State Space Models

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2.1 Local Level Model

$$
\begin{aligned}
\text{observation:} \quad & y_t = x_t + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_\epsilon^2) \\
\text{state:} \quad & x_{t+1} = x_t + \eta_t, \quad \eta_t \sim N(0, \sigma_\eta^2)
\end{aligned}
$$

2.2 Trivariate Local Level Model

$$
\begin{aligned}
\text{observation:} \quad & y_t = \begin{bmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \end{bmatrix} = x_t + \epsilon_t, \quad \epsilon_t \sim N(\mathbf{0}, \begin{bmatrix} \sigma_{\epsilon 1}^2 \\ \sigma_{\epsilon 2}^2 \\ \sigma_{\epsilon 3}^2 \end{bmatrix} I) \\
\text{state:} \quad & x_{t+1} = x_t + \eta_t, \quad \eta_t \sim N(\mathbf{0}, \Sigma_\eta)
\end{aligned}
$$

where we restrict the covariance matrix of the state disturbances, Σ_η, to the form

$$
\Sigma_\eta = \begin{bmatrix} \sigma_{\eta 1}^2 & \rho\sigma_{\eta 1}\sigma_{\eta 2} & \rho\sigma_{\eta 1}\sigma_{\eta 3} \\ \rho\sigma_{\eta 1}\sigma_{\eta 2} & \sigma_{\eta 2}^2 & \rho\sigma_{\eta 2}\sigma_{\eta 3} \\ \rho\sigma_{\eta 1}\sigma_{\eta 3} & \rho\sigma_{\eta 2}\sigma_{\eta 3} & \sigma_{\eta 3}^2 \end{bmatrix}
$$

Thus, Σ_η can be described by the vector $[\sigma_{\eta 1}^2, \sigma_{\eta 2}^2, \sigma_{\eta 3}^2, \rho]$.

2.3 Hierarchical Dynamic Poisson Model

Let t denote the day and i be the intraday index.

$$
\text{state:} \quad x_{ti} = \text{Poisson}(\lambda_{ti})
$$

where the parameter

$$
\log \lambda_{ti} = \log \lambda_t^{(D)} + \log \lambda_i^{(P)} + \log \lambda_{ti}^{(I)}
$$

consists of a daily, a periodic, and an intra-daily component:

$$
\begin{aligned}
\text{daily component:} \quad & \log \lambda_t^{(D)} = \phi_0^{(D)} + \phi_1^{(D)} \log \lambda_{t-1}^{(D)} + u_t^{(D)} && \text{(AR(1))} \\
\text{periodic component:} \quad & \log \lambda_i^{(P)} = \delta_i && \text{(B-spline)} \\
\text{intra-daily component:} \quad & \log \lambda_{ti}^{(I)} = \phi_1^{(I)} \log \lambda_{ti-1}^{(I)} + u_{ti}^{(I)} && \text{(AR(1))}
\end{aligned}
$$

3 Filtering

The object of filtering is to update our knowledge of the system each time a new observation y_t is brought in.

3.1 Kalman Filter

3.1.1 Likelihood evaluation

$$\log L(Y_n) = -\frac{nd}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}(\log|F_t| + v_t^T F_t^{-1} v_t)$$

4 Conclusion

by Etessami et al.[?]