Report Project 2 - Continuous Control

Part I - Theory

Deep Deterministic Policy Gradient - DDPG

The first unsuccessful attempt training one single agent uses a model based on the DDPG (Deep Deterministic Policy Gradient) algorithm introduced in the paper [Continuous control with deep reinforcement learning](). A brief description of the algorithm following the notation established in Project 1 follows.

Again the formal framework of a **Markov Decision Process** (MDP) is used with

- a **state space** $\mathbb{S}$ of states S_t
- an **action space** $\mathbb{A}$ with $A_t \in \mathbb{R}^N$
- **transition dynamics** $\mathbb{P}(S_{t+1}|S_t, A_t)$ that define the distribution of states that the agent can land on taking an action $A_t \in \mathbb{R}^N$ in state S_t, with initial state distribution $\mathbb{P}(S_1)$
- a **reward function** $R(S_t, A_t)$

An agent's behavior is defined by a policy, π, which maps states to a probability distribution over the actions $\pi : \mathbb{S} \to \mathbb{P}(\mathbb{A})$. The return from a state is defined as the sum of discounted future reward $R_t = \sum_{i=t}^{T} \gamma^{(i-t)} R(s_i, a_i)$ with a discounting factor $\gamma \in [0, 1]$.

The goal of reinforcement learning is to learn a policy which maximizes the expected return from the start distribution $J = \mathbb{E}_\pi(R_1)$. The discounted state visitation distribution for a policy π is denoted with ρ^π.

The action-value function describes the expected return after taking an action A_t in state S_t and thereafter following policy π:

$$Q^\pi(S_t, A_t) = \mathbb{E}_\pi(R_t|S_t, A_t)$$

The bedrock of many reinforcement learning algorithms is the Bellman equation which is given by

$$Q^\pi(S_t, A_t) = \mathbb{E}(R(S_t, A_t) + \gamma \mathbb{E}_\pi(Q^\pi(S_{t+1}, A_{t+1}))).$$

If the target policy is deterministic it can be described as a function $\mu : \mathbb{S} \leftarrow \mathbb{A}$ and the inner expectation can be avoided:

$$Q^\mu(S_t, A_t) = \mathbb{E}(R(S_t, A_t) + \gamma Q^\mu(S_{t+1}, A_{t+1})).$$

Given that the remaining expectation depends only on the environment and not on the policy, it is possible to learn Q^μ off-policy, using transitions which are generated from a different stochastic behavior policy β.

Q-Learning, which is an off-policy algorithm, uses the greedy policy $\mu(s) = \mathrm{argmax}_a Q(s, a)$. Function approximators parameterized by θ^Q are used, which are optimized by minimizing the loss:

$$L(\theta^Q) = \mathbb{E}\left(\left(Q(S_t, A_t|\theta^Q) - Y_t\right)^2\right)$$

where

$$Y_t = R(S_t, A_t) + \gamma Q(S_{t+1}, \mu(S_{t+1})|\theta^Q)$$

While Y_t is also dependent on θ^Q, this is typically ignored.

Using non-linear function approximators with a large number of parameters for learning action-value functions has previously been avoided as theoretical performance guarantees are impossible and practically learning is unstable. Q-Learning became usable in practice with the introduction of two major changes:

- the use of a **replay buffer** and
- a separate **target network** for calculating Y_t

In continuous action spaces $\mathbb{A}$ it is not straightforward to apply Q-Learning, because finding the greedy policy in each timestep requires evaluating an optimization problem, which is too slow to be practical with large, unconstrained function approximators and large action spaces.

Instead DDPG builds on the actor-critic approach introduced by the DPG algorithm introduced in Deterministic Policy Gradient Algorithms.

The DPG algorithm maintains a parameterized actor function $\mu(s|\theta^\mu)$ which specifies the current policy by deterministically mapping states to a specific action. The critic $Q(S, A)$ is learned using the Bellman equation as in Q-Learning. The actor is updated by applying the chain rule to the expected return from the start distribution J with respect to the actor parameters:

$$\begin{aligned}\nabla_{\theta^\mu} J &\approx \mathbb{E}_{\rho^\beta}(\nabla_{\theta^\mu} Q(S, A|\theta^Q)|_{S=s_t, A=\mu(s_t|\theta^\mu)}) \\ &= \mathbb{E}_{\rho^\beta}(\nabla_A Q(S, A|\theta^Q)|_{S=s_t, A=\mu(s_t)} \nabla_{\theta_\mu} \mu(s|\theta^\mu)|_{S=s_t})\end{aligned}$$

This is the policy gradient, the gradient of the policy's performance.

The first major change to the application of neural networks to DPG previously mentioned is the use of a **replay buffer**. The replay buffer is a finite sized cache $\mathcal{R}$. Transitions are sampled from the environment according to the exploration policy and the tuple (S_t, A_t, R_t, S_{t+1}) is stored in the replay buffer. When replay buffer is full the oldest samples are discarded. At each timestep the actor and the critic are updated by sampling a minibatch uniformly from the buffer. Because DDPG is an off-policy algorithm, the replay buffer can be large, allowing the algorithm to benefit from learning a set of uncorrelated transitions.

Additionally, a **target network** is deployed by using soft target updates, rather than directly copying the weights. A copy of the actor and critic networks $Q'(S, A|\theta^{Q'})$ and $\mu'(s|\theta^{\mu'})$ that is used for calculating the target values. The weights of these target networks is then updated by having them slowly track the learned networks: $\theta' \leftarrow \tau\theta + (1-\tau)\theta'$ with $\tau \ll 1$. Thus, the target values are constrained to change slowly, greatly improving the stability of the learning.

In order to do better exploration, an exploration policy μ' is constructed by adding a noise process $\mathcal{N}$ to the actor policy:

$$\mu'(s_t) = \mu(s_t|\theta_t^\mu) + \mathcal{N}$$

Algorithm 1 DDPG algorithm

Randomly initialize critic network $Q(s, a|\theta^Q)$ and actor $\mu(s|\theta^\mu)$ with weights θ^Q and θ^μ.
Initialize target network Q' and μ' with weights $\theta^{Q'} \leftarrow \theta^Q$, $\theta^{\mu'} \leftarrow \theta^\mu$
Initialize replay buffer R
for episode = 1, M **do**
  Initialize a random process $\mathcal{N}$ for action exploration
  Receive initial observation state s_1
  for t = 1, T **do**
    Select action $a_t = \mu(s_t|\theta^\mu) + \mathcal{N}_t$ according to the current policy and exploration noise
    Execute action a_t and observe reward r_t and observe new state s_{t+1}
    Store transition (s_t, a_t, r_t, s_{t+1}) in R
    Sample a random minibatch of N transitions (s_i, a_i, r_i, s_{i+1}) from R
    Set $y_i = r_i + \gamma Q'(s_{i+1}, \mu'(s_{i+1}|\theta^{\mu'})|\theta^{Q'})$
    Update critic by minimizing the loss: $L = \frac{1}{N}\sum_i (y_i - Q(s_i, a_i|\theta^Q))^2$
    Update the actor policy using the sampled policy gradient:

$$\nabla_{\theta^\mu} J \approx \frac{1}{N}\sum_i \nabla_a Q(s, a|\theta^Q)|_{s=s_i, a=\mu(s_i)} \nabla_{\theta^\mu}\mu(s|\theta^\mu)|_{s_i}$$

    Update the target networks:

$$\theta^{Q'} \leftarrow \tau\theta^Q + (1-\tau)\theta^{Q'}$$
$$\theta^{\mu'} \leftarrow \tau\theta^\mu + (1-\tau)\theta^{\mu'}$$

  end for
end for

Distributed Distributional Deep Deterministic Policy Gradients - D4PG

Distributed Distributional DDPG applies a set of improvements on DDPG to make it run in a distributional fashion.

1. **Distributional Critic**: The critic estimates the expected Q value as a random variable, a distribution Z_w parameterized by w and therefore $Q_w(S, A) = \mathbb{E}(Z_w(S, A))$. The loss that is to be minimized for learning the distribution parameter is constructed through some measure d of the distance between two distributions, the distributional TD error $L(w) = \mathbb{E}(d(\mathcal{T}_{\pi_\theta}, Z'_w(S, A), Z_w(S, A)))$, where $\mathcal{T}_{\mu_\theta}$ is the Bellman operator. The deterministic policy gradient update becomes:

$$\begin{aligned}\nabla_{\theta^\mu} J &\approx \mathbb{E}_{\rho^\beta}(\nabla_A Q_w(S, A|\theta^Q)|_{S=s_t, A=\mu(s_t)} \nabla_{\theta_\mu}\mu(s|\theta^\mu)|_{S=s_t}) \\ &= \mathbb{E}_{\rho^\beta}(\mathbb{E}(\nabla_A Z_w(S, A|\theta^Q)|_{S=s_t, A=\mu(s_t)}) \nabla_{\theta_\mu}\mu(s|\theta^\mu)|_{S=s_t})\end{aligned}$$

2.

3. **N-step returns**: When calculating the TD error, D4PG computes the N-step TD target rather than the one-step TD target to incorporate rewards of more future steps.

4. **Multiple Distributed Parallel Actors**: D4PG utilizes K independent actors, gathering experience in parallel and feeding data into the same replay buffer.

5. **Prioritized Experience Replay**: The last modification is to do sampling from the replay buffer of size R with a non-uniform probability p_i. This way, a sample i has the probability $(Rp_i)^{-1}$ to be selected, which is the importance weight.

Algorithm 1 D4PG

Input: batch size M, trajectory length N, number of actors K, replay size R, exploration constant ϵ, initial learning rates α_0 and β_0

Initialize network weights (θ, w) at random
Initialize target weights $(\theta', w') \leftarrow (\theta, w)$
Launch K actors and replicate network weights (θ, w) to each actor
for $t = 1, \ldots, T$ **do**
  Sample M transitions $(\mathbf{x}_{i:i+N}, \mathbf{a}_{i:i+N-1}, r_{i:i+N-1})$ of length N from replay with priority p_i
  Construct the target distributions $Y_i = \sum_{n=0}^{N-1} \gamma^n r_{i+n} + \gamma^N Z_{w'}(\mathbf{x}_{i+N}, \pi_{\theta'}(\mathbf{x}_{i+N}))$ TD(n)
  Compute the actor and critic updates

importance weight

Q update: $$\delta_w = \frac{1}{M} \sum_i \nabla_w \boxed{(Rp_i)^{-1}} d(Y_i, Z_w(\mathbf{x}_i, \mathbf{a}_i))$$ Q estimated as a random variable ~ distribution Z_w

Policy update: $$\delta_\theta = \frac{1}{M} \sum_i \nabla_\theta \pi_\theta(\mathbf{x}_i)\, \mathbb{E}[\nabla_\mathbf{a} Z_w(\mathbf{x}_i, \mathbf{a})]\big|_{\mathbf{a}=\pi_\theta(\mathbf{x}_i)}$$

  Update network parameters $\theta \leftarrow \theta + \alpha_t \delta_\theta$, $w \leftarrow w + \beta_t \delta_w$
  If $t = 0 \mod t_{\text{target}}$, update the target networks $(\theta', w') \leftarrow (\theta, w)$
  If $t = 0 \mod t_{\text{actors}}$, replicate network weights to the actors
end for
return policy parameters θ

Actor (Periodically updated)

repeat
  Sample action $\mathbf{a} = \pi_\theta(\mathbf{x}) + \epsilon\mathcal{N}(0, 1)$
  Execute action $\mathbf{a}$, observe reward r and state $\mathbf{x}'$
  Store $(\mathbf{x}, \mathbf{a}, r, \mathbf{x}')$ in replay
until learner finishes

Asynchronous Advantage Actor-Critic - A3C

A3C is a classical policy gradient method with a special focus on parallel training. In A3C, the critics learn the value function while multiple actors are trained in parallel and get synchronized with global parameters from time to time. A3C is designed to work well for parallel training.

Taking the state-value function as an example, the loss function for the state value is to minimize the mean squared error $J_v(w) = (G_t - V_w(S))^2$. Gradient descent can be applied to find the optimal w. This state-value function is used as the baseline in the policy gradient update.

The outline of the algorithms follows:

1. There are global parameters θ and w and similarly thread-specific parameters θ' and w'.
2. Initialize $t = 1$.
3. While $T \leq T_{\max}$:
    1. Reset the gradient by setting $d\theta = 0$ and $dw = 0$.
    2. Synchronize the thread-specific parameters with the global ones $\theta' = \theta$ and $w' = w$.
    3. Set $t_{\text{start}} = t$ and sample a starting state s_t.
    4. While s_t is not in the terminal state and $t - t_{\text{start}} \leq t_{\max}$:
        1. Pick the action $A_t \sim \pi_{\theta'}(A_t|S_t)$ and receive a new reward R_t and a new state S_{t+1}.
        2. Update $t = t + 1$ and $T = T + 1$.
    5. Initialize the variable that holds the return estimation

$$R = \begin{cases} 0 & \text{if } S_t \text{ is terminal} \\ V_{w'}(S_t) & \text{otherwise} \end{cases}$$

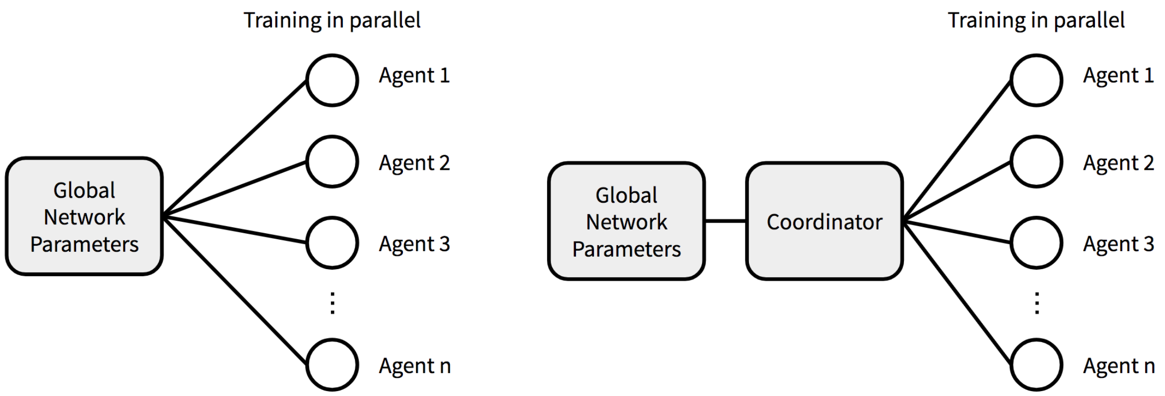
6. For $i = t-1, \dots, t_{\text{start}}$:
    1. $R \leftarrow \gamma R + R_i$, where R is a MC measure of G_i
    2. Accumulate gradients with respect to $\theta' : d\theta \leftarrow d\theta + \nabla_{\theta'} \log \pi_{\theta'}(A_i, S_i)(R - V_{w'}(S_i))$
    3. Accumulate gradients with respect to w': $dw \leftarrow dw + 2(R - V_{w'}(s_i))\nabla_{w'}(R - V_{w'}(S_i))$
7. Update asynchronously θ using $d\theta$ and w using dw.

A3C enables parallelism in multiple agent training. The gradient accumulation step can be understood as a parallelized version of the minibatch-based stochastic gradient update: the values w and θ get corrected by a little bit in the direction of each training thread independently.

Advantage Actor-Critic - A2C

A2C is a synchronous, deterministic version of A3C. In A3C each agent talks to the global parameters independently, so it is sometimes possible that the thread-specific agents would by processing with different policies and thus the aggregated update would not be optimal. To resolve this problem, a coordinator in A2C waits for all the parallel actors to finish their work before updating the global parameters and then in the next iteration parallel actors start from the same policy. The synchronized gradient update keeps the training more cohesive and makes convergence potentially faster.

A2C has been shown to utilize GPUs more efficiently and work better with large batch sizes while achieving same or better performance than A3C.



A3C (Async) **A2C (Sync)**

Part II - Practice

In this project, the three model types described in Part I were applied; DDPG and D4PG on the environment with the single agent and A2C on the environment with the 20 agents.

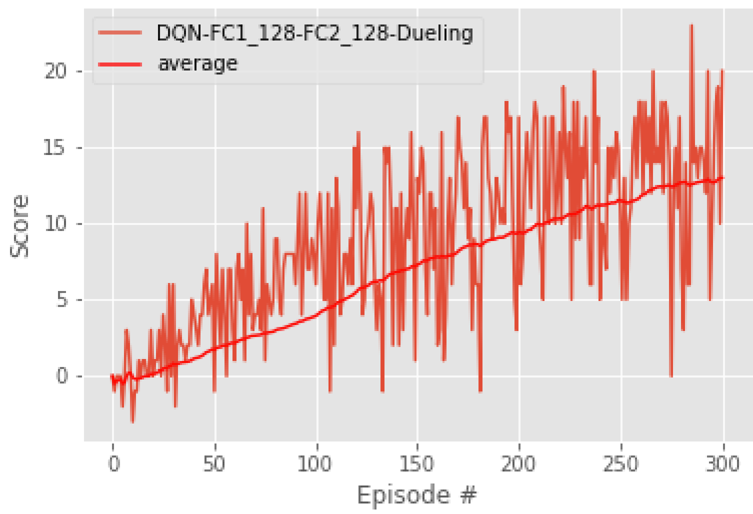
For the environment with the single agent, DDPG and D4PG are appropriate as the sequence of experiences will be highly correlated and thus the replay buffer employed by both models is useful to break these correlations.

While DDPG with the applied configuration does not seem to learn effectively in the single agent environment, the successor method D4PG successfully learns the task, albeit slowly.

For the environment with the 20 agents, A2C is an appropriate model as its structure has been shown to work well in environments where parallel batch processing is feasible.

FC1 Units	FC2 Units	Use Double Q-Learning	Use Dueling Q-Learning	Number of Episodes	Average Score
64	64	TRUE	TRUE	360	13.04
64	64	TRUE	FALSE	252	13.05
64	64	FALSE	TRUE	279	13.05
64	64	FALSE	FALSE	249	13.02
64	128	TRUE	TRUE	313	13.04
64	128	TRUE	FALSE	315	13.01
64	128	FALSE	TRUE	300	13.02
64	128	FALSE	FALSE	343	13.01
128	64	TRUE	TRUE	231	13.02
128	64	TRUE	FALSE	346	13.01
128	64	FALSE	TRUE	303	13.03
128	64	FALSE	FALSE	314	13.06
128	128	TRUE	TRUE	233	13.07
128	128	TRUE	FALSE	298	13.04
128	128	FALSE	TRUE	201	13.01
128	128	FALSE	FALSE	310	13.01

Below we show the trajectory of the best model, the trajectories for other models can be found in the sub-directory images of the project folder. The checkpoints can be found in the sub-directory pth_checkpoints.



Part III - Improvements

- **Improve model evaluation**: All model configurations were evaluated on a single run of each model. To ensure a more stringent evaluation, each model configuration should be run multiple times and evaluated on the basis of these samples.
- **Add prioritized experience replay**: Prioritized experience replay (see Prioritized Experience Replay) does not sample experience transitions uniformly from replay memory, but prioritizes important transitions by sampling them more frequently.
- **Implement Noisy DQN**: Implement the method described in Noisy Networks for Exploration, which adds parametric noise to the weights to induce stochasticity to the agent's policy, yielding better performing exploration.