A Maintenance Cost Optimization Strategy Based on Prognostics and Health Monitoring Information

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***Abstract*—Maintenance interventions impact the operational cost of equipment and systems. The use of Prognostics and Health Monitoring (PHM) information to support the implementation of an effective maintenance strategy has been a topic of great interest among researchers and industry practitioners. This paper presents a condition-based maintenance (CBM) optimization strategy for a single unit degrading system. The system under consideration is subjected to a degradation process that is modeled as a Gamma process. The system degradation level is obtained during non-periodic inspections. After each inspection, a maintenance scope is defined in order to minimize the expected maintenance cost per time unit. If a system failure is revealed during an inspection, the system is replaced and its degradation level is restored to the "as good as new" state. If the system is still functioning but with at a high degradation level, imperfect repair actions take place. Imperfect repair actions reduce the degradation level, but the system may still present a residual degradation level. In addition, imperfect repair actions may cause an increase in the system degradation rate. In the proposed strategy, the decision-maker may also decide not to perform maintenance after an inspection. The proposed maintenance strategy can dynamically define the best maintenance decision at each inspection, as well as the best time to perform the next inspection, based on system degradation level and the involved costs. Numerical experiments are carried out to illustrate the application of the proposed strategy. The impact of system degradation variability and inspection cost in the performance of the proposed strategy is analyzed.**

***Index Terms*—prognostics, health monitoring, maintenance, imperfect repair, cost optimization.**

I. Introduction

The efficiency of a maintenance strategy may have a significant impact on the operational cost of equipment and systems. The use of Prognostics and Health Monitoring (PHM) information to enable the implementation of a Condition Based Maintenance (CBM) strategy has been a topic of interest among researchers and industry practitioners in recent years. Condition Based Maintenance is a proactive maintenance strategy that helps maintenance planners to schedule maintenance interventions based on the assessment of the health condition of systems. When a CBM approach is used, optimal maintenance decisions can be made based on an objective function to be optimized. Maintenance cost and system reliability are the most common optimization criteria used to define maintenance policies.

Different degradation models have been used to describe the evolution of the system degradation level. Many authors use gamma processes to describe degradation evolution. In [1], the authors consider a system that degrades continuously and define a fixed preventive replacement zone that indicates when preventive repair actions must be performed. In [2], the authors consider that the system under study fails whenever its degradation level exceeds a dynamic failure threshold. They also investigate the economic benefits of grouping maintenance activities of different components. In [3], the authors consider both perfect and imperfect maintenance actions. They propose a model to identify the optimal number of imperfect actions that should be performed before replacing the system with a new one.

Another approach commonly used by researchers is modeling system degradation as a Wiener process. This approach can describe both monotonous and non-monotonous degradation processes. In [4], the authors propose a stochastic Remaining Useful Life (RUL) estimation method that considers the influence of imperfect maintenance interventions on system degradation level and system degradation rate. In [5], the randomness of failure threshold is taken into account to develop a method to estimate the RUL.

In [6], the authors present an investigation on degradation model selection. They present a set of numerical studies to identify in which scenarios each classical degradation model is likely to perform better than others.

The literature also contains different models for imperfect maintenance interventions. For instance, the model proposed in [7] assumes that maintenance interventions always reduce the degradation level, but may increase the degradation rate. The model presented in [8] considers that maintenance can be harmful and damage the system, causing an increase in the degradation level.

This paper presents a maintenance cost optimization strategy based on system health condition information. A gamma process is used to model the system degradation evolution. During each inspection, two decisions are made as a function of the observed degradation level and the costs involved: the maintenance scope that minimizes the expected maintenance cost per time unit, and the optimal interval until the next inspection.

The remaining sections of this paper are organized as follows. Section II presents the system degradation process model. Section III describes the proposed maintenance policy.

Section IV presents the model that describes how imperfect maintenance interventions affect both system degradation level and system degradation rate. Section V presents the results obtained in numerical experiments carried out to illustrate the application of the proposed maintenance strategy. Concluding remarks and directions for future research are presented in section VI.

II. System Degradation Process

Consider a deteriorating system. Let $X(t)$ be the system degradation level at time t. The system has a monotonic degradation process. Inspections, imperfect repairs, and replacements may occur only in equally spaced time instants, called maintenance opportunities. The interval between two consecutive maintenance opportunities is called a period. The degradation accumulated by the system in each period (i.e. between two consecutive maintenance opportunities) is described by a random variable that follows a gamma distribution. This assumption has been adopted by many authors [1], [9], [3]. Gamma processes are monotonically increasing and positive, which are desirable characteristics when modeling degradation processes. The PDF (probability density function) of the gamma distribution is presented in (1).

$$f(x|w,\theta) = \frac{x^{w-1} \cdot exp^{\frac{-x}{\theta}}}{\Gamma(w) \cdot \theta^{w}}; x, w, \theta > 0 \qquad (1)$$

where w is the shape parameter of the gamma distribution, θ is the scale parameter of the gamma distribution, and $\Gamma(w)$ is the Gamma function evaluated at w, which is computed according to (2).

$$\Gamma(w) = \int_{x=0}^{\infty} x^{w-1} \cdot exp^{-x} dx \qquad (2)$$

Fig. 1 illustrates the evolution of the system degradation level $X(t)$ over time and the probability of a system failure to occur before a certain maintenance opportunity. In Fig. 1, $t = 0$ represents the current time, and assuming that $X(0)$ and parameters w and θ of the gamma distribution that describes the increment in the degradation level are known, the probability of a failure to occur before $t = 1$ can be computed as the probability that $X(1) \geq L$, where L is the failure threshold.

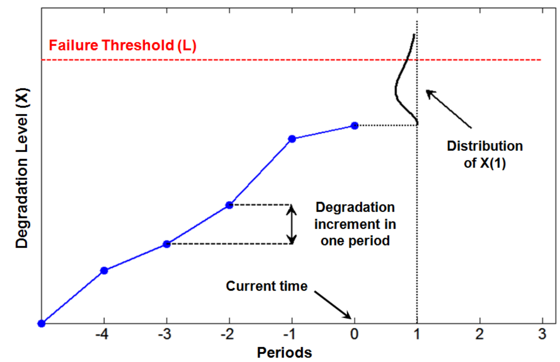

Fig. 1. System degradation evolution and failure probability estimation.

III. Maintenance Policy

The maintenance policy adopted in this paper is based on the system degradation level $X(t)$ obtained during inspections. After each inspection, two decisions are made [1]:

- Definition of the maintenance scope: perform a preventive repair, replace the system (if a failure is detected during inspection), or leave the system as it is.
- Definition of the best time to perform the next inspection.

As mentioned earlier, three different maintenance actions are considered in this paper: inspection, imperfect repair and replacement. The assumptions assumed in this paper are listed below.

- The duration of inspections, preventive repairs and replacements are small and can be ignored.
- The maintenance opportunities are equally spaced in time.
- A failure occurs whenever system degradation is equal to or greater than a fixed failure threshold level L.
- If a failure is observed during an inspection, the failed system must be replaced immediately.
- An inspection cost C_I is incurred during each inspection.
- Replacements bring the system to the "as good as new" state (i.e. the degradation level returns to zero).
- A replacement cost C_C is incurred every time a failed system is replaced.
- A degraded but still functional system can be repaired. In this case, a preventive repair cost C_P, with $C_P < C_C$, is incurred.

The scope of the maintenance to be performed after each inspection is defined in order to minimize the maintenance cost rate. Assuming that an inspection occurred at time t, let $X(t^-)$ be the system degradation level observed during the inspection. Also, assuming that a maintenance action (repair or replacement) occurred at time t, let $X(t^+)$ be the system degradation level immediately after the maintenance action. Fig. 2 shows the process used to define the maintenance scope.

A. Maintenance Cost Model

Assume that t is the current time. Also, assume that an inspection is performed at time t. If $X(t^-) \geq L$, a system failure is detected and the system is replaced. A replacement cost C_C is incurred. System degradation level returns to zero, i.e. $X(t^+) = 0$.

If $X(t^-) < L$, the system is still functioning. In this case, the cost rate associated with performing a preventive repair with cost C_P at instant t, denoted by $CPR(t)$, is computed according to (3).

$$CPR(t) = \frac{(M+1) \cdot C_I + C_P}{D} \qquad (3)$$

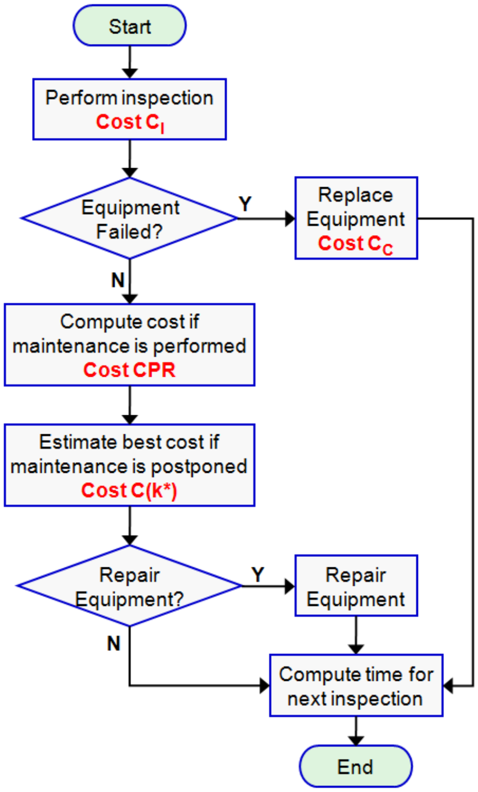

Fig. 2. Maintenance scope definition.

where C_I is the inspection cost, C_P is the preventive repair cost, D is the number of periods since the last maintenance intervention (repair or replace), and M is the number of inspections performed since the last maintenance intervention.

Then, the expected cost rate associated with performing no maintenance at instant t and schedule the next system inspection for k periods in the future, denoted by $C(k)$, is estimated according to (4).

$$C(k) = \frac{(M+2) \cdot C_I + C_P \cdot Q_F(k) + C_C \cdot P_F(k) + C_D \cdot T_D(k)}{D+k} \quad (4)$$

where C_C is the replacement cost, $P_F(k)$ is the probability that a system failure will occur until period k, $Q_F(k)$ is the complement of $P_F(k)$ (i.e. $Q_F(k) = 1 - P_F(k)$), C_D is the downtime cost per period, and $T_D(k)$ is the expected system downtime if the next inspection is performed k periods in the future, which can be computed according to (5).

$$T_D(k) = \int_0^k P(X(t) \geq L) \cdot (k-t)dt \quad (5)$$

Let k^* be the value that minimizes $C(k)$. If $C(k^*) < CPR(t)$, then no maintenance action is performed at period t. The next inspection is scheduled for k^* periods in the future, and $X(t^+) = X(t^-)$. Otherwise, a preventive repair action is performed, and the degradation level is reduced to $X(t^+) \leq X(t^-)$. Figs. 3 and 4 illustrate, respectively, a situation in which a preventive repair action is performed, and a situation in which no maintenance action takes place based on the maintenance cost rate criterion.

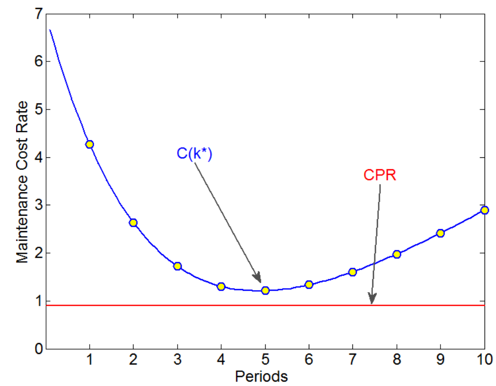

Fig. 3. Example of a situation in which a preventive repair is performed.

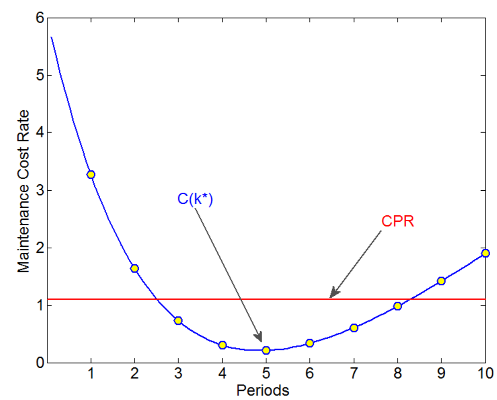

Fig. 4. Example of a situation in which no maintenance action is performed.

IV. Imperfect Maintenance Model

In this paper, the impact of imperfect maintenance interventions in system degradation is twofold: (1) it reduces the current degradation level, and (2) it increases the degradation rate.

A. Reduction in System Degradation Level

When modeling maintenance effectiveness, two extreme cases can be defined. The first case is the perfect maintenance approach, that assumes that every maintenance action brings the system under consideration to the "as good as new" state (the degradation level returns to zero). The second case is the minimal repair approach, that assumes that every maintenance action brings the system under consideration to the "as bad as old" state (the system degradation level does not change) [10].

In most real situations, maintenance interventions do not affect the system degradation level as described by the two aforementioned extreme cases. Instead, the impact of maintenance interventions in system degradation level has an intermediate behavior between minimal repair and perfect repair. In the literature, this intermediate behavior is called imperfect maintenance [11]. Fig. 5 illustrates the impact of a maintenance intervention in system degradation level when different maintenance approaches are considered.

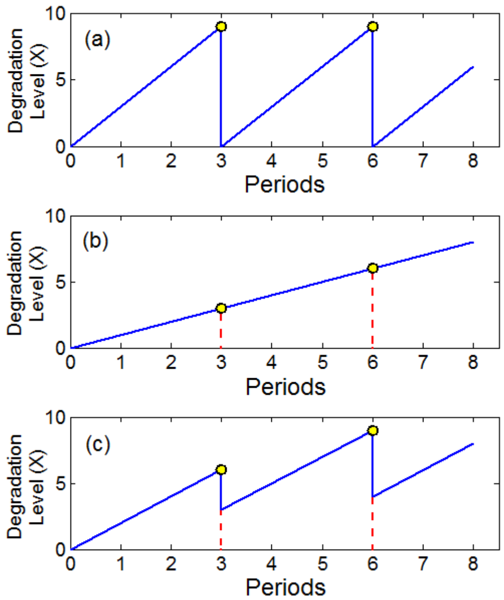

Fig. 5. Types of maintenance actions. (a) Perfect repair. (b) Minimal repair. (c) Imperfect repair. [10]

Different imperfect maintenance models have been proposed in the literature. For instance, the authors in [12] proposed a model in which maintenance interventions bring the system degradation level to the "as good as new" state with probability ρ, and bring the system degradation level to the "as bad as old" state with probability $1-\rho$. In [2], the authors proposed a model in which maintenance interventions reduce the system degradation level by a factor $1-b$, where b is called the improvement factor. For each maintenance intervention, the improvement factor is randomly chosen from a beta distribution. Other classes of imperfect maintenance models are presented in [13].

In the imperfect maintenance model adopted in this paper, whenever a preventive maintenance action is performed, the system degradation level after the maintenance, $X(t^+)$, is computed according to (6).

$$X(t^+) = X(t^-) - \Delta X \tag{6}$$

where ΔX is a random variable chosen from a triangular distribution with support $[0, X(t^-)]$.

When $\Delta X = 0$, $X(t^+) = X(t^-)$ and a minimal repair is performed. When $\Delta X = X(t^-)$, $X(t^+) = 0$ and a perfect repair is performed. The probability density function of a triangular distribution is given by (7).

$$f(x) = \begin{cases} \frac{2(x-a)}{(c-a)(b-a)} & \text{, for } a \le x \le b \\ \frac{2(x-a)}{(c-a)(b-a)} & \text{, for } b < x \le c \\ 0 & \text{, otherwise} \end{cases} \tag{7}$$

where a is the lower limit, b is the mode, and c is the upper limit of the distribution.

B. Increase in System Degradation Rate

In the model proposed in this paper, when imperfect maintenance is carried out, it not only reduces the system degradation level but also increases the system degradation rate. This assumption has been used in other models proposed in the literature [3], [7].

As described in section II, the degradation accumulated by the system in each period is modeled as a gamma distribution with shape parameter w and scale parameter θ. When an imperfect repair is performed, the parameters of the resulting gamma distribution are updated based on the following assumptions:

- The mean of the resulting gamma distribution is multiplied by a fixed degradation factor η.
- The variance of the resulting gamma distribution is not affected.

The mean μ and the variance σ^2 of a gamma distribution with shape parameter w and scale parameter θ are given by (8) and (9), respectively [14].

$$\mu = w \cdot \theta \tag{8}$$
$$\sigma^2 = w \cdot \theta^2 \tag{9}$$

Let $w(t^-)$ and $\theta(t^-)$ be parameters of the gamma distribution just before the start of a repair maintenance. Similarly, let $w(t^+)$ and $\theta(t^+)$ be parameters of the gamma distribution just after the completion of a repair maintenance. In order to simultaneously satisfy the aforementioned assumptions, it is easy to show that parameters w and θ must be updated according to (10) and (11), respectively.

$$w(t^+) = w(t^-) \cdot \eta^2 \tag{10}$$
$$\theta(t^+) = \frac{\theta(t^-)}{\eta} \tag{11}$$

Let w_0 and θ_0 be the shape and the scale parameters that describe the degradation accumulated by a new system during one period, respectively. Every time a system failure is observed during an inspection and the system is replaced, parameters w and θ are reset to w_0 and θ_0, respectively.

V. Numerical Experiments

This section presents the results obtained in numerical experiments carried out to illustrate the application of the proposed maintenance strategy. Two different simulations are presented. First, the impact of system degradation variability is investigated. System degradation variability is related to how the increment in system degradation in one period can vary, which is quantified by the variance of the system degradation gamma distribution presented in (9). Second, the impact of the relative value of the inspection cost C_I is investigated.

A. Impact of System Degradation Variability

In this first experiment, the goal is to investigate the impact of system degradation variability. The increase in the degradation level observed in one period may vary due to changes in many factors such as system load, environmental conditions, system age, human factors, etc.

A reference system, denoted by system 1, is defined. The results obtained from the simulation of system 1 will be used as a reference baseline. Table I shows the parameters used in the simulation of the reference system.

TABLE I
SIMULATION PARAMETERS FOR THE REFERENCE SYSTEM.

Parameter	Value
w_0	5
θ_0	1
η	1.08
L	120
C_I	5
C_P	30
C_C	100
C_D	25

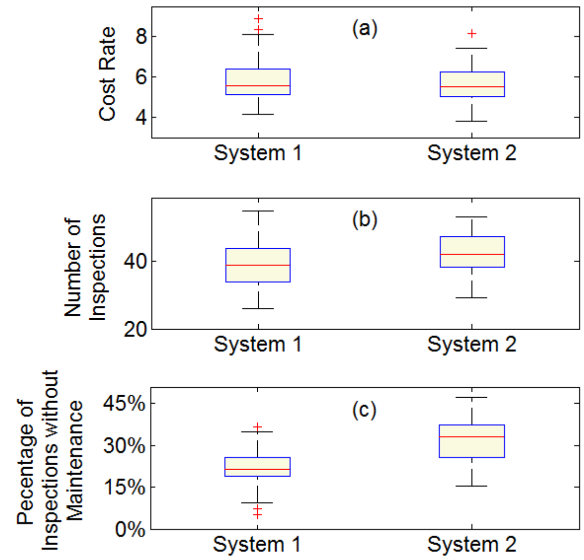

Fig. 6. Comparison between system 1 (reference system) and system 2 (with a higher degradation variability).

In this first experiment, in order to investigate the impact of system degradation variability in the performance of the proposed strategy, a comparison is made between the reference system and a system with a higher degradation variability, denoted by system 2. The initial values for the shape parameter w_0 and the scale parameter θ_0 used in the simulation of system 2 are presented in Table II. The values for all other parameters are the ones presented in Table I.

TABLE II
SIMULATION PARAMETERS FOR SYSTEM 2.

Parameter	Value
w_0	2.5
θ_0	2.0

One can see that the mean increase in system degradation level in one period for system 1, denoted by μ_1, is equal to the mean increase in system degradation level in one period for system 2, denoted by μ_2 (see (8)). However, the variance of the increase in system degradation level in one period for systems 1 and 2, denoted by σ_1^2 and σ_2^2, respectively, are different. In this example, $\sigma_1^2 < \sigma_2^2$ (see (9)).

A total of 100 simulations were carried out for each system. Fig. 6 shows a comparison between the results obtained in terms of cost rate, total number of inspections, and percentage of inspections in which no maintenance interventions were performed. Figs. 7 and 8 show the result of one simulation of the degradation evolution of reference system 1 and system 2, respectively.

System 2 presented a higher number of inspections than system 1. Also, the percentage of inspections in which no maintenance intervention (repair or replacement) was performed was much higher for system 2 in comparison with system 1. These results can be explained by the fact that the higher degradation variability of system 2 leads to a conservative approach when defining the time for the next inspection. The shorter average interval between inspections increases the chance of observing a low degradation level during the inspections, which increases the chance that no maintenance intervention will be necessary or justified in terms of cost rate. Despite the differences in the number of inspections and the scope of maintenance interventions, it was not possible to observe a significant change in the cost rate.

Simplicity is a desired characteristic for maintenance strategies. A simple and commonly used maintenance strategy is to choose a maintenance threshold of Z, with $0 < Z < L$, which defines the minimum degradation level that justifies a preventive repair. So, if during an inspection a degradation level $X < Z$ is observed, no maintenance intervention is needed. If a degradation level $Z \leq X \leq L$ is observed, a preventive repair action is performed. This decision process has been used by different researchers [14], [15], [16].

The results showed that when the system has a lower degradation variability, it is possible to define a maintenance threshold Z so that, in most cases, the simple maintenance decision process described earlier will provide the same maintenance decision obtained by the maintenance cost minimization strategy proposed in this paper. For instance, in Fig. 7, the maximum degradation level observed during an inspection in which no maintenance intervention was performed was $X(175) = 100.1$, while the minimum degradation level observed in which a repair action was performed was $X(166) = 101.3$. In this case, with a maintenance threshold $100.1 \leq Z \leq 101.3$, the maintenance decision after each inspection would be the same obtained by minimizing the cost rate.

However, when system degradation variability increases, the use of a fixed maintenance threshold of Z and the proposed

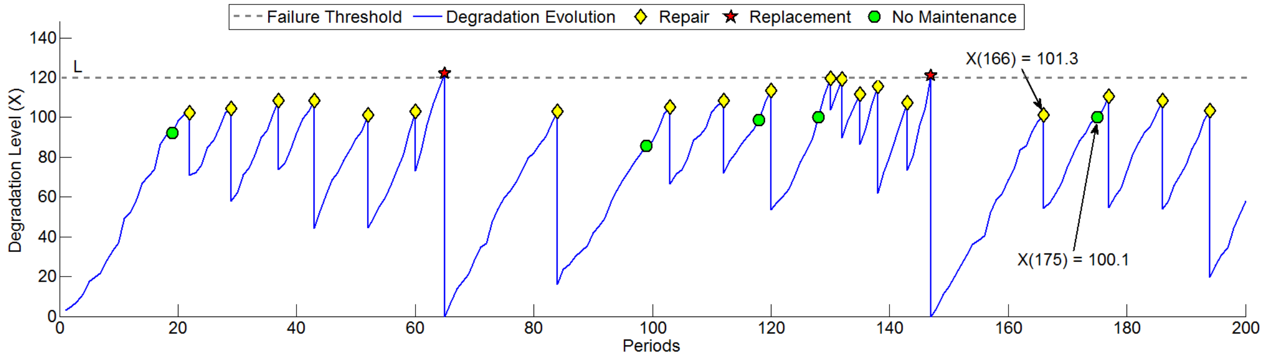

Fig. 7. Degradation evolution of the reference system 1.

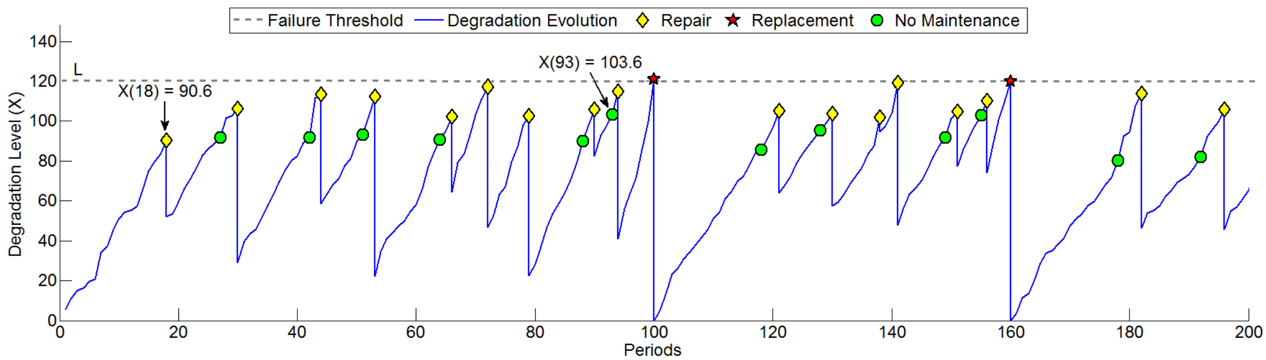

Fig. 8. Degradation evolution of system 2, a system with a higher variability.

strategy lead to different results. For instance, in Fig. 8, the maximum degradation level in which no maintenance intervention was performed was $X(93) = 103.6$. This value is higher than the minimum degradation level observed in which a repair action was performed, $X(18) = 90.6$.

B. Impact of Inspection Cost

In this second experiment, the goal is to investigate the impact of the inspection cost C_I. System 3 is defined, with an inspection cost of 15. The other parameters are the same used for system 1 as presented in Table I. A total of 100 simulations of system 3 were carried out. Fig. 9 shows the result of one simulation of the degradation evolution of system 3. Fig. 10 shows a comparison between system 3 and system 1 in terms of cost rate, total number of inspections, and percentage of inspections in which no maintenance interventions were performed.

The higher inspection cost C_I considered in the simulation of system 3 caused a decrease in the average number of inspections. It also caused a decrease in the percentage of inspections in which no maintenance intervention was performed. With a higher inspection cost, performing a maintenance intervention becomes more attractive. The average degradation level after an inspection (and the respective maintenance intervention) is smaller. Consequently, the time until the next inspection increases, which causes a decrease in the total number of system inspections.

VI. Conclusions

This paper presented a condition-based maintenance strategy to minimize the maintenance cost per time unit of a single unit system. The proposed maintenance strategy indicates the best decision on the maintenance scope based on the degradation level observed and the cost parameters.

Numerical experiments showed that the proposed strategy is more robust in comparison with the definition of a fixed preventive maintenance threshold, especially when dealing with systems with a high degradation variability.

Future research may extend the scope of this paper by introducing an algorithm to identify the parameters of the degradation level distribution based on the measurements observed during the inspections.

Acknowledgment

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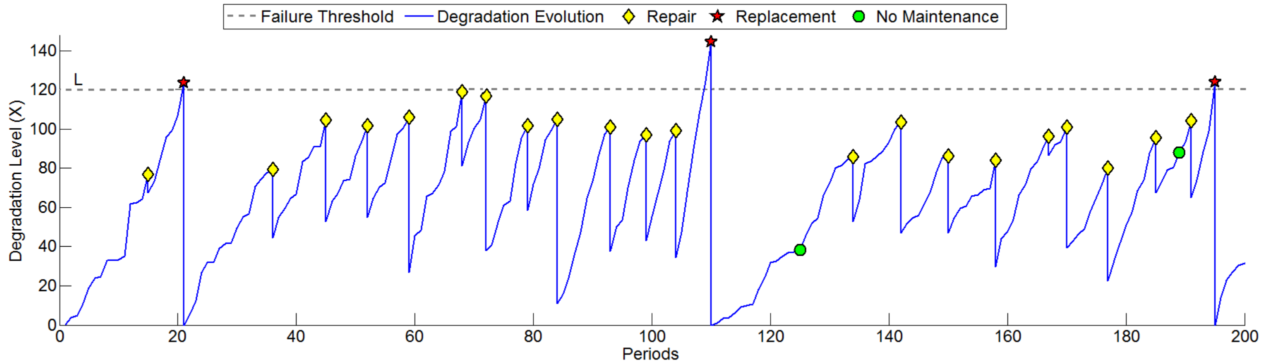

Fig. 9. Degradation evolution of system 3, a system with higher inspection cost C_I.

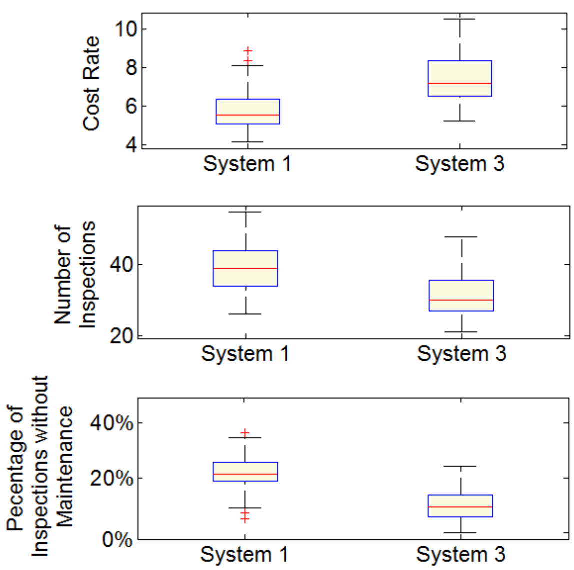

Fig. 10. Comparison between system 1 (reference system) and system 3 (a system with a higher inspection cost C_I).

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