Appendix of "Towards Neuro-symbolic Causal Rule Synthesis, Verification, and Evaluation Grounded in Legal and Safety Principles"

February 2026

1 Principles

1.1 Legal Principles

- *keep-control*: Maintain constant vehicle control
- *above-minimum-speed*: Avoid impeding traffic flow
- *below-speed-limit*: Adhere to maximum speed
- *no-stopping*: Stopping prohibited on motorways
- *keep-right*: Drive as far right as possible
- *speed-adv-overtaking*: Only overtake with sufficient speed advantage
- *overtaking-maneuver*: Ensure no danger to others when overtaking
- *no-right-overtaking*: Overtake only on the left
- *safe-distance*: Maintain safe distance from other vehicles
- *being-overtaken*: Do not accelerate when being overtaken
- *emergency-stopping*: Apply emergency brake if needed
- *traffic-light*: Follow traffic signals
- *seatbelt-mandatory*: All occupants must wear seatbelts
- *no-drinking-driving*: Driving under influence is prohibited
- *license-required*: Valid driving license required
- *vehicle-registration*: Vehicles must be registered and insured
- *no-mobile-use*: No hand-held mobile phone usage
- *helmet-mandatory*: Helmets required for motorcyclists

1.2 Safety Principles

- *stationary-no-friction-decrease*: Stationary vehicle friction cannot decrease
- *max-acceleration-limited-by-friction*: Acceleration limited by tire-road friction
- *max-deceleration-limited-by-friction*: Deceleration limited by friction
- *slip-if-force-exceeds-friction*: Skid if forces exceed friction
- *collision-without-emergency-brake*: Vehicle collides if brake not applied
- *collision-if-obstacles*: Vehicle may collide if obstacles present
- *too-fast-for-conditions*: Risk of losing control if too fast
- *braking-distance-increases-with-speed*: Stop distance grows with speed
- *turning-radius-increases-with-speed*: Reduce speed to maintain traction in turns
- *speeding-reduces-reaction-time*: Less time to react at higher speeds
- *overtake-speed-requirement*: Must have speed advantage to overtake
- *overspeed-on-curve-risk*: Skidding risk if curve speed exceeded
- *speed-limited-by-friction*: Max safe speed limited by friction
- *hydroplaning-risk*: Vehicle may hydroplane on wet roads
- *maintain-safe-distance*: Keep safe distance from vehicle ahead
- *check-blind-spots*: Always check blind spots
- *reduce-speed-in-bad-weather*: Reduce speed in rain, snow, fog
- *use-indicators*: Signal before maneuvers
- *avoid-distractions*: Avoid distractions while driving
- *vehicle-maintenance*: Ensure vehicle is properly maintained
- *emergency-brake-ready*: Be able to apply emergency brake
- *night-driving-caution*: Extra caution at night or low visibility

2 Scenario Output

```
Effect: Successfully merge into heavy traffic

After decomposition
"causes": [
    "Driver maintains control of the vehicle",
    "Vehicle is traveling at a speed that allows for safe merging",
    "Sufficient distance from other vehicles to merge safely",
    "No vehicles are overtaking on the right",
    "Traffic conditions allow for merging without impeding flow",
    "Driver adheres to traffic laws regarding merging",
    "No sudden obstacles in the merging path",
    "Driver is aware of surrounding traffic"
]

After semantic consolidation
"unique_causes": [
    "Driver maintains control of the vehicle and is aware of surrounding traffic.
     ",
    "Vehicle is traveling at a speed that allows for safe merging and adheres to
     traffic laws regarding merging.",
    "Sufficient distance from other vehicles to merge safely and no vehicles are
     overtaking on the right.",
    "Traffic conditions allow for merging without impeding flow and no sudden
     obstacles in the merging path."
]

Symbolic Translation
"rules": [
    {
        "cause": Driver maintains control of the vehicle
        "rule":    x ( collide (x)    sd-front(x)    sd-rear(x)    lane -change(x)
     )
        "thinking": "Maintaining control of the vehicle involves ensuring that
     there is a safe distance to both the preceding and following vehicles, as
     well as avoiding lane changes that could lead to instability. If these
     conditions are met, the vehicle is less likely to collide with obstacles or
     other agents."
    }
    {
        "cause": "Vehicle is traveling at a speed that allows for safe merging",
        "rule": " x (sd-front(x)  sd-rear(x)  speed-adv(x,y)    speed (x)>
     vthreshold)",
        "thinking": "For a vehicle to merge safely, it must maintain a safe
     distance from both the preceding and following vehicles, ensuring that it is
     not too close to either. Additionally, the vehicle's speed must be greater
     than a certain threshold relative to the speed of other vehicles to
     facilitate a safe merge."
    }
    {
        "cause": "Sufficient distance from other vehicles to merge safely",
        "rule": " x (sd-front(x)  sd-rear(x)  dense (x))",
        "thinking": "To merge safely, a vehicle must have a safe distance to both
     the preceding and following vehicles. This ensures that there is enough
     space to maneuver without the risk of collision, which is further supported
     by the absence of dense traffic around the vehicle."
```

```
    }
    {
        "cause": "No vehicles are overtaking on the right",
        "rule": " x  (right(i,j)   lane-change(i))",
        "thinking": "The condition states that no vehicles are overtaking on the
     right, which implies that if a vehicle i is to the right of another vehicle
     j, then vehicle i cannot be crossing a lane boundary to overtake. This
     ensures that the vehicles maintain their positions without any overtaking
     maneuvers on the right side."
    }
    {
        "cause": "Traffic conditions allow for merging without impeding flow",
        "rule": " x (sd-front(x)   sd-rear(x)   dense (x)    lane-change(x))",
        "thinking": "For merging to occur without impeding flow, the vehicle must
     have a safe distance to both the preceding and following vehicles, and
     there should not be a dense traffic condition. This ensures that the vehicle
     can change lanes safely without causing disruptions."
    }
    {
        "cause": "Driver adheres to traffic laws regarding merging",
        "rule": " x (  (lane-change(x)   collide(x))    merged(x)    sd-front(x)
     sd-rear(x))",
        "thinking": "Adhering to traffic laws regarding merging implies that the
     driver should not change lanes in a way that would cause a collision. The
     driver must have already passed the merging point (merged(x)), maintain a
     safe distance to the vehicle in front (sd-front(x)), and a safe distance to
     the vehicle behind (sd-rear(x)) to ensure safe merging."
    }
    {
        "cause": "No sudden obstacles in the merging path",
        "rule": " x ( collide (x)    near (x, obstacle)    dense (x))",
        "thinking": "The condition of having no sudden obstacles in the merging
     path implies that the vehicle must not be colliding with any obstacles and
     must not be near any obstacles that could cause a collision. Additionally,
     the vehicle should not be in a dense situation where multiple nearby
     vehicles could create sudden obstacles."
    }
    {
        "cause": "Driver is aware of surrounding traffic",
        "rule": " x ( collide (x)    dense(x)    sd-front(x)    sd-rear(x))",
        "thinking": "The awareness of surrounding traffic implies that the driver
     can perceive the density of nearby vehicles and maintain safe distances
     both in front and behind. This awareness helps prevent collisions by
     ensuring that the driver can react appropriately to the traffic situation."
    }
]


Initial Individual Evaluations:
"evaluations": [
    {
      "cause": "Driver maintains control of the vehicle and is aware of
     surrounding traffic.",
      "result": "necessary",
      "reason": "Without the driver maintaining control, the vehicle cannot
     safely merge into heavy traffic, violating safety laws regarding vehicle
     control and potential collisions."
```

```
    },
    {
      "cause": "Vehicle is traveling at a speed that allows for safe merging and adheres to traffic laws regarding merging.",
      "result": "necessary",
      "reason": "Traveling at an appropriate speed is essential for safe merging; if the vehicle is not at a safe speed, it may violate legal laws regarding speed and control, leading to unsafe conditions."
    },
    {
      "cause": "Sufficient distance from other vehicles to merge safely and no vehicles are overtaking on the right.",
      "result": "not necessary",
      "reason": "It is possible to merge into heavy traffic even if there is insufficient distance from other vehicles, as long as the vehicle adheres to speed and control laws; thus, this cause is not strictly required."
    },
    {
      "cause": "Traffic conditions allow for merging without impeding flow and no sudden obstacles in the merging path.",
      "result": "not necessary",
      "reason": "Merging can occur even if traffic conditions are not ideal, provided that the vehicle maintains control and adheres to speed laws; thus, this cause is not strictly necessary for the effect."
    }
  ]

Necessary Sets:

Necessary Set 1: Driver maintains control of the vehicle and is aware of
    surrounding traffic.
Necessary Set 2: Vehicle is traveling at a speed that allows for safe merging and
     adheres to traffic laws regarding merging.
Necessary Set 3: Sufficient distance from other vehicles to merge safely and no
    vehicles are overtaking on the right.
Necessary Set 4: Traffic conditions allow for merging without impeding flow and
    no sudden obstacles in the merging path.


Sufficient Sets:

Sufficient Set 1:
- Driver maintains control of the vehicle and is aware of surrounding traffic.
- Vehicle is traveling at a speed that allows for safe merging and adheres to
    traffic laws regarding merging.
- Sufficient distance from other vehicles to merge safely and no vehicles are
    overtaking on the right.
- Traffic conditions allow for merging without impeding flow and no sudden
    obstacles in the merging path.
```

Listing 1: Scenario: Successfully Merge into Heavy Traffic

```
Effect: Maintain a constant speed on a highway segment

After decomposition
"causes": [
    "Driver␣maintains␣vehicle␣control",
    "Vehicle␣speed␣is␣within␣legal␣limits",
    "Vehicle␣speed␣is␣above␣minimum␣required␣speed",
    "Sufficient␣friction␣between␣tires␣and␣road",
    "No␣obstacles␣on␣the␣highway␣segment",
    "No␣sudden␣changes␣in␣traffic␣conditions",
    "Driver␣is␣attentive␣and␣responsive␣to␣road␣conditions",
    "No␣emergency␣situations␣requiring␣sudden␣braking"
]

After deduplication and merging
"unique_causes": [
    "Driver␣maintains␣vehicle␣control␣and␣is␣attentive␣and␣responsive␣to␣road␣
     conditions.",
    "Vehicle␣speed␣is␣within␣legal␣limits␣and␣above␣minimum␣required␣speed.",
    "Sufficient␣friction␣between␣tires␣and␣road.",
    "No␣obstacles␣on␣the␣highway␣segment.",
    "No␣sudden␣changes␣in␣traffic␣conditions.",
    "No␣emergency␣situations␣requiring␣sudden␣braking."
]

Symbolic Translation
"rules": [
    {
        "cause": "Driver␣maintains␣vehicle␣control␣and␣is␣attentive␣and␣
     responsive␣to␣road␣conditions.",
        "rule": " x ( collide (x)␣  ␣sd-front(x)␣ ␣sd-rear(x)␣ ␣ lane -change(x))"
     ,
        "thinking": "Maintaining␣vehicle␣control␣and␣being␣attentive␣implies␣that
     ␣the␣driver␣keeps␣safe␣distances␣to␣both␣the␣preceding␣and␣following␣
     vehicles␣and␣avoids␣unnecessary␣lane␣changes.␣Awareness␣and␣responsiveness␣
     to␣road␣conditions␣reduce␣instability␣and␣lower␣the␣probability␣of␣
     collisions."
    },
    {
        "cause": "Vehicle␣speed␣is␣within␣legal␣limits␣and␣above␣minimum␣required
     ␣speed.",
        "rule": " x ( speed (x)␣  ␣legal_limit␣ ␣ speed (x)␣>␣
     minimum_required_speed)",
        "thinking": "The␣vehicle␣must␣operate␣below␣the␣legal␣maximum␣speed␣while
     ␣remaining␣above␣the␣minimum␣threshold␣required␣for␣stable␣traffic␣flow.␣
     Staying␣within␣this␣interval␣ensures␣both␣regulatory␣compliance␣and␣safe␣
     dynamic␣behavior."
    },
    {
        "cause": "Sufficient␣friction␣between␣tires␣and␣road.",
        "rule": " x ( friction (x)␣>␣0)",
        "thinking": "Adequate␣friction␣between␣the␣tires␣and␣the␣road␣surface␣
     ensures␣controllability␣during␣acceleration,␣braking,␣and␣steering.␣Positive
     ␣friction␣is␣a␣prerequisite␣for␣maintaining␣stability␣and␣preventing␣
     skidding."
    },
    {
```

```
        "cause": "No obstacles on the highway segment.",
        "rule": " x ( near (x, obstacle))",
        "thinking": "If no obstacles are present on the road segment, the vehicle is not exposed to unexpected physical barriers. The absence of nearby obstacles supports uninterrupted and safe driving."
    },
    {
        "cause": "No sudden changes in traffic conditions.",
        "rule": " x (sudden_change(x))",
        "thinking": "Stable traffic conditions imply the absence of abrupt accelerations, decelerations, or unpredictable maneuvers. This stability supports consistent vehicle control and reduces risk."
    },
    {
        "cause": "No emergency situations requiring sudden braking",
        "rule": " x ( EB (x))",
        "thinking": "If no emergency situation exists, emergency braking is not activated. The absence of emergency braking indicates that the vehicle is not facing an imminent collision or critical hazard."
    }
]


Initial Individual Evaluations:
"evaluations": [
    {
      "cause": "Driver maintains vehicle control and is attentive and responsive to road conditions.",
      "result": "necessary",
      "reason": "Without the driver maintaining control, the vehicle cannot safely maintain a constant speed, violating safety laws regarding vehicle control."
    },
    {
      "cause": "Vehicle speed is within legal limits and above minimum required speed.",
      "result": "necessary",
      "reason": "If the vehicle speed is not within legal limits, it cannot legally maintain a constant speed, violating legal laws regarding speed limits."
    },
    {
      "cause": "Sufficient friction between tires and road.",
      "result": "necessary",
      "reason": "Without sufficient friction, the vehicle cannot maintain a constant speed without risking skidding or losing control, violating safety laws related to friction."
    },
    {
      "cause": "No obstacles on the highway segment.",
      "result": "not necessary",
      "reason": "The vehicle can maintain a constant speed even if there are obstacles, as long as the driver can control the vehicle and respond appropriately, which is covered by other causes."
    },
    {
      "cause": "No sudden changes in traffic conditions.",
```

```
      "result": "not necessary",
      "reason": "The vehicle can still maintain a constant speed despite sudden changes in traffic conditions if the driver is attentive and responsive, which is covered by other causes."
    },
    {
      "cause": "No emergency situations requiring sudden braking.",
      "result": "not necessary",
      "reason": "The vehicle can maintain a constant speed even if an emergency situation arises, provided the driver is able to manage the situation without braking suddenly, which is covered by other causes."
    }
  ]

Necessary Sets:

Necessary Set 1: Driver maintains vehicle control and is attentive and responsive to road conditions.
Necessary Set 2:  Vehicle speed is within legal limits and above minimum required speed.
Necessary Set 3: Sufficient friction between tires and road.

Sufficient Sets:

Sufficient Set 1:
  - Driver maintains vehicle control and is attentive and responsive to road conditions.
- Vehicle speed is within legal limits and above minimum required speed.
- Sufficient friction between tires and road.
- No obstacles on the highway segment.

Sufficient Set 2:
- Driver maintains vehicle control and is attentive and responsive to road conditions.
- Vehicle speed is within legal limits and above minimum required speed.
- Sufficient friction between tires and road.
- No sudden changes in traffic conditions.
```

Listing 2: Scenario: Maintain a Constant Speed on a Highway Segment

3 Prompts

```
DECOMPOSE_EFFECT_PROMPT = '''
You are an expert in causal reasoning. You task is to decompose an effect into a list of its necesssary causes.

You are given 3 things as input.
Input:
{effect}
{legal_laws}
{safety_laws}

Instructions:
- **Necessary Cause:** A condition that must be present for the effect to occur. If this condition is removed, the effect becomes impossible (The "But-For" Test).
- **Direct Link:** Avoid distal or "butterfly effect" causes. Focus on the immediate safety, and legal requirements.
- Produce only valid logical causes in **strict JSON format**; that ensure that the effect can happen in real world.

The JSON output must follow this exact schema:

{{
  "causes": [
    "cause_1",
    "cause_2",
    ...
  ]
}}


Input:
{effect}

Example:
effect: Turn left while maintaing low speed.
output:
{{
  "causes": [
    "Safe distance from nearby vehicles",
    "Vehicle speed is low enough to maintain traction",
    "Steering input applied to turn left",
    "No excessive lateral acceleration",
    "Sufficient friction between tires and road",
    "Driver maintains vehicle control",
    "No obstacles blocking the left turn",
    "Vehicle not in violation of legal rules for turning"
  ]
}}

IMPORTANT: Output must be strictly JSON, without any markdown, backticks, or explanations.
'''
```

Listing 3: Decompose Effect Prompt

```
SEMANTIC_CONSOLIDATE_PROMPT='''
You␣are␣given␣a␣list␣of␣causes␣from␣a␣driving␣situtation.
Some␣of␣these␣causes␣are␣repeated␣or␣very␣similar.

Input:
{causes}

Instructions␣for␣deduplication:
-␣Only␣merge␣causes␣if␣they␣truly␣convey␣the␣exact␣same␣information.
-␣Preserve␣distinct␣types␣of␣reasoning,␣such␣as:
␣␣␣␣-␣Legal␣rules␣(speed␣limits,␣permissions)
␣␣␣␣-␣Safety␣requirements␣(reaction␣time,␣stopping␣distance)
␣␣␣␣-␣Physics␣constraints␣(friction,␣braking␣distance)
-␣Retain␣both␣legal␣and␣safety␣aspects␣even␣if␣they␣overlap␣conceptually.
-␣Each␣cause␣must␣remain␣logically␣meaningful␣and␣actionable.


The␣JSON␣output␣must␣follow␣this␣exact␣schema:

{{
␣␣"unique_causes":␣[
␣␣␣␣␣␣"unique_cause_1",
␣␣␣␣␣␣"unique_cause_2",
␣␣␣␣␣␣...
␣␣]
}}


Example:
Input:
{{
␣␣"causes":␣[
␣␣␣␣␣␣"Road␣must␣be␣free␣of␣debris,␣hazards,␣and␣obstacles",
␣␣␣␣␣␣"Road␣conditions␣must␣be␣dry␣and␣clear",
␣␣␣␣␣␣"Road␣is␣not␣under␣construction␣or␣repair",
␣␣␣␣␣␣"Speed␣must␣be␣within␣threshold"
␣␣]
}}

Output:
{{
␣␣"unique_causes":␣[
␣␣␣␣␣␣"Road␣conditions␣must␣be␣dry␣and␣clear,␣free␣from␣debris,␣hazards,␣obstacles,
    ␣or␣construction/repair␣activities.",
␣␣␣␣␣␣"Speed␣must␣be␣within␣threshold"
␣␣]
}}

IMPORTANT:␣Output␣must␣be␣strictly␣JSON,␣without␣any␣markdown,␣backticks,␣or␣
    explanations.
'''
```

Listing 4: Semantic Consolidate Prompt

```
CHECK_NECESSITY_PROMPT = '''
You␣are␣an␣expert␣in␣causal␣reasoning␣for␣Autonomous␣Vehicle␣(AV)␣systems.

```

```
You␣are␣given:
1.␣A␣main␣effect.
2.␣A␣list␣of␣causes.
3.␣A␣set␣of␣legal␣laws.
4.␣A␣set␣of␣safety␣laws.


effect:␣{effect}
causes:␣{causes}
legal_laws:␣{legal_laws}
safety_laws:␣{safety_laws}

CRITICAL␣REASONING␣RULES:

INDEPENDENT␣NECESSITY␣TEST:
-␣Evaluate␣EACH␣cause␣independently.
-␣When␣evaluating␣one␣cause,␣assume␣ONLY␣that␣this␣cause␣is␣FALSE.
-␣Do␣NOT␣assume␣any␣other␣listed␣causes␣are␣present␣or␣absent.
-␣Do␣NOT␣assume␣hidden␣conditions.
-␣Only␣rely␣on␣the␣provided␣legal␣and␣safety␣laws.

DEFINITION:
-␣A␣cause␣is␣"necessary"␣if␣the␣effect␣is␣logically␣impossible␣under␣the␣provided
    ␣laws␣when␣this␣cause␣is␣false.
-␣A␣cause␣is␣"not␣necessary"␣if␣there␣exists␣at␣least␣one␣logically␣possible␣
    situation,␣consistent␣with␣the␣laws,␣where␣the␣effect␣occurs␣while␣this␣
    cause␣is␣false.

IMPORTANT:
-␣Do␣NOT␣assume␣substitution␣by␣other␣listed␣causes.
-␣Do␣NOT␣assume␣background␣facts␣not␣stated␣in␣the␣laws.
-␣Do␣NOT␣evaluate␣optimality␣or␣quality.
-␣Safety␣laws␣must␣be␣fully␣satisfied.
-␣Your␣reasoning␣must␣strictly␣follow␣the␣provided␣laws.

Instructions:
-␣Evaluate␣each␣cause␣independently.
-␣Determine␣whether␣the␣effect␣could␣legally␣and␣safely␣occur␣if␣this␣cause␣were␣
    false.
-␣Provide␣reasoning␣grounded␣only␣in␣the␣provided␣legal␣and␣safety␣laws.


Output␣JSON␣format:

{{
␣␣"evaluations":␣[
␣␣␣␣{{
␣␣␣␣␣␣"cause":␣"exact␣cause␣text",
␣␣␣␣␣␣"result":␣"necessary"␣or␣"not␣necessary",
␣␣␣␣␣␣"reason":␣"clear␣explanation,␣strictly␣using␣legal␣and␣safety␣laws␣why␣this
    ␣cause␣is␣or␣is␣not␣required␣for␣the␣effect"
␣␣␣␣}}
␣␣]
}}

Example:

```

```
effect: Turn left while maintaining low speed.
causes:
[
    "Safe distance from nearby vehicles",
    "Vehicle speed is low enough to maintain traction",
    "Steering input applied to turn left",
    "No excessive lateral acceleration",
    "Sufficient friction between tires and road",
    "Driver maintains vehicle control",
    "No obstacles blocking the left turn",
    "Vehicle not in violation of legal rules for turning"
]


Output:
{{
  "evaluations": [
    {{
      "cause": "Safe distance from nearby vehicles",
      "result": "not necessary",
      "reason": "Maintaining low-speed left turn is physically possible without considering other vehicles; safe distance affects safety but is not strictly required for the turn to occur according to laws."
    }},
    {{
      "cause": "Vehicle speed is low enough to maintain traction",
      "result": "necessary",
      "reason": "Low speed is required by friction and vehicle control laws; without it, the effect violates safety laws."
    }},
    ...
  ]
}}

IMPORTANT: Output must be strictly valid JSON without markdown or explanations.
'''
```

Listing 5: Check Individual Necessity Prompt

```
NECESSITY_SET_PROMPT = '''
You are an expert in causal verification for an Autonomous Vehicle (AV) system.
You must strictly rely on legal laws and safety laws.

CRITICAL RULE:
You are NOT deciding how to achieve the effect.
You are verifying whether the effect could have already happened under the given conditions.

CLOSED WORLD RULE:
Any cause not explicitly listed as present must be treated as false.
No additional facts exist beyond those provided.

The effect is a factual event that already occurred.
You must check whether this event is logically possible given the causes.

You are NOT allowed to:
- Reinterpret the effect
```

```
- Simplify the effect
- Assume missing causes
- Add hidden assumptions
- Change the meaning of the effect

Definitions:

1. Present causes:
These causes are fully true and hold in the scenario.

2. Absent cause:
This cause is definitively false.
It did NOT occur and CANNOT contribute in any way.

3. Necessity test:
Assume:
- All present causes are true
- The absent cause is false
- Check if the effect could still have happened.
- If the effect can not happen without the absent cause, then the absent cause is necessary.


Instructions:

- Treat the effect as an event that already happened.
- Treat the absent cause as completely false.
- Do NOT reinterpret the effect.
- Do NOT assume additional causes.
- Decide if this event is safely and legally possible.
- Answer "yes" if the event is still possible.
- Answer "no" if the event is impossible without the absent cause.
- Your reasoning must be strictly grounded in the provided legal and safety laws.
- For necessary causes, indicate:
    - "necessary" if the effect would be impossible without it due to a law,
    - "not necessary" if the effect can still occur without violating any law.
- Provide a precise explanation citing legal as well safety aspect of laws that determine the necessity.

You are given:

Effect:
{effect}

Causes that are present:
{causes}

Cause that is explicitly NOT present (false):
{absent_cause}

Legal laws:
{legal_laws}

Safety laws:
{safety_laws}

Output strictly in JSON:
```

```
73
74  {{
75  ␣␣"result":␣"yes"␣or␣"no",
76  ␣␣"reason":␣<reason>
77  }}
78
79  IMPORTANT:␣Output␣must␣be␣strictly␣JSON,␣without␣any␣markdown,␣backticks,␣or␣
        explanations.
80  '''
```

Listing 6: Necessary Set Prompt

```
1   SUFFICIENCY_SET_PROMPT='''
2   You␣are␣an␣expert␣in␣causal␣verification␣for␣an␣Autonomous␣Vehicle␣(AV)␣system.
3   You␣must␣strictly␣rely␣on␣legal␣laws␣and␣safety␣laws.
4
5   CRITICAL␣RULE:
6   You␣are␣NOT␣deciding␣how␣to␣achieve␣the␣effect.
7   You␣are␣verifying␣whether␣the␣effect␣could␣have␣already␣happened␣under␣the␣given␣
        conditions.
8
9   Instructions:
10
11  -␣Treat␣the␣effect␣as␣an␣event␣whose␣logical␣occurrence␣must␣be␣evaluated.
12  -␣Treat␣all␣present␣causes␣as␣fully␣true.
13  -␣Treat␣all␣absent␣causes␣as␣completely␣false.
14  -␣CLOSED-WORLD␣RULE:␣Any␣cause␣not␣explicitly␣listed␣as␣present␣must␣be␣treated␣
        as␣false.␣No␣additional␣conditions␣exist.
15  -␣Do␣NOT␣reinterpret␣the␣effect.
16  -␣Do␣NOT␣simplify␣the␣effect.
17  -␣Do␣NOT␣assume␣additional␣causes.
18  -␣Do␣NOT␣rely␣on␣background␣knowledge␣beyond␣the␣provided␣legal␣and␣safety␣laws.
19  -␣Determine␣whether␣the␣present␣causes␣logically␣guarantee␣the␣effect␣under␣the␣
        legal␣and␣safety␣laws.
20  -␣The␣effect␣must␣occur␣in␣all␣logically␣possible␣interpretations␣consistent␣with
        ␣the␣provided␣laws␣and␣the␣given␣causes.
21  -␣If␣there␣exists␣any␣logically␣possible␣situation␣under␣these␣constraints␣where␣
        the␣effect␣does␣not␣occur,␣then␣the␣causes␣are␣not␣sufficient.
22  -␣Answer␣"yes"␣only␣if␣the␣effect␣is␣logically␣entailed␣by␣the␣present␣causes.
23  -␣Answer␣"no"␣if␣the␣effect␣is␣not␣logically␣guaranteed.
24  -␣For␣sufficient␣causes,␣indicate:
25  ␣␣-␣"sufficient"␣if␣the␣present␣causes␣logically␣guarantee␣the␣effect,
26  ␣␣-␣"not␣sufficient"␣if␣they␣do␣not␣logically␣guarantee␣the␣effect.
27  -␣Provide␣a␣precise␣explanation␣grounded␣strictly␣in␣the␣provided␣legal␣and␣
        safety␣laws␣that␣determine␣the␣sufficiency.
28
29  You␣are␣given:
30
31  Effect:
32  {effect}
33
34  Causes␣that␣are␣absent␣that␣are␣explicitly␣NOT␣present␣(false):
35  {causes}
36
37  Causes␣that␣are␣present:
38  {present_causes}
39
```

```
Legal␣laws:
{legal_laws}

Safety␣laws:
{safety_laws}


Output␣strictly␣in␣JSON:

{{
␣␣"result":␣"yes"␣or␣"no",
␣␣"reason":␣<reason>
}}

IMPORTANT:␣Output␣must␣be␣strictly␣JSON,␣without␣any␣markdown,␣backticks,␣or␣
    explanations.
'''
```

Listing 7: Sufficient Set Prompt

```
CONVERT_TO_SYMBOLIC_RULE_PROMPT='''
You␣are␣an␣expert␣in␣converting␣a␣condition␣into␣a␣symbolic␣rule.
You␣are␣given␣a␣single␣condition␣in␣a␣logical␣format␣of␣if␣and␣else)␣as␣an␣input.
Your␣task␣is␣to␣produce␣symbolic␣rule␣for␣the␣condition.
You␣must␣follow␣the␣grammar␣provided␣you␣below

Input:
{condition}

Instructions:
-␣Use␣the␣grammar␣provided␣below.
-␣*Do␣not*␣make␣up␣grammar␣by␣yourself.
-␣Use␣only␣the␣allowed␣predicates␣and␣operators.␣Map␣each␣natural␣language␣clause
    ␣to␣meaningful␣predicates␣rather␣than␣inventing␣new␣ones
-␣Before␣giving␣the␣final␣output␣break␣the␣condition␣into␣sub-conditions␣(
    intermediate␣logical␣steps),␣then␣map␣each␣step␣to␣a␣symbolic␣rule
-␣For␣each␣condition,␣think␣about␣what␣safety␣outcome␣or␣system␣effect␣it␣is␣
    describing.␣Then␣map␣it␣to␣the␣symbolic␣predicates␣and␣allowed␣logical␣
    operators.
-␣Do␣not␣create␣rules␣that␣are␣logically␣impossible␣(e.g.,␣a␣vehicle␣being␣both␣
    left␣and␣right␣of␣another␣at␣the␣same␣time)␣or␣that␣contradict␣safety␣logic.
-␣For␣conditions␣involving␣driver␣perception,␣vehicle␣control,␣or␣environment,␣
    make␣sure␣the␣symbolic␣rule␣reflects␣the␣resulting␣safety␣outcome␣(collision
    ␣avoidance,␣safe␣stopping,␣emergency␣brake)

Allowed␣Predicates␣and␣their␣explanations:
-␣dense(i)␣-␣i␣is␣closer␣than␣rdense(a␣distance␣threshold␣for␣crowdedness)␣to␣
    Ndense(a␣number␣of␣nearby␣vehicles␣that␣counts␣as␣dense)␣or␣more␣agents
-␣pred(i,j)␣-␣i␣is␣the␣predecessor␣of␣j
-␣right(i,j)␣-␣i␣is␣to␣the␣right␣of␣j
-␣left(i,j)␣-␣i␣is␣to␣the␣left␣of␣j
-␣in-front(i,j)␣-␣i␣is␣in␣front␣of␣j
-␣behind(i,j)␣-␣i␣is␣behind␣j
-␣merged(i)␣-␣i␣has␣passed␣a␣static␣merging␣point,␣from␣which␣on␣a␣merge␣is␣not␣
    possible␣anymore
-␣sd-front(i)␣-␣i␣has␣a␣safe␣distance␣to␣the␣preceding␣vehicle
-␣sd-rear(i)␣-␣i␣has␣a␣safe␣distance␣to␣the␣following␣vehicle
```

```
- collide(i) - i is colliding with road boundaries or any other agent or obstacle
- lane-change(i) - i is crossing a lane boundary
- near(i,j) - i is closer than dnear to j
- lane-end(i) - i has less than srem remaining to the end of the lane
- acc(i) - i accelerates with a > alim
- speed-adv(i,j) - i is faster than j and some threshold vdiff
- built-up(i) - i is within a built-up area
- motorway(i) - i is on a road type: motorway
- div-lane(i) - i is on a lane type: diverging lane
- acc-lane(i) - i is on a lane type: acceleration lane
-  speed (i) - speed of i
-  friction (i) - friction of i
- collision(i) - collision of i
- EB(i) - i has applied emergency break

Allowed rule forms ONLY:
1. Universal constraint: x    ( condition       condition )
2. Implication:  x (conclusion     condition)
3. Probabilistic rule: x(  (event(x), p%))
4. Conditional probabilistic rule: x,  v (  (event(x), p%)     condition involving
     v)

Allowed comparisons:
1. > greater than
2. < lesser than
3. = equals to
4. >= greater than equals to
5. <= lesser than equals to

Allowed logical operators:
1.     Negation
2.     Conjunction
3.     Implication

Example:
{{
    "condition": "Stopping distance is within vehicle's braking capability",
    "rule": " x (EB(x)    sd-front(x)    friction (x) > 0)",
    "thinking": "Stopping distance is within vehicles braking capability is not
     only about distance.
    It is fundamentally about whether the brakes can generate enough deceleration
      to stop the car before the obstacle.
    For which there should be enough distance with car in front and friction as
     non zero"
}}


Output should be in this format (strict JSON only):
{{
    "condition": "logical condition 1",
    "rule": "symbolic rule 1",
    "thinking": "thought process"
}}
IMPORTANT: Output must be strictly JSON, without any markdown, backticks, or
     explanations.
'''
```

Listing 8: Convert to Symbolic Rule Prompt