

Assignment-14

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Abstract—In this document, we find the matrix of T in the ordered basis \mathbf{B} And

Download all latex-tikz codes from

https://github.com/poojah15/EE5609_Assignments/tree/master/Assignment_14

1 PROBLEM STATEMENT

Let T be the linear operator on \mathbf{R}^2 defined by

$$T \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -x_2 \\ x_1 \end{pmatrix} \quad (1.0.1)$$

What is the matrix of T in the ordered basis $\mathbf{B} = \{\alpha_1, \alpha_2\}$, where $\alpha_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\alpha_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$?

2 SOLUTION

Let \mathbf{B}' be the standard ordered basis for \mathbf{R}^2 . Then,

$$T(\alpha'_1) = T \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0\alpha_1 + 1\alpha_2 \quad (2.0.1)$$

$$T(\alpha'_2) = T \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix} = -1\alpha_1 + 0\alpha_2 \quad (2.0.2)$$

Hence, the matrix of T in the standard ordered basis \mathbf{B}' is

$$T_{\mathbf{B}'} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad (2.0.3)$$

Given,

$$\alpha_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad (2.0.4)$$

$$\alpha_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (2.0.5)$$

Then,

$$\alpha'_1 = 1\alpha_1 + 2\alpha_2 \quad (2.0.6)$$

$$\alpha'_2 = 1\alpha_1 - 1\alpha_2 \quad (2.0.7)$$

So the \mathbf{P} matrix is

$$\mathbf{P} = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix} \quad (2.0.8)$$

$$\mathbf{P}^{-1} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{pmatrix} \quad (2.0.9)$$

Hence

$$[T]_{\mathbf{B}} = \mathbf{P}^{-1}[T]_{\mathbf{B}'}\mathbf{P} \quad (2.0.10)$$

$$= \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix} \quad (2.0.11)$$

$$= \begin{pmatrix} -\frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix} \quad (2.0.12)$$

$$[T]_{\mathbf{B}} = \begin{pmatrix} \frac{1}{3} & -\frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{pmatrix} \quad (2.0.13)$$

Hence, $[T]_{\mathbf{B}}$ is the required matrix for the given ordered basis \mathbf{B} .