Time Series Forecasting

Lecture 5

Chapter 2: Regression Analysis and Time Series Analysis

Institute of Engineering
Asst. Prof. Anita Prajapati, Ph.D.

14 June 2023

Spreadsheet Modeling & Decision Analysis:

Chapter 11 : Time Series Forecasting

A Practical Introduction to Management Science, 3e

by Cliff Ragsdale

Introduction to Time Series Analysis

- A *time-series* is a set of observations on a quantitative variable collected over time.
- Examples
  - Historical data on sales, inventory, customer counts, interest rates, costs, etc
- Businesses are often very interested in forecasting time series variables.
- Often, independent variables are not available to build a regression model of a time series variable.
- In time series analysis, we analyze the past behavior of a variable in order to predict its future behavior.

Some Time Series Terms

Stationary Data - a time series variable exhibiting no significant upward or downward trend over time.

Nonstationary Data - a time series variable exhibiting a significant upward or downward trend over time.

Seasonal Data - a time series variable exhibiting a repeating patterns at regular intervals over time.

Approaching Time Series Analysis

- There are many, many different time series techniques.
- It is usually impossible to know which technique will be best for a particular data set.
- It is customary to try out several different techniques and select the one that seems to work best.
- To be an effective time series modeler, you need to keep several time series techniques in your “tool box.”

Measuring Accuracy

- We need a way to compare different time series techniques for a given data set.
- Four common techniques are the:
  - mean absolute deviation,
  - mean absolute percent error,
  - the mean square error,
  - root mean square error.

$$\text{MAD} = \sum_{i=1}^{n} \frac{\left|Y_i - \hat{Y}_i\right|}{n}$$

$$\text{MAPE} = \frac{100}{n} \sum_{i=1}^{n} \frac{\left|Y_i - \hat{Y}_i\right|}{Y_i}$$

$$\text{MSE} = \sum_{i=1}^{n} \frac{\left(Y_i - \hat{Y}_i\right)^2}{n}$$

$$\text{RMSE} = \sqrt{\text{MSE}}$$

- We will focus on MSE.

Extrapolation Models

- Extrapolation models try to account for the past behavior of a time series variable in an effort to predict the future behavior of the variable.

$$\hat{Y}_{t+1} = f\left(Y_t, Y_{t-1}, Y_{t-2}, \ldots\right)$$

- We'll first talk about several extrapolation techniques that are appropriate for stationary data.

An Example

- Electra-City is a retail store that sells audio and video equipment for the home and car.
- Each month the manager of the store must order merchandise from a distant warehouse.
- Currently, the manager is trying to estimate how many VCRs the store is likely to sell in the next month.
- He has collected 24 months of data.

See file Fig11-1.xls

Moving Averages

$$\hat{Y}_{t+1} = \frac{Y_t + Y_{t-1} + Y_{t-k+1}}{k}$$

- No general method exists for determining *k*.
- We must try out several *k* values to see what works best.

Implementing the Model

See file Fig11-2.xls

A Comment on Comparing MSE Values

- Care should be taken when comparing MSE values of two different forecasting techniques.
- The lowest MSE may result from a technique that fits older values very well but fits recent values poorly.
- It is sometimes wise to compute the MSE using only the most recent values.

Forecasting With The Moving Average Model

- Forecasts for time periods 25 and 26 at time period 24:

$$\hat{Y}_{25} = \frac{Y_{24} + Y_{23}}{2} = \frac{36 + 35}{2} = 35.5$$

$$\hat{Y}_{26} = \frac{\hat{Y}_{25} + Y_{24}}{2} = \frac{35.5 + 36}{2} = 35.75$$

Weighted Moving Average

- The moving average technique assigns equal weight to all previous observations

$$\hat{Y}_{t+1} = \frac{1}{k}Y_t + \frac{1}{k}Y_{t-1} + \cdots + \frac{1}{k}Y_{t-k-1}$$

- The weighted moving average technique allows for different weights to be assigned to previous observations.

$$\hat{Y}_{t+1} = w_1 Y_t + w_2 Y_{t-1} + \cdots + w_k Y_{t-k-1}$$

- We must determine values for k and the w_i

Implementing the Model

See file Fig11-4.xls

Forecasting With The Weighted Moving Average Model

$$\hat{Y}_{25} = w_1 Y_{24} + w_2 Y_{23} = 0.291 \times 36 + 0.709 \times 35 = 35.3$$

$$\hat{Y}_{26} = w_1 \hat{Y}_{25} + w_2 Y_{24} = 0.291 \times 35.3 + 0.709 \times 36 = 35.80$$

- Forecasts for time periods 25 and 26 at time period 24:

Exponential Smoothing

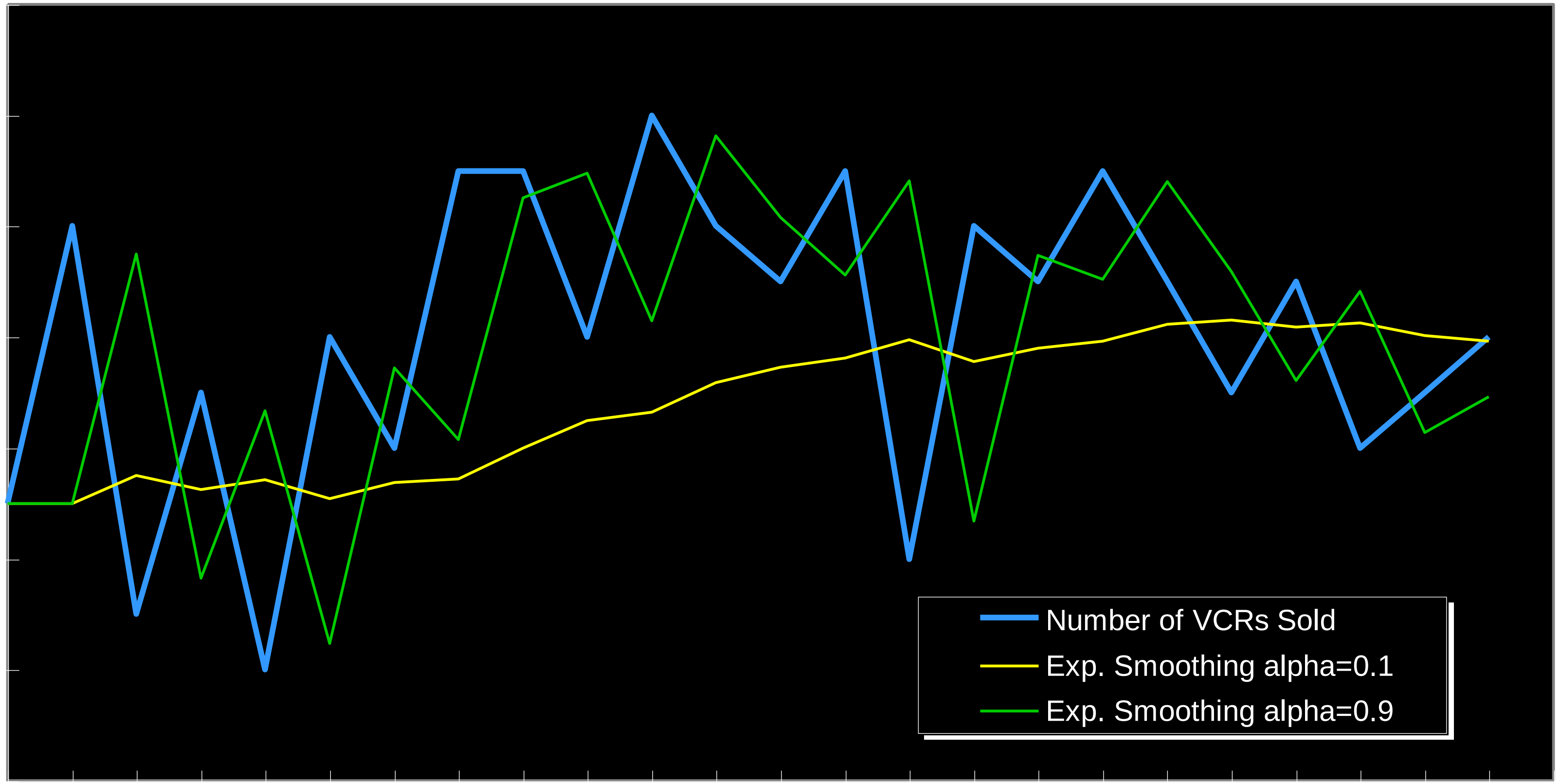
$$\hat{Y}_{t+1} = \hat{Y}_t + \alpha(Y_t - \hat{Y}_t)$$
$$\text{where } 0 \le \alpha \le 1$$

- Another averaging technique for stationary data that allows weights to assign to past data

$$\hat{Y}_{t+1} = \alpha Y_t + \alpha(1-\alpha)Y_{t-1} + \alpha(1-\alpha)^2 Y_{t-2} + \cdots + \alpha(1-\alpha)^n Y_{t-n} + \cdots$$

- It can be shown that the above equation is equivalent to:

Examples of Two Exponential Smoothing Functions



Implementing the Model

See file Fig11-8.xls

Forecasting With The Exponential Smoothing Model

Forecasts for time periods 25 and 26 at time period 24:

$$\hat{Y}_{25} = \hat{Y}_{24} + \alpha(Y_{24} - \hat{Y}_{24}) = 35.74 + 0.268(36 - 35.74) = 35.81$$

$$\hat{Y}_{26} = \hat{Y}_{25} + \alpha(Y_{25} - \hat{Y}_{25}) \approx \hat{Y}_{25} + \alpha(\hat{Y}_{25} - \hat{Y}_{25}) = \hat{Y}_{25} = 35.81$$

Note that,

$$\hat{Y}_{t} = 35.81, \text{ for } t = 25, 26, 27, \ldots$$

Time Series Forecasting (Cont…)

Lecture 6

Chapter 2: Regression Analysis and Time Series Analysis

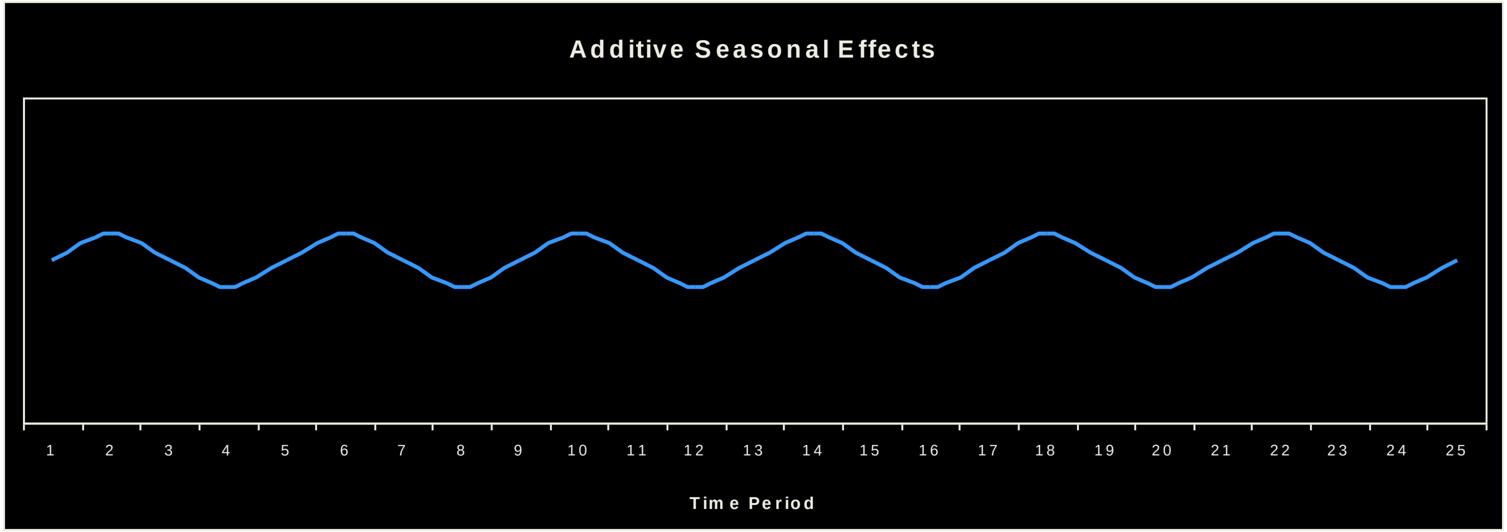
Institute of Engineering
Asst. Prof. Anita Prajapati, Ph.D.

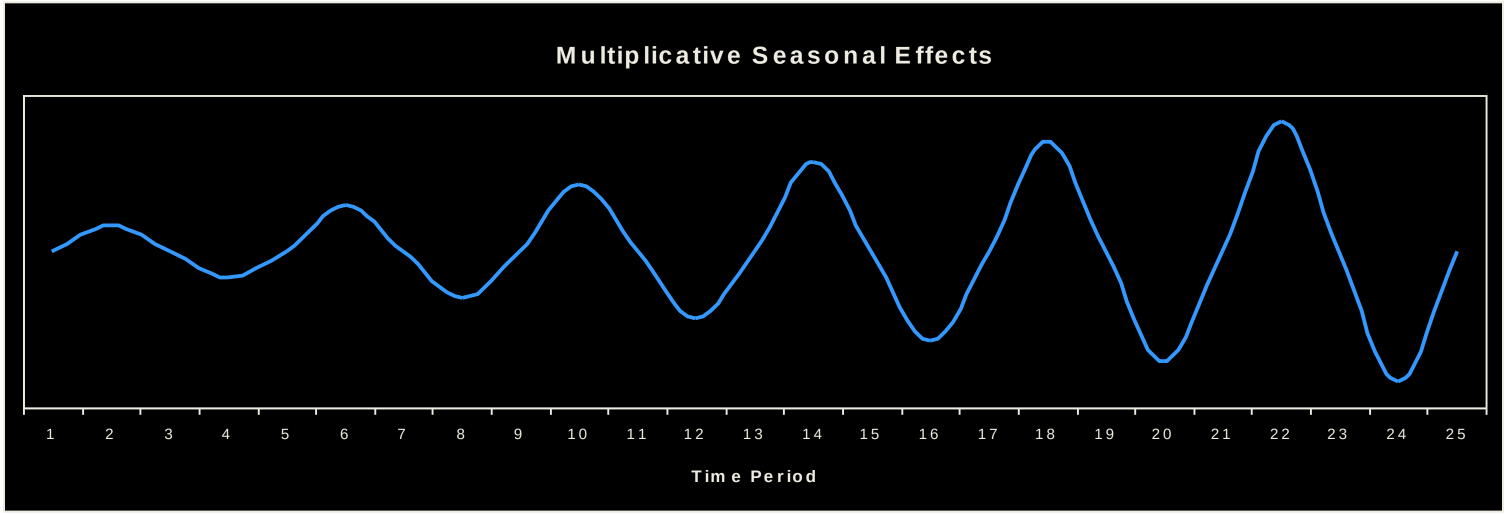
16 June 2023

Seasonality

- Seasonality is a regular, repeating pattern in time series data.
- May be additive or multiplicative in nature...

Stationary Seasonal Effects





Stationary Data With Additive Seasonal Effects

$$\hat{Y}_{t+n} = E_t + S_{t+n-p}$$

where

$$E_t = \alpha(Y_t - S_{t-p}) + (1-\alpha)E_{t-1}$$

$$S_t = \beta(Y_t - E_t) + (1-\beta)S_{t-p}$$

$$0 \le \alpha \le 1$$

$$0 \le \beta \le 1$$

p represents the number of seasonal periods

- E_t is the expected level at time period t.
- S_t is the seasonal factor for time period t.

Implementing the Model

See file Fig11-13.xls

Forecasting With The Additive Seasonal Effects Model

$$\hat{Y}_{24+n} = E_{24} + S_{24+n-4}$$

$$\hat{Y}_{25} = E_{24} + S_{21} = 354.44 + 8.45 = 363.00$$

$$\hat{Y}_{26} = E_{24} + S_{22} = 354.44 - 17.82 = 336.73$$

- Forecasts for time periods 25 - 28 at time period 24:

Stationary Data With Multiplicative Seasonal Effects

where

$$\hat{Y}_{t+n} = E_t \times S_{t+n-p}$$

$$E_t = \alpha(Y_t/S_{t-p}) + (1-\alpha)E_{t-1}$$

$$S_t = \beta(Y_t/E_t) + (1-\beta)S_{t-p}$$

p represents the number of seasonal periods

- E_t is the expected level at time period *t*.
- S_t is the seasonal factor for time period *t*.

Implementing the Model

See file Fig11-16.xls

Forecasting With The Multiplicative, Seasonal Effects Model

Forecasts for time periods 25 - 28 at time period 24:

$$\hat{Y}_{25} = E_{24} \times S_{21} = 353.95 \times 1.015 = 359.26$$
$$\hat{Y}_{26} = E_{24} \times S_{22} = 354.44 \times 0.946 = 334.83$$
$$\hat{Y}_{27} = E_{24} \times S_{23} = 354.44 \times 1.133 = 401.03$$
$$\hat{Y}_{28} = E_{24} \times S_{24} = 354.44 \times 0.912 = 322.80$$

Trend Models

- Trend is the long-term sweep or general direction of movement in a time series.

- We'll now consider some nonstationary time series techniques that are appropriate for data exhibiting upward or downward trends.

An Example

- WaterCraft Inc. is a manufacturer of personal water crafts (also known as jet skis).
- The company has enjoyed a fairly steady growth in sales of its products.
- The officers of the company are preparing sales and manufacturing plans for the coming year.
- Forecasts are needed of the level of sales that the company expects to achieve each quarter.
- For quarterly sales data,

see file Fig11-19.xls

Double Moving Average

Involves the average of average

Moving average $$M_t = (Y_t + Y_{t-1} + \cdots + Y_{t-k+1}) / k$$

Double moving average $$D_t = (M_t + M_{t-1} + \cdots + M_{t-k+1}) / k$$

Double moving average forecasting function $$\hat{Y}_{t+n} = E_t + nT_t$$

where

$$E_t = 2M_t - D_t$$

$$T_t = 2(M_t - D_t)/(k-1)$$

E_t is the expected base level at time period t.
T_t is the expected trend at time period t.

Implementing the Model

See file Fig11-20.xls

Forecasting With The Double Moving Average Model

$$\hat{Y}_{20+n} = E_{20} + nT_{20}$$

$$\hat{Y}_{21} = E_{20} + 1T_{20} = 2385.33 + 1 \times 139.9 = 2525.23$$

$$\hat{Y}_{22} = E_{20} + 2T_{20} = 2385.33 + 2 \times 139.9 = 2665.13$$

$$\hat{Y}_{23} = E_{20} + 3T_{20} = 2385.33 + 3 \times 139.9 = 2805.03$$

$$\hat{Y}_{24} = E_{20} + 4T_{20} = 2385.33 + 4 \times 139.9 = 2944.94$$

Forecasts for time periods 21 through 24 at time period 20:

Double Exponential Smoothing (Holt's Method)

where

$$\hat{Y}_{t+n} = E_t + nT_t$$

$$E_t = \alpha Y_t + (1-\alpha)(E_{t-1} + T_{t-1})$$

$$T_t = \beta(E_t - E_{t-1}) + (1-\beta)\, T_{t-1}$$

$$0 \le \alpha \le 1 \text{ and } 0 \le \beta \le 1$$

E_t is the expected base level at time period t.
T_t is the expected trend at time period t.

Implementing the Model

See file Fig11-22.xls

Forecasting With Holt's Model

Forecasts for time periods 21 through 24 at time period 20:

$$\hat{Y}_{21} = E_{20} + 1T_{20} = 2336.8 + 1 \times 152.1 = 2488.9$$

$$\hat{Y}_{22} = E_{20} + 2T_{20} = 2336.8 + 2 \times 152.1 = 2641.0$$

$$\hat{Y}_{23} = E_{20} + 3T_{20} = 2336.8 + 3 \times 152.1 = 2793.1$$

$$\hat{Y}_{24} = E_{20} + 4T_{20} = 2336.8 + 4 \times 152.1 = 2945.2$$

Holt-Winter's Method For Additive Seasonal Effects

$$\hat{Y}_{t+n} = E_t + nT_t + S_{t+n-p}$$

where

$$E_t = \alpha\left(Y_t - S_{t-p}\right) + (1-\alpha)(E_{t-1} + T_{t-1})$$

$$T_t = \beta\left(E_t - E_{t-1}\right) + (1-\beta)T_{t-1}$$

$$S_t = \gamma\left(Y_t - E_t\right) + (1-\gamma)S_{t-p}$$

$$0 \le \alpha \le 1$$

$$0 \le \beta \le 1$$

$$0 \le \gamma \le 1$$

Implementing the Model

See file Fig11-25.xls

Forecasting With Holt-Winter's Additive Seasonal Effects Method

$$\hat{Y}_{20+n} = E_{20} + nT_{20} + S_{20+n-4}$$

$$\hat{Y}_{21} = E_{20} + 1 \times T_{20} + S_{17} = 2253.3 + 1 \times 154.3 + 262.66 = 2670.3$$

Forecasts for time periods 21 through 24 at time period 20:

$$\hat{Y}_{24} = E_{20} + 4 \times T_{20} + S_{20} = 2253.3 + 4 \times 154.3 + 386.12 = 3256.6$$

Holt-Winter's Method For Multiplicative Seasonal Effects

where

$$E_t = \alpha\left(Y_t / S_{t-p}\right) + (1-\alpha)(E_{t-1} + T_{t-1})$$

$$\hat{Y}_{t+n} = \left(E_t + nT_t\right)S_{t+n-p}$$

$$S_t = \gamma\left(Y_t / E_t\right) + (1-\gamma)S_{t-p}$$

$$0 \le \alpha \le 1$$

$$0 \le \beta \le 1$$

$$0 \le \gamma \le 1$$

Implementing the Model

See file Fig11-28.xls

Forecasting With Holt-Winter's Multiplicative Seasonal Effects Method

$$\hat{Y}_{20+n} = \left(E_{20} + nT_{20}\right)S_{20+n-4}$$

$$\hat{Y}_{21} = (E_{20} + 1T_{20})S_{17} = (2217.6 + 1 \times 137.3)1.152 = 2712.8$$

$$\hat{Y}_{22} = (E_{20} + 2T_{20})S_{18} = (2217.6 + 2 \times 137.3)0.849 = 2115.9$$

$$\hat{Y}_{23} = (E_{20} + 3T_{20})S_{19} = (2217.6 + 3 \times 137.3)1.103 = 2900.3$$

$$\hat{Y}_{24} = (E_{20} + 4T_{20})S_{20} = (2217.6 + 4 \times 137.3)1.190 = 3292.5$$

Forecasts for time periods 21 through 24 at time period 20:

The Linear Trend Model

$$\hat{Y}_t = b_0 + b_1 X_{1_t}$$

$$\text{where } X_{1_t} = t$$

For example:

$$X_{1_1} = 1, X_{1_2} = 2, X_{1_3} = 3, \ldots$$

Implementing the Model

See file Fig11-31.xls

Forecasting With The Linear Trend Model

$$\hat{Y}_{21} = b_0 + b_1 X_{1_{21}} = 375.1 + 92.6255 \times 21 = 2320.3$$

$$\hat{Y}_{22} = b_0 + b_1 X_{1_{22}} = 375.1 + 92.6255 \times 22 = 2412.9$$

$$\hat{Y}_{23} = b_0 + b_1 X_{1_{23}} = 375.1 + 92.6255 \times 23 = 2505.6$$

$$\hat{Y}_{24} = b_0 + b_1 X_{1_{24}} = 375.1 + 92.6255 \times 24 = 2598.2$$

Forecasts for time periods 21 through 24 at time period 20:

The TREND() Function

TREND(Y-range, X-range, X-value for prediction)

where:

Y-range is the spreadsheet range containing the dependent Y variable,
X-range is the spreadsheet range containing the independent X variable(s),
X-value for prediction is a cell (or cells) containing the values for the independent X variable(s) for which we want an estimated value of Y.

Note: The TREND( ) function is dynamically updated whenever any inputs to the function change. However, it does not provide the statistical information provided by the regression tool. It is best two use these two different approaches to doing regression in conjunction with one another.

Summary of the Calculation and Use of Seasonal Indices

1. Create a trend model and calculate the estimated value   for each observation in the sample.
2. For each observation, calculate the ratio of the actual value to the predicted trend value:

   (For additive effects, compute the difference:
3. For each season, compute the average of the ratios calculated in step 2. These are the seasonal indices.
4. Multiply any forecast produced by the trend model by the appropriate seasonal index calculated in step 3. (For additive seasonal effects, add the appropriate factor to the forecast.)

Refining the Seasonal Indices

- Note that Solver can be used to *simultaneously* determine the optimal values of the seasonal indices *and* the parameters of the trend model being used.

- There is no guarantee that this will produce a better forecast, but it should produce a model that fits the data better in terms of the MSE.

See file Fig11-39.xls

Crystal Ball (CB) Predictor

- CB Predictor is an add-in that simplifies the process of performing time series analysis in Excel.
- A trial version of CB Predictor is available on the CD-ROM accompanying this book.
- For more information on CB Predictor see:

  http://www.decisioneering.com

Combining Forecasts

It is also possible to combine forecasts to create a composite forecast.
Suppose we used three different forecasting methods on a given data set.

- Denote the predicted value of time period t using each method as follows:

$$F_{1_t}, F_{2_t}, \text{ and } F_{3_t}$$

- We could create a composite forecast as follows:

$$\hat{Y}_t = b_0 + b_1 F_{1_t} + b_2 F_{2_t} + b_3 F_{3_t}$$

Summary

- Several methods for forecasting future values of a time series variables
- Discussed time series methods for stationary data, non-stationary data and seasonal patterns
- Fit the model to the past behavior of a time series and use the models to project future values
- Helps to be aware of different forecasting techniques and the types of problems for which they are intended
- Often useful to try several techniques and then compare them based on measures of forecast accuracy
- CD predictor makes it easy to apply several time series techniques to a data set and compare their accuracy
- Combining the forecast form different procedures

Assignments

Chapter 11 from Ragsdale book

Question no.: 5,7,10,15,17,20,21, 22,31,32,47,48,57

End of Chapter 2