

Assignment-10

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Abstract—In this document, we present the solution to $\mathbf{AX} = c\mathbf{X}$ where, x_3 is arbitrary. Therefore,

Download all latex-tikz codes from

https://github.com/poojah15/EE5609_Assignments/tree/master/Assignment_10

$$\mathbf{X} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} t \quad (2.0.6)$$

Hence, the given matrix has single eigenvector and is not diagonalizable.

1 PROBLEM STATEMENT

Let

$$\mathbf{A} = \begin{pmatrix} 5 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & 1 & 5 \end{pmatrix} \quad (1.0.1)$$

For which \mathbf{X} does there exist a scalar c such that $\mathbf{AX} = c\mathbf{X}$

2 SOLUTION

Given

$$\mathbf{A} = \begin{pmatrix} 5 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & 1 & 5 \end{pmatrix} \quad (2.0.1)$$

The given matrix has single eigenvalue as it is the lower triangular matrix and has equal diagonal elements. Hence $c_1 = c_2 = c_3 = 5$. To find the corresponding eigenvector, consider the following

$$(\mathbf{A} - c\mathbf{I})\mathbf{X} = 0 \quad (2.0.2)$$

$$\Rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (2.0.3)$$

Solving the homogeneous system of linear equations by performing rref, we get

$$\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow[R_3 \leftrightarrow R_2]{R_2 \leftrightarrow R_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \quad (2.0.4)$$

Hence we get,

$$x_1 = 0, x_2 = 0, x_3 = t \quad (2.0.5)$$