

Assignment-13

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Abstract—In this document, we find whether the given function T from \mathbb{R}^2 into \mathbb{R}^2 is a linear transformation or not.

Download all latex-tikz codes from

https://github.com/poojah15/EE5609_Assignments/tree/master/Assignment_13

1 PROBLEM STATEMENT

Verify whether $T(x_1, x_2) = (x_1 - x_2, 0)$ is a linear transformation or not.

2 SOLUTION

Let \mathbf{V} and \mathbf{W} be the vector spaces. The function $T : \mathbf{V} \rightarrow \mathbf{W}$ is called a linear transformation of \mathbf{V} into \mathbf{W} if for all \mathbf{u} and \mathbf{v} in \mathbf{V} and for any scalar k in field \mathbf{F} ,

$$T(k\mathbf{u} + \mathbf{v}) = kT(\mathbf{u}) + T(\mathbf{v}) \quad (2.0.1)$$

Given,

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad (2.0.2)$$

Consider,

$$T(k\mathbf{x} + \mathbf{y}) = \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} (k\mathbf{x} + \mathbf{y}) \quad (2.0.3)$$

$$= \begin{pmatrix} k & -k \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} \mathbf{y} \quad (2.0.4)$$

$$= k \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} \mathbf{y} \quad (2.0.5)$$

$$T(k\mathbf{x} + \mathbf{y}) = kT(\mathbf{x}) + T(\mathbf{y}) \quad (2.0.6)$$

Therefore, the given function T is a linear transformation.