W Boson Pair Production via Gluon Fusion

A Dimension-8 Effective Field Theory Study

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Abstract

The $ggWW$ quartic couplings are absent in the Standard Model for 4-vertex contact interactions. The process of $gg \to W^+W^-$ receives a single contribution from a SMEFT dimension-6 operator. The process can also arise from SMEFT operators of dimension-8. We study the dimension-8 operators associated to the production of W^+W^- via gluon fusion, $gg \to WW$. An exhaustive list of transition amplitudes associated to these dimension-8 operators is reported for the first time. Further, we report the transition amplitude accounting for decay into leptons, i.e $gg \to W^+(l^+\nu)W(l'^-\bar{\nu_{l'}})$. The latter amplitudes are implemented into MCFM so that numerical predictions demonstrating the potential resultant phenomenology can be generated.

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Preface

Chapter 1 briefly summaries topics from the Standard Model and and gives an introduction to the approach of Effective Field Theory. Understanding of these topics is required to understand the approach and motivation of the following Chapters. This chapter is a review of material from research papers, textbooks and explanation from my project supervisor. Chapter 2 is the main body of work carried out throughout the year and computes the dimension-8 amplitudes contributing to W pair production via gluon fusion. The operators used in the paper are contributions from Adam Martin's paper [1]. I rewrote these into the $\mathcal{O}_{1,2,3,4}$ form that were used in this paper. The Feynman rules in this chapter were then provided by both my supervisor, Andrea Banfi and Adam Martin. The work from 2.2.1 to the conclusion of the Chapter 2 is my original work. Andrea told me the method of computing the transition amplitudes from the Feynman rules, after which, I initially computed these by hand and verified them with the Mathematica notebook **ggWW_amplitudes.nb**.[1] The notebook is both collaboration of work with Adam. The work in Chapter 3 only began in late March (and thus it is not comprehensively detailed in this report). I am not very familiar with the Spinor Helicity Formalism yet but I was able to verify the results with the Mathematica notebook **ggWW_decay.nb**. This notebook was originally provided by Adam and edited and adapted by myself to be more suitable for MCFM, the program we are using for numerical results [2, 3]. I transcribed these amplitudes into the Fortran language used by MCFM with help from Andrea. We then implemented these into the MCFM files **gg_ww_dim8.f90** and **gg_WW_int.f** included in the appendix. All figures in the project are original work. The results presented in Chapters 2 and 3 are, to the best of the knowledge of my supervisor and I, original to the field. They are the main body of the aforementioned paper, currently being prepared for publication to the Journal of High Energy Physics (JHEP).

[1] The Appendix includes a GitHub repository to find the two notebooks

Chapter 1

Introduction

This chapter aims to give the reader a basic understanding of the concepts used in the following chapters. It briefly summaries topics from the Standard Model and gives an introduction to the approach of Effective Field Theory. A comprehensive understanding of Quantum Field Theory is not required to follow this study. Thus, only a basic introduction to the field equations is given.

There are two distinct ways in which our understanding of the fundamental interactions of the universe can progress. We may discover new fields outside of the Standard Model; or we may discover new interactions between the existing Standard Model fields. There is currently no confirmed evidence of the existence of fields beyond the Standard Model [4]. In this project we explore new possible interactions between Standard Model Fields. Using the methods of Effective Field Theory, we will look at possible production of a W^+W^- boson pair from a gluon pair. This interaction is not allowed by the existing Standard Model.

1.1 The Standard Model

1.1.1 Field Representation Example: Electrodynamics

The Standard Model is a gauge theory. Gauge theories are built on a Lagrangian that is invariant under certain (gauge, Lorentz) transformations. Fields are used to represent particles. An example is given of the photon in Electrodynamics. The electrodynamic four potential is $A^\mu = (\phi, \vec{A})$ where ϕ is the electric potential and $\vec{A}$ is the magnetic vector potential with natural units ($c = \hbar = 1$) eV. The electrodynamic field strength tensor is then defined as:

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \tag{1.1}$$

The term ∂^μ is the 4-gradient function whose contravariant components are: $\partial^\mu = (\partial_t, -\partial_x, -\partial_y, -\partial_z)$. From this tensor, we can construct the following Lorentz invariants:[1]

$$F^{\mu\nu}F_{\mu\nu} = 2\left(B^2 - E^2\right) , \qquad \tilde{F}^{\mu\nu}F_{\mu\nu} = 8\left(B \cdot E\right) , \tag{1.2}$$

In Cartesian coordinates, B and E are the 3-vectors for the magnetic and electric field, with components: $B = (B_x, B_y, B_z)$ and $E = (E_x, E_y, E_z)$. It is of interest to note that $F^{\mu\nu}F_{\mu\nu}$ is a true scalar and $\tilde{F}^{\mu\nu}F_{\mu\nu}$ is a pseudoscalar, i.e it switches sign under parity inversion. In our system of units, field strength tensors have dimension of energy to the second power, which we call "dimension-2".

[1] In the notation of this paper: $\tilde{X}_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}X^{\rho\sigma}$

1.1.2 Field Representation of W Bosons and Gluons

In general, bosons are described by tensor fields and fermions are described by spinor fields. Similar to EM fields, gluons and W bosons have corresponding fields $G^a_{\mu\nu}$ and $W^I_{\mu\nu}$. Here, a and I are integer indices labelling the fields with $I \in \{1, 2, 3\}$ and $a \in \{1, .., 8\}$. These are functions of their respective potentials, W^I_μ and G^a_μ as such:

$$\begin{aligned} W^I_{\mu\nu} &= \partial_\mu W^I_\nu - \partial_\nu W^I_\mu - g_w\, \epsilon^{IJK} W^J_\mu W^K_\nu\,, \\ G^a_{\mu\nu} &= \partial_\mu G^a_\nu - \partial_\nu G^a_\mu - g_s\, f^{abc} G^b_\mu G^c_\nu\,, \end{aligned} \tag{1.3}$$

with gauge coupling constant g_w and g_s respectively. The quantity f^{abc} is the structure constant of the gauge group under which the gluon fields transform.[2] The symbol f^{abc} is real and, like ϵ^{IJK} (the Levi-Civita symbol), it is antisymmetric. The derivation of these results can be found in [5]. Further details are not relevant or included within this report. The fields whose quanta are the W bosons are a linear combinations of the fields W^1 and W^2 (but notably not W^3), as follows [6]:

$$W^{\pm} = \frac{1}{\sqrt{2}}(W^1 \mp iW^2) \tag{1.4}$$

1.1.3 Lagrangian Formulation of Standard Model

The Lagrangian can be broken down into terms that involve mass and kinetic contributions (i.e 'movement' and 'mass' associated to the field) and interaction contributions (the coupling between different fields). Any term contributing the Lagrangian must be both gauge and reference frame invariant. We break $\mathcal{L}$ down into the kinetic term $\mathcal{L}_0$ and the interaction term, $\mathcal{L}_I$:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_I \tag{1.5}$$

The Mass and Kinetic Terms

A free particle can be represented by a mass term and a kinetic term which denotes to the 'motion' of the fields. We express these terms by the $\mathcal{L}_0$ contribution to the Lagrangian. For the SM gauge fields $W^a_{\mu\nu}$ and $G^a_{\mu\nu}$, the kinetic term is given by:

$$\mathcal{L}_0 = -\frac{1}{2}\mathrm{tr} W_{\mu\nu}W^{\mu\nu} - \frac{1}{2}\mathrm{tr} G_{\mu\nu}G^{\mu\nu} \tag{1.6}$$

Mass terms are forbidden by gauge invariance. W bosons acquire their masses via interactions with the scalar Higgs field, via the so-called Brout-Englert-Higgs mechanism [7].

The Interaction Terms

Interactions within the Standard Model follow from gauge invariance and are local. This means that in the Lagrangian they appear as products of the fields at the same spacetime point [5]. Any such product is commonly refered to as an "operator", even though at this point it is just a product of classical fields. This is because after quantisation of the fields, these products will be become Hermitian operators used in Quantum Field Theory.

[2] It is defined by the commutator: $[t_a, t_b] = if^{abc}t_c$, where t_i are the generators of the group.

1.2 Standard Model Effective Field Theory

1.2.1 Beyond Standard Model Contributions to the Lagrangian

The Standard Model Lagrangian does not include all possible interaction terms compatible with gauge invariance, thus some could arise from interactions beyond the Standard Model. We can look to find deviations from SM in higher dimensional operators. This method of predicting physics is called Standard Model Effective Field Theory. All operators within this method are constructed from SM fields. The contributions of these operators to the Lagrangian are said to be suppressed by some power of $1/\Lambda$. We call Λ the typical scale of new physics. This scale is a constant and has units of energy. The suppression is to the appropriate power such that it ensures its corresponding operator's contribution to the Lagrangian remains of dimension-4. For example, a operator of dimension-6 will be suppressed by $1/\Lambda^2$ and thus its overall contribution is dimension-4. Effective Field Theory can only be used up to energies of order Λ. At energies beyond Λ, there is no justification for neglecting operators of any dimension, since they are no longer suppressed. Operators of arbitrarily high dimension become important. Since there are an infinite number of them, there is no longer utility in the Effective Field Theory approach. We see no clear contributions from SMEFT operators at the current scale at which we probe physics (at the LHC $\sim$ 10 TeV). From this, clearly Λ must be sufficiently large, $\Lambda \gg$100 GeV. In the limit, $\Lambda \to \infty$, the Standard Model is recovered because all BSM contributions are suppressed to zero. Another key point to note, all interactions that are proportional to an odd power of $1/\Lambda$ violate B and/or L [8] and thus we only look for operators of even power. We also include a dimensionless coefficient factor for each operator, $\mathcal{O}_i$, which we label similarly to c_i. Each operator has its own dimensionless coefficient. These coefficients are assumed to be of order $\sim$1 and are free parameters. The Lagrangian is therefore the sum of all operators contributing at each allowed dimension:

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_i \frac{c_i}{\Lambda^2}\mathcal{O}(dim-6)_i + \sum_i \frac{c_i}{\Lambda^4}\mathcal{O}(dim-8)_i + \dots \tag{1.7}$$

A reference containing a full list of SMEFT dim-6 operators is provided [9]. At this date, many SMEFT predictions have been made at dim-6 for example [10, 11]. However, so far these are not comprehensive as some contributing operators are absent and many SMEFT interactions only occur at dim-8 or above.

We can break a dimension-8 Lagrangian density down to CP-even and CP-odd operators. Here, O_i are the CP-even operators, whereas $\tilde{O}_i$ are the CP-odd ones.

$$\mathcal{L}_{dim-8} = \sum_i \frac{c_i}{\Lambda^4}\mathcal{O}(dim-8)_i + \sum_i \frac{\tilde{c}_i}{\Lambda^4}\tilde{\mathcal{O}}(dim-8)_i \tag{1.8}$$

We choose to disregard CP-odd operators as they do not interfere with SM contributions at the order we consider. This deviation will be insignificant by comparison and so we choose to disregard it. This is further explained in section 1.2.4. The process of $gg \to WW$ can only occur at dim-6 SMEFT via the interaction of the gluons with the Higgs field [9]. This project seeks to find impact of dim-8 operators on the $gg \to WW$ process. In fact, dim-8 operators produce contact interactions between gluons and W bosons which are not present if we limit ourselves to dim-6 operators.

1.2.2 Effective Field Theory - an Example

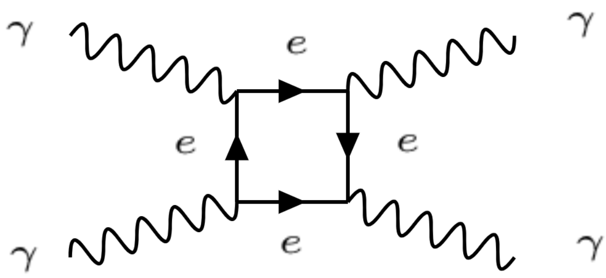

Figure 1.1: Feynman diagram showing the SM process of $\gamma\gamma \to \gamma\gamma$ process via an electron box

Take the example of light-by-light scattering ie: $\gamma\gamma \to \gamma\gamma$ for which there is plenty of evidence [12]. This process can be explained by the Standard Model via an electron box process.[3] The process is shown in fig. 1.1. However, at energies much below M_e, the mass of the electron, the process looks very much like a contact interaction EFT, suppressed by $1/M_e^4$. Of course, unlike the methods of SMEFT, this can be derived using the principles of QED. A brilliant reference detailing these derivations describing light-by-light scattering is provided [13]. Ultimately, it is found that there is a leading order correction term, $\mathcal{L}_\Delta$, that looks like:

$$\mathcal{L}_\Delta = \frac{\alpha^2}{90M_e^4}\left\{(F^{\mu\nu}F_{\mu\nu})^2 + \frac{7}{16}(F^{\mu\nu}\tilde{F}_{\mu\nu})^2\right\} \tag{1.9}$$

Here, α, is the well defined fine structure constant ($\alpha \approx 1/137$), which is dimensionless. Without any knowledge of the QED derivation, upon inspection, $\mathcal{L}_\Delta$ looks very much like a SMEFT contribution from dim-8 operators. The Field Strength Tensor operators have energy dimension-8 and are suppressed by an energy scale, $1/M_e$, to the fourth power. There are also dimensionless coefficients for each. At energies much below M_e this box process looks exactly like a contact interaction i.e the intermediate electron box states shrink to a single spacetime point. At energies approaching M_e, we could construct further order Effective Theories to describe potential deviations in behaviour. Although reiterating, in this particular case, these corrections could also be derived from QED principles. The point is that without any understanding of the box process, one could arrive at $\mathcal{L}_\Delta$ using SMEFT methods. That is to say, we could predict potential behaviour without having the specific details of an underlying theory. It can be seen that the method of Effective Field Theory is a powerful tool. A downside to the SMEFT method, however, would be that the values of the energy scale and coefficients would be unknown. These would need to be measured experimentally. This serves as an example of the potential of EFT methods.

1.2.3 Transition Amplitudes

The total Lagrangian for a given process can be used to compute the process's corresponding transition amplitude, M. When we take M^2, this gives a probability density for the final particle states and thus includes final state observables. An example of such an observable is the invariant mass distribution of a pair of W bosons, M_{WW}.[4] The transition amplitude is a function of the initial state and final state and is notably dependant on the

[3] This box process can also occur via muon and tauon boxes. However, these have much larger masses and thus have much larger suppression

[4] This cannot be observed directly since W bosons decay into undetectable particles, i.e neutrinos. However it is possible to devise observables that are strongly correlated with M_{WW}

initial (precursor) and final (product) polarisations states and the centre of mass energy of the system. The partial amplitude $\mathcal{M}^{(i)}$ can be computed for each contributing operator $\mathcal{O}_i$. This is done using Feynman rules which is discussed later in 2.1.2. The total transition amplitude for a given state is given by the sum of the partial amplitudes of each contributing operator[5]:

$$M_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} = \sum_i \mathcal{M}^{(i)}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4}\,, \tag{1.10}$$

. The initial polarisation states are denoted λ_1, λ_2 and the final polarisation states are denoted λ_3, λ_4. In the case where we have no information of the initial or final polarisation states we must sum over all possible polarisation state permutations as such:

$$M = \sum_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} M_{\lambda_1,\lambda_2,\lambda_3,\lambda_4}\,, \tag{1.11}$$

1.2.4 Interference

Although contact interactions between the W and G boson fields are not included in the SM, interactions for $gg \to WW$ can arise in the Standard Model via loops of quarks (or anti-quarks) as shown in fig. 1.2.

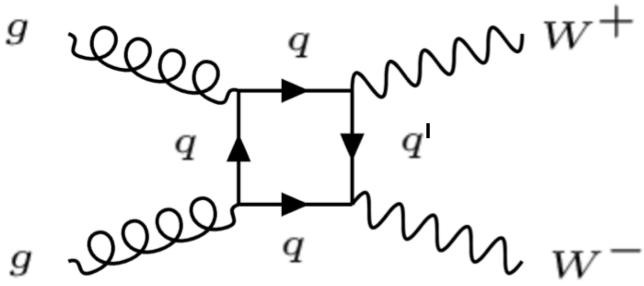

Figure 1.2: Feynman diagram showing the SM process of $gg \to WW$ process via a quark box

The contact 4-vertex level interaction (namely quartic coupling) arises from dim-8 operators. The dim-8 operators will be explicitly discussed in 2.1. We can write the transition amplitude as the sum of SM and SMEFT contributions.

$$M = \mathcal{M}_{SM} + \mathcal{M}_{EFT} \tag{1.12}$$

In the case of this project $\mathcal{M}_{SM}$ refers to the partial amplitude[6] corresponding to $gg \to WW$ via the box process shown in fig. 1.2. Because we square the amplitude to find the probability density we have an expansion of partial sums squared. When this squared sum is expanded, we will see terms which are products of different partial amplitudes. These are interference terms. For example, for a transition amplitude for a process where there is only one SM contribution, one additional dimension-6 contribution and one additional dimension-8 contribution, i.e:

$$M = \mathcal{M}_{SM} + \frac{c_{dim-6}}{\Lambda^2}\mathcal{M}_{dim-6} + \frac{c_{dim-8}}{\Lambda^4}\mathcal{M}_{dim-8}\,, \tag{1.13}$$

[5]The amplitudes shown are for a four particle process, which is the case for $gg \to WW$, which we are studying. This is why it is a function of four polarisation states, λ_i

[6]This contribution was originally the total amplitude before considering higher dimensional operators. Now, since it is not the only contribution, we refer to it as a partial amplitude

Taking the square of M gives:

$$M^2 = \mathcal{M}_{SM}^2 + 2\frac{c_{dim-6}}{\Lambda^2}\mathcal{M}_{SM}\mathcal{M}_{dim-6} + \frac{c_{dim-6}^2}{\Lambda^4}\mathcal{M}_{dim-6}^2 + 2\frac{c_{dim-8}}{\Lambda^4}\mathcal{M}_{SM}\mathcal{M}_{dim-8} + \dots \tag{1.14}$$

The leading order additional term in $1/\Lambda$ is the interference term of the dim-6 contribution with the SM contribution. These are of order $1/\Lambda^2$. There are two next leading order terms in $1/\Lambda$. These are the dim-6 contribution squared and the interference term of the dim-8 contribution with the SM term. These are of order $1/\Lambda^4$. The point of this is to show that if one wants to accurately make SMEFT predictions at order $1/\Lambda^4$, dim-8 contributions must also be considered. Any dimension-8 terms that do not interfere with the Standard Model can only contribute at highest order $1/\Lambda^6$ via interference with dim-6 terms and thus will have a very small contribution. This is why we choose to disregard CP-odd operators for this study.

Concluding Remarks

It is important to note that naive dimensional considerations of eq. (1.14) leads to the fact that the contribution of dimension-8 operators should grow with energy. Therefore, we expect that distributions in observables probing high energies will display spectacular deviations from the Standard Model. Failure to observe such deviations can then be used to put constraints on the size of the coefficient c_{dim-8}, or better, assuming it to be of order 1, to give an idea of the scale Λ.

Chapter 2

Amplitudes for WW production via gluon fusion at dimension-8

In this chapter we will compute the contributions to the amplitude for $gg \to WW$ induced by the relevant set of dimension-8 operators.

2.1 The Dimension-8 Operators

2.1.1 The Valid Combinations

We get our dimension-8 operators from [1]. In that paper the operators are expressed as the following combinations of fields $G^{L,R}_{\mu\nu}$ and $W^{L,R}_{\mu\nu}$. The fields can be expressed in Left and Right forms [1]:

$$X^{L,R}_{\mu\nu} = \frac{1}{2}(X_{\mu\nu} \pm i\tilde{X}_{\mu\nu}) \tag{2.1}$$

In this form, these are the valid combinations listed in the paper:

$$\begin{aligned}
1 &: (G^{L,R\,a})_{\mu\nu}(G^{L,R\,a})_{\rho\sigma}(W^{L,R\,I})^{\mu\nu}(W^{L,R\,I})^{\rho\sigma} \\
2 &: (G^{L,R\,a})_{\mu\nu}(G^{R,L\,a})_{\rho\sigma}(W^{L,R\,I})^{\mu\nu}(W^{R,L\,I})^{\rho\sigma} \\
3 &: (G^{L,R\,a})_{\mu\nu}(G^{L,R\,a})^{\mu\nu}(W^{L,R\,I})_{\rho\sigma}(W^{L,R\,I})^{\rho\sigma} \\
4 &: (G^{L,R\,a})_{\mu\nu}(G^{L,R\,a})^{\mu\nu}(W^{R,L\,I})_{\rho\sigma}(W^{R,L\,I})^{\rho\sigma}
\end{aligned} \tag{2.2}$$

To construct transition amplitudes it is more useful to have these operators expanded and expressed in the more conventional way. The CP-even operators can be found by simply substituting the expressions for "Left" and "Right" fields and expanding the four bracket product. For example, combination 1:

$$({G^a}_{\mu\nu} \pm i{\tilde{G}^a}_{\mu\nu})({G^a}_{\rho\sigma} \pm i{\tilde{G}^a}_{\rho\sigma})(W^{I,\mu\nu} \pm i\tilde{W}^{I,\mu\nu})(W^{I,\rho\sigma} \pm i\tilde{W}^{I,\rho\sigma})\,, \tag{2.3}$$

Expands to give 16 terms:

$$\begin{array}{ll}
+\,{G^a}_{\mu\nu}{G^a}_{\rho\sigma}W^{I,\mu\nu}W^{I,\rho\sigma} & \pm i{G^a}_{\mu\nu}{G^a}_{\rho\sigma}\tilde{W}^{I,\mu\nu}W^{I,\rho\sigma} \\
\pm i{G^a}_{\mu\nu}{G^a}_{\rho\sigma}W^{I,\mu\nu}\tilde{W}^{I,\rho\sigma} & -\,{G^a}_{\mu\nu}{G^a}_{\rho\sigma}\tilde{W}^{I,\mu\nu}\tilde{W}^{I,\rho\sigma} \\
\pm i{\tilde{G}^a}_{\mu\nu}{G^a}_{\rho\sigma}W^{I,\mu\nu}W^{I,\rho\sigma} & -\,{\tilde{G}^a}_{\mu\nu}{G^a}_{\rho\sigma}\tilde{W}^{I,\mu\nu}W^{I,\rho\sigma} \\
-\,{\tilde{G}^a}_{\mu\nu}{G^a}_{\rho\sigma}W^{I,\mu\nu}\tilde{W}^{I,\rho\sigma} & \mp i{\tilde{G}^a}_{\mu\nu}{G^a}_{\rho\sigma}\tilde{W}^{I,\mu\nu}\tilde{W}^{I,\rho\sigma} \\
\pm i{G^a}_{\mu\nu}{\tilde{G}^a}_{\rho\sigma}W^{I,\mu\nu}W^{I,\rho\sigma} & -\,{G^a}_{\mu\nu}{\tilde{G}^a}_{\rho\sigma}\tilde{W}^{I,\mu\nu}W^{I,\rho\sigma} \\
-\,{G^a}_{\mu\nu}{\tilde{G}^a}_{\rho\sigma}W^{I,\mu\nu}\tilde{W}^{I,\rho\sigma} & \mp i{G^a}_{\mu\nu}{\tilde{G}^a}_{\rho\sigma}\tilde{W}^{I,\mu\nu}\tilde{W}^{I,\rho\sigma} \\
-\,{\tilde{G}^a}_{\mu\nu}{\tilde{G}^a}_{\rho\sigma}W^{I,\mu\nu}W^{I,\rho\sigma} & \mp i{\tilde{G}^a}_{\mu\nu}{\tilde{G}^a}_{\rho\sigma}\tilde{W}^{I,\mu\nu}W^{I,\rho\sigma} \\
\mp i{\tilde{G}^a}_{\mu\nu}{\tilde{G}^a}_{\rho\sigma}W^{I,\mu\nu}\tilde{W}^{I,\rho\sigma} & +\,{\tilde{G}^a}_{\mu\nu}{\tilde{G}^a}_{\rho\sigma}\tilde{W}^{I,\mu\nu}\tilde{W}^{I,\rho\sigma}
\end{array} \tag{2.4}$$

The operators coloured in red are pseudoscalars (are odd under parity inversion), thus are all CP-odd and are discarded for the previously stated reasons.[1] All operators coloured blue (four of which) are different representations of the same operator. This can be easily verified with tensor algebra. Note that each of the representations do not all have the same coefficient (but also importantly do not overall sum to zero). For example: $\tilde{G}^a{}_{\mu\nu}G^a{}_{\rho\sigma}\tilde{W}^{I,\mu\nu}W^{I,\rho\sigma} = 4G^a{}_{\mu\nu}G^a{}_{\rho\sigma}W^{I,\mu\nu}W^{I,\rho\sigma}$.[2] The same is true of all operators coloured green. These are true scalars and so are CP-even. We find two unique CP-even operators in case 1 and name them:

$$\mathcal{O}_1 = G^a_{\mu\nu}G^{a,\rho\sigma}W^{I,\mu\nu}W^I_{\rho\sigma}\,, \qquad \mathcal{O}_4 = G^a_{\mu\nu}G^{a,\rho\sigma}\tilde{W}^{I,\mu\nu}\tilde{W}^I_{\rho\sigma}\,, \tag{2.5}$$

The expansions of 2, 3, 4 are performed in the exact same way but the are omitted for the sake of brevity. Case 2 yields the same CP-even operators. Cases 3 and 4 yield two more unique CP-even operators in a similar way:

$$\mathcal{O}_2 = G^a_{\mu\nu}G^{a,\mu\nu}W^{I,\rho\sigma}W^I_{\rho\sigma}\,, \qquad \mathcal{O}_3 = G^a_{\mu\nu}\tilde{G}^{a,\mu\nu}W^{I,\rho\sigma}\tilde{W}^I_{\rho\sigma}\,, \tag{2.6}$$

These are not the only dim-8 operators relevant to process $gg \to WW$. In fact, two additional terms arise as a result of interaction with with the Higgs field. These are sourced from the paper [1].

$$\mathcal{O}_5 = G^a_{\mu\rho}G^{a,\rho\nu}(D^\mu H)^\dagger(D_\nu H)\,, \qquad \mathcal{O}_6 = G^a_{\mu\nu}G^{a,\mu\nu}(D^\rho H)^\dagger(D_\rho H)\,. \tag{2.7}$$

The following is a basic background explanation for these. For what we are concerned, the Higgs field is given by a scalar doublet. Fixing the gauge to the so called "unitary" gauge, this reads:

$$H(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \tag{2.8}$$

The quantity v is a constant quantity of units energy, with $v = 246$ GeV and $h(x)$ is a real scalar field. We also refer to D_μ, which is the Standard Model covariant derivative and is given by:

$$D_\mu := \partial_\mu + i\frac{g_1}{2}YB_\mu + i\frac{g_2}{2}\sigma_I W^I_\mu + i\frac{g_3}{2}\lambda_\alpha G^\alpha_\mu \tag{2.9}$$

Only the W term is relevant for this exercise. Here, σ_I are the Pauli Matrices. We can express the relevant gauge covariant derivative in terms of W^+ and W^- as defined in eq. (1.4) with some manipulation. As far as we are concerned:

$$D_\mu := \partial_\mu + \frac{g_w}{\sqrt{2}}\begin{pmatrix} \dots & W^+_\mu \\ W^-_\mu & \dots \end{pmatrix} \tag{2.10}$$

We take $h = 0$ since we are considering the process $gg \to WW$ not $gg \to WWh$, thus:

$$D_\mu H = \frac{g_w}{2}\begin{pmatrix} \dots & W^+_\mu \\ W^-_\mu & \dots \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{1}{2}g_w v\begin{pmatrix} W^+_\mu \\ \dots \end{pmatrix} = M_W\begin{pmatrix} W^+_\mu \\ \dots \end{pmatrix}\,, \tag{2.11}$$

because: $M_W = \frac{1}{2}g_w v$. Therefore we also have by eq. (1.4):

[1] Note that we chose to disregard the 1/2 factor for each $X^{L,R}_{\mu\nu}$ and $X^{R,L}_{\mu\nu}$ since any overall constant factor can be encapsulated within the free parameters, c_i

[2] Again, we choose to disregard these factors for the same reason

$$(D_\mu H)^\dagger = M_W \begin{pmatrix} W_\mu^- & \dots \end{pmatrix} \tag{2.12}$$

These "..." terms correspond to terms of field W_μ^3 which is not included within either W^+ or W^-. It therefore, is not relevant to the $gg \to W^+W^-$ process. Effectively we have:

$$(D^\mu H)^\dagger (D_\nu H) = M_W^2 \begin{pmatrix} W^{\mu -} & \dots \end{pmatrix} \begin{pmatrix} W_\nu^+ \\ \dots \end{pmatrix} = M_W^2 (W^{\mu -} W_\nu^+ + \dots) = M_W^2 W^{\mu -} W_\nu^+ \tag{2.13}$$

Substituting these into the expressions for operators $\mathcal{O}_5$ and $\mathcal{O}_5$ gives:

$$\mathcal{O}_5 = M_W^2 G^a_{\mu\rho} G^{a,\rho\nu} W^{\mu -} W_\nu^+ \,, \qquad \mathcal{O}_6 = M_W^2 G^a_{\mu\nu} G^{a,\mu\nu} W^{\rho -} W_\rho^+ \,. \tag{2.14}$$

The full group of contributing CP-even operators in appropriate form can now be given:

$$\begin{aligned} \mathcal{O}_1 &= G^a_{\mu\nu} G^{a,\rho\sigma} W^{I,\mu\nu} W^I_{\rho\sigma} \,, & \mathcal{O}_2 &= G^a_{\mu\nu} G^{a,\mu\nu} W^{I,\rho\sigma} W^I_{\rho\sigma} \,, \\ \mathcal{O}_3 &= G^a_{\mu\nu} \tilde{G}^{a,\mu\nu} W^{I,\rho\sigma} \tilde{W}^I_{\rho\sigma} \,, & \mathcal{O}_4 &= G^a_{\mu\nu} G^{a,\rho\sigma} \tilde{W}^{I,\mu\nu} \tilde{W}^I_{\rho\sigma} \,, \\ \mathcal{O}_5 &= M_W^2 G^a_{\mu\rho} G^{a,\rho\nu} W^{\mu -} W_\nu^+ \,, & \mathcal{O}_6 &= M_W^2 G^a_{\mu\nu} G^{a,\mu\nu} W^{\rho -} W_\rho^+ \,. \end{aligned} \tag{2.15}$$

2.1.2 The Feynman Rules

Feynman rules are objects which we can use as tools to compute the partial transition amplitudes for a given operator.[3] A resource to how these Feynman rules are computed is provided [5]. The Feynman rules in this project were computed by Andrea Banfi and Adam Martin. Further details as to how these objects are computed is not given but it is important to know how to use them. Contracting a Feynman rule,[4] $\mathcal{M}^{(i)}_{\mu_1\mu_2\mu_3\mu_4}$, with the polarisation vectors of the involved particles computes the corresponding (partial) transition amplitude for the chosen polarisation states:[5]

$$\mathcal{M}^{(i)}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} = \mathcal{M}^{(i)}_{\mu_1\mu_2\mu_3\mu_4} \varepsilon^{\mu_1}_{\lambda_1}(p_1) \varepsilon^{\mu_2}_{\lambda_2}(p_2) \varepsilon^{\mu_3}_{\lambda_3}(p_3) \varepsilon^{\mu_4}_{\lambda_4}(p_4) \tag{2.16}$$

Since we have no knowledge of the polarisation states of the gluons or W bosons, this amplitude can then be summed over for all permutations of the polarisation states and we find the total transition amplitude:

$$\mathcal{M}^{(i)}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} = \mathcal{M}^{(i)}_{\mu_1\mu_2\mu_3\mu_4} \varepsilon^{\mu_1}_{\lambda_1}(p_1) \varepsilon^{\mu_2}_{\lambda_2}(p_2) \varepsilon^{\mu_3}_{\lambda_3}(p_3) \varepsilon^{\mu_4}_{\lambda_4}(p_4) \tag{2.17}$$

The Feynman rules were contributed by Andrea Banfi and Adam Martin. In unitary gauge, the Feynman rules corresponding to each operator are:

[3] The use of Feynman rules is not the only method for calculating the transition amplitudes. However, this is the method chosen in this project.

[4] It is important to note that the Feynman rules have traditional indices μ_i

[5] The saturation shown is for a four particle process, which is the case for $gg \to WW$, which we are studying

$$
\begin{aligned}
&\mathcal{O}_1 : 8i\,\frac{c_1^{(GW)}}{\Lambda^4}\delta_{a_1a_2}\left[(p_1^{\mu_3}p_3^{\mu_1}-\eta^{\mu_1\mu_3}(p_1p_3))(p_2^{\mu_4}p_4^{\mu_2}-\eta^{\mu_2\mu_4}(p_2p_4))\right.\\
&\qquad\qquad\left.+\,(p_1^{\mu_4}p_4^{\mu_1}-\eta^{\mu_1\mu_4}(p_1p_4))(p_2^{\mu_3}p_3^{\mu_2}-\eta^{\mu_2\mu_3}(p_2p_3))\right]\\
&\mathcal{O}_2 : 16i\,\frac{c_2^{(GW)}}{\Lambda^4}\delta_{a_1a_2}(p_1^{\mu_2}p_2^{\mu_1}-\eta^{\mu_1\mu_2}(p_1p_2))(p_3^{\mu_4}p_4^{\mu_3}-\eta^{\mu_3\mu_4}(p_3p_4))\\
&\mathcal{O}_3 : 16i\,\frac{c_3^{(GW)}}{\Lambda^4}\delta_{a_1a_2}\epsilon^{\mu_1\mu_2}{}_{\alpha\beta}\,\epsilon^{\mu_3\mu_4}{}_{\gamma\delta}\,p_1^{\alpha}p_2^{\beta}p_3^{\gamma}p_4^{\delta}\\
&\mathcal{O}_4 : 8i\,\frac{c_4^{(GW)}}{\Lambda^4}\delta_{a_1a_2}\left[\,\epsilon^{\mu_1\mu_3}{}_{\alpha\beta}\,\epsilon^{\mu_2\mu_4}{}_{\gamma\delta}\,p_1^{\alpha}p_3^{\beta}p_2^{\gamma}p_4^{\delta}+\epsilon^{\mu_1\mu_4}{}_{\alpha\beta}\,\epsilon^{\mu_2\mu_3}{}_{\gamma\delta}\,p_1^{\alpha}p_4^{\beta}p_2^{\gamma}p_3^{\delta}\right]\\
&\mathcal{O}_5 : i\,\frac{c_5^{(GW)}}{\Lambda^4}\delta_{a_1a_2}M_W^2\left[\,((p_1p_2)\eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4}+\eta^{\mu_1\mu_2}p_1^{\mu_3}p_2^{\mu_4}-\eta^{\mu_1\mu_3}p_1^{\mu_2}p_2^{\mu_4}-\eta^{\mu_2\mu_4}p_1^{\mu_3}p_2^{\mu_1})\right.\\
&\qquad\qquad\left.+\,((p_1p_2)\eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3}+\eta^{\mu_1\mu_2}p_1^{\mu_4}p_2^{\mu_3}-\eta^{\mu_1\mu_4}p_1^{\mu_2}p_2^{\mu_3}-\eta^{\mu_2\mu_3}p_1^{\mu_4}p_2^{\mu_1})\right]\\
&\mathcal{O}_6 : 4i\,\frac{c_6^{(GW)}}{\Lambda^4}\delta_{a_1a_2}M_W^2(p_1^{\mu_2}p_2^{\mu_1}-(p_1p_2)\eta^{\mu_1\mu_2})\eta^{\mu_3\mu_4}
\end{aligned}
\tag{2.18}
$$

We denote by $M_{\lambda_1,\lambda_2,\lambda_3,\lambda_4}$ the scattering amplitude corresponding to polarisations $\lambda_1,\lambda_2,\lambda_3,\lambda_4$, with $\lambda_1,\lambda_2=\pm$ and $\lambda_3,\lambda_4=\pm,0$. This is given by:

$$
M_{\lambda_1,\lambda_2,\lambda_3,\lambda_4}=i\frac{\delta_{a_1a_2}}{\Lambda^4}\sum_i c_i^{(GW)}\mathcal{M}^{(i)}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4}\,, \tag{2.19}
$$

where $\mathcal{M}^{(i)}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4}$ is the partial amplitude contribution of operator $\mathcal{O}_i$ to the total amplitude. The terms a_1 and a_2 refer to the gluon colour charges. The Kronecker-delta term, $\delta_{a_1a_2}$, ensures that only gluons of the same colour charge can fuse in the process. If this factor were not included, the vertex could have non-zero colour charge. This would require the W bosons to carry colour in order to conserve colour charge, which is forbidden.

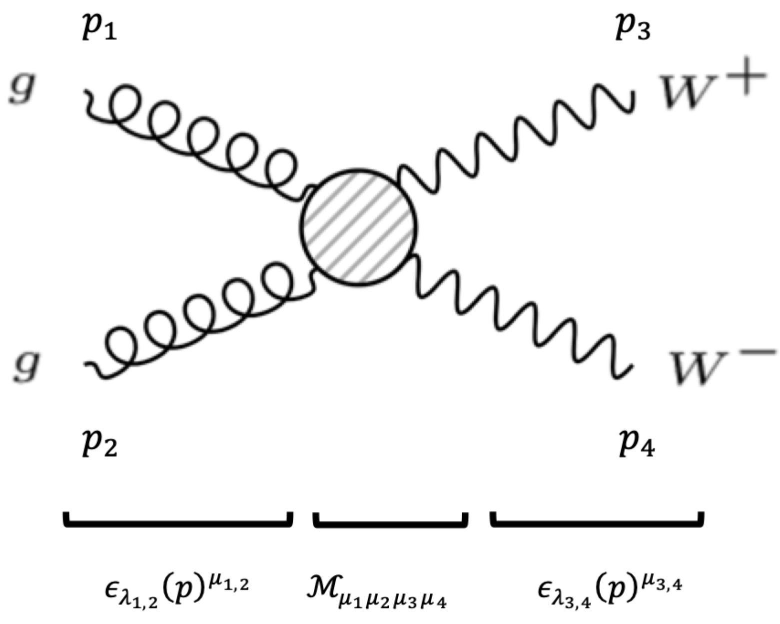

Figure 2.1: Feynman diagram showing the components of the amplitude, M, for $gg\to WW$ via dim-8 contributions

To calculate $\mathcal{M}^{(i)}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4}$ we must take the Feynman rule, $\mathcal{M}^{(i)}_{\mu_1\mu_2\mu_3\mu_4}$, and contract with the polarisation vectors of the particles. These Feynman rules are constructed under the convention that all particles are incoming which is why we use the complex conjugate of polarisation vectors for 3 and 4 as such:

$$\mathcal{M}^{(i)}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} = \mathcal{M}^{(i)}_{\mu_1\mu_2\mu_3\mu_4}\varepsilon^{\mu_1}_{\lambda_1}(p_1)\varepsilon^{\mu_2}_{\lambda_2}(p_2)\varepsilon^{\mu_3}_{\lambda_3}(p_3)^*\varepsilon^{\mu_4}_{\lambda_4}(p_4)^* \tag{2.20}$$

2.2 Computation of the Amplitudes

2.2.1 Relativistic Kinematics

Here we will discuss the generalised kinematics of the process. It is important that in our definitions of the momenta that we ensure any possible final state can be written. The process can be defined to occur on plane defined by a constant azimuthal angle ϕ with respect to both the gluon pair and the W boson pair. A kinematic diagram of the interaction is shown:

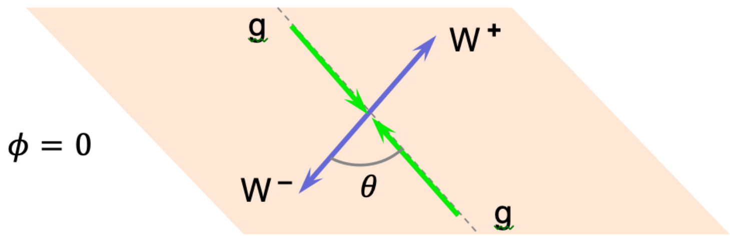

Figure 2.2: Kinematic diagram showing of the $gg \to WW$ process in the CoM frame

Momenta

We work in the centre of mass frame of the gluon pair and in the azimuthal $\phi = 0$ plane, since this is arbitrary. The 4-momenta of the gluons, 1 and 2, and W bosons, 3 and 4 are given by:[6]

$$\begin{aligned} p_1 =& \frac{\sqrt{s}}{2}(1,0,0,1)\,, & p_2 =& \frac{\sqrt{s}}{2}(1,0,0,-1)\,, \\ p_3 =& \frac{\sqrt{s}}{2}(1,0,\beta\sin(\theta),\beta\cos(\theta))\,, & p_4 =& \frac{\sqrt{s}}{2}(1,0,-\beta\sin(\theta),-\beta\cos(\theta))\,, \end{aligned} \tag{2.21}$$

Polarisation Vectors

The gluons can only have polarisation states $\lambda = +,-$ since they are massless. The W bosons however can have polarisation states $\lambda = +,0,-$. The polarisation vectors are functions of the particle momenta. The polarisation vectors for these momenta are given by [5]:[7]

[6]The quantity s is the Mandelstam variable for the centre of mass energy: $s = (p_1+p_2)^2 = (p_3+p_4)^2$ and $\beta = \frac{p}{E}$, E is the energy of each particle and p is the 3-momentum magnitude for the W bosons in the CoM frame. It can be easily verified from the given definitions that these substitution quantities are equal

[7]We choose to use the $\gamma = -\phi$ convention. Details of which are provided within the reference.

$$
\begin{aligned}
\varepsilon_{\pm}(p_1) &= -\frac{1}{\sqrt{2}}(0,\pm 1, i, 0)\,, & \varepsilon_{\pm}(p_2) &= -\frac{1}{\sqrt{2}}(0,\pm 1, -i, 0)\,,\\
\varepsilon_{\pm}(p_3) &= -\frac{1}{\sqrt{2}}(0,\pm 1, i\cos(\theta), -i\sin(\theta))\,, & \varepsilon_{\pm}(p_4) &= -\frac{1}{\sqrt{2}}(0,\pm 1, -i\cos(\theta), i\sin(\theta))\,,\\
\varepsilon_{0}(p_3) &= \frac{\sqrt{s}}{2M_W}(\beta, 0, \sin(\theta), \cos(\theta))\,, & \varepsilon_{0}(p_4) &= \frac{\sqrt{s}}{2M_W}(\beta, 0, -\sin(\theta), -\cos(\theta))\,.
\end{aligned}
\tag{2.22}
$$

2.2.2 Amplitudes

The amplitudes were calculated by computing eq. (2.20) in Mathematica using the Feyncalc package [14, 15]. The Mathematica notebook, **gg_WW_amplitudes.nb**, is included in the GitHub repository within the appendix. We make the substitution: $\cos\theta \to \frac{t-u}{\beta s}$ and therefore also:[8] $\sin\theta \to \sqrt{\frac{\beta^2 s^2-(t-u)^2}{\beta^2 s^2}}$

M1

$$
\begin{aligned}
\mathcal{M}^{(1)}_{\pm\pm\pm\pm} &= \frac{(\beta-1)^2\left(\beta^2 s^2+(t-u)^2\right)}{4\beta^2}\\
\mathcal{M}^{(1)}_{\mp\pm\pm\pm} &= \mathcal{M}^{(1)}_{\pm\mp\pm\pm} = \mathcal{M}^{(1)}_{\pm\pm\mp\pm} = \mathcal{M}^{(1)}_{\pm\pm\pm\mp} = \frac{M_W^2\left((t-u)^2-\beta^2 s^2\right)}{\beta^2 s}\\
\mathcal{M}^{(1)}_{\mp\mp\pm\pm} &= \frac{(\beta+1)^2\left(\beta^2 s^2+(t-u)^2\right)}{4\beta^2}\\
\mathcal{M}^{(1)}_{\pm\mp\pm\mp} &= \frac{\left(\beta^2+1\right)(\beta s+t-u)^2}{4\beta^2}\\
\mathcal{M}^{(1)}_{\pm\mp\mp\pm} &= \frac{\left(\beta^2+1\right)(\beta s-t+u)^2}{4\beta^2}\\
\mathcal{M}^{(1)}_{\pm\pm 0\pm} &= \mathcal{M}^{(1)}_{\pm\pm\pm 0} = \frac{iM_W(\beta-1)(t-u)\sqrt{s^2\beta^2-(t-u)^2}}{\sqrt{2s}\beta^2}\\
\mathcal{M}^{(1)}_{\mp\pm 0\pm} &= \mathcal{M}^{(1)}_{\pm\mp\pm 0} = -\frac{iM_W(\beta s+t-u)\sqrt{(\beta^2 s^2-(t-u)^2}}{\sqrt{2s}\beta^2}\\
\mathcal{M}^{(1)}_{\pm\mp 0\pm} &= \mathcal{M}^{(1)}_{\mp\pm\pm 0} = \frac{iM_W(\beta s-t+u)\sqrt{(\beta^2 s^2-(t-u)^2}}{\sqrt{2s}\beta^2}\\
\mathcal{M}^{(1)}_{\pm\pm 0\mp} &= \mathcal{M}^{(1)}_{\pm\pm\mp 0} = -\frac{iM_W(\beta+1)(t-u)\sqrt{\beta^2 s^2-(t-u)^2}}{\sqrt{2s}\beta^2}\\
\mathcal{M}^{(1)}_{\pm\pm 00} &= \mathcal{M}^{(1)}_{\pm\mp 00} = \frac{2M_W^2\left((t-u)^2-\beta^2 s^2\right)}{s\beta^2}
\end{aligned}
\tag{2.23}
$$

[8] The quantities t and u are also Mandelstam variables: $t=(p_1-p_3)^2=(p_4-p_1)^2$, $u=(p_1-p_4)^2=(p_3-p_2)^2$. Mandelstam variables encode the energy, momentum, and angles of particles in a scattering process and are Lorentz-invariant.

M2

$$\begin{aligned}\mathcal{M}^{(2)}_{\pm\pm\pm\pm} =& \mathcal{M}^{(2)}_{\pm\pm\mp\mp} = 2\left(\beta^2+1\right)s^2 \\ \mathcal{M}^{(2)}_{\pm\pm\pm\pm} =& -8M_W^2 s\end{aligned} \tag{2.24}$$

M3

$$\begin{aligned}\mathcal{M}^{(3)}_{\pm\pm\pm\pm} =& 4\beta s^2 \\ \mathcal{M}^{(3)}_{\pm\pm\mp\mp} =& -4\beta s^2\end{aligned} \tag{2.25}$$

M4

$$\begin{aligned}
\mathcal{M}^{(4)}_{\pm\pm\pm\pm} =& -\frac{(\beta-1)^2\left(\beta^2 s^2+(t-u)^2\right)}{4\beta^2} \\
\mathcal{M}^{(4)}_{\mp\pm\pm\pm} =& \mathcal{M}^{(4)}_{\pm\mp\pm\pm} = \frac{M_W^2\left((t-u)^2-\beta^2 s^2\right)}{\beta^2 s} \\
\mathcal{M}^{(4)}_{\pm\pm\mp\pm} =& \mathcal{M}^{(4)}_{\pm\pm\pm\mp} = -\frac{M_W^2\left((t-u)^2-\beta^2 s^2\right)}{\beta^2 s} \\
\mathcal{M}^{(4)}_{\pm\pm\mp\mp} =& -\frac{(\beta+1)^2\left(\beta^2 s^2+(t-u)^2\right)}{4\beta^2} \\
\mathcal{M}^{(4)}_{\pm\mp\pm\mp} =& \frac{\left(\beta^2+1\right)(\beta s+t-u)^2}{4\beta^2} \\
\mathcal{M}^{(4)}_{\pm\mp\mp\pm} =& \frac{\left(\beta^2+1\right)(\beta s-t+u)^2}{4\beta^2} \\
\mathcal{M}^{(4)}_{\pm\pm 0\pm} =& \mathcal{M}^{(4)}_{\pm\pm\pm 0} = -\frac{iM_W(\beta-1)(t-u)\sqrt{\beta^2 s^2-(t-u)^2}}{\sqrt{2s}\beta^2} \\
\mathcal{M}^{(4)}_{\mp\pm 0\pm} =& \mathcal{M}^{(4)}_{\pm\mp\pm 0} = -\frac{iM_W(\beta s+t-u)\sqrt{\beta^2 s^2-(t-u)^2}}{\sqrt{2s}\beta^2} \\
\mathcal{M}^{(4)}_{\pm\mp 0\pm} =& \mathcal{M}^{(4)}_{\mp\pm\pm 0} = \frac{iM_W(\beta s+t-u)\sqrt{\beta^2 s^2-(t-u)^2}}{\sqrt{2s}\beta^2} \\
\mathcal{M}^{(4)}_{\pm\pm 00} =& -\frac{2M_W^2\left((t-u)^2-\beta^2 s^2\right)}{s\beta^2} \\
\mathcal{M}^{(4)}_{\pm\mp 00} =& \frac{2M_W^2\left((t-u)^2-\beta^2 s^2\right)}{s\beta^2}
\end{aligned} \tag{2.26}$$

M5

$$
\begin{aligned}
\mathcal{M}^{(5)}_{\pm\pm\pm\pm} =& \frac{M_W^2 \left(3\beta^2 s^2 + (t-u)^2\right)}{8\beta^2 s} \\
\mathcal{M}^{(5)}_{\mp\pm\pm\pm} =& \mathcal{M}^{(5)}_{\pm\mp\pm\pm} = -\frac{M_W^2 (\beta^2 s^2 - (t-u)^2)}{4\beta^2 s} \\
\mathcal{M}^{(5)}_{\pm\pm\mp\pm} =& \mathcal{M}^{(5)}_{\pm\pm\pm\mp} = \frac{M_W^2 \left((t-u)^2 - \beta^2 s^2\right)}{8\beta^2 s} \\
\mathcal{M}^{(5)}_{\pm\pm\pm\pm} =& \frac{M_W^2 \left(3\beta^2 s^2 + (t-u)^2\right)}{8\beta^2 s} \\
\mathcal{M}^{(5)}_{\pm\mp\pm\mp} =& \frac{M_W^2 (\beta s + t - u)^2}{4\beta^2 s} \\
\mathcal{M}^{(5)}_{\pm\mp\mp\pm} =& \frac{M_W^2 (\beta s - t + u)^2}{4\beta^2 s} \\
\mathcal{M}^{(5)}_{\pm\pm 0\pm} =& \mathcal{M}^{(5)}_{\pm\pm\pm 0} = \mathcal{M}^{(5)}_{\pm\pm 0\mp} = \mathcal{M}^{(5)}_{\pm\pm\mp 0} = -\frac{i M_W \sqrt{\beta^2 s^2 - (t-u)^2} \left(\beta^2 s + t - u\right)}{8\sqrt{2s}\beta^2} \\
\mathcal{M}^{(5)}_{\mp\pm 0\pm} =& \mathcal{M}^{(5)}_{\mp\pm\pm 0} = -\frac{i M_W \sqrt{\beta^2 s^2 - (t-u)^2} (\beta s + t - u)}{4\sqrt{2s}\beta^2} \\
\mathcal{M}^{(5)}_{\pm\mp 0\pm} =& \mathcal{M}^{(5)}_{\mp\pm\pm 0} = \frac{i M_W \sqrt{\beta^2 s^2 - (t-u)^2} (\beta s - t + u)}{4\sqrt{2s}\beta^2} \\
\mathcal{M}^{(5)}_{\pm\pm 00} =& \frac{(t-u)^2 - 2\beta^2 s(s - t + u) - \beta^4 s^2}{16\beta^2} \\
\mathcal{M}^{(5)}_{\pm\mp 00} =& \frac{1}{8\beta^2} \left((t-u)^2 - \beta^2 s^2\right)
\end{aligned}
\tag{2.27}
$$

M6

$$
\begin{aligned}
\mathcal{M}^{(6)}_{\pm\pm\pm\pm} =& \mathcal{M}^{(6)}_{\pm\pm\mp\mp} = -2M_W^2 s \\
\mathcal{M}^{(6)}_{\pm\pm 00} =& \frac{1}{2} \left(\beta^2 + 1\right) s^2
\end{aligned}
\tag{2.28}
$$

Concluding Remarks

These reported amplitudes can be utilised in numerical studies by suitably multiplying by the amplitude of $W \to l\nu$ and summing over W polarisations. This, however, does not match the convention used by the program we will be using, MCFM. This uses the Spinor Helicity Formalism. Therefore we choose to use the latter and keep these results as a sanity check and as a reference result for other researchers. In the next section we reformulate these amplitudes in terms of spinor helicity products.

Chapter 3

Numerical results via W decay into leptons

In this chapter we focus on the computation of $gg \to WW \to l\nu_l l'\nu_{l'}$ with the intention of generating numerical results with the included dim-8 amplitudes which can be verified experimentally.

3.1 WW decay into Leptons

W bosons are not directly observed in particle detectors. In order to generate useful numerical results to compare to experimental findings we must also account for the W boson decay. If the predicted $gg \to WW$ dim-8 process does occur with non-zero dimensionless constants c_i then we should see a resultant deviation in lepton production cross sections in processes such as the $p\bar{p}$ collisions studied at the LHC. Protons are bound together by gluons and so this is a possible method of detection of the $gg \to WW$ process.

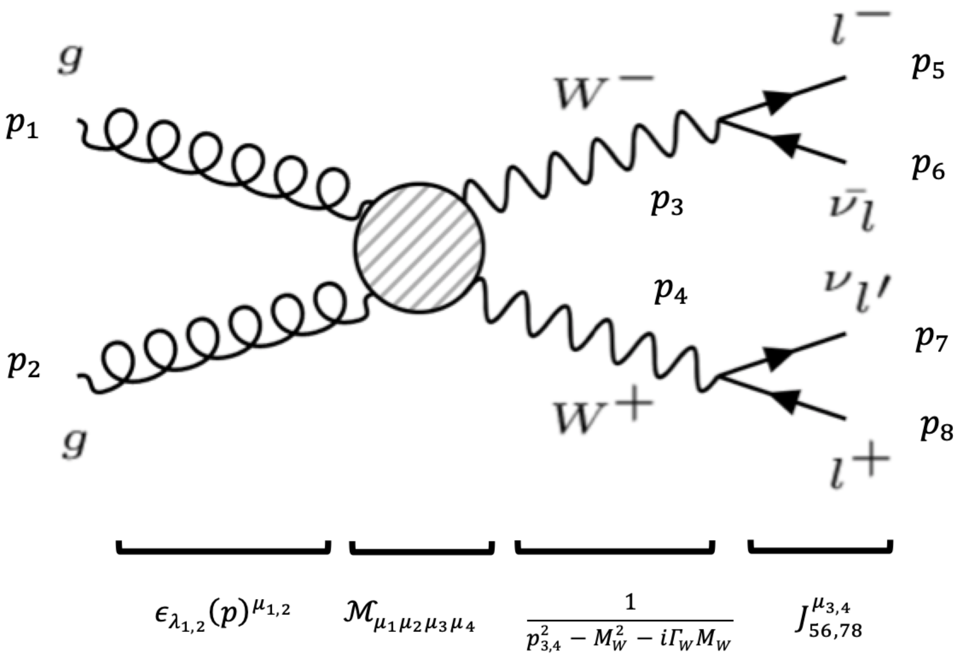

Figure 3.1: Feynman diagram showing the $gg \to W^+(l^+\nu)W^-(l^-\bar{\nu})$ process and terms that are used to compute M

3.1.1 Reformulation of the Amplitudes

W bosons only interact with left-handed lepton and right-handed antileptons and so the overall amplitude becomes only dependant on λ_1 and λ_2 (because final λ states are fixed). For this overall process, we must contract the Feynman rule with the polarisation vectors of the gluons and with the leptonic currents, J_{56} and J_{78}. Thus, the Amplitude associated to a given operator i is now given by:

$$\mathcal{M}^{(i)}_{\lambda_1,\lambda_2} = P\mathcal{M}^{(i)}_{\mu_1\mu_2\mu_3\mu_4}\varepsilon^{\mu_1}_{\lambda_1}(p_1)\varepsilon^{\mu_2}_{\lambda_2}(p_2)J^{\mu_3}_{56}J^{\mu_4}_{78}\,, \tag{3.1}$$

where P is the propagator factor associated with the intermediate W boson states and relevant constants. It is given by:

$$P = \frac{g_w^2}{2((p_5+p_6)^2 - M_W^2 - i\Gamma_W M_W)((p_7+p_8)^2 - M_W^2 - i\Gamma_W M_W)} \tag{3.2}$$

3.1.2 Amplitudes using Spinor Helicity Formalism

There is a formalism that allows to compute the amplitudes corresponding to given helicities of fermions in the final state in a very efficient way. This is called the Spinor Helicity Formalism. We choose to use the Spinor Helicity Formalism (SHF) to represent the particles. We do this in order to most easily implement the amplitude into the program, MCFM, which has many inbuilt features for SHF. MCFM will allow us to generate numerical results for the new amplitudes. The details of SHF are beyond the scope of this report and thus are omitted. These are well established results that are detailed in many academic references. For SHF I have used the academic references [16, 17]. In this formalism, the expressions polarisation vectors and leptonic currents are given as follows:

$$\begin{aligned}
\varepsilon^{\mu_1}_{+}(p_1) &= \frac{[1\,|\gamma^{\mu_1}|\,b_1\rangle}{\sqrt{2}\,\langle b_1\,1\rangle} & \varepsilon^{\mu_1}_{-}(p_1) &= \frac{\langle 1\,|\gamma^{\mu_1}|\,b_1]}{\sqrt{2}\,[b_1\,1]}\,,\\
\varepsilon^{\mu_2}_{+}(p_2) &= \frac{[2\,|\gamma^{\mu_2}|\,b_2\rangle}{\sqrt{2}\,\langle b_2\,2\rangle} & \varepsilon^{\mu_2}_{-}(p_2) &= \frac{\langle 2\,|\gamma^{\mu_2}|\,b_2]}{\sqrt{2}\,[b_2\,2]}\,,\\
J^{\mu_3}_{5,6} &= \langle 5\,|\gamma^{\mu_3}|\,6]\,, & J^{\mu_4}_{7,8} &= \langle 7\,|\gamma^{\mu_4}|\,8]\,.
\end{aligned} \tag{3.3}$$

The amplitudes are, again, computed on Mathematica with both the FeynCalc [14, 15] and the Spinors@Mathematica packages [18]. The notebook used to compute these is **gg_WW_decay.nb**. This is included within the appendix fig. A.0.3. It is timesaving to note:[1] $\mathcal{M}^{(i)}_{--} = \overline{\mathcal{M}}^{(i)}_{++}$ and $\mathcal{M}^{(i)}_{-+} = \overline{\mathcal{M}}^{(i)}_{+-}$ and therefore only $\mathcal{M}^{(i)}_{++}$ and $\mathcal{M}^{(i)}_{+-}$ are computed. The SHF amplitudes are not written into this report but are easily comprehensible from the aforementioned notebook file. In order to interpret the results there is some notation to note:

$$\langle i\,j\rangle = \sqrt{s_{i,j}}e^{i\phi_{i,j}}\,, \qquad [i\,j] = \sqrt{s_{i,j}}e^{-i\phi_{i,j}}\,, \tag{3.4}$$

where $s_{i,j} = 2p_i\cdot p_j$. This is the magnitude of a complex number with phase $\phi_{i,j}$. The phase $\phi_{i,j}$ is more complicated and so details are not included in the report. The key takeaway is that $\langle i\,j\rangle$ and $[i\,j]$ are complex numbers. **Please view gg_WW_decay.nb to find the calculated amplitudes.**

[1]The overline represents taking the complex conjugate.

3.2 Implementation into MCFM

MCFM is a particle physics program [2, 3]. The computed amplitudes have been programmed into a relevant existing MCFM file, **gg_WW_int.f**. An additional module file **gg_w_dim8.f90** has also been written. This module is responsible for defining the amplitude functions used in the implementation within **gg_WW_int.f**. These additional reported partial amplitudes are then simply appended to the list of existing partial amplitudes already included within the file corresponding to the $gg \to WW$ process. These files are included in the appendix and the originality of the work is detailed. It should be noted that in the MCFM implementation, the momenta 5,6,7,8 defined in fig. 3.1 are relabelled 3,4,5,6 in order to fit with the MCFM notation. The intermediate W boson 4-momenta are defined in terms of their decay product 4-momenta.

We have performed checks such as ensuring $\mathcal{O}_6$ has the same structure as the dim-6 Higgs contribution to $gg \to WW$ and we will use this fix normalisation and conventions. The next stage of this project will be to produce plots. We are very close to being able to run our implementation within MCFM (essentially just press 'run'). The last step will be to produce sensitivity plots showing what fraction of the parameter space can be excluded given data distribution according to the Standard Model.

Chapter 4

Conclusion

What I have discussed in this report is the core of how to perform research beyond the Standard Model via the methods of Standard Model Effective Field Theory. A concise summary of Standard Model field theory has been given in Chapter 1. Further, the methods and merits of Standard Model Effective Field Theory have been described. In Chapter 2, a full list of dimension-8 operators contributing to the $gg \to WW$ process have been given and explained. The partial amplitudes corresponding to these operators have been computed using their Feynman rules have been reported for the first time. In Chapter 3, these amplitudes have been reformulated into the Spinor Helicity Formalism. These were then implemented into the files of MCFM. With a bit more work, the program can be used to simulate the $gg \to WW$ and we should be able to predict the resultant phenomenology which will be dependant on the unknown parameters c_i and the unknown scale of new physics Λ.

The next step of this project is to produce sensitivity plots showing what fraction of the parameter space can be excluded, given data distribution according to the Standard Model. We can then perform phenomenology studies on the numerical results. We hope to see non-zero coefficients for these partial amplitudes in future experimental data. This was the most work that could be completed for the report given the time constraints.

Upon reflection, this project has been both extremely fascinating as well as challenging. When Andrea first told me I would have the opportunity to contribute to such a cutting edge research project I was extremely excited. I was also quite unsure at the beginning how I would be able to contribute in any meaningful way to research on the frontier of Particle Physics. Andrea, has done a wonderful job of illuminating many intricacies of the topic to me. If I could give advice to myself; or any other student beginning a project; it would be this: don't be afraid to ask stupid questions. I learned a lot more in project meetings in the second half of the year than the first simply by asking more questions (which admittedly sometimes had obvious answers). Growing in confidence within the project allowed me to both contribute more and gain more. I wish we had been able to make that little bit more progress on the project because the conclusion stage is always the most revealing and satisfying. However, it is definitely sensible to take ones time with research such as this, so that results can be presented with confidence that they are correct and as detailed as possible. I hope to continue contributing to the project.

Acknowledgements

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Appendix A

Code

A.0.1 gg_ww_dim.f90

This file is entirely original and will be added to the MCFM directory **src/HWW** It is responsible for defining the amplitude functions that we have reported and will be used in the implementation. These functions were written in the Fortran 90 language using MCFM notation by myself. These were then collaboratively constructed into this module with my project supervisor. To help interpret the file: A1pp defines the function defining $\mathcal{M}^{(1)}_{+,+}$; A3pm defines to the function defining $\mathcal{M}^{(3)}_{+,-}$ etc. The terms za(i,j) is the MCFM language denoting $\langle i\, j\rangle$ and zb(i,j) denoting $[i\, j]$ in SHF. It should be noted that in the MCFM implementation, the momenta 5,6,7,8 labels we have used are relabelled 3,4,5,6 in order to fit with their notation.

```
module gg_ww_dim8
  implicit none
  include 'types.f'

  private

  public :: A1pp,A2pp,A3pp,A4pp,A5pp,A6pp
  public :: A1pm,A3pm,A4pm,A5pm

contains

  function A1pp(za,zb) result(res)
    complex(dp) :: za(:,:), zb(:,:)
    complex(dp) :: res

    res = ((za(1,4)*za(2,3)-za(1,3)*za(2,4))*&
         &(za(1,6)*za(2,5)-za(1,5)*za(2,6))*&
         &(zb(4,2)**2*zb(6,1)**2+zb(4,1)**2*zb(6,2)**2))/za(1,2)**2

  end function A1pp

  function A2pp(za,zb) result(res)
    complex(dp) :: za(:,:), zb(:,:)
    complex(dp) :: res

    res = -(zb(2,1)**2*((za(3,5)*zb(5,4)+za(3,6)*zb(6,4))*&
         &(za(3,5)*zb(6,3)+za(4,5)*zb(6,4))+zb(5,3)*zb(6,4)*&
```

```
       &(za(3,5)*zb(5,3)+za(4,5)*zb(5,4)+za(3,6)*zb(6,3)+za(4,6)*zb(6,4)))

end function A2pp

function A3pp(za,zb) result(res)
  complex(dp) :: za(:,:), zb(:,:)
  complex(dp) :: res

  res =(za(1,5)**2*za(2,3)**2*(zb(3,2)*zb(4,1)-zb(3,1)*zb(4,2))*&
       &(zb(5,2)*zb(6,1)-zb(5,1)*zb(6,2))+za(2,5)*(za(2,5)*za(1,3)**2*&
       &(zb(3,2)*zb(4,1)-zb(3,1)*zb(4,2))*(zb(5,2)*zb(6,1)-zb(5,1)*zb(6,2))&
       &+za(1,6)*za(2,4)*za(1,3)*(zb(4,2)*zb(6,1)-zb(4,1)*zb(6,2))**2&
       &-za(1,4)*za(1,6)*za(2,3)*(zb(4,2)*zb(6,1)-zb(4,1)*zb(6,2))**2)&
       &+za(1,5)*(za(1,4)*za(2,3)*za(2,6)*(zb(4,2)*zb(6,1)-zb(4,1)&
       &*zb(6,2))**2-za(1,3)*(za(2,4)*za(2,6)*(zb(4,2)*zb(6,1)&
       &-zb(4,1)*zb(6,2))**2+2*za(2,3)*za(2,5)*(zb(3,2)*zb(4,1)&
       &-zb(3,1)*zb(4,2))*(zb(5,2)*zb(6,1)-zb(5,1)*zb(6,2)))))/za(1,2)**2

end function A3pp

function A4pp(za,zb) result(res)
  complex(dp) :: za(:,:), zb(:,:)
  complex(dp) :: res

  res = (-za(2,3)*za(2,5)*zb(4,2)*(za(1,3)*zb(3,2)+za(1,4)*zb(4,2))&
       &*zb(6,1)*(za(1,5)*zb(5,1)+za(1,6)*zb(6,1))+za(1,5)*za(2,3)&
       &*zb(4,2)*(za(1,3)*zb(3,2)+za(1,4)*zb(4,2))*zb(6,1)&
       &*(za(2,5)*zb(5,1)+za(2,6)*zb(6,1))-za(2,3)*za(2,5)*zb(4,1)&
       &*(za(1,3)*zb(3,1)+za(1,4)*zb(4,1))*zb(6,2)*(za(1,5)*zb(5,2)&
       &+za(1,6)*zb(6,2))+za(1,3)*za(2,5)*zb(4,1)*(za(2,3)*zb(3,1)&
       &+za(2,4)*zb(4,1))*zb(6,2)*(za(1,5)*zb(5,2)+za(1,6)*zb(6,2))&
       &+za(1,5)*za(2,3)*zb(4,1)*(za(1,3)*zb(3,1)+za(1,4)*zb(4,1))&
       &*zb(6,2)*(za(2,5)*zb(5,2)+za(2,6)*zb(6,2))-za(1,3)*za(1,5)&
       &*zb(4,1)*(za(2,3)*zb(3,1)+za(2,4)*zb(4,1))*zb(6,2)*(za(2,5)&
       &*zb(5,2)+za(2,6)*zb(6,2))+za(1,2)*zb(2,1)*(za(1,3)*zb(3,2)&
       &+za(1,4)*zb(4,2))*zb(5,3)*(za(2,5)*zb(5,1)+za(2,6)*zb(6,1))&
       &*zb(6,4)+za(1,2)*zb(2,1)*(za(2,3)*zb(3,1)+za(2,4)*zb(4,1))&
       &*zb(5,3)*(za(1,5)*zb(5,2)+za(1,6)*zb(6,2))*zb(6,4)+za(1,2)&
       &*za(2,3)*zb(2,1)*zb(4,1)*(za(1,5)*zb(5,2)+za(1,6)*zb(6,2))&
       &*(za(3,5)*zb(6,3)+za(4,5)*zb(6,4))-za(1,2)*za(1,5)*za(2,3)&
       &*zb(2,1)*zb(4,1)*zb(6,2)*(za(3,5)*zb(5,3)+za(4,5)*zb(5,4)&
       &+za(3,6)*zb(6,3)+za(4,6)*zb(6,4))-za(1,2)*zb(2,1)*(za(3,5)&
       &*zb(5,4)+za(3,6)*zb(6,4))*(-za(2,5)*(za(1,3)*zb(3,2)+za(1,4)&
       &*zb(4,2))*zb(6,1)+za(1,5)*(za(2,3)*zb(3,2)+za(2,4)*zb(4,2))&
       &*zb(6,1)+za(2,5)*(za(1,3)*zb(3,1)+za(1,4)*zb(4,1))*zb(6,2)&
       &-za(1,5)*(za(2,3)*zb(3,1)+za(2,4)*zb(4,1))*zb(6,2)-2*za(1,2)&
       &*zb(2,1)*(za(3,5)*zb(6,3)+za(4,5)*zb(6,4)))-za(1,2)*zb(2,1)&
       &*(zb(5,3)*zb(6,4)*((za(2,3)*zb(3,2)+za(2,4)*zb(4,2))&
       &*(za(1,5)*zb(5,1)+za(1,6)*zb(6,1))+(za(1,3)*zb(3,1)&
       &+za(1,4)*zb(4,1))*(za(2,5)*zb(5,2)+za(2,6)*zb(6,2))&
       &-2*za(1,2)*zb(2,1)*(za(3,5)*zb(5,3)+za(4,5)*zb(5,4)&
```

```
        &+za(3,6)*zb(6,3)+za(4,6)*zb(6,4)))+za(2,3)*zb(4,2)&
        &*((za(1,5)*zb(5,1)+za(1,6)*zb(6,1))*(za(3,5)*zb(6,3)&
        &+za(4,5)*zb(6,4))-za(1,5)*zb(6,1)*(za(3,5)*zb(5,3)&
        &+za(4,5)*zb(5,4)+za(3,6)*zb(6,3)+za(4,6)*zb(6,4)))&
        &+za(1,3)*zb(4,1)*((za(2,5)*zb(5,2)+za(2,6)*zb(6,2))&
        &*(za(3,5)*zb(6,3)+za(4,5)*zb(6,4))-za(2,5)*zb(6,2)&
        &*(za(3,5)*zb(5,3)+za(4,5)*zb(5,4)+za(3,6)*zb(6,3)&
        &+za(4,6)*zb(6,4))))+za(1,3)*zb(4,2)*((za(1,6)*za(2,5)-za(1,5)&
        &*za(2,6))*(za(2,3)*zb(3,2)+za(2,4)*zb(4,2))*zb(6,1)**2&
        &+za(1,2)*zb(2,1)*(za(2,6)*zb(6,1)*(za(3,5)*zb(6,3)+za(4,5)*zb(6,4))&
        &-za(2,5)*(za(3,5)*(zb(5,3)*zb(6,1)-zb(5,1)*zb(6,3))+zb(6,1)&
        &*(za(3,6)*zb(6,3)+za(4,6)*zb(6,4))+za(4,5)*(zb(5,4)&
        &*zb(6,1)-zb(5,1)*zb(6,4))))))/za(1,2)**2

end function A4pp

function A5pp(za,zb) result(res)
  complex(dp) :: za(:,:), zb(:,:)
  complex(dp) :: res

  res = (za(1,2)*za(1,3)*za(2,5)*zb(2,1)*zb(4,1)*zb(6,2)&
        &-za(1,2)*zb(2,1)*(za(1,3)*za(2,5)*zb(4,2)*zb(6,1)&
        &+za(1,5)*za(2,3)*zb(4,1)*zb(6,2)))/za(1,2)**2

end function A5pp

function A6pp(za,zb) result(res)
  complex(dp) :: za(:,:), zb(:,:)
  complex(dp) :: res

  res = zb(2,1)**2*za(3,5)*zb(6,4)

end function A6pp


!-----------------------

function A1pm(za,zb) result(res)
  complex(dp) :: za(:,:), zb(:,:)
  complex(dp) :: res

  res = -(2*(za(1,3)*za(2,4)-za(1,4)*za(2,3))*za(2,5)*zb(6,2)*zb(4,1)**2&
        &*(za(2,5)*(zb(5,1)-zb(5,2))+za(2,6)*(zb(6,1)-zb(6,2)))&
        &+2*za(2,3)*(za(1,5)*za(2,6)-za(1,6)*za(2,5))*zb(4,2)*zb(6,1)**2&
        &*(za(2,3)*(zb(3,1)-zb(3,2))+za(2,4)*(zb(4,1)-zb(4,2))))/&
        &(za(1,2)*zb(2,1))

end function A1pm

function A3pm(za,zb) result(res)
  complex(dp) :: za(:,:), zb(:,:)
```

```fortran
  complex(dp) :: res

  res = za(1,5)*za(2,3)*(za(2,3)*(za(2,6)*(zb(4,1)-zb(4,2))*zb(6,1)&
       &*(zb(3,2)*zb(6,1)-zb(3,1)*zb(6,2))+za(2,5)*(zb(3,2)*zb(4,1)&
       &*zb(5,1)*(zb(6,1)-zb(6,2))+zb(3,1)*(zb(4,2)*(zb(5,2)-zb(5,1))&
       &*zb(6,1)+zb(4,1)*(zb(5,1)*zb(6,2)-zb(5,2)*zb(6,1)))))-za(2,4)&
       &*(zb(4,1)-zb(4,2))*(za(2,6)*zb(6,1)*(zb(4,1)*zb(6,2)-zb(4,2)*zb(6,1))&
       &+za(2,5)*zb(4,1)*(zb(5,2)*zb(6,1)-zb(5,1)*zb(6,2))))+za(2,5)&
       &*(za(2,3)*(zb(4,2)*zb(6,1)-zb(4,1)*zb(6,2))*(za(1,6)*zb(6,1)&
       &*(za(2,3)*(zb(3,2)-zb(3,1))+za(2,4)*(zb(4,2)-zb(4,1)))&
       &+za(1,4)*zb(4,1)*(za(2,5)*(zb(5,1)-zb(5,2))+za(2,6)&
       &*(zb(6,1)-zb(6,2))))+za(1,3)*(za(2,4)*zb(4,1)*(zb(6,1)-zb(6,2))&
       &*(za(2,5)*(zb(4,1)*zb(5,2)-zb(4,2)*zb(5,1))+za(2,6)&
       &*(zb(4,1)*zb(6,2)-zb(4,2)*zb(6,1)))+za(2,3)*(za(2,5)&
       &*(zb(3,2)*zb(4,1)*zb(5,1)*(zb(6,2)-zb(6,1))+zb(3,1)&
       &*(zb(4,2)*(zb(5,1)-zb(5,2))*zb(6,1)+zb(4,1)&
       &*(zb(5,2)*zb(6,1)-zb(5,1)*zb(6,2))))-za(2,6)&
       &*(zb(3,2)*zb(4,1)-zb(3,1)*zb(4,2))*zb(6,1)*(zb(6,1)-zb(6,2)))))&
       &/(za(1,2)*zb(2,1))


end function A3pm

function A4pm(za,zb) result(res)
  complex(dp) :: za(:,:), zb(:,:)
  complex(dp) :: res

  res = (-za(2,3)*za(2,5)*zb(4,2)*zb(6,1)*(za(2,3)*zb(3,1)+za(2,4)*zb(4,1))&
       &*(za(1,5)*zb(5,1)+za(1,6)*zb(6,1))+za(1,5)*za(2,3)*zb(4,2)*zb(6,1)&
       &*(za(2,3)*zb(3,1)+za(2,4)*zb(4,1))*(za(2,5)*zb(5,1)+za(2,6)*zb(6,1))&
       &-za(2,3)*za(2,5)*zb(4,1)*zb(6,2)*(za(1,3)*zb(3,1)+za(1,4)*zb(4,1))&
       &*(za(2,5)*zb(5,1)+za(2,6)*zb(6,1))+za(1,3)*za(2,5)*zb(4,1)*zb(6,2)&
       &*(za(2,3)*zb(3,1)+za(2,4)*zb(4,1))*(za(2,5)*zb(5,1)+za(2,6)*zb(6,1))&
       &+za(2,3)*za(2,5)*zb(4,1)*zb(6,2)*(za(1,3)*zb(3,1)+za(1,4)*zb(4,1))&
       &*(za(2,5)*zb(5,2)+za(2,6)*zb(6,2))-za(1,3)*za(2,5)*zb(4,1)*zb(6,2)&
       &*(za(2,3)*zb(3,1)+za(2,4)*zb(4,1))*(za(2,5)*zb(5,2)+za(2,6)*zb(6,2))&
       &+2*za(1,2)*zb(2,1)*zb(5,3)*zb(6,4)*(za(2,3)*zb(3,1)+za(2,4)*zb(4,1))&
       &*(za(2,5)*zb(5,1)+za(2,6)*zb(6,1))+za(1,2)*za(2,5)*zb(2,1)&
       &*(zb(6,1)+zb(6,2))*(za(2,3)*zb(3,1)+za(2,4)*zb(4,1))&
       &*(za(3,5)*zb(5,4)+za(3,6)*zb(6,4))+za(1,2)*za(2,3)*zb(2,1)*zb(4,1)&
       &*(za(2,5)*zb(5,1)+za(2,6)*zb(6,1))*(za(3,5)*zb(6,3)+za(4,5)*zb(6,4))&
       &-za(1,2)*za(2,3)*za(2,5)*zb(2,1)*zb(4,1)*zb(6,2)&
       &*(za(3,5)*zb(5,3)+za(4,5)*zb(5,4)+za(3,6)*zb(6,3)+za(4,6)*zb(6,4))&
       &+za(2,3)*zb(4,2)*((za(1,6)*za(2,5)-za(1,5)*za(2,6))*zb(6,1)**2&
       &*(za(2,3)*zb(3,2)+za(2,4)*zb(4,2))+za(1,2)*zb(2,1)&
       &*(za(2,6)*zb(6,1)*(za(3,5)*zb(6,3)+za(4,5)*zb(6,4))&
       &-za(2,5)*(za(3,5)*(zb(5,3)*zb(6,1)-zb(5,1)*zb(6,3))&
       &+zb(6,1)*(za(3,6)*zb(6,3)+za(4,6)*zb(6,4))+za(4,5)&
       &*(zb(5,4)*zb(6,1)-zb(5,1)*zb(6,4))))))/(za(1,2)*zb(2,1))

end function A4pm
```

```
  function A5pm(za,zb) result(res)
    complex(dp) :: za(:,:), zb(:,:)
    complex(dp) :: res

    res = -(za(2,3)*za(2,5)*zb(4,2)*zb(6,1)+za(2,3)*za(2,5)*zb(4,1)*zb(6,2))

  end function A5pm

end module gg_ww_dim8
```

A.0.2 gg_WW_int.f

The following code is **to be added** into the **gg_WW_int.f** file within **src/HWW/gg_WW_int.f** file in the MCFM directory. **This is not the full file** but additions to the existing file. These additions were collaboratively made with my project supervisor. The first index of the Adim8 array refers to the operator number. The second and third indices refer to the gluon helicities, with $2 \to +$ and $1 \to -$. It can be seen that as previously stated in 3.1.2: $\mathcal{M}^{(i)}_{--} = \overline{\mathcal{M}}^{(i)}_{++}$ and $\mathcal{M}^{(i)}_{-+} = \overline{\mathcal{M}}^{(i)}_{+-}$. It is timesaving to use the inbuilt Fortran conjugate function to define $\mathcal{M}^{(i)}_{--}$ and $\mathcal{M}^{(i)}_{-+}$. These defined amplitudes can easily be added to the summed amplitude included in the file.

```
c---   dimension 8 operators
       Adim8(1,2,2)= A1pp(za,zb)*im*(four,zero)
       Adim8(1,2,1)= A1pm(za,zb)*im*ctwo
       Adim8(1,1,2)= conjg(Adim8(1,2,1))
       Adim8(1,1,1)= conjg(Adim8(1,2,2))

       Adim8(2,2,2)= A2pp(za,zb)*im*(8._dp,zero)
       Adim8(2,2,1)= czip
       Adim8(2,1,2)= czip
       Adim8(2,1,1)= conjg(Adim8(2,2,2))

       Adim8(3,2,2)= A3pp(za,zb)*im*(8._dp,zero)
       Adim8(3,2,1)= A3pm(za,zb)*im*(8._dp,zero)
       Adim8(3,1,2)= conjg(Adim8(3,2,1))
       Adim8(3,1,1)= conjg(Adim8(3,2,2))

       Adim8(4,2,2)= A4pp(za,zb)*im*(four,zero)
       Adim8(4,2,1)= A4pm(za,zb)*im*(four,zero)
       Adim8(4,1,2)= conjg(Adim8(4,2,1))
       Adim8(4,1,1)= conjg(Adim8(4,2,2))

       Adim8(5,2,2)= A5pp(za,zb)*im*wmass**2
       Adim8(5,2,1)= A5pm(za,zb)*im*wmass**2
       Adim8(5,1,2)= conjg(Adim8(5,2,1))
       Adim8(5,1,1)= conjg(Adim8(5,2,2))

       Adim8(6,2,2)= A6pp(za,zb)*im*(four,zero)*wmass**2
       Adim8(6,2,1)= czip
       Adim8(6,1,2)= czip
       Adim8(6,1,1)= conjg(Adim8(6,2,2))
```

```
do h1=1,2
   do h2=1,2
      Adim8(:,h1,h2)=Adim8(:,h1,h2)*cdim8(:)
   enddo
enddo
```

A.0.3 Mathematica Notebooks

The Mathematica notebooks **ggWW_amplitudes.nb** and **ggWW_decay.nb** are too long and clunky to be explicitly included in the Appendix. They are available in the following GitHub repository: *https://github.com/hpsussex/Project*

Please again note that the results of ggWW_decay were not explicitly written into this report for the sake of brevity. They are easily interpretable in the notebook.