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PEOPLE FOR THE ETHICAL TREATMENT OF SPECTRAL LINES

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ABSTRACT

A description of the mathematicl model to estimate the counts in a *Chandra* gratings observation around the O VIII the and O VII triplet.

Keywords: spectroscopy, X-rays, methods – statistical

1. DATA

The data are lists of events collected over a duration $\mathcal{T}$ in a *source* region and in a separate *background* region which has a geometric area $\frac{1}{r}$ of the source region. The events are marked by their observed wavelength, w,

$$Y_S = \{\mathrm{w}_s, s = 1..N_S\} \tag{1}$$

$$Y_B = \{\mathrm{w}_b, b = 1..N_B\} \tag{2}$$

There are $\approx$ 100 separate observations of Capella at high spectral resolution with *Chandra*, in various detector/grating combinations (ACIS-S+HETGS, ACIS-S+LETGS, HRC-S+LETGS). Each of these combinations provides their own quirks. ACIS-S+HETGS has the best resolution, and two arms (HEG and MEG), but has very low effective area over the wavelength region of interest. HRC-S+LETGS has much higher effective area, but also has much higher particle background. ACIS-S+LETGS has intermediate effective area, but has only two observations.

The wavelength region of interest, and some prominent O lines in that region, are shown in Table 1. The lines are located at intrinsic wavelengths $\omega_l, l = 1..7$, which could be altered when a new atomic emissivity replicate curve $\epsilon_l^{(Z)}$ is constructed.

Table 1. Spectral lines of interest

component	wavelength ω [Å]	species	transiton
0	18.5-22.5	continuum	Bremss, RRC, DR
1	18.9671	O VIII	$^2\mathrm{P}_{3/2} \rightarrow {}^2\mathrm{S}_{1/2}$ (resonance)
2	18.9726	O VIII	$^2\mathrm{P}_{1/2} \rightarrow {}^2\mathrm{S}_{1/2}$ (resonance)
3	21.6020	O VII	$^1\mathrm{P}_1 \rightarrow {}^1\mathrm{S}_0$ (w) (1s.2p to 1s2 resonance)
4	21.8040	O VII	$^3\mathrm{P}_2 \rightarrow {}^1\mathrm{S}_0$ (x) (intercombination)
5	21.8070	O VII	$^3\mathrm{P}_1 \rightarrow {}^1\mathrm{S}_0$ (y) (intercombination)
6	22.1010	O VII	$^3\mathrm{S}_1 \rightarrow {}^1\mathrm{S}_0$ (z) (forbidden)
7	18.6270	O VII	$^1\mathrm{P}_1 \rightarrow {}^1\mathrm{S}_0$ (1s.3p to 1s2 resonance)

2. MODEL

The Differential Emission Measure (DEM) is usually written

$$g(n_e, T) = n_e^2(T)\ \frac{dV(T)}{d\ln T}\ , \tag{3}$$

where n_e is the electron number density and T is the temperature of the plasma. The DEM is often written as a sole function of T (see §4).

The flux from the l^{th} component is the product of the emissivity ϵ and the DEM,

$$\mathrm{f}_l^{(Z)}(\omega) = \epsilon_l^{(Z)}(\omega; n_e, T)\ g(n_e, T)\,, \tag{4}$$

where (Z) represents the $(Z)^{th}$ sample from the atomic replicates sample. When seen through an instrument with effective area $\mathrm{A}_{eff}(\omega)$ and line response function $\mathrm{R}(\omega \rightarrow \mathrm{w})$,[1] for a duration of $\mathcal{T}$, and integrated over T, the estimated counts at observed wavelength w,

$$\mathrm{c}^{(Z)}(\mathrm{w}, n_e) = \mathcal{T} \int d\mathrm{ln}T \sum_l \int d\omega\ \mathrm{f}_l^{(Z)}(\omega; n_e, T)\ \mathrm{A}_{eff}(\omega)\ \mathrm{R}(\omega \rightarrow \mathrm{w})\,. \tag{6}$$

The indices $l = 0..7$, represent the components that are included, with 0 representing the continuum, and the others representing the lines listed in Table 1. The continuum component ϵ_0 is assumed to be independent of n_e, is identical for all (Z), and is supplied as a 2-D table on a grid of (ω, T), while the line emissivities are delta-functions at the wavelengths $\omega_l^{(Z)}$, close to (but not necessarily identical) to those listed in Table 1.

$$\epsilon_0^{(Z)}(\omega; n_e, T) \equiv \mathcal{E}_0(\omega, T) \tag{7}$$

$$\epsilon_{l>0}^{(Z)}(\omega; n_e, T) \equiv \delta(\omega - \omega_l^{(Z)})\ \cdot\ \mathcal{E}_l^{(Z)}(n_e, T) \tag{8}$$

3. PROBABILITY MODEL

If $\theta = \{\theta_S, \theta_B\}$ are the model parameters for the DEM (see §4) and the background respectively,

$$\begin{aligned} p(\theta|Y_S, Y_B) &\propto p(Y_S, Y_B|\theta_S, \theta_B)\ p(\theta) \\ &\propto p(Y_S|\theta_S, \theta_B)\ p(\theta_S)\ p(\theta_B|Y_B)\ p(\theta_B)\,. \end{aligned} \tag{9}$$

For the background, a useful approximation is to use the $\mathrm{A}_{eff}(\omega)$ to define the shape, and use the normalization as the sole parameter. Representing this as $\mathrm{A}(\omega)$, the estimated background intensity,

$$b(\mathrm{w}) = norm_B \cdot \int d\omega\ \mathrm{R}(\omega \rightarrow \mathrm{w})[\mathrm{A}(\omega)]\,. \tag{10}$$

For HRC-S+LETGS, the dependence on $\mathrm{A}(\omega)$ is negligible, and a flat $b(\mathrm{w}) = norm_B$ is sufficient.

For the likelihood,

$$\begin{aligned} p(Y_S|\theta_S, \theta_B) &= \Pi_{s=1}^{S}\ \mathrm{Poisson}(1|\mathrm{c}(\mathrm{w}_s; \theta_S) + b(\mathrm{w}_s)/r) \\ &= \Pi_{s=1}^{S}\ (\mathrm{c}(\mathrm{w}_s; \theta_S) + b(\mathrm{w}_s)/r)\ e^{-(\mathrm{c}(\mathrm{w}_s; \theta_S) + b(\mathrm{w}_s)/r)} \end{aligned} \tag{11}$$

where the atomic replicates index (Z) has been dropped for legibility from c.

4. DEM PARAMETERIZATION

There are effectively 7 data points (6 lines and the continuum), so we are limited to estimating at most 6 parameters. Since choosing one of (Z) is a prime driver of this exercise, that implies the DEM must be parameterized by at most 5 parameters. Some possibilities are listed below.

— logNormal : $norm$, $\mathrm{ln}T_0$, $\sigma_{\mathrm{ln}T}$

$$g(n_e, T) \equiv N(\mathrm{ln}T; \mathrm{ln}T_0, \sigma_{\mathrm{ln}T}) \propto exp\left[-\frac{(\mathrm{ln}T - \mathrm{ln}T_0)^2}{2\sigma_{\mathrm{ln}T}^2}\right] \tag{12}$$

— gamma : $norm$, α, β

$$g(n_e, T) \equiv \gamma(\mathrm{ln}T; \alpha, \beta) \propto \mathrm{ln}T^{\alpha} e^{-\beta \mathrm{ln}T} \tag{13}$$

[1] The line response function for the HRC-S+LETGS can be approximated as a β-profile,

$$\mathrm{R}(\omega \rightarrow \mathrm{w}) \equiv \frac{1}{\left[1 + \left(\frac{\omega - \mathrm{w}}{0.05}\right)^2\right]^{2.5}} \tag{5}$$

for a bin width of $\delta\mathrm{w} = 0.0067$Å.

— scaled gamma : $norm$, α, β, γ

$$x = T^{\gamma}$$
$$g(n_e, T) \equiv \gamma(x; \alpha, \beta) \propto x^{\alpha} e^{-\beta x} \tag{14}$$
$$\tag{15}$$

— Guennou models : Gaussian, tophat, AR (T^{α}, $T < T_{\mathrm{MAX}}$, power law + half a Gaussian for $T > T_{\mathrm{MAX}}$

5. UNITS

The counts c are in counts per detector bin, and the widths of the detector bins are set within the R($\omega \rightarrow$ w).

The DEM (as in Equation 3 has units of [cm^{-3} lnK^{-1}]. DEMs are often also reported in units of [cm^{-3} K^{-1}, by replacing the $d$ln$T$ by dT. In the solar case, the column emission measure is of interest rather than the volume emission measure, with units of [cm^{-5} ...].

The emissivities $\mathcal{E}$ are usually in [$ergs\ cm^3\ s^{-1}$] (in [$ph\ cm^3\ s^{-1}$] in AtomDB). Therefore, the continuum fluxes f_0 are in [$ergs\ cm^{-2}\ s^{-1}\ bin^{-1}$] and the line fluxes $\mathrm{f}_{l>0}$ are in [$ergs\ cm^{-2}\ s^{-1}$].