Chapter 5

Network Modeling

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5.0 Introduction

A number of practical decision problems in business fall into a category known as **network flow problems**. These problems share a common characteristic—they can be described or displayed in a graphical form known as a **network**. This chapter focuses on several types of network flow problems: transshipment problems, shortest path problems, maximal flow problems, transportation/assignment problems, and generalized network flow problems. Although specialized solution procedures exist for solving network flow problems, we will consider how to formulate and solve these problems as LP problems. We will also consider a different type of network problem known as the **minimum spanning tree problem**.

5.1 The Transshipment Problem

Let's begin our study of network flow problems by considering the transshipment problem. As you will see, most of the other types of network flow problems all can be viewed as simple variations of the transshipment problem. So, once you understand how to formulate and solve transshipment problems, the other types of problems will be easy to solve. The following example illustrates the transshipment problem.

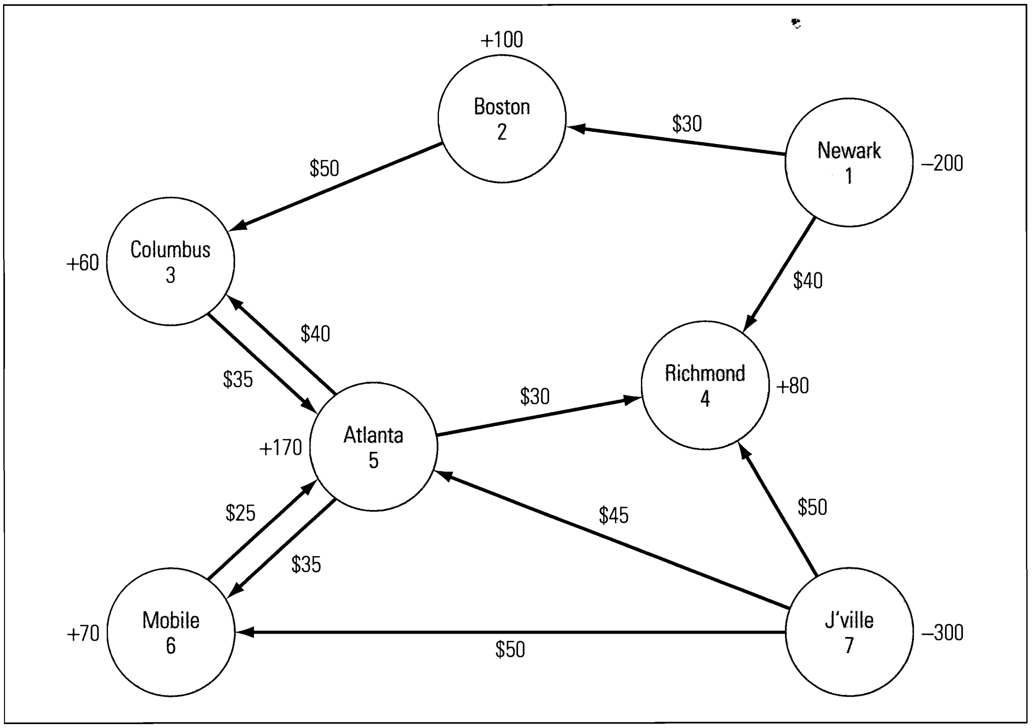
> The Bavarian Motor Company (BMC) manufactures expensive luxury cars in Hamburg, Germany, and exports cars to sell in the United States. The exported cars are shipped from Hamburg to ports in Newark, New Jersey, and Jacksonville, Florida. From these ports, the cars are transported by rail or truck to distributors located in Boston, Massachusetts; Columbus, Ohio; Atlanta, Georgia; Richmond, Virginia; and Mobile, Alabama. Figure 5.1 shows the possible shipping routes available to the company along with the transportation cost for shipping each car along the indicated path.
>
> Currently, 200 cars are available at the port in Newark and 300 are available in Jacksonville. The numbers of cars needed by the distributors in Boston, Columbus, Atlanta, Richmond, and Mobile are 100, 60, 170, 80, and 70, respectively. BMC wants to determine the least costly way of transporting cars from the ports in Newark and Jacksonville to the cities where they are needed.

5.1.1 CHARACTERISTICS OF NETWORK FLOW PROBLEMS

Figure 5.1 illustrates a number of characteristics common to all network flow problems. All network flow problems can be represented as a collection of nodes connected by

FIGURE 5.1

Network representation of the BMC transshipment problem



arcs. The circles in Figure 5.1 are called **nodes** in the terminology of network flow problems, and the lines connecting the nodes are called **arcs**. The arcs in a network indicate the valid paths, routes, or connections between the nodes in a network flow problem. When the lines connecting the nodes in a network are arrows that indicate a direction, the arcs in the network are called **directed arcs**. This chapter discusses directed arcs primarily but, for convenience, refers to them as arcs.

The notion of **supply nodes** (or sending nodes) and **demand nodes** (or receiving nodes) is another common element of network flow problems illustrated in Figure 5.1. The nodes representing the port cities of Newark and Jacksonville are both supply nodes because each has a supply of cars to send to other nodes in the network. Richmond represents a demand node because it demands to receive cars from the other nodes. All the other nodes in this network are transshipment nodes. **Transshipment nodes** can both send to and receive from other nodes in the network. For example, the node representing Atlanta in Figure 5.1 is a transshipment node because it can *receive* cars from Jacksonville, Mobile, and Columbus, and it also can send cars to Columbus, Mobile, and Richmond.

The net supply or demand for each node in the network is indicated by a positive or negative number next to each node. **Positive numbers** represent the *demand* at a given node, and **negative numbers** represent the *supply* available at a node. For example, the value +80 next to the node for Richmond indicates that the number of cars needs to increase by 80—or that Richmond has a *demand* for 80 cars. The value −200 next to the node for Newark indicates that the number of cars can be reduced by 200—or that Newark has a *supply* of 200 cars. A transshipment node can have either a net supply or demand, but not both. In this particular problem, all the transshipment nodes have demands. For example, the node representing Mobile in Figure 5.1 has a demand for 70 cars.

5.1.2 THE DECISION VARIABLES FOR NETWORK FLOW PROBLEMS

The goal in a network flow model is to determine how many items should be moved (or flow) across each of the arcs. In our example, BMC needs to determine the least costly method of transporting cars along the various arcs shown in Figure 5.1 to distribute cars where they are needed. Thus, each of the arcs in a network flow model represents a decision variable. Determining the optimal flow for each arc is the equivalent of determining the optimal value for the corresponding decision variable.

It is customary to use numbers to identify each node in a network flow problem. In Figure 5.1, the number 1 identifies the node for Newark, 2 identifies the node for Boston, and so on. You can assign numbers to the nodes in any manner, but it is best to use a series of consecutive integers. The node numbers provide a convenient way to identify the decision variables needed to formulate the LP model for the problem. For each arc in a network flow model, you need to define one decision variable as:

X_{ij} = the number of items shipped (or flowing) *from* node i *to* node j

The network in Figure 5.1 for our example problem contains 11 arcs. Therefore, the LP formulation of this model requires the following 11 decision variables:

X_{12} = the number of cars shipped *from* node 1 (Newark) *to* node 2 (Boston)
X_{14} = the number of cars shipped *from* node 1 (Newark) *to* node 4 (Richmond)
X_{23} = the number of cars shipped *from* node 2 (Boston) *to* node 3 (Columbus)
X_{35} = the number of cars shipped *from* node 3 (Columbus) *to* node 5 (Atlanta)
X_{53} = the number of cars shipped *from* node 5 (Atlanta) *to* node 3 (Columbus)
X_{54} = the number of cars shipped *from* node 5 (Atlanta) *to* node 4 (Richmond)
X_{56} = the number of cars shipped *from* node 5 (Atlanta) *to* node 6 (Mobile)
X_{65} = the number of cars shipped *from* node 6 (Mobile) *to* node 5 (Atlanta)
X_{74} = the number of cars shipped *from* node 7 (Jacksonville) *to* node 4 (Richmond)
X_{75} = the number of cars shipped *from* node 7 (Jacksonville) *to* node 5 (Atlanta)
X_{76} = the number of cars shipped *from* node 7 (Jacksonville) *to* node 6 (Mobile)

5.1.3 THE OBJECTIVE FUNCTION FOR NETWORK FLOW PROBLEMS

Each unit that flows from node i to node j in a network flow problem usually incurs some cost, c_{ij}. This cost might represent a monetary payment, a distance, or some other type of penalty. The objective in most network flow problems is to minimize the total cost, distance, or penalty that must be incurred to solve the problem. Such problems are known as ***minimum cost network flow problems***.

In our example problem, different monetary costs must be paid for each car shipped across a given arc. For example, it costs \$30 to ship each car from node 1 (Newark) to node 2 (Boston). Because X_{12} represents the number of cars shipped from Newark to Boston, the total cost incurred by cars shipped along this path is determined by $\$30X_{12}$. Similar calculations can be done for the other arcs in the network. Because BMC is interested in minimizing the total shipping costs, the objective function for this problem is expressed as:

$$\begin{aligned}\text{MIN:}\quad & +30X_{12} + 40X_{14} + 50X_{23} + 35X_{35} + 40X_{53} + 30X_{54} \\ & + 35X_{56} + 25X_{65} + 50X_{74} + 45X_{75} + 50X_{76}\end{aligned}$$

5.1.4 THE CONSTRAINTS FOR NETWORK FLOW PROBLEMS

Just as the number of arcs in the network determines the number of variables in the LP formulation of a network flow problem, the number of nodes determines the number of constraints. In particular, there must be one constraint for each node. A simple set of rules, known as the *Balance-of-Flow Rules*, applies to constructing the constraints for minimum cost network flow problems. These rules are summarized as follows:

For Minimum Cost Network Flow Problems Where:	Apply This Balance-of-Flow Rule at Each Node:
Total Supply > Total Demand	Inflow − Outflow ≥ Supply or Demand
Total Supply < Total Demand	Inflow − Outflow ≤ Supply or Demand
Total Supply = Total Demand	Inflow − Outflow = Supply or Demand

Note that if the total supply in a network flow problem is less than the total demand, then it will be impossible to satisfy all the demand. The balance-of-flow rule listed for this case assumes that you want to determine the least costly way of distributing the available supply—knowing that it is impossible to satisfy all the demand.

So to apply the correct balance-of-flow rule, we first must compare the total supply in the network to the total demand. In our example problem, there is a total supply of 500 cars and a total demand for 480 cars. Because the total supply exceeds the total demand, we will use the first balance-of-flow rule to formulate our example problem. That is, at each node, we will create a constraint of the form:

$$\text{Inflow} - \text{Outflow} \geq \text{Supply or Demand}$$

For example, consider node 1 (Newark) in Figure 5.1. No arcs flow into this node but two arcs (represented by X_{12} and X_{14}) flow out of the node. According to the balance-of-flow rule, the constraint for this node is:

$$\text{Constraint for node 1:} \qquad -X_{12} - X_{14} \geq -200$$

Notice that the supply at this node is represented by −200 following the convention we established earlier. If we multiply both sides of this equation by −1, we see that it is equivalent to $+X_{12} + X_{14} \leq +200$. (Note that multiplying an inequality by −1 reverses the direction of the inequality.) This constraint indicates that the total number of cars flowing out of Newark must not exceed 200. So, if we include either form of this constraint in the model, we can ensure that no more than 200 cars will be shipped from Newark.

Now consider the constraint for node 2 (Boston) in Figure 5.1. Because Boston has a demand for 100 cars, the balance-of-flow rule requires that the total number of cars coming into Boston from Newark (via X_{12}) minus the total number of cars being shipped out of Boston to Columbus (via X_{23}) must leave at least 100 cars in Boston. This condition is imposed by the constraint:

$$\text{Constraint for node 2:} \qquad +X_{12} - X_{23} \geq +100$$

Note that this constraint makes it possible to leave more than the required number of cars in Boston (e.g., 200 cars could be shipped into Boston and only 50 shipped out, leaving 150 cars in Boston). However, because our objective is to minimize costs, we can be sure that an excess number of cars will never be shipped to any city, because that would result in unnecessary costs being incurred.

Using the balance-of-flow rule, the constraints for each of the remaining nodes in our example problem are represented as:

Constraint for node 3:	$+X_{23} + X_{53} - X_{35} \geq +60$
Constraint for node 4:	$+X_{14} + X_{54} + X_{74} \geq +80$
Constraint for node 5:	$+X_{35} + X_{65} + X_{75} - X_{53} - X_{54} - X_{56} \geq +170$
Constraint for node 6:	$+X_{56} + X_{76} - X_{65} \geq +70$
Constraint for node 7:	$-X_{74} - X_{75} - X_{76} \geq -300$

Again, each constraint indicates that the flow into a given node minus the flow out of that same node must be greater than or equal to the supply or demand at the node. So, if you draw a graph of a network flow problem like the one in Figure 5.1, it is easy to write out the constraints for the problem by following the balance-of-flow rule. Of course, we also need to specify the following nonnegativity condition for all the decision variables because negative flows should not occur on arcs:

$$X_{ij} \geq 0 \text{ for all } i \text{ and } j$$

5.1.5 IMPLEMENTING THE MODEL IN A SPREADSHEET

The formulation for the BMC transshipment problem is summarized as:

$$\text{MIN:} \quad \left.\begin{array}{l} +30X_{12} + 40X_{14} + 50X_{23} + 35X_{35} + 40X_{53} \\ +30X_{54} + 35X_{56} + 25X_{65} + 50X_{74} + 45X_{75} \\ +50X_{76} \end{array}\right\} \text{total shipping cost}$$

Subject to:

$-X_{12} - X_{14} \geq -200$	} flow constraint for node 1
$+X_{12} - X_{23} \geq +100$	} flow constraint for node 2
$+X_{23} + X_{53} - X_{35} \geq +60$	} flow constraint for node 3
$+X_{14} + X_{54} + X_{74} \geq +80$	} flow constraint for node 4
$+X_{35} + X_{65} + X_{75} - X_{53} - X_{54} - X_{56} \geq +170$	} flow constraint for node 5
$+X_{56} + X_{76} - X_{65} \geq +70$	} flow constraint for node 6
$-X_{74} - X_{75} - X_{76} \geq -300$	} flow constraint for node 7
$X_{ij} \geq 0$ for all i and j	} nonnegativity conditions

A convenient way to implement this type of problem is shown in Figure 5.2 (and in the file Fig5-2.xls on your data disk). In this spreadsheet, cells B6 through B16 are used to represent the decision variables for our model (or the number of cars that should flow between each of the cities listed). The unit cost of transporting cars between each city is listed in column G. The objective function for the model is then implemented in cell G18 as follows:

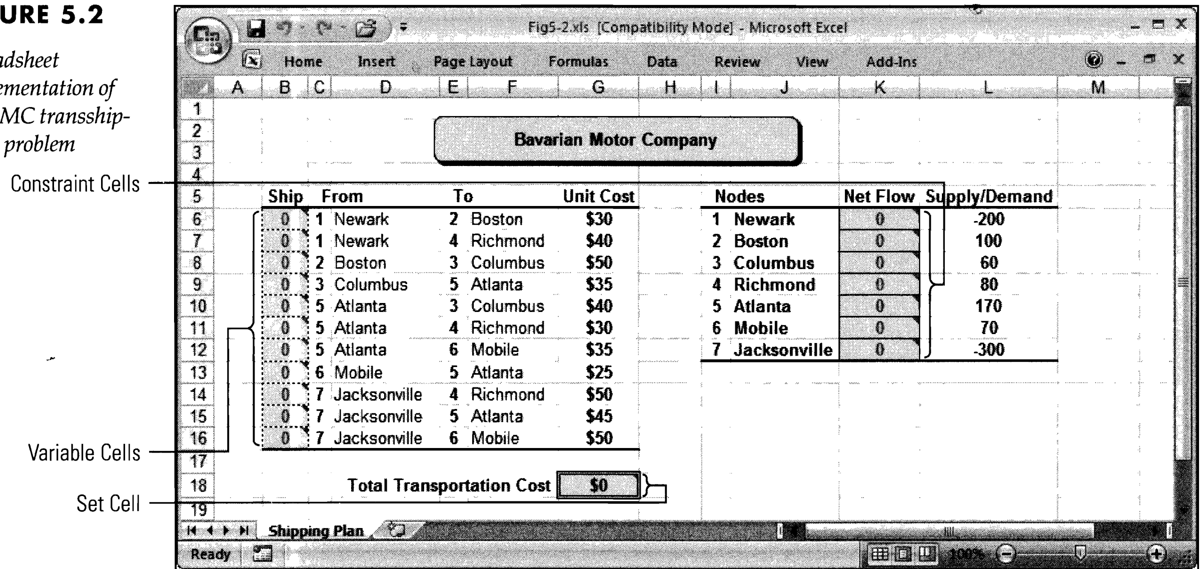
Formula for cell G18: =SUMPRODUCT(B6:B16,G6:G16)

To implement the LHS formulas for the constraints in this model, we need to compute the total inflow minus the total outflow for each node. This is done in cells K6 through K12 as follows:

Formula for cell K6: =SUMIF($E$6:$E$16,I6,$B$6:$B$16)−
(Copy to cells K7 through K12.) SUMIF($C$6:$C$16,I6,$B$6:$B$16)

FIGURE 5.2

Spreadsheet implementation of the BMC transshipment problem



Key Cell Formulas

Cell	Formula	Copied to
D6	=VLOOKUP(C6,$I$6:$J$12,2)	D7:D16 and F6:F16
G18	=SUMPRODUCT(B6:B16,G6:G16)	--
K6	=SUMIF($E$6:$E$16,I6,$B$6:$B$16)-SUMIF($C$6:$C$16,I6,$B$6:$B$16)	K7:K12

The first SUMIF function in this formula compares the values in the range E6 through E16 to the value in I6 and, if a match occurs, sums the corresponding value in the range B6 through B16. Of course, this gives us the total number of cars flowing ***into*** Newark (which in this case will always be zero because none of the values in E6 through E16 match the value in I6). The next SUMIF function compares the values in the range C6 through C16 to the value in I6 and, if a match occurs, sums the corresponding values in the range B6 through B16. This gives us the total number of cars flowing ***out of*** Newark (which in this case will always equal the values in cells B6 and B7 because these are the only arcs flowing out of Newark). Copying this formula to cells K7 though K12 allows us to easily calculate the total inflow minus the total outflow for each of the nodes in our problem. The RHS values for these constraint cells are shown in cells L6 though L12.

Figure 5.3 shows the Solver parameters and options required to solve this model. The optimal solution to the problem is shown in Figure 5.4.

5.1.6 ANALYZING THE SOLUTION

Figure 5.4 shows the optimal solution for BMC's transshipment problem. The solution indicates that 120 cars should be shipped from Newark to Boston ($X_{12} = 120$), 80 cars from Newark to Richmond ($X_{14} = 80$), 20 cars from Boston to Columbus ($X_{23} = 20$), 40 cars from Atlanta to Columbus ($X_{53} = 40$), 210 cars from Jacksonville to Atlanta ($X_{75} = 210$), and 70 cars from Jacksonville to Mobile ($X_{76} = 70$). Cell G18 indicates that

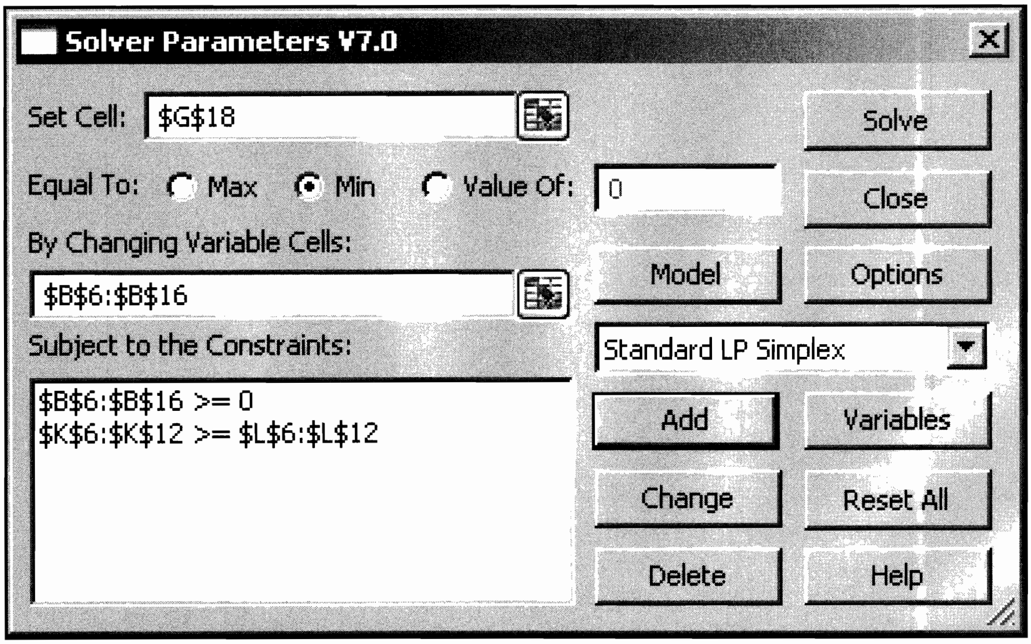


FIGURE 5.3

Solver parameters for the BMC transshipment problem

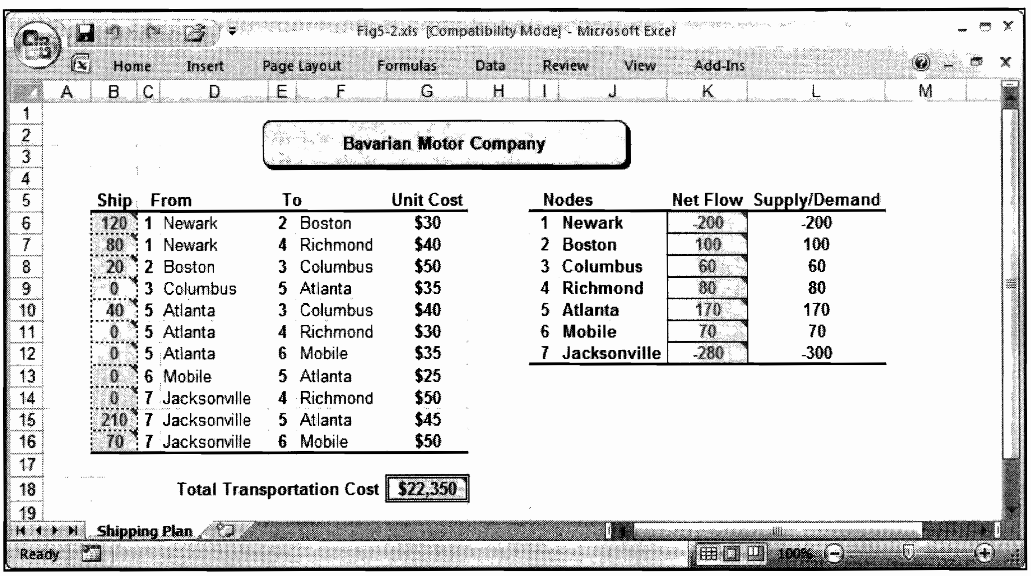


FIGURE 5.4

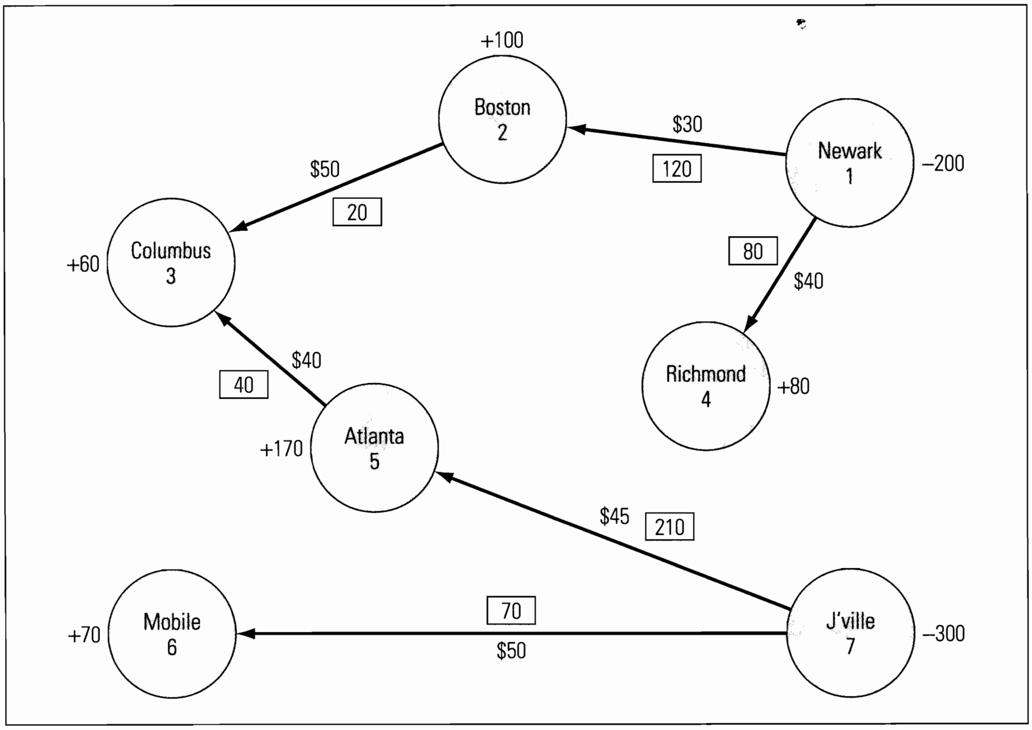
Optimal solution to the BMC transshipment problem

the total cost associated with this shipping plan is $22,350. The values of the constraint cells in K6 and K12 indicate, respectively, that all 200 cars available at Newark are being shipped and only 280 of the 300 cars available at Jacksonville are being shipped. A comparison of the remaining constraint cells in K7 through K11 with their RHS values in L7 through L11 reveals that the demand at each of these cities is being met by the net flow of cars through each city.

This solution is summarized graphically, as shown in Figure 5.5. The values in the boxes next to each arc indicate the optimal flows for the arcs. The optimal flow for all the other arcs in the problem, which are not shown in Figure 5.5, is 0. Notice that the amount flowing into each node minus the amount flowing out of each node is equal

FIGURE 5.5

Network representation of the optimal solution for the BMC transshipment problem



to the supply or demand at the node. For example, 210 cars are being shipped from Jacksonville to Atlanta. Atlanta will keep 170 of the cars (to satisfy the demand at this node) and send the extra 40 to Columbus.

5.2 The Shortest Path Problem

In many decision problems, we need to determine the shortest (or least costly) route or path through a network from a starting node to an ending node. For example, many cities are developing computerized models of their highways and streets to help emergency vehicles identify the quickest route to a given location. Each street intersection represents a potential node in a network, and the streets connecting the intersections represent arcs. Depending on the day of the week and the time of day, the time required to travel various streets can increase or decrease due to changes in traffic patterns. Road construction and maintenance also affect traffic flow patterns. So, the quickest route (or shortest path) for getting from one point in the city to another can change frequently. In emergency situations, lives or property can be lost or saved depending on how quickly emergency vehicles arrive where they are needed. The ability to quickly determine the shortest path to the location of an emergency situation is extremely useful in these situations. The following example illustrates another application of the shortest path problem.

The American Car Association (ACA) provides a variety of travel-related services to its members, including information on vacation destinations, discount hotel reservations, emergency road assistance, and travel route planning. This last service, travel route planning, is one of its most popular services. When members of the ACA are planning to take a driving trip, they call the organization's toll-free 800 number and indicate what cities they will be traveling from and to. The ACA then

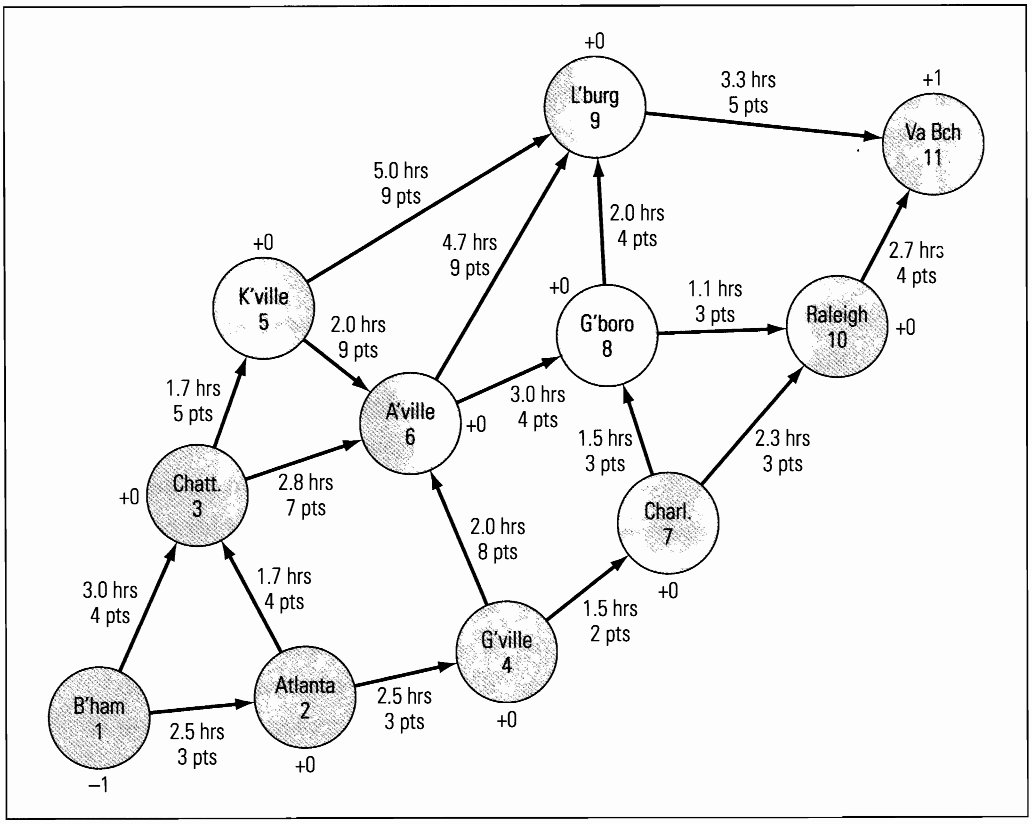


FIGURE 5.6

Network of possible routes for the ACA's shortest path problem

determines an optimal route for traveling between these cities. The ACA's computer databases of major highways and interstates are kept up-to-date with information on construction delays and detours and estimated travel times along various segments of roadways.

Members of the ACA often have different objectives in planning driving trips. Some are interested in identifying routes that minimize travel times. Others, with more leisure time on their hands, want to identify the most scenic route to their desired destination. The ACA wants to develop an automated system for identifying an optimal travel plan for its members.

To see how the ACA could benefit by solving shortest path problems, consider the simplified network shown in Figure 5.6 for a travel member who wants to drive from Birmingham, Alabama to Virginia Beach, Virginia. The nodes in this graph represent different cities and the arcs indicate the possible travel routes between the cities. For each arc, Figure 5.6 lists both the estimated driving time to travel the road represented by each arc and the number of points that route has received on the ACA's system for rating the scenic quality of the various routes.

Solving this problem as a network flow model requires the various nodes to have some supply or demand. In Figure 5.6, node 1 (Birmingham) has a supply of 1, node 11 (Virginia Beach) has a demand of 1, and all other nodes have a demand (or supply) of 0. If we view this model as a transshipment problem, we want to find either the quickest way or the most scenic way of shipping 1 unit of flow from node 1 to node 11. The route this unit of supply takes corresponds to either the shortest path or the most scenic path through the network, depending on which objective is being pursued.

5.2.1 AN LP MODEL FOR THE EXAMPLE PROBLEM

Using the balance-of-flow rule, the LP model to minimize the driving time in this problem is represented as:

$$\begin{aligned}\text{MIN:}\quad & +2.5X_{12} + 3X_{13} + 1.7X_{23} + 2.5X_{24} + 1.7X_{35} + 2.8X_{36} + 2X_{46} + 1.5X_{47} + 2X_{56} + 5X_{59} \\ & + 3X_{68} + 4.7X_{69} + 1.5X_{78} + 2.3X_{7,10} + 2X_{89} + 1.1X_{8,10} + 3.3X_{9,11} + 2.7X_{10,11}\end{aligned}$$

Subject to:

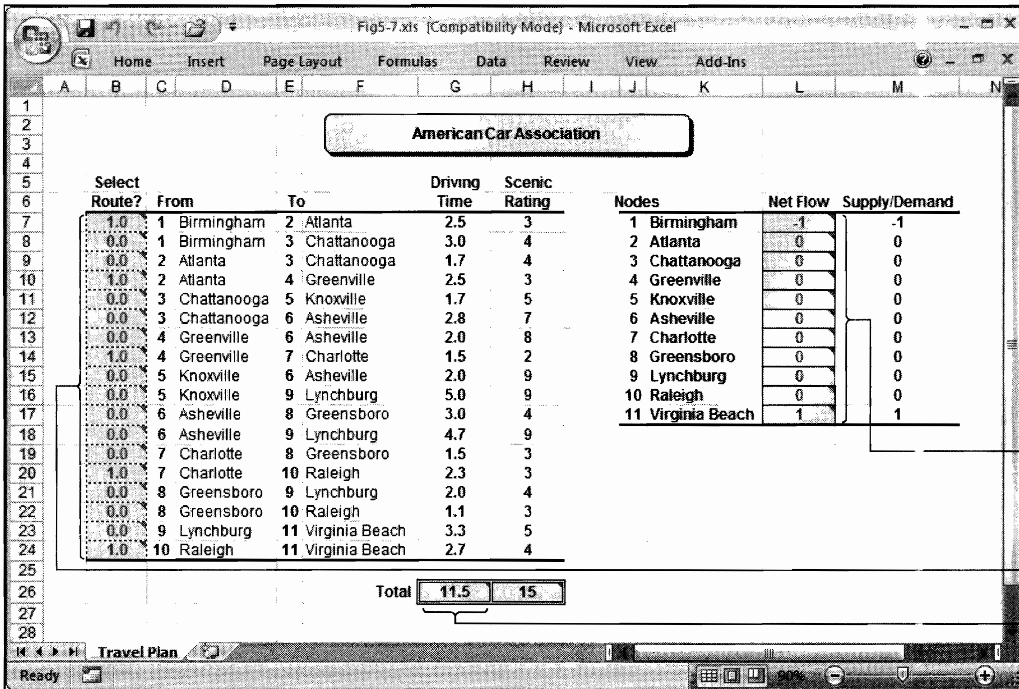
$$\begin{array}{lrl}
-X_{12} - X_{13} & = -1 & \text{\} flow constraint for node 1} \\
+X_{12} - X_{23} - X_{24} & = 0 & \text{\} flow constraint for node 2} \\
+X_{13} + X_{23} - X_{35} - X_{36} & = 0 & \text{\} flow constraint for node 3} \\
+X_{24} - X_{46} - X_{47} & = 0 & \text{\} flow constraint for node 4} \\
+X_{35} - X_{56} - X_{59} & = 0 & \text{\} flow constraint for node 5} \\
+X_{36} + X_{46} + X_{56} - X_{68} - X_{69} & = 0 & \text{\} flow constraint for node 6} \\
+X_{47} - X_{78} - X_{7,10} & = 0 & \text{\} flow constraint for node 7} \\
+X_{68} + X_{78} - X_{89} - X_{8,10} & = 0 & \text{\} flow constraint for node 8} \\
+X_{59} + X_{69} + X_{89} - X_{9,11} & = 0 & \text{\} flow constraint for node 9} \\
+X_{7,10} + X_{8,10} - X_{10,11} & = 0 & \text{\} flow constraint for node 10} \\
+X_{9,11} + X_{10,11} & = +1 & \text{\} flow constraint for node 11} \\
X_{ij} \geq 0 \text{ for all } i \text{ and } j & & \text{\} nonnegativity conditions}
\end{array}$$

Because the total supply equals the total demand in this problem, the constraints should be stated as equalities. The first constraint in this model ensures that the 1 unit of supply available at node 1 is shipped to node 2 or node 3. The next nine constraints indicate that anything flowing to nodes 2 though node 10 must also flow out of these nodes because each has a demand of 0. For example, if the unit of supply leaves node 1 for node 2 (via X_{12}), the second constraint ensures that it will leave node 2 for node 3 or node 4 (via X_{23} or X_{24}). The last constraint indicates that the unit ultimately must flow to node 11. Thus, the solution to this problem indicates the quickest route for getting from node 1 (Birmingham) to node 11 (Virginia Beach).

5.2.2 THE SPREADSHEET MODEL AND SOLUTION

The optimal solution to this problem shown in Figure 5.7 (and in the file Fig5-7.xls on your data disk) was obtained using the Solver parameters and options shown in Figure 5.8. Notice that this model includes calculations of both the total expected driving time (cell G26) and total scenic rating points (cell H26) associated with any solution. Either of these cells can be chosen as the objective function according to the client's desires. However, the solution shown in Figure 5.7 minimizes the expected driving time.

The optimal solution shown in Figure 5.7 indicates that the quickest travel plan involves driving from node 1 (Birmingham) to node 2 (Atlanta), then to node 4 (Greenville), then to node 7 (Charlotte), then to node 10 (Raleigh), and finally to node 11 (Virginia Beach). The total expected driving time along this route is 11.5 hours. Also note that this route receives a rating of 15 points on the ACA's scenic rating scale.



Select Route?	From		To		Driving Time	Scenic Rating
1.0	1	Birmingham	2	Atlanta	2.5	3
0.0	1	Birmingham	3	Chattanooga	3.0	4
0.0	2	Atlanta	3	Chattanooga	1.7	4
1.0	2	Atlanta	4	Greenville	2.5	3
0.0	3	Chattanooga	5	Knoxville	1.7	5
0.0	3	Chattanooga	6	Asheville	2.8	7
0.0	4	Greenville	6	Asheville	2.0	8
1.0	4	Greenville	7	Charlotte	1.5	2
0.0	5	Knoxville	6	Asheville	2.0	9
0.0	5	Knoxville	9	Lynchburg	5.0	9
0.0	6	Asheville	8	Greensboro	3.0	4
0.0	6	Asheville	9	Lynchburg	4.7	9
0.0	7	Charlotte	8	Greensboro	1.5	3
1.0	7	Charlotte	10	Raleigh	2.3	3
0.0	8	Greensboro	9	Lynchburg	2.0	4
0.0	8	Greensboro	10	Raleigh	1.1	3
0.0	9	Lynchburg	11	Virginia Beach	3.3	5
1.0	10	Raleigh	11	Virginia Beach	2.7	4
				Total	11.5	15

Nodes		Net Flow	Supply/Demand
1	Birmingham	-1	-1
2	Atlanta	0	0
3	Chattanooga	0	0
4	Greenville	0	0
5	Knoxville	0	0
6	Asheville	0	0
7	Charlotte	0	0
8	Greensboro	0	0
9	Lynchburg	0	0
10	Raleigh	0	0
11	Virginia Beach	1	1

FIGURE 5.7

Spreadsheet model and solution showing the route that minimizes estimated driving time for the ACA's shortest path problem

Key Cell Formulas

Cell	Formula	Copied to
D7	=VLOOKUP(C7,$J$7:$K$17,2)	D8:D24 and F7:F24
G26	=SUMPRODUCT(G7:G24,$B$7:$B$24)	H26
L7	=SUMIF($E$7:$E$24,J7,$B$7:$B$24)-SUMIF($C$7:$C$24,J7,$B$7:$B$24)	L8:L17

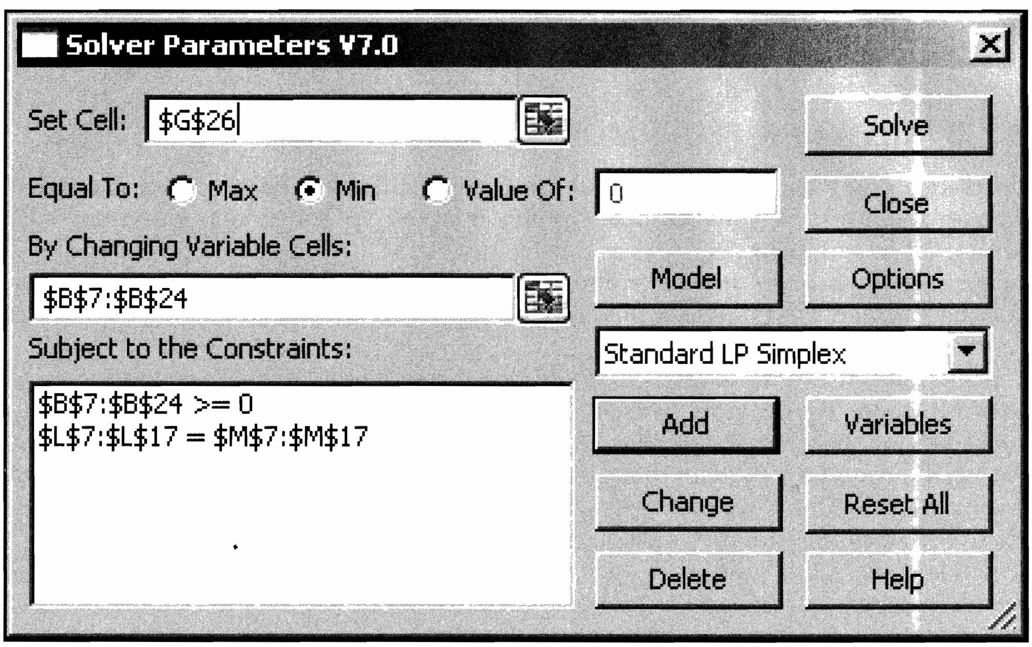
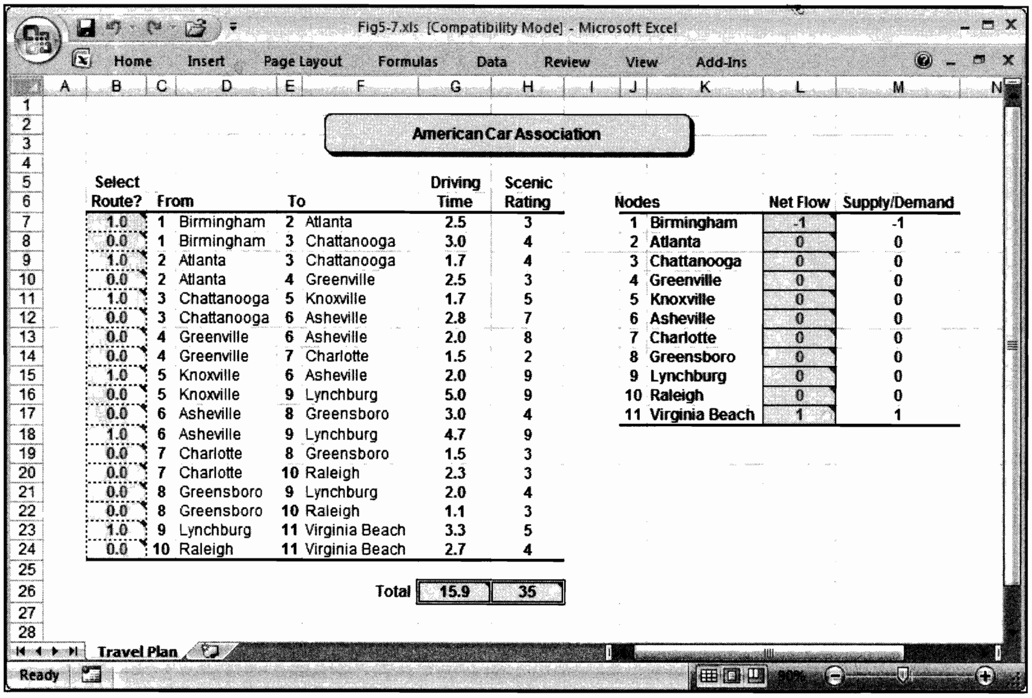


FIGURE 5.8

Solver parameters for the ACA's shortest path problem

FIGURE 5.9

Solution showing the most scenic route



American Car Association

Select Route?	From	To	Driving Time	Scenic Rating
1.0	1 Birmingham	2 Atlanta	2.5	3
0.0	1 Birmingham	3 Chattanooga	3.0	4
1.0	2 Atlanta	3 Chattanooga	1.7	4
0.0	2 Atlanta	4 Greenville	2.5	3
1.0	3 Chattanooga	5 Knoxville	1.7	5
0.0	3 Chattanooga	6 Asheville	2.8	7
0.0	4 Greenville	6 Asheville	2.0	8
0.0	4 Greenville	7 Charlotte	1.5	2
1.0	5 Knoxville	6 Asheville	2.0	9
0.0	5 Knoxville	9 Lynchburg	5.0	9
0.0	6 Asheville	8 Greensboro	3.0	4
1.0	6 Asheville	9 Lynchburg	4.7	9
0.0	7 Charlotte	8 Greensboro	1.5	3
0.0	7 Charlotte	10 Raleigh	2.3	3
0.0	8 Greensboro	9 Lynchburg	2.0	4
0.0	8 Greensboro	10 Raleigh	1.1	3
1.0	9 Lynchburg	11 Virginia Beach	3.3	5
0.0	10 Raleigh	11 Virginia Beach	2.7	4
		Total	15.9	35

Nodes	Net Flow	Supply/Demand
1 Birmingham	-1	-1
2 Atlanta	0	0
3 Chattanooga	0	0
4 Greenville	0	0
5 Knoxville	0	0
6 Asheville	0	0
7 Charlotte	0	0
8 Greensboro	0	0
9 Lynchburg	0	0
10 Raleigh	0	0
11 Virginia Beach	1	1

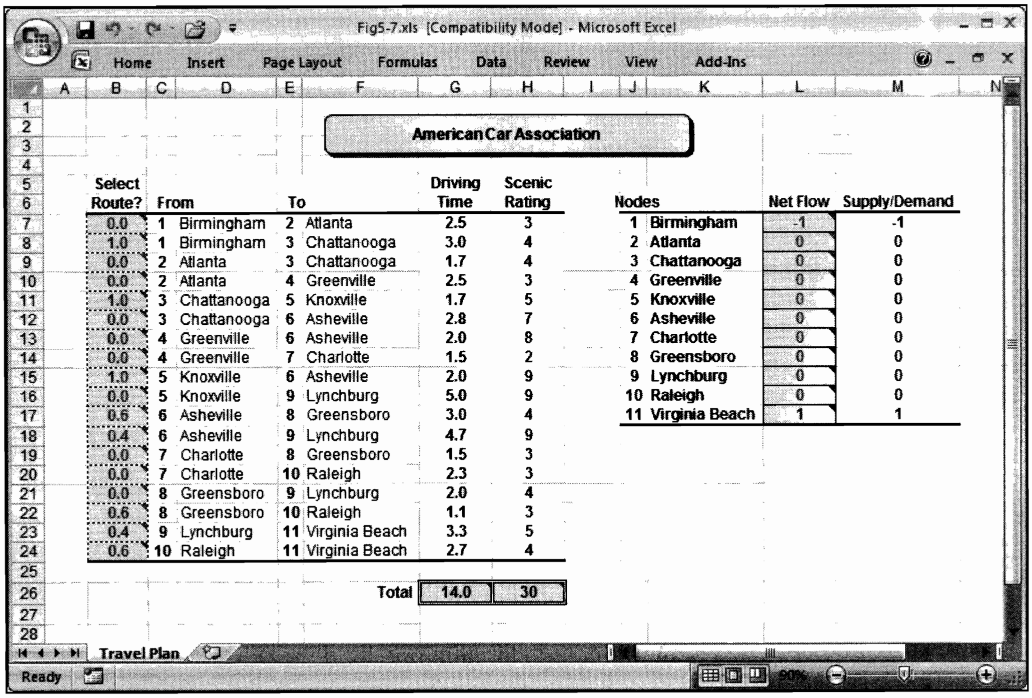
Using this spreadsheet, we also can determine the most scenic route by instructing Solver to maximize the value in cell H26. Figure 5.9 shows the optimal solution obtained in this case. This travel plan involves driving from Birmingham to Atlanta, to Chattanooga, to Knoxville, to Asheville, to Lynchburg, and finally, to Virginia Beach. This itinerary receives a rating of 35 points on the ACA's scenic rating scale but takes almost 16 hours of driving time.

5.2.3 NETWORK FLOW MODELS AND INTEGER SOLUTIONS

Up to this point, each of the network flow models that we have solved generated integer solutions. If you use the simplex method to solve any minimum cost network flow model having integer constraint RHS values, then the optimal solution automatically assumes integer values. This property is helpful because the items flowing through most network flow models represent discrete units (such as cars or people).

Sometimes, it is tempting to place additional constraints (or side constraints) on a network model. For example, in the ACA problem, suppose that the customer wants to get to Virginia Beach in the most scenic way possible within 14 hours of driving time. We can easily add a constraint to the model to keep the total driving time G26 less than or equal to 14 hours. If we then re-solve the model to maximize the scenic rating in cell H26, we obtain the solution shown in Figure 5.10.

Unfortunately, this solution is useless because it produces fractional results. Thus, if we add *side constraints* to network flow problems that do not obey the balance-of-flow rule, we can no longer ensure that the solution to the LP formulation of the problems



Select Route?	From		To		Driving Time	Scenic Rating
0.0	1	Birmingham	2	Atlanta	2.5	3
1.0	1	Birmingham	3	Chattanooga	3.0	4
0.0	2	Atlanta	3	Chattanooga	1.7	4
0.0	2	Atlanta	4	Greenville	2.5	3
1.0	3	Chattanooga	5	Knoxville	1.7	5
0.0	3	Chattanooga	6	Asheville	2.8	7
0.0	4	Greenville	6	Asheville	2.0	8
0.0	4	Greenville	7	Charlotte	1.5	2
1.0	5	Knoxville	6	Asheville	2.0	9
0.0	5	Knoxville	9	Lynchburg	5.0	9
0.6	6	Asheville	8	Greensboro	3.0	4
0.4	6	Asheville	9	Lynchburg	4.7	9
0.0	7	Charlotte	8	Greensboro	1.5	3
0.0	7	Charlotte	10	Raleigh	2.3	3
0.0	8	Greensboro	9	Lynchburg	2.0	4
0.6	8	Greensboro	10	Raleigh	1.1	3
0.4	9	Lynchburg	11	Virginia Beach	3.3	5
0.6	10	Raleigh	11	Virginia Beach	2.7	4
				Total	14.0	30

Nodes		Net Flow	Supply/Demand
1	Birmingham	-1	-1
2	Atlanta	0	0
3	Chattanooga	0	0
4	Greenville	0	0
5	Knoxville	0	0
6	Asheville	0	0
7	Charlotte	0	0
8	Greensboro	0	0
9	Lynchburg	0	0
10	Raleigh	0	0
11	Virginia Beach	1	1

FIGURE 5.10

Example of a non-integer solution to a network problem with side constraints

will be integral. If integer solutions are needed for such problems, the integer programming techniques discussed in Chapter 6 must be applied.

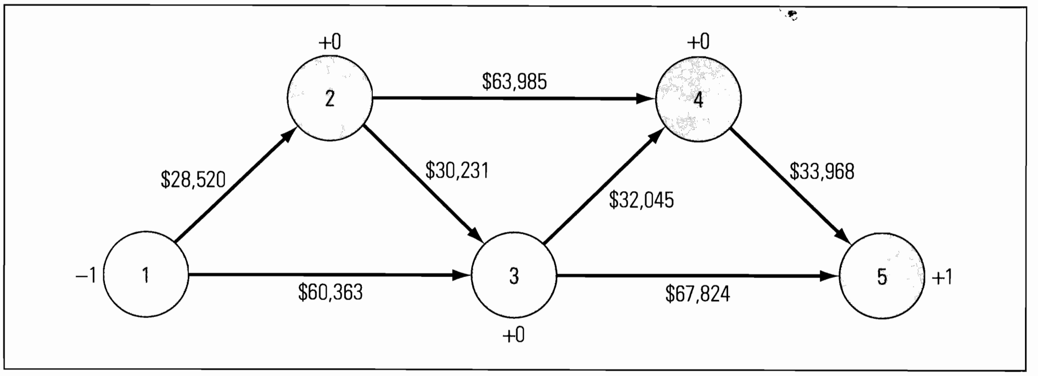
5.3 The Equipment Replacement Problem

The equipment replacement problem is a common type of business problem that can be modeled as a shortest path problem. This type of problem involves determining the least costly schedule for replacing equipment over a specified length of time. Consider the following example.

> Jose Maderos is the owner of Compu-Train, a small company that provides hands-on software education and training for businesses in and around Boulder, Colorado. Jose leases the computer equipment used in his business and he likes to keep the equipment up-to-date so that it will run the latest, state-of-the-art software in an efficient manner. Because of this, Jose wants to replace his equipment at least every two years.
>
> Jose is currently trying to decide between two different lease contracts his equipment supplier has proposed. Under both contracts Jose would be required to pay $62,000 initially to obtain the equipment he needs. However, the two contracts differ in terms of the amount Jose would have to pay in subsequent years to replace his equipment. Under the first contract, the price to acquire new equipment would increase by 6% per year, but he would be given a trade-in credit of 60% for any equipment that is one year old and 15% for any equipment that is two years old. Under the second contract, the price to acquire new equipment would increase by just 2% per year, but he would only be given a trade-in credit of 30% for any equipment that is one year old and 10% for any equipment that is two years old.

FIGURE 5.11

Network representation of Compu-Train's first contract alternative for their equipment replacement problem



Jose realizes that no matter what he does, he will have to pay $62,000 to obtain the equipment initially. However, he wants to determine which contract would allow him to minimize the remaining leasing costs over the next five years and when he should replace his equipment under the selected contract.

Each of the two contracts Jose is considering can be modeled as a shortest path problem. Figure 5.11 shows how this would be accomplished for the first contract under consideration. Each node corresponds to a point in time during the next five years when Jose can replace his equipment. Each arc in this network represents a choice available to Jose. For example, the arc from node 1 to node 2 indicates that Jose can keep the equipment he initially acquires for one year and then replace it (at the beginning of year 2) for a net cost of $28,520 ($\$62{,}000 \times 1.06 - 0.6 \times \$62{,}000 = \$28{,}520$). Alternatively, the arc from node 1 to node 3 indicates that Jose can keep his initial equipment for two years and replace it at the beginning of year 3 for a net cost of $60,363 ($\$62{,}000 \times 1.06^2 - 0.15 \times \$62{,}000 = \$60{,}363$).

The arc from node 2 to node 3 indicates that if Jose replaces his initial equipment at the beginning of year 2, he can keep the new equipment for one year and replace it at the beginning of year 3 at a net cost of $30,231 ($\$62{,}000 \times 1.06^2 - 0.60 \times (\$62{,}000 \times 1.06) = \$30{,}231$). The remaining arcs and costs in the network can be interpreted in the same way. Jose's decision problem is to determine the least costly (or shortest) way of getting from node 1 to node 5 in this network.

5.3.1 THE SPREADSHEET MODEL AND SOLUTION

The LP formulation of Jose's decision problem can be generated from the graph in Figure 5.11 using the balance-of-flow rule in the same manner as the previous network flow problems. The spreadsheet model for this problem was implemented as shown in Figure 5.12 (and in the file Fig5-12.xls on your data disk) and solved using the settings shown in Figure 5.13. To assist Jose in comparing the two different alternatives he faces, notice that an area of the spreadsheet in Figure 5.12 has been reserved to represent assumptions about the annual increase in leasing costs (cell G5), and the trade-in values for one- and two-year old equipment (cells G6 and G7). The rest of the spreadsheet model uses these assumed values to compute the various costs. This enables us to change any of the assumptions and re-solve the model very easily.

The optimal solution to this problem shows that under the provisions of the first contract, Jose should replace his equipment at the beginning of each year at a total cost of $124,764. This amount is in addition to the $62,000 he has to pay up front at the beginning of year 1.

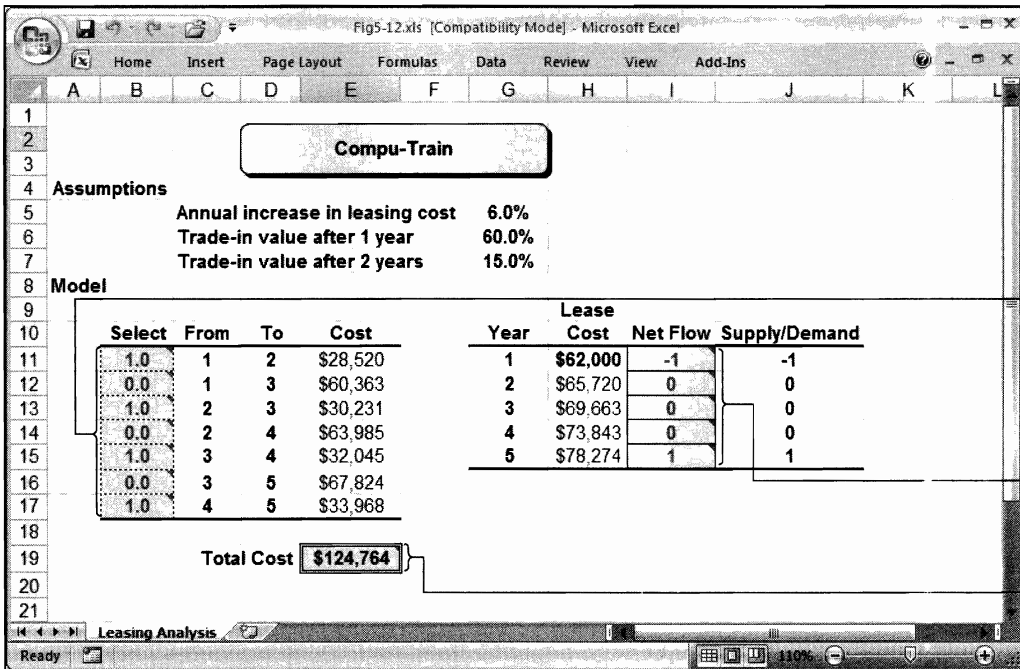


FIGURE 5.12

Spreadsheet model and solution for Compu-Train's first lease contract alternative

Key Cell Formulas

Cell	Formula	Copied to
E19	=SUMPRODUCT(E11:E17,B11:B17)	--
I11	=SUMIF($D$11:$D$17,G11,$B$11:$B$17) -SUMIF($C$11:$C$17,G11,$B$11:$B$17)	I12:I15
H12	=H11*(1+$G$5)	H13:H15
E11	=VLOOKUP(D11,G$11:J$15,2)-(IF(D11-C11=1,G$6,G$7) *VLOOKUP(C11,G$11:J$15,2))	E12:E17

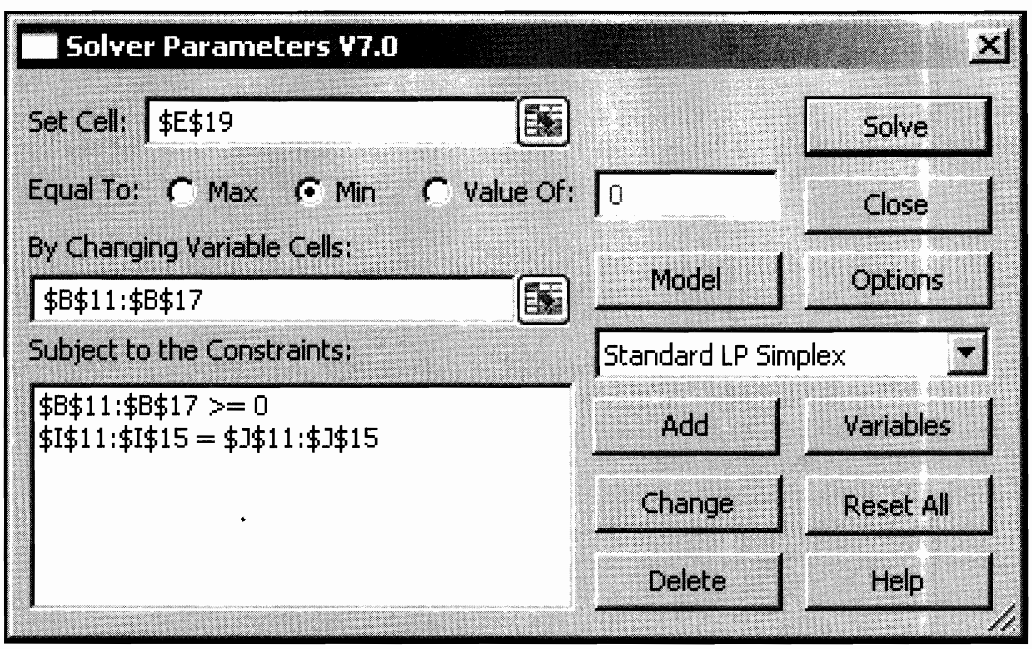


FIGURE 5.13

Solver parameters for Compu-Train's equipment replacement problem

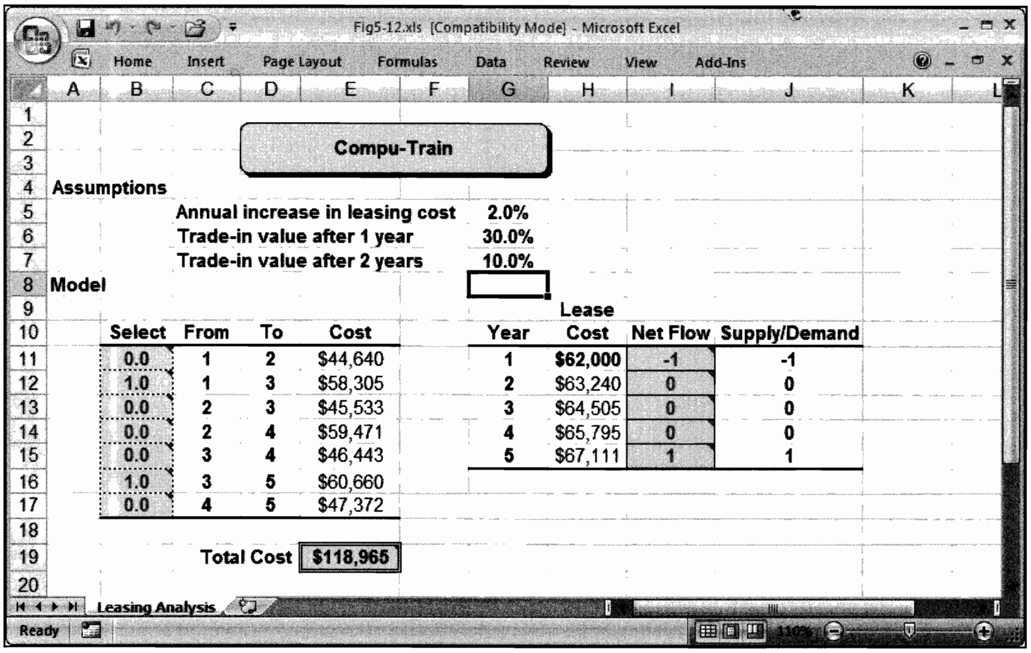


FIGURE 5.14

Solution for Compu-Train's second lease contract alternative

To determine the optimal replacement strategy and costs associated with the second contract, Jose could simply change the assumptions at the top of the spreadsheet and re-solve the model. The results of this are shown in Figure 5.14.

The optimal solution to this problem shows that under the provisions of the second contract, Jose should replace his equipment at the beginning of years 3 and 5 at a total cost of $118,965. Again, this amount is in addition to the $62,000 he has to pay up front at the beginning of year 1. Although the total costs under the second contract are lower than under the first, under the second contract Jose would be working with older equipment during years 2 and 4. Thus, although the solution to these two models makes the financial consequences of the two different alternatives clear, Jose still must decide for himself whether the benefits of the financial cost savings under the second contract outweigh the non-financial costs associated with using slightly out-of-date equipment during years 2 and 4. Of course, regardless of which contract Jose decides to go with, he will get to reconsider whether or not to upgrade his equipment at the beginning of each of the next 4 years.

Summary of Shortest Path Problems

You can model any shortest path problem as a transshipment problem by assigning a supply of 1 to the starting node, a demand of 1 to the ending node, and a demand of 0 to all other nodes in the network. Because the examples presented here involved only a small number of paths through each of the networks, it might have been easier to solve these problems simply by enumerating the paths and calculating the total distance of each one. However, in a problem with many nodes and arcs, an automated LP model is preferable to a manual solution approach.

5.4 Transportation/Assignment Problems

Chapter 3 presented an example of another type of network flow problem known as the transportation/assignment problem. The example involved the Tropicsun Company—a grower and distributor of fresh citrus products. The company wanted to determine the least expensive way to transport freshly picked fruit from three citrus groves to three processing plants. The network representation of the problem is repeated in Figure 5.15.

The network shown in Figure 5.15 differs from the earlier network flow problems in this chapter because it contains no transshipment nodes. Each node in Figure 5.15 is either a sending node or a receiving node. The lack of transshipment nodes is the key feature that distinguishes transportation/assignment problems from other types of network flow problems. As you saw in Chapter 3, this property allows you to set up and solve transportation/assignment problems conveniently in a matrix format in the spreadsheet. Although it is possible to solve transportation/assignment problems in the same way in which we solved transshipment problems, it is much easier to implement and solve these problems using the matrix approach described in Chapter 3.

Sometimes, transportation/assignment problems are *sparse* or not fully interconnected (meaning that not all the supply nodes have arcs connecting them to all the demand nodes). These "missing" arcs can be handled conveniently in the matrix approach to implementation by assigning arbitrarily large costs to the variable cells representing these arcs so that flow on these arcs becomes prohibitively expensive. However, as the number of missing arcs increases, the matrix approach to implementation becomes less and less computationally efficient compared to the procedure described in this chapter.

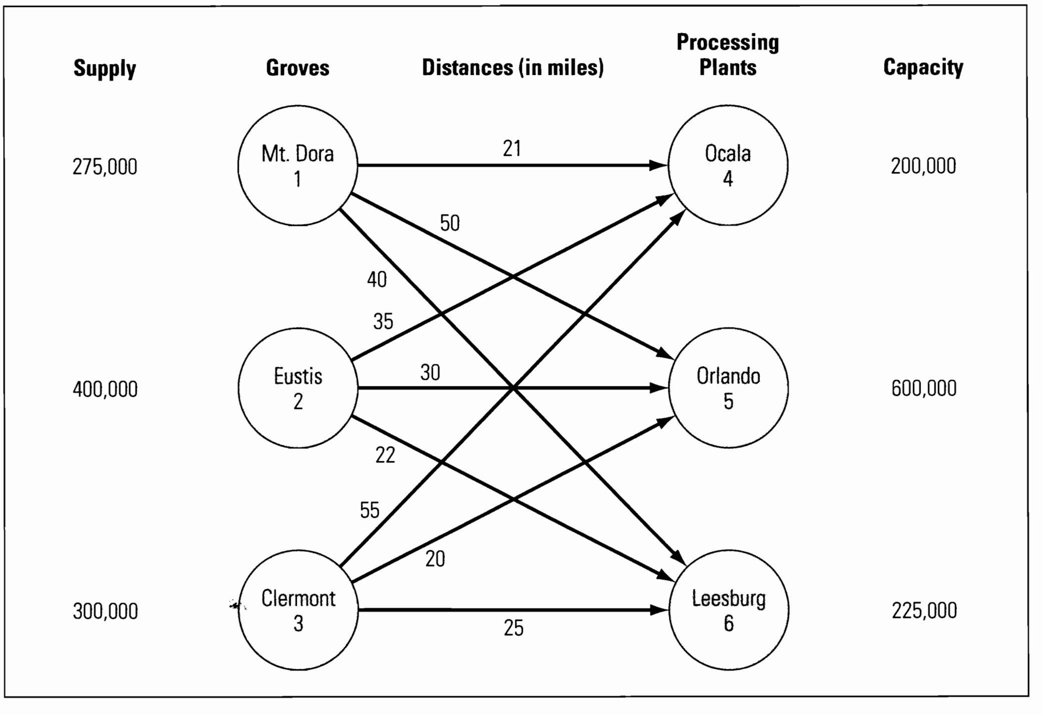


FIGURE 5.15

Network representation of Tropicsun's transportation/assignment problem

5.5 Generalized Network Flow Problems

In all of the network problems we have considered so far, the amount of flow that exited an arc was always the same as the amount that entered the arc. For example, if we put 40 cars on a train in Jacksonville and sent them to Atlanta, the same 40 cars came off the train in Atlanta. However, there are numerous examples of network flow problems in which a gain or loss occurs on flows across arcs. For instance, if oil or gas is shipped through a leaky pipeline, the amount of oil or gas arriving at the intended destination will be less than the amount originally placed in the pipeline. Similar loss-of-flow examples occur as a result of evaporation of liquids, spoilage of foods and other perishable items, or imperfections in raw materials entering production processes that result in a certain amount of scrap. Many financial cash flow problems can be modeled as network flow problems in which flow gains (or increases) occur in the form of interest or dividends as money flows through various investments. The following example illustrates the modeling changes required to accommodate these types of problems.

Nancy Grant is the owner of Coal Bank Hollow Recycling, a company that specializes in collecting and recycling paper products. Nancy's company uses two different recycling processes to convert newspaper, mixed paper, white office paper, and cardboard into paper pulp. The amount of paper pulp extracted from the recyclable materials and the cost of extracting the pulp differs depending on which recycling process is used. The following table summarizes the recycling processes:

	Recycling Process 1		Recycling Process 2	
Material	**Cost per ton**	**Yield**	**Cost per Ton**	**Yield**
Newspaper	$13	90%	$12	85%
Mixed Paper	$11	80%	$13	85%
White Office Paper	$9	95%	$10	90%
Cardboard	$13	75%	$14	85%

For instance, every ton of newspaper subjected to recycling process 1 costs $13 and yields 0.9 tons of paper pulp. The paper pulp produced by the two different recycling processes goes through other operations to be transformed into pulp for newsprint, packaging paper, or print stock quality paper. The yields associated with transforming the recycled pulp into pulp for the final products are summarized in the following table:

	Newsprint Pulp		Packaging Paper Pulp		Print Stock Pulp	
Pulp Source	**Cost per Ton**	**Yield**	**Cost per Ton**	**Yield**	**Cost per Ton**	**Yield**
Recycling Process 1	$5	95%	$6	90%	$8	90%
Recycling Process 2	$6	90%	$8	95%	$7	95%

For instance, a ton of pulp exiting recycling process 2 can be transformed into 0.95 tons of packaging paper at a cost of $8.

Nancy currently has 70 tons of newspaper, 50 tons of mixed paper, 30 tons of white office paper, and 40 tons of cardboard. She wants to determine the most efficient way of converting these materials into 60 tons of newsprint pulp, 40 tons of packaging paper pulp, and 50 tons of print stock pulp.

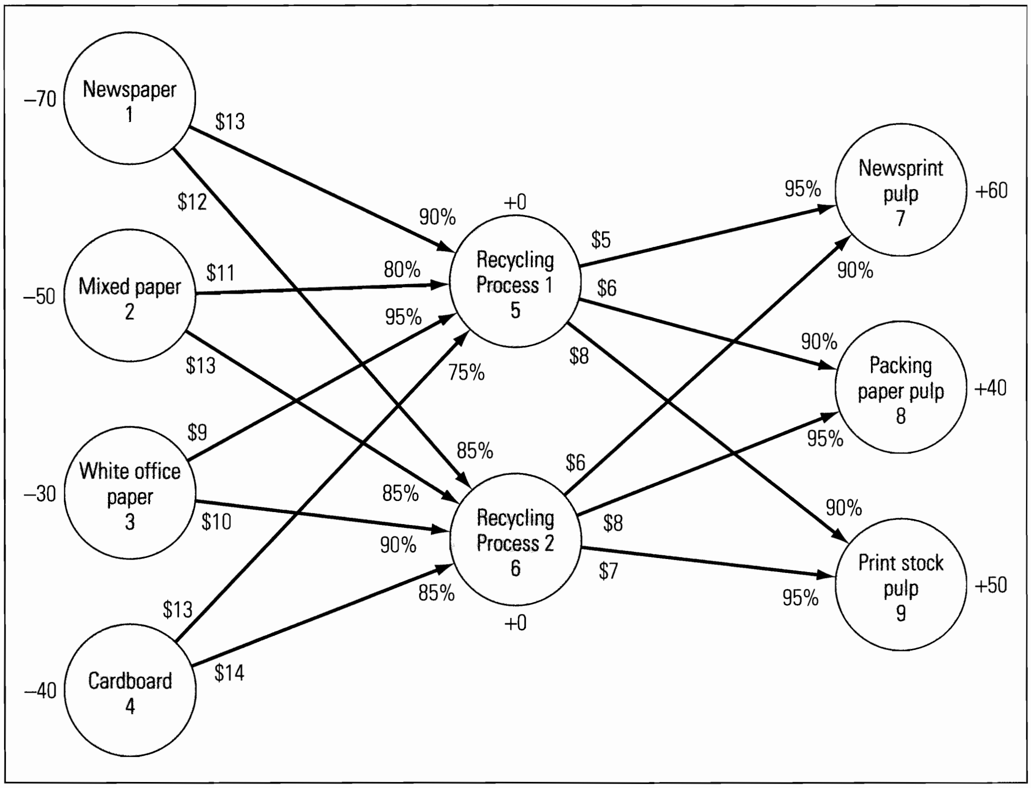


FIGURE 5.16

Graphical representation of Coal Bank Hollow Recycling's generalized network flow problem

Figure 5.16 shows how Nancy's recycling problem can be viewed as a generalized network flow problem. The arcs in this graph indicate the possible flow of recycling material through the production process. On each arc, we have listed both the cost of flow along the arc and the reduction factor that applies to flow along the arc. For instance, the arc from node 1 to node 5 indicates that each ton of newspaper going to recycling process 1 costs $13 and yields 0.90 tons of paper pulp.

5.5.1 FORMULATING AN LP MODEL FOR THE RECYCLING PROBLEM

To formulate the LP model for this problem algebraically, we defined the decision variable X_{ij} to represent the tons of product flowing from node i to node j. The objective is then stated in the usual way as follows:

$$\text{MIN:} \quad 13X_{15} + 12X_{16} + 11X_{25} + 13X_{26} + 9X_{35} + 10X_{36} + 13X_{45} + 14X_{46} + 5X_{57} + 6X_{58} + 8X_{59} + 6X_{67} + 8X_{68} + 7X_{69}$$

The constraints for this problem may be generated using the balance-of-flow rule for each node. The constraints for the first four nodes (representing the supply of newspaper, mixed paper, white office paper, and cardboard, respectively) are given by:

$$-X_{15} - X_{16} \geq -70 \quad \text{\} flow constraint for node 1}$$
$$-X_{25} - X_{26} \geq -50 \quad \text{\} flow constraint for node 2}$$
$$-X_{35} - X_{36} \geq -30 \quad \text{\} flow constraint for node 3}$$
$$-X_{45} - X_{46} \geq -40 \quad \text{\} flow constraint for node 4}$$

These constraints simply indicate that the amount of product flowing out of each of these nodes may not exceed the supply available at each node. (Recall that the constraint given for node 1 is equivalent to $+X_{15} + X_{16} \leq +70$.)

Applying the balance-of-flow rule at nodes 5 and 6 (representing the two recycling processes) we obtain:

$+0.9X_{15} + 0.8X_{25} + 0.95X_{35} + 0.75X_{45} - X_{57} - X_{58} - X_{59} \geq 0$ } flow constraint for node 5
$+0.85X_{16} + 0.85X_{26} + 0.9X_{36} + 0.85X_{46} - X_{67} - X_{68} - X_{69} \geq 0$ } flow constraint for node 6

To better understand the logic of these constraints, we will rewrite them in the following algebraically equivalent manner:

$+0.9X_{15} + 0.8X_{25} + 0.95X_{35} + 0.75X_{45} \geq +X_{57} + X_{58} + X_{59}$ } equivalent flow constraint for node 5
$+0.85X_{16} + 0.85X_{26} + 0.9X_{36} + 0.85X_{46} \geq +X_{67} + X_{68} + X_{69}$ } equivalent flow constraint for node 6

Notice that the constraint for node 5 requires that the amount being shipped from node 5 (given by $X_{57} + X_{58} + X_{59}$) cannot exceed the net amount that would be available at node 5 (given by $0.9X_{15} + 0.8X_{25} + 0.95X_{35} + 0.75X_{45}$). Thus, here the yield factors come into play in determining the amount of product that would be available from the recycling processes. A similar interpretation applies to the constraint for node 6.

Finally, applying the balance-of-flow rule to nodes 7, 8, and 9 we obtain the constraints:

$+0.95X_{57} + 0.90X_{67} \geq 60$ } flow constraint for node 7
$+0.9X_{58} + 0.95X_{68} \geq 40$ } flow constraint for node 8
$+0.9X_{59} + 0.95X_{69} \geq 50$ } flow constraint for node 9

The constraint for node 7 ensures that the final amount of product flowing to node 7 ($0.95X_{57} + 0.90X_{67}$) is sufficient to meet the demand for pulp at this node. Again, similar interpretations apply to the constraints for nodes 8 and 9.

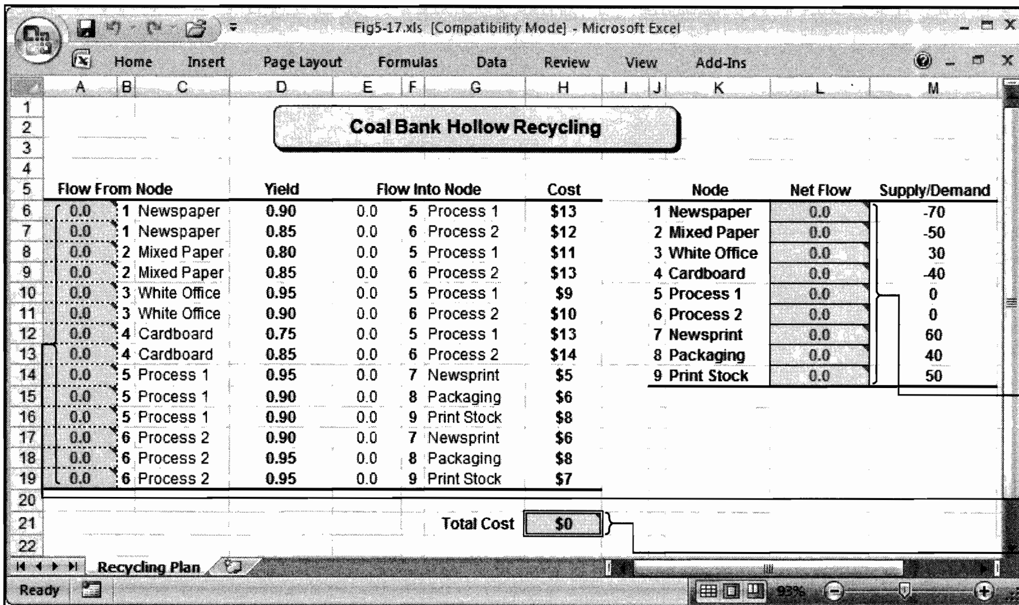
5.5.2 IMPLEMENTING THE MODEL

The model for Coal Bank Hollow Recycling's generalized network flow problem is summarized as:

MIN: $13X_{15} + 12X_{16} + 11X_{25} + 13X_{26} + 9X_{35} + 10X_{36} + 13X_{45} + 14X_{46} + 5X_{57} + 6X_{58} + 8X_{59} + 6X_{67} + 8X_{68} + 7X_{69}$

Subject to:

$-X_{15} - X_{16} \geq -70$ } flow constraint for node 1
$-X_{25} - X_{26} \geq -50$ } flow constraint for node 2
$-X_{35} - X_{36} \geq -30$ } flow constraint for node 3
$-X_{45} - X_{46} \geq -40$ } flow constraint for node 4
$+0.9X_{15} + 0.8X_{25} + 0.95X_{35} + 0.75X_{45} - X_{57} - X_{58} - X_{59} \geq 0$ } flow constraint for node 5
$+0.85X_{16} + 0.85X_{26} + 0.9X_{36} + 0.85X_{46} - X_{67} - X_{68} - X_{69} \geq 0$ } flow constraint for node 6
$+0.95X_{57} + 0.90X_{67} \geq 60$ } flow constraint for node 7
$+0.9X_{58} + 0.95X_{68} \geq 40$ } flow constraint for node 8
$+0.9X_{59} + 0.95X_{69} \geq 50$ } flow constraint for node 9
$X_{ij} \geq 0$ for all i and j } nonnegativity conditions



Flow From Node		Yield	Flow Into Node		Cost	Node	Net Flow	Supply/Demand
0.0	1 Newspaper	0.90	0.0	5 Process 1	$13	1 Newspaper	0.0	-70
0.0	1 Newspaper	0.85	0.0	6 Process 2	$12	2 Mixed Paper	0.0	-50
0.0	2 Mixed Paper	0.80	0.0	5 Process 1	$11	3 White Office	0.0	30
0.0	2 Mixed Paper	0.85	0.0	6 Process 2	$13	4 Cardboard	0.0	-40
0.0	3 White Office	0.95	0.0	5 Process 1	$9	5 Process 1	0.0	0
0.0	3 White Office	0.90	0.0	6 Process 2	$10	6 Process 2	0.0	0
0.0	4 Cardboard	0.75	0.0	5 Process 1	$13	7 Newsprint	0.0	60
0.0	4 Cardboard	0.85	0.0	6 Process 2	$14	8 Packaging	0.0	40
0.0	5 Process 1	0.95	0.0	7 Newsprint	$5	9 Print Stock	0.0	50
0.0	5 Process 1	0.90	0.0	8 Packaging	$6			
0.0	5 Process 1	0.90	0.0	9 Print Stock	$8			
0.0	6 Process 2	0.90	0.0	7 Newsprint	$6			
0.0	6 Process 2	0.95	0.0	8 Packaging	$8			
0.0	6 Process 2	0.95	0.0	9 Print Stock	$7			
				Total Cost	$0			

FIGURE 5.17

Spreadsheet model for Coal Bank Hollow Recycling's generalized network flow problem

Key Cell Formulas

Cell	Formula	Copied to
C6	=VLOOKUP(B6,$J$6:$K$14,2)	C7:C19 and G6:G19
E6	=D6*A6	E7:E19
H21	=SUMPRODUCT(H6:H19,A6:A19)	--
L6	=SUMIF($F$6:$F$19,J6,$E$6:$E$19) -SUMIF($B$6:$B$19,J6,$A$6:$A$19)	L7:L14

In all the other network flow models we have seen up to this point, all the coefficients in all the constraints were implicitly always +1 or −1. This is not true in the above model. Thus, we must give special attention to the coefficients in the constraints as we implement this model in the spreadsheet. One approach to implementing this problem is shown in Figure 5.17 (and the file Fig5-17.xls on your data disk).

The spreadsheet in Figure 5.17 is very similar to those of the other network flow problems we have solved. Cells A6 through A19 represent the decision variables (arcs) for our model, and the corresponding unit cost associated with each variable is listed in the range from H6 through H19. The objective function is implemented in cell H21 as:

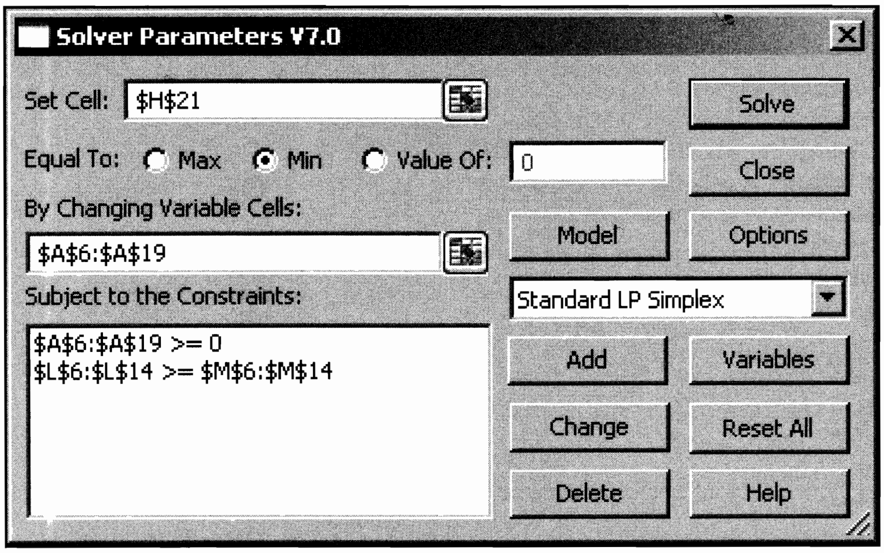
Formula for cell H21: =SUMPRODUCT(H6:H19,A6:A19)

To implement the LHS formulas for our constraints, we no longer can simply sum the variables flowing into each node and subtract the variables flowing out of the nodes. Instead, we first need to multiply the variables flowing into a node by the appropriate yield factor. With the yield factors entered in column D, the yield-adjusted flow for each arc is computed in column E as follows:

Formula for cell E6: =A6*D6
(Copy to cells E7 through E19.)

FIGURE 5.18

Solver parameters for the recycling problem



Now, to implement the LHS formulas for each node in cells L6 through L14, we will sum the yield-adjusted flows into each node and subtract the raw flow out of each node. This may be done as follows:

Formula for cell L6:	= SUMIF($F$6:$F$19,J6,$E$6:$E$19)–
(Copy to cells L7 through L14.)	SUMIF($B$6:$B$19,J6,$A$6:$A$19)

Notice that the first SUMIF function in this formula sums the appropriate yield-adjusted flows in column E while the second SUMIF sums the appropriate raw flow values from column A. Thus, although this formula is very similar to the ones used in earlier models, there is a critical difference here that must be carefully noted and understood. The RHS values for these constraint cells are listed in cells M6 through M14.

5.5.3 ANALYZING THE SOLUTION

The Solver parameters used to solve this problem are shown in Figure 5.18 and the optimal solution is shown in Figure 5.19.

In this solution, 43.4 tons of newspaper, 50 tons of mixed paper, and 30 tons of white office paper are assigned to recycling process 1 (i.e., $X_{15} = 43.4$, $X_{25} = 50$, $X_{35} = 30$). This recycling process then yields a total of 107.6 tons of pulp (i.e., $0.9 \times 43.3 + 0.8 \times 50 + 0.95 \times 30 = 107.6$) of which 63.2 tons are allocated to the production of newsprint pulp ($X_{57} = 63.2$) and 44.4 tons are allocated to the production of pulp for packaging paper ($X_{58} = 44.4$). This allows us to meet the demand for 60 tons of newsprint pulp ($0.95 \times 63.2 = 60$) and 40 tons of packaging paper ($0.90 \times 44.4 = 40$).

The remaining 26.6 tons of newspaper are combined with 35.4 tons of cardboard in recycling process 2 (i.e., $X_{16} = 26.6$, $X_{46} = 35.4$). This results in a yield of 52.6 tons of pulp (i.e., $0.85 \times 26.6 + 0.85 \times 35.4 = 52.6$), which is all devoted to the production of 50 tons of print stock quality pulp ($0.95 \times 52.6 = 50$).

It is important for Nancy to note that this production plan calls for the use of all her supply of newspaper, mixed paper, and white office paper, but leaves about 4.6 tons of cardboard left over. Thus, she should be able to lower her total costs further by acquiring more newspaper, mixed paper, or white office paper. It would be wise for her to see if she could trade her surplus cardboard to another recycler for the material that she lacks.

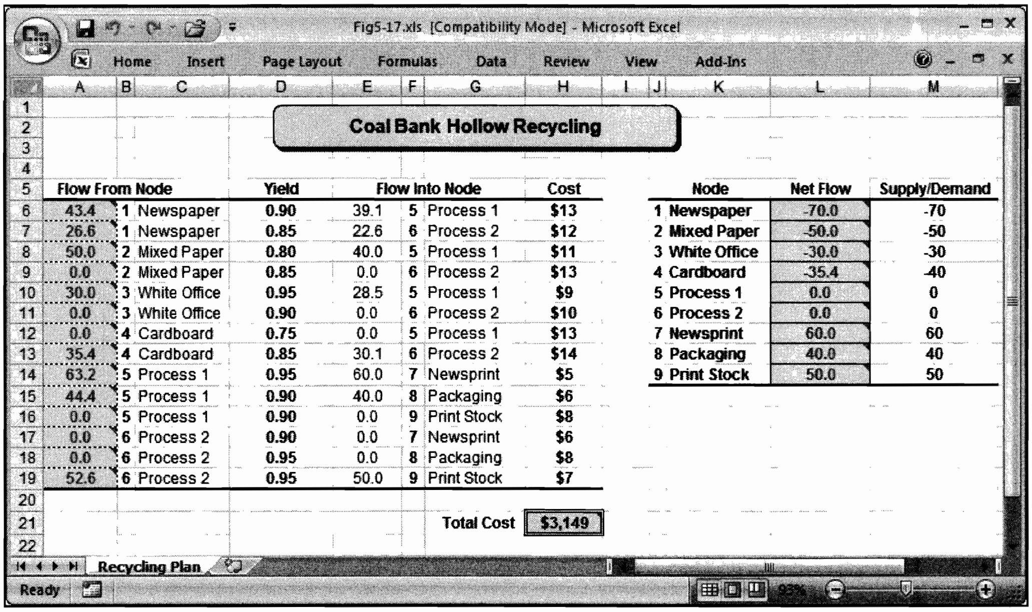


Fig5-17.xls [Compatibility Mode] - Microsoft Excel

Coal Bank Hollow Recycling

Flow	From Node	Yield	Flow	Into Node	Cost
43.4	1 Newspaper	0.90	39.1	5 Process 1	$13
26.6	1 Newspaper	0.85	22.6	6 Process 2	$12
50.0	2 Mixed Paper	0.80	40.0	5 Process 1	$11
0.0	2 Mixed Paper	0.85	0.0	6 Process 2	$13
30.0	3 White Office	0.95	28.5	5 Process 1	$9
0.0	3 White Office	0.90	0.0	6 Process 2	$10
0.0	4 Cardboard	0.75	0.0	5 Process 1	$13
35.4	4 Cardboard	0.85	30.1	6 Process 2	$14
63.2	5 Process 1	0.95	60.0	7 Newsprint	$5
44.4	5 Process 1	0.90	40.0	8 Packaging	$6
0.0	5 Process 1	0.90	0.0	9 Print Stock	$8
0.0	6 Process 2	0.90	0.0	7 Newsprint	$6
0.0	6 Process 2	0.95	0.0	8 Packaging	$8
52.6	6 Process 2	0.95	50.0	9 Print Stock	$7
				Total Cost	$3,149

Node	Net Flow	Supply/Demand
1 Newspaper	-70.0	-70
2 Mixed Paper	-50.0	-50
3 White Office	-30.0	-30
4 Cardboard	-35.4	-40
5 Process 1	0.0	0
6 Process 2	0.0	0
7 Newsprint	60.0	60
8 Packaging	40.0	40
9 Print Stock	50.0	50

FIGURE 5.19

Optimal solution to Coal Bank Hollow Recycling's generalized network flow problem

5.5.4 GENERALIZED NETWORK FLOW PROBLEMS AND FEASIBILITY

In generalized network flow problems, the gains and/or losses associated with flows across each arc *effectively* increase and/or decrease the supply available in the network. For example, consider what happens in Figure 5.16 if the supply of newspaper is reduced to 55 tons. Although it *appears* that the total supply in the network (175 tons) still exceeds the total demand (150 tons), if we try to solve the modified problem, Solver will tell us that the problem has no feasible solution. (You may verify this on your own.) So we are not able to satisfy all of the demand due to the loss of material that occurs in the production process.

The point being made here is that with generalized network flow problems, you cannot always tell before solving the problem if the total supply is adequate to meet the total demand. As a result, you cannot always know which balance-of-flow rule to apply. When the issue is unclear, it is safest[1] to first assume that all the demand can be met and (according to the balance-of-flow rule) use constraints of the form: Inflow − Outflow ≥ Supply or Demand. If the resulting problem is infeasible (and there are no errors in the model!), then we know all the demand cannot be satisfied and (according to the balance-of-flow rule) use constraints of the form: Inflow − Outflow ≤ Supply or Demand. In this later case, the solution will identify the least costly way of using the available supply to meet as much of the demand as possible.

Figures 5.20 and 5.21 show, respectively, the Solver parameters and optimal solution for this revised recycling problem with 55 tons of newspaper. Note that this solution uses all of the available supply of each of the recycling materials. Although the solution satisfies all the demand for newsprint pulp and packaging paper pulp, it falls almost 15 tons short of the total demand for print stock pulp.

[1] See question 3 at the end of this chapter for more on this issue.

FIGURE 5.20

Solver parameters for modified recycling problem

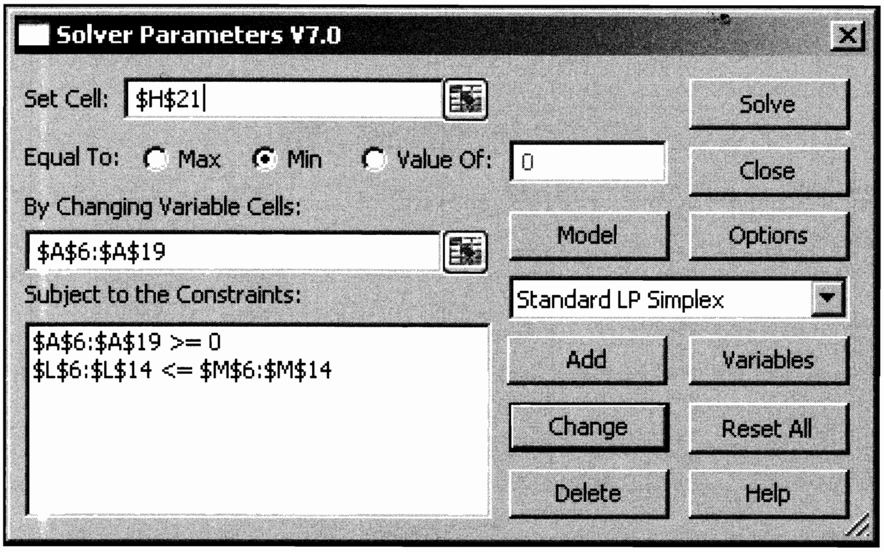
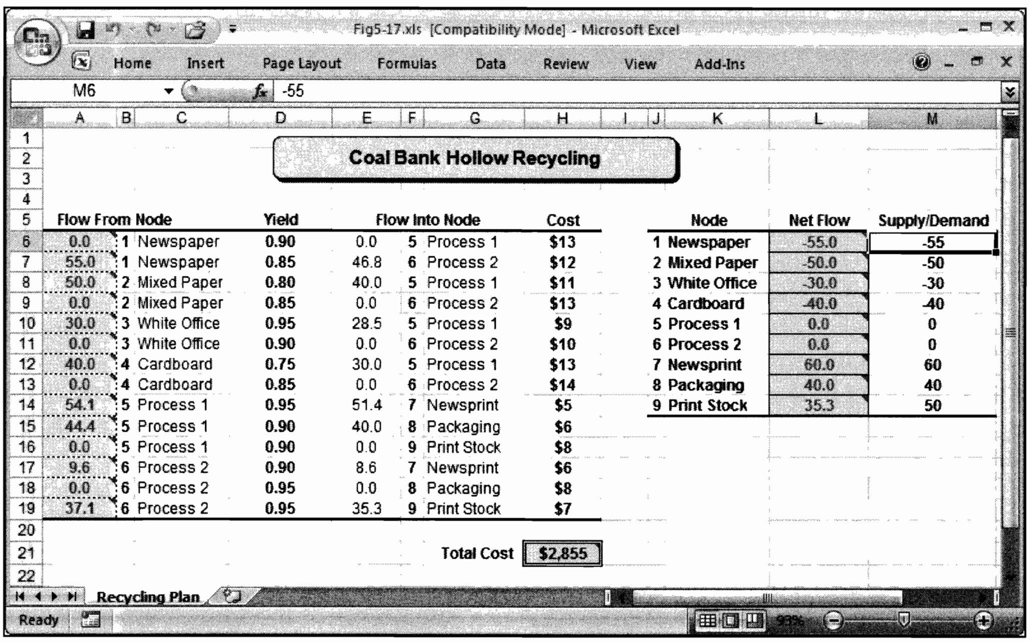


FIGURE 5.21

Optimal solution to modified recycling problem



Flow	From Node	Yield	Flow	Into Node	Cost
0.0	1 Newspaper	0.90	0.0	5 Process 1	$13
55.0	1 Newspaper	0.85	46.8	6 Process 2	$12
50.0	2 Mixed Paper	0.80	40.0	5 Process 1	$11
0.0	2 Mixed Paper	0.85	0.0	6 Process 2	$13
30.0	3 White Office	0.95	28.5	5 Process 1	$9
0.0	3 White Office	0.90	0.0	6 Process 2	$10
40.0	4 Cardboard	0.75	30.0	5 Process 1	$13
0.0	4 Cardboard	0.85	0.0	6 Process 2	$14
54.1	5 Process 1	0.95	51.4	7 Newsprint	$5
44.4	5 Process 1	0.90	40.0	8 Packaging	$6
0.0	5 Process 1	0.90	0.0	9 Print Stock	$8
9.6	6 Process 2	0.90	8.6	7 Newsprint	$6
0.0	6 Process 2	0.95	0.0	8 Packaging	$8
37.1	6 Process 2	0.95	35.3	9 Print Stock	$7
				Total Cost	$2,855

Node	Net Flow	Supply/Demand
1 Newspaper	-55.0	-55
2 Mixed Paper	-50.0	-50
3 White Office	-30.0	-30
4 Cardboard	-40.0	-40
5 Process 1	0.0	0
6 Process 2	0.0	0
7 Newsprint	60.0	60
8 Packaging	40.0	40
9 Print Stock	35.3	50

Important Modeling Point

For generalized network flow problems, the gains and/or losses associated with flows across each arc *effectively* increase and/or decrease the supply available in the network. As a result, it is sometimes difficult to tell in advance whether the total supply is adequate to meet the total demand in a generalized network flow problem. When in doubt, it is best to assume that the total supply is capable of satisfying the total demand and use Solver to prove (or refute) this assumption.

5.6 Maximal Flow Problems

The maximal flow problem (or max flow problem) is a type of network flow problem in which the goal is to determine the maximum amount of flow that can occur in the network. In a maximal flow problem, the amount of flow that can occur over each arc is limited by some capacity restriction. This type of network might be used to model the flow of oil in a pipeline (in which the amount of oil that can flow through a pipe in a unit of time is limited by the diameter of the pipe). Traffic engineers also use this type of network to determine the maximum number of cars that can travel through a collection of streets with different capacities imposed by the number of lanes in the streets and speed limits. The following example illustrates a max flow problem.

5.6.1 AN EXAMPLE OF A MAXIMAL FLOW PROBLEM

The Northwest Petroleum Company operates an oil field and refinery in Alaska. The crude obtained from the oil field is pumped through the network of pumping substations shown in Figure 5.22 to the company's refinery located 500 miles from the oil field. The amount of oil that can flow through each of the pipelines, represented by the arcs in the network, varies due to differing pipe diameters. The numbers next to the arcs in the network indicate the maximum amount of oil that can flow through the various pipelines (measured in thousands of barrels per hour). The company wants to determine the maximum number of barrels per hour that can flow from the oil field to the refinery.

The max flow problem appears to be very different from the network flow models described earlier because it does not include specific supplies or demands for the nodes. However, you can solve the max flow problem in the same way as a transshipment

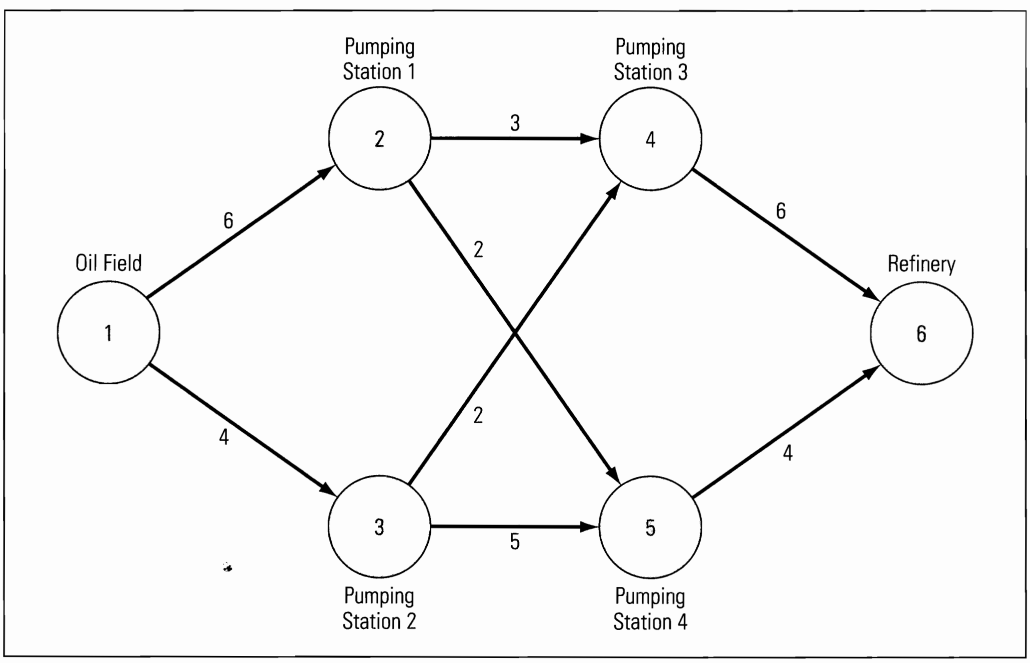


FIGURE 5.22

Network representation of Northwest Petroleum's oil refinery operation

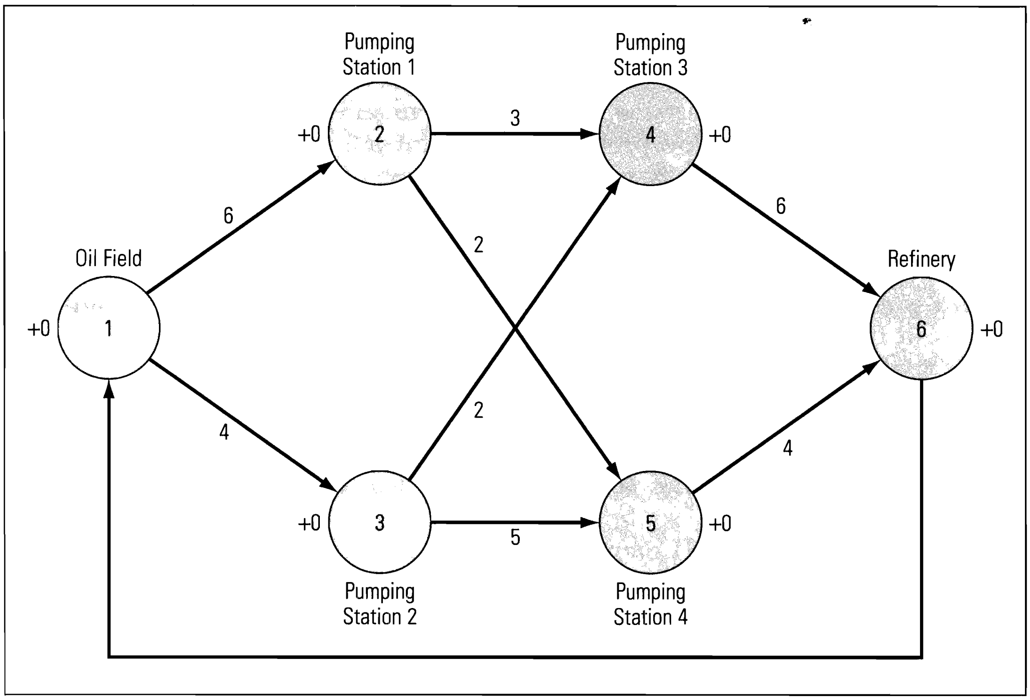


FIGURE 5.23

Network structure of Northwest Petroleum's max flow problem

problem if you add a return arc from the ending node to the starting node, assign a demand of 0 to all the nodes in the network, and attempt to maximize the flow over the return arc. Figure 5.23 shows these modifications to the problem.

To understand the network in Figure 5.23, suppose that k units are shipped from node 6 to node 1 (where k represents some integer). Because node 6 has a supply of 0, it can send k units to node 1 only if these units can be returned through the network to node 6 (to balance the flow at node 6). The capacities on the arcs limit how many units can be returned to node 6. Therefore, the maximum flow through the network corresponds to the largest number of units that can be shipped from node 6 to node 1 and then returned through the network to node 6 (to balance the flow at this node). We can solve an LP model to determine the maximal flow by maximizing the flow from node 6 to node 1, given appropriate upper bounds on each arc and the usual balance-of-flow constraints. This model is represented as:

$$
\begin{aligned}
\text{MAX:} \quad & X_{61} \\
\text{Subject to:} \quad & +X_{61} - X_{12} - X_{13} = 0 \\
& +X_{12} - X_{24} - X_{25} = 0 \\
& +X_{13} - X_{34} - X_{35} = 0 \\
& +X_{24} + X_{34} - X_{46} = 0 \\
& +X_{25} + X_{35} - X_{56} = 0 \\
& +X_{46} + X_{56} - X_{61} = 0
\end{aligned}
$$

with the following bounds on the decision variables:

$$
\begin{array}{lll}
0 \le X_{12} \le 6 & 0 \le X_{25} \le 2 & 0 \le X_{46} \le 6 \\
0 \le X_{13} \le 4 & 0 \le X_{34} \le 2 & 0 \le X_{56} \le 4 \\
0 \le X_{24} \le 3 & 0 \le X_{35} \le 5 & 0 \le X_{61} \le \infty
\end{array}
$$

5.6.2 THE SPREADSHEET MODEL AND SOLUTION

This model is implemented in the spreadsheet shown in Figure 5.24 (and in the file Fig5-24.xls on your data disk). This spreadsheet model differs from the earlier network models in a few minor, but important, ways. First, column G in Figure 5.24 represents the upper bounds for each arc. Second, the objective function (or set cell) is represented by cell B16, which contains the formula:

Formula in cell B16: =B14

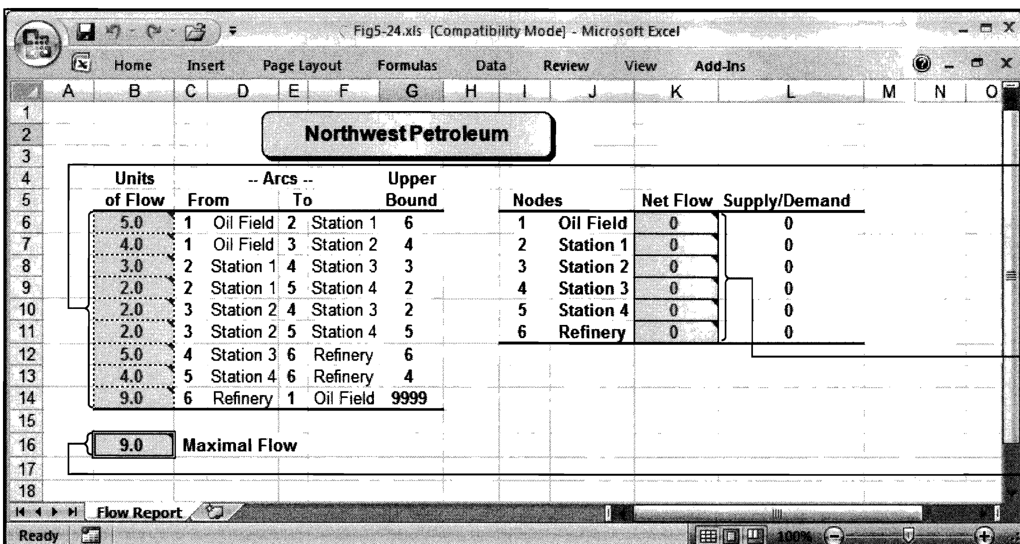
Cell B14 represents the flow from node 6 to node 1 (or X_{61}). This cell corresponds to the variable that we want to maximize in the objective function of the LP model. The Solver parameters and options shown in Figure 5.25 are used to obtain the optimal solution shown in Figure 5.24.

Because the arcs leading to node 6 (X_{46} and X_{56}) have a total capacity for 10 units of flow, it might be surprising to learn that only 9 units can flow through the network. However, the optimal solution shown in Figure 5.24 indicates that the maximal flow through the network is just 9 units.

The optimal flows identified in Figure 5.24 for each arc are shown in the boxes next to the capacities for each arc in Figure 5.26. In Figure 5.26, the arc from node 5 to node 6 is at its full capacity of 4 units, whereas the arc from node 4 to node 6 is 1 unit below its full capacity of 6 units. Although the arc from node 4 to node 6 can carry 1 additional unit of flow, it is prevented from doing so because all the arcs flowing to node 4 (X_{24} and X_{34}) are at full capacity.

FIGURE 5.24

Spreadsheet model and solution to Northwest Petroleum's max flow problem



Northwest Petroleum

Units of Flow	From		To		Upper Bound	Nodes		Net Flow	Supply/Demand
5.0	1	Oil Field	2	Station 1	6	1	Oil Field	0	0
4.0	1	Oil Field	3	Station 2	4	2	Station 1	0	0
3.0	2	Station 1	4	Station 3	3	3	Station 2	0	0
2.0	2	Station 1	5	Station 4	2	4	Station 3	0	0
2.0	3	Station 2	4	Station 3	2	5	Station 4	0	0
2.0	3	Station 2	5	Station 4	5	6	Refinery	0	0
5.0	4	Station 3	6	Refinery	6				
4.0	5	Station 4	6	Refinery	4				
9.0	6	Refinery	1	Oil Field	9999				

9.0 Maximal Flow

Variable Cells

Constraint Cells

Set Cell

Key Cell Formulas

Cell	Formula	Copied to
B16	=B14	—
D6	=VLOOKUP(C6,$I$6:$J$11,2)	D7:D14 and F6:F14
K6	=SUMIF($E$6:$E$14,I6,$B$6:$B$14)-SUMIF($C$6:$C$14,I6,$B$6:$B$14)	K7:K11

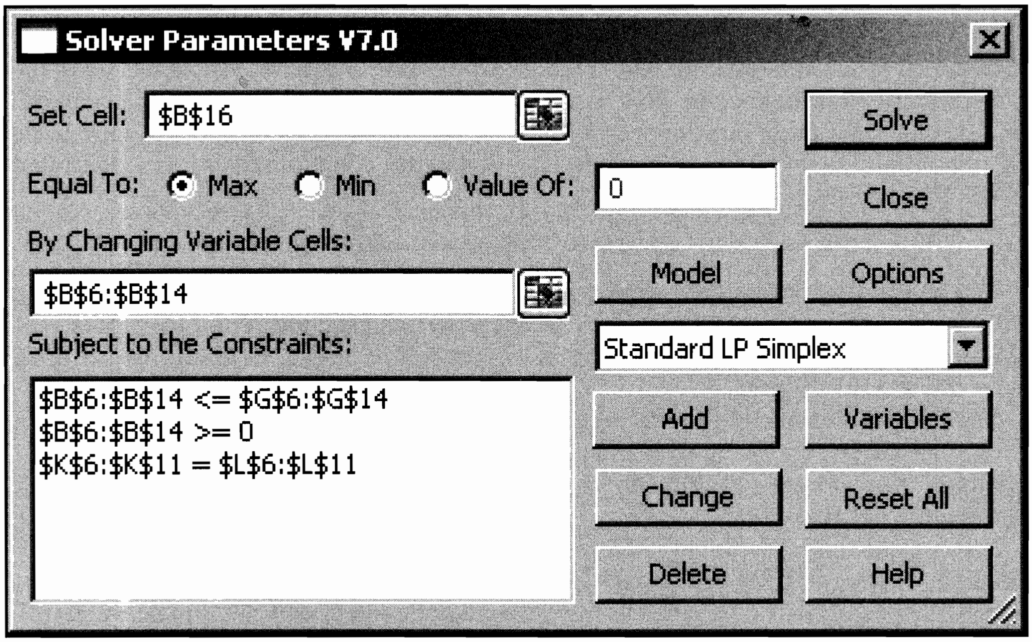


FIGURE 5.25

Solver parameters for Northwest Petroleum's max flow problem

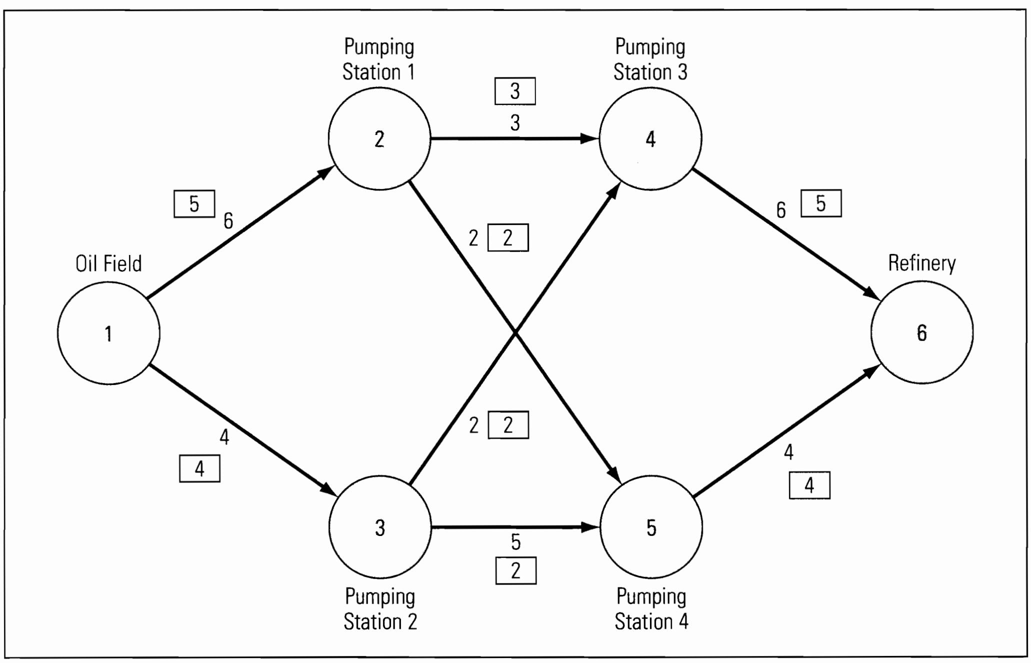


FIGURE 5.26

Network representation of the solution to Northwest Petroleum's max flow problem

A graph like Figure 5.26, which summarizes the optimal flows in a max flow problem, is helpful in identifying where increases in flow capacity would be most effective. For example, from this graph, we can see that even though X_{24} and X_{34} are both at full capacity, increasing their capacity will not necessarily increase the flow through the network. Increasing the capacity of X_{24} would allow for an increased flow through the network because an additional unit could then flow from node 1 to node 2 to node 4 to node 6. However, increasing the capacity of X_{34} would not allow for an increase in the total flow because the arc from node 1 to node 3 is already at full capacity.

5.7 Special Modeling Considerations

A number of special conditions can arise in network flow problems that require a bit of creativity to model accurately. For example, it is easy to impose minimum or maximum flow restrictions on individual arcs in the networks by placing appropriate lower and upper bounds on the corresponding decision variables. However, in some network flow problems, minimum or maximum flow requirements may apply to the *total* flow emanating from a given node. For example, consider the network flow problem shown in Figure 5.27.

Now suppose that the total flow into node 3 must be at least 50 and the total flow into node 4 must be at least 60. We could enforce these conditions easily with the following constraints:

$$X_{13} + X_{23} \geq 50$$
$$X_{14} + X_{24} \geq 60$$

Unfortunately, these constraints do not conform to the balance-of-flow rule and would require us to impose *side constraints* on the model. An alternative approach to modeling this problem is shown in Figure 5.28.

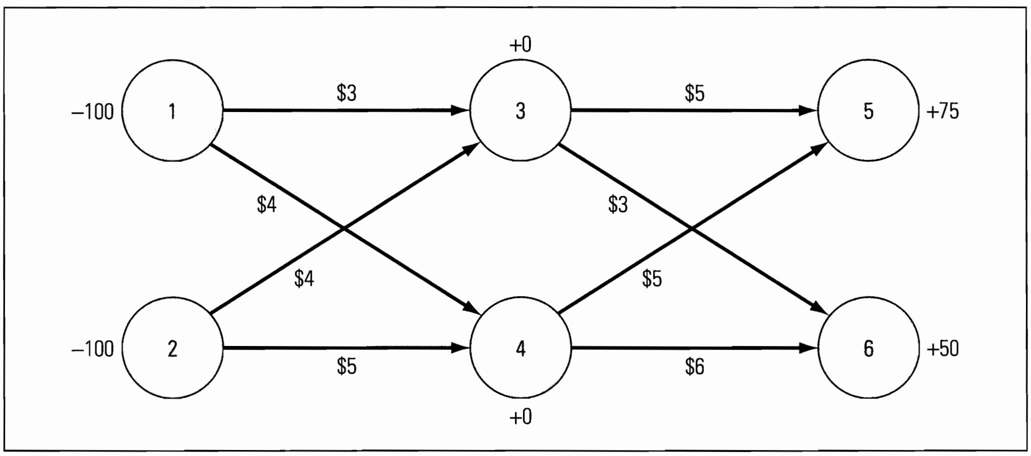


FIGURE 5.27

Example network flow problem

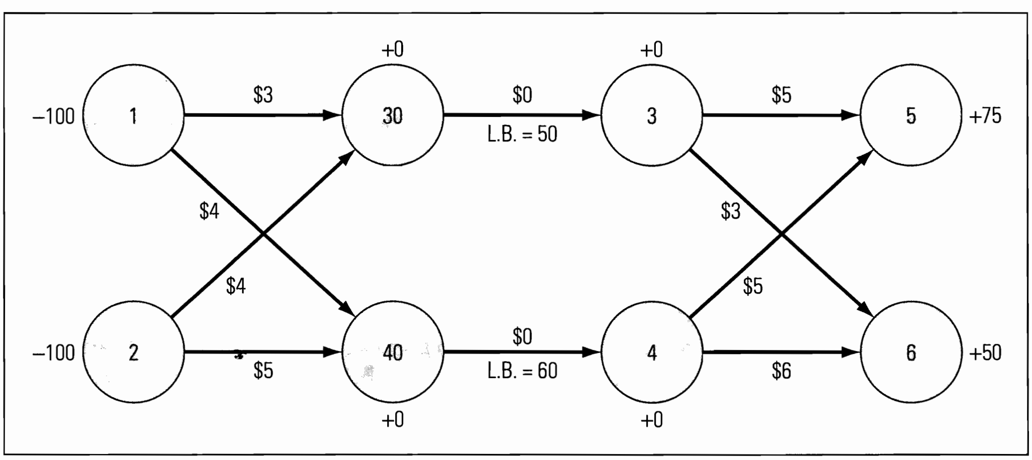
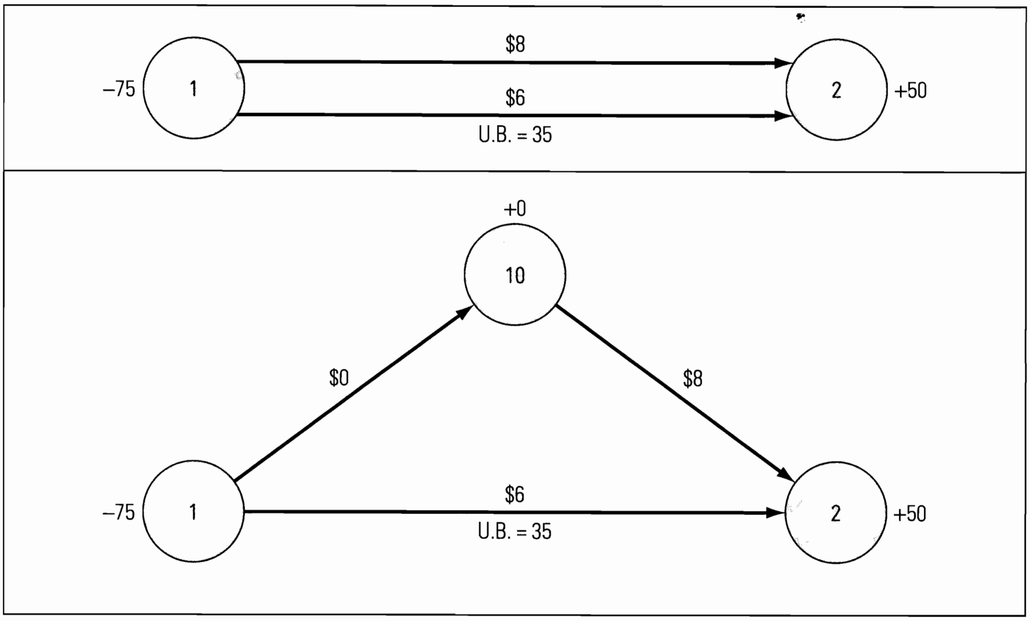


FIGURE 5.28

Revised network flow problem with lower bounds on the total flow into nodes 3 and 4

FIGURE 5.29

Alternative networks allowing two different types of flow between two nodes



Two additional nodes and arcs were inserted in Figure 5.28. Note that the arc from node 30 to node 3 has a lower bound (L.B.) of 50. This will ensure that at least 50 units flow into node 3. Node 3 then must distribute this flow to nodes 5 and 6. Similarly, the arc connecting node 40 to node 4 ensures that at least 60 units will flow into node 4. The additional nodes and arcs added to Figure 5.28 are sometimes referred to as ***dummy nodes*** and ***dummy arcs***.

As another example, consider the network in the upper portion of Figure 5.29 in which the flow between two nodes can occur at two different costs. One arc has a cost of $6 per unit of flow and an upper bound (U.B.) of 35. The other arc has a cost of $8 per unit of flow with no upper bound on the amount of flow allowed. Note that the minimum cost solution is to send 35 units of flow from node 1 to node 2 across the $6 arc and 15 units from node 1 to node 2 across the $8 arc.

To model this problem mathematically, we would like to have two arcs called X_{12} because both arcs go from node 1 to node 2. However, if both arcs are called X_{12}, there is no way to distinguish one from the other! A solution to this dilemma is shown in the lower portion of Figure 5.29 in which we inserted a dummy node and a dummy arc. Thus, there are now two distinct arcs flowing into node 2: X_{12} and $X_{10,2}$. Flow from node 1 to node 2 across the $8 arc now must first go through node 10.

As a final example, note that upper bounds (or capacity restrictions) on the arcs in a network flow might *effectively* limit the amount of supply that can be sent through the network to meet the demand. As a result, in a network flow problem with flow restrictions (upper bounds) on the arcs, it is sometimes difficult to tell in advance whether the total demand can be met—even if the total supply available exceeds the total demand. This again creates a potential problem in knowing which balance-of-flow rule to use. Consider the example in Figure 5.30.

The upper portion of Figure 5.30 shows a network with a total supply of 200 and total demand of 155. Because the total supply appears to exceed the total demand, we are inclined to apply the balance-of-flow rule that would generate constraints of the form: Inflow − Outflow ≥ Supply or Demand. This balance of flow rule requires the total

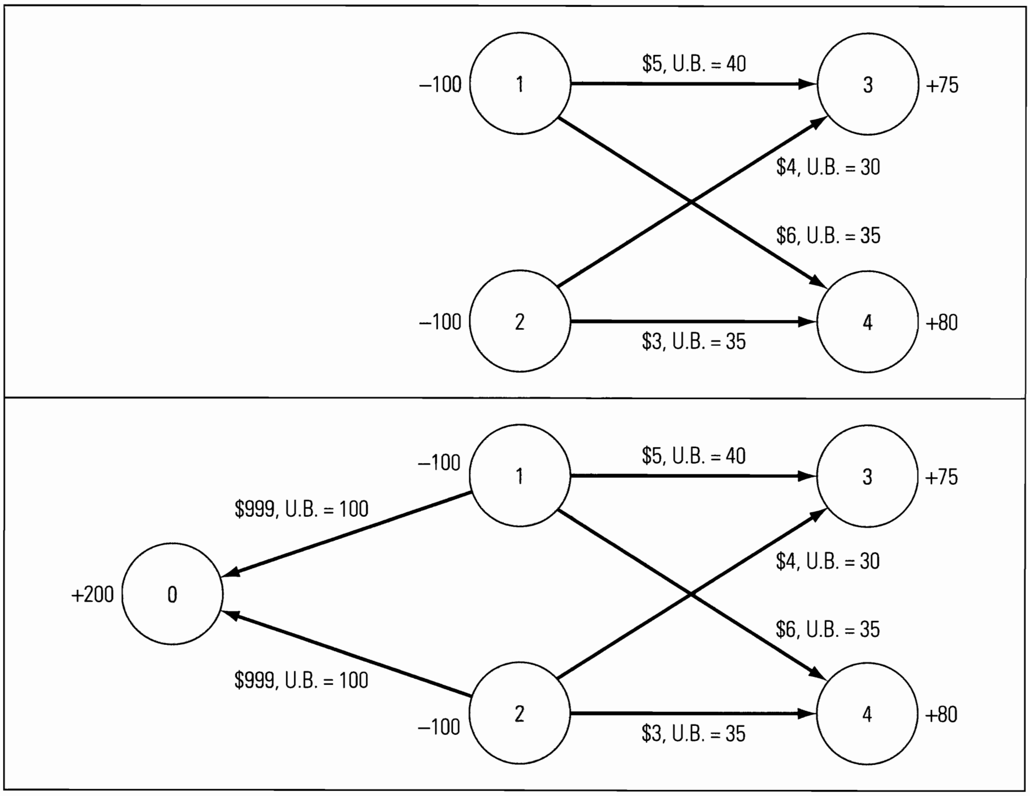


FIGURE 5.30

Example of using a dummy demand node

inflow to nodes 3 and 4 to be greater than or equal to their demands of 75 and 80, respectively. However, the upper bounds on the arcs leading into node 3 limit the total flow into this node to 70 units. Similarly, the total flow into node 4 is limited to 70. As a result, there is no feasible solution to the problem. In this case, we cannot resolve the infeasibility by reversing the constraints to be of the form: Inflow − Outflow ≤ Supply or Demand. Although this allows for less than the total amount demanded to be sent to nodes 3 and 4, it now *requires* all the supply to be sent out of nodes 1 and 2. Clearly, some of the 200 units of supply available from nodes 1 and 2 will have nowhere to go if the total flow into nodes 3 and 4 cannot exceed 140 units (as required by the upper bounds on the arcs).

A solution to this predicament is shown in the bottom half of Figure 5.30. Here, we added a dummy demand node (node 0) that is connected directly to nodes 1 and 2 with arcs that impose very large costs on flows to the dummy node. Note that the demand at this dummy node is equal to the total supply in the network. Now, the total demand exceeds the total supply so the balance-of-flow rule mandates we use constraints of the form: Inflow − Outflow ≤ Supply or Demand. Again, this allows for less than the total amount demanded to be sent to nodes 0, 3, and 4 but *requires* all the supply to be sent out of nodes 1 and 2. Due to the large costs associated with flows from nodes 1 and 2 to the dummy demand node, Solver will ensure that as much of the supply as possible is first sent to nodes 3 and 4. Any remaining supply at nodes 1 and 2 would then be sent to the dummy node. Of course, flows to the dummy node actually represent excess supply or inventory at nodes 1 and 2 that would not actually be shipped anywhere or incur any costs. But using a dummy node in this manner allows us to model and solve the problem accurately.

Dummy nodes and arcs can be helpful in modeling a variety of situations that occur naturally in network problems. The techniques illustrated here are three "tricks of the trade" in network modeling and might prove useful in some of the problems at the end of this chapter.

5.8 Minimal Spanning Tree Problems

Another type of network problem is known as the minimal spanning tree problem. This type of problem cannot be solved as an LP problem, but is solved easily using a simple manual algorithm.

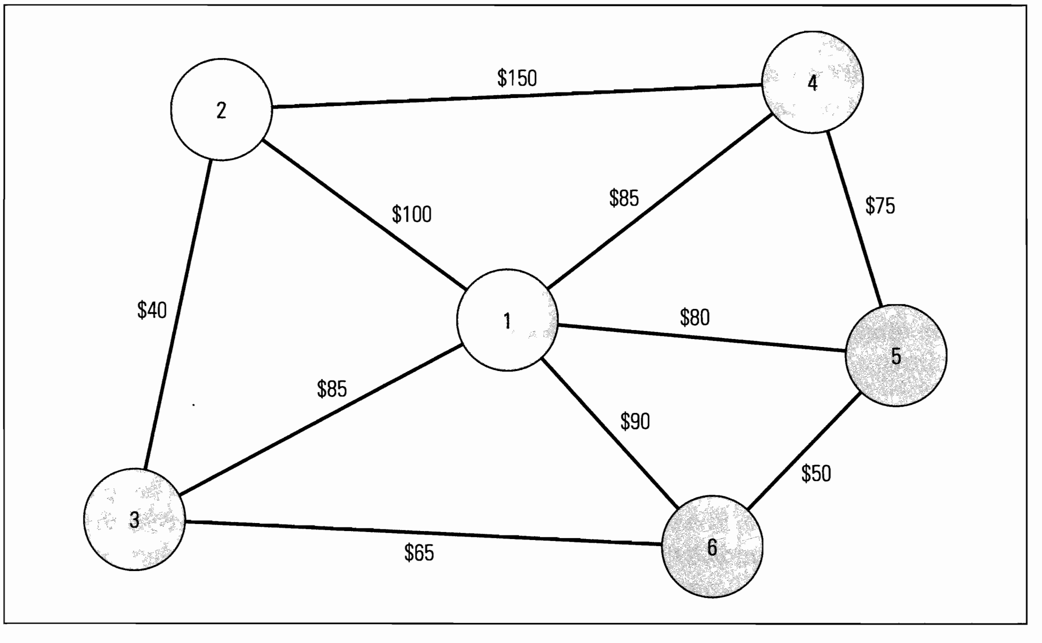
For a network with n nodes, a **spanning tree** is a set of $n - 1$ arcs that connects all the nodes and contains no loops. A minimum spanning tree problem involves determining the set of arcs that connects all the nodes in a network while minimizing the total length (or cost) of the selected arcs. Consider the following example.

> Jon Fleming is responsible for setting up a local area network (LAN) in the design engineering department of Windstar Aerospace Company. A LAN consists of a number of individual computers connected to a centralized computer or file server. Each computer in the LAN can access information from the file server and communicate with the other computers in the LAN.
>
> Installing a LAN involves connecting all the computers together with communications cables. Not every computer has to be connected directly to the file server, but there must be some link between each computer in the network. Figure 5.31 summarizes all the possible connections that Jon could make. Each node in this figure represents one of the computers to be included in the LAN. Each line connecting the nodes represents a possible connection between pairs of computers. The dollar amount on each line represents the cost of making the connection.

The arcs in Figure 5.31 have no specific directional orientation, indicating that information can move in either direction across the arcs. Also note that the communication

FIGURE 5.31

Network representation of Windstar Aerospace's minimal spanning tree problem



links represented by the arcs do not exist yet. Jon's challenge is to determine which links to establish. Because the network involves $n = 6$ nodes, a spanning tree for this problem consists of $n - 1 = 5$ arcs that results in a path existing between any pair of nodes. The objective is to find the minimal (least costly) spanning tree for this problem.

5.8.1 AN ALGORITHM FOR THE MINIMAL SPANNING TREE PROBLEM

You can apply a simple algorithm to solve minimal spanning tree problems. The steps to this algorithm are:

1. Select any node. Call this the current subnetwork.
2. Add to the current subnetwork the cheapest arc that connects any node within the current subnetwork to any node not in the current subnetwork. (Ties for the cheapest arc can be broken arbitrarily.) Call this the current subnetwork.
3. If all the nodes are in the subnetwork, stop; this is the optimal solution. Otherwise, return to step 2.

5.8.2 SOLVING THE EXAMPLE PROBLEM

You can program this algorithm easily or, for simple problems, execute it manually. The following steps illustrate how to execute the algorithm manually for the example problem shown in Figure 5.31.

Step 1. If we select node 1 in Figure 5.31, then node 1 is the current subnetwork.

Step 2. The cheapest arc connecting the current subnetwork to a node not in the current subnetwork is the $80 arc connecting nodes 1 and 5. This arc and node 5 are added to the current subnetwork.

Step 3. Four nodes (nodes 2, 3, 4, and 6) remain unconnected—therefore, return to step 2.

Step 2. The cheapest arc connecting the current subnetwork to a node not in the current subnetwork is the $50 arc connecting nodes 5 and 6. This arc and node 6 are added to the current subnetwork.

Step 3. Three nodes (nodes 2, 3, and 4) remain unconnected—therefore, return to step 2.

Step 2. The cheapest arc connecting the current subnetwork to a node not in the current subnetwork is the $65 arc connecting nodes 6 and 3. This arc and node 3 are added to the current subnetwork.

Step 3. Two nodes (nodes 2 and 4) remain unconnected—therefore, return to step 2.

Step 2. The cheapest arc connecting the current subnetwork to a node not in the current subnetwork is the $40 arc connecting nodes 3 and 2. This arc and node 2 are added to the current subnetwork.

Step 3. One node (node 4) remains unconnected—therefore, return to step 2.

Step 2. The cheapest arc connecting the current subnetwork to a node not in the current subnetwork is the $75 arc connecting nodes 5 and 4. This arc and node 4 are added to the current subnetwork.

Step 3. All the nodes are now connected. Stop; the current subnetwork is optimal.

Figure 5.32 shows the optimal (minimal) spanning tree generated by this algorithm. The algorithm described here produces the optimal (minimal) spanning tree regardless

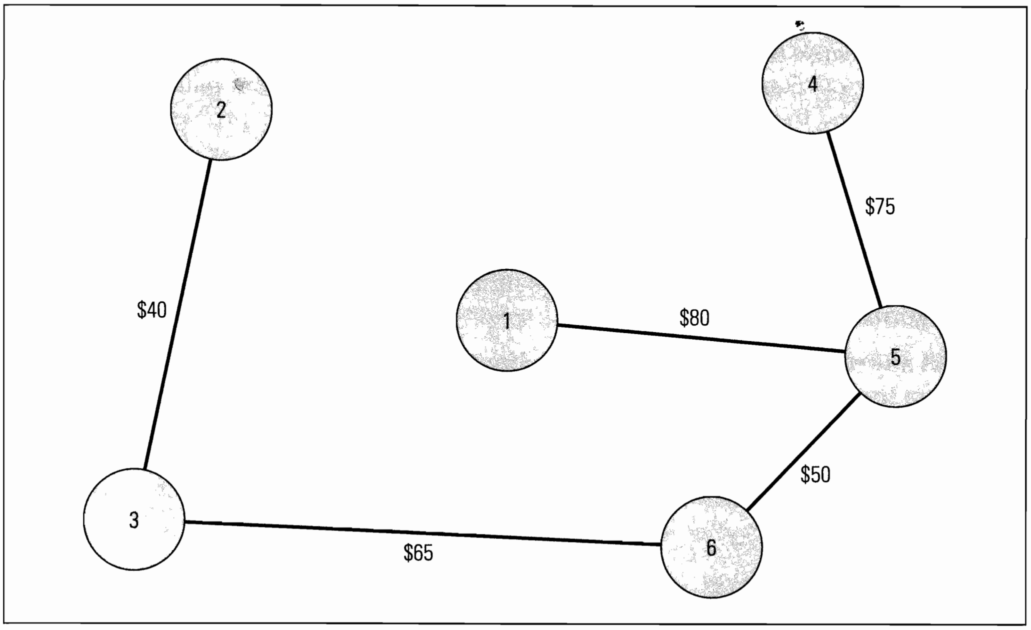


FIGURE 5.32

Optimal solution to Windstar Aerospace's minimal spanning tree problem

of which node is selected initially in step 1. You can verify this by solving the example problem again starting with a different node in step 1.

5.9 Summary

This chapter presented several business problems modeled as network flow problems, including transshipment problems, shortest path problems, maximal flow problems, transportation/assignment problems, and generalized network flow models. It also introduced the minimal spanning tree problem and presented a simple algorithm for solving this type of problem manually.

Although special algorithms exist for solving network flow problems, you can also formulate and solve them as LP problems. The constraints in an LP formulation of a network flow problem have a special structure that enables you to implement and solve these models easily in a spreadsheet. Although there might be more efficient ways of solving network flow problems, the methods discussed in this chapter are often the most practical. For extremely complex network flow problems, you might need to use a specialized algorithm. Unfortunately, you are unlikely to find this type of software at your local software store. However, various network optimization packages can be found in the technical/scientific directories on the Internet.

5.10 References

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THE WORLD OF MANAGEMENT SCIENCE

Yellow Freight System Boosts Profits and Quality with Network Optimization

One of the largest motor carriers in the United States, Yellow Freight System, Inc. of Overland Park, Kansas, uses network modeling and optimization to assist management in load planning, routing empty trucks, routing trailers, dropping or adding direct service routes, and strategic planning of terminal size and location. The system, called SYSNET, operates on a network of Sun workstations optimizing over a million network flow variables. The company also uses a tactical planning room equipped with graphical display tools that allow planning meetings to be conducted interactively with the system.

The company competes in the less–than–truckload (LTL) segment of the trucking market. That is, they contract for shipments of any size, regardless of whether the shipment fills the trailer. To operate efficiently, Yellow Freight must consolidate and transfer shipments at 23 break–bulk terminals located throughout the United States. At these terminals, shipments might be reloaded into different trailers depending on the final destination. Each break–bulk terminal serves several end–of–line terminals, in a hub–and–spoke network. Normally, shipments are sent by truck to the break–bulk dedicated to the origination point. Local managers occasionally try to save costs by loading direct, which means bypassing a break–bulk and sending a truckload of consolidated shipments directly to the final destination. Before SYSNET, these decisions were made in the field without accurate information on how they would affect costs and reliability in the entire system.

Since its implementation in 1989, SYSNET has scored high with upper management. Often, the first response to a new proposal is, "Has it been run through SYSNET?" The benefits attributed to the new system include:

- an increase of 11.6% in freight loaded directly, saving $4.7 million annually
- better routing of trailers, saving $1 million annually
- savings of $1.42 million annually by increasing the average number of pounds loaded per trailer
- reduction in claims for damaged merchandise
- a 27% reduction in the number of late deliveries
- tactical planning projects with SYSNET in 1990 that identified $10 million in annual savings

Equally important has been the effect on the management philosophy and culture at Yellow Freight. Management now has greater control over network operations; tradition, intuition, and "gut feel" have been replaced with formal analytical tools; and Yellow Freight is better able to act as a partner with customers in total quality management and just–in–time inventory systems.

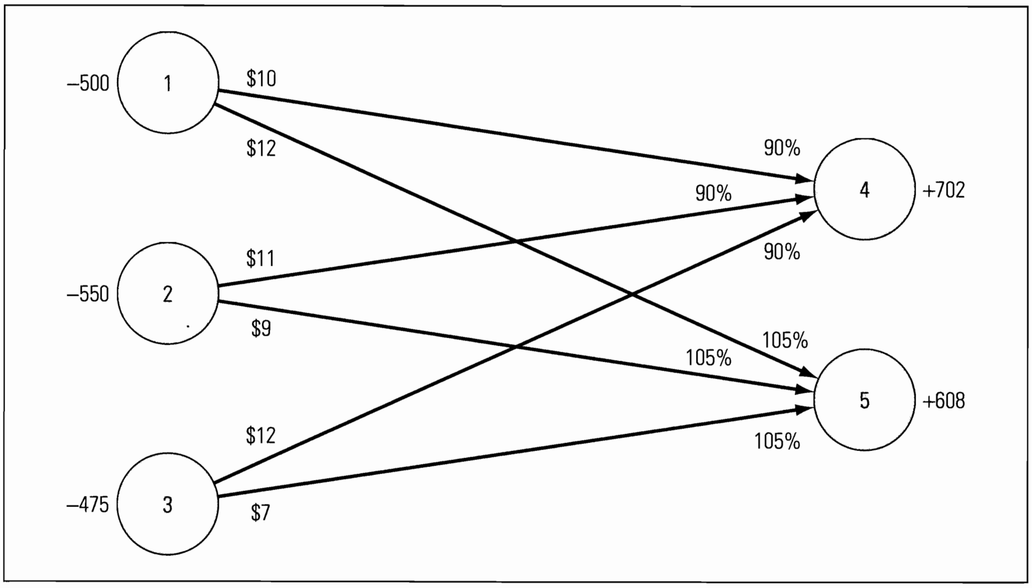
Source: Braklow, John W., William W. Graham, Stephen M. Hassler, Ken E. Peck and Warren B. Powell, "Interactive Optimization Improves Service and Performance for Yellow Freight System," *Interfaces*, 22:1, January–February 1992, pages 147–172.

Questions and Problems

1. This chapter followed the convention of using negative numbers to represent the supply at a node and positive numbers to represent the demand at a node. Another convention is just the opposite—using positive numbers to represent supply and negative numbers to represent demand. How would the balance-of-flow rule presented in this chapter need to be changed to accommodate this alternate convention?
2. To use the balance-of-flow rule presented in this chapter, constraints for supply nodes must have negative RHS values. Some LP software packages cannot solve problems in which the constraints have negative RHS values. How should the balance-of-flow rules be modified to produce LP models that can be solved with such software packages?
3. Consider the revised Coal Bank Hollow recycling problem discussed in section 5.5.4 of this chapter. We said that it is safest to assume the supply in a generalized network flow problem is capable of meeting the demand (until Solver proves otherwise).
   a. Solve the problem in Figure 5.17 (and file Fig5-17.xls on your data disk) assuming 80 tons of newspaper is available and that the supply is NOT adequate to meet the demand. How much of each of the raw recycling materials is used? How much demand for each product is met? What is the cost of this solution?
   b. Solve the problem again assuming that the supply is adequate to meet the demand. How much of each of the raw recycling materials is used? How much demand for each product is met? What is the cost of this solution?
   c. Which one is better? Why?
   d. Suppose there are 55 tons of newspaper available. Figure 5.21 shows the least cost solution for distributing the supply in this case. In that solution, the demand for newsprint pulp and packaging pulp is met but we are almost 15 tons short on print stock pulp. How much can this shortage be reduced (without creating shortages of the other products) and how much extra would it cost to do so?
4. Consider the generalized transportation problem shown in Figure 5.33. How can this problem be transformed into an equivalent transportation problem? Draw the network for the equivalent problem.

FIGURE 5.33

Graph of a generalized network flow problem



5. Draw the network representation of the following network flow problem.

MIN: $+7X_{12} + 6X_{14} + 3X_{23} + 4X_{24} + 5X_{32} + 9X_{43} + 8X_{52} + 5X_{54}$

Subject to:
$-X_{12} - X_{14} = -5$
$+X_{12} + X_{52} + X_{32} - X_{23} - X_{24} = +4$
$-X_{32} + X_{23} + X_{43} = +8$
$+X_{14} + X_{24} + X_{54} - X_{43} = +0$
$-X_{52} - X_{54} = -7$
$X_{ij} \geq 0$ for all i and j

6. Draw the network representation of the following network flow problem. What kind of network flow problem is this?

MIN: $+2X_{13} + 6X_{14} + 5X_{15} + 4X_{23} + 3X_{24} + 7X_{25}$

Subject to:
$-X_{13} - X_{14} - X_{15} = -8$
$-X_{23} - X_{24} - X_{25} = -7$
$+X_{13} + X_{23} = +5$
$+X_{14} + X_{24} = +5$
$+X_{15} + X_{25} = +5$
$X_{ij} \geq 0$ for all i and j

7. Refer to the equipment replacement problem discussed in section 5.3 of this chapter. In addition to the lease costs described for the problem, suppose that it costs Compu-Train \$2,000 extra in labor costs whenever they replace their existing computers with new ones. What effect does this have on the formulation and solution of the problem? Which of the two leasing contracts is optimal in this case?
8. Suppose the X's in the following table indicate locations where fire sprinkler heads need to be installed in an existing building. The S indicates the location of the water source to supply these sprinklers. Assume that pipe can be run only vertically or horizontally (not diagonally) between the water source and the sprinkler heads.

	1	**2**	**3**	**4**	**5**	**6**	**7**
1		X					
2	X	X	X		X	X	X
3	X		X		X	X	X
4	X	X	X			X	X
5	X	X	X		X		X
6							
7	X						
8				S			X

   a. Create a spanning tree showing how water can be brought to all the sprinkler heads using a minimal amount of pipe.
   b. Suppose that it takes 10 feet of pipe to connect each cell in the table to each adjacent cell. How much pipe does your solution require?
9. SunNet is a residential Internet Service Provider (ISP) in the central Florida area. Presently, the company operates one centralized facility that all of its clients call into for Internet access. To improve service, the company is planning to open three satellite offices in the cities of Pine Hills, Eustis, and Sanford. The company has identified five different regions to be serviced by these three offices. The following table summarizes the number of customers in each region, the service capacity at each

office, and the average monthly per customer cost of providing service to each region from each office. Table entries of "n.a." indicate infeasible region-to-service-center combinations. SunNet would like to determine how many customers from each region to assign to each service center to minimize cost.

Region	Pine Hills	Eustis	Sanford	Customers
1	$6.50	$7.50	n.a.	30,000
2	$7.00	$8.00	n.a.	40,000
3	$8.25	$7.25	$6.75	25,000
4	n.a.	$7.75	$7.00	35,000
5	n.a.	$7.50	$6.75	33,000
Capacity	60,000	70,000	40,000	

a. Draw a network flow model to represent this problem.
b. Implement your model in Excel and solve it.
c. What is the optimal solution?

10. Acme Manufacturing makes a variety of household appliances at a single manufacturing facility. The expected demand for one of these appliances during the next four months is shown in the following table along with the expected production costs and the expected capacity for producing these items.

	Month			
	1	2	3	4
Demand	420	580	310	540
Production Cost	$49.00	$45.00	$46.00	$47.00
Production Capacity	500	520	450	550

Acme estimates that it costs $1.50 per month for each unit of this appliance carried in inventory at the end of each month. Currently, Acme has 120 units in inventory on hand for this product. To maintain a level workforce, the company wants to produce at least 400 units per month. They also want to maintain a safety stock of at least 50 units per month. Acme wants to determine how many of each appliance to manufacture during each of the next four months to meet the expected demand at the lowest possible total cost.

a. Draw a network flow model for this problem.
b. Create a spreadsheet model for this problem and solve it using Solver.
c. What is the optimal solution?
d. How much money could Acme save if they were willing to drop the restriction about producing at least 400 units per month?

11. Sunrise Swimwear manufactures ladies' swimwear in January through June of each year that is sold through retail outlets in March through August. The following table summarizes the monthly production capacity and retail demand (in 1000s), and inventory carrying costs (per 1000).

			Carrying Cost	
Month	Capacity	Demand	First Month	Other Months
January	16	—	$110	$55
February	18	—	$110	$55
March	20	14	$120	$55
April	28	20	$135	$55
May	29	26	$150	$55
June	36	33	$155	$55
July	—	28	—	—
August	—	10	—	—

a. Draw a network flow representation of this problem.
b. Implement a spreadsheet model for this problem.
c. What is the optimal solution?

12. Jacobs Manufacturing produces a popular custom accessory for pickup trucks at plants in Huntington, West Virginia and Bakersfield, California, and ships them to distributors in Dallas, Texas; Chicago, Illinois; Denver, Colorado; and Atlanta, Georgia. The plants in Huntington and Bakersfield have, respectively, the capacity to produce 3,000 and 4,000 units per month. For the month of October, costs of shipping a carton of 10 units from each plant to each distributor are summarized in the following table:

	Shipping Cost per Container			
	Dallas	**Chicago**	**Denver**	**Atlanta**
Huntington	$19	$15	$14	$12
Bakersfield	$16	$18	$11	$13

Jacobs has been notified that these shipping rates will each increase by $1.50 on November 1. Each distributor has ordered 1,500 units of Jacobs' product for October and 2,000 units for November. In any month, Jacobs can send each distributor up to 500 units more than they have ordered if Jacobs provides a $2 per unit discount on the excess (which the distributor must hold in inventory from one month to the next). In October, the per unit cost of production in Huntington and Bakersfield are $12 and $16, respectively. In November, Jacobs expects the cost of production at both plants to be $14 per unit. The company wants to develop production and distribution plan for the months of October and November that would allow them to meet the expected demand from each distributor at the minimum cost.

a. Draw a network flow model for this problem.
b. Implement your model in a spreadsheet and solve it.
c. What is the optimal solution?

13. A construction company wants to determine the optimal replacement policy for the earth mover it owns. The company has a policy of not keeping an earth mover for more than five years, and has estimated the annual operating costs and trade-in values for earth movers during each of the five years they might be kept as:

	Age in Years				
	0–1	**1–2**	**2–3**	**3–4**	**4–5**
Operating Cost	$8,000	$9,100	$10,700	$9,200	$11,000
Trade-in Value	$14,000	$9,000	$6,000	$3,500	$2,000

Assume that new earth movers currently cost $25,000 and are increasing in cost by 4.5% per year. The company wants to determine when it should plan on replacing its current, 2-year-old earth mover. Use a 5-year planning horizon.

a. Draw the network representation of this problem.
b. Write out the LP formulation of this problem.
c. Solve the problem using Solver. Interpret your solution.

14. The Ortega Food Company needs to ship 100 cases of hot tamales from its warehouse in San Diego to a distributor in New York City at minimum cost. The costs associated with shipping 100 cases between various cities are:

	To					
From	**Los Angeles**	**Denver**	**St. Louis**	**Memphis**	**Chicago**	**New York**
San Diego	5	13	—	45	—	105
Los Angeles	—	27	19	50	—	95
Denver	—	—	14	30	32	—
St. Louis	—	14	—	35	24	—
Memphis	—	—	35	—	18	25
Chicago	—	—	24	18	—	17

a. Draw the network representation of this problem.
b. Write out the LP formulation of this problem.
c. Solve the problem using Solver. Interpret your solution.

15. A cotton grower in south Georgia produces cotton on farms in Statesboro and Brooklet, ships it to cotton gins in Claxton and Millen, where it is processed, and then sends it to distribution centers in Savannah, Perry, and Valdosta, where it is sold to customers for $60 per ton. Any surplus cotton is sold to a government warehouse in Hinesville for $25 per ton. The cost of growing and harvesting a ton of cotton at the farms in Statesboro and Brooklet is $20 and $22, respectively. There are presently 700 and 500 tons of cotton available in Statesboro and Brooklet, respectively. The cost of transporting the cotton from the farms to the gins and the government warehouse is shown in the following table:

	Claxton	**Millen**	**Hinesville**
Statesboro	$4.00	$3.00	$4.50
Brooklet	$3.5	$3.00	$3.50

The gin in Claxton has the capacity to process 700 tons of cotton at a cost of $10 per ton. The gin in Millen can process 600 tons at a cost of $11 per ton. Each gin must use at least one half of its available capacity. The cost of shipping a ton of cotton from each gin to each distribution center is summarized in the following table:

	Savannah	**Perry**	**Valdosta**
Claxton	$10	$16	$15
Millen	$12	$18	$17

Assume that the demand for cotton in Savannah, Perry, and Valdosta is 400, 300, and 450 tons, respectively.
a. Draw a network flow model to represent this problem.
b. Implement your model in Excel and solve it.
c. What is the optimal solution?

16. The blood bank wants to determine the least expensive way to transport available blood donations from Pittsburg and Staunton to hospitals in Charleston, Roanoke, Richmond, Norfolk, and Suffolk. The supply and demand for donated blood is shown in Figure 5.34 along with the unit cost of shipping along each possible arc.
a. Create a spreadsheet model for this problem.
b. What is the optimal solution?
c. Suppose that no more than 1000 units of blood can be transported over any one arc. What is the optimal solution to this revised problem?

17. A furniture manufacturer has warehouses in cities represented by nodes 1, 2, and 3 in Figure 5.35. The values on the arcs indicate the per unit shipping costs required to transport living room suites between the various cities. The supply of living room

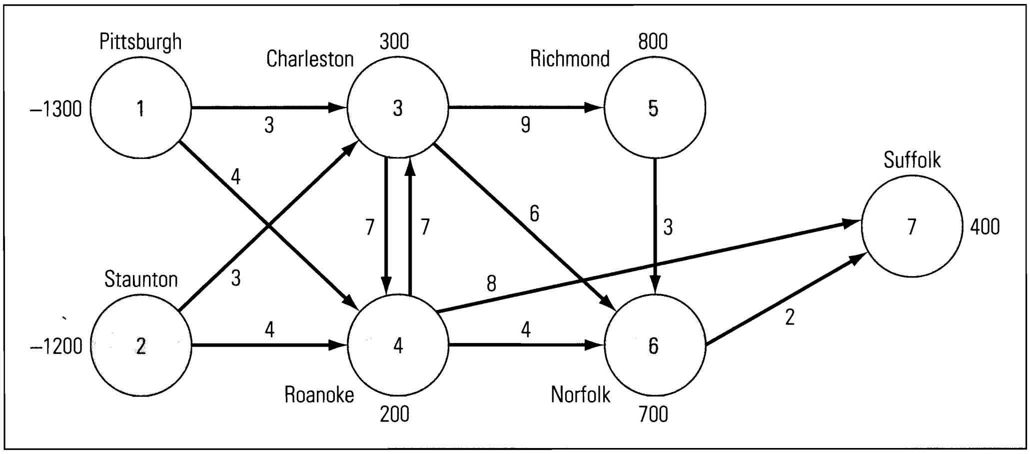


FIGURE 5.34

Network flow model for the blood bank problem

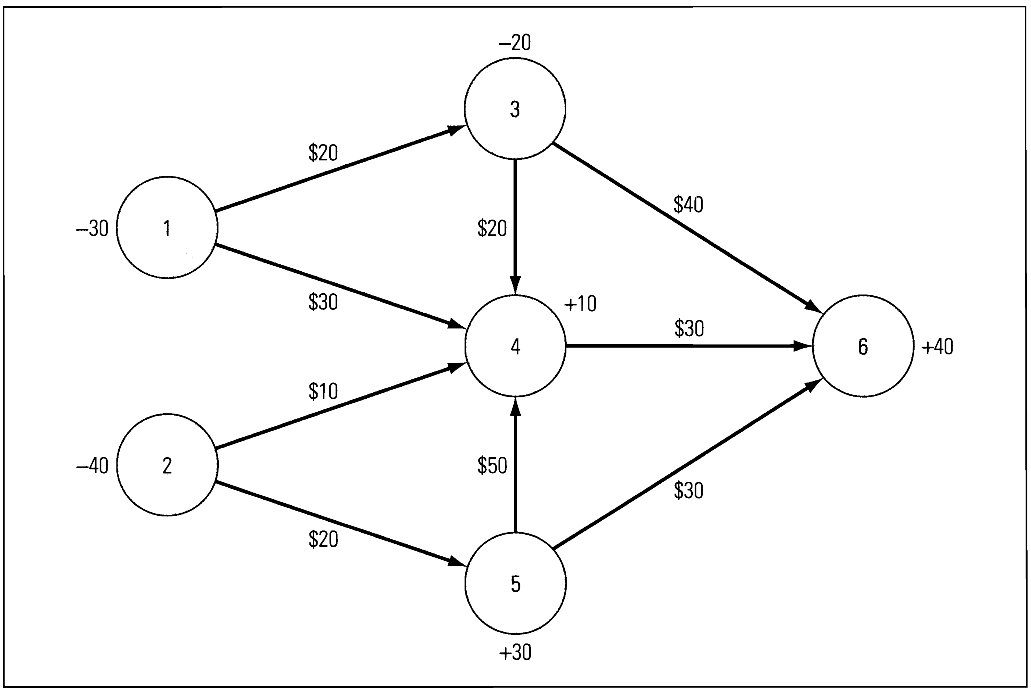


FIGURE 5.35

Network flow model for the furniture manufacturing problem

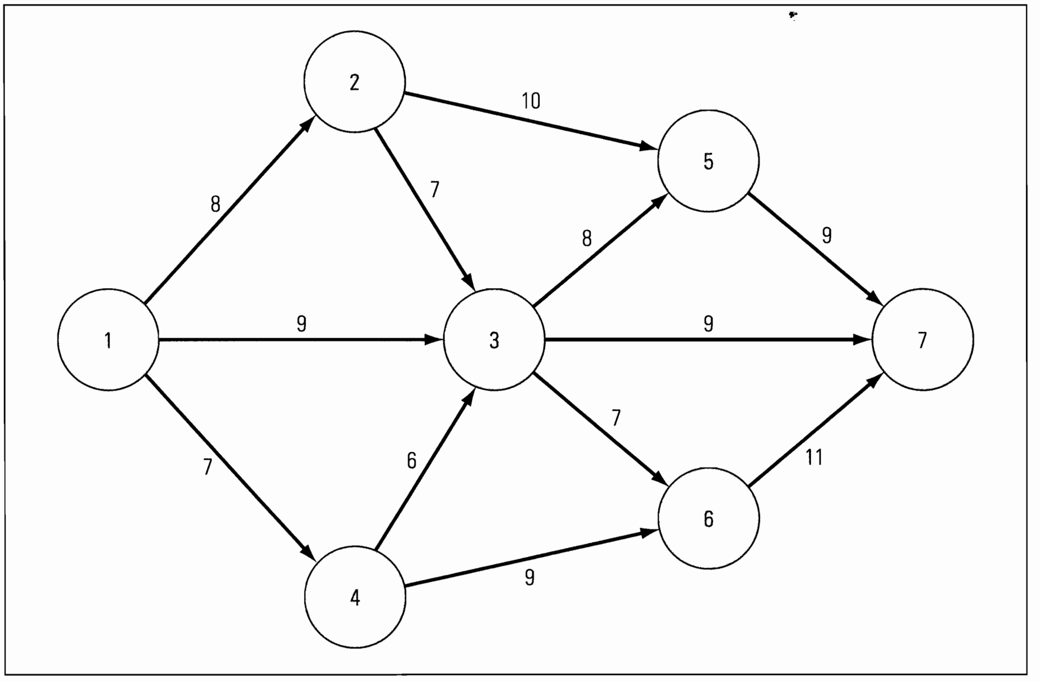
suites at each warehouse is indicated by the negative number next to nodes 1, 2, and 3. The demand for living room suites is indicated by the positive number next to the remaining nodes.

a. Identify the supply, demand, and transshipment nodes in this problem.
b. Use Solver to determine the least costly shipping plan for this problem.

18. The graph in Figure 5.36 represents various flows that can occur through a sewage treatment plant with the numbers on the arcs representing the maximum flow (in tons of sewage per hour) that can be accommodated. Formulate an LP model to determine the maximum tons of sewage per hour that can be processed by this plant.
19. A company has three warehouses that supply four stores with a given product. Each warehouse has 30 units of the product. Stores 1, 2, 3, and 4 require 20, 25, 30,

FIGURE 5.36

Network flow model for the sewage treatment plant



and 35 units of the product, respectively. The per unit shipping costs from each warehouse to each store are:

	Store			
Warehouse	**1**	**2**	**3**	**4**
1	5	4	6	5
2	3	6	4	4
3	4	3	3	2

a. Draw the network representation of this problem. What kind of problem is this?
b. Formulate an LP model to determine the least expensive shipping plan to fill the demands at the stores.
c. Solve the problem using Solver.
d. Suppose that shipments are not allowed between warehouse 1 and store 2 or between warehouse 2 and store 3. What is the easiest way to modify the spreadsheet so that you can solve this modified problem? What is the optimal solution to the modified problem?

20. A used-car broker needs to transport his inventory of cars from locations 1 and 2 in Figure 5.37 to used-car auctions being held at locations 4 and 5. The costs of transporting cars along each of the routes are indicated on the arcs. The trucks used to carry the cars can hold a maximum of 10 cars. Therefore, the maximum number of cars that can flow over any arc is 10.
a. Formulate an LP model to determine the least costly method of distributing the cars from locations 1 and 2 so that 20 cars will be available for sale at location 4, and 10 cars will be available for sale at location 5.
b. Use Solver to find the optimal solution to this problem.

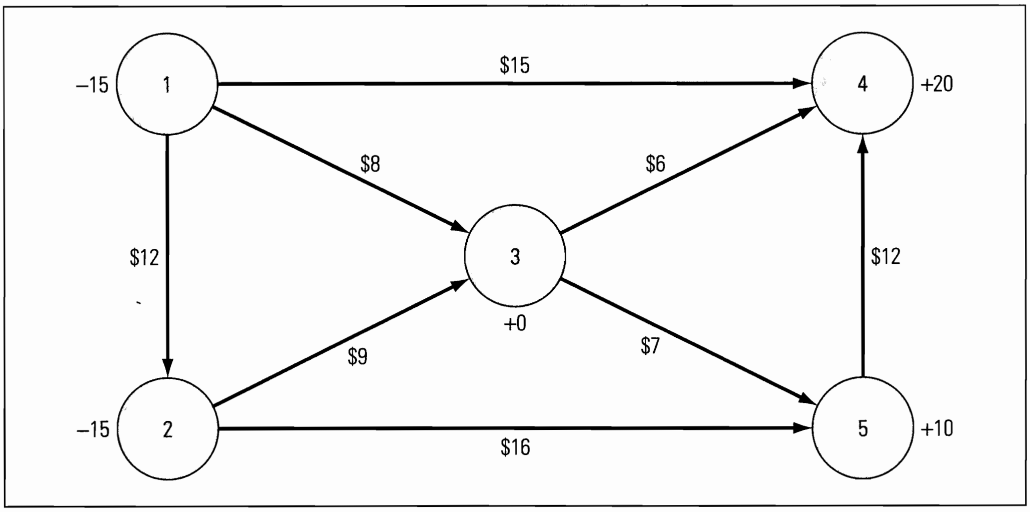


FIGURE 5.37

Network flow model for the used car problem

21. An information systems consultant who lives in Dallas must spend the majority of the month of March onsite with a client in San Diego. Her travel schedule for the month is as follows:

Leave Dallas	Leave San Diego
Monday, March 2	Friday, March 6
Monday, March 9	Thursday, March 12
Tuesday, March 17	Friday, March 20
Monday, March 23	Wednesday, March 25

The usual round-trip ticket price between Dallas and San Diego is $750. However, the airline offers a 25% discount if the dates on a round-trip ticket cover less than 7 nights and include a weekend. A 35% discount is offered for round-trip tickets covering 10 or more nights, and a 45% discount is available for round-trip tickets covering 20 or more nights. The consultant can purchase four round-trip tickets in any manner that allows her to leave Dallas and San Diego on the days indicated.

a. Draw a network flow model for this problem.
b. Implement the problem in a spreadsheet and solve it.
c. What is the optimal solution? How much does this save for four full-cost round-trip tickets?

22. The Conch Oil Company needs to transport 30 million barrels of crude oil from a port in Doha, Qatar in the Persian Gulf to three refineries throughout Europe. The refineries are in Rotterdam, Netherlands; Toulon, France; and Palermo, Italy, and require 6 million, 15 million, and 9 million barrels, respectively. The oil can be transported to the refineries in three different ways. First, oil may be shipped from Qatar to Rotterdam, Toulon, and Palermo on supertankers traveling around Africa at costs of $1.20, $1.40, and $1.35 per barrel, respectively. Conch is contractually obligated to send at least 25% of its oil via these supertankers. Alternatively, oil can be shipped from Doha to Suez, Egypt, at a cost of $0.35 per barrel, then through the Suez Canal to Port Said at a cost of $0.20 per barrel, then from Port

Said to Rotterdam, Toulon, and Palermo at per barrel costs of \$0.27, \$0.23, and \$0.19, respectively. Finally, up to 15 million barrels of the oil shipped from Doha to Suez can then be sent via pipeline Damietta, Egypt, at \$0.16 per barrel. From Damietta, it can shipped to Rotterdam, Toulon, and Palermo at costs of \$0.25, \$0.20, and \$0.15, respectively.

a. Draw a network flow model for this problem.
b. Implement your model in a spreadsheet and solve it.
c. What is the optimal solution?

23. Omega Airlines has several nonstop flights between Atlanta and Los Angeles every day. The schedules of these flights are shown in the following table.

Flight	Departs Atlanta	Arrives in L.A.	Flight	Departs L.A.	Arrive in Atlanta
1	6 am	8 am	1	5 am	9 am
2	8 am	10 am	2	6 am	10 am
3	10 am	Noon	3	9 am	1 pm
4	Noon	2 pm	4	Noon	4 pm
5	4 pm	6 pm	5	2 pm	6 pm
6	6 pm	8 pm	6	5 pm	9 pm
7	7 pm	9 pm	7	7 pm	11 pm

Omega wants to determine the optimal way of assigning flight crews to these different flights. The company wants to ensure that the crews always return to the city from which they left each day. FAA regulations require at least one hour of rest for flight crews between flights. However, flight crews become irritated if they are forced to wait for extremely long periods of time between flights, so Omega wants to find an assignment of flight schedules that minimizes these waiting periods.

a. Draw a network flow model for this problem.
b. Implement the problem in a spreadsheet and solve it.
c. What is the optimal solution? What is the longest period of time that a flight crew has to wait between flights, according to your solution?
d. Are there alternate optimal solutions to this problem? If so, do any alternate optimal solutions result in a smaller maximum waiting period between flights?

24. A residential moving company needs to move a family from city 1 to city 12 in Figure 5.38 where the numbers on the arcs represents the driving distance in miles between cities.

a. Create a spreadsheet model for this problem.
b. What is the optimal solution?
c. Suppose that the moving company gets paid by the mile and, as a result, wants to determine the longest path from city 1 to city 12. What is the optimal solution?
d. Now suppose that travel is permissible in either direction between cities 6 and 9. Describe the optimal solution to this problem.

25. Joe Jones wants to establish a construction fund (or sinking fund) to pay for a new bowling alley he is having built. Construction of the bowling alley is expected to take six months and cost \$300,000. Joe's contract with the construction company requires him to make payments of \$50,000 at the end of the second and fourth months, and a final payment of \$200,000 at the end of the sixth month when the

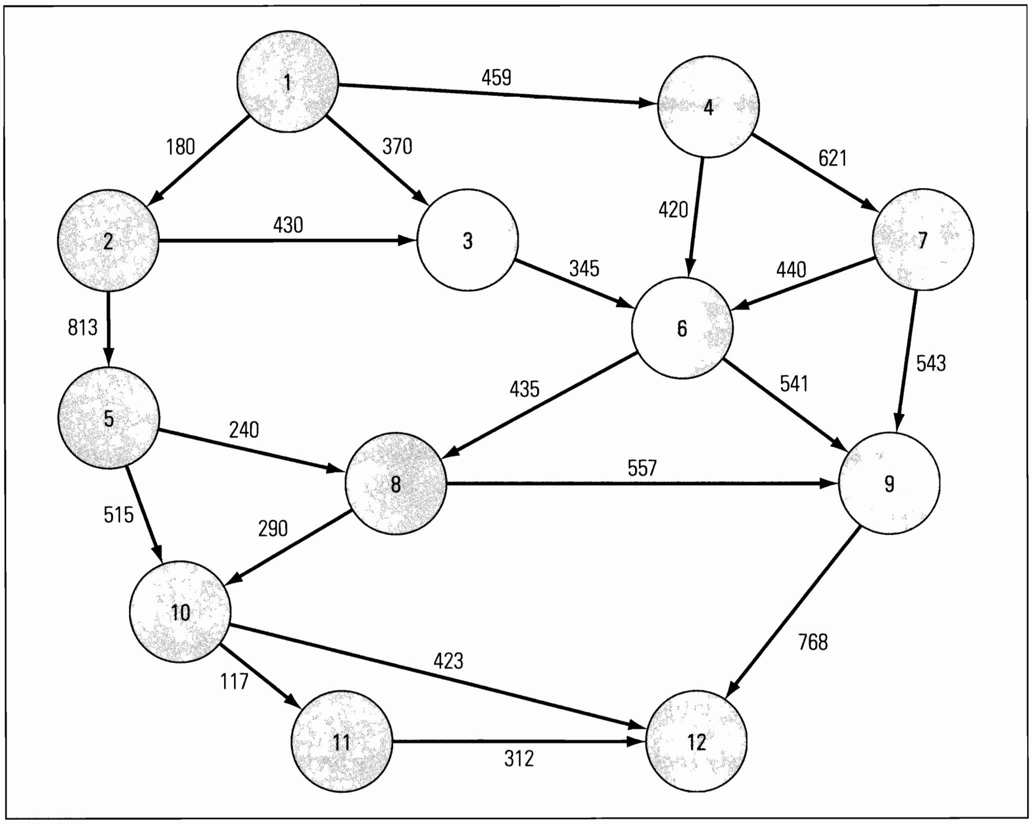


FIGURE 5.38

Network flow model for the moving company problem

bowling alley is completed. Joe has identified four investments that he can use to establish the construction fund; these investments are summarized in the following table:

Investment	Available in Month	Months to Maturity	Yield at Maturity
A	1, 2, 3, 4, 5, 6	1	1.2%
B	1, 3, 5	2	3.5%
C	1, 4	3	5.8%
D	1	6	11.0%

The table indicates that investment A will be available at the beginning of each of the next six months, and funds invested in this manner mature in one month with a yield of 1.2%. Similarly, funds can be placed in investment C only at the beginning of months 1 and/or 4, and mature at the end of three months with a yield of 5.8%. Joe wants to minimize the amount of money he must invest in month 1 to meet the required payments for this project.

a. Draw a network flow model for this problem.
b. Create a spreadsheet model for this problem and solve it.
c. What is the optimal solution?

26. Telephone calls for the YakLine, a discount long distance carrier, are routed through a variety of switching devices that interconnect various network hubs in different

cities. The maximum number of calls that can be handled by each segment of their network is shown in the following table:

Network Segments	Calls (in 1,000s)
Washington, DC to Chicago	800
Washington, DC to Kansas City	650
Washington, DC to Dallas	700
Chicago to Dallas	725
Chicago to Denver	700
Kansas City to Denver	750
Kansas City to Dallas	625
Denver to San Francisco	900
Dallas to San Francisco	725

YakLine wants to determine the maximum number of calls that can go from their east coast operations hub in Washington, DC to their west coast operations hub in San Francisco.

a. Draw a network flow model for this problem.
b. Create a spreadsheet model for this problem and solve it.
c. What is the optimal solution?

27. Union Express has 60 tons of cargo that needs to be shipped from Boston to Dallas. The shipping capacity on each of the routes Union Express planes fly each night is shown in the following table:

Nightly Flight Segments	Capacity (in tons)
Boston to Baltimore	30
Boston to Pittsburgh	25
Boston to Cincinnati	35
Baltimore to Atlanta	10
Baltimore to Cincinnati	5
Pittsburgh to Atlanta	15
Pittsburgh to Chicago	20
Cincinnati to Chicago	15
Cincinnati to Memphis	5
Atlanta to Memphis	25
Atlanta to Dallas	10
Chicago to Memphis	20
Chicago to Dallas	15
Memphis to Dallas	30
Memphis to Chicago	15

Will Union Express be able to move all 60 tons from Boston to Dallas in one night?

a. Draw a network flow model for this problem.
b. Create a spreadsheet model for the problem and solve it.
c. What is the maximum flow for this network?

28. E-mail messages sent over the Internet are broken up into electronic packets that may take a variety of different paths to reach their destination where the original message is reassembled. Suppose that the nodes in the graph shown in Figure 5.39 represent a series of computer hubs on the Internet and the arcs represent connections between them. Suppose that the values on the arcs represent the number of packets per minute (in 1,000,000s) that can be transmitted over each arc.

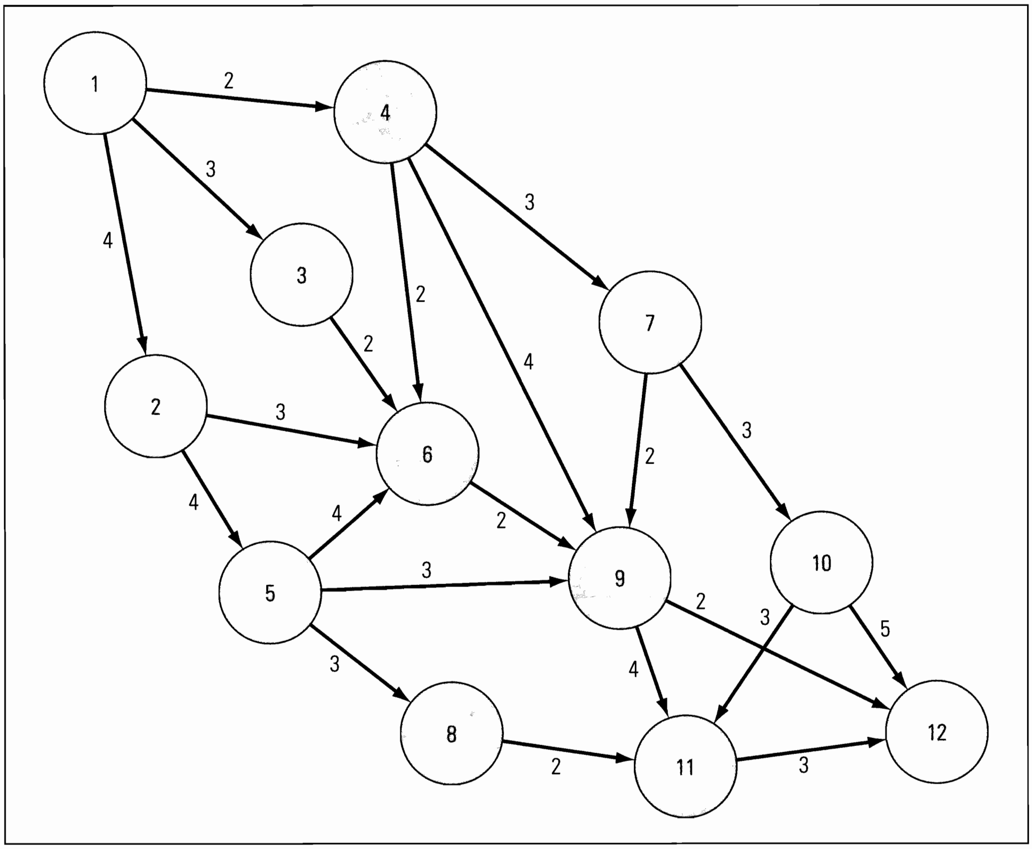


FIGURE 5.39

Network hubs and interconnections for the e-mail problem

a. Implement a network flow model to determine the maximum number of packets that can flow from node 1 to node 12 in one minute.
b. What is the maximum flow?

29. The Britts & Straggon company manufactures small engines at three different plants. From the plants, the engines are transported to two different warehouse facilities before being distributed to three wholesale distributors. The per unit manufacturing cost at each plant is shown in the following table in addition to the minimum required and maximum available daily production capacities.

Plant	Manufacturing Cost	Minimum Required Production	Maximum Production Capacity
1	$13	150	400
2	$15	150	300
3	$12	150	600

The unit cost of transporting engines from each plant to each warehouse is shown below.

Plant	Warehouse 1	Warehouse 2
1	$4	$5
2	$6	$4
3	$3	$5

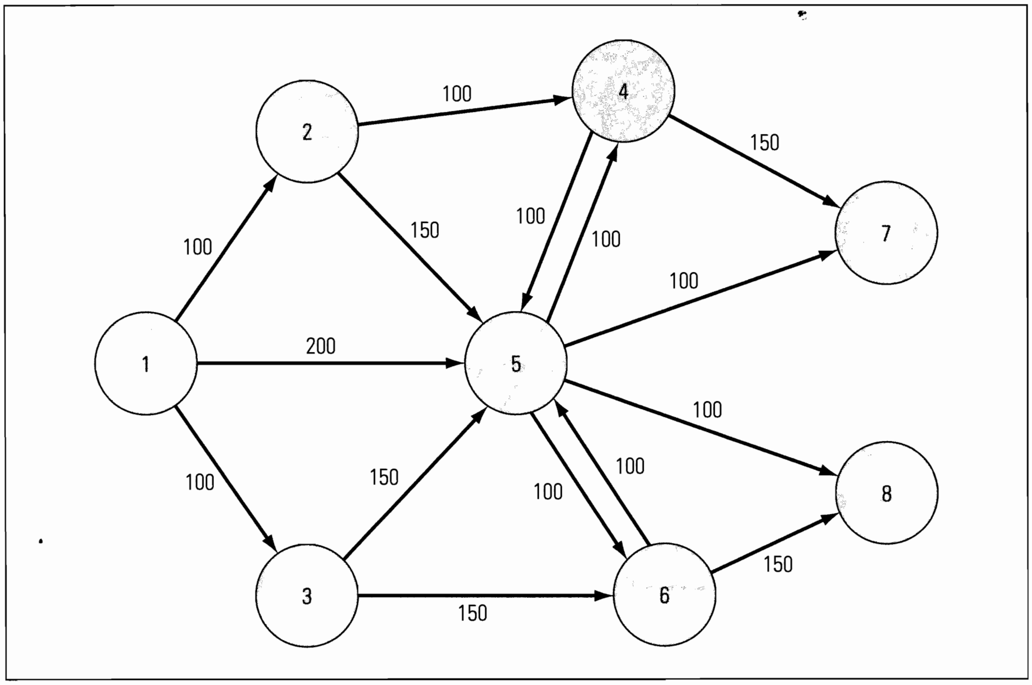


FIGURE 5.40

Network flow model for the airport terminal problem

The unit cost of shipping engines from each warehouse to each distributor is shown in the following table along with the daily demand for each distributor.

Warehouse	Distributor 1	Distributor 2	Distributor 3
1	$6	$4	$3
2	$3	$5	$2
Demand	300	600	100

Each warehouse can process up to 500 engines per day.

a. Draw a network flow model to represent this problem.
b. Implement your model in Excel and solve it.
c. What is the optimal solution?

30. A new airport being built will have three terminals and two baggage pickup areas. An automated baggage delivery system has been designed to transport the baggage from each terminal to the two baggage pickup areas. This system is depicted graphically in Figure 5.40, where nodes 1, 2, and 3 represent the terminals, and nodes 7 and 8 represent the baggage pickup areas. The maximum number of bags per minute that can be handled by each part of the system is indicated by the value on each arc in the network.
    a. Formulate an LP model to determine the maximum number of bags per minute that can be delivered by this system.
    b. Use Solver to find the optimal solution to this problem.
31. Bull Dog Express runs a small airline that offers commuter flights between several cities in Georgia. The airline flies into and out of small airports only. These airports have limits on the number of flights Bull Dog Express can make each day. The airline can make five round-trip flights daily between Savannah and Macon, four round-trip flights daily between Macon and Albany, two round-trip flights daily between

Macon and Atlanta, two round-trip flights daily between Macon and Athens, two round-trip flights daily between Athens and Atlanta, and two round-trip flights daily from Albany to Atlanta. The airline wants to determine the maximum number of times connecting flights from Savannah to Atlanta can be offered in a single day.

a. Draw the network representation of this problem.
b. Formulate an LP model for this problem. What kind of problem is this?
c. Use Solver to determine the optimal solution to this problem.

32. The U.S. Department of Transportation (DOT) is planning to build a new interstate to run from Detroit, Michigan, to Charleston, South Carolina. Several different routes have been proposed. They are summarized in Figure 5.41, where node 1 represents Detroit and node 12 represents Charleston. The numbers on the arcs indicate the estimated construction costs of the various links (in millions of dollars). It is estimated that all of the routes will require approximately the same total driving time to make the trip from Detroit to Charleston. Thus, the DOT is interested in identifying the least costly alternative.
    a. Formulate an LP model to determine the least costly construction plan.
    b. Use Solver to determine the optimal solution to this problem.
33. A building contractor is designing the ductwork for the heating and air conditioning system in a new, single-story medical building. Figure 5.42 summarizes the possible connections between the primary air handling unit (node 1) and the various air outlets to be placed in the building (nodes 2 through 9). The arcs in the network represent possible ductwork connections, and the values on the arcs represent the feet of ductwork required.

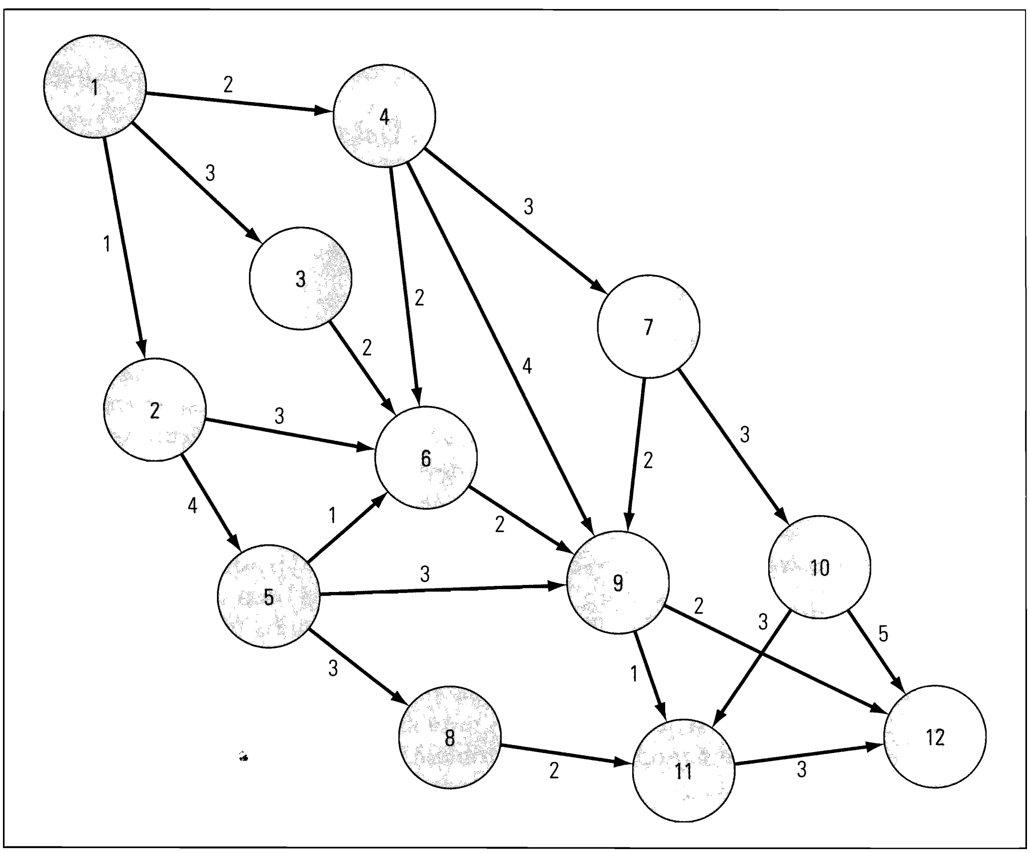
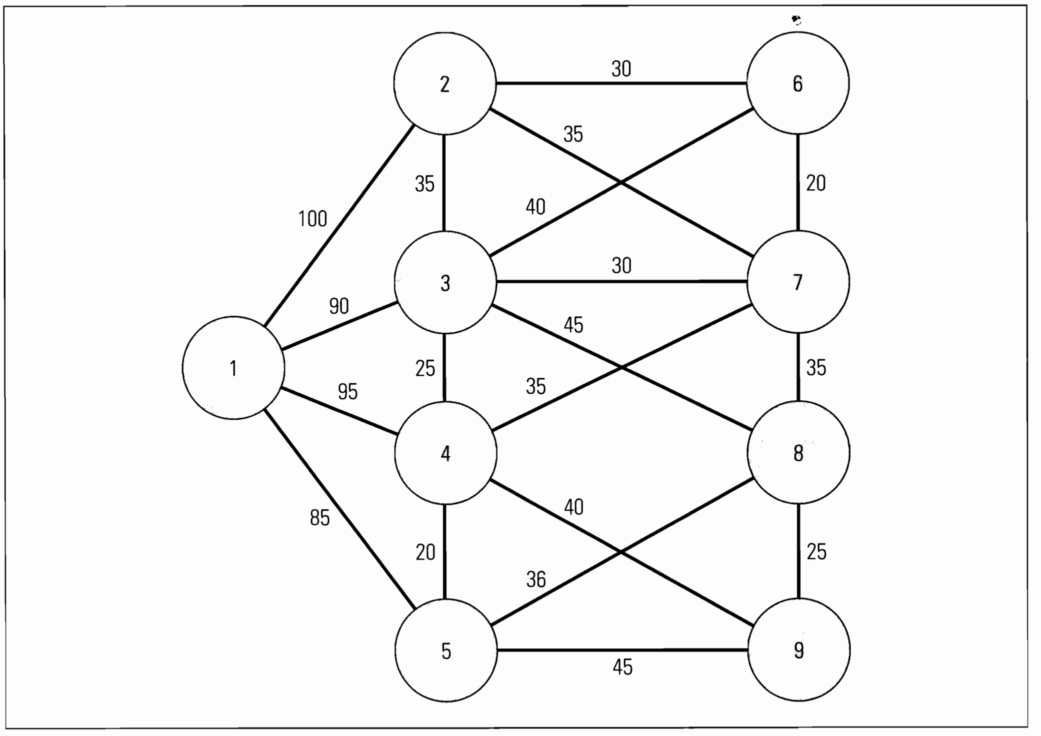


FIGURE 5.41

Possible routes for the interstate construction problem

FIGURE 5.42

Network representation of the ductwork problem



Starting at node 1, use the minimal spanning tree algorithm to determine how much ductwork should be installed to provide air access to each vent while requiring the least amount of ductwork.

34. The manager of catering services for the Roanoker Hotel has a problem. The banquet hall at the hotel is booked each evening during the coming week for groups who have reserved the following numbers of tables:

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Tables Reserved	400	300	250	400	350

The hotel has 500 tablecloths that can be used for these banquets. However, the tablecloths used at each banquet will have to be cleaned before they can be used again. A local cleaning service will pick up the soiled tablecloths each evening after the banquet. It offers overnight cleaning for \$2 per tablecloth, or 2-day service for \$1 per tablecloth (*i.e.*, a tablecloth picked up Monday night can be ready Tuesday for \$2 or ready for use Wednesday for \$1). There are no tablecloth losses and all tablecloths must be cleaned. Due to the cleaner's capacity restrictions, the overnight service can be performed on up to only 250 tablecloths, and overnight service is not available on tablecloths picked up Friday night. All cloths used on Friday must be ready for use again by Monday. The hotel wants to determine the least costly plan for having its tablecloths cleaned.

a. Draw a network flow model for this problem. (*Hint:* Express the supplies and demands as minimum required and maximum allowable flows over selected arcs.)
b. Create a spreadsheet model for this problem and solve it. What is the optimal solution?

Hamilton & Jacobs

CASE 5.1

Hamilton & Jacobs (H&J) is a global investment company, providing start-up capital to promising new ventures around the world. Due to the nature of its business, H&J holds funds in a variety of countries and converts between currencies as needs arise in different parts of the world. Several months ago, the company moved $16 million into Japanese yen (JPY) when one U.S. dollar (USD) was worth 75 yen. Since that time, the value of the dollar has fallen sharply, where it now requires almost 110 yen to purchase one dollar.

Besides its holdings of yen, H&J also currently owns 6 million European EUROs and 30 million Swiss Francs (CHF). H&J's chief economic forecaster is predicting that all of the currencies it is presently holding will continue to gain strength against the dollar for the rest of the year. As a result, the company would like to convert all its surplus currency holdings back to U.S. dollars until the economic picture improves.

The bank H&J uses for currency conversions charges different transaction fees for converting between various currencies. The following table summarizes the transaction fees (expressed as a percentage of the amount converted) for US dollars (USD), Australian dollars (AUD), British pounds (GBP), European Euros (EURO), Indian Rupees (INR), Japanese yen (JPY), Singapore dollars (SGD), and Swiss Francs (CHF).

Transaction Fee Table

FROM/TO	USD	AUD	GBP	EUR	INR	JPY	SGD	CHF
USD	—	0.10%	0.50%	0.40%	0.40%	0.40%	0.25%	0.50%
AUD	0.10%	—	0.70%	0.50%	0.30%	0.30%	0.75%	0.75%
GBP	0.50%	0.70%	—	0.70%	0.70%	0.40%	0.45%	0.50%
EUR	0.40%	0.50%	0.70%	—	0.05%	0.10%	0.10%	0.10%
INR	0.40%	0.30%	0.70%	0.05%	—	0.20%	0.10%	0.10%
JPY	0.40%	0.30%	0.40%	0.10%	0.20%	—	0.05%	0.50%
SGD	0.25%	0.75%	0.45%	0.10%	0.10%	0.05%	—	0.50%
CHF	0.50%	0.75%	0.50%	0.10%	0.10%	0.50%	0.50%	—

Because it costs differing amounts to convert between various currencies, H&J determined that converting existing holdings directly into US dollars might not be the best strategy. Instead, it might be less expensive to convert existing holdings to an intermediate currency before converting the result back to US dollars. The following table summarizes the current exchange rates for converting from one currency to another.

Exchange Rate Table

From/To	USD	AUD	GBP	EUR	INR	JPY	SGD	CHF
USD	1	1.29249	0.55337	0.80425	43.5000	109.920	1.64790	1.24870
AUD	0.77370	1	0.42815	0.62225	33.6560	85.0451	1.27498	0.96612
GBP	1.80710	2.33566	1	1.45335	78.6088	198.636	2.97792	2.25652
EUR	1.24340	1.60708	0.68806	1	54.0879	136.675	2.04900	1.55263
INR	0.02299	0.02971	0.01272	0.01849	1	2.5269	0.03788	0.02871
JPY	0.00910	0.01176	0.00503	0.00732	0.39574	1	0.01499	0.01136
SGD	0.60683	0.78433	0.33581	0.48804	26.3972	66.7031	1	0.75775
CHF	0.80083	1.03507	0.44316	0.64407	34.8362	88.0275	1.31969	1

The exchange rate table indicates, for instance, that one Japanese yen can be converted into 0.00910 US dollars. So 100,000 yen would produce $910 US. However, the bank's 0.40% fee for this transaction would reduce the net amount received to

$910 × (1− 0.004) = $906.36. So H&J wants your assistance in determining the best way to convert all its non-US currency holdings back into US dollars.

a. Draw a network flow diagram for this problem.
b. Create a spreadsheet model for this problem and solve it.
c. What is the optimal solution?
d. If H&J converted each non-US currency it owns directly into US dollars, how many US dollars would it have?
e. Suppose that H&J wants to perform the same conversion but also leave $5 million in Australian dollars. What is the optimal solution in this case?

CASE 5.2 Old Dominion Energy

The United States is the biggest consumer of natural gas, and the second largest natural gas producer in the world. According to the U.S. Energy Information Administration (EIA), in 2001 the U.S. consumed 22.7 trillion cubic feet of natural gas. Stemming from phased deregulation, the transportation and delivery of natural gas from wellheads has grown since the '80s and there are now more than 278,000 miles of gas pipeline nationwide (see: http://www.platts.com/features/usgasguide/pipelines.shtml). With more electric power companies turning to natural gas as a cleaner-burning fuel, natural gas is expected to grow even more quickly over the next 20 years.

To ensure an adequate supply of natural gas, gas storage facilities have been built in numerous places along the pipeline. Energy companies can buy gas when prices are low and store it in these facilities for use or sale at a later date. Because energy consumption is influenced greatly by the weather (which is not entirely predictable), imbalances often arise in the supply and demand for gas in different parts of the country. Gas traders constantly monitor these market conditions and look for opportunities to sell gas from storage facilities when the price offered at a certain location is high enough. This decision is complicated because it costs different amounts of money to transport gas through different segments of the nationwide pipeline, and the capacity available in different parts of the pipeline is changing constantly. Thus, when traders see an opportunity to sell at a favorable price, they must quickly see how much capacity is available in the network and create deals with individual pipeline operators for the necessary capacity to move gas from storage to the buyer.

Bruce McDaniel is a gas trader for Old Dominion Energy (ODE), Inc. The network in Figure 5.43 represents a portion of the gas pipeline where ODE does business. The values next to each arc in this network are of the form (x,y) where x is the cost per thousand cubic feet (cf) of transporting gas along the arc, and y is the available transmission capacity of the arc in thousands of cubic feet. Note that the arcs in this network are bidirectional (i.e., gas can flow in either direction at the prices and capacities listed).

Bruce currently has 100,000 cf of gas in storage at Katy. Industrial customers in Joliet are offering $4.35 per thousand cf for up to 35,000 cf of gas. Buyers in Leidy are offering $4.63 per thousand cf for up to 60,000 cf of gas. Create a spreadsheet model to help Bruce answer the following questions.

a. Given the available capacity in the network, how much gas can be shipped from Katy to Leidy? From Katy to Joliet?
b. How much gas should Bruce offer to sell to Joliet and Leidy if he wants to maximize profits?
c. Is Bruce able to meet all the demand from both customers? If not, why not?
d. If Bruce wanted to try to pay more to obtain additional capacity on some of the pipelines, which ones should he investigate and why?

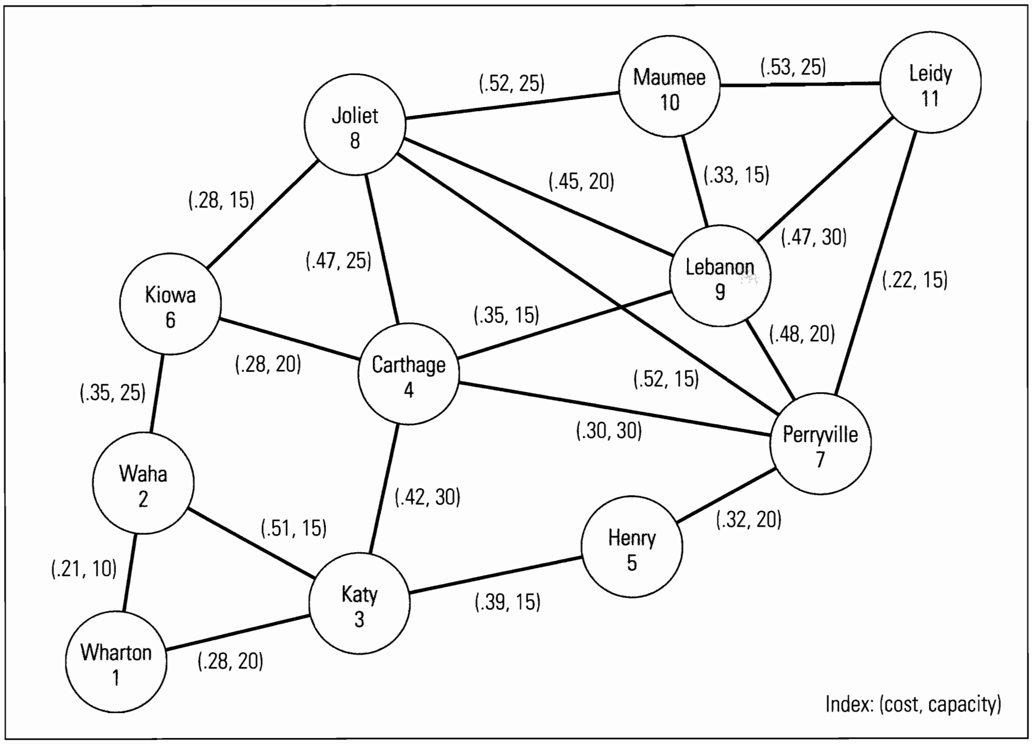


FIGURE 5.43

Gas pipeline network for Old Dominion Energy

US Express

CASE 5.3

US Express is an overnight package delivery company based in Atlanta, Georgia. Jet fuel is one of the largest operating costs incurred by the company and they want your assistance in managing this cost. The price of jet fuel varies considerably at different airports around the country. As a result, it seems that it might be wise to "fill up" on jet fuel at airports where it is least expensive. However, the amount of fuel an airliner burns depends, in part, on the weight of the plane—and excess fuel makes an airplane heavier and, therefore, less fuel-efficient. Similarly, more fuel is burned on flights from the east coast to the west coast (going against the jet stream) than from the west coast to the east coast (going with the jet stream).

The following table summarizes the flight schedule (or rotation) flown nightly by one of the company's planes. For each flight segment, the table summarizes the minimum required and maximum allowable amount of fuel on board at takeoff and the cost of fuel at each point of departure. The final column provides a linear function relating fuel consumption to the amount of fuel on board at takeoff.

Segment	Depart	Arrive	Minimum Fuel Level at Takeoff (in 1000s)	Maximum Fuel Level at Takeoff (in 1000s)	Cost per Gallon	Fuel used in Flight with G Gallons (in 1000s) on Board at Takeoff
1	Atlanta	San Francisco	21	31	$0.92	$3.20 + 0.45 \times G$
2	San Francisco	Los Angeles	7	20	$0.85	$2.25 + 0.65 \times G$
3	Los Angeles	Chicago	18	31	$0.87	$1.80 + 0.35 \times G$
4	Chicago	Atlanta	16	31	$1.02	$2.20 + 0.60 \times G$

For instance, if the plane leaves Atlanta for San Francisco with 25,000 gallons on board, it should arrive in San Francisco with approximately $25 - (3.2 + 0.45 \times 25) = 10.55$ thousand gallons of fuel.

The company has many other planes that fly different schedules each night, so the potential cost savings from efficient fuel purchasing is quite significant. But before turning you loose on all of their flight schedules, the company wants you to create a spreadsheet model to determine the most economical fuel purchasing plan for the previous schedule. (*Hint:* Keep in mind that the most fuel you would purchase at any departure point is the maximum allowable fuel level for takeoff at that point. Also, assume that whatever fuel is on board when the plane returns to Atlanta at the end of the rotation will still be on board when the plane leaves Atlanta the next evening.)

a. Draw the network diagram for this problem.
b. Implement the model for this problem in your spreadsheet and solve it.
c. How much fuel should US Express purchase at each departure point and what is the cost of this purchasing plan?

CASE 5.4 The Major Electric Corporation

Henry Lee is the Vice President of Purchasing for the consumer electronics division of the Major Electric Corporation (MEC). The company recently introduced a new type of video camcorder that has taken the market by storm. Although Henry is pleased with the strong demand for this product in the marketplace, it has been a challenge to keep up with MEC's distributors' orders of this camcorder. His current challenge is how to meet requests from MEC's major distributors in Pittsburgh, Denver, Baltimore, and Houston who have placed orders of 10,000, 20,000, 30,000, and 25,000 units, respectively, for delivery in two months (there is a one-month manufacturing and one-month shipping lead time for this product).

MEC has contracts with companies in Hong Kong, Korea, and Singapore who manufacture camcorders for the company under the MEC label. These contracts require MEC to order a specified minimum number of units each month at a guaranteed per unit cost. The contracts also specify the maximum number of units that may be ordered at this price. The following table summarizes these contracts:

	Monthly Purchasing Contract Provisions		
Supplier	**Unit Cost**	**Minimum Required**	**Maximum Allowed**
Hong Kong	$375	20,000	30,000
Korea	$390	25,000	40,000
Singapore	$365	15,000	30,000

MEC also has a standing contract with a shipping company to transport product from each of these suppliers to ports in San Francisco and San Diego. The cost of shipping from each supplier to each port is given in the following table along with the minimum required and maximum allowable number of shipping cartons each month:

	Monthly Shipping Contract Provisions					
	San Francisco Requirements			San Diego Shipping Requirements		
Supplier	**Cost per Container**	**Minimum Containers**	**Maximum Containers**	**Cost per Container**	**Minimum Containers**	**Maximum Containers**
Hong Kong	$2,000	5	20	$2,300	5	20
Korea	$1,800	10	30	$2,100	10	30
Singapore	$2,400	5	25	$2,200	5	15

Under the terms of this contract, MEC guarantees it will send at least 20 but no more than 65 shipping containers to San Francisco each month, and at least 30 but no more than 70 shipping containers to San Diego each month.

Each shipping container can hold 1,000 video cameras and ultimately will be trucked from the seaports on to the distributors. Again, MEC has a standing contract with a trucking company to provide trucking services each month. The cost of trucking a shipping container from each port to each distributor is summarized in the following table.

Unit Shipping Cost per Container

	Pittsburgh	Denver	Baltimore	Houston
San Francisco	$1,100	$850	$1,200	$1,000
San Diego	$1,200	$1,000	$1,100	$900

As with the other contracts, to obtain the prices given above, MEC is required to use a certain minimum amount of trucking capacity on each route each month and may not exceed certain maximum shipping amounts without incurring cost penalties. These minimum and maximum shipping restrictions are summarized in the following table.

Minimum Required and Maximum Allowable Number of Shipping Containers per Month

	Pittsburgh		Denver		Baltimore		Houston	
	Min	Max	Min	Max	Min	Max	Min	Max
San Francisco	3	7	6	12	10	18	5	15
San Diego	4	6	5	14	5	20	10	20

Henry is left with the task of sorting through all this information to determine the least purchasing and distribution plan to fill the distributor's requests. But because he and his wife have tickets to the symphony for this evening, he has asked you to take a look at this problem and give him your recommendations at 9:00 tomorrow morning.

a. Create a network flow model for this problem. (*Hint:* Consider inserting intermediate nodes in your network to assist in meeting the minimum monthly purchase restrictions for each supplier and the minimum monthly shipping requirements for each port.)
b. Implement a spreadsheet model for this problem and solve it.
c. What is the optimal solution?