`matlab` language skills and debugging

Aarthi Sridhar, Ph.D., Jenni Rinker, Ph.D., Kim Bourne, Ph.D., and Henri P. Gavin, Ph.D.

January 28, 2025

1 Orientation

Matlab and its open-source derivative Octave provide a programming language and computing environments that are efficient at calculations involving vectors and matrices. As a *computing environment*, Matlab / Octave allow users to interactively carry out calculations by manually entering code or calculations in the Command window. The Command window looks like a terminal window with a `>>` prompt. As a *programming language*, `matlab` allows users to write algorithms into scripts or functions saved as `.m`-files. The Workspace window conveniently displays the dimensions and values of variables available within the Command window.

Code in the `matlab` language *looks* like matrix equations. For example, if `A` and `B` are matrices of compatible dimensions, they can be multiplied just like this ... `C = A*B`. This and other features of the `matlab` language present opportunities for writing terse code, and often eliminate the need for writing `for`-loops, especially when computing with matrices.

2 Download and Install

Octave for Windows , Octave for OSX , Octave for Linux , or Matlab

2.1 Set-up the Octave Environment

- Edit > Set Path > Add Folder > Folder with Subfolders ...
  navigate to your m-files folder and add it to the path ... Save > Close
  The folder names in the path should not contain parentheses or periods.
- Window > for now, uncheck: Show Command History, Show File Browser, Show Workspace
  CTRL+0: switch to the Command window. CTRL+4: switch to the Editor window.
- Edit > Preferences > General , Command , Editor ... adjust comment, font and colors

2.2 Set-up the Matlab Environment

- Home > Environment > Set Path > Add with Subfolders ...
  navigate to your m-files folder and add it to the path ... Save
  The folder names in the path should not contain parentheses or periods.
- Home > Environment > Preferences > Numeric Dislay > Compact
- Home > Environment > Preferences > MATLAB > Appearance > Theme > Dark
- How to change the Matlab layout (recommended)

2.3 Other editors

Code in the `matlab` language is written in "`.m`-files," which are plain text files and may be edited outside of MATLAB or Octave. You may prefer to edit your `.m`-files in notepad++, Sublime, VS Code, or some other text editor. This VS code plugin allows you to edit and run your `.m`-files from within VS Code, without first opening MATLAB.

3 matlab language tutorial videos

- https://www.youtube.com/watch?v=TqwSlEsbObg (94 minutes)
- https://www.youtube.com/watch?v=T_ekAD7U-wU (266 minutes)

Refer to the indexed Time Stamps in the descriptions.

4 Useful tips

1. **Comment your code.** Start one-line comments with `%` or `...` Text and code following the `%` or `...` and going to the end of the line is not executed. It's fine and good to comment every line.

   Blocks of text or code starting with `%{` as the first characters of the line at the top of the block and ending with `%}` as the first characters of the line at the bottom of the block are not executed.

2. **Suppress output.** End every line with a `;` to keep intermediate results from being displayed to the MATLAB Command window. To display the value of any variable of interest to the Command window, remove the `;` from the end of the line.

3. **Re-run a previous command.** Scroll through your previous command-line inputs by selecting the Command window and hitting the up arrow and the down arrow on your keyboard.

4. **Clear the Command Window.** Type `clc` into the Command window.

5. **Clear variables from the Workspace.** Type `clear <varname1> <varname2> ...` to clear certain variables from the workspace, and the memory allocated to them. Doing so, you will see these variables disappear from the Workspace window. Clear all variables by typing `clear all`

6. **Break up long lines of code in your `.m`-files.** If a line of code is longer than 80 characters (one page width of text), you may continue an expression onto subsequent lines by splitting the line with an ellipsis (`...`). This is not a strict rule, and there are some instances in which a line of code should be very long. Note that when splitting a line with an ellipsis, no blank space may follow the ellipsis.

   An ellipses is unnecessary with matrices; you may enter matrix row by row on separate lines like this ...

   ```
   M = [ 1 2 3
         4 5 6 ]
   ```

7. **Abort.** Hit Ctrl-C in the command window to stop any calculation for any reason.

5 Assigning values to scalar, vector, matrix, and cell variables

Use square brackets `[ ]` to begin and end vectors and matrices. Commas or blank space separate elements in a single row. Semicolons or new-lines separate rows.

1. Scalar `a = 4`

2. Row vector `r = [ 1 2 3 ]` or `r = [ 1 , 2 , 3 ]` or `r = [ 1 ; 2 ; 3 ]'`

3. Column vector `c = [ 1 2 3 ]'` or `c = [ 1 ; 2 ; 3 ]` or `c = [ 1 , 2 , 3 ]'`

4. Matrix `M = [ 1 3 5 ; 2 4 6 ]`

5. Blank space can have meaning.

`a = [ 1 2 3-1 ]`	is the (1×3) vector	`[ 1 2 2 ]`
`a = [ 1 2 3 -1 ]`	is the (1×4) vector	`[ 1 2 3 -1 ]`
`a = [ 1 2 3- 1 ]`	is the (1×3) vector	`[ 1 2 2 ]`
`a = [ 1 2 3 - 1 ]`	is the (1×3) vector	`[ 1 2 2 ]`
`a = [ 1 , 2 , 3 , - 1 ]`	is the (1×4) vector	`[ 1 2 3 -1 ]`

6. Cell Array (a set of dissimilar quantities, such as a set including character strings and numerical quantities)
   `S = { 'hello' , [ 3 5 ; 7 11 ] , [ 2 4 6 8 ] , 'appreciate' }`
   `S{5} = 'today'`

7. Dimensions of variables are shown in the Workspace window
   or can be found by Command window commands:
   `[nr,nc] = size(X)` ... `nr = size(X,1)` ... `nc = size(X,2)` ... `n = length(x)`

6 Shortcuts to assign values to vectors and matrices

1. **Define vectors of uniformly spaced values using the colon**
   `v = [ start : increment : end ]` represents a *row* vector from `start` up to and including `end` with `increment` between sequential values.
   For example `v = [ 3 : 3 : 24 ]` represents `v = [ 3 6 9 12 15 18 21 24 ]` and `v = [ 2 : 10 ]` represents `v = [ 2 3 4 5 6 7 8 9 10 ]` and
   `v = [ 10 : -1 : 2 ]` represents `v = [ 10 9 8 7 6 5 4 3 2 ]`

2. **Define vectors using `linspace` and `logspace`**
   `v = linspace( start , end , N )` represents a *row* vector of `N` values from `start` up to and including `end` with a uniform increment between values.
   `v = logspace( start , end , N )` represents a *row* vector of `N` values from `10^start` up to and including `10^end` with log-increment between values.
   For example `v = linspace(1,4,5)`
   represents the *row* vector `v = [ 1.00 , 1.75 , 2.50 , 3.25 , 4.00 ]`
   and `v = logspace(-1,1,3)`
   represents the *row* vector `v = [ 0.1 , 1.0 , 10.0 ]`

3. **Common Values**
   The matlab language provides a number of commonly used built-in numerical values, such as `pi` (3.14159...), `e` (2.71828...), `i` or `j` ($\sqrt{-1}$), `Inf` (∞, `1/0`), `eps` (2.22×10^{-16}), and `NaN` (not a number, `0/0`). There is no need to write `pi = 3.14159`. Avoid re-assigning other values to built-in variables.

4. **Vector or Matrix of zeros**
   `zeros(m,n)` creates an `m`-by-`n` matrix of zeros

5. **Vector or matrix of ones**
   `ones(m,n)` creates an `m`-by-`n` matrix of ones

6. **Identity matrix**
   `I = eye(n)` creates an `n`-by-`n` identity matrix.
   The identity matrix is all zeros except for the main diagonal which has values of `1`.

7. **Diagonal matrix**
   `D = diag(v)` creates a diagonal matrix whose diagonal elements are the elements of the vector `v`
   `D = diag(v,1)` creates a diagonal matrix whose first diagonal above the main diagonal are the elements of the vector `v`
   `D = diag(v,-2)` creates a diagonal matrix whose second diagonal below the main diagonal are the elements of the vector `v`

8. **Matrices of Random Values**
   `M = rand(m,n)` creates a `m`-by-`n` matrix of random values uniformly distributed between `0` and `1`.
   `M = randn(m,n)` creates a `m`-by-`n` matrix of random values normally distributed with mean of `0` and standard deviation of `1`.
   `M = randi(min,max,m,n)` creates a `m`-by-`n` matrix of random integers uniformly distributed between `min` and `max`.

9. **Assigning values to other kinds of matrices**
   Other built-in shortcuts used to assign values to certain kinds of matrices include: `tril`, `triu`, `pascal`, `magic`, `hadamard`, `compan`, `vander`, `toeplitz`, and `hankel`.
   You can read about these, and any other function, by typing (for example) `help tril` at the Command window prompt `>>`

7 Vector and Matrix operations

1. **Select elements from vector**
   `v(i)` selects the `i`th entry of `v`, where `1` $\leq$ `i` $\leq$ the length of `v`
   `v(idx)` , where `idx` is a vector of integers from `1` to the length of `v`, selects the elements of `v` whose indices correspond to `idx` . For example,
   if `v = [5 1 3 7]` and `idx = [ 1 4 3 ]` then `v(idx)` represents `[5 7 3]`

2. **Select to end of vector**
   `v(i:end)` selects the `i`th entry to the last entry of `v` .
   `v(i:end-1)` selects up to the second-to-last entry of `v` .
   This works for any index number from `1` up to the length of the vector.

3. **Select elements from matrix**
   This follows the same principle as vectors, but indices for row and column are separated by a comma.
   `M(i,j)` is the scalar in the `i`th row and `j`th column of matrix `M`.
   `M(i,:)` represents the vector in the `i`th row of matrix `M`, a *row* vector.
   `M(:,j)` represents the vector in the `j`th column of matrix `M`, a *column* vector.

   If `idc = [3:2:9]` then `M(:,idc)` represents the odd column of matrix `M` from column `3` to column `9`.

   If `idr = [2:2:4]` and `idc = [3:2:9]` then `M(idr,idc)` represents the values of `M` in the even rows from row `2` to row `4` and the odd columns from column `3` to column `9`.

4. **Transpose**
   If `v` is a row vector, `v'` is a column vector. If `v` is a column vector, `v'` is a row vector.

   If `M` is a `m`-by-`n` matrix then `M'` represents a `n`-by-`m` matrix.
   Row `i` of `M'` is the transpose of column `i` of `M`.

   If some values in `v` or `M` are complex, the transpose can also involve taking the complex-conjugate of the values in `v` or `M`. If `v` is a complex row (column) vector, then `v'` represents the column (row) vector containing the complex conjugates of the elements of `v`. If `v` is a complex row (column) vector, then `v.'` represents the the column (row) vector with the same elements of `v`.

   If `v` and `M` are real, the transposes `v'` and `v.'` are equivalent, and the transposes `M` and `M.'` are equivalent.

5. **Tiling Matrices**
   If `A = zeros(3); B = ones(3,4);` and `D = randn(4);` then `M = [A, B ; -B', D]` is a 7-by-7 matrix in which the first three rows of the first three columns are all zero, the first three rows of columns 4 through 7 are all ones, the last four rows of the first three columns are all negative ones, and the last four rows of the last four columns are random.

```
M = [   0.00   0.00   0.00   1.00   1.00   1.00   1.00
        0.00   0.00   0.00   1.00   1.00   1.00   1.00
        0.00   0.00   0.00   1.00   1.00   1.00   1.00
       -1.00  -1.00  -1.00  -0.21   2.10   0.89  -0.18
       -1.00  -1.00  -1.00   0.78  -0.13  -0.90  -0.17
       -1.00  -1.00  -1.00   1.51  -1.10  -0.68   0.65
       -1.00  -1.00  -1.00  -0.29  -0.11  -1.57  -1.08 ]
```

6. **Addition, Subtraction, Multiplication, and Division of a matrix (or a vector) with a scalar**
   If `M` is a matrix and `s` is a scalar, then:
   `A = M+s` is a matrix of the sum of each element of `M` and `s` ... `A(i,j) = M(i,j)+s`
   `A = M-s` is a matrix of the difference of each element of `M` and `s` ... `A(i,j) = M(i,j)-s`
   `A = M*s` is a matrix of the product of each element of `M` and `s` ... `A(i,j) = M(i,j)*s`
   `A = M/s` is a matrix of the ratio of each element of `M` over `s` ... `A(i,j) = M(i,j)/s`

7. **Vector Addition and Subtraction**
If `u` and `v` are vectors the same dimension (both row vectors or both column vectors, and of the same length) then `a = u + v` represents a vector of the sum of each element of `u` with the corresponding element of `v`, and has the same dimensions as `u` and `v`

$$a_i = u_i + v_i$$

If `u` and `v` are both column vectors (or both row vectors) of different dimensions, they cannot be added or subtracted, and Matlab / Octave will return the error:
`Matrix dimensions must agree.`
This is a very common error in Matlab programming, and it's useful to know what it means. To fix matrix dimension errors, check the dimensions of the involved variables, either by looking at the Workspace window or entering the line `[n_row,n_col] = size(X)` (without a semi-colon ';') in the Command window or in your `.m`-file.

If `u` is a column vector of length `m` and `v` is a row vector of length `n`, then `M = u + v` represents a *matrix* of dimension `m`-by-`n`, where

$$M_{ij} = u_i + v_j$$

8. **Matrix Addition and Subtraction**
If `P` and `Q` are both `m`-by-`n` matrices then `M = P + Q` is also a `m`-by-`n` matrix where

$$M_{ij} = P_{ij} + Q_{ij}$$

If `P` and `Q` are matrices of different dimension, they cannot be added, and Matlab / Octave will return the error: `Matrix dimensions must agree.`

9. **Vector Multiplication**
In `matlab` there are four kinds of vector multiplication: `*`, `*`, `.*`, and `cross`.

If `u` and `v` are both column vectors of length `n`, then `w = u' * v` is a scalar-valued *inner product* (or *dot product*)

$$w = \sum_{i=1}^{n} u_i v_i$$

If `u` is a column vector of dimension `m`-by-`1` and `v` is a column vector of dimension `n`-by-`1`, then `M = u * v'` is a matrix-valued *outer product*

$$M_{ij} = u_i v_j$$

The matrix `M` has dimension `m`-by-`n`.

If `u` and `v` are vectors of the same dimension (`1`-by-`n` *or* `n`-by-`1`), then `w = u .* v` is a vector of the same dimension with elements

$$w_i = u_i v_i$$

This is called "element by element" multiplication.

If `u` and `v` are both vectors of dimension 3, then `cross (u,v)` is the *cross product* of `u` and `v` defined by the determinant

$$u \times v = \det\left(\begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{bmatrix}\right)$$

10. **Matrix Multiplication**
    In the `matlab` language there are three kinds of matrix multiplication: `*`, `*`, and `.*`

    If `P` has dimension `p`-by-`q` and `Q` has dimension `q`-by-`r` then `M = P * Q` is the *inner product* (or *dot product*) ans is a `p`-by-`r` matrix.

    $$M_{ij} = \sum_{k=1}^{q} P_{ik} Q_{kj}$$

    Note that the *inner dimension* of the two matrices being multiplied must be equal.

    If `P` has dimension `p`-by-`q` and `Q` has dimension `q`-by-`r` and `p` is not the same as `r`, then `M = Q * P` cannot be computed since the inner dimensions do not agree, and Matlab / Octave will display the error message: `Matrix dimensions must agree.`

    If `u` is a column vector of dimension `m`-by-`1` and `v` is a column vector of dimension `n`-by-`1`, then `M = u * v'` is a matrix-valued *outer product*

    $$M_{ij} = u_i v_j$$

    The matrix `M` has dimension `m`-by-`n`.

    If `P` and `Q` are both `m`-by-`n` matrices then `M = P .* Q` is also a `m`-by-`n` matrix where

    $$M_{ij} = P_{ij} Q_{ij}$$

    This is called "element by element" multiplication.

11. **Matrix (and Vector) "Exponentiation"**

    If `A` is a square matrix, then `M = A^2` is the same as `M = A*A`

    If `A` is a matrix (or a vector) of any dimension, then `M = A.^2` is the same as `M = A.*A`

12. **Matrix (and Vector) "Division"**

    In the `matlab` languange element-wise division by a matrix (or vector) is done via: `./`

    If `P` and `Q` are matrices (or vectors) with the same dimension, then `M = P ./ Q` is a matrix (or vector) of the same dimension where

    $$M_{ij} = P_{ij} / Q_{ij}$$

    This also works if the numerator is a scalar: `M = a ./ Q` is a matrix (or vector) of the same dimension as `Q` where

    $$M_{ij} = a / Q_{ij}$$

8 Solving Linear Equations

Given a full rank matrix A and a vector b in which A and b have the same number of rows, the system of linear equations $Ax = b$ may be solved for x using `x = A \ b` . This is called a *matrix left divide*.

1. If A is square (and full rank), `x = A \ b` is conceptually the same as $x = A^{-1}b$, or in the `matlab` language, `x = inv(A) * b`
2. If A is tall (and full rank), there are more (unique) equations (rows of A) than unknowns (rows of x), meaning there could be no unique solution, in which case `x = A \ b` returns the value of x that minimizes the sum of the squares of the difference between Ax and b, which is $(Ax - b)'(Ax - b)$. This is used when fitting an equation to data points.
3. If A is wide (and full rank), there are more unknowns than equations, and there can be an infinite number of solutions, in which case and `x = A \ b` returns the value of x that minimizes $x'x$ such that $Ax - b = 0$.

9 `<` , `>` , `<=` , `>=` , `==` , `~=` , `any` , `all` , `find` (with vectors and matrices)

1. The comparison operators ... `<` , `>` , `<=` , `>=` , `==` , `~=` return a `1` if the comparison is `true` and return a `0` otherwise.
   Examples with vectors ... for `a = [ 1 : 3 : 11 ];` and `b = [ 3 : 2 : 10 ];`
   `x = a < 5;` evaluates to `x = [ 1 1 0 0 ]`
   `x = a == 7;` evaluates to `x = [ 0 0 1 0 ]`
   `x = a ~= 5;` evaluates to `x = [ 1 1 1 1 ]`
   `x = a <= 0;` evaluates to `x = [ 0 0 0 0 ]`
   `x = a <= b;` evaluates to `x = [ 1 1 1 0 ]`
   `x = a == b;` evaluates to `x = [ 0 0 1 0 ]`

   Examples with matrices ... for `A = a'*b` and `B = pascal(4)`
   `X = A < 5;` evaluates to

   ```
   X = [ 1 0 0 0
         0 0 0 0
         0 0 0 0
         0 0 0 0 ]
   ```

   `X = A == 7;` evaluates to

   ```
   X = [ 0 0 1 0
         0 0 0 0
         0 0 0 0
         0 0 0 0 ]
   ```

   `X = A ~= 5;` evaluates to

   ```
   X = [ 1 0 1 1
         1 1 1 1
         1 1 1 1
         1 1 1 1 ]
   ```

   `X = A <= 0;` evaluates to

```
X = [ 0 0 0 0
      0 0 0 0
      0 0 0 0
      0 0 0 0 ]
```

`X = A <= 9*B;` evaluates to

```
X = [ 1 1 1 1
      0 0 0 1
      0 0 1 1
      0 0 1 1 ]
```

2. `q = any(x);` returns `1` if any element of `x` is non-zero, `0` otherwise
3. `q = all(x);` returns `1` if all elements of `x` are non-zero, `0` otherwise
4. `idx = find(x);` finds the indices of vector `x` that are non-zero.
   `[IDr,IDc] = find(X);` finds the row,column indices of matrix `X` that are non-zero.
   Examples with vectors ... for `a = [ 1 : 3 : 11 ];` and `b = [ 3 : 2 : 10 ];`
   `idx = find(a < 5);` evaluates to `idx = [ 1 2 ]`
   `idx = find(a == 7);` evaluates to `idx = 3`
   `idx = find(a ~= 5);` evaluates to `idx = [ 1 2 3 4 ]`
   `idx = find(a <= 0);` evaluates to `idx = [](0x1)`
   `idx = find(a <= b);` evaluates to `idx = [ 1 2 3 ]`
   `idx = find(a == b);` evaluates to `idx = 3`

   Examples with matrices ... for `A = a'*b` and `B = pascal(4)`
   `[IDr,IDc] = find(A < 5);` evaluates to `IDr = 1` and `IDc = 1`
   `[IDr,IDc] = find(A == 7);` evaluates to `IDr = 1` and `IDc = 3`
   `[IDr,IDc] = find(A ~= 5);` evaluates to
   `IDr = [ 1 2 3 4 2 3 4 1 2 3 4 1 2 3 4 ]`
   `IDc = [ 1 1 1 1 2 2 2 3 3 3 3 4 4 4 4 ]`
   `[IDr,IDc] = find(A <= 0);` evaluates to `IDr = [](0x1)` and `IDc = [](0x1)`
   `[IDr,IDc] = find(A <= 9*B);` evaluates to
   `IDr = [ 1 1 1 3 4 1 2 3 4 ];`
   `IDc = [ 1 2 3 3 3 4 4 4 4 ];`
5. Find the indices of the elements of vector `u` that are greater than `u_limit`
   ```
   idx_u = find( u > u_limit );
   ```
   Find the fraction of elements in the vector `u` that are greater than `u_limit`
   ```
   P = sum( u > u_limit ) / length( u );
   ```
6. Use `find` to confirm mathematical equalities, for example for any square matrix `A`,
   ```
   A = rand(5);
   find(A^2 - A*A)  evaluates to  [](0x1)
   ```

10 `&&` , `||` , `&` , `|` (with scalars and vectors)

The logical operators `&&` (AND) and `||` (OR) return a `1` if the condition is `true` and return a `0` otherwise.

1. `&&` is used as AND to combine comparison operators or logical operators.
   For example, for scalars `a,b,c,d`, if both `a` is less than `b` AND `c` is less than `d`,
   `x = (a < b) && (c < d)`
   evaluates to
   `x = 1`

2. `||` is used as OR to combine comparison operators or logical operators.
   For example, for scalars `a,b,c,d`, if either `a` is less than `b` OR `c` is less than `d`,
   `x = (a < b) || (c < d)`
   evaluates to
   `x = 1`

3. `&` is used as a bitwise AND to combine vector-valued logical statements.
   For example for vectors of equal length `a,b,c,d`
   `x = (a < b) & (c < d)`
   evaluates to a vector with each element corresponding to the elements of
   `(a<b)` AND `(c<d)`.
   If `a = [ 1 2 3 4 ];` `b = [ 4 3 2 1 ];` `c = [ 4 1 2 3 ];` and `d = [ 1 2 3 4 ];`
   `(a<b)` evaluates to `[ 1 1 0 0 ]` and `(c<d)` evaluates to `[ 0 1 1 0 ]` so
   `x = (a<b) & (c<d)` evaluates to `x = [ 0 1 0 0 ]`

4. `|` is used as a bitwise OR to combine vector-valued logical statements.
   For example for vectors of equal length `a,b,c,d`
   `x = (a < b) | (c < d)`
   evaluates to a vector with each element corresponding to the elements of
   `(a<b)` OR `(c<d)`.
   With the same vectors in the previous example ...
   `(a<b)` evaluates to `[ 1 1 0 0 ]` and `(c<d)` evaluates to `[ 0 1 1 0 ]` so
   `x = (a<b) | (c<d)` evaluates to `x = [ 1 1 1 0 ]`

11 the `matlab` language has many built-in math functions, such as ...

`max(x)`, `min(x)`, `sort(x)`, `sum(x)`, `diff(x)`,
`exp(x)`, `log(x)` `log10(x)`, `log2(x)`,
`sin(r)`, `cos(r)`, `tan(r)`, `cot(r)`, `sec(r)`, `csc(r)`,
`sind(d)` `cosd(d)`, `tand(d)`, `cotd(d)`, `secd(d)`, `cscd(d)`, ... `d` in degrees
`sinh(r)`, `cosh(r)`, `tanh(r)`, `coth(r)`, `sech(r))`, `csch(r)`,
`asin(t)` `acos(t)`, `atan(t)` `acot(t)`, `asec(t)`, `acsc(t)`,
`asind(t)`, `acosd(t)`, `atand(t)`, `acotd(t)`, `asecd(t)`, `acscd(t)` ... result in degrees
`asinh(t)`, `acosh(t)`, `atanh(t)`, `acoth(t)`, `asech(t)`, `acsch(t)`,
`rank(A)`, `lu(A)`, `qr(A)`, `chol(A)`, `eig(A)`, `qz(A)`, `svd(A)`, `schur(A)`, `expm(A)`,

Usage information for any function can be found by typing, for example, `help rank`.

12 max , min , sort , sum , diff (with vectors and matrices)

1. `[a_max,idx] = max(x)`
   ... the most positive value of vector `x` and its index
   `[A_max,idx] = max(X)`
   ... the max value of each column of matrix `X` and the index of each column

2. `[a_min,idx] = min(x)`
   ... the most negative value of vector `x` and its index
   `[A_min,idx] = min(X)`
   ... the min value of each column of matrix `X` and the index of each column

3. `[a_sort,idx] = sort(x)`
   ... sorted values of vector `x` in numerically increasing order and the sorted index
   `[A_sort,IDX] = sort(X)`
   ... sorted columns of matrix `X` and the sorted index of each column

   Example with a vector ... `a = [ 5 7 5 3 9 1 4 ];`
   `[sort_a, idx] = sort(a)` evaluates to
   `sort_a = [ 1 3 4 5 5 7 9 ]`
   `idx    = [ 6 4 7 1 3 2 5 ]`
   `find(a(idx) - sort_a)` evaluates to `[](1x0)`

4. `sum_x = sum(x)` ... the sum of the elements of the vector `x`
   `sum_A = sum(X)` ... the sum down the columns of matrix `X`
   `sum_A = sum(X,2)` ... the sum across the rows of matrix `X`

   Example with a matrix ... `M = magic(5);`

   ```
   M = [ 17   24    1    8   15
         23    5    7   14   16
          4    6   13   20   22
         10   12   19   21    3
         11   18   25    2    9 ]
   ```

   `sc = sum(M)` evaluates to `sc = [ 65 65 65 65 65 ]`
   `sc = sum(M,2)` evaluates to

   ```
     sc = [ 65
            65
            65
            65
            65 ]
   ```

5. `diff_x = diff(x)` ... the differences between adjacent elements of the vector `x`
   `diff_A = diff(X)` ... the adjacent differences down the columns of matrix `X`
   `diff_A = diff(X,2)` ... the adjacent differences across the rows of matrix `X`

   Example with a vector ... `a = [ 5 7 5 3 9 1 4 ]`
   `da = diff(a);` evaluates to `da = [ 2 -2 -2 6 -8 3 ]`

13 Conditional program flow and loops

13.1 `if else`

```
 if (condition is true)
   (run this code)
 else
   (run alternative code)
 end                          % close   if-else   with   end
```

13.2 `switch case otherwise`

```
switch (option)
  case (optionA)
    ( this code runs when option = optionA )
  case (optionB)
    ( this code runs when option = optionB )
  otherwise
    ( this code runs when option is neither option A nor optionB )
end                          % close   switch-case-otherwise   with   end
```

13.3 `while` loops

```
 while (condition is true)
   (run this code)
 end                          % close   while   loops with   end
```

For example, using nested `while`-loops to evaluate

$$S(n) = \sum_{k=1}^{n} (-1)^{k-1}/(2k-1)$$

for three values of $n =$ `[ 5 , 10 , 15 ];`

```
 n = [ 5 , 10 , 15 ];
 S = zeros( 1 , length(n) );
 i = 1;
 while ( i <= length(n) )
   k = 1;
   while ( k <= n(i) )
     % accumulate the sum for the i-th value of vector n and summing over values of k
     S(i) = S(i) + (-1)^(k-1) / ( 2*k-1 )
     k = k+1;
   end
   i = i+1;
 end
```

Using a `while`-loop, the index variables (`i` and `k`) need to be initialized and incremented using additional lines of code. The command `break` exits a loop before the end condition (e.g., `i` equals `n`) is met.

13.4 `for` loops

```
 for i = 1 : n
   (run this code)
 end                          % close   for    loops with   end
```

For example, using nested `for`-loops to evaluate the same sum as above

```
 n = [ 5 , 10 , 15 ];
 S = zeros( 1 , length(n) );
 for i = 1 : length(n)       % loop over the values in the vector n
   for k = 1 : n(i)            % what does n(i) represent?
     % accumulate the sum for the i-th value of vector n and summing over values of k
     S(i) = S(i) + (-1)^(k-1) / ( 2*k-1 )
   end
 end
```

14 Loops should be avoided whenever possible

1. Evaluate $M = P \cdot Q$ where P is a $p \times n$ matrix and Q is a $n \times q$ matrix.
Don't do this

```
for i = 1 : p
  for j = 1 : q
    M(i,j) = 0;
    for k = 1 : n
      M(i,j) = M(i,j) + P(i,k) * Q(k,j);
    end
  end
end
```

Just do this

```
A = P * Q;
```

2. Evaluate $y_i = \sin(\pi x_i)$ for $x = [\ 0 \quad 0.01 \quad 0.02 \quad \ldots \quad 10\ ]$
Don't do this

```
x = [ 0 : 0.01 : 10 ];
N = length(x);
for i = 1 : N
  y(i)  =  sin(pi*x(i));
end
```

Just do this

```
dx=0.01; N=10/dx; x=[0:N]*dx;     % all values of x in a vector *
y = sin(pi*x);                    % evaluate in one line without a loop
```

3. Evaluate $y_i = \displaystyle\sum_{k=1}^{5} a_k \ \cos(\pi \ (2k-1) \ x_i))$
for $a = [\ 5 \quad -4 \quad 3 \quad -2 \quad 1\ ]$ and $x = [\ 0 \quad 0.01 \quad 0.02 \quad \ldots \quad 10\ ]$
Don't do this

```
a = [ 5 , -4 , 3 , -2 , 1 ];
x = [ 0 : 0.01 : 10 ];
M = length(a);
N = length(x);
y = zeros(1,N);
for i = 1 : N
  for k = 1 : M
    y(i)  =  y(i) + a(k) * cos(pi*(2*k-1)*x(i));
  end
end
```

Just do this

```
a = [ 5 , -4 , 3 , -2 , 1 ];      % coefficient values,          * row vector *
dx=0.01; N=10/dx; x=[0:N]*dx;     % all values of x in a         * row vector *
k = [ 1 : 5 ]';                   % integers 1 , 2 , 3 , 4 , 5   * col vector *
y = a * cos(pi*(2*k-1)*x);        % evaluate in one line without for-loops
```

- `pi*(2*k-1)*x` is an outer product resulting in *matrix* with 5 rows and 1001 columns,
- `cos(pi*(2*k-1)*x)` is also a *matrix* with 5 rows and 1001 columns, and
- `a * cos(pi*(2*k-1)*x)` is

a vector-matrix inner product resulting in a *row vector* with 1001 columns.

15 Scripts and functions

To perform lots of operations in a single go, write your operations into a .m-file as scripts or functions. You may run a script by typing the name of the script (without the .m file name extension) into the Command window or by entering F-5 in the Editor window. With scripts, numerical values for all variables are already in the Workspace or are provided within the script. With functions, none of the internal variables are stored in the Workspace. Basically, scripts are an assembly of lines you would type into the Command window, whereas functions compute a set of zero or more output results using a set of zero or more input values. To define a function in a .m-file, begin the function with a line like:

```
function [outputA , outputB]  = functionName( inputA , inputB )
```

Save the function into a .m-file named `functionName.m` ... the same as the function named at the beginning of the file. The name of the .m-file **must** match the name of the main function in the file. To call the function (either in the Command window or within other functions) enter the line of code following the `function` keyword in the function's .m-file. For example to call the example function above, type the following lines (either in a different function, a script, or in the Command window)

```
inputA = 7;
inputB = 8;
[ outputA , outputB ] = functionName(inputA , inputB);
```

More than one function can be defined in a single .m-file. Other functions defined in the .m-file can only be helper functions to the first function included in `functionName.m` and are not accessible outside of the .m-file. For example, consider an .m-file called `mainFunction.m` containing code for functions `mainFunction`, `helper1`, `helper2`, etc. If I type `mainFunction` into the Command window, then function `mainFunction` will run and can make use of functions `helper1` and `helper2` written within the .m-file `mainFunction.m`. However, if I type `helper1` in the Command window I will get an error stating that the function `helper1` is undefined, since the Command window cannot *see* the function `helper1` within `mainFunction.m`.

As you write, download and use more and more .m-functions, you will develop a library of your own .m-files that may be of use in a variety of contexts. It is convenient to store all your general purpose .m-files in one place where MATLAB / Octave knows to look for them. Section 2 explains how to set the Path to your own .m-file library in MATLAB / Octave.

You may wish to save .m-files for a particular project elsewhere, like in a separate `myProject` directory (aka "folder"). That's fine and good. When you start MATLAB just click the `Browse for folder` icon on the upper left of your screen and navigate to your `myProject` directory (aka "folder"). Then you can work with the .m-files in `myProject` while also having access to your library of .m-files listed in the Path.

15.1 Example of a script and a function

In this example a function will evaluate the sum

$$y_i = \sum_{k=1}^{m} a_k \ \cos\left(\pi \ (2k-1) \ x_i\right)$$

for any set of m coefficients a_k and any set of values of x_i, and a script will call the function for a specific set of values for a_k and x_i.

Here is the function, written in a `.m`-file called `sumCos.m`.

```
function y = sumCos(a,x)
% y = sumCos(a,x)
% evaluate a sum of cosines with coefficients a and at values of x
% the variables a and x must both be row-vectors
k = [ 1 : length(a) ]';           % integers 1,2, ... m  in a column vector
y = a * cos(pi*(2*k-1)*x);        % evaluate in one line without for-loops
```

And here is a script in a `.m`-file called `sumCosCalc.m` that uses the function `sumCos` within the `.m`-file `sumCos.m`

```
% sumCosCalc.m
a = [  5 , -4 , 3 , -2 , 1 ];
x = [ 1 : 0.01 : 10 ];
y = sumCos(a,x);
```

This script can be executed by entering the command `sumCosCalc` within the Command window or by F-5 in the Editor window displaying the script file `sumCosCalc.m`. The script file `sumCosCalc.m` and the function file `sumCos.m` must be either in the present working directory or in a directory listed in the Path.

15.2 Editing and running `.m`-files using keyboard shortcuts

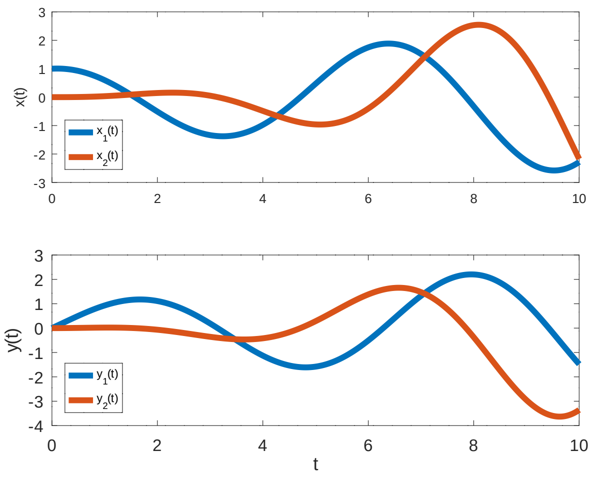
Familiarity with keyboard shortcuts facilitates programming. Here are some useful keyboard shortcuts with Matlab / Octave. Or you may choose to develop expertise with another editor with `matlab` plugins. (e.g., VS Code, Sublime or even some other text editor that makes the mouse obsolete).

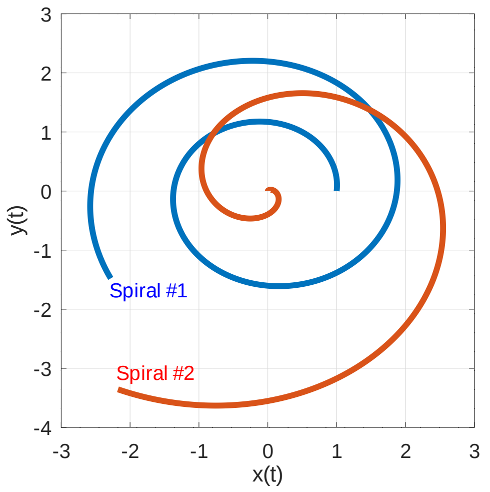
	Matlab	Octave
Comment selected code	Ctrl-R	Ctrl-R
Un-comment selected code	Ctrl-T	Ctrl-Shift-R
Auto-indent code	Ctrl-I	Edit>Format>Indent Code
Go to line (n)	Ctrl-L n	Ctrl-L n
Find (and replace)	Ctrl-F	Ctrl-F
Undo	Ctrl-Z	Ctrl-Z
Save	Ctrl-S	Ctrl-S
Run the `.m`-script	F-5	F-5
Run the selected code	F-9	F-9
Stop code from running	Ctrl-C	Ctrl-C

16 2-D and 3-D Plots

To start with, here are two commented examples of 2-D plotting of parametric equations.

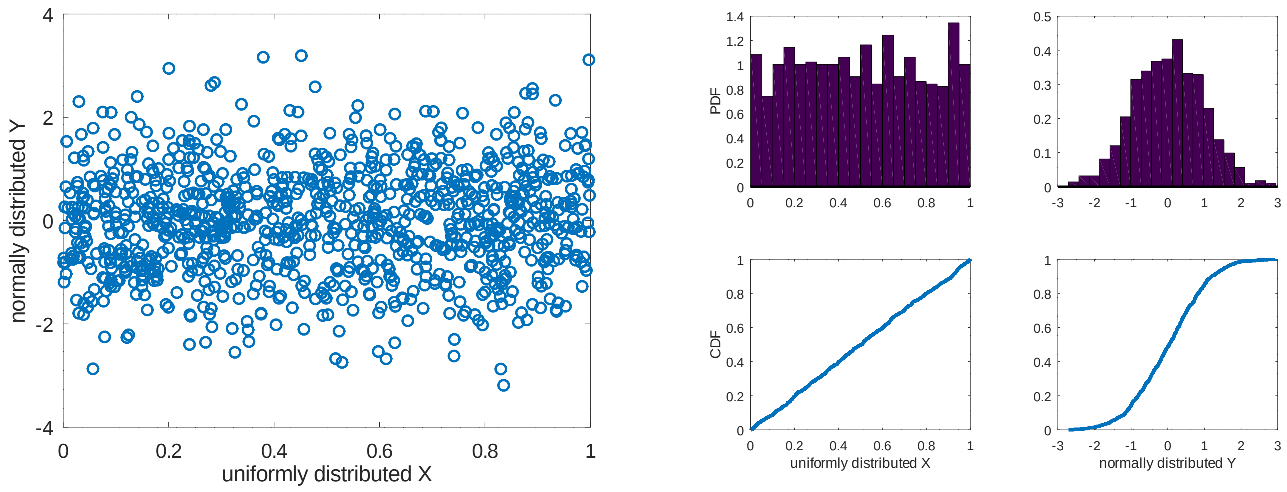
```
Plots = 1;                       % 1: draw plots, 0: don't
pdfPlots = 0;                    % 1: export plots to .pdf files, 0: don't

t  = [ 0 : 0.01 : 10];           % points in the form :  [start : stepSize : end]
x1 =  cos(t) .* exp(t/10);       % compute some data for the 1st curve ... why use  .*  here ?
y1 =  sin(t) .* exp(t/10);
x2 =  sin(t) .* (t/5).^2;        % compute some data for the 2nd curve ... why use  .^  here ?
y2 =  cos(t) .* (t/5).^2;


if Plots

  figure(1);                        % plot x vs t and y vs t
  lw = 5;                           % line width value
  clf                               % clear the plot
  subplot(2,1,1)                    % 1st of 2 sub-plots in this figure
   plot(t,x1, 'LineWidth', lw)      % plot the first curve
   hold on                          % hold the first plot on the axes before plotting the next curve
   plot(t,x2, 'LineWidth', lw)      % plot the second curve
   ylabel('x(t)')                   % y-axis label
   legend('x_1(t)','x_2(t)')
   legend('location','SouthWest')
  subplot(2,1,2)                    % 2nd of 2 sub-plots in this figure
   plot(t,y1, 'LineWidth', lw)      % plot the first curve
   hold on                          % hold the first plot on the axes before plotting the next curve
   plot(t,y2, 'LineWidth', lw)      % plot the second curve
   ylabel('y(t)')                   % y-axis label
   xlabel('t')                      % x-axis label
   legend('y_1(t)','y_2(t)')
   legend('location','SouthWest')
  set(gca, 'FontSize', 14)        % increase the font size
  if pdfPlots, print('MSDB-1.pdf', '-dpdfcrop'); end   % in matlab just use '-dpdf'


  figure(2);                       % plot y vs x
  clf                              % clear the plot
  plot(x1,y1, 'LineWidth', 5)      % plot the first curve
  hold on                          % hold the first plot on the axes before plotting the next curve
  plot(x2, y2, 'LineWidth', 5)     % plot the second curve
  xlabel('x(t)')                   % axis lables
  ylabel('y(t)')
  axis('square')                   % equal spacing in x and y axes
  grid on                          % add a grid to the plot
  %legend('spriral #1', 'spiral #2')
  text(-2.3,-1.7, 'Spiral #1', 'FontSize', 17, 'color', 'b');  % put some text at -2.3,-1.7
  text(-2.2,-3.1, 'Spiral #2', 'FontSize', 17, 'color', 'r');  % put some text at -2.2,-3.1
  set(gca, 'FontSize', 17)       % increase the font size
  if pdfPlots, print('MSDB-2.pdf', '-dpdfcrop'); end   % in matlab just use '-dpdf'

end        % Plots
```





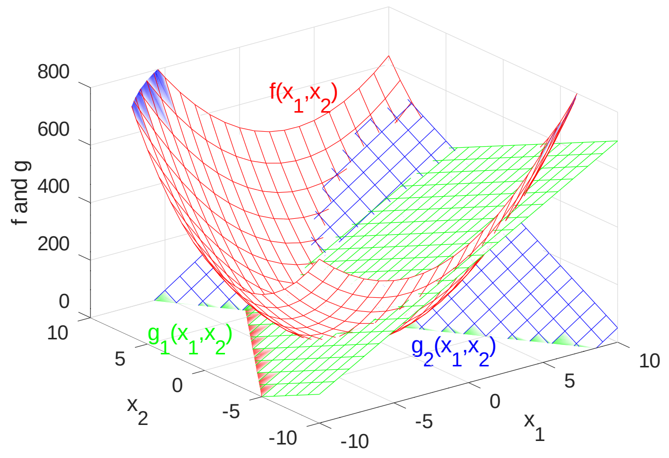
And here are commented examples of plotting random data.

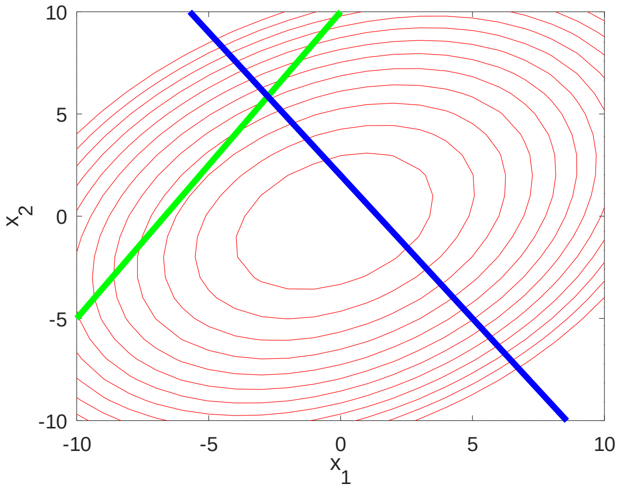
```
Plots = 1;                         % 1: draw plots , 0: don't
pdfPlots = 0;                      % 1: export plots to .pdf files , 0: don't

N   = 1000;
rU  = rand(N,1);      % a vector of N random numbers uniformly distributed from 0 to 1
rN  = randn(N,1);     % a vector of N random numbers normally distributed , mean=0, std.dev=1

if Plots

  figure(3)                        % scatter plot of rN vs rU
  clf
  plot(rU,rN, 'o', 'MarkerSize', 9)
  xlabel('uniformly distributed X')
  ylabel('normally distributed Y')
  set(gca, 'FontSize', 17)         % increase the font size
  if pdfPlots, print('MSDB-3.pdf', '-dpdfcrop'); end     % in matlab just use '-dpdf'

  figure(4)                        % PDF and CDF of rU and rN
  nBin = 20;                       % number of bins in the histogram
  clf
  subplot(2,2,1)
   hist( rU, nBin, nBin/(max(rU)-min(rU)) );             % plot the histogram (pdf) of rU
   ylabel('PDF')
  subplot(2,2,2)
   hist( rN, nBin, nBin/(max(rN)-min(rN)) );             % plot the histogram (pdf) of rN
  subplot(2,2,3)
   stairs( sort(rU), ([1:N]-0.5)/N, 'LineWidth', 3 );    % plot the ogive (cdf) of rU
   xlabel('uniformly distributed X')
   ylabel('CDF')
  subplot(2,2,4)
   stairs( sort(rN), ([1:N]-0.5)/N, 'LineWidth', 3 );    % plot the ogive (cdf) of rN
   xlabel('normally distributed Y')
  if pdfPlots, print('MSDB-4.pdf', '-dpdfcrop'); end     % in matlab just use '-dpdf'

end          % Plots
```



And here are two commented examples of 3-D plotting surfaces and contours of surfaces.

```
Plots = 1;                          % 1: draw plots , 0: don't
pdfPlots = 0;                       % 1: export plots to .pdf files , 0: don't

%% generate some data for plotting
x1 = [ -10 : 10 ];                                   % a vector of x1 values
x2 = [ -10 : 10 ];                                   % a vector of x2 values
[X1,X2] = meshgrid(x1,x2);                           % all combinations of x1,x2 values

f = X1 + 2*X2 - 3*X1.*X2 + 4*X1.^2 + 5*X2.^2;        % objective function ... why .* ? why .^ ?
g1 = 30*X1 - 20*X2 + 200;                            % constraint inequality 1
g2 = 35*X1 + 25*X2 -  50;                            % constraint inequality 2

if Plots
  figure(5)                             % plot the surfaces of f(x1,x2), g1(x1,x2), and g2(x1,x2)
  clf
  cc = zeros(size(X1,1),size(X1,2),3);
  cr = cc;        cg = cc;        cb = cc;
  cr(:,:,1) = 1; cg(:,:,2) = 1; cb(:,:,3) = 1;    % red, green, blue color codes
  mesh(x1,x2,f , cr);                   % plot the 3D surface of f(x1,x2)  in red
  hold on
  mesh(x1,x2,g1, cg);                   % plot the 3D surface of g1(x1,x2) in green
  mesh(x1,x2,g2, cb);                   % plot the 3D surface of g2(x1,x2) in blue
  xlabel('x_1')
  ylabel('x_2')
  zlabel('f and g')
  text(  2,10,600,'f(x_1,x_2)',    'color', 'r', 'FontSize', 19)    % text label for f
  text(-10, 5, 20,'g_1(x_1,x_2)', 'color', 'g', 'FontSize', 19)    % text label for g1
  text(  0,-5, 20,'g_2(x_1,x_2)', 'color', 'b', 'FontSize', 19)    % text label for g2
  axis( [ -10 10 -10 10 0 800 ])         % limit the span of the plot axes
  set(gca, 'FontSize', 17)               % increase the font size
  if pdfPlots, print('MSDB-5.pdf', '-dpdfcrop'); end  % in matlab just use '-dpdf'

  figure(6)                              % plot contours of figure 5
  clf
  contour(x1,x2,f, [-100:50:600], '-r')                 % plot the contours of f  in red
  hold on
  contour(x1,x2,g1, [0.1 -0.1], '-g', 'LineWidth', 5) % plot the zero contour of g1 in green
  contour(x1,x2,g2, [0.1 -0.1], '-b', 'LineWidth', 5) % plot the zero contour of g2 in blue
  set(gca, 'FontSize', 17)               % increase the font size
  xlabel('x_1')
  ylabel('x_2')
  if pdfPlots, print('MSDB-6.pdf', '-dpdfcrop'); end  % in matlab just use '-dpdf'
end         % Plots
```





17 Importing and Exporting Data

It is useful to know how to import data into MATLAB code, in order to analyze the data, maybe make some plots, and then to export the processed data in a particular format. The `.mat` data format is MATLAB's own file format for exporting and importing MATLAB arrays and structures. For example, to save the three MATLAB variables called `A` , `testData` , and `results` into a single file called `myWork.mat` :

```
save  myWork.mat  A testData  results
```

And to reload the contents of a `.mat` file called `myWork.mat` back into MATLAB,

```
load  myWork.mat
```

The `save` and `load` commands can import and export data in ASCII (plain text) or binary format, and is compatible with multiple versions of MATLAB. ASCII files with numerical data stored in multiple tab or space delimited columns of equal length, preceded by some lines of header text, in which the first character of each header line is the comment character `%` may also be imported into a matrix using `load`. For example, to import the numerical data in an ASCII file called `test-20201017.173529.dat`

```
 % filename   test-20201017.173529.dat
 % Sat Oct 17 2020 17:35:29
 % Experimental Data
 % Time   Sensor 1    Sensor 2    Sensor 3
   0.01    5.02       6.2384      -7.2304
   0.02    8.22       7.2484       1.3324
   0.03    8.42      -6.2284      -4.2311
   0.04    7.32       9.2584       3.4378
   0.05    5.18       1.1371       9.3470
```

into a five-row and four-column matrix, use ...

```
 data = load('test-20201017.173529.dat');
```

Note here that every row and every column have the same number of values and that all the information in the header is ignored.

To import/export numerical data from/to an ASCII file of delimited columns, use `dlmread` or `dlmwrite`. And to import/export numerical data from/to an ASCII file of comma-delimited columns, (i.e., `.csv` files) use `csvread` or `csvwrite`. In `dlmread` and `csvread`, missing values are replaced by `0`. For example, to import the numerical data in a file called `test-20201017.173529.dat` with comma-separated columns

```
  filename  test-20201017.173529.dat
  Sat Oct 17 2020 17:35:29
  Experimental Data
  Time  Sensor 1   Sensor 2   Sensor 3
  0.01 , 5.02  ,   6.2384   , -7.2304
  0.02 , 8.22  ,   7.2484   ,  1.3324
  0.03 , 8.42  ,            , -4.2311
  0.04 , 7.32  ,   9.2584   ,  3.4378
  0.05 , 5.18  ,   1.1371   ,
```

into a five-row and four-column matrix, skipping the first four lines (rows) of plain text data, skipping over zero columns, and replacing the missing values with zeros, use ...

```
data = dlmread( 'test-20201017.173529.dat' , ',' , 4 , 0 );
```

The most general purpose function for importing ASCII data (text and numerical values) is `textscan`. The `textscan` function scans a data file line by line and saves the results in to a Matlab structure, having one structure element for each of the format-specified variables. For example, to import the numerical data in contained in the ASCII file called `test-20201017.173529.dat`

```
% filename  test-20201017.173529.dat
% Sat Oct 17 2020 17:35:29
% Experimental Data
% Time  Sensor 1   Sensor 2   Sensor 3
  0.01   5.02      6.2384     -7.2304
  0.02   8.22      7.2484      1.3324
  0.03   8.42     -6.2284     -4.2311
  0.04   7.32      9.2584      3.4378
  0.05   5.18      1.1371      9.3470
```

into a structure of four vectors (one vector for each column), and skipping the first four lines of header information, use ...

```
fid = fopen( 'test-20201017.173529.dat' , 'r' );    % file pointer
data = textscan( fid , '%f %f %f %f' , 'HeaderLines', 4 );
fclose(fid);
```

in which `data` is a structure with four elements: `data{1}` is a column vector of the first column of the file, `data{2}` is a column vector of the second column of the file, and so on. An alternative usage of the `textscan` command simply skips over all text from `%` to the end of the line, and doesn't require knowing the number of header lines.

```
data = textscan( fid , '%f %f %f %f' , 'CommentStyle', '%' );
```

The matlab language supports the C-standard file input/output functions like `fopen()`, `fclose()`, `fprintf()`, and `fscanf()`.

To automate the processing of a number of data files stored in a particular directory, use the `dir` command to create a list of the data files in the directory. For example, to import all the data from a set of five-column data files that have header lines and other comments starting with the # character, into a structure of structures of five column vectors,

```
dataDir = '/home/userID/CoolProject/SpecialData/';
files = dir(dataDir);        % file names in dataDir
nFiles = size(files,1);      % number files in the SpecialData folder

for ff = 3:nFiles   % --- loop over all files in dataDir skipping . and ..
    fn = files(ff).name;                          % file name
    fid = fopen( fullfile(dataDir,fn) , 'r' );    % file pointer
    dataIn{ff} = textscan( fid, '%f %f %f %f %f', 'CommentStyle', '#' );
    fclose(fid);
end
```

18 Skills

1. Expertise with a powerful text editor, with many keyboard shortcuts, (e.g., VS Code, Sublime or even some other text editor that makes the mouse obsolete) is a useful skill. The same editor can be used for codeing in any programming language.

2. Understand the differences between a scalar (`1`-by-`1`), a row-vector (`1`-by-`n`), a column-vector (`n`-by-`1`), a matrix (`m`-by-`n`), and a multi-dimensionally array (`m`-by-`n`-by-`r`-by-...) By default vectors are row-vectors, unless pre-initialized by a command like:
   `my_vector = zeros(100,1);`

3. Understand the need-for, and proper application of element-wise multiplications and vector multiplication:
   `X = A  * B` ... vector multiplication
   `X = A .* B` ... element wise multiplication

dimensions of A	1 × 1	m × n	m × k	m × n
dimensions of B	m × n	1 × 1	k × n	m × n
use:	`*`	`*`	`*`	`.*`
dimensions of X	m × n	m × n	m × n	m × n

4. Understand the need-for, and proper application of left divide vs. right divide in matrix equations:
   `X = A \ B` ... left divide solves ... $AX = B$ ... for ... X

dimensions of A	1 × 1	m × n	m × n	m × n
dimensions of B	m × n	1 × 1	m × k	m × n
use:	`\`	`.\`	`\`	`.\`
dimensions of X	m × n	m × n	n × k	m × n

   `X = B / A` ... right divide solves ... $XA = B$ ... for ... X

dimensions of A	1 × 1	m × n	m × n	m × n
dimensions of B	m × n	1 × 1	k × n	m × n
use:	`/`	`./`	`/`	`./`
dimensions of X	m × n	m × n	k × m	m × n

5. Understand the need-for, and proper application of element-wise exponentiation and vector exponentiation:
   `X = A  ^ B` ... vector exponentiation
   `X = A .^ B` ... element wise exponentiation

dimensions of A	1 × 1	m × n	n × n	m × n
dimensions of B	m × n	1 × 1	1 × 1	m × n
use:	`.^`	`.^`	`^` or `.^`	`.^`
dimensions of X	m × n	m × n	n × n	m × n

6. Understand the need-for, and proper application of scripts vs. functions.

7. Understanding how to concatenate row-vectors into a matrix and column-vectors into a matrix, or, more generally, how to tile sub-matrices into a larger matrix. Try to answer the following questions with pencil and paper.

   Given ...

```
a = [ 1 2 3 4 ]
b = [ 5 6 7 8 ]
x = [ 5 6 7 8 9 ]
```

   ... what are ...

```
c = [ a  , b  ]
d = [ a' ; b' ]
C = [ a  ; b  ]
D = [ a' , b' ]
X = [ a  , b' ]  ... or ... X = [ a ; b' ] ... or ... [ a ; x ] ... etc.
```

   Given ...

```
A = [ 1 2 3 ; 4 5 6 ; 7 8 9 ]
B = [ 10 11 ;  12 13 ;  14 15 ]
C = [ 16 17 18 ; 19 20 21 ]
D = [ 22 23 ; 24 25 ]
```

   ... what is ...

```
H = [ A B ; C D ]
```

8. Use descriptive names for variables, functions, and scripts

9. Start variable names with a letter, not a number.

10. Comment every line

11. Organize your code into sections in this order:

    (a) within scripts, initialize variables with numerical values, or `NaN` (not a number)

    (b) within functions, pull out vector- or structure- input variables into scalar or matrix variables meaningful names

    (c) calculations

    (d) plots

12. Start programming problems by writing pseudo-code on paper.

19 Debugging

New computer programs almost never work as intended. The ability to locate problematic lines of code within a program is a helpful skill. Fortunately, MATLAB / Octave error messages indicate which lines of code are involved in each error.

Errors in `matlab` often result from trying to operate on matrices of incompatible dimension, divide by zero, index a matrix beyond its dimensions, omit the `end` on a loop, solve an unsolvable matrix equation, etc. Here are eleven common errors and their associated error messages. Clicking on error messages displayed in the Command window opens the Editor window at the line of the error.

1. Attempting to use C language indexing, as in `A = rand(5); b = A[3][4];`

```
error: parse error near line 11 of file /path/to/some_m_file

  syntax error

>>> b = A[3][4]
```

2. Attempting to operate on matrices of incompatible dimensions, as in
`A = rand(5,3); B = rand(4,7); C = A*B`

```
error: operator *: nonconformant arguments (op1 is 5x3, op2 is 4x7)
error: called from
    some_m_file at line 22 column 1
```

3. Attempting to evaluate an element of a element vector outside its dimensions, as in
`x = rand(1,4); b = x(5);`

```
error: x(5): out of bound 4 (dimensions are 1x4)
error: called from
    some_m_file at line 33 column 1
```

4. Attempting to evaluate element `i` of vector `a` where `i` is $\sqrt{-1}$, as in
`x = rand(1,4); b = x(i);`

```
error: a(0+1i): subscripts must be real (forgot to initialize i or j?)
error: called from
    some_m_file at line 44 column 1
```

5. Attempting to compare two values with `=` instead of `==`, as in
`if a = b`, which evaluates to the value of `b`. If `b` is zero, `(a=b)` evaluates to `false`, and the `if` code will not run.

```
warning: suggest parenthesis around assignment used as truth value
         near line 55, column 7 in file /path/to/some_m_file.m
```

6. Attempting to assign one matrix to another matrix of differing dimension, as in `A = rand(5); A(1:2,3:4) = rand(3,2)`

```
error: =: nonconformant arguments  (op1 is 2x2, op2 is 3x2)
error: called from
    some_m_file at line 66 column 8
```

7. Accidentally missing a close parenthesis, as in `a = sum(rand(4)` in a file with 76 lines

```
error: parse error near line 77 of file /path/to/some_m_file.m

  syntax error
```

   This error message does not identify the line of code with the error.

8. Accidentally missing a close bracket, as in `a = [ 2, 4, 6, 8` in a file with 87 lines

```
error: parse error near line 88 of file /path/to/some_m_file.m

  syntax error
```

   This error message does not identify the line of code with the error.

9. Accidentally omitting the `end` to close a `for` , `if` or `while` statement in a file with 98 lines

```
error: parse error near line 99 of file /path/to/some_m_file.m

  syntax error
```

   This error message does not identify the line of code with the error.

10. Attempting to solve an under-determined system of equations `A*x = b` , as in `a = randn(6,1); b = randn(6,1); A = a*a'; x = A \ b;`

```
warning: matrix singular to machine precision
warning: called from
    some_m_file at line 1010 column 1
```

   Why is the matrix `A = a*a'` not invertible in this example?

11. Attempting to divide by zero, as in `x = 1/0`

```
 x = Inf
```

   No error or warning is provided in this case.

Even if the program runs, it might give nonsensical or otherwise unintended results. These errors can be more difficult to track down.

Here are tips on how to find the lines of code that could be the source of these errors.

1. Read the error messages carefully. Matlab / Octave will often identify the lines of code involved with attempting operations on matrices with incompatable dimensions, and will indicate the dimensions of the matrices in question.
2. Clicking on error messages displayed in the Command window opens the Editor window at the line of the error.
3. Remove semi-colons (`;`) from lines in question to view variable values in the Command window. Do the values shown make sense?
4. Check the dimensions and numerical values of a variable using the `size` function or displaying the value of the variable by omitting the semicolon (`;`) at the end of the line computing the variable's value.
5. Use the built-in `help` function to understand how a function is meant to be used.
6. Save variables into a `.mat` file in order to read them back in again, after you've taken a break from `matlab` programming.