Assignment-8

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Abstract—**In this document, we present the solution to the QR factorization with an example.**

Download all python codes from

https://github.com/poojah15/EE5609_Assignments/tree/master/Assignment_8

Download all latex-tikz codes from

https://github.com/poojah15/EE5609_Assignments/tree/master/Assignment_8

1 Problem Statement

Given a matrix $\mathbf{A} = \begin{pmatrix} 3 & -2 \\ 4 & -2 \end{pmatrix}$, find its **QR** decomposition

2 Solution

Given

$$\mathbf{A} = \begin{pmatrix} 3 & -2 \\ 4 & -2 \end{pmatrix} \tag{2.0.1}$$

Let us use the Gram-Schmidt approach to obtain QR decomposition of **A**. Consider column vectors say $\mathbf{a}_1$ and $\mathbf{a}_2$ of **A** which is given by

$$\mathbf{a}_1 = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{2.0.2}$$

$$\mathbf{a}_2 = \begin{pmatrix} -2 \\ -2 \end{pmatrix} \tag{2.0.3}$$

$$\mathbf{u}_1 = \mathbf{a}_1 = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{2.0.4}$$

$$\mathbf{e}_1 = \frac{\mathbf{u}_1}{\|\mathbf{u}_1\|} = \begin{pmatrix} \frac{3}{5} \\ \frac{4}{5} \end{pmatrix} \tag{2.0.5}$$

$$\mathbf{u}_2 = \mathbf{a}_2 - \left(\mathbf{a}_2^T.\mathbf{e}_1\right)\mathbf{e}_1 \tag{2.0.6}$$

$$= \begin{pmatrix} -2 \\ -2 \end{pmatrix} - \left(-\frac{14}{5}\right)\begin{pmatrix} \frac{3}{5} \\ \frac{4}{5} \end{pmatrix} \tag{2.0.7}$$

$$= \begin{pmatrix} -2 \\ -2 \end{pmatrix} - \begin{pmatrix} -\frac{42}{25} \\ -\frac{56}{25} \end{pmatrix} = \begin{pmatrix} -\frac{8}{25} \\ \frac{6}{25} \end{pmatrix} \tag{2.0.8}$$

$$\mathbf{e}_2 = \frac{\mathbf{u}_2}{\|\mathbf{u}_2\|} = \begin{pmatrix} -\frac{4}{5} \\ \frac{3}{5} \end{pmatrix} \tag{2.0.9}$$

The matrix **Q** and **R** is given by,

$$\mathbf{Q} = \begin{pmatrix} \mathbf{e}_1 & \mathbf{e}_2 \end{pmatrix} = \begin{pmatrix} \frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{pmatrix} \tag{2.0.10}$$

$$\mathbf{R} = \begin{pmatrix} \mathbf{a}_1^T.\mathbf{e}_1 & \mathbf{a}_2^T.\mathbf{e}_1 \\ 0 & \mathbf{a}_2^T.\mathbf{e}_2 \end{pmatrix} = \begin{pmatrix} 5 & -\frac{14}{5} \\ 0 & \frac{2}{5} \end{pmatrix} \tag{2.0.11}$$

Hence, the **QR** decomposition of matrix **A** is as follows:

$$\begin{pmatrix} 3 & -2 \\ 4 & -2 \end{pmatrix} = \begin{pmatrix} \frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{pmatrix}\begin{pmatrix} 5 & -\frac{14}{5} \\ 0 & \frac{2}{5} \end{pmatrix} \tag{2.0.12}$$