Assignment-1

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***Abstract*—This assignment finds whether the lines passing through the given points are parallel or not.**

Download all python codes from

https://github.com/poojah15/EE5609_Assignments

1 Problem Statement

To show that the line passing through the points $\begin{pmatrix}4\\7\\8\end{pmatrix}, \begin{pmatrix}2\\3\\4\end{pmatrix}$ is parallel to the line through the points $\begin{pmatrix}-1\\-2\\1\end{pmatrix}, \begin{pmatrix}1\\2\\5\end{pmatrix}$

2 Theory

Let the lines be parallel and the first two points pass through $\mathbf{n}^T\mathbf{x} = c1$. i.e.

$$\mathbf{n}^T\mathbf{x}_1 = c_1 => \mathbf{x}_1^T\mathbf{n} = c_1 \tag{2.0.1}$$

$$\mathbf{n}^T\mathbf{x}_2 = c_2 => \mathbf{x}_2^T\mathbf{n} = c_2 \tag{2.0.2}$$

and the second two points pass through $\mathbf{n}^T\mathbf{x} = c2$ Then

$$\mathbf{n}^T\mathbf{x}_3 = c_3 => \mathbf{x}_3^T\mathbf{n} = c_3 \tag{2.0.3}$$

$$\mathbf{n}^T\mathbf{x}_4 = c_4 => \mathbf{x}_4^T\mathbf{n} = c_4 \tag{2.0.4}$$

Putting all the equations together, we obtain

$$\begin{pmatrix}\mathbf{x}_1^T\\\mathbf{x}_2^T\\\mathbf{x}_3^T\\\mathbf{x}_4^T\end{pmatrix}\mathbf{n} = \begin{pmatrix}c_1\\c_2\\c_3\\c_4\end{pmatrix} \tag{2.0.5}$$

Now if this equation has a solution, then $\mathbf{n}$ exists and the lines will be parallel.

3 Example

Given the points, $\mathbf{A} = \begin{pmatrix}4\\7\\8\end{pmatrix}, \mathbf{B} = \begin{pmatrix}2\\3\\4\end{pmatrix}, and \quad \mathbf{C} = \begin{pmatrix}-1\\-2\\1\end{pmatrix}, \mathbf{D} = \begin{pmatrix}1\\2\\5\end{pmatrix}$

Applying the row reduction procedure on the coefficient matrix:

$$\begin{pmatrix}4 & 7 & 8\\2 & 3 & 4\\-1 & -2 & 1\\1 & 2 & 5\end{pmatrix} \tag{3.0.1}$$

$$\xleftrightarrow[R_4\leftarrow R_3+R_4]{R_2\leftarrow R_1-2R_2} \begin{pmatrix}4 & 7 & 8\\0 & 1 & 0\\-1 & -2 & 1\\0 & 0 & 6\end{pmatrix} \tag{3.0.2}$$

$$\xleftrightarrow[R_3\leftarrow R_3-6R_4]{R_1\leftarrow R_1-7R_2} \begin{pmatrix}4 & 0 & 8\\0 & 1 & 0\\-1 & -2 & 0\\0 & 0 & 6\end{pmatrix} \tag{3.0.3}$$

$$\xleftrightarrow[R_1\leftarrow R_1-8R_4]{R_4\leftarrow R_4/6} \begin{pmatrix}4 & 0 & 0\\0 & 1 & 0\\-1 & -2 & 1\\0 & 0 & 1\end{pmatrix} \tag{3.0.4}$$

$$\xleftrightarrow[R_3\leftarrow R_3+R_4]{R_3\leftarrow(-R_3-2R_2)} \begin{pmatrix}4 & 0 & 0\\0 & 1 & 0\\1 & 0 & 0\\0 & 0 & 1\end{pmatrix} \tag{3.0.5}$$

$$\xleftrightarrow{R_1\leftarrow R_1-4R_3} \begin{pmatrix}0 & 0 & 0\\0 & 1 & 0\\1 & 0 & 0\\0 & 0 & 1\end{pmatrix} \tag{3.0.6}$$

Here, the number of non-zero rows are three and hence the rank of the matrix is 3 which implies that the solution exists. Therefore the lines passing through $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}, \mathbf{D}$ are parallel.