14D009 Social and Economic Networks

Project: Spread of (Mis)information

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Abstract

This report describes a simulation-based approach to study the spread of (mis)information and obstacles to wisdom of crowds in social learning for a particular syntactic network. I consider a variant of different models which are based on a belief update model proposed and studied by Acemoglu, Ozdaglar, and ParandehGheibi [1]. In particular, I test the speed of convergence and its related consensus value in the presence of misinformation, both with and without forceful agents. I also simulate the effect of multiple stubborn agents with different beliefs. Finally, I propose a novel extension to the model and show the impact this extension has on the convergence and consensus value of the tested network.

1 Motivation

The growing importance of (online) social networks such as Facebook as relevant source of news to many people as well as recent political events have provoked a widespread discussion on the role of *fake news* in society. Numerous examples in the real-world demonstrate that despite of the relative ease to access information in today's world, gaps of consensus between different groups keep existing over long terms. Quite often, social groups are swayed by misleading leaders, news agencies, and other players. Prominent examples from recent history are the birther movement (on Obama's birth certificate) and the global warming controversy.

Over the past years several belief update models have been studied in terms of their belief convergence characteristics. They show that both network topology and the belief update model are critical determinants of factors such as convergence speed and consensus value. In my project I chose to simulate a particular model proposed and studied by Acemoglu, Ozdaglar, and ParandehGheibi [1] which in my opinion models the belief update process in a more realistic way than many other models do. I also run simulations on a novel extension to their model which - as I will show - can explain long-term differences in group beliefs very well.

2 The Network

I decided to work with the same syntactic network throughout the project. This allowed me to explore a wider range of different models and compare the results on the same ground. I chose a particular network for this purpose which is shown in *Figure 1*. The network consists of 12 nodes (agents), connected by weighted, undirected edges. The weights range between 1 (weak connection) and 10 (strong connection) and are uniformly distributed. The network has several characteristics which make it particularly attractive to study:

- The network is fully connected and therefore satisfies the *no man is an island* assumption. We know from results by Acemoglu et al. [1] that the network agents will eventually reach consensus (assuming no stubborn agents).
- The network consists of two, very weakly connected components/clusters, connected by a single weight 1 edge. The smaller component forms a star with strongly weighted edges. The larger component consists of a well-connected core and several peripheral nodes.
- The network has an exponential degree distribution. Most agents have very few neighbors. This feature allows to test what influence the degree of the mis-informing source has on the convergence speed and consensus value.

3 The Model

I simulate the following model, proposed and studied by Acemoglu et al [1]: Consider a social network G with a set of n agents N. Each agent receives an initial signal which is independently drawn from a standard normal with mean $\theta = 0$. Meetings between pairs of agents are modeled as a stochastic process. As Acemoglu et al pointed out, an advantage of introducing this stochastic meetings is that it limits duplication of information which is one of the main critique points of the general DeGroot model [2]. At each iteration, I

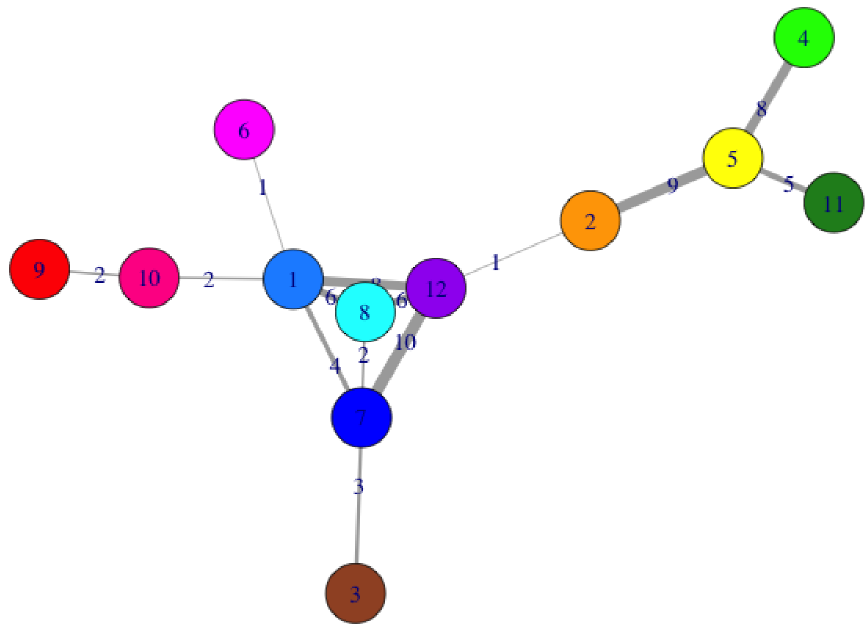

Figure 1: Test network with 12 agents and edge weights

randomly select an edge in the network. The underlying probability distribution of the draw is described by the edge weights, i.e. the probability of an edge to be selected equals its weight, divided by the sum of all weights in the network. For example, the probability of a meeting between agents 4 and 5, p_{45}, is:

$$p_{45} = \frac{g_{45}}{\sum_{1 \leq i < j \leq n} g_{ij}} = \frac{8}{67}$$

Note that this way of simulating meetings diverges from the way Acemoglu et al describe in their paper. There, meetings are modeled as a rate n Poisson process.

Naive Agents

In the absence of forceful agents, the two involved agents always adopt their average opinion. Let $x_i(t)$ denote the belief of agent i at time t. Then,

$$\begin{aligned} x_i(t+1) &= \frac{x_i(t) + x_j(t)}{2} \\ x_j(t+1) &= \frac{x_i(t) + x_j(t)}{2} \end{aligned} \tag{1}$$

Forceful Agents

Another model variant adds the following extra element: We make the reasonable assumption that whoever spreads misinformation does so deliberately and therefore will not (easily) change his belief when meeting with other agents. Acemoglu et al. add this assumption in the form of *forceful agents*: Whenever a forceful agent meets, we observe one out of 3 possible outcomes with probabilities α, β, and γ (such that $\alpha + \beta + \gamma = 1$):

α) Unilateral influence of the forceful agent on the other (necessarily non-forceful) agent. The extend to which the forceful agent changes the other's belief is captured with $\epsilon \in (0, 1/2]$:

$$\begin{aligned} x_i(t+1) &= \epsilon x_i(t) + (1-\epsilon)x_j(t) \\ x_j(t+1) &= x_j(t) \end{aligned} \tag{2}$$

β) Both agents adopt their mean opinion:

$$\begin{aligned} x_i(t+1) &= \frac{x_i(t) + x_j(t)}{2} \\ x_j(t+1) &= \frac{x_i(t) + x_j(t)}{2} \end{aligned} \tag{3}$$

γ) Both agents stick to their own opinion:

$$\begin{aligned} x_i(t+1) &= x_i(t) \\ x_j(t+1) &= x_j(t) \end{aligned} \tag{4}$$

Note that this, again, simplifies the model from the Acemoglu et al. paper to some extend: In the paper, the outcome probabilities α, β, and γ are potentially different for every pair of agents. Whenever two forceful agents meet, I assume that both stick to their own belief. Likewise, whenever two "regular" agents meet, I assume that both adopt their mean opinion.

4 Simulation: Belief Convergence

This section describes the belief convergence of the network shown in *Figure 1* under different situations. I plot the belief of every agent (represented by their unique color) at every moment (i.e. iteration) in time. The dashed black line represents the mean of everybody's initial beliefs.

4.1 No misinformation

As a start, I simulate belief convergence without any initial misinformation, thus every agent gets assigned a random initial belief drawn from a standard normal. *Figure 2* shows the belief convergence under the stochastic meeting belief update process after 300 and 2000 iterations. We can observe that the two weakly connected clusters of the network converge rather quickly to their local mean. After 2000 iterations the the full network has converged to the mean of all initial beliefs.

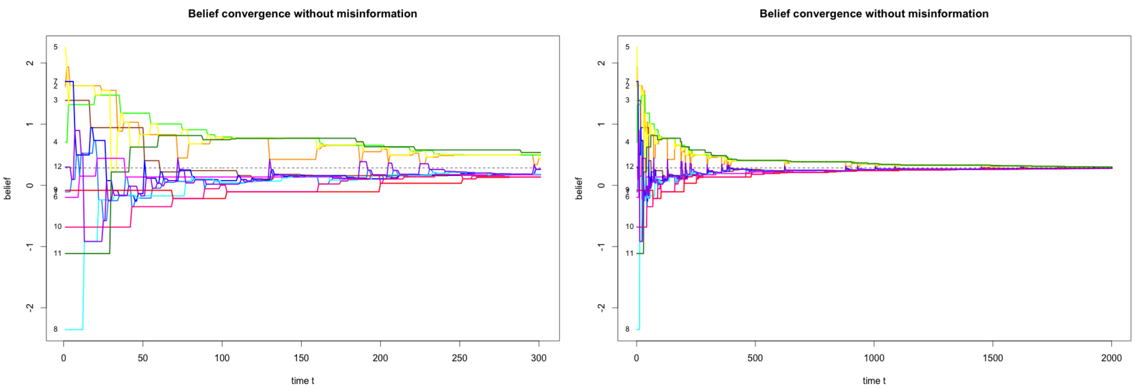

Figure 2: Belief convergence without misinformation after 300 and 2000 iterations

4.2 Misinformation with naive agents

Next, I simulate belief convergence in the presence of misinformation. In particular, I assign a "fake" initial value of 5 to agent 4 and leave all other initial values untouched. The convergence plots in *Figure 3* show that the network again converges to the mean of everybody's initial beliefs. The convergence speed is slowed down though, simply because the initial difference in beliefs was larger.

4.3 Misinformation with forceful agents

The situation becomes more interesting once we replace the naively misinformed agent by a forceful agent. Tested with parameters $\alpha = 0.8$, $\beta = 0.2$, $\gamma = 0$, and $\epsilon = 0.4$, and leaving everything else untouched, we can observe two significant changes in the new belief convergence plots in *Figure 4*. First, the convergence to a belief consensus takes significantly longer. The forceful agent 4 strongly influences the agents in his own cluster (nodes 2, 5, 11). Outside information is only rarely shared through the cluster bridge between nodes 2 and 12 and in many cases gets overruled by agent 4's forceful belief.

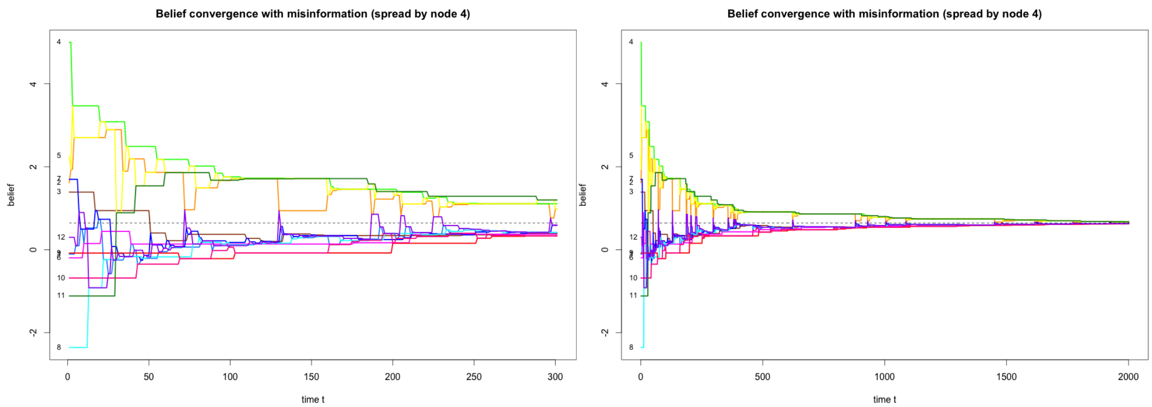

Figure 3: Belief convergence with misinformation by naive agent after 300 and 2000 iterations

The second effect we can observe from the plots is that the network no longer converges to the mean initial belief! Since the forceful agent only changes his belief in 1 out of 5 meetings, he pulls the other nodes in the network towards his own belief. The extend to which the forceful agent influences the consensus belief mainly depends on the stochastic realization of the process. Especially early meetings play an important role.

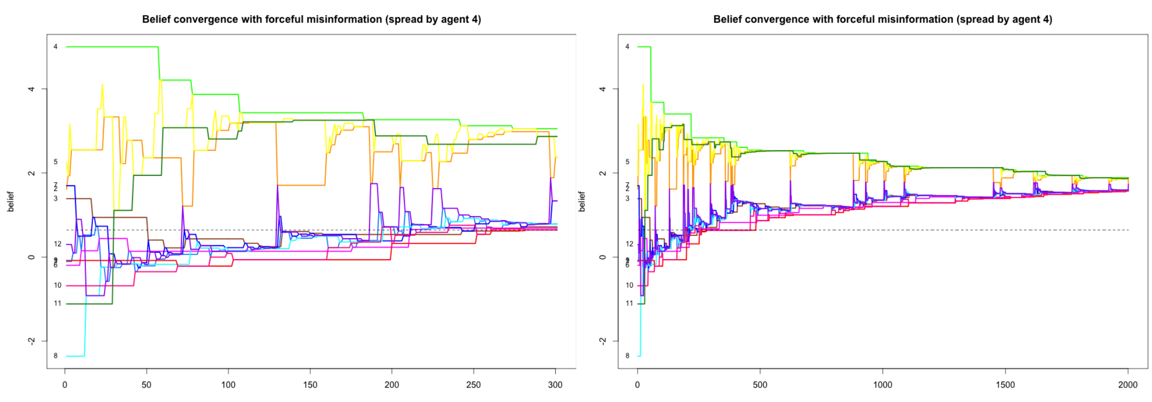

Figure 4: Belief convergence with misinformation by forceful agent after 300 and 2000 iterations

4.4 Misinformation with multiple stubborn agents

Finally, I simulate how stubborn agents influence the belief convergence. Stubborn agents are known to never change their own belief. Trivially, if all stubborn agents in the network start with the same initial belief, the network will eventually converge to this belief. If, however, at least two stubborn agents insist on different initial beliefs, the network cannot converge.

In the following simulation I tested the belief convergence for the latter scenario (stubborn agents with contradicting beliefs). In particular, I consider 2 stubborn agents - node 4 and node 6 - with initial beliefs -5 and 5, respectively. The belief convergence plots in *Figure 5* demonstrate how the 2 stubborn agents dominate their respective component. Occasional meetings between agents 2 and 12 temporarily move the beliefs closer, but the stubborn agents inevitably cancel out this movements.

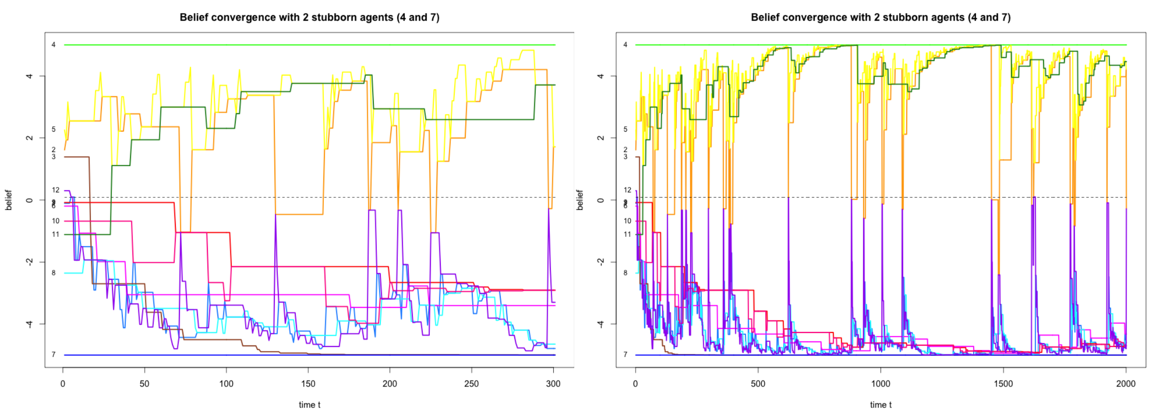

Figure 5: Belief convergence with contradicting misinformation by 2 stubborn agents after 300 and 2000 iterations

5 Simulation: Moment Convergence

So far, all simulations that involved misinformation put the "fake" value on agent 4. The following plots in *Figure 6* and *Figure 7* consider all 12 agents as initial source of fake news in turn. For each case, the graphs plot the standard deviation of beliefs over time. To make the results comparable, the same realization of stochastic meetings is used in all simulations.

Figure 6 shows how the standard deviation of beliefs evolves over time without and with misinformation. As expected, the standard deviation converges to zero in all cases. Also not surprisingly, the convergence usually takes longer in the case of misinformation. This is especially true if the initial misinformation is assigned to one of the agents in the smaller cluster which consists of nodes 2, 4, 5, and 11. Their influence seems to be lower compared to other nodes in the network.

Figure 7 shows how the standard deviation of beliefs evolves over time in the presence of forceful or stubborn agents. In all cases I assume that only forceful/stubborn agents spread misinformation. We can see that the network converges in all scenarios involving an forceful agent. The convergence speed is slowed down as compared to *Figure 6*. Again, the convergence rate seems to be lower if any of the nodes in the smaller cluster which consists of nodes 2, 4, 5, and 11 is assumed to be the forceful agent.

In the stubborn agent plot I always placed 2 stubborn agents with contradicting beliefs per simulation. The first stubborn agent starts with belief 5, and the other agent (always the agent with the next higher number) starts with belief -5. As we have seen earlier, the network does not converge to a consensus in this scenario. Thus, the standard deviation remains strictly greater than 0.

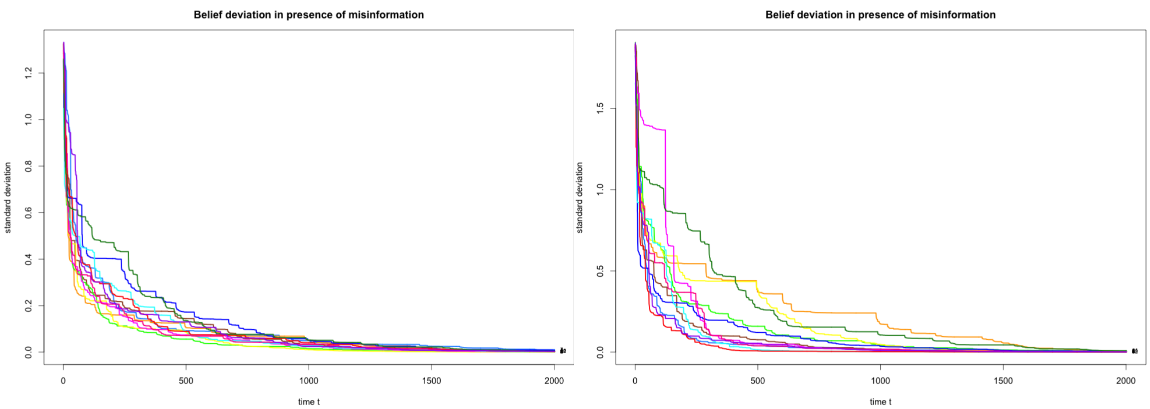

Figure 6: Standard deviation of belies without and with misinformation

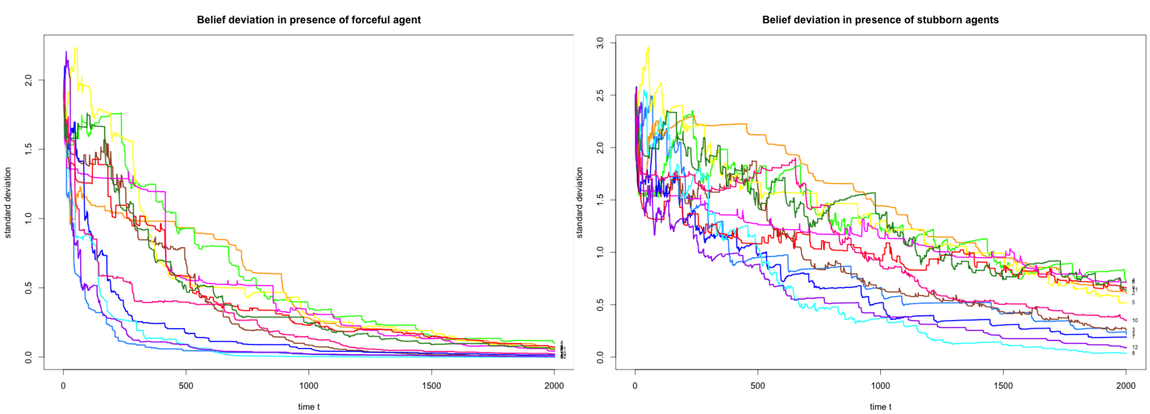

Figure 7: Standard deviation of belies with forceful and stubborn agents

6 Novel Extension

The models considered so far assumed that two unforceful agents would reach consensus (namely, the mean of their beliefs) no matter by how much they disagree. This is arguably a rather poor simplification, especially when agents are suspicious about potential fake information in the network. An agent will likely be more willed to trust another agent with similar beliefs. Also, agents might not often be eager to change their beliefs by a lot.

I therefore propose an extension to the existing models - with and without forceful agents - that to my knowledge has not yet been considered in the literature: Whenever two unforceful agents meet, either let them reach consensus on their mean belief, or let them both stick to their own belief. This also includes a forceful agent meeting an unforceful agent and not forcing its belief (β option). Let the meeting outcome be a random process with the probability of reaching consensus being inversely related to the difference on their own beliefs, i.e. the higher the difference, the lower the probability of reaching consensus.

In the following simulations I chose the probability of consensus to be a function of the absolute difference $x \geq 0$:

$$p(x) = e^{-x}$$

Note that this function equals the density function of an exponential distribution with $\lambda = 1$. The function satisfies necessary conditions: The value is a probability ($\in [0, 1]$) with a value of 1 if the difference is 0, and the function monotonically decreases as the absolute difference x increases. Furthermore, the probability of convergence is non-zero everywhere. The function works well for initial beliefs that are drawn from a standard normal. For example, the probability of reaching consensus with a belief difference of 1 is roughly 0.37. The function can still be used for other initial belief probability distributions, but the absolute difference should be first scaled in that case.

The belief convergence plots for the same network under this new model are shown in *Figure 8* and *Figure 9*. *Figure 8* plots the belief convergence with misinformation assigned to an naive agent 4 after 2000 and 5000 iterations. We can compare the result with the earlier simulation shown in *Figure 3* and conclude that the new model extension slows down convergence significantly. In the earlier simulation, the network reached consensus after around 2000 iterations. In the new model, consensus is only reached after roughly 4000 iterations. The consensus value is the same in both simulations.

Figure 9 plots the belief convergence with misinformation assigned to a forceful agent 4 after 2000 and 10000 iterations. This time, we can compare the result with the earlier simulation shown in *Figure 4*. Again, the new model extension slows down convergence significantly. In the earlier simulation, the network reached consensus after around 4000 iterations. In the new model, consensus is only reached after roughly 8000 iterations. In both models, the consensus value diverged from the mean of everybody's initial belief.

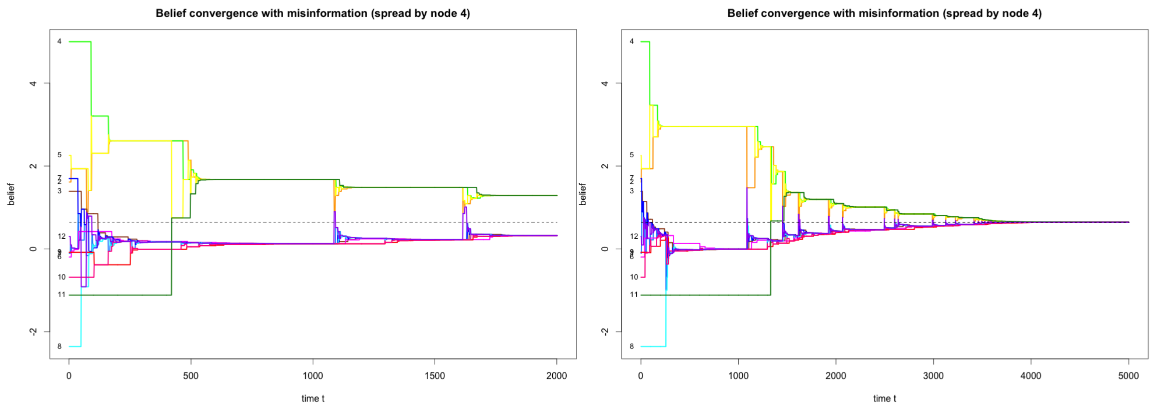

Figure 8: New belief convergence with misinformation by naive agent after 2000 and 5000 iterations

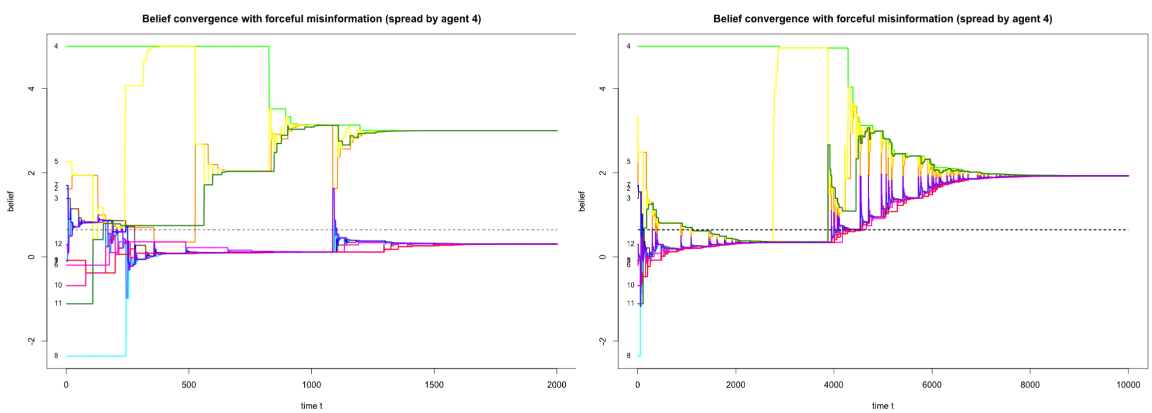

Figure 9: New belief convergence with misinformation by forceful agent after 2000 and 10000 iterations

The main reason for the prolonged disagreement in the new simulation is that large belief differences are likely maintained for long periods under the new model. All but agent 4 reached consensus after roughly 2000 iterations. The large disagreement between agent 4 and the rest made it very unlikely in the following to reach consensus in any meeting (eg. the probability for a difference of 5 is < 0.007). At one point, agent 5 changes his opinion and adapts agent 4's belief but for a long time is unable to convince any other nodes. When this finally happens, most agents move closer to agent 4's belief. This increases the chances of finally convincing agent 4 to change his belief. When this happened, it finally triggered the convergence of the whole network.

7 Conclusion

The simulated models do well to explain the existence of groups with different beliefs over long periods of time. While the network in all tested models (except for the model with stubborn agents) eventually reaches consensus, several factors can delay this event significantly. In particular, the presence of forceful agents slows down convergence and influences the consensus value. The novel model extension which formulates meeting convergence as a random process with the probability of reaching consensus being inversely related to the difference on their own belief also has been shown to slow convergence by a factor of 2.

References

[1] D. Acemoglu, A. Ozdaglar, and A. ParandehGheibi. Spread of (mis)information in social networks. *Games and Economic Behavior*, 70:194–227, 2010.

[2] M. H. DeGroot. Reaching a consensus. *Journal of the American Statistical Association*, 69(345):118–121, 1974.