Assignment-15

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***Abstract*—In this document, we find the dual basis of the given basis B**

Download all latex-tikz codes from

https://github.com/poojah15/EE5609_Assignments/tree/master/Assignment_15

1 Problem Statement

Let $\mathbf{B} = \{\alpha_1, \alpha_2, \alpha_3\}$ be the basis for $\mathbf{C}^3$ defined by

$$\alpha_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \alpha_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \alpha_3 = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} \tag{1.0.1}$$

Find the dual basis of $\mathbf{B}$.

2 Solution

Let $\{f_1, f_2, f_3\}$ be the dual basis of $\mathbf{B}$ such that,

$$f_i(\alpha_j) = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \tag{2.0.1}$$

and

$$f_i(\alpha_1, \alpha_2, \alpha_3) = \sum_{j=1}^{3} \delta_{ij}\alpha_j \tag{2.0.2}$$

Given,

$$\mathbf{B} = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ -1 & 1 & 0 \end{pmatrix} \tag{2.0.3}$$

Then,

$$\mathbf{B}^{-1} = \begin{pmatrix} 1 & -1 & 0 \\ 1 & -1 & 1 \\ -\frac{1}{2} & 1 & -\frac{1}{2} \end{pmatrix} \tag{2.0.4}$$

So the elements of dual basis are rows of matrix $\mathbf{B}^{-1}$. Therefore we get,

$$(\delta_{11}, \delta_{12}, \delta_{13}) = (1, -1, 0) \tag{2.0.5}$$

$$(\delta_{21}, \delta_{22}, \delta_{23}) = (1, -1, 1) \tag{2.0.6}$$

$$(\delta_{31}, \delta_{32}, \delta_{33}) = (-\frac{1}{2}, 1, -\frac{1}{2}) \tag{2.0.7}$$

Using (2.0.2), we get

$$\begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 0 \\ 1 & -1 & 1 \\ -\frac{1}{2} & 1 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} \tag{2.0.8}$$

$$= \begin{pmatrix} \alpha_1 - \alpha_2 \\ \alpha_1 - \alpha_2 + \alpha_3 \\ -\frac{1}{2}\alpha_1 + \alpha_2 - \frac{1}{2}\alpha_3 \end{pmatrix} \tag{2.0.9}$$

Hence, $\{f_1, f_2, f_3\}$ is the required dual basis for $\mathbf{B}$.