2. Let $\{\vec{v_1}, \vec{v_2}, \dots, \vec{v_k}\}$ be a set of k linearly dependent vectors in $\mathbb{R}^n$

Let A be an arbitrary matrix $A^{n \times n}$.

$\{\vec{v_1}, \dots \vec{v_k}\}$ being linearly dependent implies that there exists an index i and scalars α_j's such that

$$\vec{v_i} = \sum_{j \neq i} \alpha_j \vec{v_j}.$$

Multiplying by A on both sides:

$$A\vec{v_i} = A\left(\sum_{j \neq i} \alpha_j \vec{v_j}\right)$$

Using the properties of vector-matrix multiplication,

$$A\vec{v_i} = \sum_{j \neq i} A(\alpha_j \vec{v_j}) = \sum_{j=i} \alpha_j (A v_j)$$

$\therefore$ the set of vectors $\{A\vec{v_1}, A\vec{v_2}, \dots, A\vec{v_n}\}$ is linearly dependent again by definition.

4. a) $$\begin{bmatrix} q_x \\ q_y \end{bmatrix} = \begin{bmatrix} R_{xx} & R_{xy} \\ R_{yx} & R_{yy} \end{bmatrix} \begin{bmatrix} P_x \\ P_y \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

$$\begin{bmatrix} q_x \\ q_y \end{bmatrix} = \begin{bmatrix} R_{xx}P_x + R_{xy}P_y \\ R_{yx}P_x + R_{yy}P_y \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

You know the vector $\vec{q}$ and the vector $\vec{y}$; you don't know the $\vec{T}$ and $R_{xx}, R_{xy}, R_{yx}, R_{yy}$ for a total of <u>6 unknowns</u>

You need <u>6 equations</u> to solve for all unknowns thus you need at least <u>3 pairs</u> of common points $\vec{p}$ and $\vec{q}$.

b)
$$\begin{aligned}
R_{xx}P_{x_1} + R_{xy}P_{y_2} + T_{x_1} &= q_{x_1} \\
R_{yx}P_{x_1} + R_{yy}P_{y_2} + T_{y_1} &= q_{y_1} \\
R_{xx}P_{x_2} + R_{xy}P_{y_3} + T_{x_2} &= q_{x_2} \\
R_{yx}P_{x_2} + R_{yy}P_{y_3} + T_{y_2} &= q_{y_2} \\
R_{xx}P_{x_3} + R_{xy}P_{y_4} + T_{x_3} &= q_{x_3} \\
R_{yx}P_{x_3} + R_{yy}P_{y_4} + T_{y_3} &= q_{y_3}
\end{aligned}$$

c) If $\vec{P}_1, \vec{P}_2, \vec{P}_3$ are colinear that means that $(\vec{P}_2 - \vec{P}_1) = k(\vec{P}_3 - \vec{P}_1)$ for some $k \in \mathbb{R}$.

$$\vec{P}_2 - \vec{P}_1 = k\vec{P}_3 - k\vec{P}_1$$

$$\vec{P}_2 = k\vec{P}_3 + (1-k)\vec{P}_1$$

$\therefore$ since we can write $\vec{P}_2$ as a linear combo of $\vec{P}_3$ and $\vec{P}_1$ we do not have enough information to solve for all and therefore can stitch the pictures in more than one way (for ex. rotation since points are colinear or reflection)

3.

a) $0.5T_1 + 0.5T_2 = P_1$
$0.5T_2 + 0.5T_3 = P_2$
$0.5T_3 + 0.5T_4 = P_3$
$0.5T_4 + 0.5T_5 = P_4$
$0.5T_5 + 0.5T_6 = P_5$
$0.6T_6 + 0.5T_1 = P_6$

$$\Rightarrow \begin{bmatrix} 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0 & 0 & 0 & 0.5 \end{bmatrix}$$

$$\begin{bmatrix} 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0 & 0 & -0.5 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0 & 0.5 & 0 & 0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0.5 \\ 0.5 & 0 & -0.5 & 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 0.5 & 0 & -0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Since the last row is linearly dependent and there are not enough pivots, the matrix is not invertible and therefore does not have a unique solution.

A counterexample would be if all odds' ordered plates that are \$6 each whereas evens' ordered plates that are \$8 each. This is indistinguishable from the reverse case where odds order \$8 each and evens' order \$6 each since the only information you have for both cases is that all plates have \$5 each.

b) $0.5T_1 + 0.5T_2 = P_1$
$0.5T_2 + 0.5T_3 = P_2$
$0.5T_3 + 0.5T_4 = P_3$
$0.5T_4 + 0.5T_5 = P_4$
$0.5T_5 + 0.5T_1 = P_5$

$$\Rightarrow \begin{bmatrix} 0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0 & 0 & 0.5 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & -0.5 & 0 & 0 & 0.5 \end{bmatrix} \rightarrow \begin{bmatrix} 0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0.5 & 0 & 0.5 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & -0.5 & 0.5 \end{bmatrix} \rightarrow \begin{bmatrix} 0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 & 0.5 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0.5 \end{bmatrix} \rightarrow \begin{bmatrix} 0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0.5 \end{bmatrix} \rightarrow$$

$$\begin{bmatrix} 0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0.5 \end{bmatrix} \rightarrow \begin{bmatrix} 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0.5 \end{bmatrix}$$

$\therefore$ since this matrix has pivots in all columns, it is invertible and therefore has a unique solution allowing us to find individual tips from the tip plates.

c) For $n \% 2 == 1$, you can figure out everyone's tips since if it is even there are not enough pivots covering the last equation to be a linear combo of another whereas if it n is odd then there are enough pivots leading to a unique soln.

5)

I worked with Jonathan Kim (3031867008) and I also went to OH Monday 2pm - 4pm and worked with Nathan Bentolila (303184210 0)