Modeling and Solving LP Problems in a Spreadsheet

Lecture 8

Chapter 3: Optimization

Institute of Engineering
Asst. Prof. Anita Prajapati, Ph.D.

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Chapter 3

Modeling and Solving LP Problems in a Spreadsheet

Spreadsheet Modeling & Decision Analysis:

A Practical Introduction to Management Science

3d edition

by Cliff Ragsdale

Introduction

- Solving LP problems graphically is only possible when there are two decision variables
- Few real-world LP have only two decision variables
- Fortunately, we can now use spreadsheets to solve LP problems

Spreadsheet Solvers

The company that makes the Solver in Excel, Lotus 1-2-3, and Quattro Pro is Frontline Systems, Inc.

Check out their web site:
http://www.frontsys.com

Other packages for solving MP problems:

AMPL	LINDO
CPLEX	MPSX

The Steps in Implementing LP Model in a Spreadsheet

1. Organize the data for the model on the spreadsheet.

2. Reserve separate cells in the spreadsheet to represent each decision variable in the model.

3. Create a formula in a cell in the spreadsheet that corresponds to the objective function.

4. For each constraint, create a formula in a separate cell in the spreadsheet that corresponds to the left-hand side (LHS) of the constraint.

Let's Implement a Model for the Blue Ridge Hot Tubs Example...

MAX:	$350X_1 + 300X_2$	} profit
S.T.:	$1X_1 + 1X_2 <= 200$	} pumps
	$9X_1 + 6X_2 <= 1566$	} labor
	$12X_1 + 16X_2 <= 2880$	} tubing
	$X_1, X_2 >= 0$	} non-negativity

Implementing the Model

See file Fig3-1.xls

How Solver Views the Model

Target cell - the cell in the spreadsheet that represents the *objective function*

Changing cells - the cells in the spreadsheet representing the *decision variables*

Constraint cells - the cells in the spreadsheet representing the *LHS formulas* on the constraints

Goals For Spreadsheet Design

- Communication - A spreadsheet's primary business purpose is that of communicating information to managers.
- Reliability - The output a spreadsheet generates should be correct and consistent.
- Auditability - A manager should be able to retrace the steps followed to generate the different outputs from the model in order to understand the model and verify results.
- Modifiability - A well-designed spreadsheet should be easy to change or enhance in order to meet dynamic user requirements.

Spreadsheet Design Guidelines

- Organize the data, then build the model around the data.
- Do not embed numeric constants in formulas.
- Things which are logically related should be physically related.
- Use formulas that can be copied.
- Column/rows totals should be close to the columns/rows being totaled.
- The English-reading eye scans left to right, top to bottom.
- Use color, shading, borders and protection to distinguish changeable parameters from other model elements.
- Use text boxes and cell notes to document various elements of the model.

Make vs. Buy Decisions: The Electro-Poly Corporation

Electro-Poly is a leading maker of slip-rings.
A $750,000 order has just been received.

	Model 1	Model 2	Model 3
Number ordered	**3,000**	**2,000**	**900**
Hours of wiring/unit	**2**	**1.5**	**3**
Hours of harnessing/unit	**1**	**2**	**1**
Cost to Make	**$50**	**$83**	**$130**
Cost to Buy	**$61**	**$97**	**$145**

- The company has 10,000 hours of wiring capacity and 5,000 hours of harnessing capacity.

Defining the Decision Variables

M_1 = Number of model 1 slip rings to make in-house

M_2 = Number of model 2 slip rings to make in-house

M_3 = Number of model 3 slip rings to make in-house

B_1 = Number of model 1 slip rings to buy from competitor

B_2 = Number of model 2 slip rings to buy from competitor

B_3 = Number of model 3 slip rings to buy from competitor

Defining the Objective Function

Minimize the total cost of filling the order.

MIN: $50M_1 + 83M_2 + 130M_3 + 61B_1 + 97B_2 + 145B_3$

Defining the Constraints

Demand Constraints

$M_1 + B_1 = 3{,}000$ } model 1

$M_2 + B_2 = 2{,}000$ } model 2

$M_3 + B_3 = 900$ } model 3

Resource Constraints

$2M_1 + 1.5M_2 + 3M_3 <= 10{,}000$ } wiring

$1M_1 + 2.0M_2 + 1M_3 <= 5{,}000$ } harnessing

Non-negativity Conditions

$M_1, M_2, M_3, B_1, B_2, B_3 >= 0$

Implementing the Model

See file Fig3-17.xls

An Investment Problem: Retirement Planning Services, Inc.

A client wishes to invest $750,000 in the following bonds.

Company	Return	Years to Maturity	Rating
Acme Chemical	8.65%	11	1-Excellent
DynaStar	9.50%	10	3-Good
Eagle Vision	10.00%	6	4-Fair
Micro Modeling	8.75%	10	1-Excellent
OptiPro	9.25%	7	3-Good
Sabre Systems	9.00%	13	2-Very Good

Investment Restrictions

- No more than 25% can be invested in any single company.
- At least 50% should be invested in long-term bonds (maturing in 10+ years).
- No more than 35% can be invested in DynaStar, Eagle Vision, and OptiPro.

Defining the Decision Variables

X_1 = amount of money to invest in Acme Chemical

X_2 = amount of money to invest in DynaStar

X_3 = amount of money to invest in Eagle Vision

X_4 = amount of money to invest in MicroModeling

X_5 = amount of money to invest in OptiPro

X_6 = amount of money to invest in Sabre Systems

Defining the Objective Function

Maximize the total annual investment return.

$$\text{MAX: } .0865X_1 + .095X_2 + .10X_3 + .0875X_4 + .0925X_5 + .09X_6$$

Defining the Constraints

Total amount is invested

$$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 = 750{,}000$$

No more than 25% in any one investment

$$X_i <= 187{,}500, \text{ for all } \boldsymbol{i}$$

50% long term investment restriction.

$$X_1 + X_2 + X_4 + X_6 >= 375{,}000$$

35% Restriction on DynaStar, Eagle Vision, and OptiPro.

$$X_2 + X_3 + X_5 <= 262{,}500$$

Non-negativity conditions

$$X_i >= 0 \text{ for all } \boldsymbol{i}$$

Implementing the Model

See file Fig3-20.xls

A scheduling problem: Air-Express

- Anyone responsible for creating work schedules for a number of employees can appreciate the difficulties in this task. It can be very difficult to develop a ***feasible*** schedule, much less than an ***optimal*** schedule.

- LP models have been devised to solve these problems.

A scheduling problem: Air-Express

Air-Express is an express shipping service that guarantees overnight delivery of packages anywhere in the continental United states. The Co. has various operations centers, called hubs, at airports in major cities across the country. Packages are received at hubs from other locations and then shipped to intermediate hubs or to their final destinations.

A scheduling problem: Air-Express

Day of the week	Workers reqd
Sunday	18
Monday	27
Tuesday	22
Wednesday	26
Thursday	25
Friday	21
Saturday	19

shift	Days Off	Wage
1	Sunday & Monday	$680
2	Monday & Tuesday	705
3	Tuesday & Wednesday	705
4	Wednesday & Thursday	705
5	Thursday & Friday	705
6	Friday & Saturday	680
7	Saturday & Sunday	655

As per negotiation, Saturday and Sunday are off days and the Co. has to pay extra $ 25 per day for workers on their work on these days.

A scheduling problem: Air-Express

Model

Min:

$$680x_1 + 705x_2 + 705x_3 + 705x_4 + 705x_5 + 680x_6 + 655x_7$$

$$s.t.$$

$$0x_1 + 1x_2 + 1x_3 + 1x_4 + 1x_5 + 1x_6 + 0x_7 \geq 18$$

$$0x_1 + 0x_2 + 1x_3 + 1x_4 + 1x_5 + 1x_6 + 1x_7 \geq 27$$

$$1x_1 + 0x_2 + 0x_3 + 1x_4 + 1x_5 + 1x_6 + 1x_7 \geq 22$$

$$1x_1 + 1x_2 + 0x_3 + 0x_4 + 1x_5 + 1x_6 + 1x_7 \geq 26$$

$$1x_1 + 1x_2 + 1x_3 + 0x_4 + 0x_5 + 1x_6 + 1x_7 \geq 25$$

$$1x_1 + 1x_2 + 1x_3 + 1x_4 + 0x_5 + 0x_6 + 1x_7 \geq 21$$

$$1x_1 + 1x_2 + 1x_3 + 1x_4 + 1x_5 + 0x_6 + 0x_7 \geq 19$$

$$x_1, \qquad x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0$$

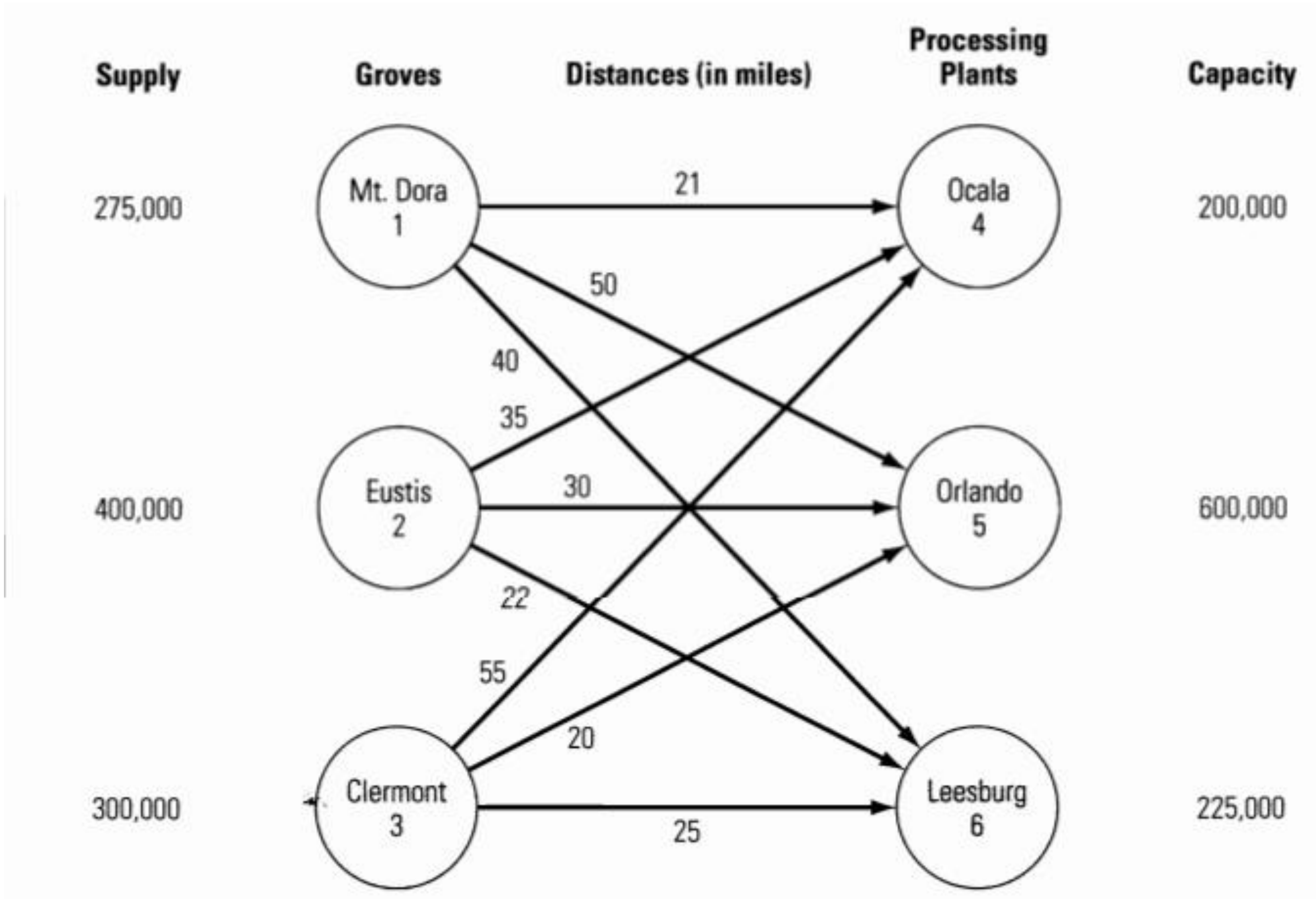
Transportation & Assignment Problems

Tropicsun is a leading grower and distributor of fresh citrus products with three large citrus groves scattered around central Florida in the cities of Mt. Dora, Eustis, and Clermont. Tropicsun currently has 275,000 bushels of citrus at the grove in Mt. Dora, 400,000 bushels at the grove in Eustis, and 300,000 bushels at the grove in Clermont. Tropicsun has citrus processing plants in Ocala, Orlando, and Leesburg with processing capacities to handle 200,000, 600,000, and 225,000 bushels, respectively. Tropicsun contracts with a local trucking company to transport its fruit from the groves to the processing plants. The trucking company charges a flat rate for every mile that each bushel of fruit must be transported. Each mile a bushel of fruit travels is known as a bushel-mile. The following table summarizes the distances (in miles) between the groves and processing plants:

	Distances (in miles) Between Groves and Plants		
Grove	**Ocala**	**Orlando**	**Leesburg**
Mt. Dora	21	50	40
Eustis	35	30	22
Clermont	55	20	25

Tropicsun wants to determine how many bushels to ship from each grove to each processing plant to minimize the total number of bushel-miles the fruit must be shipped.

A Transportation Problem: Tropicsun



Defining the Decision Variables

X_{ij} = # of bushels shipped from node *i* to node *j*

Specifically, the nine decision variables are:

X_{14} = # of bushels shipped from Mt. Dora (node 1) to Ocala (node 4)

X_{15} = # of bushels shipped from Mt. Dora (node 1) to Orlando (node 5)

X_{16} = # of bushels shipped from Mt. Dora (node 1) to Leesburg (node 6)

X_{24} = # of bushels shipped from Eustis (node 2) to Ocala (node 4)

X_{25} = # of bushels shipped from Eustis (node 2) to Orlando (node 5)

X_{26} = # of bushels shipped from Eustis (node 2) to Leesburg (node 6)

X_{34} = # of bushels shipped from Clermont (node 3) to Ocala (node 4)

X_{35} = # of bushels shipped from Clermont (node 3) to Orlando (node 5)

X_{36} = # of bushels shipped from Clermont (node 3) to Leesburg (node 6)

Defining the Objective Function

Minimize the total number of bushel-miles.

$$\begin{aligned}\text{MIN: } & 21X_{14} + 50X_{15} + 40X_{16} + \\ & 35X_{24} + 30X_{25} + 22X_{26} + \\ & 55X_{34} + 20X_{35} + 25X_{36}\end{aligned}$$

Defining the Constraints

Capacity constraints

$X_{14} + X_{24} + X_{34} <= 200{,}000$ } Ocala

$X_{15} + X_{25} + X_{35} <= 600{,}000$ } Orlando

$X_{16} + X_{26} + X_{36} <= 225{,}000$ } Leesburg

Supply constraints

$X_{14} + X_{15} + X_{16} = 275{,}000$ } Mt. Dora

$X_{24} + X_{25} + X_{26} = 400{,}000$ } Eustis

$X_{34} + X_{35} + X_{36} = 300{,}000$ } Clermont

Nonnegativity conditions

$X_{ij} >= 0$ for all ***i* and *j***

Implementing the Model

See file Fig3-24.xls

A Blending Problem: The Agri-Pro Company

Agri-Pro has received an order for 8,000 pounds of chicken feed to be mixed from the following feeds.

	Percent of Nutrient in			
Nutrient	**Feed 1**	**Feed 2**	**Feed 3**	**Feed 4**
Corn	**30%**	**5%**	**20%**	**10%**
Grain	**10%**	**30%**	**15%**	**10%**
Minerals	**20%**	**20%**	**20%**	**30%**
Cost per pound	**$0.25**	**$0.30**	**$0.32**	**$0.15**

- The order must contain at least 20% corn, 15% grain, and 15% minerals.

Defining the Decision Variables

X_1 = pounds of feed 1 to use in the mix

X_2 = pounds of feed 2 to use in the mix

X_3 = pounds of feed 3 to use in the mix

X_4 = pounds of feed 4 to use in the mix

Defining the Objective Function

Minimize the total cost of filling the order.

$$\text{MIN:} \quad 0.25X_1 + 0.30X_2 + 0.32X_3 + 0.15X_4$$

Defining the Constraints

Produce 8,000 pounds of feed

$$X_1 + X_2 + X_3 + X_4 = 8{,}000$$

Mix consists of at least 20% corn

$$(0.3X_1 + 0.5X_2 + 0.2X_3 + 0.1X_4)/8000 >= 0.2$$

Mix consists of at least 15% grain

$$(0.1X_1 + 0.3X_2 + 0.15X_3 + 0.1X_4)/8000 >= 0.15$$

Mix consists of at least 15% minerals

$$(0.2X_1 + 0.2X_2 + 0.2X_3 + 0.3X_4)/8000 >= 0.15$$

Nonnegativity conditions

$$X_1, X_2, X_3, X_4 >= 0$$

A Comment About Scaling

- Notice that the coefficient for X_2 in the 'corn' constraint is $0.05/8000 = 0.00000625$
- As Solver solves our problem, intermediate calculations must be done that make coefficients large or smaller.
- Storage problems may force the computer to use approximations of the actual numbers.
- Such 'scaling' problems sometimes prevents Solver from being able to solve the problem accurately.
- Most problems can be formulated in a way to minimize scaling errors...

Re-Defining the Decision Variables

X_1 = ***thousands of pounds*** of feed 1 to use in the mix

X_2 = ***thousands of pounds*** of feed 2 to use in the mix

X_3 = ***thousands of pounds*** of feed 3 to use in the mix

X_4 = ***thousands of pounds*** of feed 4 to use in the mix

Re-Defining the Objective Function

Minimize the total cost of filling the order.

$$\text{MIN:}\quad 250X_1 + 300X_2 + 320X_3 + 150X_4$$

Re-Defining the Constraints

Produce 8,000 pounds of feed

$X_1 + X_2 + X_3 + X_4 = 8$

Mix consists of at least 20% corn

$(0.3X_1 + 0.5X_2 + 0.2X_3 + 0.1X_4)/8 >= 0.2$

Mix consists of at least 15% grain

$(0.1X_1 + 0.3X_2 + 0.15X_3 + 0.1X_4)/8 >= 0.15$

Mix consists of at least 15% minerals

$(0.2X_1 + 0.2X_2 + 0.2X_3 + 0.3X_4)/8 >= 0.15$

Nonnegativity conditions

$X_1, X_2, X_3, X_4 >= 0$

A Comment About Scaling

- Earlier the largest coefficient in the constraints was 8,000 and the smallest is 0.05/8 = 0.00000625.
- Now the largest coefficient in the constraints is 8 and the smallest is 0.05/8 = 0.00625.
- The problem is now more evenly scaled.

A Production Planning Problem: The Upton Corporation

Upton is planning the production of their heavy-duty air compressors for the next 6 months.

	Month					
	1	**2**	**3**	**4**	**5**	**6**
Unit Production Cost	**$240**	**$250**	**$265**	**$285**	**$280**	**$260**
Units Demanded	**1,000**	**4,500**	**6,000**	**5,500**	**3,500**	**4,000**
Maximum Production	**4,000**	**3,500**	**4,000**	**4,500**	**4,000**	**3,500**
Minimum Production	**2,000**	**1,750**	**2,000**	**2,250**	**2,000**	**1,750**

- Beginning inventory = 2,750 units
- Safety stock = 1,500 units
- Unit carrying cost = 1.5% of unit production cost
- Maximum warehouse capacity = 6,000 units

Defining the Decision Variables

P_i = number of units to produce in month i, $i=1$ *to* 6

B_i = beginning inventory month i, $i=1$ *to* 6

Defining the Objective Function

Minimize the total cost production & inventory costs.

$$
\begin{aligned}
\text{MIN:}\ & 240P_1 + 250P_2 + 265P_3 + 285P_4 + 280P_5 + 260P_6 + \\
& 3.6(B_1+B_2)/2 + 3.75(B_2+B_3)/2 + 3.98(B_3+B_4)/2 + \\
& 4.28(B_4+B_5)/2 + 4.20(B_5+B_6)/2 + 3.9(B_6+B_7)/2
\end{aligned}
$$

Note: The beginning inventory in any month is the same as the ending inventory in the previous month.

Defining the Constraints

Production levels

$2{,}000 <= P_1 <= 4{,}000$ } month 1
$1{,}750 <= P_2 <= 3{,}500$ } month 2
$2{,}000 <= P_3 <= 4{,}000$ } month 3
$2{,}250 <= P_4 <= 4{,}500$ } month 4
$2{,}000 <= P_5 <= 4{,}000$ } month 5
$1{,}750 <= P_6 <= 3{,}500$ } month 6

Ending Inventory (EI = BI + P - D)

$1{,}500 <= B_1 + P_1 - 1{,}000 <= 6{,}000$ } month 1
$1{,}500 <= B_2 + P_2 - 4{,}500 <= 6{,}000$ } month 2
$1{,}500 <= B_3 + P_3 - 6{,}000 <= 6{,}000$ } month 3
$1{,}500 <= B_4 + P_4 - 5{,}500 <= 6{,}000$ } month 4
$1{,}500 <= B_5 + P_5 - 3{,}500 <= 6{,}000$ } month 5
$1{,}500 <= B_6 + P_6 - 4{,}000 <= 6{,}000$ } month 6

Defining the Constraints *(cont'd)*

Beginning Balances

$$B_1 = 2750$$
$$B_2 = B_1 + P_1 - 1{,}000$$
$$B_3 = B_2 + P_2 - 4{,}500$$
$$B_4 = B_3 + P_3 - 6{,}000$$
$$B_5 = B_4 + P_4 - 5{,}500$$
$$B_6 = B_5 + P_5 - 3{,}500$$
$$B_7 = B_6 + P_6 - 4{,}000$$

Notice that the B_i can be computed directly from the P_i. Therefore, only the P_i need to be identified as changing cells.

Implementing the Model

See file Fig3-31.xls

Assignment

Question Number : 14,15, 18,21,22,24, 25, 29,30

End of Chapter 3