Chapter 7

Goal Programming and Multiple Objective Optimization

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7.0 Introduction

Chapter 6 discussed the modeling techniques that apply to optimization problems that require integer solutions. This chapter presents two other modeling techniques that are sometimes helpful in solving optimization problems. The first technique—goal programming—involves solving problems containing not one specific objective function, but rather a collection of goals that we would like to achieve. As we will see, a goal can be viewed as a constraint with a flexible, or soft, Right-Hand Side (RHS) value.

The second technique—multiple objective optimization—is closely related to goal programming and applies to problems containing more than one objective function. In business and government, different groups of people frequently pursue different objectives. Therefore, it is quite possible that a variety of objective functions can be proposed for the same optimization problem.

Both techniques require an *iterative solution procedure* in which the decision maker investigates a variety of solutions to find one that is most satisfactory. Thus, unlike the LP and ILP procedures presented earlier, we cannot formulate a multiple objective or goal programming problem and solve one optimization problem to identify the optimal solution. In these problems, we might need to solve several variations of the problem before we find an acceptable solution.

We will begin with the topic of goal programming. Then, we will investigate multiple objective optimization and see how the concepts and techniques of goal programming can be applied to these problems as well.

7.1 Goal Programming

The optimization techniques presented in the preceding chapters always have assumed that the constraints in the model are **hard constraints**, or constraints that *cannot* be violated. For example, labor constraints indicated that the amount of labor used to produce a variety of products could not exceed some fixed amount (such as 1,566 hours). As another example, monetary constraints indicated that the amount of money invested in a number of projects could not exceed some budgeted amount (such as $850,000).

Hard constraints are appropriate in many situations; however, these constraints might be too restrictive in other situations. For example, when you buy a new car, you probably have in mind a maximum purchase price that you do not want to exceed. We might call this your goal. However, you probably will find a way to spend more than this amount if it is impossible to acquire the car you really want for your goal amount. So, the goal you have in mind is *not* a hard constraint that cannot be violated. We might view it more accurately as a **soft constraint**—representing a target that you would like to achieve.

Numerous managerial decision-making problems can be modeled more accurately using goals rather than hard constraints. Often, such problems do not have one explicit objective function to be maximized or minimized over a constraint set but, instead, can be stated as a collection of goals that also might include hard constraints. These types of problems are known as **goal programming** (GP) problems.

Balancing Objectives for Enlightened Self-Interest

As he stood on a wooded hillside watching water cascade over what once was a coal mine, Roger Holnback, executive director of the Western Virginia Land Trust, described what that section of land could have looked like if a typical subdivision was being built in the area. "They'd figure out a way to use this bottom land for development," he said, pointing out how neatly a row of houses could fit in below the hill. "They maximize the lots they build to whatever the zoning says."

But because of an agreement between developers Bill Ellenbogen and Steve Bodtke and the Western Virginia Land Trust and New River Land Trust, nearly half of a 225-acre subdivision on Coal Bank Ridge will be preserved through a conservation easement. "Our goal was to do a nice development while protecting the surrounding areas," Ellenbogen said. Conservation easements are agreements between landowners and land trusts to restrict development while allowing the owner to keep the property and continue to use it. The trusts monitor use of the land to make sure that it complies with the conditions of the easement.

Ellenbogen doesn't try to hide the fact that he's a businessman, and as a developer he needs to make a profit. But making a profit and preserving the scenic views and rural character of the area are not mutually exclusive goals. "We think it adds tremendous value," he said. "People live in this community because of the beauty of the land. If you destroy that beauty, people won't want to live here. I call it enlightened self-interest." "The question is, 'How can I make money and still have a livable community?'" Holnback said. "It's a simple concept."

Adapted from: "Developers see conservation as smart business," *The Roanoke Times*, Dec. 20, 2003.

7.2 A Goal Programming Example

The technique of linear programming can help a decision maker analyze and solve a GP problem. The following example illustrates the concepts and modeling techniques used in GP problems.

Davis McKeown is the owner of a resort hotel and convention center in Myrtle Beach, South Carolina. Although his business is profitable, it is also highly seasonal; the summer months are the most profitable time of year. To increase profits during the rest of the year, Davis wants to expand his convention business but, to do so, he needs to expand his conference facilities. Davis hired a marketing research firm to determine the number and sizes of conference rooms that would be required by the conventions he wants to attract. The results of this study indicated that Davis's facilities should include at least 5 small (400 square foot) conference rooms, 10 medium (750 square foot) conference rooms, and 15 large (1,050 square foot) conference rooms. Additionally, the marketing research firm indicated that if the expansion consisted of a total of 25,000 square feet, Davis would have the largest convention center among his competitors—which would be desirable for advertising purposes. While discussing his expansion plans with an architect, Davis learned that he can expect to pay $18,000 for each small conference room in the expansion, $33,000 for each medium conference room, and $45,150 for each large conference room. Davis wants to limit his expenditures on the convention center expansion to approximately $1,000,000.

7.2.1 DEFINING THE DECISION VARIABLES

In this problem, the fundamental decision facing the hotel owner is how many small, medium, and large conference rooms to include in the conference center expansion. These quantities are represented by X_1, X_2, and X_3, respectively.

7.2.2 DEFINING THE GOALS

This problem is somewhat different from the problems presented earlier in this book. Rather than one specific objective, this problem involves several goals, which are stated (in no particular order) as:

Goal 1: The expansion should include approximately 5 small conference rooms.
Goal 2: The expansion should include approximately 10 medium conference rooms.
Goal 3: The expansion should include approximately 15 large conference rooms.
Goal 4: The expansion should consist of approximately 25,000 square feet.
Goal 5: The expansion should cost approximately $1,000,000.

Notice that the word "approximately" appears in each goal. This word reinforces that these are soft goals rather than hard constraints. For example, if the first four goals could be achieved at a cost of $1,001,000, it is very likely that the hotel owner would not mind paying an extra $1,000 to achieve such a solution. However, we must determine if we can find a solution that exactly meets all of the goals in this problem and, if not, what trade-offs can be made among the goals to determine an acceptable solution. We can formulate an LP model for this GP problem to help us make this determination.

7.2.3 DEFINING THE GOAL CONSTRAINTS

The first step in formulating an LP model for a GP problem is to create a goal constraint for each goal in the problem. A **goal constraint** allows us to determine how close a given solution comes to achieving the goal. To understand how these constraints should be formulated, let's begin with the three goal constraints associated with the number of small, medium, and large conference rooms in the expansion.

If we wanted to make sure that *exactly* 5 small, 10 medium, and 15 large conference rooms were included in the planned expansion, we would include the following hard constraints in our GP model:

$$X_1 = 5$$
$$X_2 = 10$$
$$X_3 = 15$$

However, the goals stated that the expansion should include *approximately* 5 small conference rooms, *approximately* 10 medium conference rooms, and *approximately* 15 large conference rooms. If it is impossible to achieve all the goals, the hotel owner might consider a solution involving only 14 large conference rooms. The hard constraints would not allow for such a solution; they are too restrictive. However, we can modify them easily to allow for departures from the stated goals, as:

$$
\begin{aligned}
&X_1 + d_1^- - d_1^+ = 5 && \}\text{ small rooms}\\
&X_2 + d_2^- - d_2^+ = 10 && \}\text{ medium rooms}\\
&X_3 + d_3^- - d_3^+ = 15 && \}\text{ large rooms}\\
&\text{where } d_i^-, d_i^+ \geq 0 \text{ for all } i
\end{aligned}
$$

The RHS value of each goal constraint (the values 5, 10, and 15 in the previous constraints) is the **target value** for the goal because it represents the level of achievement that the decision maker wants to obtain for the goal. The variables d_i^- and d_i^+ are called **deviational variables** because they represent the amount by which each goal deviates from its target value. The d_i^- represents the amount by which each goal's target value is *underachieved*, and the d_i^+ represents the amount by which each goal's target value is *overachieved*.

To illustrate how deviational variables work, suppose that we have a solution where $X_1 = 3$, $X_2 = 13$, and $X_3 = 15$. To satisfy the first goal constraint listed previously, its deviational variables would assume the values $d_1^- = 2$ and $d_1^+ = 0$ to reflect that the goal of having 5 small conference rooms is *underachieved* by 2. Similarly, to satisfy the second goal constraint, its deviational variables would assume the values $d_2^- = 0$ and $d_2^+ = 3$ to reflect that the goal of having 10 medium conference rooms is *overachieved* by 3. Finally, to satisfy the third goal constraint, its deviational variables would assume the values $d_3^- = 0$ and $d_3^+ = 0$, to reflect that the goal of having 15 medium conference rooms is *exactly* achieved.

We can formulate the goal constraints for the remaining goals in the problem in a similar manner. Because each small, medium, and large conference room requires 400, 750, and 1,050 square feet, respectively, and the hotel owner wants the total square footage of the expansion to be 25,000, the constraint representing this goal is:

$$400X_1 + 750X_2 + 1{,}050X_3 + d_4^- - d_4^+ = 25{,}000 \qquad \}\text{ square footage}$$

Because each small, medium, and large conference room results in building costs of \$18,000, \$33,000, and \$45,150, respectively, and the hotel owner wants to keep the cost of the expansion at approximately \$1,000,000, the constraint representing this goal is:

$$18{,}000X_1 + 33{,}000X_2 + 45{,}150X_3 + d_5^- - d_5^+ = 1{,}000{,}000 \qquad \}\text{ building cost}$$

The deviational variables in each of these goal constraints represent the amounts by which the actual values obtained for the goals deviate from their respective target values.

7.2.4 DEFINING THE HARD CONSTRAINTS

As noted earlier, not all of the constraints in a GP problem have to be goal constraints. A GP problem also can include one or more hard constraints typically found in LP problems. In our example, if \$1,000,000 was the absolute maximum amount that the hotel owner was willing to spend on the expansion, this could be included in the model as a hard constraint. (As we'll see, it is also possible to change a soft constraint into a hard constraint during the analysis of a GP problem.)

7.2.5 GP OBJECTIVE FUNCTIONS

Although it is fairly easy to formulate the constraints for a GP problem, identifying an appropriate objective function can be quite tricky and usually requires some mental effort. Before formulating the objective function for our sample problem, let's consider some of the issues and options involved in this process.

The objective in a GP problem is to determine a solution that achieves all the goals as closely as possible. The *ideal* solution to any GP problem is one in which each goal is achieved exactly at the level specified by its target value. (In such an ideal solution, all the deviational variables in all the goal constraints would equal 0.) Often, it is not possible to achieve the ideal solution because some goals might conflict with others. In such a case, we want to find a solution that deviates as little as possible from the ideal solution. One possible objective for our example GP problem is:

$$\text{Minimize the sum of the deviations:} \qquad \text{MIN:} \quad \sum_i (d_i^- + d_i^+)$$

With this objective, we attempt to find a solution to the problem where all the deviational variables are 0—or where all the goals are met exactly. But if such a solution is not possible, will this objective always produce a desirable solution? The answer is "probably not."

The previous objective has several shortcomings. First, the deviational variables measure entirely different things. In our example problem, $d_1^-, d_1^+, d_2^-, d_2^+, d_3^-$, and d_3^+ all measure rooms of one size or another, whereas d_4^-, and d_4^+ are measures of square footage, and d_5^-, and d_5^+ are financial measures of building costs. One obvious criticism of the previous objective is that it is unclear how to interpret any numerical value that the objective assumes (7 rooms + 1,500 dollars = 1,507 units of what?).

One solution to this problem is to modify the objective function so that it measures the sum of *percentage deviations* from the various goals. This is accomplished as follows, where t_i represents the target value for goal i:

$$\text{Minimize the sum of the percentage deviations:} \qquad \text{MIN:} \quad \sum_i \frac{1}{t_i}(d_i^- + d_i^+)$$

In our example problem, suppose that we arrive at a solution where the first goal is underachieved by 1 room ($d_1^- = 1$) and the fifth goal is overachieved by \$20,000 ($d_5^+ =$ 20,000) and all other goals are achieved exactly (all other d_i^- and d_i^+ equal 0). Using the sum of percentage deviations objective, the optimal objective function value is:

$$\frac{1}{t_1}d_1^- + \frac{1}{t_5}d_5^+ = \frac{1}{5} \times 1 + \frac{1}{1{,}000{,}000} \times 20{,}000 = 20\% + 2\% = 22\%$$

Note that the percentage deviation objective can be used only if all the target values for all the goals are non-zero; otherwise a division by zero error will occur.

Another potential criticism of the previous objective functions concerns how they evaluate deviations. In the previous example, where the objective function value is 22%, the objective function implicitly assumes that having 4 small conference rooms (rather than 5) is 10 times worse than being \$20,000 over the desired building cost budget. That is, the budget overrun of \$20,000 would have to increase 10 times to \$200,000 before the percentage deviation on this goal equaled the 20% deviation caused by being one room below the goal of having 5 small conference rooms. Is having one fewer conference room really as undesirable as having to pay \$200,000 more than budgeted? Only the decision maker in this problem can answer this question. It would be nice to provide the decision maker a way to evaluate and change the implicit trade-offs among the goals if he or she wanted to do so.

Both of the previous objective functions view a deviation from any goal in any direction as being equally undesirable. For example, according to both of the previous objective functions, a solution resulting in a building cost of \$900,000 (if $X_5 = 900{,}000$ and

$d_5^- = 100{,}000$) is as undesirable as a solution with a building cost of \$1,100,000 (if $X_5 = 1{,}100{,}000$ and $d_5^+ = 100{,}000$). But, the hotel owner probably would prefer to pay \$900,000 for the expansion rather than \$1,100,000. So, while overachieving the building cost goal is an undesirable occurrence, underachieving this goal is probably desirable or at least neutral. On the other hand, underachieving the goal related to the number of small conference rooms might be viewed as undesirable, whereas overachieving this goal might be viewed as desirable or possibly neutral. Again, it would be nice to provide the decision maker a way to represent which deviations are desirable and undesirable in the objective function.

One solution to the previous criticisms is to allow the decision maker to assign weights to the deviational variables in the objective function of a GP problem, to better reflect the importance and desirability of deviations from the various goals. So, a more useful type of objective function for a GP problem is:

$$\text{Minimize the weighted sum of the deviations: MIN: } \sum_i (w_i^- d_i^- + w_i^+ d_i^+)$$

or

$$\text{Minimize the weighted sum of the percentage deviations: MIN: } \sum_i \frac{1}{t_i}(w_i^- d_i^- + w_i^+ d_i^+)$$

In these weighted objective functions, the w_i^- and w_i^+ represent numeric constants that can be assigned values to weight the various deviational variables in the problem. A variable that represents a highly undesirable deviation from a particular goal is assigned a relatively large weight—making it highly undesirable for that variable to assume a value larger than 0. A variable that represents a neutral or desirable deviation from a particular goal is assigned a weight of 0 or some value lower than 0 to reflect that it is acceptable or even desirable for the variable to assume a value greater than 0.

Unfortunately, no standard procedure is available for assigning values to the w_i^- and w_i^+ in a way that guarantees that you will find the most desirable solution to a GP problem. Rather, you need to follow an iterative procedure in which you try a particular set of weights, solve the problem, analyze the solution, and then refine the weights and solve the problem again. You might need to repeat this process many times to find a solution that is the most desirable to the decision maker.

7.2.6 DEFINING THE OBJECTIVE

In our example problem, assume that the decision maker considers it undesirable to underachieve any of the first three goals related to the number of small, medium, and large conference rooms, but is indifferent about overachieving these goals. Also assume that the decision maker considers it undesirable to underachieve the goal of adding 25,000 square feet, but equally undesirable to overachieve this goal. Finally, assume that the decision maker finds it undesirable to spend more than \$1,000,000, but is indifferent about spending less than this amount. In this case, if we want to minimize the weighted percentage deviation for our example problem, we use the following objective:

$$\text{MIN:} \quad \frac{w_1^-}{5} d_1^- + \frac{w_2^-}{10} d_2^- + \frac{w_3^-}{15} d_3^- + \frac{w_4^-}{25{,}000} d_4^- + \frac{w_4^+}{25{,}000} d_4^+ + \frac{w_5^+}{1{,}000{,}000} d_5^+$$

Notice that this objective omits (or assigns weights of 0 to) the deviational variables about which the decision maker is indifferent. Thus, this objective would not penalize a solution where, for example, 7 small conference rooms were selected (and therefore $d_1^+ = 2$) because we assume that the decision maker would not view this as an undesirable deviation from the goal of having 5 small conference rooms. On the other hand, this objective would penalize a solution where 3 small conference rooms were selected (and therefore $d_1^- = 2$) because this represents an undesirable deviation from the goal of having 5 small conference rooms. To begin our analysis of this problem, we will assume that $w_1^- = w_2^- = w_3^- = w_4^- = w_4^+ = w_5^+ = 1$ and all other weights are 0.

7.2.7 IMPLEMENTING THE MODEL

To summarize, the LP model for our example GP problem is:

$$\text{MIN:} \quad \frac{w_1^-}{5} d_1^- + \frac{w_2^-}{10} d_2^- + \frac{w_3^-}{15} d_3^- + \frac{w_4^-}{25{,}000} d_4^- + \frac{w_4^+}{25{,}000} d_4^+ + \frac{w_5^+}{1{,}000{,}000} d_5^+$$

Subject to:

$$\begin{aligned}
X_1 + d_1^- - d_1^+ &= 5 && \}\text{ small rooms} \\
X_2 + d_2^- - d_2^+ &= 10 && \}\text{ medium rooms} \\
X_3 + d_3^- - d_3^+ &= 15 && \}\text{ large rooms} \\
400X_1 + 750X_3 + 1{,}050X_3 + d_4^- - d_4^+ &= 25{,}000 && \}\text{ square footage} \\
18{,}000X_1 + 33{,}000X_2 + 45{,}150X_3 + d_5^- - d_5^+ &= 1{,}000{,}000 && \}\text{ building cost} \\
d_i^-, d_i^+ \geq 0 \text{ for all } i & && \}\text{ non-negativity conditions} \\
X_i \geq 0 \text{ for all } i & && \}\text{ non-negativity conditions} \\
X_i \text{ must be integers} & &&
\end{aligned}$$

Because this is an ILP model, it can be implemented in a spreadsheet in the usual way. One approach for doing this is shown in Figure 7.1 (and in the file Fig7-1.xls on your data disk).

The first section of the spreadsheet lists basic data about the square footage and costs of the different conference rooms. The next section represents the decision variables, deviational variables, and goal constraints for the problem. Specifically, cells B9 through D9 correspond to X_1, X_2, and X_3—the number of small, medium, and large conference rooms to be included in the expansion. Cells E9 and F9 contain the following formulas, which calculate the total square footage and total building cost for any combination of small, medium, and large conference rooms:

Formula for cell E9: =SUMPRODUCT(B9:D9,B5:D5)

Formula for cell F9: =SUMPRODUCT(B9:D9,B6:D6)

Cells B10 through F11 correspond to the deviational variables in our algebraic model. These cells indicate the amount by which each goal is underachieved or overachieved. The LHS formulas for the goal constraints are implemented in cells B12 through F12. Specifically, in cell B12 we enter the following formula and then copy it to cells C12 through F12:

Formula for cell B12: =B9+B10−B11
(Copy to C12 through F12.)

The target (or RHS) values for the goal constraints are listed in cells B13 through F13.

To implement the objective function, we first implemented formulas to convert the values of the deviational variables into percent format by dividing each deviational variable represented in cells B10 through F11 by the appropriate target value. This is done as follows:

Formula for cell B16: =B10/B$13
(Copy to B16 through F17.)

Next, weights for each of the deviational variables are entered in cells B20 through F21. Because solving a GP problem is an iterative process in which you probably will need to change the weights for the objective, it is best to place the weights in a separate location on the spreadsheet.

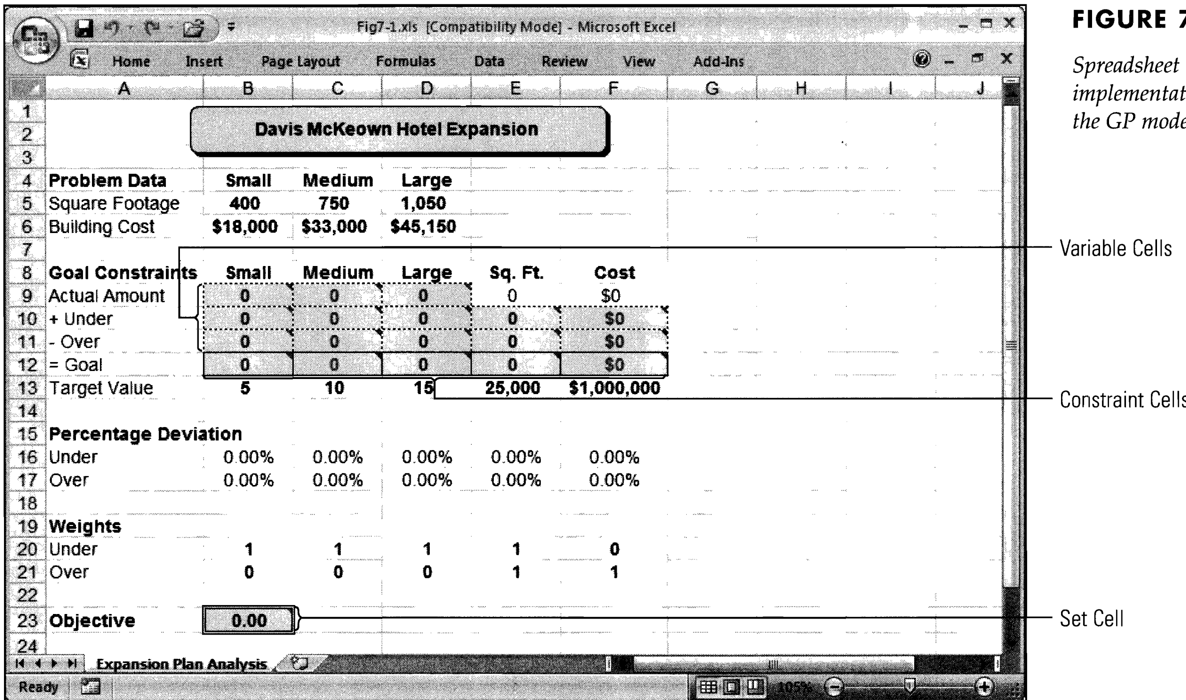


Fig7-1.xls [Compatibility Mode] - Microsoft Excel

	A	B	C	D	E	F
2		Davis McKeown Hotel Expansion				
4	**Problem Data**	**Small**	**Medium**	**Large**		
5	Square Footage	**400**	**750**	**1,050**		
6	Building Cost	**$18,000**	**$33,000**	**$45,150**		
8	**Goal Constraints**	**Small**	**Medium**	**Large**	**Sq. Ft.**	**Cost**
9	Actual Amount	0	0	0	0	$0
10	+ Under	0	0	0	0	$0
11	- Over	0	0	0	0	$0
12	= Goal	0	0	0	0	$0
13	Target Value	5	10	15	25,000	$1,000,000
15	**Percentage Deviation**					
16	Under	0.00%	0.00%	0.00%	0.00%	0.00%
17	Over	0.00%	0.00%	0.00%	0.00%	0.00%
19	**Weights**					
20	Under	1	1	1	1	0
21	Over	0	0	0	1	1
23	**Objective**	0.00				

Key Cell Formulas

Cell	Formula	Copied to
B12	=B9+B10-B11	C12:F12
B16	=B10/B$13	B16:F17
E9	=SUMPRODUCT(B9:D9,B5:D5)	--
F9	=SUMPRODUCT(B9:D9,B6:D6)	--
B23	=SUMPRODUCT(B16:F17,B20:F21)	--

FIGURE 7.1

Spreadsheet implementation of the GP model

Finally, cell B23 contains the following formula, which implements the objective function for the problem:

Formula for cell B23: =SUMPRODUCT(B16:F17,B20:F21)

7.2.8 SOLVING THE MODEL

The model can be solved using the Solver parameters shown in Figure 7.2. The solution obtained using these settings is shown in Figure 7.3.

7.2.9 ANALYZING THE SOLUTION

As shown in Figure 7.3, this solution includes exactly 5 small, 10 medium, and 15 large rooms in the expansion. Thus, there is no deviation at all from the target values for the first three goals, which would please the decision maker. However, considering the fourth and fifth goals, the current solution overachieves the targeted square footage level by 250 square feet (or 1%) and is over the building cost goal by $97,250 (or 9.73%).

FIGURE 7.2

Solver parameters for the GP model

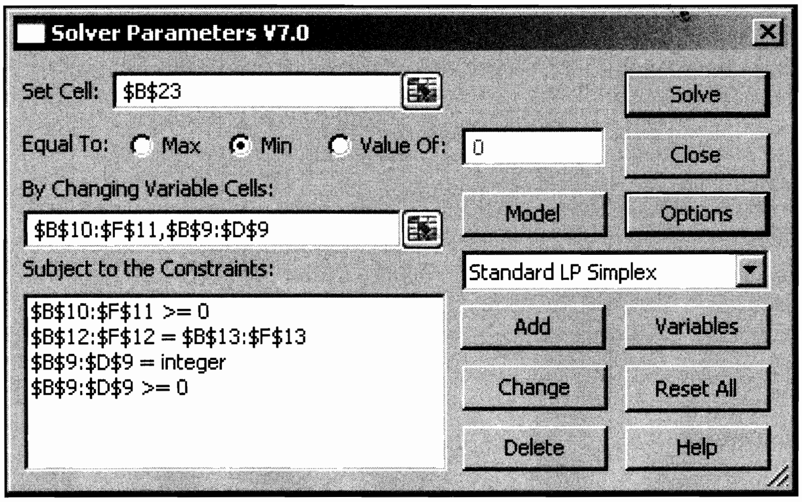
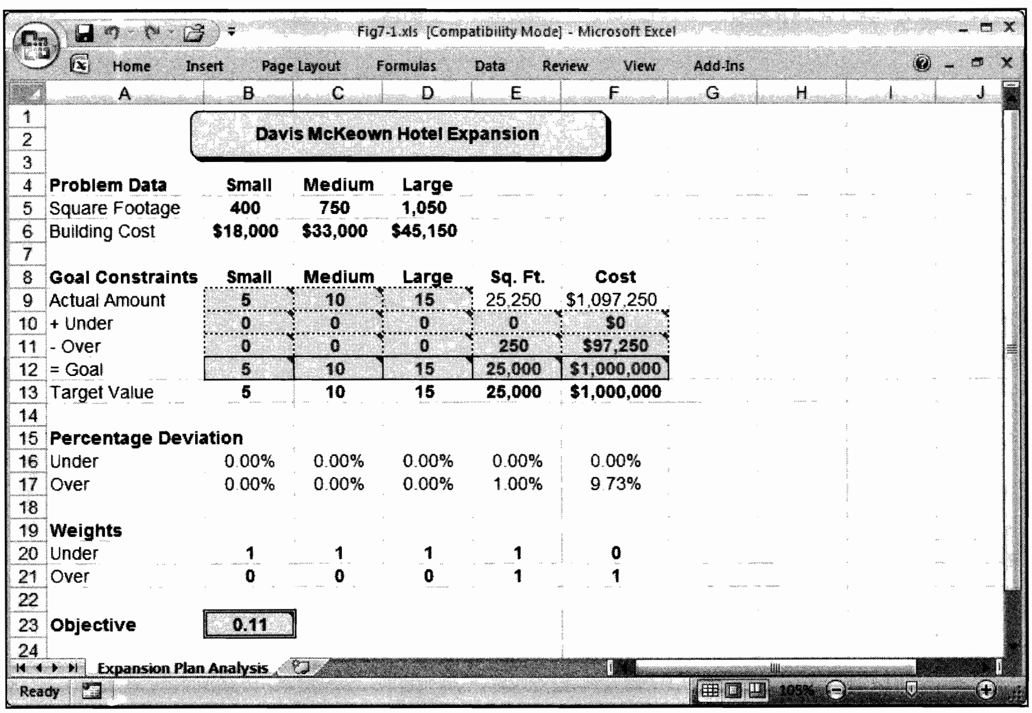


FIGURE 7.3

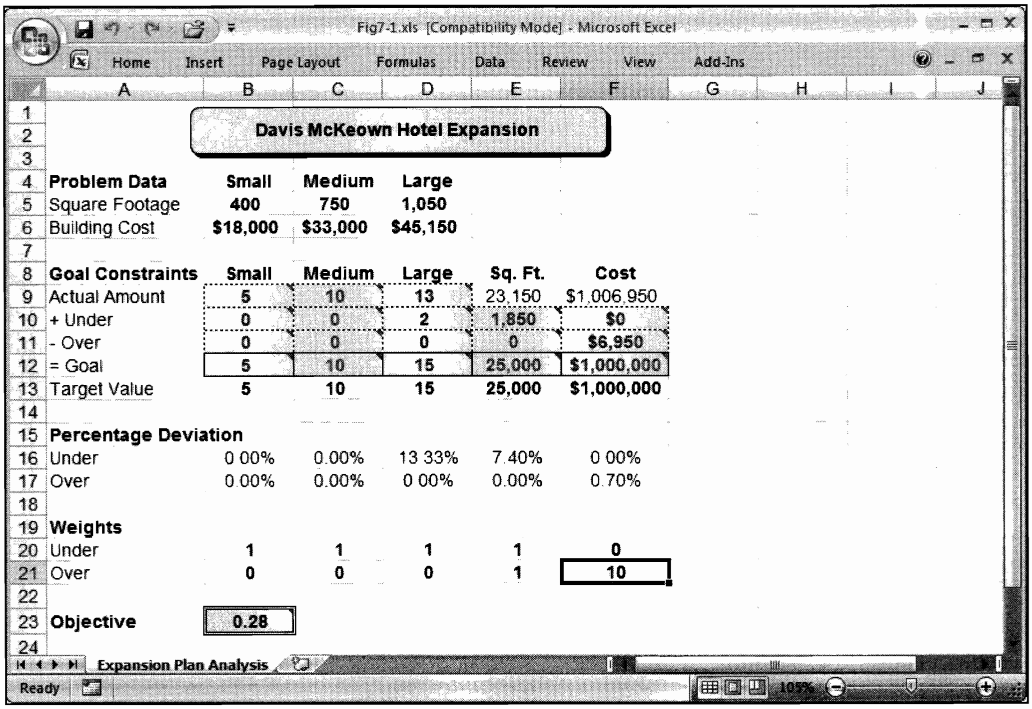
First solution to the GP model



	A	B	C	D	E	F
4	Problem Data	Small	Medium	Large		
5	Square Footage	400	750	1,050		
6	Building Cost	$18,000	$33,000	$45,150		
8	Goal Constraints	Small	Medium	Large	Sq. Ft.	Cost
9	Actual Amount	5	10	15	25,250	$1,097,250
10	+ Under	0	0	0	0	$0
11	- Over	0	0	0	250	$97,250
12	= Goal	5	10	15	25,000	$1,000,000
13	Target Value	5	10	15	25,000	$1,000,000
15	Percentage Deviation					
16	Under	0.00%	0.00%	0.00%	0.00%	0.00%
17	Over	0.00%	0.00%	0.00%	1.00%	9.73%
19	Weights					
20	Under	1	1	1	1	0
21	Over	0	0	0	1	1
23	Objective	0.11				

7.2.10 REVISING THE MODEL

Although the decision maker might not mind being 1% over the square footage goal, exceeding the building cost goal by almost $100,000 most likely would be a concern. The decision maker might want to find another solution that comes closer to achieving the building cost goal. This can be done by adjusting the weights in the problem so that a larger penalty is assigned to overachieving the building cost goal. That is, we can



Davis McKeown Hotel Expansion

Problem Data	Small	Medium	Large		
Square Footage	400	750	1,050		
Building Cost	$18,000	$33,000	$45,150		
Goal Constraints	**Small**	**Medium**	**Large**	**Sq. Ft.**	**Cost**
Actual Amount	5	10	13	23,150	$1,006,950
+ Under	0	0	2	1,850	$0
- Over	0	0	0	0	$6,950
= Goal	5	10	15	25,000	$1,000,000
Target Value	5	10	15	25,000	$1,000,000
Percentage Deviation					
Under	0.00%	0.00%	13.33%	7.40%	0.00%
Over	0.00%	0.00%	0.00%	0.00%	0.70%
Weights					
Under	1	1	1	1	0
Over	0	0	0	1	10
Objective	0.28				

FIGURE 7.4

Second solution to the GP model

increase the value in cell F21 representing w_5^+. Again, there is no way to tell exactly how much larger this value should be. As a rule of thumb, we might change its value by one order of magnitude, or from 1 to 10. If we make this change in the spreadsheet and re-solve the problem, we obtain the solution shown in Figure 7.4.

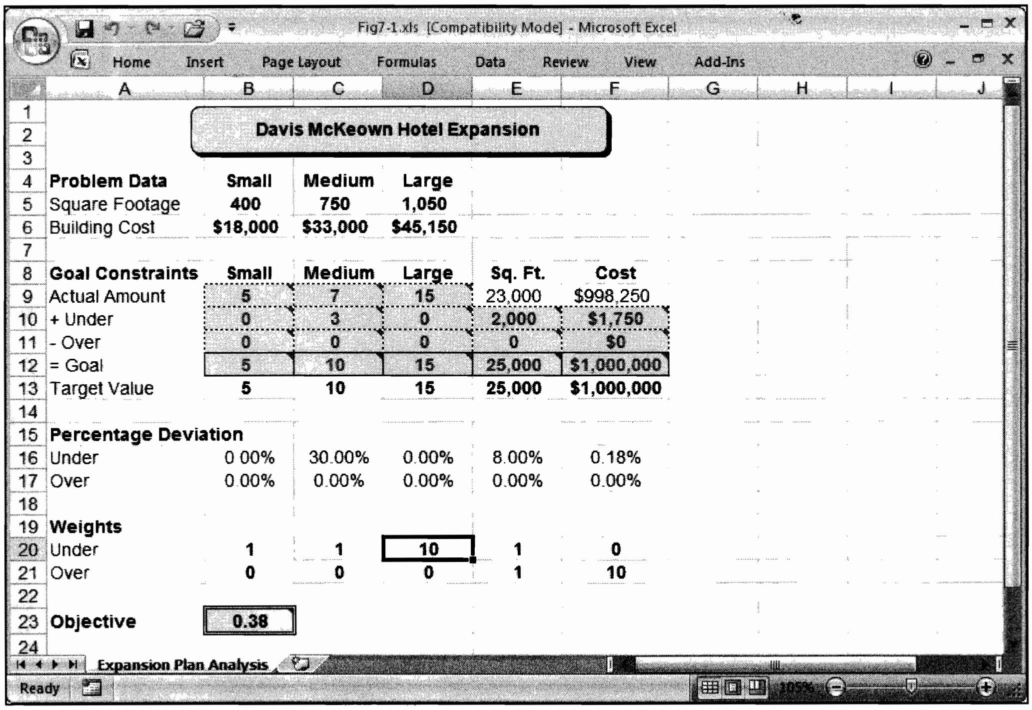
In Figure 7.4, notice that increasing the penalty for overachieving the building cost goal from 1 to 10 reduced the overachievement of this goal from $97,250 to $6,950. We are now within 1% of the target value for the building cost goal. However, to obtain this improved level of achievement on the building cost goal, we had to give up two large conference rooms, resulting in a 13.33% underachievement for this goal. If the decision maker considers this unacceptable, we can increase the penalty on this deviational variable from 1 to 10 and re-solve the problem. Figure 7.5 shows the resulting solution.

7.2.11 TRADE-OFFS: THE NATURE OF GP

In Figure 7.5, the target number of large conference rooms is met exactly, but the desired number of medium rooms is now underachieved by 3. Depending on the preferences of the decision maker, we could continue to fine-tune the weights in the problem until we reach a solution that is most satisfactory to the decision maker. The nature of GP involves making trade-offs among the various goals until a solution is found that gives the decision maker the greatest level of satisfaction. Thus, unlike the other applications of LP presented earlier, the use of LP in GP does not indicate immediately the best possible solution to the problem (unless the decision maker initially specifies an appropriately weighted objective function). Rather, it provides a method by which a decision maker can explore several possible solutions and try to find the solution that comes closest to satisfying the goals under consideration. Figure 7.6 provides a summary of the steps involved in solving a GP problem.

FIGURE 7.5

Third solution to the GP model



	A	B	C	D	E	F
1						
2		Davis McKeown Hotel Expansion				
3						
4	**Problem Data**	**Small**	**Medium**	**Large**		
5	Square Footage	400	750	1,050		
6	Building Cost	$18,000	$33,000	$45,150		
7						
8	**Goal Constraints**	**Small**	**Medium**	**Large**	**Sq. Ft.**	**Cost**
9	Actual Amount	5	7	15	23,000	$998,250
10	+ Under	0	3	0	2,000	$1,750
11	- Over	0	0	0	0	$0
12	= Goal	5	10	15	25,000	$1,000,000
13	Target Value	5	10	15	25,000	$1,000,000
14						
15	**Percentage Deviation**					
16	Under	0.00%	30.00%	0.00%	8.00%	0.18%
17	Over	0.00%	0.00%	0.00%	0.00%	0.00%
18						
19	**Weights**					
20	Under	1	1	10	1	0
21	Over	0	0	0	1	10
22						
23	**Objective**	0.38				
24						

FIGURE 7.6

Summary of the steps involved in formulating and solving a GP problem

Summary of Goal Programming

1. Identify the decision variables in the problem.
2. Identify any hard constraints in the problem and formulate them in the usual way.
3. State the goals of the problem along with their target values.
4. Create constraints using the decision variables that would achieve the goals exactly.
5. Transform the above constraints into goal constraints by including deviational variables.
6. Determine which deviational variables represent undesirable deviations from the goals.
7. Formulate an objective that penalizes the undesirable deviations.
8. Identify appropriate weights for the objective.
9. Solve the problem.
10. Inspect the solution to the problem. If the solution is unacceptable, return to step 8 and revise the weights as needed.

7.3 Comments about Goal Programming

Some additional comments should be made before we leave the topic of GP. First, it is important to note that different GP solutions cannot be compared simply on the basis of their optimal objective function values. The user changes the weights in the objective functions from iteration to iteration; therefore, comparing their values is not appropriate because they measure different things. The objective function in a GP problem serves more of a mechanical purpose, allowing us to explore possible solutions. Thus, we should compare the solutions that are produced—not the objective function values.

Second, in some GP problems, one or more goals might be viewed as being infinitely more important than the other goals. In this case, we could assign arbitrarily large weights to deviations from these goals to ensure that undesirable deviations from them never occur. This is sometimes referred to as *preemptive* GP because certain goals preempt others in order of importance. If the target values for these goals can be achieved, the use of preemptive weights effectively makes these goals hard constraints that should never be violated.

Third, we can place hard constraints on the amount by which we can deviate from a goal. For example, suppose that the owner of the hotel in our example problem wants to eliminate from consideration any solution that exceeds the target building cost by more than \$50,000. We could build this requirement into our model easily with the hard constraint:

$$d_5^+ \leq 50{,}000$$

Fourth, the concept of deviational variables is not limited to GP. These types of variables can be used in other problems that are quite different from GP problems. So, understanding deviational variables can prove useful in other types of mathematical programming situations.

Finally, another type of objective function, called the MINIMAX objective, is sometimes helpful in GP when you want to minimize the maximum deviation from any goal. To implement the MINIMAX objective, we must create one additional constraint for each deviational variable as follows, where Q is the MINIMAX variable:

$$d_1^- \leq Q$$
$$d_1^+ \leq Q$$
$$d_2^- \leq Q$$
and so on . . .

The objective is to minimize the value of Q, stated as:

$$\text{MIN: } Q$$

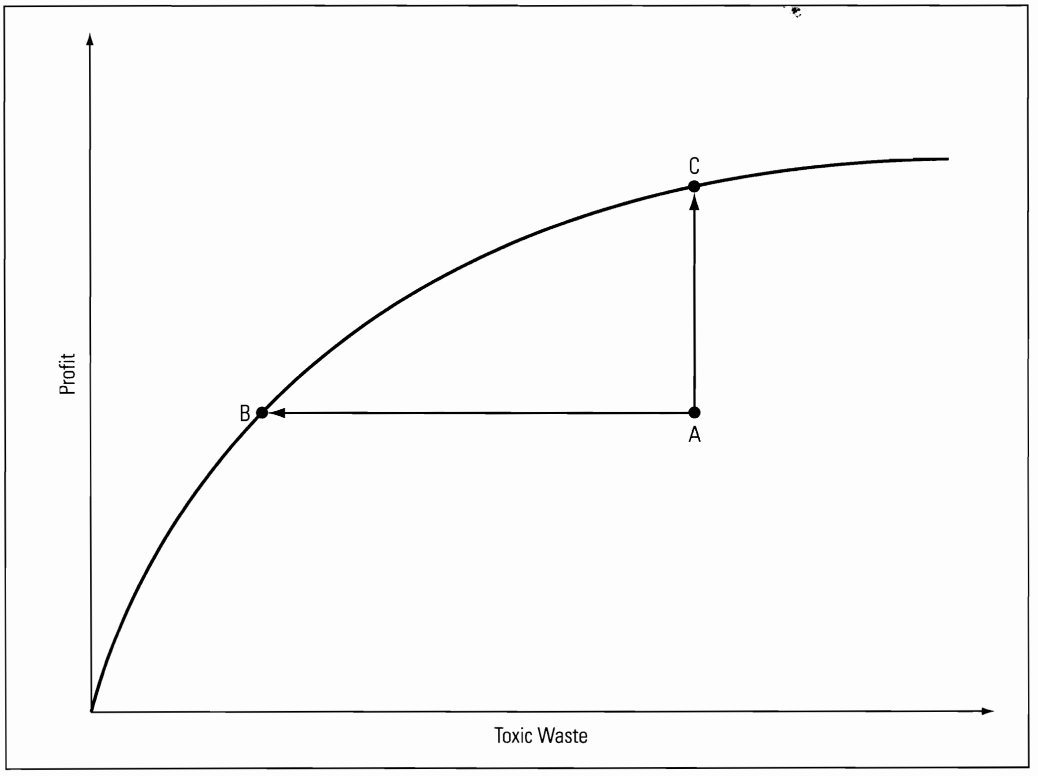
Because the variable Q must be greater than or equal to the values of all the deviational variables, and because we are trying to minimize it, Q will always be set equal to the maximum value of the deviational variables. At the same time, this objective function tries to find a solution where the maximum deviational variable (and the value of Q) is as small as possible. Therefore, this technique allows us to minimize the maximum deviation from all the goals. As we will see shortly, this type of objective is especially valuable if a GP problem involves hard constraints.

7.4 Multiple Objective Optimization

We now consider how to solve LP problems involving multiple objective functions. These problems are called **multiple objective linear programming** (MOLP) problems.

FIGURE 7.7

Illustration of trade-offs between objectives and dominated decision solution alternatives



Most of the LP and ILP problems discussed in previous chapters involved one objective function. These objective functions typically sought to maximize profits or minimize costs. However, another objective function could be formulated for most of these problems. For example, if a production process creates a toxic pollutant that is dangerous to the environment, a company might want to minimize this toxic by-product. But this objective is likely to be in direct conflict with the company's other objective of maximizing profits. Increasing profit will likely always result in the creation of additional toxic waste. Figure 7.7 shows a hypothetical example of the potential trade-offs between profit and the production of toxic waste. Each point on the curve in this graph corresponds to a possible level of profit and the minimum amount of toxic waste that must be produced to achieve this level of profit. Clearly, reaching higher levels of profit (which is desirable) is associated with incurring greater levels of toxic waste production (which is undesirable). So the decision maker must decide what level of trade-off between profit and toxic waste is most desirable.

Another important MOLP issue to note in Figure 7.7 is the concept of dominated and non-dominated solutions. Accepting a solution that offers the combination of profit and toxic waste indicated by point A is clearly undesirable. There is another alternative (i.e., point B on the graph) that offers less toxic waste production for the same level of profit. Also, there is another alternative (i.e., point C on the graph) that offers more profit for the same level of toxic waste. So points B and C both would be preferable to (or dominate) point A. Indeed, all the points along the curve connecting point B to point C dominate point A. In MOLP, a decision alternative is **dominated** if there is another alternative that produces a better value for at least one objective without worsening the value of the other objectives. Clearly, rational decision makers should want to consider only decision alternatives that are non-dominated. The technique for MOLP presented in this chapter guarantees that the solutions presented to the decision maker are non-dominated.

Fortunately, MOLP problems can be viewed as special types of GP problems where, as part of solving the problem, we also must determine target values for each goal or objective. Analyzing these problems effectively also requires that we use the MINIMAX objective described earlier.

7.5 An MOLP Example

The following example illustrates the issues involved in an MOLP problem. Although this example involves only three objectives, the concepts and techniques presented apply to problems involving any number of objectives.

Lee Blackstone is the owner of the Blackstone Mining Company, which operates two different coal mines in Wythe and Giles counties in Southwest Virginia. Due to increased commercial and residential development in the primary areas served by these mines, Lee is anticipating an increase in demand for coal in the coming year. Specifically, her projections indicate a 48-ton increase in the demand for high-grade coal, a 28-ton increase in the demand for medium-grade coal, and a 100-ton increase in the demand for low-grade coal. To handle this increase in demand, Lee must schedule extra shifts of workers at the mines. It costs $40,000 per month to run an extra shift of workers at the Wythe County mine and $32,000 per month at the Giles mine. Only one additional shift can be scheduled each month at each mine. The amount of coal that can be produced in a month's time at each mine by a shift of workers is summarized in the following table.

Type of Coal	Wythe Mine	Giles Mine
High grade	12 tons	4 tons
Medium grade	4 tons	4 tons
Low grade	10 tons	20 tons

Unfortunately, the methods used to extract coal from these mines produce toxic water that enters the local groundwater aquifers. At the Wythe mine, approximately 800 gallons of toxic water per month will be generated by running an extra shift, whereas the mine in Giles County will generate about 1,250 gallons of toxic water. Although these amounts are within EPA guidelines, Lee is concerned about the environment and doesn't want to create any more pollution than is absolutely necessary. Additionally, although the company follows all OSHA safety guidelines, company records indicate that approximately 0.20 life-threatening accidents occur per shift each month at the Wythe mine, whereas 0.45 accidents occur per shift each month at the Giles mine. Lee knows that mining is a hazardous occupation, but she cares about the health and welfare of her workers and wants to keep the number of life-threatening accidents to a minimum.

7.5.1 DEFINING THE DECISION VARIABLES

In this problem, Lee has to determine the number of months to schedule an extra shift at each of the company's mines. Thus, we can define the decision variables as:

X_1 = number of months to schedule an extra shift at the Wythe county mine

X_2 = number of months to schedule an extra shift at the Giles county mine

7.5.2 DEFINING THE OBJECTIVES

This problem is different from the other types of LP problems we have considered in that three different objective functions are possible. Lee might be interested in minimizing costs, minimizing the production of toxic waste water, or minimizing the expected number of life-threatening accidents. These three different objectives would be formulated as follows:

Minimize:	$\$40X_1 + \$32X_2$	} Production costs (in \$1,000s)
Minimize:	$800X_1 + 1250X_2$	} Toxic water produced (in gallons)
Minimize:	$0.20X_1 + 0.45X_2$	} Life-threatening accidents

In an LP model, Lee would be forced to decide which of these three objectives is most important or most appropriate, and use that single objective in the model. However, in an MOLP model, Lee can consider how both of these objectives (and any others she might want to formulate) can be incorporated into the analysis and solution of the problem.

7.5.3 DEFINING THE CONSTRAINTS

The constraints for this problem are formulated in the same way as for any LP problem. The following three constraints ensure that required amounts of high-grade, medium-grade, and low-grade coal are produced.

$12X_1 + 4X_2 \geq 48$	} High-grade coal required
$4X_1 + 4X_2 \geq 28$	} Medium-grade coal required
$10X_1 + 20X_2 \geq 100$	} Low-grade coal required

7.5.4 IMPLEMENTING THE MODEL

To summarize, the MOLP formulation of this problem is represented as:

Minimize:	$\$40X_1 + \$32X_2$	} Production costs (in \$1,000s)
Minimize:	$800X_1 + 1250X_2$	} Toxic water produced (in gallons)
Minimize:	$0.20X_1 + 0.45X_2$	} Life-threatening accidents
Subject to:	$12X_1 + 4X_2 \geq 48$	} High-grade coal required
	$4X_1 + 4X_2 \geq 28$	} Medium-grade coal required
	$10X_1 + 20X_2 \geq 100$	} Low-grade coal required
	$X_1, X_2 \geq 0$	} non-negativity conditions

This model is implemented in a spreadsheet in the usual way except that three different cells represent the three objective functions. One approach to implementing this model is shown in Figure 7.8 (and in the file Fig7-8.xls on your data disk).

In Figure 7.8, cells B5 and C5 represent the decision variables X_1 and X_2, respectively. The coefficients for the various objective functions are entered in cells B8 through C10. Next, the coefficients for the constraints are entered in cells B13 through C15. The objectives are then implemented in cells D8 through D10 as follows,

Formula for cell D8: =SUMPRODUCT(B8:C8,$B$5:$C$5)
(Copy to D9 through D10.)

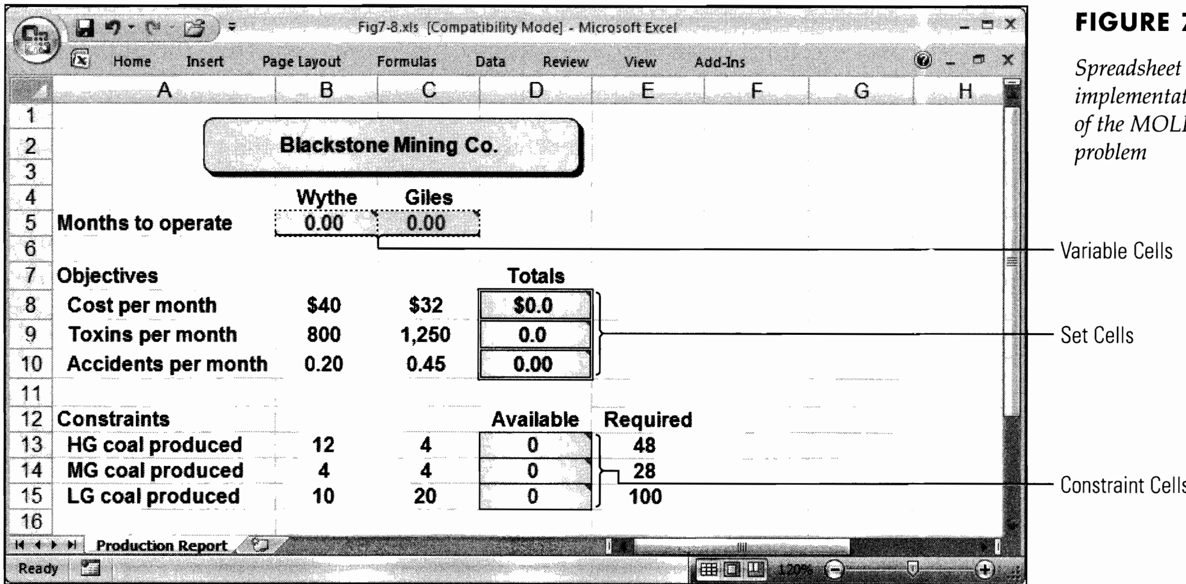


FIGURE 7.8

Spreadsheet implementation of the MOLP problem

Key Cell Formulas

Cell	Formula	Copied to
D8	=SUMPRODUCT(B8:C8,$B$5:$C$5)	D9:D10 and D13:D15

Next, the coefficients for the constraints are entered in cells B13 through C15. The LHS formulas for the constraints are then entered in cells D13 through D15.

Formula for cell D13: =SUMPRODUCT(B13:C13,$B$5:$C$5)
(Copy to D14 through D15.)

The RHS values for these constraints are given by cells E13 through E15.

7.5.5 DETERMINING TARGET VALUES FOR THE OBJECTIVES

An LP problem can have only one objective function, so how can we include three objectives in our spreadsheet model? If these objectives had target values, we could treat them the same way as the goals in our example earlier in this chapter. That is, the objectives in this problem can be stated as the following goals if we have appropriate values for t_1, t_2, and t_3:

Goal 1: The total production cost should be approximately t_1.
Goal 2: The gallons of toxic water produced should be approximately t_2.
Goal 3: The number of life-threatening accidents should be approximately t_3.

Unfortunately, the problem did not provide explicit values for t_1, t_2, and t_3. However, if we solve our model to find the solution that minimizes the first objective (total production cost), the optimal value of this objective function would be a reasonable value to use as t_1 in the first goal. Similarly, if we solve the problem two more times minimizing the second and third objectives, respectively, the optimal objective function values for these solutions would provide reasonable values to use as t_2 and t_3 in the second and third goals. We could then view our MOLP problem in the format of a GP problem.

Figure 7.9 shows the Solver parameters required to determine the minimum production cost that could be realized in this problem. Note that this involves minimizing the value of cell D8. Figure 7.10 shows the optimal solution obtained by solving this LP problem. Notice that the best possible (minimum) production cost for this problem is \$244 (in \$1,000s) and this solution can be obtained by running an extra shift at the Wythe County mine for 2.5 months and at the Giles County mine for 4.5 months. Thus, a

FIGURE 7.9

Solver parameters to minimize production costs

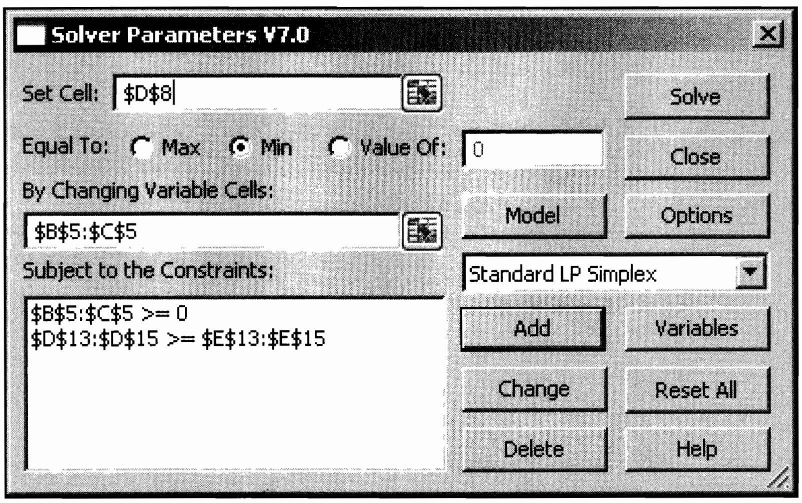
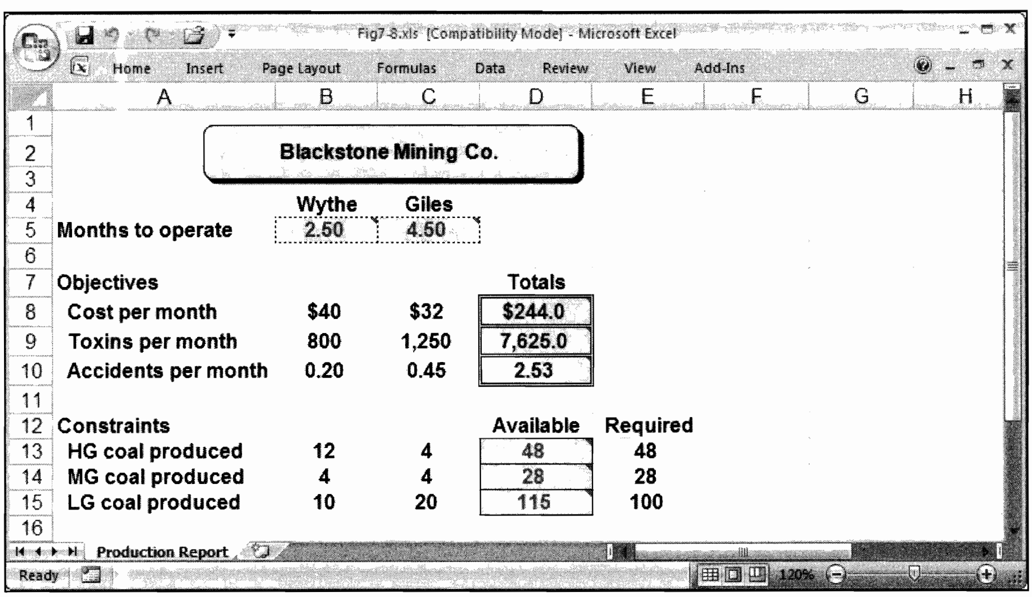


FIGURE 7.10

Optimal solution when minimizing production costs



	A	B	C	D	E
4		Wythe	Giles		
5	Months to operate	2.50	4.50		
7	Objectives			Totals	
8	Cost per month	$40	$32	$244.0	
9	Toxins per month	800	1,250	7,625.0	
10	Accidents per month	0.20	0.45	2.53	
12	Constraints			Available	Required
13	HG coal produced	12	4	48	48
14	MG coal produced	4	4	28	28
15	LG coal produced	10	20	115	100

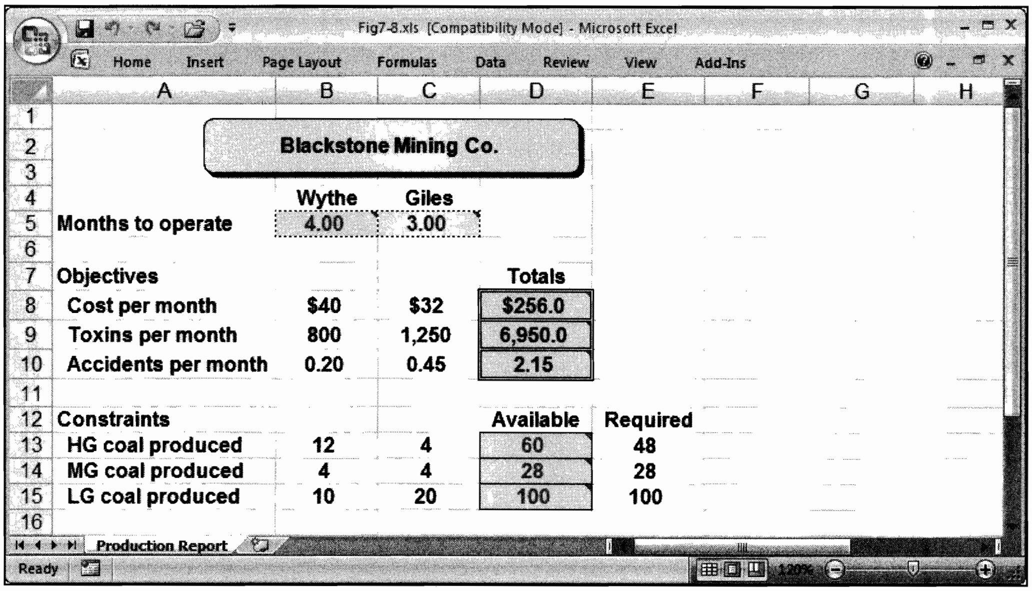


FIGURE 7.11

Optimal solution when minimizing the amount of toxic water generated

reasonable value for t_1 is \$244. It is impossible to obtain a solution to this problem with a production cost lower than this amount.

Figure 7.11 shows the solution obtained if we minimize the generation of toxic groundwater pollutants (obtained by minimizing the value in cell D9). This production schedule requires that we run an extra shift at the Wythe County mine for 4.0 months and at the Giles County mine for 3.0 months and generates a total of 6,950 gallons of toxic water. Thus, a reasonable value for t_2 is 6,950. It is impossible to obtain a solution to this problem that produces less toxic water.

Finally, Figure 7.12 shows the solution obtained if we minimize the expected number of life-threatening accidents (obtained by minimizing the value in cell D10). This production schedule requires that we run an extra shift at the Wythe County mine for 10 months and not run any extra shifts at the Giles mine. A total of 2 life-threatening accidents can be expected with this schedule. Thus, a reasonable value for t_3 is 2. It is impossible to obtain a solution to this problem with a lower number of expected life-threatening accidents.

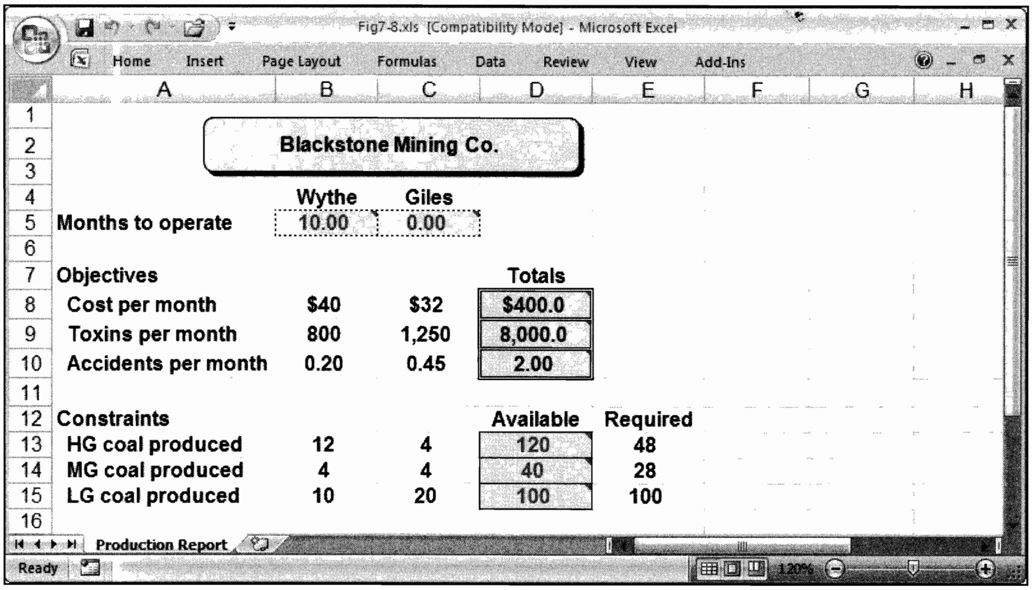
7.5.6 SUMMARIZING THE TARGET SOLUTIONS

Figure 7.13 summarizes the solutions shown in Figures 7.10, 7.11, and 7.12 and shows where each of the solutions occurs in terms of the feasible region for this problem.

Two important points should be observed here. First, Figure 7.13 clearly shows that the objectives in this problem conflict with one another. Solution 1 has the lowest production cost (\$244,000) but also has the highest expected number of accidents (2.53). Conversely, solution 3 has the lowest expected number of accidents (2.0) but generates the highest production costs (\$400,000) and also the highest creation of toxic water (8,000 gallons). This is not surprising, but does reinforce that this problem involves a trade-off between the three objectives. No single feasible point simultaneously optimizes all of the objective functions. To improve the value of

FIGURE 7.12

Optimal solution when minimizing the expected number of life threatening accidents



one objective, we must sacrifice the value of the others. This characteristic is common to most MOLP problems. Thus, the purpose of MOLP (and of GP) is to study the trade-offs among the objectives to find a solution that is the most desirable to the decision maker.

Second, the graph in Figure 7.13 shows the solutions only at three corner points of the feasible region for this problem. Because we already have determined the levels of cost, toxic water production, and expected accident rates offered by these three solutions, if none of these solutions are acceptable, the decision maker might wish to explore some of the other *non-corner point* feasible solutions shown in Figure 7.13. As we will see, this poses a tricky problem.

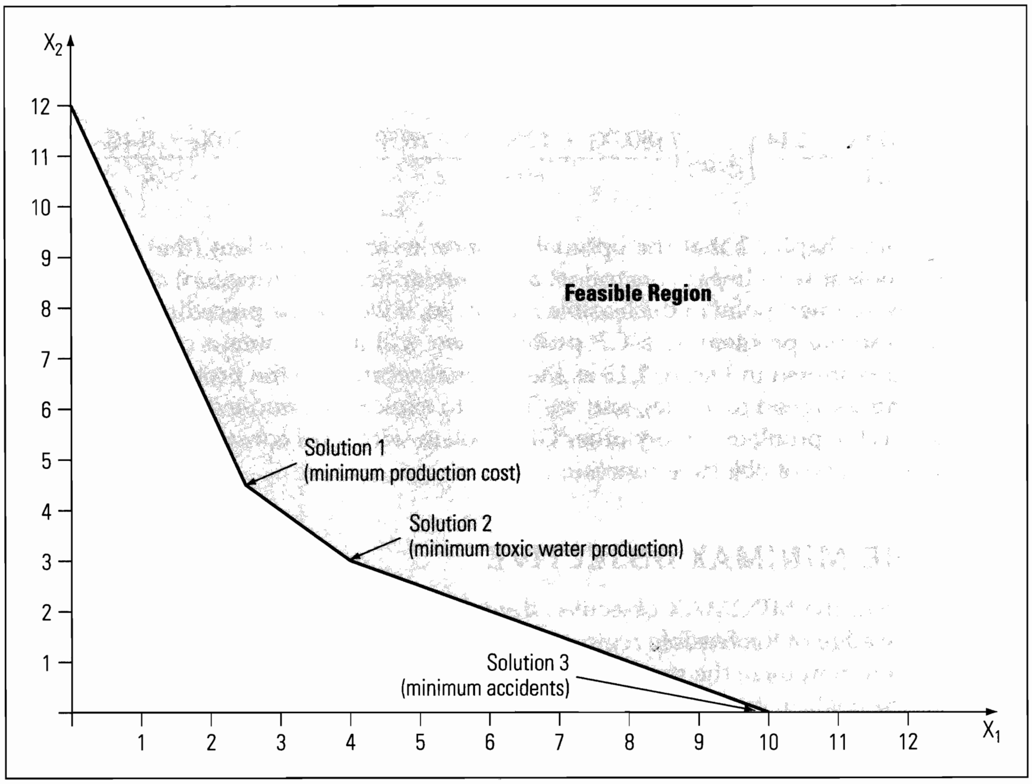
7.5.7 DETERMINING A GP OBJECTIVE

Now that we have target values for the three objectives in our problem, we can formulate a weighted GP objective to allow the decision maker to explore possible solutions. Earlier in this chapter, we discussed several GP objectives and illustrated the use of an objective that minimized the weighted percentage deviation from the goals' target values. Let's consider how to formulate this same type of objective for the current problem.

We can restate the objectives of this problem as the goals:

Goal 1: The total production cost should be approximately \$244.
Goal 2: The gallons of toxic water produced should be approximately 6,950.
Goal 3: The number of life-threatening accidents should be approximately 2.0.

We now know that the actual total production cost never can be smaller than its target (optimum) value of \$244, so the percentage deviation from this goal may be computed as:



Solution	Months of Operation at Wythe Mine (X_1)	Months of Operation at Giles Mine (X_2)	Production Cost	Gallons of Toxic Pollutants Produced	Expected Number of Life Threatening Accidents
1	2.5	4.5	$244	7,625	2.53
2	4.0	3.0	$256	6,950	2.15
3	10.0	0.0	$400	8,000	2.00

FIGURE 7.13

Summary of the solutions minimizing each of the three possible objectives

$$\frac{\text{actual value} - \text{target value}}{\text{target value}} = \frac{(40X_1 + 32X_2) - 244}{244}$$

Similarly, the actual amount of toxic water generated never can be less than its target (optimum) value of 6,950, so the percentage deviation from this goal is calculated as:

$$\frac{\text{actual value} - \text{target value}}{\text{target value}} = \frac{(800X_1 + 1250X_2) - 6950}{6950}$$

Finally, the actual expected number of life-threatening accidents never can be less than its target (optimum) value of 2, so the percentage deviation from this goal is calculated as:

$$\frac{\text{actual value} - \text{target value}}{\text{target value}} = \frac{(0.20X_1 + 0.45X_2) - 2}{2}$$

These percentage deviation calculations are all linear functions of the decision variables. Thus, if we form an objective function as a weighted combination of these percentage deviation functions, we obtain the following linear objective function:

$$\text{MIN: } w_1\left(\frac{(40X_1 + 32X_2) - 244}{244}\right) + w_2\left(\frac{(800X_1 + 1250X_2) - 6950}{6950}\right) + w_3\left(\frac{(0.20X_1 + 0.45X_2) - 2}{2}\right)$$

Recall from Chapter 2 that the optimal solution to an LP problem (that is, an optimization problem with linear constraints and a linear objective function) *always* occurs at an extreme (corner) point of the feasible region. So, if we use the preceding objective to solve our example problem as a GP problem, we will *always* obtain one of the four extreme points shown in Figure 7.13 as the optimal solution to the problem, regardless of the weights assigned to w_1, w_2, and w_3. Thus, to explore the non-extreme feasible solutions to this GP problem (or any other GP problem with hard constraints), we need to use a different type of objective function.

7.5.8 THE MINIMAX OBJECTIVE

As it turns out, the MINIMAX objective, described earlier, can be used to explore the points on the edge of the feasible region—in addition to corner points. To illustrate this, let's attempt to minimize the maximum weighted percentage deviation from the target values for the goals in our example problem using the objective:

$$\text{MIN: the maximum of } w_1\left(\frac{(40X_1 + 32X_2) - 244}{244}\right), w_2\left(\frac{(800X_1 + 1250X_2) - 6950}{6950}\right),$$

$$\text{and } w_3\left(\frac{(0.20X_1 + 0.45X_2) - 2}{2}\right)$$

We implement this objective by establishing a MINIMAX variable Q that we minimize with the objective:

$$\text{MIN: Q}$$

subject to the additional constraints:

$$w_1\left(\frac{(40X_1 + 32X_2) - 244}{244}\right) \leq Q$$

$$w_2\left(\frac{(800X_1 + 1250X_2) - 6950}{6950}\right) \leq Q$$

$$w_3\left(\frac{(0.20X_1 + 0.45X_2) - 2}{2}\right) \leq Q$$

The first constraint indicates that the weighted percentage deviation from the target production cost must be less than or equal to Q. The second constraint indicates that the weighted percentage deviation from the target level of toxic water production also must be less than or equal to Q. The third constraint indicates that the weighted percentage deviation from the target expected number of life-threatening accidents also must be less than or equal to Q. Thus, as we minimize Q, we also are minimizing the weighted percentage deviations from the target values for each of our goals. In this way, the maximum weighted deviation from any of the goals is minimized—or we have MINImized the MAXimum deviation (hence the term MINIMAX).

7.5.9 IMPLEMENTING THE REVISED MODEL

The revised GP model of our investment problem is summarized as:

MIN: Q
Subject to:

$$
\begin{array}{lll}
12X_1 + 4X_2 \geq 48 & & \} \text{ High-grade coal required} \\
4X_1 + 4X_2 \geq 28 & & \} \text{ Medium-grade coal required} \\
10X_1 + 20X_2 \geq 100 & & \} \text{ Low-grade coal required} \\
w_1(40X_1 + 32X_2 - 244)/\ 244 \leq Q & & \} \text{ goal 1 MINIMAX constraint} \\
w_2(800X_1 + 1250X_2 - 6950)/\ 6950 \leq Q & & \} \text{ goal 2 MINIMAX constraint} \\
w_3(0.20X_1 + 0.45X_2 - 2)/\ 2 \leq Q & & \} \text{ goal 3 MINIMAX constraint} \\
X_1, X_2 \geq 0 & & \} \text{ non-negativity conditions} \\
w_1, w_2, w_3 \text{ are positive constants} & &
\end{array}
$$

The spreadsheet shown earlier in Figure 7.8 can be modified easily to implement this new model. The revised spreadsheet is shown in Figure 7.14 (and in the file Fig7-14.xls on your data disk).

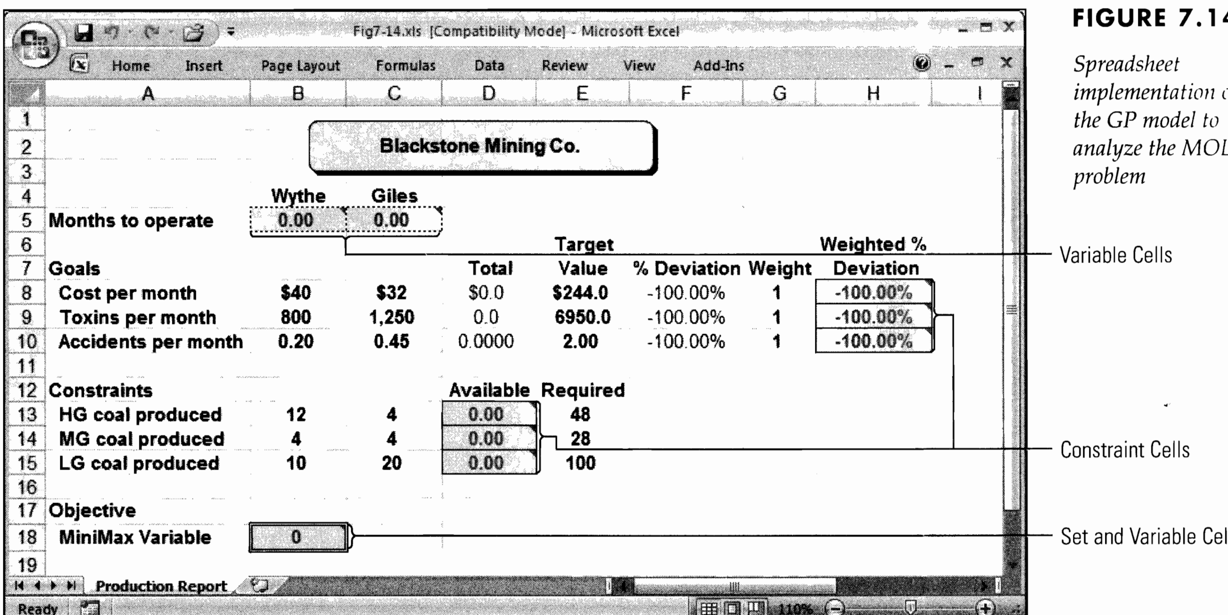


FIGURE 7.14

Spreadsheet implementation of the GP model to analyze the MOLP problem

Key Cell Formulas

Cell	Formula	Copied to
D8	=SUMPRODUCT(B8:C8,$B$5:$C$5)	D9:D10 and D13:D15
F8	=(D8–E8)/E8	F9:F10
H8	=F8*G8	H9:H10

In Figure 7.14, cells E8 through E10 contain the target values for the goals. The percentage deviations from each goal are calculated in cells F8 through F10 as follows:

Formula for cell F8: =(D8-E8)/E8
(Copy to cells F9 through F10.)

Arbitrary weights for the deviations from the goals were entered in cells G8 through G10. Cells H8 through H10 contain the following formulas, which calculate the weighted percentage deviation from the goals:

Formula for cell H8: =F8*G8
(Copy to cells H9 through H10.)

The formulas in cells H8 through H10 are equivalent to the LHS formulas of the MINIMAX constraints for each of the goals in our model. Finally, cell B18 is reserved to represent the MINIMAX variable Q. Notice that this cell is a changing cell (because it represents the variable Q) *and* the set cell (because it also represents the objective to be minimized).

7.5.10 SOLVING THE MODEL

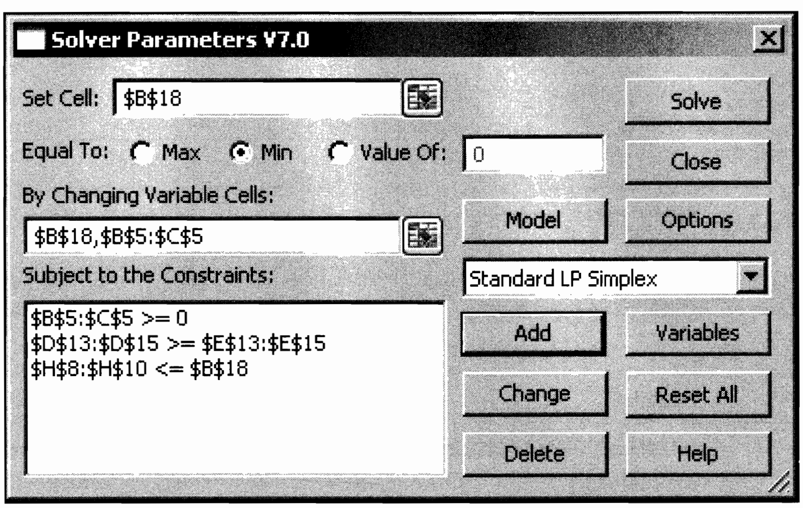
The Solver parameters shown in Figure 7.15 were used to solve the model shown in Figure 7.14. The solution obtained for this model is shown in Figure 7.16.

Notice that the solution shown in Figure 7.16 ($X_1 = 4.23$, $X_2 = 2.88$) *does not* occur at an extreme point of the feasible region shown earlier in Figure 7.13. Also notice that this solution is within approximately 7.2% of achieving the target solution for goals 1 and 3 and is less than 1% from the target value for goal 2. Thus, the decision maker in this problem might find this solution more appealing than the other solutions occurring at the extreme points of the feasible region. Using other weights would produce different solutions. Figure 7.17 shows several representative solutions indicated on the original feasible region for this problem.

Figure 7.17 illustrates that as the relative weight on the first goal (w_1) increases, the solution is driven closer to achieving the target value for this goal (which occurs at the point $X_1 = 2.5$, $X_2 = 4.5$, as shown in Figure 7.13). As the relative weight on

FIGURE 7.15

Solver parameters and options for the GP implementation of the MOLP problem



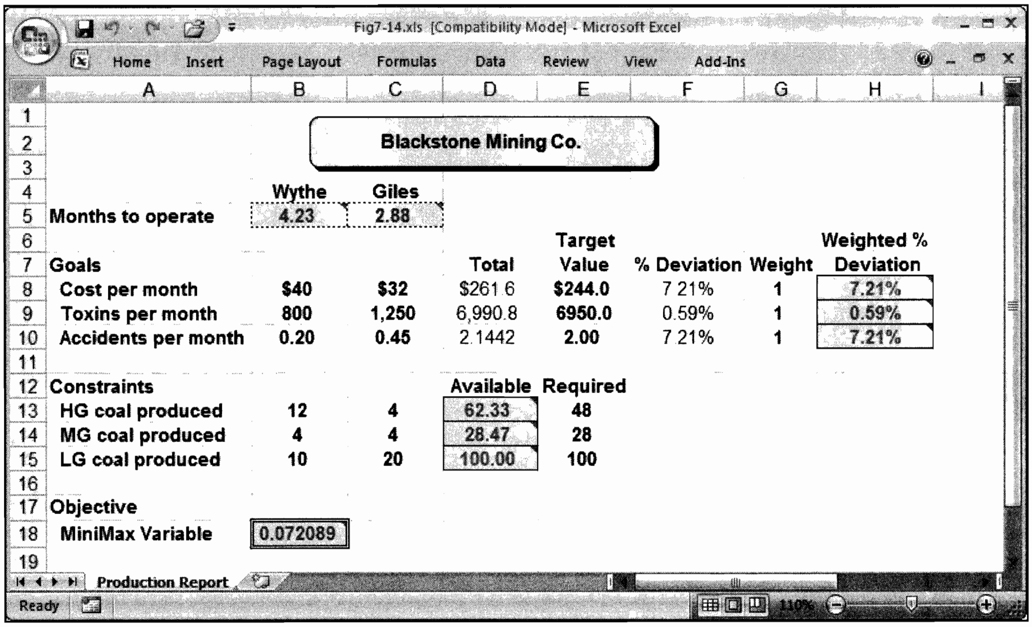


Fig7-14.xls [Compatibility Mode] - Microsoft Excel

Blackstone Mining Co.

	Wythe	Giles					
Months to operate	4.23	2.88					
				Target			Weighted %
Goals			Total	Value	% Deviation	Weight	Deviation
Cost per month	$40	$32	$261.6	$244.0	7.21%	1	7.21%
Toxins per month	800	1,250	6,990.8	6950.0	0.59%	1	0.59%
Accidents per month	0.20	0.45	2.1442	2.00	7.21%	1	7.21%
Constraints			Available	Required			
HG coal produced	12	4	62.33	48			
MG coal produced	4	4	28.47	28			
LG coal produced	10	20	100.00	100			
Objective							
MiniMax Variable	0.072089						

Production Report

FIGURE 7.16

Solution to the MOLP problem obtained through GP

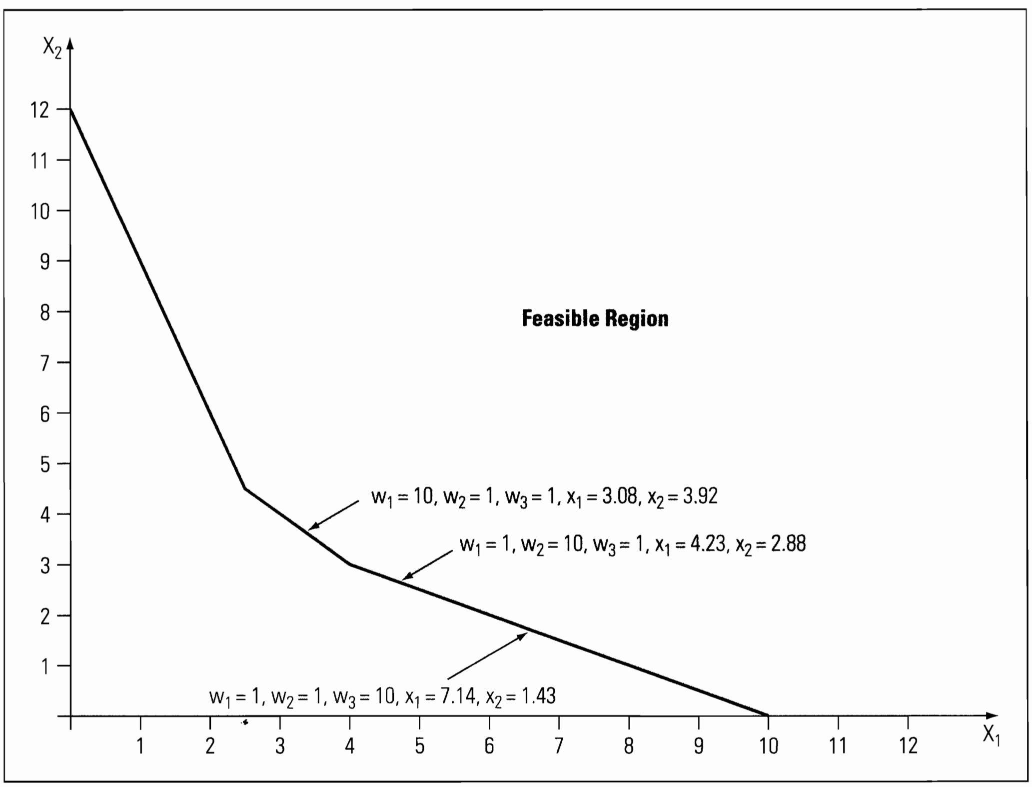


FIGURE 7.17

Graph of other solutions obtained using the MINIMAX objective the second goal (w_2) increases, the solution is driven closer to achieving the target value for this goal (which occurs at the point $X_1 = 4.0$, $X_2 = 3.0$). Finally, as the relative weight on the third goal (w_3) increases, the solution is driven closer to achieving the target value for this goal (which occurs at the point $X_1 = 10.0$, $X_2 = 0.0$). Thus, by adjusting the weights, the decision maker can explore several solutions that do not necessarily occur at the corner points of the original feasible region to the problem.

7.6 Comments on MOLP

Figure 7.18 provides a summary of the steps involved in solving an MOLP problem. Although the MOLP example in this chapter was somewhat simple, the same basic process applies in virtually any MOLP problem, regardless of the number of objectives or the complexity of the problem.

One advantage of using the MINIMAX objective to analyze MOLP problems is that the solutions generated are always ***Pareto optimal***. That is, given any solution generated using this approach, we can be certain that no other feasible solution allows an increase in any objective without decreasing at least one other objective. (There are one or two exceptions to this statement, but they go beyond the scope of this text.)

Although the MINIMAX objective is helpful in the analysis of MOLPs, its usefulness is not limited to these problems. Like deviational variables, the MINIMAX technique can prove useful in other types of mathematical programming situations.

In the example MOLP problem presented here, all of the goals were derived from minimization objectives. Because of this, we knew that the actual value for any goal never could be less than its derived target value, and we used the following formula to calculate the percentage deviation for each goal constraint:

$$\frac{\text{actual value} - \text{target value}}{\text{target value}}$$

FIGURE 7.18

Summary of the steps involved in formulating and solving an MOLP problem

Summary of Multiple Objective Optimization

1. Identify the decision variables in the problem.
2. Identify the objectives in the problem and formulate them in the usual way.
3. Identify the constraints in the problem and formulate them in the usual way.
4. Solve the problem once for each of the objectives identified in step 2 to determine the optimal value of each objective.
5. Restate the objectives as goals using the optimal objective values identified in step 4 as the target values.
6. For each goal, create a deviation function that measures the amount by which any given solution fails to meet the goal (either as an absolute or a percentage).
7. For each of the deviation functions identified in step 6, assign a weight to the deviation function and create a constraint that requires the value of the weighted deviation function to be less than the MINIMAX variable Q.
8. Solve the resulting problem with the objective of minimizing Q.
9. Inspect the solution to the problem. If the solution is unacceptable, adjust the weights in step 7 and return to step 8.

For goals derived from maximization objectives, we know that the actual value of the goal never can be greater than its derived target value, and the percentage deviation for such goals should be calculated as:

$$\frac{\text{target value} - \text{actual value}}{\text{target value}}$$

If the target value of a goal is zero, it is not possible to use weighted percentage deviations in the solution to the MOLP (because division by zero is not permissible). In this case, you can simply use weighted deviations.

7.7 Summary

This chapter presented two separate but closely related issues in optimization—GP and MOLP. GP provides a way of analyzing potential solutions to a decision problem that involves soft constraints. Soft constraints can be stated as goals with target values. These goals can be translated into constraints through the use of deviational variables, which measure the amount by which a given solution deviates from a particular goal. The objective in GP problems is to minimize some weighted function of the deviational variables. By adjusting the weights on the deviational variables, a variety of potential solutions can be analyzed.

MOLP provides a way to analyze LP problems involving multiple objectives that conflict with one another. Although an MOLP problem is somewhat different from a standard GP problem, the objectives can be restated as goals after identifying appropriate target values for the objectives. The MINIMAX objective is helpful in analyzing the possible solutions to an MOLP problem.

Solving a GP or MOLP problem is not as simple as solving a single LP problem. Rather, a sequence of problems must be solved to allow the decision maker to analyze the trade-offs among the various goals and objectives at different possible solutions. Thus, both of these procedures are highly iterative and interactive.

7.8 References

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THE WORLD OF MANAGEMENT SCIENCE

Truck Transport Corporation Controls Costs and Disruptions While Relocating a Terminal

The Truck Transport Corporation, having decided to move its East St. Louis terminal, knew that relationships with customers and suppliers (independent truckers) were critical factors for continued profitable operations. Therefore, evaluating five

(Continued)

potential new sites, management considered driver and customer preferences in addition to costs in making its final selection.

At Truck Transport Corporation, the traditional approach to evaluating a new site is to include the candidate site in a transportation LP model with the four other terminals and twelve major customers, and find the solution that minimizes total transportation costs. This minimum cost solution is then compared with those for the other candidates to choose the most efficient site. An assignment problem is solved to assign independent truckers to terminals to minimize travel costs from the truckers' homes.

Some of the drivers, however, have strong preferences not to be assigned to particular terminals, usually on the basis of personal relationships with terminal managers. Some customers also have similar preferences. In a competitive market, failure to consider these preferences might cause the drivers or customers to do business elsewhere.

The linear goal programming model used to evaluate the sites combined the transportation problem and the trucker assignment problem. The constraints defined the following deviational variables, in declining order of priority: shortages in number of trips to major customers, shortages in number of trips assigned to each driver, number of driver preferences violated, number of customer preferences violated, increase in transportation costs from drivers' homes, and increase in transportation costs to the customers.

The model was validated by evaluating the East St. Louis site and comparing results to historical costs. The site that ultimately was selected fully satisfied the requirements for number of shipments and preferences. Total transportation costs for all drivers were projected to increase only $3,200, and customer transportation costs were projected to increase $1,400. The East St. Louis terminal was moved with no changes in the usual patterns of driver turnover or business with customers, and no complaints from drivers about decreased profitability because of the new site.

Source: Schneiderjans, Marc J., N. K. Kwak and Mark C. Helmer, "An Application of Goal Programming to Resolve a Site Location Problem." Interfaces, vol. 12, no. 3, June 1982, pp. 65–70.

Questions and Problems

1. What is the difference between an objective function and a goal?
2. Is there an optimal solution to a GP or MOLP problem? Explain.
3. Read the feature at the end of section 7.1 in this chapter titled "Balancing Objectives for Enlightened Self-Interest." What objectives were the real estate developers in this article considering in their plans for the Coal Bank Ridge development? Describe how the objectives you identify might conflict or support each other.
4. As this chapter was being written, Hurricane Katrina decimated the gulf coast of the United States between Mobile, Alabama, and New Orleans, Louisiana. The aftermath of this storm left the city of New Orleans flooded, both with water and human victims of the storm. Responding to this disaster is a logistical nightmare and presents governmental decision makers with an extremely difficult challenge.

a. Identify several key objectives that the decision makers managing this problem must consider simultaneously.
b. Identify the key resources that the decision makers must allocate.
c. How do the objectives and the resources interrelate?
d. Do the objectives you identified conflict or compete with one another in terms of resource usage?
e. How might the techniques presented in this chapter help decision makers determine how to allocate resources to achieve the objectives?

5. Refer to the MOLP example presented in this chapter.
   a. What weights could be used to generate the solution at $X_1 = 2.5, X_2 = 4.5$?
   b. What weights could be used to generate the solution at $X_1 = 4.0, X_2 = 3.0$?
   c. What weights could be used to generate the solution at $X_1 = 10.0, X_2 = 0.0$?
   d. What weights could be used to generate solutions along the edge of the feasible region that runs from the point $X_1 = 0, X_2 = 12.0$ to the point $X_1 = 2.5, X_2 = 4.5$?
6. Suppose that the first goal in a GP problem is to make $2X_1 + 5X_2$ approximately equal to 25.
   a. Using the deviational variables d_1^- and d_1^+, what constraint can be used to express this goal?
   b. If we obtain a solution where $X_1 = 4$ and $X_2 = 3$, what values do the deviational variables assume?
   c. Consider a solution where $X_1 = 4, X_2 = 3, d_1^- = 6$, and $d_1^+ = 4$. Can this solution ever be optimal? Why or why not?
7. Consider the following MOLP:

$$
\begin{array}{ll}
\text{MAX:} & 4X_1 + 2X_2 \\
\text{MIN:} & X_1 + 3X_2 \\
\text{Subject to:} & 2X_1 + X_2 \le 18 \\
& X_1 + 4X_2 \le 12 \\
& X_1 + X_2 \ge 4 \\
& X_1, X_2 \ge 0
\end{array}
$$

   a. Graph the feasible region for this problem.
   b. Calculate the value of each objective at each extreme point.
   c. What feasible points in this problem are Pareto optimal?
8. It has been suggested that one way to solve MOLP problems is to create a composite objective function as a linear combination of all the objectives in the problem. For example, in the previous problem, we might weight the first objective by 0.75 and the second by 0.25 to obtain the composite objective, MAX: $2.75X_1 + 0.75X_2$. (Note that the second objective in the previous problem is equivalent to MAX: $-X_1 - 3X_2$.) We then use this as the objective in an LP model to generate possible solutions. What problem, if any, do you see with this approach?
9. Refer to the MOLP problem presented in this chapter. The solutions shown in Figures 7.9, 7.10, and 7.11 each result in more than the required amount of one or more types of coal being produced, as summarized in the following table.

Solution Shown in:		**Excess Production of**	
	High-Grade Coal	**Medium-Grade Coal**	**Low-Grade Coal**
Figure 7.9	0 tons	0 tons	15 tons
Figure 7.10	12 tons	0 tons	0 tons
Figure 7.11	72 tons	12 tons	0 tons

a. Formulate an LP model that could be solved to find the solution that minimizes the maximum amount of excess coal produced. (*Hint:* Use a MINIMAX objective rather than a MAX( ) function.)
b. Implement your model in a spreadsheet and solve it.
c. What is the optimal solution?
d. Revise your model to find the solution that minimizes the maximum percentage of excess coal produced. What is the optimal solution?

10. The CFO for the Shelton Corporation has $1.2 million to allocate to the following budget requests from 5 departments:

Dept 1	Dept 2	Dept 3	Dept 4	Dept 5
$450,000	$310,000	$275,000	$187,500	$135,000

Because the total budget requests exceed the available $1.2 million, not all the requests can be satisfied. Suppose that the CFO considers the requests for departments 2 and 3 to be twice as important as those from departments 4 and 5, and the request from department 1 to be twice as important as those from departments 2 and 3. Further suppose that the CFO wants to make sure each department receives at least 70% of the requested amount.

a. Formulate a GP model for this problem.
b. Implement your model and solve it. What is the optimal solution?
c. Suppose the CFO is willing to allocate more than $1.2 million to these budgets but regards exceeding the $1.2 million figure as being twice as undesirable as not meeting the budget request of department 1. What is the optimal solution?
d. Suppose the CFO regards all deviations from the original budget amounts (including the $1.2 million available) to be equally undesirable. What solution minimizes the maximum percentage deviation from the budgeted amounts?

11. The Reeves Corporation wants to assign each of their thirteen corporate clients to exactly one of their three salespersons. The estimated annual sales potential (in $1,000,000s) for each of the clients is summarized in the following table:

Client	A	B	C	D	E	F	G	H	I	J	K	L	M
Est. Sales	$67	$84	$52	$70	$74	$62	$94	$63	$73	$109	$77	$36	$114

Reeves wants to each salesperson to be assigned at least three customers and no more than six customers. The company wants to assign customers to the sales force in such a way that the estimated annual sales potential for each salesperson's set of customers is as equal as possible.

a. Formulate a GP model for this problem. (*Hint:* There should be a goal for the estimated annual sales potential for each salesperson.)
b. Assume that the company wants to minimize the sum of the absolute deviations from each goal. Implement your model in a spreadsheet and solve it.

12. Blue Ridge Hot Tubs manufactures and sells two models of hot tubs: the Aqua-Spa and the Hydro-Lux. Howie Jones, the owner and manager of the company, needs to decide how many of each type of hot tub to produce during his next production cycle. Howie buys prefabricated fiberglass hot tub shells from a local supplier and adds the pump and tubing to the shells to create his hot tubs. (This supplier has the capacity to deliver as many hot tub shells as Howie needs.) Howie installs the same type of pump into both hot tubs. He will have

only 200 pumps available during his next production cycle. From a manufacturing standpoint, the main difference between the two models of hot tubs is the amount of tubing and labor required. Each Aqua-Spa requires 9 hours of labor and 12 feet of tubing. Each Hydro-Lux requires 6 hours of labor and 16 feet of tubing. Howie expects to have 1,566 production labor hours and 2,880 feet of tubing available during the next production cycle. Howie earns a profit of $350 on each Aqua-Spa he sells and $300 on each Hydro-Lux he sells. He is confident that he can sell all the hot tubs he produces. The production of each Aqua-Spa generates 15 pounds of a toxic resin, whereas each Hydro-Lux produces 10 pounds of toxic resin. Howie has identified two different objectives that could apply to his problem: He can maximize profit or he can minimize the production of toxic resin. Suppose Howie considers the maximization of profit as half as important as the minimization of toxic resin.

a. Formulate an MOLP model for Howie's decision problem.
b. Implement your model in a spreadsheet and solve it.
c. What is the solution to Howie's MOLP problem?
d. The feasible region for this problem was shown in Figure 2.7. Identify on this graph the Pareto optimal solutions for Howie's MOLP problem.

13. The owner of the Weiner-Meyer meat processing plant wants to determine the best blend of meats to use in the next production run of hamburgers. Three sources of meat can be used. The following table summarizes relevant characteristics of these meats:

	Meat 1	Meat 2	Meat 3
Cost per pound	$0.75	$0.87	$0.98
% Fat	15%	10%	5%
% Protein	70%	75%	80%
% Water	12%	10%	8%
% Filler	3%	5%	7%

A local elementary school has ordered 500 pounds of meat for $1.10 per pound. The only requirement is that the meat consist of at least 75% protein and at most 10% each of water and filler. Ordinarily, the owner would produce the blend of meats that achieved this objective in the least costly manner. However, with the concern of too much fat in school lunches, the owner also wants to produce a blend that minimizes the fat content of the meat produced.

a. Formulate an MOLP for this problem.
b. Implement your formulation in a spreadsheet and individually optimize the two objectives under consideration.
c. How much profit must be forfeited to fill this order using the mix that minimizes the fat content?
d. Solve this problem with the objective of minimizing the maximum percentage deviation from the target values of the goals. What solution do you obtain?
e. Assume that the owner considers minimizing the fat content twice as important as maximizing profit. What solution does this imply?

14. A new Italian restaurant called the Olive Grove is opening in several locations in the Memphis area. The marketing manager for these stores has a budget of $150,000 to use in advertising and promotions for the new stores. The manager can run magazine ads at a cost of $2,000 each that result in 250,000 exposures each. TV ads result in approximately 1,200,000 exposures each, but cost $12,000 each. The manager

wants to run at least five TV ads and ten magazine ads, while maximizing the number of exposures generated by the advertising campaign. But the manager also wants to spend no more than $120,000 on magazine and TV advertising so that the remaining $30,000 could be used for other promotional purposes. However, the manager would spend more than $120,000 on advertising if it resulted in a substantial increase in advertising coverage.

a. Formulate a GP model for this problem assuming that the marketing manager has the following goals:

   Goal 1: Exposures should be maximized.
   Goal 2: No more than $120,000 should be spent on advertising.

(Note that you will have to determine an appropriate target value for the first goal.) Assume that the marketing manager wants to minimize the maximum percentage deviation from either goal.

b. Implement your model in a spreadsheet and solve it.
c. What is the solution you obtain?
d. What changes do you make to your model if the manager wants to spend less on advertising than your solution suggests?

15. The city of Abingdon is determining its tax rate structure for the coming year. The city needs to generate $6 million in tax revenue via taxes of property, sales, prepared food, and utilities. The following table summarizes how much tax revenue would be generated from each segment of the population by the 1% increase in each tax category. (For instance, a 2% tax on prepared food would generate $240,000 in tax revenue from upper income residents.)

	Revenues (in $1000s) per 1% Tax Rate			
Income Group	**Sales**	**Property**	**Food**	**Utility**
Low	$200	$ 600	$ 50	$ 80
Middle	$250	$ 800	$100	$100
Upper	$400	$1200	$120	$120

City commissioners have specified that the tax rate for each revenue category must be between 1% and 3% and that the tax rate on prepared food cannot exceed half the sales tax rate. Ideally, the commissioners have a goal of making up the $6 million tax budget with $1.5 million from low income residents, $2.1 million from middle income residents, and $2.4 million from high income residents. If that is not possible, the commissioners would like a solution that minimizes the maximum percentage deviation from these tax revenue goals for each income group.

a. Create a spreadsheet model for this problem.
b. What is the optimal solution?

16. The Royal Seas Company runs a three-night cruise to the Caribbean from Port Canaveral. The company wants to run TV ads promoting its cruises to high-income men, high-income women, and retirees. The company has decided to consider airing ads during prime-time, afternoon soap operas, and during the evening news. The number of exposures (in millions) expected to be generated by each type of ad in each of the company's target audiences is summarized in the following table:

	Prime Time	**Soap Operas**	**Evening News**
High-income men	6	3	6
High-income women	3	4	4
Retirees	4	7	3

Ads during prime-time, the afternoon soaps, and the news hour cost \$120,000, \$85,000, and \$100,000, respectively. Royal Seas wants to achieve the following goals:

Goal 1: To spend approximately \$900,000 of TV advertising
Goal 2: To generate approximately 45 million exposures among high-income men
Goal 3: To generate approximately 60 million exposures among high-income women
Goal 4: To generate approximately 50 million exposures among retirees

a. Formulate a GP model for this problem. Assume overachievement of the first goal is equally as undesirable as underachievement of the remaining goals on a percentage deviation basis.
b. Implement your model in a spreadsheet and solve it.
c. What is the optimal solution?
d. What solution allows the company to spend as close to \$900,000 as possible without exceeding this amount?
e. Assume that the company can spend no more than \$900,000. What solution minimizes the maximum percentage underachievement of all the goals?
f. Which of the two preceding solutions would you most prefer? Why?

17. Virginia Tech operates its own power generating plant. The electricity generated by this plant supplies power to the university and to local businesses and residences in the Blacksburg area. The plant burns three types of coal, which produces steam that drives the turbines that generate the electricity. The Environmental Protection Agency (EPA) requires that for each ton of coal burned, the emissions from the coal furnace smokestacks contain no more than 2,500 parts per million (ppm) of sulfur and no more than 2.8 kilograms (kg) of coal dust. However, the managers of the plant are concerned about the environment and want to keep these emissions to a minimum. The following table summarizes the amounts of sulfur, coal dust, and steam that result from burning a ton of each type of coal.

Coal	Sulfur (in ppm)	Coal Dust (in kg)	Pounds of Steam Produced
1	1,100	1.7	24,000
2	3,500	3.2	36,000
3	1,300	2.4	28,000

The three types of coal can be mixed and burned in any combination. The resulting emission of sulfur or coal dust and the pounds of steam produced by any mixture are given as the weighted average of the values shown in the table for each type of coal. For example, if the coals are mixed to produce a blend that consists of 35% of coal 1, 40% of coal 2, and 25% of coal 3, the sulfur emission (in ppm) resulting from burning one ton of this blend is:

$$0.35 \times 1{,}100 + 0.40 \times 3{,}500 + 0.25 \times 1{,}300 = 2{,}110$$

The manager of this facility wants to select a blend of coal to burn while considering the following objectives:

Objective 1: Maximize the pounds of steam produced.
Objective 2: Minimize sulfur emissions.
Objective 3: Minimize coal dust emissions.

a. Formulate an MOLP model for this problem and implement your model in a spreadsheet.
b. Determine the best possible value for each objective in the problem.

c. Determine the solution that minimizes the maximum percentage deviation from the optimal objective function values. What solution do you obtain?
d. Suppose management considers maximizing the amount of steam produced five times as important as achieving the best possible values for the other objectives. What solution does this suggest?

18. The Waygate Corporation makes five different types of metal casing for personal computers. The company is in the process of replacing its machinery with three different new models of metal stamping machines: the Robo-I, Robo-II, and Robo-III. The unit costs of each machine are $18,500, $25,000, and $35,000, respectively. Each machine can be programmed to produce any of the five casings. Once the machine is programmed, it produces each type of casing at the following rates:

	Casings per Hour				
	Type 1	**Type 2**	**Type 3**	**Type 4**	**Type 5**
Robo-I	100	130	140	210	80
Robo-II	265	235	170	220	120
Robo-III	200	160	260	180	220

The company has the following goals:

Goal 1: To spend no more than approximately $400,000 on the purchase of new machines
Goal 2: To have the ability to produce approximately 3,200 units of type 1 casings per hour
Goal 3: To have the ability to produce approximately 2,500 units of type 2 casings per hour
Goal 4: To have the ability to produce approximately 3,500 units of type 3 casings per hour
Goal 5: To have the ability to produce approximately 3,000 units of type 4 casings per hour
Goal 6: To have the ability to produce approximately 2,500 units of type 5 casings per hour

a. Formulate a GP model for this problem. Assume that all percentage deviations from all goals are equally undesirable.
b. Implement your model in a spreadsheet and solve it.
c. What is the optimal solution?
d. What is the solution that minimizes the maximum percentage deviation from all the goals?
e. Assume that the company can spend no more than $400,000. What is the solution that minimizes the maximum percentage deviation from all the remaining goals?

19. The central Florida high school basketball tournament pits teams from four different counties against one another. The average distance (in miles) between tournament locations in each country is given in the following table.

	Average Distance (in miles) Between Counties			
	Orange	**Seminole**	**Osceola**	**Volusia**
Orange	—	30	45	60
Seminole	30	—	50	20
Osceola	45	50	—	75
Volusia	60	20	75	—

Games are officiated by certified refereeing crews from each county. Orange, Seminole, Osceola, and Volusia Counties have 40, 22, 20, and 26 certified crews, respectively. During the tournament, officiating crews cannot work games in their home counties. They are paid $0.23 per mile in travel costs (in addition to a $50 per game officiating fee). (Assume that each officiating crew travels to games in a single vehicle.) Additionally, crews from one county cannot work more than 50% of the games in any other single county. It is anticipated that Orange, Seminole, Osceola, and Volusia Counties will host 28, 24, 16, and 20 games, respectively.

a. Create a spreadsheet model to determine the least costly plan for allocating officiating crews from the various counties.
b. What is the optimal solution and associated travel cost for the referees?
c. Suppose it is desired to spend no more than $700 referee travel expenses for these games? Is that possible? If not, determine the solution that minimizes the maximum percentage deviation from the 50% officiating requirement for each county while requiring no more than $700 in travel costs.

20. The Chick'n-Pick'n fast-food chain is considering how to expand its operations. Three types of retail outlets are possible: a lunch counter operation designed for office buildings in downtown areas, an eat-in operation designed for shopping malls, and a stand-alone building with drive-through and sit-down facilities. The following table summarizes the number of jobs, start-up costs, and annual returns associated with each type of operation:

	Lunch Counter	Mall	Stand-alone
Jobs	9	17	35
Costs	$150,000	$275,000	$450,000
Returns	$ 85,000	$125,000	$175,000

The company has $2,000,000 available to pay start-up costs for new operations in the coming year. Additionally, there are five possible sites for lunch counter operations, seven possible mall locations, and three possible stand-alone locations. The company wants to plan its expansion in a way that maximizes annual returns and the number of jobs created.

a. Formulate an MOLP for this problem.
b. Determine the best possible value for each objective in the problem.
c. Implement your model in a spreadsheet and solve it to determine the solution that minimizes the maximum percentage deviation from the optimal objective function values. What solution do you obtain?
d. Suppose management considers maximizing returns three times as important as maximizing the number of jobs created. What solution does this suggest?

21. A private foundation has offered $3 million to allocate to cities to help fund programs that aid the homeless. Grant proposals were received from cities A, B, and C seeking assistance of $750,000, $1.2 million, and $2.5 million, respectively. In the grant proposals, cities were requested to quantify the number of assistance units that would be provided using the funds (an assistance unit is a night on a bed in a shelter or a free meal). Cities A, B, and C reported they could provide 485,000, 850,000, and 1.5 million assistance units, respectively, with the funds requested during the coming year. The directors of the foundation have two objectives. They want to maximize the number of assistance units obtained with the $3 million. However, they also want to help each of the cities by funding as much of their individual requests as possible (this might be done by maximizing the minimum percentage of funding received by any city).

a. Formulate an MOLP for this problem.
b. Determine the best possible value for each objective in the problem.
c. Implement your model in a spreadsheet and solve it to determine the solution that minimizes the maximum percentage deviation from the optimal objective function values. What solution do you obtain?

22. The marketing manager for Glissen Paint is working on the weekly sales and marketing plan for the firm's industrial and contractor sales staff. Glissen's sales representatives contact two types of customers: existing customers and new customers. Each contact with an existing customer normally takes 3 hours of the salesperson's time (including travel time) and results in an average sale of $425. Contacts with new customers generally take a bit longer, on average 4 hours, and result in an average sale of $350. The company's salespeople are required to work 40 hours a week, but often work more to achieve their sales quotas (on which their bonuses are based). The company has a policy limiting the number of hours a salesperson can work to 50 hours per week. The sales manager wants to set customer contact quotas for the salespeople that will achieve the following goals (listed in order of importance):

    Goal 1: Each salesperson should achieve an average weekly sales level of $6,000.
    Goal 2: Each salesperson should contact at least 10 existing customers per week.
    Goal 3: Each salesperson should contact at least 5 new customers per week.
    Goal 4: Each salesperson should limit overtime to no more than 5 hours per week.

a. Formulate this problem as a GP with an objective of minimizing the sum of the weighted undesirable percentage deviation from the goals.
b. Implement your model in a spreadsheet and solve it assuming equal weights on each goal. What solution do you obtain?

23. A paper recycling company converts newspaper, mixed paper, white office paper, and cardboard into pulp for newsprint, packaging paper, and print-stock quality paper. The recycler is currently trying to determine the best way of filling an order for 500 tons of newsprint pulp, 600 tons of packaging paper pulp, and 300 tons of print-stock quality pulp. The following table summarizes the yield for each kind of pulp recovered from each ton of recycled material.

	Recycling Yield		
	Newsprint	**Packaging**	**Print Stock**
Newspaper	85%	80%	—
Mixed Paper	90%	80%	70%
White Office Paper	90%	85%	80%
Cardboard	80%	70%	—

For instance, a ton of newspaper can be recycled using a technique that yields 0.85 tons of newsprint pulp. Alternatively, a ton of newspaper can be recycled using a technique that yields 0.80 tons of packaging paper. Similarly, a ton of cardboard can be recycled to yield 0.80 tons of newsprint or 0.70 tons of packaging paper pulp. Note that newspaper and cardboard cannot be converted to print-stock pulp using the techniques available to the recycler. As each material is recycled, it also produces a toxic sludge that the recycler must dispose of. The amount of toxic sludge (in pounds) created by processing a ton of each of the raw materials into each of the types of pulp is summarized in the following table. For instance, each ton of newspaper that is processed into newspaper pulp creates 125 pounds of sludge.

	Sludge (lbs.)		
	Newsprint	Packaging	Print Stock
Newspaper	125	100	0
Mixed Paper	50	100	150
White Office Paper	50	75	100
Cardboard	100	150	0

The cost of processing each ton of raw material into the various types of pulp is summarized in the following table along with the amount of each of the four raw materials that can be purchased, and their costs.

	Processing Costs per Ton			Purchase Cost Per Ton	Tons Available
	Newsprint	Packaging	Print Stock		
Newspaper	$6.50	$11.00	—	$15	600
Mixed Paper	$9.75	$12.25	$9.50	$16	500
White Office Paper	$4.75	$ 7.75	$8.50	$19	300
Cardboard	$7.50	$ 8.50	—	$17	400

These processing costs include the cost of disposing of sludge. However, the managers of the recycling facility would prefer to minimize the amount of sludge created, and the total cost of filling the order.

a. Formulate an MOLP model for this problem and implement your model in a spreadsheet.
b. Determine the best possible value for each objective in the problem.
c. Determine the solution that minimizes the maximum percentage deviation from the optimal objective function values. What solution do you obtain?
d. Suppose that management considers minimizing costs to be twice as important as minimizing the amount of sludge produced. What solution does this suggest?

24. A trust officer at Pond Island Bank needs to determine what percentage of the bank's investible funds to place in each of the following investments.

Investment	Yield	Maturity	Risk
A	11.0%	8	5
B	8.0%	1	2
C	8.5%	7	1
D	10.0%	6	5
E	9.0%	2	3

The column labeled Yield represents each investment's annual yield. The column labeled Maturity indicates the number of years funds must be placed in each investment. The column labeled Risk indicates an independent financial analyst's assessment of each investment's risk. In general, the trust officer wants to maximize the weighted average yield on the funds placed in these investments while minimizing the weighted average maturity and the weighted average risk.

a. Formulate an MOLP model for this problem and implement your model in a spreadsheet.
b. Determine the best possible value for each objective in the problem.
c. Determine the solution that minimizes the maximum percentage deviation from the optimal objective function values. What solution do you obtain?
d. Suppose management considers minimizing the average maturity to be twice as important as minimizing average risk, and maximizing average yield to be twice as important as minimizing average maturity. What solution does this suggest?

25. A major city in the northeast wants to establish a central transportation station from which visitors can ride buses to four historic landmarks. The city is arranged in a grid, or block, structure with equally spaced streets running north and south and equally spaced avenues running east and west. The coordinates of any corner of any block in the city can be identified by the street and avenue numbers intersecting at that particular corner. The following table gives the coordinates for the four historic landmarks:

Landmark	Street	Avenue
1	7	3
2	3	1
3	1	6
4	6	9

The transportation planners want to build the transportation station at the location in the city that minimizes the total travel distance (measured rectangularly) to each landmark. For example, if they built the station at 6th Street and 2nd Avenue, the total distance to each landmark will be:

Landmark	Distance
1	$\lvert 7-6\rvert + \lvert 3-2\rvert = 1 + 1 = 2$
2	$\lvert 3-6\rvert + \lvert 1-2\rvert = 3 + 1 = 4$
3	$\lvert 1-6\rvert + \lvert 6-2\rvert = 5 + 4 = 9$
4	$\lvert 6-6\rvert + \lvert 9-2\rvert = 0 + 7 = 7$
	Total Distance = 22

a. Plot the locations of the various historical landmarks on a graph where the X-axis represents avenue numbers (starting at 0) and the Y-axis represents street numbers (starting at 0).
b. Formulate an LP model to determine the corner at which the central transportation station should be located. (*Hint:* Let the decision variables represent the street location (X_1) and avenue location (X_2) of the station and use deviational variables to measure the absolute street distance and absolute avenue distance from each landmark to X_1 and X_2. Minimize the sum of the deviational variables.)

26. KPS Communications is planning to bring wireless Internet access to the town of Ames, Iowa. Using a geographic information system, KPS has divided Ames into the following 5 by 5 grid. The values in each block of the grid indicate the expected annual revenue (in $1,000s) that KPS will receive if it provides wireless Internet service to the geographic area represented by each block.

Expected Annual Revenue by Area (in $1,000s)

$34	$43	$62	$42	$34
$64	$43	$71	$48	$65
$57	$57	$51	$61	$30
$32	$38	$70	$56	$40
$68	$73	$30	$56	$44

KPS can build wireless towers in any block in the grid at a cost of $150,000 per tower. Each tower can provide wireless service to the block it is in and to all adjacent blocks. (Blocks are considered to be adjacent if they share a side. Blocks touching only at corner point are not considered adjacent.) KPS wants to determine how many towers to build, and where to build them, to maximize profits in the first year of operations. (Note: If a block can receive wireless service from two different towers, the revenue for that block should be counted only once.)

a. Create a spreadsheet model for this problem and solve it.
b. What is the optimal solution and how much money will KPS make in the first year?
c. To be the dominant player in this market, KPS also is considering providing wireless access to all of Ames—even if it is less profitable to do so in the short term. Modify your model as necessary to determine the tower location plan that maximizes the wireless coverage in Ames. What is the optimal solution and how much profit will it provide?
d. Clearly, there is a trade-off between the objective in part b of maximizing profit and the objective in part c of maximizing wireless coverage. Determine the solution that minimizes the maximum percentage deviation from the optimal objective function values from parts b and c.
e. Suppose KPS considers maximizing profit to be twice as important as maximizing coverage. What solution does this suggest?

27. A car dealer specializing in late model used cars collected the following data on the selling price and mileage of five cars of the same make and model year at an auto auction:

Mileage	Price
43,890	\$12,500
35,750	\$13,350
27,300	\$14,600
15,500	\$15,750
8,900	\$17,500

Because there seems to be a strong relationship between mileage and price, the dealer wants to use this information to predict this type of car's market value on the basis of its mileage. The dealer thinks that the car's selling price can be predicted as:

$$\text{Estimated price} = A + B \times \text{mileage}$$

A and B represent numeric constants (which might be positive or negative). Using the data collected at last week's auction, the dealer wants to determine appropriate values for A and B that minimize the following quantity:

$$\begin{aligned}\text{MIN:}\quad & |A + B \times 43890 - 12500| + |A + B \times 35750 - 13350| + \\ & |A + B \times 27300 - 14600| + |A + B \times 15500 - 15750| + \\ & |A + B \times 8900 - 17500|\end{aligned}$$

Notice that this objective seeks to find values of A and B that minimize the sum of the absolute value of the deviations between the actual prices of the cars and the estimated prices.

a. Create an LP model using deviational variables whose solution provides the best values for A and B using the stated criteria. That is, what values of A and B minimize the sum of the absolute deviations between the actual and estimated selling prices?
b. Implement your model in a spreadsheet and solve it.
c. Using the values of A and B determined by your solution, what should the estimated selling price be for each car?

28. Refer to the previous question. Suppose that the car dealer wanted to find values for A and B that minimized the maximum absolute deviation between the actual and estimated selling price for each car. What values of A and B achieve this objective?

29. A job in a machine shop must undergo five operations—A, B, C, D, and E. Each operation can be performed on either of two machines. The following table summarizes the time required for each machine to perform each operation:

	A	B	C	D	E
Machine 1	7	8	4	4	9
Machine 2	5	3	9	6	8

Formulate a model that can be solved to determine the job routing that minimizes the maximum amount of time used on either machine. That is, if t_i is the total time used on machine i, find the solution that minimizes the maximum of t_1 and t_2.

CASE 7.1 Removing Snow in Montreal

Based on: James Campbell and Andre Langevin, "The Snow Disposal Assignment Problem," *Journal of the Operational Research Society*, 1995, pp. 919–929.

Snow removal and disposal are important and expensive activities in Montreal and many northern cities. Although snow can be cleared from streets and sidewalks by plowing and shoveling, in prolonged subfreezing temperatures, the resulting banks of accumulated snow can impede pedestrian and vehicular traffic and must be removed.

To allow timely removal and disposal of snow, a city is divided into several sectors and snow removal operations are carried out concurrently in each sector. In Montreal, accumulated snow is loaded into trucks and hauled away to disposal sites (e.g., rivers, quarries, sewer chutes, surface holding areas). The different types of disposal sites can accommodate different amounts of snow due to the physical size of the disposal facility. The annual capacities for five different snow disposal sites are given below (in 1,000s of cubic meters).

	Disposal Site				
	1	2	3	4	5
Capacity	350	250	500	400	200

The snow transported to various disposal sites often is contaminated by salt and deicing chemicals. When the snow melts, these contaminants ultimately wind up in lakes, rivers, and the local water supply. The different disposal sites are equipped to remove different amounts of contaminants from the snow they receive. The percentage of contaminants that can be removed from the snow delivered to each disposal site is given below. The amount of contaminant contained in removed snow is relatively constant across sectors.

	Disposal Site				
	1	2	3	4	5
Contaminant Removed	30%	40%	20%	70%	50%

The cost of removing and disposing of snow depends mainly on the distance it must be trucked. For planning purposes, the City of Montreal uses the straight-line distance between the center of each sector to each of the various disposal sites as an approximation of the cost involved in transporting snow between these locations. The following table summarizes these distances (in kilometers) for ten sectors in the city.

	Disposal Site				
Sector	**1**	**2**	**3**	**4**	**5**
1	3.4	1.4	4.9	7.4	9.3
2	2.4	2.1	8.3	9.1	8.8
3	1.4	2.9	3.7	9.4	8.6
4	2.6	3.6	4.5	8.2	8.9
5	1.5	3.1	2.1	7.9	8.8
6	4.2	4.9	6.5	7.7	6.1
7	4.8	6.2	9.9	6.2	5.7
8	5.4	6	5.2	7.6	4.9
9	3.1	4.1	6.6	7.5	7.2
10	3.2	6.5	7.1	6	8.3

Using historical snowfall data, the city can estimate the annual volume of snow requiring removal in each sector as four times the length of streets in the sectors in meters (i.e., it is assumed that each linear meter of street generates 4 cubic meters of snow to remove over an entire year). The following table estimates the snow removal requirements (in 1000s of cubic meters) for each sector in the coming year.

Estimated Annual Snow Removal Requirements

1	2	3	4	5	6	7	8	9	10
153	152	154	138	127	129	111	110	130	135

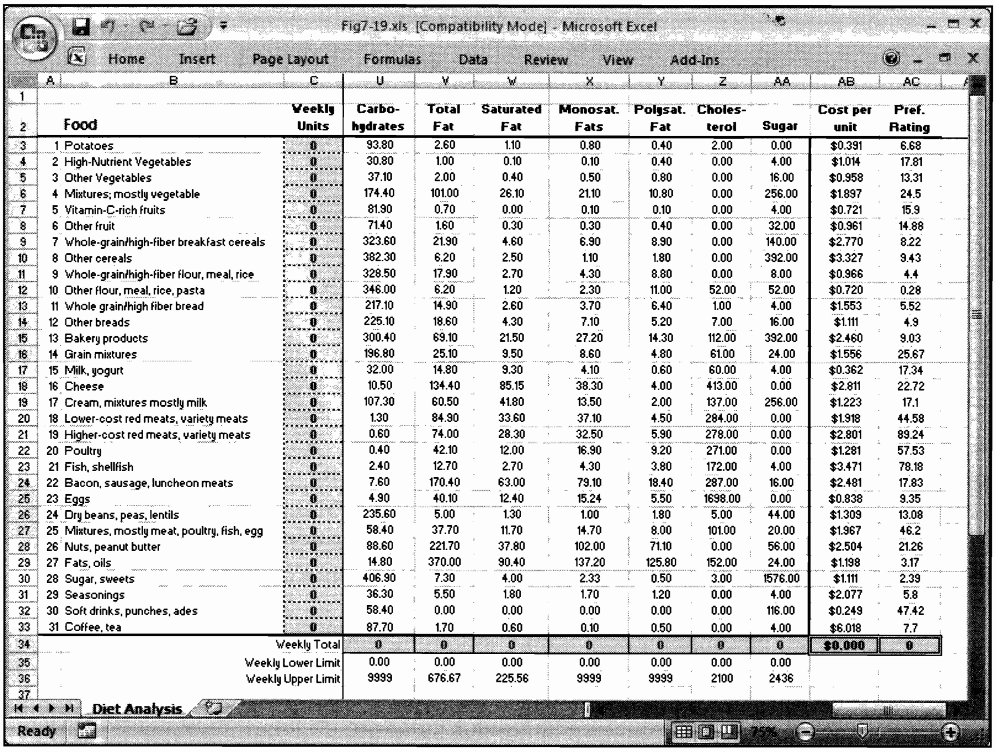
a. If Montreal wants to pursue the objective of minimizing the distance the snow must be moved (and therefore the cost of removing snow), how much snow should it plan to move from each sector to each disposal site?
b. If it costs $35 to move 1000 cubic meters of snow one kilometer, how much should Montreal plan on spending on the transportation for the removal of snow?
c. If Montreal wants to pursue the objective of maximizing the amount of contaminant that is removed from transported snow, how much snow should it plan to move from each sector to each disposal site and what transportation cost is associated with this solution?
d. Suppose Montreal wants to minimize the maximum percentage deviation from the optimal value for each of the two objectives mentioned earlier. What is the optimal solution and how far is each objective function from its optimal value?
e. Suppose the removal of contaminants is regarded as five times more important than transportation cost minimization. What solution minimizes the maximum weighted percentage deviation for each objective? How far is each objective from its optimal value?
f. What other suggestions might you have for Montreal as it attempts to deal with these two conflicting objectives?

Planning Diets for the Food Stamp Program

CASE 7.2

Based on: S. Taj, "A Mathematical Model for Planning Policies for Food Stamps." *Applications of Management Science*, Vol. 7, 25–48, 1993.

The United States Department of Agriculture (USDA) is responsible for managing and administering the national food stamp program. This program has the responsibility of



Food	Weekly Units	Carbo-hydrates	Total Fat	Saturated Fat	Monosat. Fats	Polysat. Fat	Choles-terol	Sugar	Cost per unit	Pref. Rating
1 Potatoes	0	93.80	2.60	1.10	0.80	0.40	2.00	0.00	$0.391	6.68
2 High-Nutrient Vegetables	0	30.80	1.00	0.10	0.10	0.40	0.00	4.00	$1.014	17.81
3 Other Vegetables	0	37.10	2.00	0.40	0.50	0.80	0.00	16.00	$0.958	13.31
4 Mixtures; mostly vegetable	0	174.40	101.00	26.10	21.10	10.80	0.00	256.00	$1.897	24.5
5 Vitamin-C-rich fruits	0	81.90	0.70	0.00	0.10	0.10	0.00	4.00	$0.721	15.9
6 Other fruit	0	71.40	1.60	0.30	0.30	0.40	0.00	32.00	$0.961	14.88
7 Whole-grain/high-fiber breakfast cereals	0	323.60	21.90	4.60	6.90	8.90	0.00	140.00	$2.770	8.22
8 Other cereals	0	382.30	6.20	2.50	1.10	1.80	0.00	392.00	$3.327	9.43
9 Whole-grain/high-fiber flour, meal, rice	0	328.50	17.90	2.70	4.30	8.80	0.00	8.00	$0.966	4.4
10 Other flour, meal, rice, pasta	0	346.00	6.20	1.20	2.30	11.00	52.00	52.00	$0.720	0.28
11 Whole grain/high fiber bread	0	217.10	14.90	2.60	3.70	6.40	1.00	4.00	$1.553	5.52
12 Other breads	0	225.10	18.60	4.30	7.10	5.20	7.00	16.00	$1.111	4.9
13 Bakery products	0	300.40	69.10	21.50	27.20	14.30	112.00	392.00	$2.460	9.03
14 Grain mixtures	0	196.80	25.10	9.50	8.60	4.80	61.00	24.00	$1.556	25.67
15 Milk, yogurt	0	32.00	14.80	9.30	4.10	0.60	60.00	4.00	$0.362	17.34
16 Cheese	0	10.50	134.40	85.15	38.30	4.00	413.00	0.00	$2.811	22.72
17 Cream, mixtures mostly milk	0	107.30	60.50	41.80	13.50	2.00	137.00	256.00	$1.223	17.1
18 Lower-cost red meats, variety meats	0	1.30	84.90	33.60	37.10	4.50	284.00	0.00	$1.918	44.58
19 Higher-cost red meats, variety meats	0	0.60	74.00	28.30	32.50	5.90	278.00	0.00	$2.801	89.24
20 Poultry	0	0.40	42.10	12.00	16.90	9.20	271.00	0.00	$1.281	57.53
21 Fish, shellfish	0	2.40	12.70	2.70	4.30	3.80	172.00	4.00	$3.471	78.18
22 Bacon, sausage, luncheon meats	0	7.60	170.40	63.00	79.10	18.40	287.00	16.00	$2.481	17.83
23 Eggs	0	4.90	40.10	12.40	15.24	5.50	1698.00	0.00	$0.838	9.35
24 Dry beans, peas, lentils	0	235.60	5.00	1.30	1.00	1.80	5.00	44.00	$1.309	13.08
25 Mixtures, mostly meat, poultry, fish, egg	0	58.40	37.70	11.70	14.70	8.00	101.00	20.00	$1.967	46.2
26 Nuts, peanut butter	0	88.60	221.70	37.80	102.00	71.10	0.00	56.00	$2.504	21.26
27 Fats, oils	0	14.80	370.00	90.40	137.20	125.80	152.00	24.00	$1.198	3.17
28 Sugar, sweets	0	406.90	7.30	4.00	2.33	0.50	3.00	1576.00	$1.111	2.39
29 Seasonings	0	36.30	5.50	1.80	1.70	1.20	0.00	4.00	$2.077	5.8
30 Soft drinks, punches, ades	0	58.40	0.00	0.00	0.00	0.00	0.00	116.00	$0.249	47.42
31 Coffee, tea	0	87.70	1.70	0.60	0.10	0.50	0.00	4.00	$6.018	7.7
Weekly Total		0	0	0	0	0	0	0	$0.000	0
Weekly Lower Limit		0.00	0.00	0.00	0.00	0.00	0.00	0.00		
Weekly Upper Limit		9999	676.67	225.56	9999	9999	2100	2436		

FIGURE 7.19

Data for the USDA diet planning problem

providing vouchers to low income families that can be used in place of cash to purchase food at grocery stores. In determining the cash value of the vouchers issued, the USDA must consider how much it costs to obtain a nutritional, well-balanced diet for men and women in various age groups. As a first step in this process, the USDA identified and analyzed 31 different food groups and determined the contributions that a serving from each group makes to 24 different nutritional categories. A partial listing of this information is given in Figure 7.19 (and in the file Fig7-19.xls on your data disk).

The last two rows in this spreadsheet indicate the minimum and/or maximum nutrients required per week for men between the ages of 20 and 50. (Maximum values of 9999 indicate that no maximum value applies to that particular nutritional requirement.)

The USDA uses this information to design a diet (or weekly consumption plan) that meets the indicated nutritional requirements. The last two columns in Figure 7.19 represent two different objectives that can be pursued in creating a diet. First, we might want to identify the diet that meets the nutritional requirements at a minimum cost. Although such a diet might be very economical, it also might be very unsatisfactory to the tastes of the people who are expected to eat it. To help address this issue, the USDA conducted a survey to assess people's preferences for different food groups. The last column in Figure 7.19 summarizes these preference ratings, with higher scores indicating more desirable foods and lower scores indicating less desirable foods. Thus, another objective that could be pursued would be that of determining the diet that meets the nutritional requirements and produces the highest total preference rating. However, this solution is likely to be quite expensive. Assume that the USDA has asked you to help them analyze this situation using MOLP.

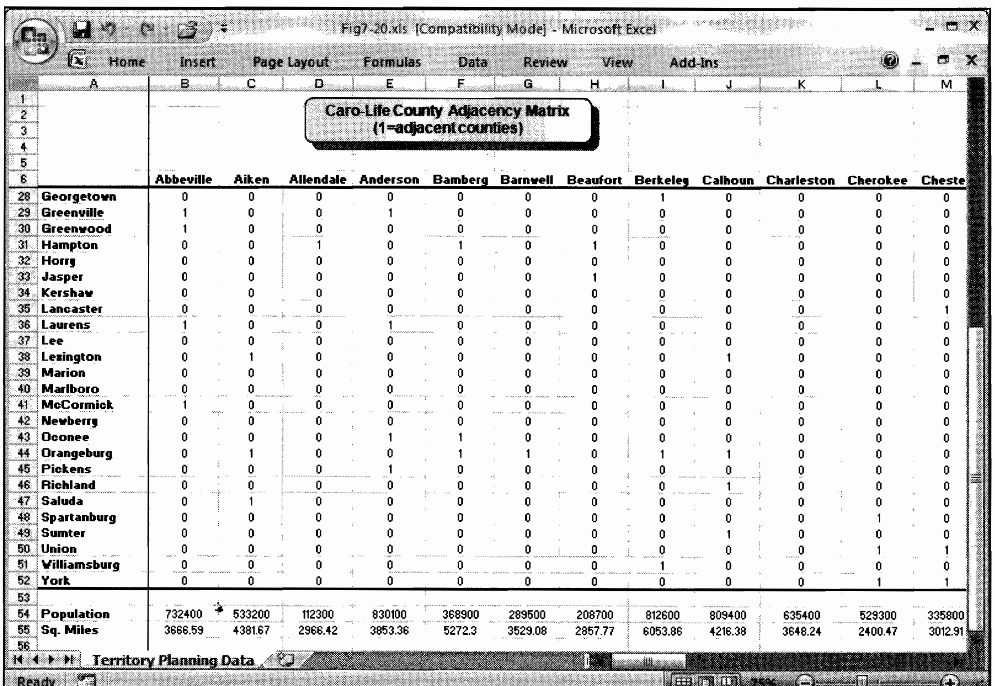
a. Find the weekly diet that meets the nutritional requirements in the least costly manner. What is the lowest possible minimum cost? What preference rating does this solution have?

b. Find the weekly diet that meets the nutritional requirements with the highest preference rating. What preference rating does this solution have? What cost is associated with this solution?
c. Find the solution that minimizes the maximum percentage deviation from the optimum values for each individual objective. What cost and preference rating is associated with this solution?
d. Suppose that deviations from the optimal cost value are weighted twice as heavily as those from the optimal preference value. Find the solution that minimizes the maximum weighted percentage deviations. What cost and preference rating is associated with this solution?
e. What other factors or constraints might you want to include in this analysis if you had to eat the resulting diet?

Sales Territory Planning at Caro-Life

CASE 7.3

Caro-Life is a financial services firm that specializes in selling life, auto, and home insurance to residential consumers in North Carolina. The company is planning to expand and offer its services in South Carolina. The company wants to open ten offices throughout the state to ensure that all residents of the state can access at least one office in either their county of residence or an adjacent county. The set of counties adjacent to the county containing each office will be regarded as the sales territory for that office. (Note that a county is considered to be adjacent to itself.) Figure 7.20 (and the file Fig7-20.xls on your data disk) shows a portion of an Excel spreadsheet with a matrix indicating county adjacencies throughout the state, and the estimated population and geographic size (in square miles) for each potential sales territory. (Values of 1 in the matrix indicate counties that are adjacent to one another.)



	Abbeville	Aiken	Allendale	Anderson	Bamberg	Barnwell	Beaufort	Berkeley	Calhoun	Charleston	Cherokee	Cheste
Georgetown	0	0	0	0	0	0	0	1	0	0	0	0
Greenville	1	0	0	1	0	0	0	0	0	0	0	0
Greenwood	1	0	0	0	0	0	0	0	0	0	0	0
Hampton	0	0	1	0	1	0	1	0	0	0	0	0
Horry	0	0	0	0	0	0	0	0	0	0	0	0
Jasper	0	0	0	0	0	0	1	0	0	0	0	0
Kershaw	0	0	0	0	0	0	0	0	0	0	0	0
Lancaster	0	0	0	0	0	0	0	0	0	0	0	1
Laurens	1	0	0	1	0	0	0	0	0	0	0	0
Lee	0	0	0	0	0	0	0	0	0	0	0	0
Lexington	0	1	0	0	0	0	0	0	1	0	0	0
Marion	0	0	0	0	0	0	0	0	0	0	0	0
Marlboro	0	0	0	0	0	0	0	0	0	0	0	0
McCormick	1	0	0	0	0	0	0	0	0	0	0	0
Newberry	0	0	0	0	0	0	0	0	0	0	0	0
Oconee	0	0	0	1	1	0	0	0	0	0	0	0
Orangeburg	0	1	0	0	1	1	0	1	1	0	0	0
Pickens	0	0	0	1	0	0	0	0	0	0	0	0
Richland	0	0	0	0	0	0	0	0	1	0	0	0
Saluda	0	1	0	0	0	0	0	0	0	0	0	0
Spartanburg	0	0	0	0	0	0	0	0	0	0	1	0
Sumter	0	0	0	0	0	0	0	0	1	0	0	0
Union	0	0	0	0	0	0	0	0	0	0	1	1
Williamsburg	0	0	0	0	0	0	0	1	0	0	0	0
York	0	0	0	0	0	0	0	0	0	0	1	1
Population	732400	533200	112300	830100	368900	289500	208700	812600	809400	635400	529300	335800
Sq. Miles	3666.59	4381.67	2966.42	3853.36	5272.3	3529.08	2857.77	6053.86	4216.38	3648.24	2400.47	3012.91

FIGURE 7.20

Data for the Caro-Life sales territory planning problem

Sales of insurance products in a given area tend to be highly correlated with the number of people living in the area. As a result, agents assigned to the various offices want their sales territories to contain as many people as possible (to maximize sales potential). On the other hand, territories containing large amounts of people also may consist of a large geographic area that might require agents to travel a lot. So the goal of having a territory with lots of people sometimes conflicts with the goal of having a territory that is compact in size. It is important for Caro-Life to design its sales territories in as equitable a manner as possible (i.e., where the territories are similar in terms of geographic size and sales potential).

a. Assume that Caro-Life wants to maximize the average sales potential of its ten offices. Where should it locate offices and what is the population and geographic area associated with each office?
b. Assume that Caro-Life wants to minimize the average geographic area covered by each of its ten offices. Where should it locate offices and what is the population and geographic area associated with each office?
c. Determine the solution that minimizes the maximum percentage deviation from the optimal objective function values identified in parts a and b. According to this solution, where should Caro-Life locate its offices and what is the population and geographic area associated with each office?
d. Suppose that Caro-Life considers maximizing average sales potential of its territories to be twice as important as minimizing the average geographic size of its territories. Find the solution that minimizes the maximum weighted percentage deviations. According to this solution, where should Caro-Life locate its offices and what is the population and geographic area associated with each office?
e. What other issues would you suggest that Caro-Life take into account in modeling this decision problem?