1. Blown away! E.A. Bergshoeff

You are sailing, together with a few friends, on a sloop in the 'Paterwoldse meer'. In the middle of the lake, the wind suddenly drops. Is it possible to reach the shore by blowing into the sail?
You may make the following simplifications. Imagine the sail to be a thin rigid wall. Instead of blowing against this wall you can imagine throwing tennis balls at it (as a substitute for air molecules). Assume the collisions to be completely elastic and neglect the friction between the boat and the air/water. Finally, assume that 30 percent of the tennis balls falls, after colliding with the wall, back on the boat.

2. Neutron Star N. Kalantar

The binding energy of a nucleus is given empirically by the following formula, if one assumes it has a spherical shape:

$$BE(A,Z) = a_v A - a_s A^{2/3} - a_c Z(Z-1)A^{-1/3} - a_a(A-2Z)^2 A^{-1}$$

Here the pairing effect has been neglected, Z is the proton number of the nucleus and A the atomic number. The coefficients are given by:

$$a_v = 15.85 \text{ MeV}, \, a_s = 18.34 \text{ MeV}, \, a_c = 0.71 \text{ MeV} \;\; \text{and} \;\; a_a = 23.21 \text{ MeV}.$$

(a) Assume the nucleus is spherical and its radius is given by $R = r_0 A^{1/3}$, with $r_0 = 1.2 \cdot 10^{-15}$ m. Also, assume that the nuclear force is short-ranged. Explain the origins of the first three terms in the above formula.

This formula gives the binding energy of a nucleus with no more than a few hundreds of nucleons ($A \approx 250$). Let us now construct a star, with nothing but neutrons (a neutronstar), in which the strong and the gravitational forces are the only relevant ones. The formula mentioned above can then be used for the star as well; except for the fact that, because of the large mass, gravity has to be taken into account.

(b) Given that the radius of the star is equal to the radius in part (a),and that the mass of the star is AM_n, where $M_n = 1.67 \cdot 10^{-27}$ kg is the mass of a neutron, calculate the extra term caused by gravity in the formula for the energy.

(c) The gravitational constant equals $G = 6.67 \cdot 10^{-11}$ Jmkg^{-2} and 1 MeV $= 1.602 \cdot 10^{-13}$ J. Calculate the minimal radius and mass of the neutronstar.

(d) The radius of a typical neutronstar is in the order of magnitude of 10 km. Explain the agreement, or the lack of it, considering the big extrapolation used.

3. Solar Birth M. Spaans

About 5 billion years ago, the Sun formed from a cloud of gas, by contraction under the influence of gravity. During this first period, before nuclear fusion processes had started, the sun shone as a black body described by Planck's law,

$$u(\omega, T) = \frac{\hbar\omega^3}{\pi^2 c^3 \left(e^{\beta\hbar\omega} - 1\right)}.$$

Here u denotes the energy density corresponding to the angular frequency ω and the temperature T. The temperature of the surface of the Sun is constant.

(a) Determine, at an arbitrary frequency, the flux received from the sun by an observer, in erg s^{-1} cm^{-2} Hz^{-1}. Take the observer to be pointlike.

As the Sun formed, it emitted radiation. Below questions are intended to study this production of radiation. To this end consider a ball of ideal self-gravitating atomic hydrogen gas with radius r, density $\rho(r)$ and total mass $M_\odot = 2.0 \cdot 10^{33}$ g. The current radius of the Sun is $r_\odot = 7.0 \cdot 10^{10}$ cm, and the constant temperature of its center $T_c = 2.0 \cdot 10^6$ K.

(b) What radius r_{vir} (the virial radius) does the ball of gas have if the current conditions in the center of the Sun are typical for its equilibrium? Compare these results to the current radius of the Sun.

(c) What is the average binding energy of a particle in the ball of gas?

Assume the work done by the gravitational field is the dominant source of energy as the ball of gas is contracting.

(d) Determine the density profile for an isothermal ball of gas.

(e) How much energy will the contraction of the ball produce.

Assume all of the energy produced is dispersed in the form of light.

(f) Estimate the time it will take the ball to emit all of this energy.

4. Breaking the Waves H. Jordens

Waves in the sea almost always arrive perpendicularly to the coast, irrespective of their original directions. This phenomenon can be understood by taking into account that, with decreasing depth, the velocity of the waves will decrease as well. This will cause waves to change direction. The change in direction is described by a law similar to the one for refraction of light (Snell's law).

(a) Show that, near the coast, waves will travel in a direction perpendicular to shore if we assume that the velocity decreases to 0 there.

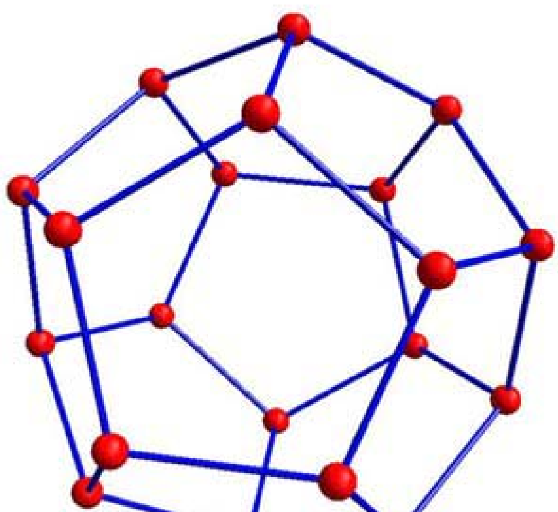
(b) Calculate the path a wave travels if its speed v is proportional to its distance to the beach y.

5. Wiring the Greek Way F.J. van Steenwijk

Consider a resistor network that is made up of 30 resistors R, arranged to form the edges of a dodecahedron (see the figure).

(a) Determine the equivalent resistor between two opposite vertices.

(b) Do the same for two neighbouring vertices.



6. Tooling Around in Space G. 't Hooft

A spaceship is flying around the earth, at a height of 1730 km above the earth's surface and in a circular orbit. The period of revolution T is 2 hours. An astronaut goes out for a spacewalk close to the spaceship. She loses a tool, which flies away at a speed of 1 kilometer per hour. Where will the tool be after some time, according to Kepler's laws? We neglect the influence of the Moon, the Sun, the other planets and the rotation of the Earth.
The astronaut uses a coordinate system with herself in the origin, chosen in such a way that the center of the Earth is on the negative z-axis, the x-axis is along the direction of motion of the spaceship and the y-axis is perpendicular to the other two axes. The coordinate system rotates with the spaceship around the Earth. Reasonable approximations may be used and lengthy calculations are to be avoided.

1. The tool flies away in the positive y-direction.
   (a) Discuss the orbit it describes with respect to the spaceship. Sketch what graphs you expect for the coordinates $x(t), y(t)$ and $z(t)$.
   (b) Give a good approximation for the formulae for these coordinates.
   (c) Where will the tool be after half an hour, an hour, one and a half hour and two hours respectively?
2. The tool flies away in the positive z-direction.
   (a) Sketch the orbit of the tool you expect to see in the coordinate system.
   (b) Use Kepler's laws to deduce the shape of the orbit in the (x, z) plane. Now give, in more detail, the expected formulae for $x(t)$ and $z(t)$.
   (c) Where will the tool be after half an hour, an hour, one and a half hour and two hours respectively?
3. The tool flies away in the positive x-direction. This is the most difficult case.
   (a) Sketch the orbit you expect in the coordinate system [hint: use the result of part 2.].
   (b) First, make an assumption about the parameters of this orbit and discuss what the period T_{tool} of the tool will be in comparison with the period T of the spaceship.
   (c) What is the formula for this orbit?
   (d) Where will the tool be after two hours?
4. The tool flies away in an arbitrary direction.
   (a) Discuss the orbit you expect.
   (b) Will the tool come back to the astronaut? How does this depend on the initial velocity of the tool?

Finally some formulae: According to Kepler the orbit of an object around a central mass is an ellipsis with a long axis a, a short axis b and the focal points at a distance c from the center. The central mass is one of the focal points. The following relations hold:

$$\begin{aligned} GM_{\oplus}\left(\frac{T}{2\pi}\right)^2 &= a^3; \\ GM_{\oplus}\left(\frac{1}{r}-\frac{1}{2a}\right)^2 &= \frac{1}{2}v^2 \end{aligned}$$

Where G is Newton's constant, $M_{\oplus}$ the mass of the Earth, r the distance from the object to the Earth's center and v its velocity.
In an infinitesimal amount of time δt the radius from the center of the Earth to an object in orbit around it sweeps out a surface $\mathcal{O}$ which is proportional to δt:

$$\mathcal{O} = C\delta t,$$

Where the constant C depends only on the orbit, not on the time t.
Further you have these numbers at your disposal:

$$\begin{aligned} R_{\oplus} &: \quad 6370 \text{ km} \\ GM_{\oplus} &: \quad 405.000 \text{ km}^3\text{s}^{-2} \end{aligned}$$

7. 日本大阪 **O. Scholten**

At the LEPS facility at the SPRING8 laboratory near Osaka, Japan, a beam of high energy photons (gamma-rays) is created by Compton backscattering of UV Ar-laser light (λ_{uv}=350 nm) off electrons with an energy of 8 GeV.

(a) Use energy and momentum conservation to give the expression for the energy of the backscattered photons as function of the scattering angle.

(b) Show that for exact backscattering ($\theta_\gamma = 180°$) the mass of the electron cannot be ignored and calculate the maximum energy of the backscattered photons.

(c) Calculate, to a good approximation, the angle $\theta_{1/10}$ at which the energy of the (almost) backscattered photons is reduced to a tenth of the maximum energy.

8. A Matter of Life and Death B. Hoenders

Around 1900 the German physicist P. Drude computed the force acting on a conducting metal plate (or halfspace), when it's illuminated by a plane electromagnetic wave. His arguments were the following: the electric fieldvector $\mathbf{E}$ will cause a current σE, where σ is the conductance of the metal. Together with the displacement current, this current will give rise to a Lorentz-force because of the presence of the magnetic field. If we restrict ourselves to plane waves perpendicular to the metal, a force per volume element will arise perpendicular to the surface of the plate.
Via an exact analysis of this problem, using the electromagnetic impuls (proportional to the Poyntingvector), one is able to obtain the right result for the force.
Unfortunately, this solution shows that Drude's solution was incomplete, and that his answer differed by a factor 1/2 from the exact answer. The mistake in his reasoning was pointed out, in his time, by Hertz and Poincaré: besides the force of the magnetic field on the current, there has to be, due to symmetry, a force from the electric field on the "magnetic current". We could dub this the "electric Lorentz force".
Find, using this result, the missing pieces of Drude's incomplete analysis.
P.S. Three years after his mistake, Drude committed suicide.

9. The Magic Marble of Mystery W. K. Ma

A bored frosh decides to take up the enterprise of launching a marble (a uniform solid sphere) with radius R and mass M up a 30 degree incline with coefficient of friction μ. At $t = 0$, the marble's linear velocity is v_0 (parallel to the incline), and its angular velocity is zero.

(a) Show that the marble first rolls without slipping when $t = \frac{v_0}{g}\frac{4}{2+7\sqrt{3}\mu}$.

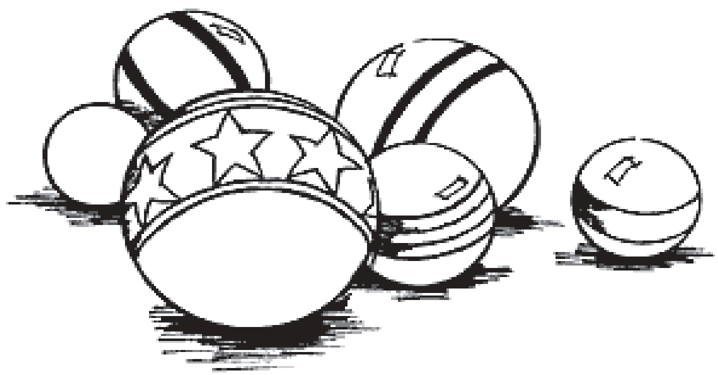
(b) How high is it then, compared to its initial height?

(c) How much heat has been produced then?

(d) How many revolutions has it made then?

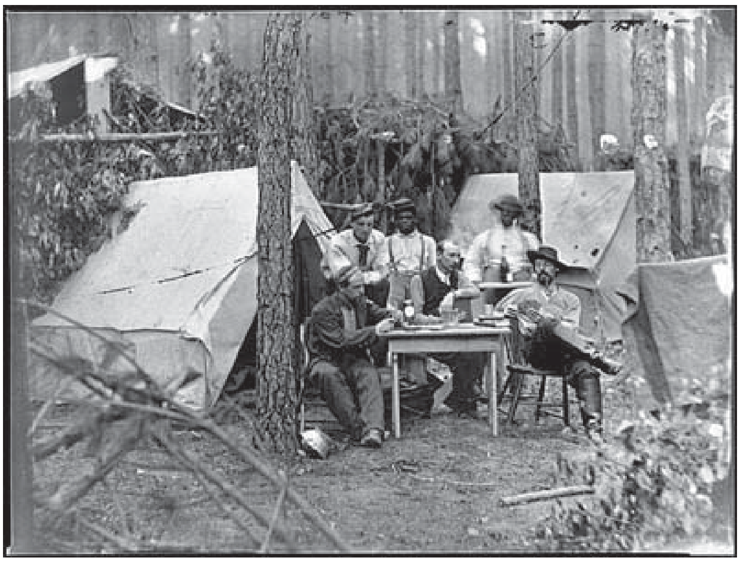
(e) Draw a qualitative graph showing both $v(t)$ and $R\omega(t)$ between $t = 0$ and the time the marble reaches maximum height.

(f) How many revolutions has the marble made when it is at its highest point?



10. Logic for spies L. P. Kok

Thirteen playing cards are distributed among three spies, aptly named A,B and C. Spy A receives six cards, as does spy B. Spy C gets the one remaining card. Spy A and B can communicate with each other, but C can eavesdrop on everything. How can spy A and B get to know the full distribution of cards, and even tell each other loudly that they do, without spy C being able to deduce it as well?



11. A Rotating Disc J. F. Schröder

A thin disc is composed of two homogenous semidiscs. One semidisc has density ρ, the other a density 2ρ. The disc is rolling on a horizontal plane without sliding. The rotation axis coincides with the axis of the cilinder. Determine the Lagrangian of this system.