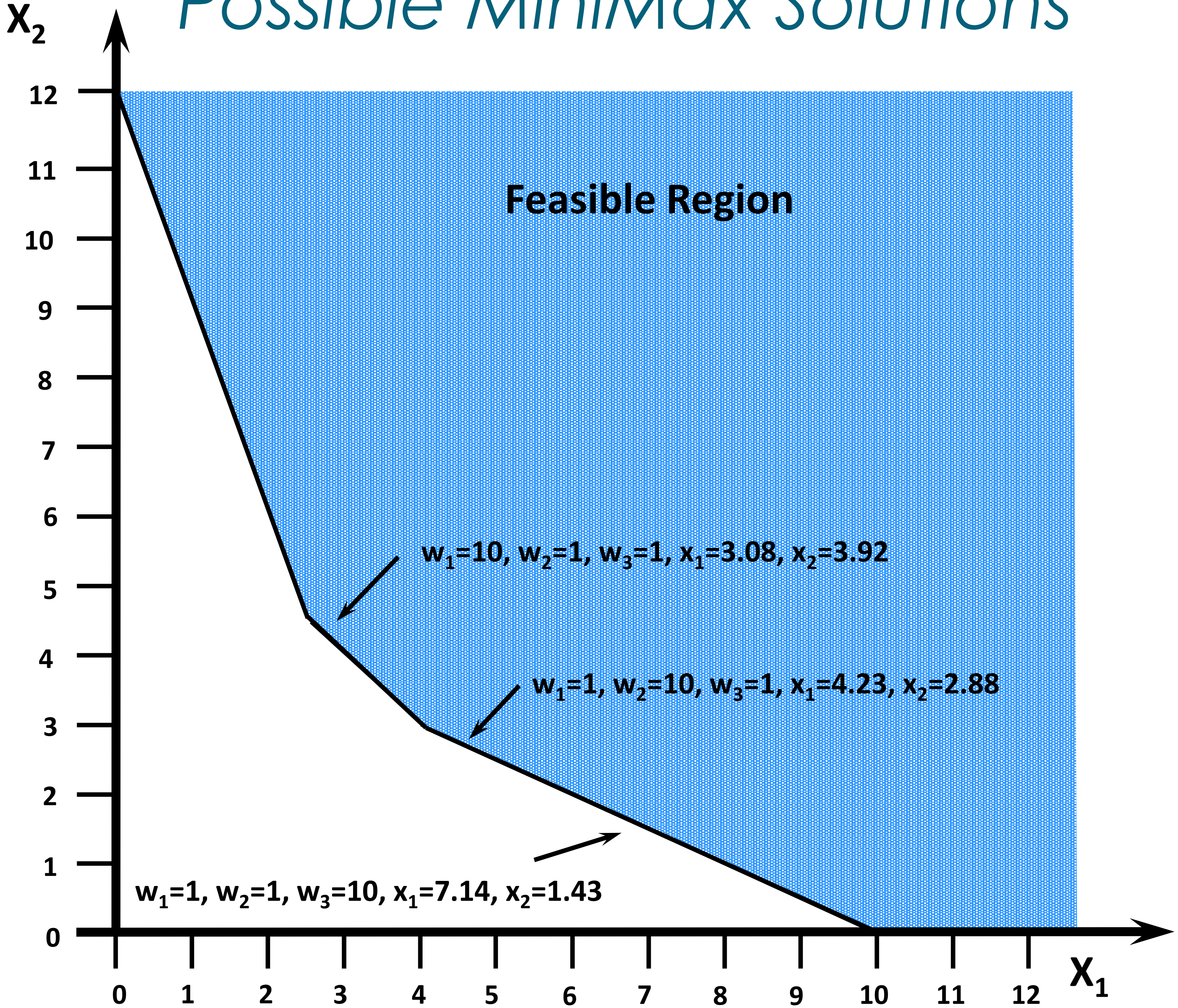
Implementing the Model

See file Fig7-13.xls

	A	B	C	D	E	F	G	H
1	0.248407643							
2			Blackstone Mining Co.					
3								
4		Wythe	Giles					
5	Months to operate	6.03	1.99					
6					Target			Weighted %
7	Goals			Total	Value	% Deviation	Weight	Deviation
8	Cost per month	$40	$32	$304.6	$244.0	24.84%	1	24.84%
9	Toxins per month	800	1,250	7,304.5	6950.0	5.10%	1	5.10%
10	Accidents per month	0.20	0.45	2.0994	2.00	4.97%	5	24.84%
11								
12	Constraints			Available	Required			
13	HG coal produced	12	4	80.25	48			
14	MG coal produced	4	4	32.05	28			
15	LG coal produced	10	20	100.00	100			
16								
17	Objective							
18	MiniMax Variable	0.248408						

Minimize: B18
By changing: B5:C5 & B18
Subject to: D13:D15.>=E13:E15
B5:C5>=0
H8:H10<=B18

Possible MiniMax Solutions



Comments About MOLP

- Solutions obtained using the MiniMax objective are Pareto Optimal.
- Deviational variables and the MiniMax objective are also useful in a variety of situations not involving MOLP or GP.
- For minimization objectives the percentage deviation is: (actual - target)/target
- For maximization objectives the percentage deviation is: (target - actual)/target
- If a target value is zero, use the weighted deviations rather than weighted % deviations.

Summary of MOLP

1. Identify the decision variables in the problem.

2. Identify the objectives in the problem and formulate them in the usual way.

3. Identify the constraints in the problem and formulate them in the usual way.

4. Solve the problem once for each of the objectives identified in step 2 to determine the optimal value of each objective.

5. Restate the objectives as goals using the optimal objective values identified in step 4 as the target values.

6. For each goal, create a deviation function that measures the amount by which any given solution fails to meet the goal (either as an absolute or a percentage).

7. For each of the deviation functions identified in step 6, assign a weight to the deviation function and create a constraint that requires the value of the weighted deviation function to be less than the MINIMAX variable Q.

8. Solve the resulting problem with the objective of minimizing Q.

9. Inspect the solution to the problem. If the solution is unacceptable, adjust the weights in step 7 and return to step 8.

Assignment 6

Question No. : 10, 11, 13, 16, 17,18,19, 23

Assignment-6 due on Wednesday 19th July 2023

End of Chapter 7