Life Supporting Orbits in a Binary Star System

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ABSTRACT
Binary star systems are common in nature, outnumbering even single star systems. Like our own solar system, these too may also have planets and other debris orbiting them in a myriad ways. This report aims to explore possible ways in which a planet can stably exist in such a system, the possible orbits it may have, and whether such systems could house planets suitable for life. This is an example of a 3-Body problem, which are notorious for having complex analytical solutions. I will use numerical methods to calculate solutions for the system of interest. By changing the starting conditions for the planet and stars, multiple stable orbits can be found - some chaotic and some suitable for life.

1 INTRODUCTION

Binary stars are systems such that a pair of stars are gravitationally bound and orbiting each other. This can give rise to interesting interactions between the stars. A most interesting example is the transfer of mass between the stars allowing the formation of pulsars, Bhattacharya & van den Heuvel (1991). Binary systems are necessarily more complex to resolve than single star systems. For instance, if we consider a planet orbiting a single star, Kepler's laws predict that the planet will move in an ellipse around the star. However, if we add a second star to form a binary, then the problem becomes an example of the infamous 3 body problems which are notoriously hard to solve. In the 18th Century, Euler and Lagrange found solutions to specific examples of 3 body problems, and much later a general analytical solution would be found. In 1912 Karl Fritiof Sundman proved that an analytical solution to 3 body problems exists, in the form of a power series (Barrow-Green (2010)). This solution is not suitable for astronomical use however, due to how slowly its solution converges for reasonable accuracy. Nowadays we turn to numerical methods to solve these problems with great precision. In reality, binary stars consist a majority of star systems, so being able to resolve their motion is of great importance. We as a species have always wondered whether life exists on other planets, and if it is discovered that a very limited number of suitable orbits are available around binary stars then we can constrain our search significantly. This paper aims to investigate some binary orbits and consider whether they could be suitable for life.

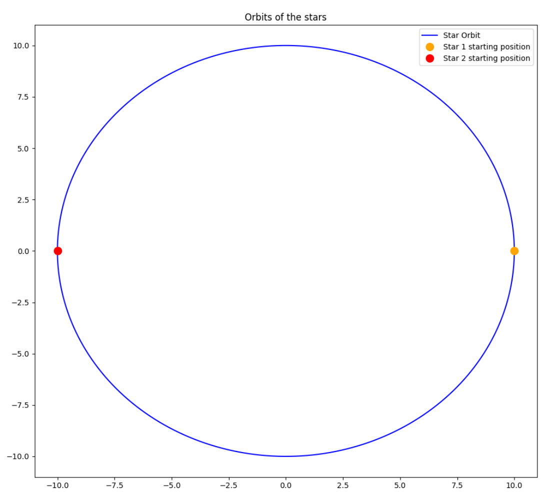

Figure 1. The movement of the stars. They both move anticlockwise on the path shown, always being exactly opposite each other.

2 CREATING AND SOLVING THE SYSTEM

2.1 Assumptions

For simplicity, I use the following assumptions about the system -

- The system acts under non-relativistic Newtonian gravity.
- The pair of stars are constrained to a plane, being essentially on rails.
- The stars have a circular orbit.
- The stars have an equal mass.
- The stars don't interact with each other, ie no mass transfer, accretion discs, braking, etc.
- The planet is much lighter than the stars so it has no effect on their orbits.

2.2 Equations of Planetary Motion

Using the given assumptions, I create the model as follows. Taking the stars to move in circles, their orbits can be simply expressed using simple sine and cosine functions in Cartesian coordinates, equations 1a, 1b, 1c, 1d. The orbit of the pair of stars can be seen in figure 1.

$$X_1 = R\cos\Omega t \tag{1a}$$

$$Y_1 = R \sin \Omega t \tag{1b}$$

$$X_2 = -R \cos \Omega t \tag{1c}$$

$$Y_2 = -R \sin \Omega t \tag{1d}$$

Where X_1, Y_1 are the x and y coordinates of the respective stars, R is the radius of orbit, Ω the frequency of orbit, and t time.

Now I consider the location of the orbiting planet, at a position (x, y). Newtons law of gravitation requires the distance between the bodies, so it is useful to define the distance between the planet and each of the stars squared as their own equations - 2a and 2b.

$$r_1^2 = (x - X_1)^2 + (y - Y_1)^2 \tag{2a}$$

$$r_2^2 = (x - X_2)^2 + (y - Y_2)^2 \tag{2b}$$

Where r_1 and r_2 are the distances between the planet and stars 1, 2 respectively, and x, y are the x and y coordinates of the planet at a given time.

I now may define the equations of motion for the planet - 3a, 3b as second order differential equations, using the quantities defined above.

$$\ddot{x} = -GM(\frac{x - X_1}{r_1^3} + \frac{x - X_2}{r_2^3}) \tag{3a}$$

$$\ddot{y} = -GM(\frac{y - Y_1}{r_1^3} + \frac{y - Y_2}{r_2^3}) \tag{3b}$$

Where $\ddot{x}$ and $\ddot{y}$ are the x and y accelerations of the planet, G is the gravitational constant, and M the mass of the stars.

It is important to note that there is a relationship between the constants, equation 4.

$$GM = R^3\Omega^2 \tag{4}$$

2.3 Solutions to the Equations

In order to solve the equations for planetary motion, they must be transformed into a different form for the numerical algorithm I will use - the Runge-Kutta Methods Press et al. (2007).

A suitable form is a pair of coupled first-order differential equations. The transformation is as follows - I first define v_x and v_y, the x and y velocities as the derivatives of x and y, equations 5a and 5b.

$$v_x = \dot{x} \tag{5a}$$

$$v_y = \dot{y} \tag{5b}$$

Using these equations for the velocities, I can now redefine the equations of motion in terms of the derivative of velocity - acceleration.

$$\dot{v_x} = -GM(\frac{x - X_1}{r_1^3} + \frac{x - X_2}{r_2^3}) \tag{6a}$$

$$\dot{v_y} = -GM(\frac{y - Y_1}{r_1^3} + \frac{y - Y_2}{r_2^3}) \tag{6b}$$

Where $\ddot{x}$ and $\ddot{y}$ are the x and y accelerations of the planet, G is the gravitational constant, and M the mass of the stars.

A final requirement is for the equations to be dimensionless. I use the following unit transformations to achieve this -

- Use units where the gravitational constant, G, is equal to 1.
- Measure distances is units of R.
- Measure mass in units of M.
- Measure time in units of $\frac{1}{\Omega}$.

An alternative method for removing Ω in equations 1 is by using the relation between the constants, $GM = R^3\Omega^2$, to set Ω equal to 1. For example, $M = 1000M$ and $R = 10R$, using units of $G = 1$.

2.4 Coordinate Transfomation

Some orbits may look chaotic in the reference frame used insofar, the inertial frame. However changing into an alternate reference frame may make some complex orbits look comparatively simple - a reference frame rotating with the stars.

To transform the coordinates, I apply the following equations to x and y to get two new coordinates, ξ (equation 7a) and η (equation 7b).

$$\xi = x \cos \Omega t + y \sin \Omega t \tag{7a}$$

$$\eta = -x \sin \Omega t + y \cos \Omega t \tag{7b}$$

2.5 Errors

To reduce errors in the calculation, I chose a step size for time that produced smooth plots and also took a reasonable amount of time to calculate. When increasing the step size, the main time increase was plotting the graph, not calculating the solution to the differential equations. When increasing the amount of steps from 500000 to 1000000 the calculation only took approximately 0.035 seconds longer. When the graph was finally plotted, having such a large amount of points also caused it to lag severely making it hard to change it to a suitable size to allow all the detail to be seen clearly. However, this is only cosmetic when plotting and makes no difference to the final graph.

Another method to reduce errors is to specify errors parameters in the function used to solve the differential equations. I chose not to change this from the default as by default the tolerance is already small enough for my needs.

3 RESULTS

Different orbits may be discovered from changing the initial conditions for the differential equations. For each of the orbits, I have provided the initial conditions used for comparison.

3.1 Planet Far From the Stars

3.1.1 Circular Orbit

If I place the planet far enough away from the stars, the pair of rotating stars appear to the planet as a single point with the mass

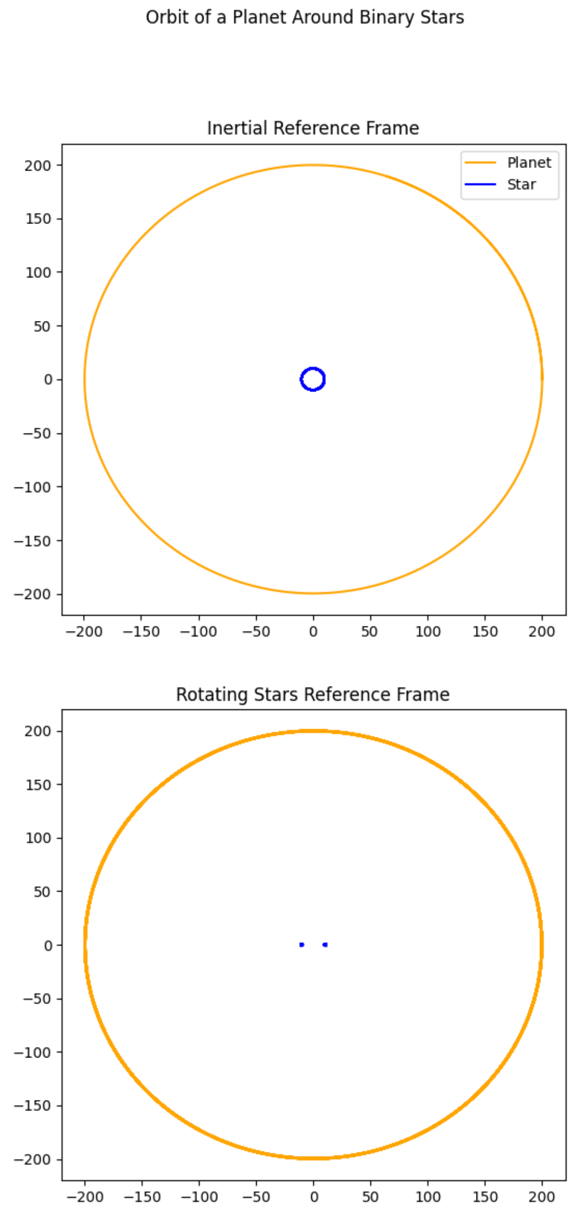

Figure 2. Planet far away from the stars. R=10R and M=100M for the stars, with initial conditions of x0=200R, vy0=1, y0=vx0=0.

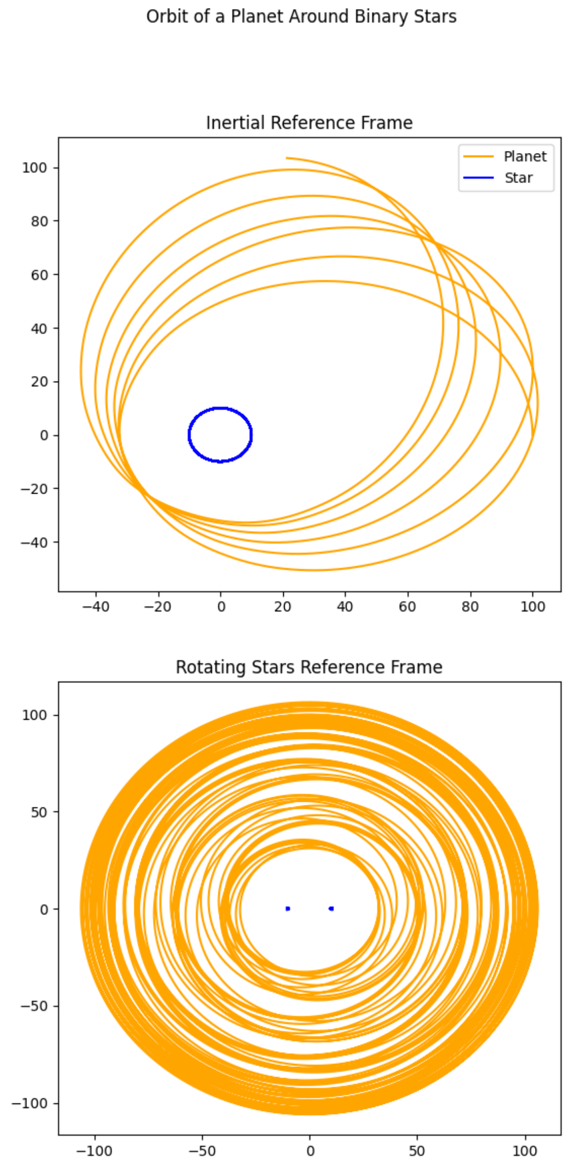

Figure 3. Planet far away from the stars. R=10R and M=100M for the stars, with initial conditions of x0=100R, vy0=1, y0=vx0=0.

of both stars added together. Thus a standard circular or elliptical orbit is obtained as if the planet was orbiting a single star. Figure 2. In both reference frames, the orbit looks the same.

3.1.2 Precessing Orbit

Placing the planet slightly closer to the stars causes a procession of the orbit - its ellipse moves slowly around the stars, in figure 3. This is due to the force difference between the stars now becoming non-negligible. A major difference between this situation and figure 2 is in the rotating stars reference frame - here the orbit looks quite chaotic, and appears to orbit the stars many times whereas in figure 2 it appears to only orbit once. The reason for this is that the planet does orbit the stars many times in figure 2 - but because it is a circle, the orbit stays the same. This is not the case for figure 3 - it precesses around the stars. While the planet may only make a few orbits around the stars, since it is so far away, the stars orbit each other many hundreds of times. This gives rise to the many apparent orbits of the planet in the rotating frame.

3.2 Prograde Close Orbits

Here I have created orbits for the planet moving in the same direction as the stars, however I have moved it much closer. To create stable orbits here, the velocity must be much higher.

3.2.1 Planet Orbiting a Star

Figure 4 shows an example of a planet orbiting one of the stars. The orbit of the planet appears thick due to precession - the other star causes its orbit to shift slightly. This can also be seen in the rotating frame - the radius of the planets orbit around one of the stars varies slightly over time.

It is possible to find an orbit with much less orbit shift but with similar behavior. By moving the stars to have an orbit of radius 100R, an extremely tight orbit may be seen. At these distances the effect of the other planet is minor, so I have used a simple relation between the force of gravity and centripetal force to calculate a more precise velocity, shown in figure 5 and using formulae 8. The detail of the orbit is hard to resolve due to its tightness, so in figure 6 I have zoomed in on the rotating star reference frame. More specifically, around the star that the planet orbits. Here we can see that indeed the planet orbits the star with much less spread in its orbit as expected. The orbit of the planet in the rotating frame is slightly elliptical however.

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \tag{8a}$$

$$\Rightarrow v = \sqrt{\frac{Gm}{r}} \tag{8b}$$

Where M,m are the masses of the stars and planet respectively, v is the velocity of the planet, r is the distance of the planet from the star and G is the gravitational constant.

3.2.2 Wide Planet Orbit

Figure 7 shows a more complicated yet clear pattern - the star appears to swing around, in a wide orbit, about one of the stars with its distance changing dramatically. This distance change can be more clearly seen in the rotating frame - at times it is close, being only 1R away, whereas at other times it can be as far as 7R away. The orbit appears to be a combination of a lop-sided figure of 8 and a slightly contorted ellipse.

3.3 Retrograde Orbits

Here are some orbits where the planet is set off in the opposite direction to the stars. These orbits are much more erratic than the prograde orbits in the inertial frame, however orderly patterns are still observed in the rotating frame.

3.3.1 Chaotic Retrograde 1

In figure 8, the orbit in the inertial frame is very messy. While there are some recognisable patterns outside the orbits of the stars, they seem to change as the planet orbits. The rotating frame is a different story. There is a clear pattern - it looks like something produced by a spirograph Desvignes (1886). It has the same quality that figure 7 has, where the planet can be far away or extremely close to the star, however this time the closest distance is much more extreme.

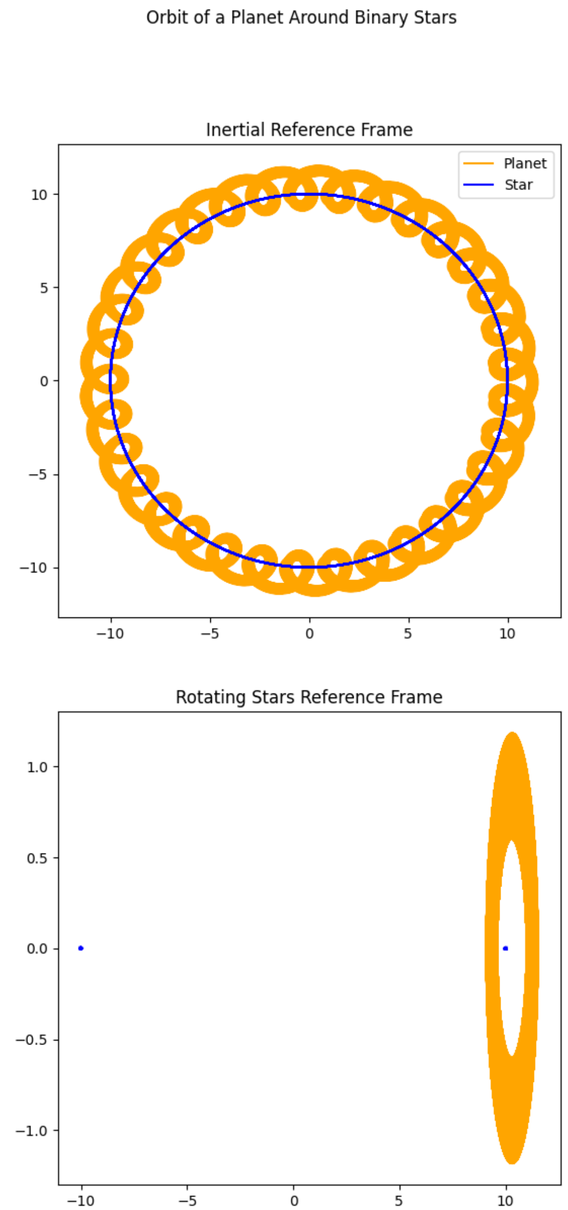

Figure 4. Forward Close Orbit. R=10R and M=1000M for the stars, with initial conditions of x0=11R, vy0=41, y0=vx0=0.

3.3.2 Chaotic Retrograde 2

A more slightly more orderly retrograde orbit is shown in figure 9. It does have some of the same qualities as figure 8 in that there is sharp loops outside the stars, however this time they are a more constant shape. In the rotating frame, the orbit has some similarities to figure 7. A major difference this time is that there are multiple orbits of different radii all with the same shape. Here, the orbit changes slightly over time. This can also be seen in the inertial frame - the spacing between the outward loops is different for each pair.

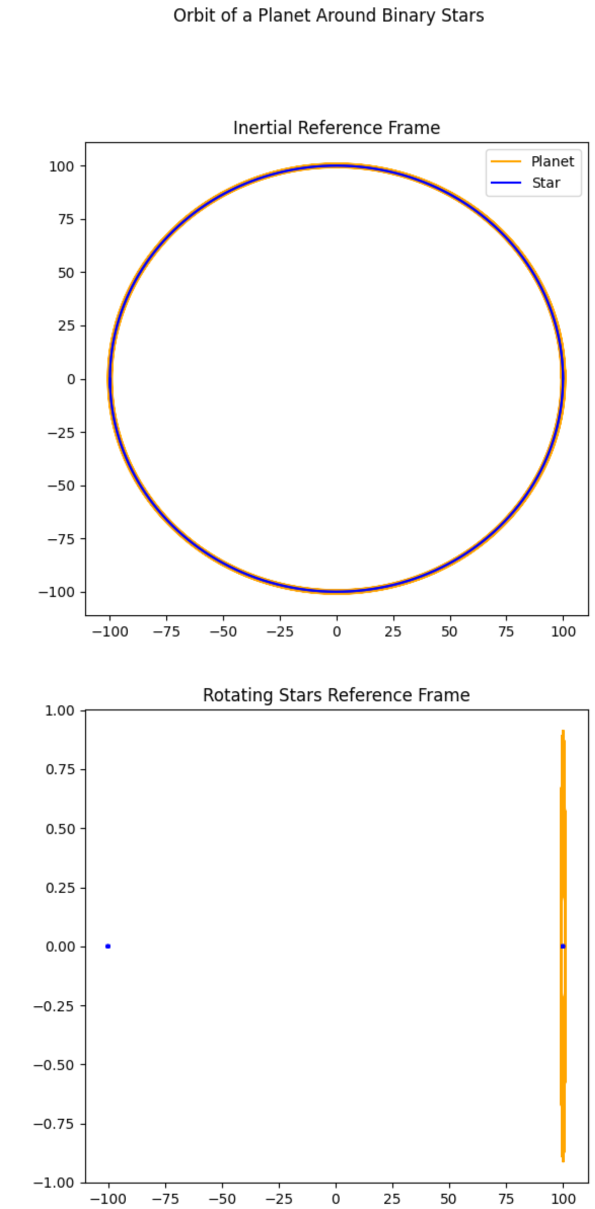

Figure 5. Forward Close Orbit. R=100R and M=1000M for the stars, with initial conditions of x0=11R, vy0=31.622, y0=vx0=0.

3.4 Finding the Orbits

When finding examples of these orbits, I found many examples where the planet left the system quickly without an orbit to be found. The main causes of this are setting the velocity too high or too low. Choosing too low of a velocity causes the planet to move too close to one of the stars and end up catapulted out of the system, as figure 10 demonstrates. Conversely, choosing a velocity that is too high along with low star masses launches the planet straight out of the system. The gravitational attraction isn't strong enough to keep the planet in the system at all, an example of which can be

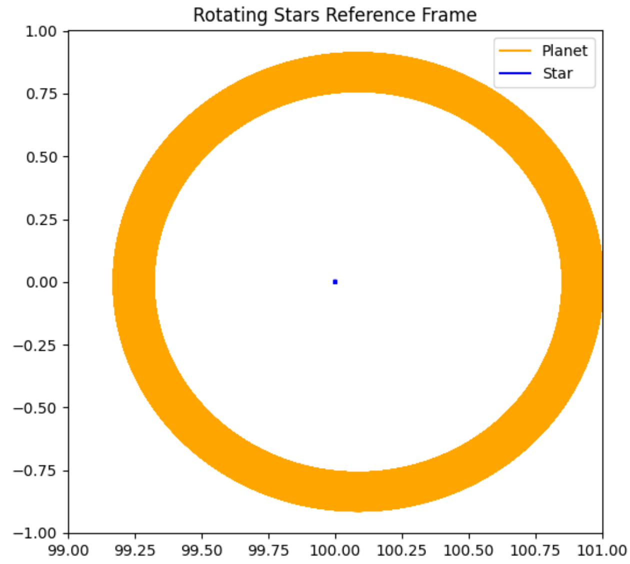

Figure 6. 6 zoomed in on the rotating stars reference frame.

seen in figure 11. Starting the planet with any x velocity, or any non-zero y position leads to the planet being ejected. All of the cases I found were due to the planet moving too close to the star and being ejected. It is possible that stable orbits exist with these starting conditions, however I haven't been able to find any.

4 DISCUSSION

4.1 Comparison with Single Star Orbits

Figures 2, 4, and 5 bear resemblance to a planet orbiting a single star, however for different reasons. For 2, it is because the planet is so far away from the stars that the stars appear as a single entity. The orbit does appear slightly thicker in the rotating stars reference frame than in the inertial frame however. This is due to the fact that while the planet may only orbit the stars once in the inertial frame, it appears to orbit many hundreds of times in the rotating frame - thus any deviation from a perfect circle will cause the orbit to differ noticeably. This can even bee seen slightly in the inertial plot - around the starting point of $(200, 0)$ where the planet begins a second orbit, a slight thickness can be seen as the orbit changes ever so slightly. An alternate explanation of the thickness could be errors due to the numerical methods used and using too large of a step size. This is unlikely however, as when I created the same plots but with a much smaller step size the same behavior emerged. In the cases of 4, and 5 the similarity is only in the rotating frame, where we can clearly see the planet orbiting a single one of the stars. Opposed to 2, we can see a more noticeable shift in the orbit. This is more apparent in 4. This is easily explained by the fact that both stars are much closer to the planet in this case, so the existence of the second star has a much greater effect than before. Another effect of the second star is that the orbit becomes slightly elliptical around the first. This can be seen even in 2 as explained, however here is it much more pronounced. If I were to move the two stars further apart and look for a case where the planet orbits a single one of the stars, I would expect these two effects to be much less significant. This expected behavior is indeed observed,

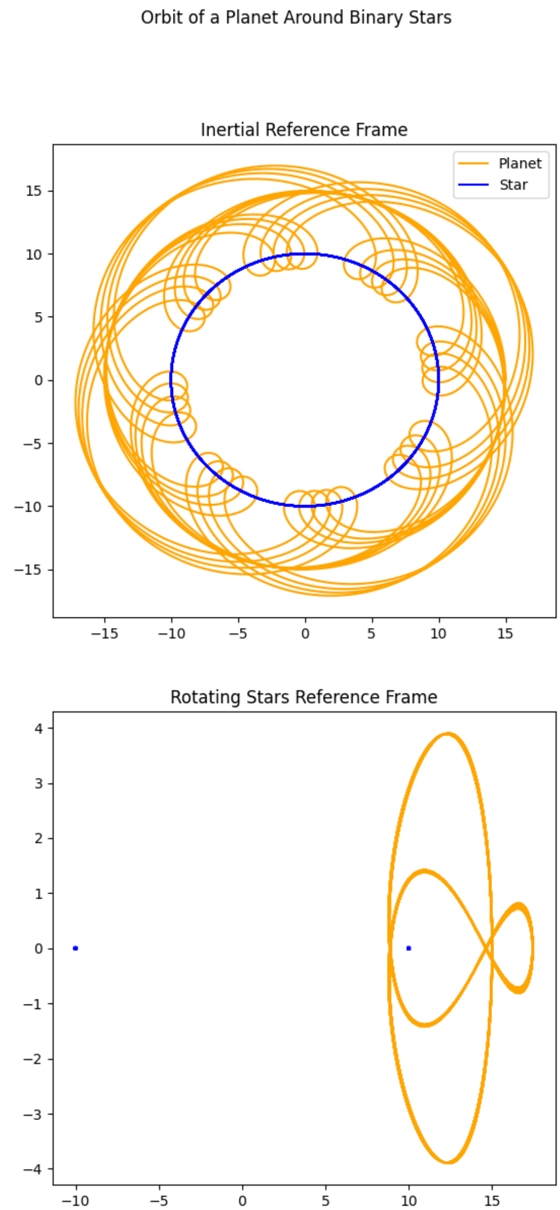

Figure 7. Forward Close Orbit. R=10R and M=1000M for the stars, with initial conditions of x0=15R, vy0=23, y0=vx0=0.

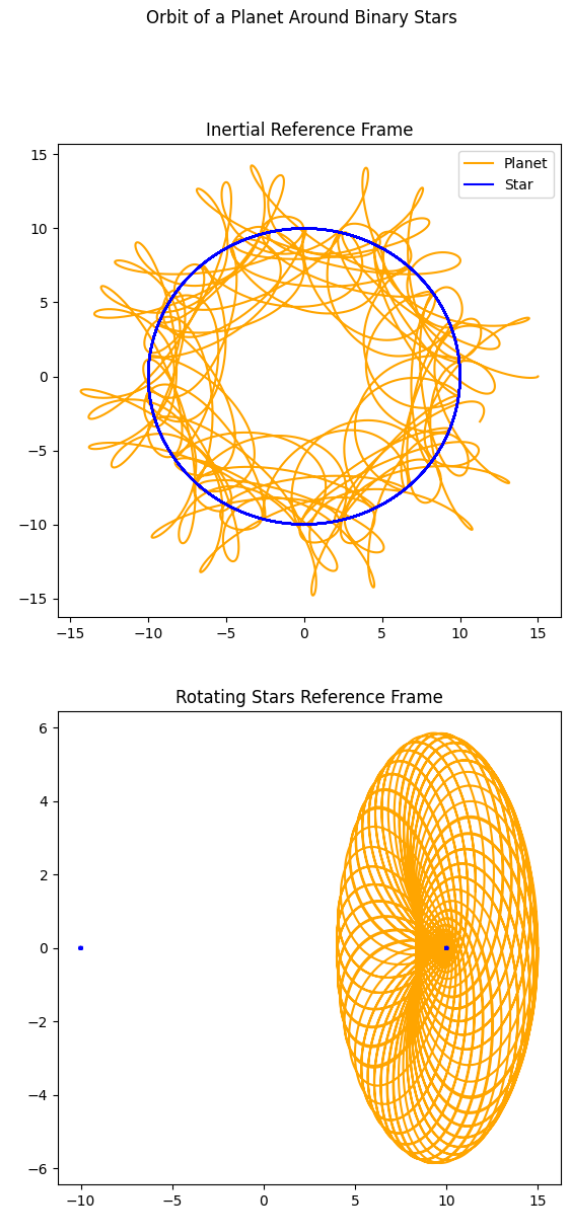

Figure 8. Retrograde Close Orbit. R=10R and M=1000M for the stars, with initial conditions of x0=15R, vy0=-1, y0=vx0=0.

as shown in figures 5, and 6. The orbit of the planet is indeed both elliptical and slightly varying over its orbit, but to a smaller degree.

4.2 Intermediate Distance

For the intermediate distance orbit around both stars in figure 3, the effect of the second star on the orbit is very pronounced. The orbit clearly precesses around both stars and is clearly elliptical. We can clearly see the many hundreds of orbits of the planet in the rotating frame, detail which was mostly lost with the planet much further away. This provides evidence for the slight thickening of the orbit in 2 being due to the orbit not being perfectly circular, opposed to a build up of errors. I believe this as in the rotating frame you can clearly see parts of the orbit where the planet doesn't retrace its orbit and the line is as thin as it can be.

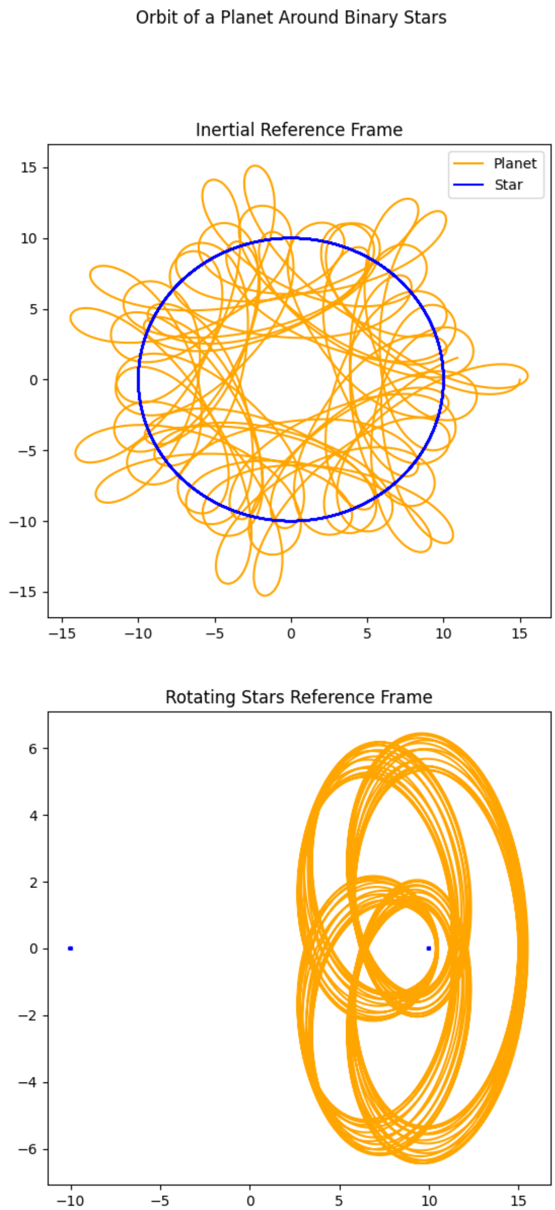

Figure 9. Retrograde Close Orbit. R=10R and M=1000M for the stars, with initial conditions of x0=15R, vy0=-5, y0=vx0=0.

4.3 Close Orbits

4.3.1 Prograde Orbit

Figure 7 exhibits a clear, regular orbital pattern. Upon close inspection, this orbit shows the same effect as 4, at least in the inertial frame. The planet can be seen orbiting the star however this time the orbit is much wider, and the points where it swings around the star are much more spaced out with each successive full orbit. The rotating frame is a much different story however. Here the only similarity to orbit 4 is having a large ellipse, however orbit 7 has an extra figure of 8 feature.

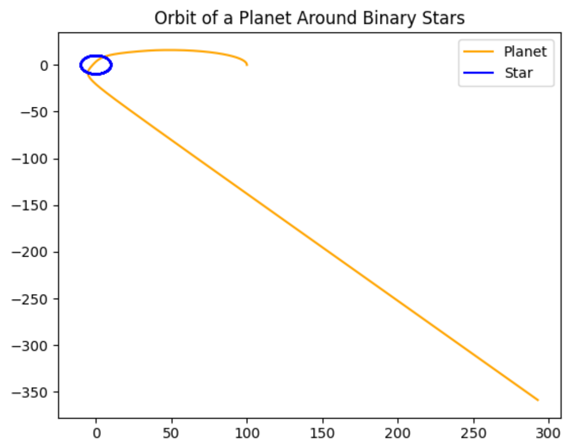

Figure 10. Planet ejected from system. R=10R and M=1000M for the stars, with initial conditions of x0=100R, vy0=1, y0=vx0=0.

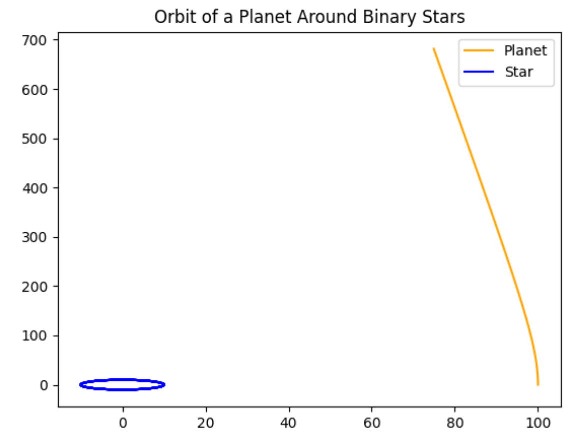

Figure 11. Planet ejected from system. R=10R and M=100M for the stars, with initial conditions of x0=100R, vy0=7, y0=vx0=0.

4.3.2 Retrograde Orbits

The retrograde orbits are the most interesting in terms of how chaotic they appear in one frame compared to another. Both orbits 8, and 9 appear very chaotic with numerous features not seen in the other orbits. The most noticeable of these is that here the planet enters much further into the region inside the stars. This might explain the more chaotic nature - closer to the centre of the stars orbit the planet is subject to the full extent of both stars gravitational fields at once. Another interesting feature is the planets in these orbits have much larger ranges of distances that the planet is away from the stars, that is - at points the planet is very close to the stars and at others, very far. Figure 8 shows this exceptionally well.

4.4 Prospects for Life

In order for life to exist on a planet, it must lie within that stars "Goldilocks Zone" or "Habitable Zone" - the region around a star where a planet may exist and maintain liquid water at atmospheric pressure. Around a single star where orbits are slightly elliptical, planets can safely remain inside this region at all points in its orbit. This heavily limits the amount of planets that may harbour human life. It is well known that the large majority of star systems are binary systems, making the our solar system a rarity. This proposes a question - is it possible for a planet to remain in an orbit that would be suitable for life in a binary star system? From the simulations and orbits I have found the answer is yes, but for specific circumstances. A type of orbit that could most definitely support life are ones like in figure 5. Here the stars are far enough apart (and systems where they are further apart are possible) that the planet could reside in the Goldilocks Zone for all its orbit. Other examples could be orbits 2, and 3 however for the elliptical orbit the ellipse must not be too wide other wise the planet risks straying out of the Goldilocks zone. I think it unlikely that life could exist on a planet retrograde with the stars, due to the massive range of distances observed in these simulations. There are many other factors that determine whether a planet could house life other than orbits, so just because an orbit may appear suitable, doesn't mean that life could actually exist on said planet. If one of the stars happened to be a pulsar its intense radiation beam could easily disrupt the evolution of life on a planet - but this doesn't mean it would be impossible, as explored in Patruno & Kama (2017). Another likely problem arises from orbits that move within the pair of stars. In reality binary stars interact with each other, namely through mass transfer. The transfer of hot gas between stars would surely destroy life on any planet to come in contact with it, making close orbits unlikely for life. As a result, the only orbit I would expect to see in reality would be a far orbit with the planet orbiting the two stars as if it were one, much like our orbit on Earth.

5 CONCLUSIONS

In this paper, I have simulated a simplification of a 3-Body problem in the context of a planet orbiting binary stars. Both stars are assumed to have equal masses, move on rails in a circular orbit, and have no effect on each other. The planet is assumed to be massless and thus have no effect on the orbits of the stars. By varying the starting conditions of the planet and stars, different types of orbits were found. Some resembled single star systems, either by orbiting both stars at once at a large distance, or by closely orbiting one of the two stars. I also discovered some more exotic systems, notably ones with retrograde orbits. These had complex patterns when viewed from an inertial frame, however had clear patterns when viewed rotating with the stars. Some of the orbits I discovered could support life - ones where the orbits resembled that of a single star system, and the planet remained in a stable elliptical or circular orbit around both of the stars. The complex retrograde orbits had the planet both extremely close to the star and far from the star in the same orbit, thus making them unsuitable to house life.

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APPENDIX A: NUMERICAL METHOD

To solve the differential equations in this paper I used scipy.integrate.odeint in python. It takes in a function or functions stored in an array to solve, along with initial conditions, a time variable, and an optional argument to pull in any constants used in the functions. You can also specify the error you want inside the function, in case you want a more precise result or a result faster. odeint uses LSODA to solve differential equations, which is a similar method to Runge-Kutta. Like Runge-Kutta, it uses intermediate calculations to more accurately determine the next point on the curve. However LSODA keeps these intermediate calculations instead of discarding them, for further use. The odeint library is used in a variety of programming languages to solve differential equations, not just python.

APPENDIX B: USES OF LSODA, ODEINT, AND DIFFERENTIAL EQUATIONS

LSODA or the odeint library have been used widely in academia, for example:

· Electronic instabilities in a lattice by de la Pena et al. (2017).

· Self-Interacting Dark Matter by Boddy et al. (2014).

· The Dynamics of Ball Bearings by Meeks & Tran (1996)

with many further examples. Differential equations are integral to physics and engineering, and other disciplines. They appear in quantum mechanics (most notably the wave equation), in modelling finance, modelling heat flow, and more. There are many real world uses of differential equations, and being able to solve them accurately allows the development of important technologies for the progress of humanity.

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