Assignment-3

Pooja H

***Abstract*—In this work, we evaluate the matrix equation, to get the value of 'k'. In addition, we find the characteristic equation for the given matrix.**

Download all latex-tikz codes from

https://github.com/poojah15/EE5609_Assignments/tree/master/Assignment_3/New_version

Download the python code from

https://github.com/poojah15/EE5609_Assignments/tree/master/Assignment_3

1 Problem Statement

If

$$\mathbf{A} = \begin{pmatrix} 3 & -2 \\ 4 & -2 \end{pmatrix} \tag{1.0.1}$$

and

$$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{1.0.2}$$

find k so that

$$\mathbf{A}^2 = k\mathbf{A} - 2\mathbf{I} \tag{1.0.3}$$

2 Solution

For a general square matrix $\mathbf{A}$ of size n x n, the characteristic equation in variable λ is defined by,

$$\det(\mathbf{A} - \lambda\mathbf{I}) = 0 \tag{2.0.1}$$

where, $\mathbf{I}$ is the identity matrix of size n x n. Hence, given $\mathbf{A} = \begin{pmatrix} 3 & -2 \\ 4 & -2 \end{pmatrix}$, the characteristic equation is computed as follows:

$$\left| \begin{pmatrix} 3 & -2 \\ 4 & -2 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right| = 0 \tag{2.0.2}$$

$$\implies \begin{vmatrix} 3-\lambda & -2 \\ 4 & -2-\lambda \end{vmatrix} = 0 \tag{2.0.3}$$

By expanding the above determinant we get,

$$(3-\lambda)(-2-\lambda) + 8 = 0 \tag{2.0.4}$$

$$\implies -6 + \lambda^2 + 2\lambda - 3\lambda + 8 = 0 \tag{2.0.5}$$

$$\implies \lambda^2 - \lambda + 2 = 0 \tag{2.0.6}$$

$$\implies \lambda^2 = \lambda - 2 \tag{2.0.7}$$

Here, (2.0.7) is the required characteristic equation and hence, matrix $\mathbf{A}$ satisfies the characteristic equation according to the Cayley-Hamilton Theorem. By comparing the coefficients of the equations in (1.0.3) and (2.0.7), we can infer that the value of $k = 1$.