Wind Turbine Aeroelasticity: dynamic inflow models

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Sources of Dynamic Inflow

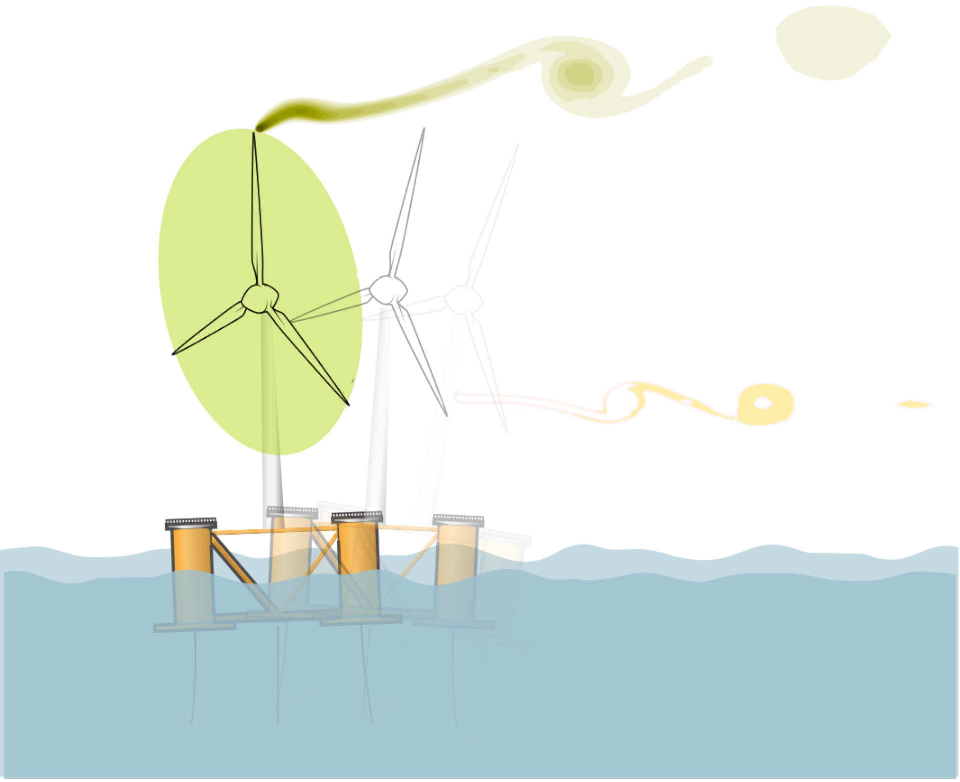

Figure: Representation of dynamic inflow due to the motion of a Floating Horizontal Axis Wind Turbine.

Introduction to Dynamic Inflow

Dynamic inflow is an expression of the unsteadiness of the flow at streamtube scale.

1. As the load on the actuator changes in time, the flow accelerates/decelerates.
2. A new equilibrium is found after sometime, which will equal to a steady-state solution.
3. However, the time scale to reach this equilibrium can be very long, and relevant for e.g. aerodynamic damping.

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Navier-Stokes equations

The incompressible continuity equation in differential form is

$$\rho \vec{\nabla} \cdot \vec{v} = 0$$

The incompressible Navier-Stokes equation for momentum in differential form is:

$$\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = -\frac{\vec{\nabla} p}{\rho} + \frac{\vec{f}}{\rho} + \nu \nabla^2 \vec{v}$$

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Dynamic inflow focus on changes between states of equilibrium

In actuator disk momentum theory

$$C_t = 4a\left(1-a\right)$$

However, in unsteady flow

$$C_t = 4a\left(1-a\right) + k_{(C_t)}\frac{\mathrm{da}}{\mathrm{dt}}$$

There are two approaches to solve the unsteady equation

- derive a form of the momentum calculation that calculates $\frac{\mathrm{da}}{\mathrm{dt}}$
- calculate the transition between two (quasi)-steady equilibrium points: from $C_{t_1} = 4a_1\left(1-a_1\right)$ to $C_{t_2} = 4a_2\left(1-a_2\right)$

Today we will explore models that follow these two approaches.

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The Pitt-Peters dynamic inflow model

The linear Pitt-Peters dynamic-inflow model is shown in the equation below for the case of axial flow in x-direction

- for an annulus at radius r_j with area A_j
- for a loading on the annulus T
- where v_x is the perturbation velocity at the rotor (induced velocity)

$$\left[\frac{8}{3\pi} r_j \rho A_j \frac{\mathrm{d v_x}}{\mathrm{dt}} + 2\rho A_j v_x \left(U_\infty + v_x\right)\right] = T$$

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The Pitt-Peters dynamic inflow model: application to heavy loaded cases

The equation for the Pitt-Peters dynamic-inflow model can be rewritten as

$$\frac{\mathrm{dv_x}}{\mathrm{dt}} = \frac{3\pi U_\infty^2}{16 r_j}\left[\frac{T}{1/2\rho A_j U_\infty^2} - 4\frac{v_x\left(U_\infty + v_x\right)}{U_\infty^2}\right]$$

and further as

$$\frac{\mathrm{dv_x}}{\mathrm{dt}} = \frac{3\pi U_\infty^2}{16 r_j}\left[C_{t_{load}} - C_{t_{(v_x/U_\infty)}}\right]$$

where $C_{T_{load}}$ in the non-dimensional thrust applied to the annulus and $C_{T_{(v_x/U_\infty)}}$ is the expression for an equivalent quasi-steady loading based on the induction v_x/U_∞ and actuator disk momentum theory.

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The Øye dynamic inflow model

The Øye dynamic-inflow model differs from the Pitt-Peters model. Instead of expressing a linear form of the momentum equation, it expresses a filtering of the quasi-steady values through two first-order differential equations

$$v_{int} + \tau_1 \frac{\mathrm{dv_{int}}}{\mathrm{dt}} = v_{qs} + 0.6\tau_1 \frac{\mathrm{dv_{qs}}}{\mathrm{dt}}$$

$$v_x + \tau_2 \frac{\mathrm{dv_x}}{\mathrm{dt}} = v_{int}$$

$$\tau_1 = \frac{1.1}{1 - 1.3a} \frac{R}{U_\infty}$$

$$\tau_2 = \left(\left(\frac{r_j}{R} \right)^2 \right) \tau_1$$

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The Larsen-Madsen model

In this model, as in Ã ye's model, dynamic inflow is modelled using a low pass filtering of the steady state induced velocities.

$$a_{t_{i+1}} = a_{t_i} e^{\frac{-\Delta t}{\tau}} + a_{qs} \left(1 - e^{\frac{-\Delta t}{\tau}}\right)$$

$$\tau = 0.5 \frac{R}{U_{wake}} \simeq 0.5 \frac{R}{a U_\infty}$$

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Modelling dynamic inflow with vortex models

The application of potential flow allows us to calculate the velocity field at any time if we know the vorticity field.

Vorticity is created by a non-conservative force field and convected with the flow.

We will setup a simple model of the dynamic wake vorticity, based on a semi-infinite vortex tube and vortex rings.

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The vorticity equation

We can then present the vorticity equation (Equation 1).

$$\frac{d\vec{\omega}}{\partial t} = \frac{\partial\vec{\omega}}{\partial t} + \left(\vec{v}\cdot\vec{\nabla}\right)\vec{\omega} = \underbrace{\vec{\nabla}\times\frac{\vec{f}}{\rho}}_{\text{source/sink}} + \underbrace{\left(\vec{\omega}\cdot\vec{\nabla}\right)\vec{v}}_{\text{vorticity turning and stretching}} + \underbrace{\nu\nabla^2\vec{\omega}}_{\text{vorticity diffusion}} \tag{1}$$

The material derivative of the vorticity is composed of three terms:

1. a source/sink term, that is generated by a non-conservative force field ($\vec{\nabla}\times\vec{f}\neq 0$).
2. a vorticity turning/stretching term $\left(\vec{\omega}\cdot\vec{\nabla}\right)\vec{v}$, responsible for conserving angular momentum.
3. a vorticity diffusion term $\nu\nabla^2\vec{\omega}$.

Notice that of the three terms, the only term that can create vorticity in a flow without vorticity ($\vec{\omega}=0$) is the source/sink term.

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Representation of an actuator disk with a force field

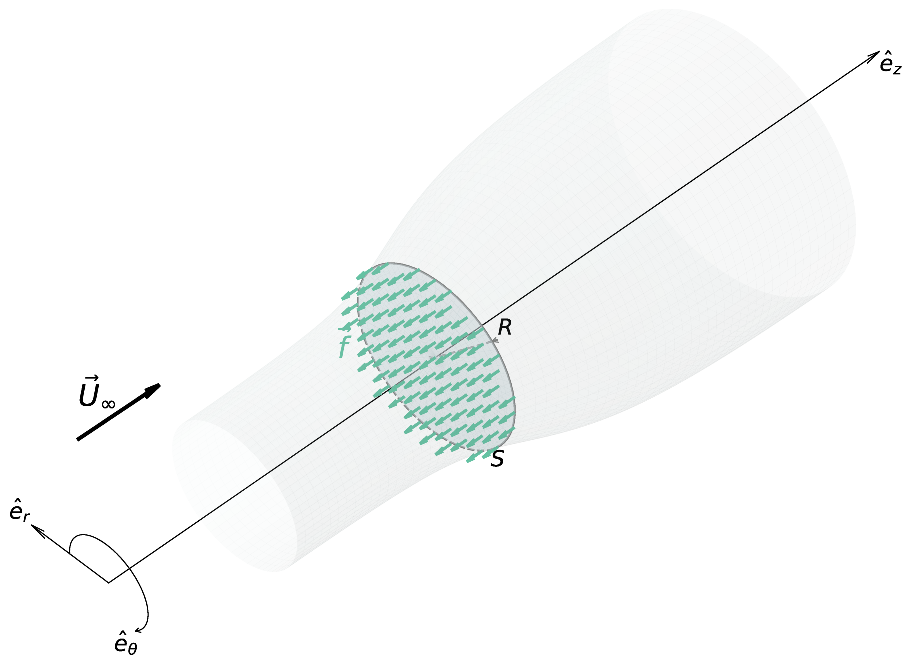

Figure: Representation of an actuator disk of S radius R perpendicular to the unperturbed flow velocity $\vec{U}_\infty$, with a force field $\vec{f}$ in axial/flow direction with an uniform distribution ($|f_z| = const$). The semi-transparent surface represents the boundary of the streamtube of the particles that cross the actuator disk.

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How vorticity is created by this force field

To understand the vorticity field, we will see how vorticity is generated. In an inviscid flow, we can apply the vorticity equation (Equation 1) to our case, resulting in:

$$\frac{\mathrm{d}\vec{\omega}}{\mathrm{d}t} = \vec{\nabla} \times \frac{\vec{f}}{\rho} = \frac{1}{\rho}\vec{\nabla} \times \vec{f} = \frac{1}{\rho}\frac{1}{r}\begin{vmatrix} \hat{e}_r & r\hat{e}_\theta & \hat{e}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ f_r & r f_\theta & f_z \end{vmatrix} = \frac{1}{\rho}\begin{bmatrix} \frac{1}{r}\frac{\partial f_z}{\partial \theta} - \frac{1}{r}\frac{\partial\, r f_\theta}{\partial z} \\ -\frac{\partial f_z}{\partial r} + \frac{\partial f_r}{\partial z} \\ \frac{1}{r}\frac{\partial\, r f_\theta}{\partial r} - \frac{1}{r}\frac{\partial f_r}{\partial \theta} \end{bmatrix} \tag{2}$$

with $\vec{f} = [0; 0; f_z]$ and f_z constant over thw surface S, Equation 2 simplifies to $\frac{\mathrm{d}\vec{\omega}}{\mathrm{d}t} = \left[0; -\frac{\partial f_z}{\rho \partial r}; 0\right]$.
Because $-\frac{\partial f_z}{\rho \partial r}$ is only different from zero at the edge of the actuator, the actuator continuously generates vortex rings of azimuthal vorticity ω_θ, creating a vortex tube. A schematic of this can be seen in Figure

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Representation of an actuator disk with a force field and wake composed of vortex tube/vortex rings

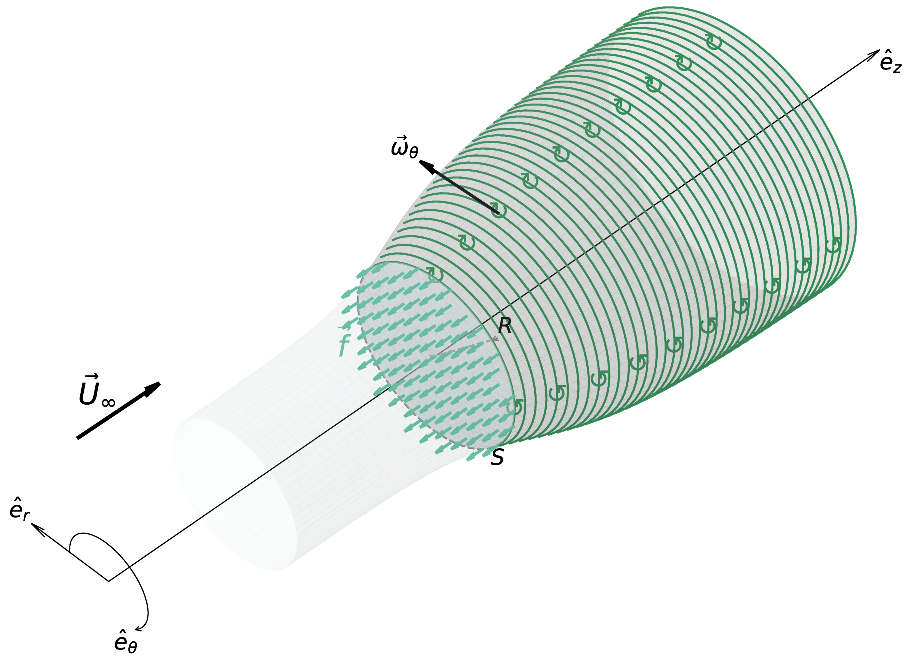

Figure: Representation of an actuator disk of S radius R perpendicular to the unperturbed flow velocity $\vec{U}_\infty$, with a force field $\vec{f}$ in axial/flow direction with an uniform distribution ($|f_z| = const$) and vorticity field generated at the edge of the actuator. The semi-transparent surface represents the boundary of the streamtube of the particles that cross the actuator disk.

Modelling the velocity induced by a vortex ring

To model the flow field of a rotor and its wake, it can be useful to represent the wake by vorticity elements for which we know analytical solutions.

One of these useful solutions is the in-plane symmetric vortex ring. Although the expression "vortex ring" is often used to mean any closed vortex line, we will consider here a line of vorticity located in a single plane, and that all points can be defined at a constant distance R_0 from a point that we will call the center of the vortex ring.

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Representation of a vortex ring

We will represent the vortex ring in cylindrical coordinates, with the ring in a plane perpendicular to the z-axis.

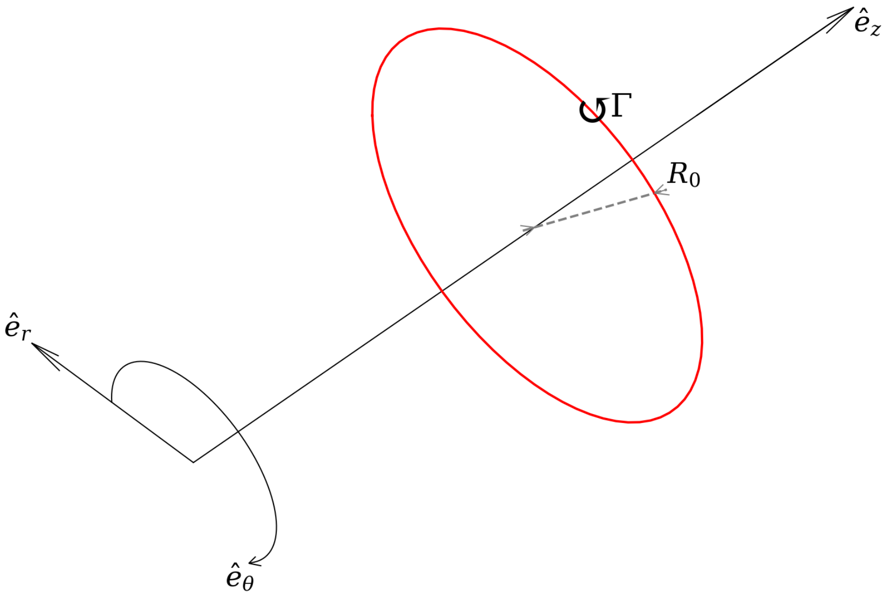

Figure: Representation of a vortex ring of circulation Γ and radius R_0 in cylindrical coordinates. The vortex ring is a plane perpendicular to the z-axis and centred in the axis.

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Velocity induced by a vortex ring

The solution of the velocity induced by the vortex ring at a point of coordinates (r, z) is given by Equations

$$v_\theta = 0 \tag{3}$$

$$v_r = \frac{k\Gamma}{4\pi R_0} \frac{z}{R_0} \left(\frac{R_0}{r}\right)^{\frac{3}{2}} \left(\frac{2-k^2}{2-2k^2} E_{(k^2)} - K_{(k^2)}\right) \tag{4}$$

$$v_z = \frac{k\Gamma}{4\pi R_0} \left(\frac{R_0}{r}\right)^{\frac{1}{2}} \left(\left(\frac{R_0}{2r} \frac{k^2}{1-k^2} - \frac{2-k^2}{2-2k^2}\right) E_{(k^2)} + K_{(k^2)}\right) \tag{5}$$

with

$$k^2 = \frac{4rR_0}{(R_0+r)^2 + z^2} \tag{6}$$

and K and E are the complete elliptic integrals of the 1^{st} and 2^{nd} kind.

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At $r = 0$, the radial velocity is null and the axial velocity is given by

$$v_z = \frac{\Gamma}{2} \frac{R_0^2}{\left(R_0^2 + z^2\right)^{\frac{3}{2}}}$$

At the ring itself, we need to assume a vortex core radius r_{core}, where the velocity acts as a highly viscous flow. At the center of the core the the radial velocity is null and the axial velocity is given by

$$v_z = \frac{\Gamma}{4\pi R_0} \left(\log \frac{8R_0}{r_{core}} - \frac{1}{4}\right)$$

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Representation of the velocity induced by a vortex ring

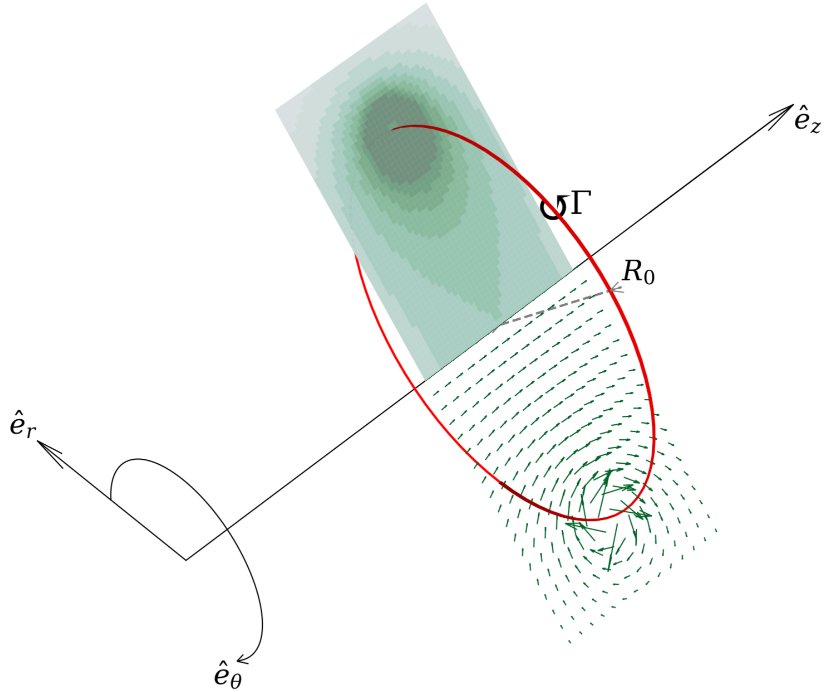

Figure: Representation of a vortex ring of circulation Γ and radius R_0 and the velocity field induced by the vortex ring. Coloured surface indicates the velocity magnitude (non-dimensioned).