Project 1- Coin Flips

EE 511:Spring 2020

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Question 1

Statement: Tossing a fair coin 50 times, each flips can be a Bernoulli trial. The Bernoulli trial has two outcomes: head and tail, which means a Bernoulli trial means each head and tail has 50 and 50 percent. Figure 1 shows the number of heads and tails of those 50 flip. 0 represent tail, and 1 represent head. There are 27 heads of 50 flips. The longest sequence of heads is 5. The longest sequence of heads is the max of the heads run lengths. The code and result shows in Figure2.

Conclusion: The number of tails and heads are almost equal. This is also the meaning of the Bernoulli trial. As the number of flips increasing, the percentage of head and tail will close to 50 %.

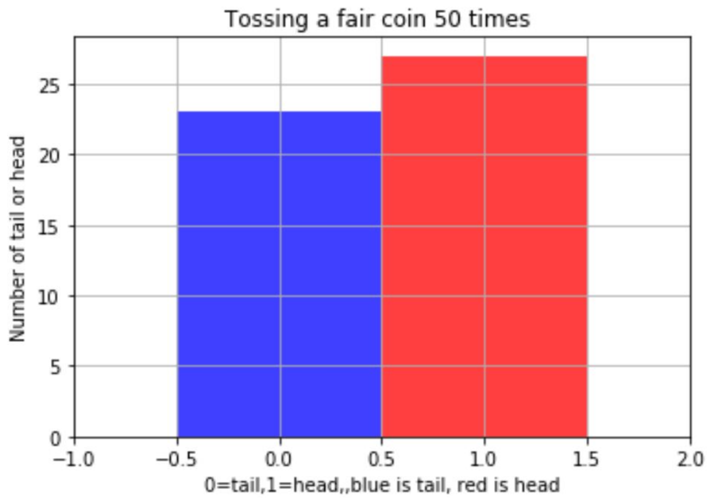
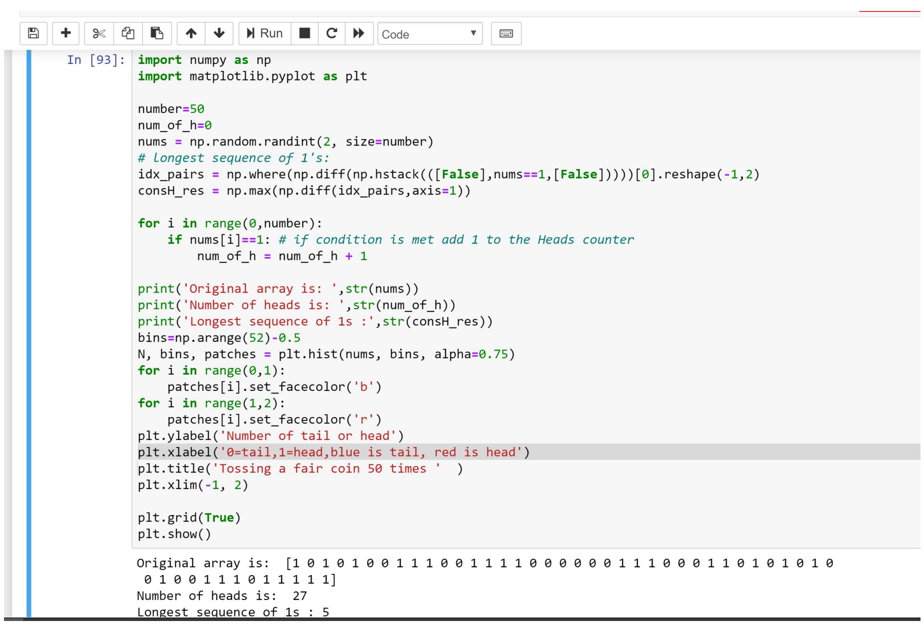


Figure 1: The number of heads and tails of those 50 flip.



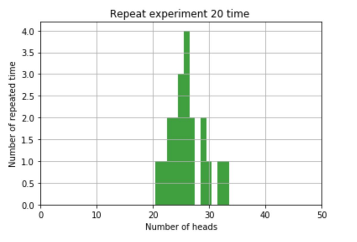
```
In [93]: import numpy as np
import matplotlib.pyplot as plt

number=50
num_of_h=0
nums = np.random.randint(2, size=number)
# longest sequence of 1's:
idx_pairs = np.where(np.diff(np.hstack(([False],nums==1,[False]))))[0].reshape(-1,2)
consH_res = np.max(np.diff(idx_pairs,axis=1))

for i in range(0,number):
    if nums[i]==1: # if condition is met add 1 to the Heads counter
        num_of_h = num_of_h + 1

print('Original array is: ',str(nums))
print('Number of heads is: ',str(num_of_h))
print('Longest sequence of 1s :',str(consH_res))
bins=np.arange(52)-0.5
N, bins, patches = plt.hist(nums, bins, alpha=0.75)
for i in range(0,1):
    patches[i].set_facecolor('b')
for i in range(1,2):
    patches[i].set_facecolor('r')
plt.ylabel('Number of tail or head')
plt.xlabel('0=tail,1=head,blue is tail, red is head')
plt.title('Tossing a fair coin 50 times '  )
plt.xlim(-1, 2)

plt.grid(True)
plt.show()

Original array is:  [1 0 1 0 1 0 0 1 1 1 0 0 1 1 1 1 0 0 0 0 0 0 1 1 1 0 0 0 1 1 0 1 0 1 0 1 0
 0 1 0 0 1 1 1 0 1 1 1 1 1]
Number of heads is:  27
Longest sequence of 1s : 5
```

Figure 2: Code and Result for Berniulli trial(Tossing a fair coin).

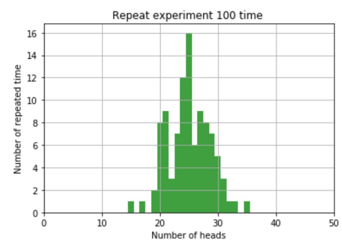
Question 1a

In this question, the 50 flips will repeated 20,100,200, and 1000 times. The Figure 3 shows the histogram for each showing the number of heads of 50 flips.

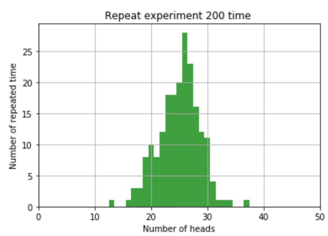
The Figure 4 shows the python code for each experiments.



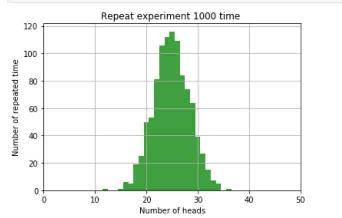
(a) Repeated 20 times.



(b) Repeated 100 times.



(c) Repeated 200 times.



(d) Repeated 1000 times.

Figure 3: The histogram for each showing the number of heads of 50 flips.

```
import numpy as np
import matplotlib.pyplot as plt
toss_number=50
repeat_num=20 #20,100,200,1000
num_of_h=0
head_nums_arr = [] #create a array
for j in range(0,repeat_num):
    num_of_h=0;
    nums = np.random.randint(2, size=toss_number)
    for i in range(0,toss_number):
        if nums[i]==1: # if condition is met add 1 to the Heads counter
            num_of_h = num_of_h + 1
    head_nums_arr.append(num_of_h)

#print(' array is: ',str(head_nums_arr))
# the histogram of the data
bins=np.arange(52)-0.5
plt.hist(head_nums_arr, bins, facecolor='g', alpha=0.75)
plt.xlabel('Number of heads')
plt.ylabel('Number of repeated time')
plt.title('Repeat experiment %i time '  %repeat_num )
plt.xlim(0, 50)
plt.grid(True)
plt.show()
```

Figure 4: The python code for each experiments.

Conclusion: Comment on the limit of the histogram: Repeat the experiment many times, the distribution become binomial distribution. Binomial distribution is the sum of each i.i.d Bernoulli trials. The probability of each head is 0.5, and there are 50 flips from last experiment. The expectation of binomial distribution is E[x]=np=50*0.5=25. When the number of repeated increasing, the graph will look like a binomial distribution with expectation of 25.

Question 2

Statement: This experiment is looks like the first experiment, but it is not a fair coin for this experiment. The probability of head is 0.8. The experiment will toss a biased coin 200 times. Figure 5 shows a histogram for this outcomes. Figure 6 shows the code and result for this experiment. The code is generating a random number between 0 and 1. When the number greater than 0.8, the trial will become tail(0) as the result. When the number smaller than 0.8, this trial will become head(1) as the result.

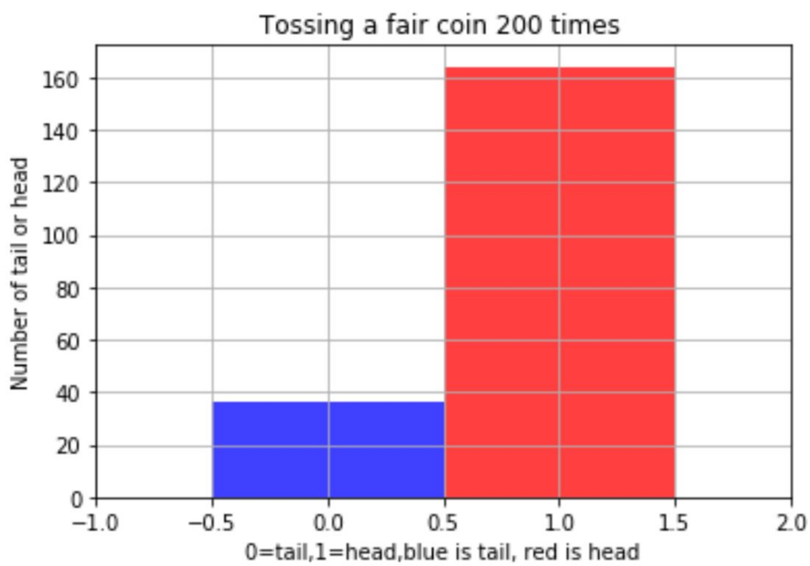


Figure 5: The number of heads and tails of those 200 flips with biased coin.

```python
import numpy as np
import matplotlib.pyplot as plt

number=200
num_of_h=0
nums = [] #create a array


for i in range(0,number):
    if np.random.rand()<0.8: # if condition is under 0.8, it become the Heads counter
        nums.append(1)  # 1 is head
        num_of_h=num_of_h+1
    else:
        nums.append(0)

# Longest sequence of 1's:
nums_head = np.array(nums)
idx_pairs = np.where(np.diff(np.hstack(([False],nums_head==1,[False]))))[0].reshape(-1,2)
consH_res = np.max(np.diff(idx_pairs,axis=1))


print('Original array is: ',str(nums_head))
print('Number of heads is: ',str(num_of_h))
print('Longest sequence of 1s :',str(consH_res))
bins=np.arange(52)-0.5
N, bins, patches = plt.hist(nums, bins, alpha=0.75)
for i in range(0,1):
    patches[i].set_facecolor('b')
for i in range(1,2):
    patches[i].set_facecolor('r')
plt.ylabel('Number of tail or head')
plt.xlabel('0=tail,1=head,blue is tail, red is head')
plt.title('Tossing a fair coin 200 times '  )
plt.xlim(-1, 2)

plt.grid(True)
plt.show()
```

```
Original array is:  [0 1 1 1 1 1 1 0 1 1 1 0 1 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
 1 1 0 0 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 1 0 1 1 0 1 1 1 0 1 1 1 0 0 0 1 1 1
 1 1 0 1 1 0 0 1 0 1 1 1 1 1 0 0 1 1 1 1 1 1 1 0 1 0 0 1 1 1 1 1 1 1 0 1 1
 1 1 1 0 1 1 1 1 1 1 0 1 1 1 1 1 1 0 1 1 0 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 0
 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1 0 1 1 1 0 1 1 0 0 1 1 1
 1 1 1 1 1 0 1 1 1 1 1 1 1 1 1]
Number of heads is:  164
Longest sequence of 1s : 24
```

Figure 6: Code and Result for a biased coin.

Conclusion: The result shows that there are 162 heads of 200 flips. The longest sequence of heads are 23. From the graph, the blue represent tail, and the red represent head. The expectation of head are E[x]=np=200*0.8= 160. The result is close to 160, that means that the experiment is successful.

Question 3

Statement: This experiment will toss a fair coin 200 times. The python code will record the heads run lengths for each. Figure 7 shows the heads run lengths for Tossing a fair coin 200 times. Figure 8 shows the code and result for those 200 times.

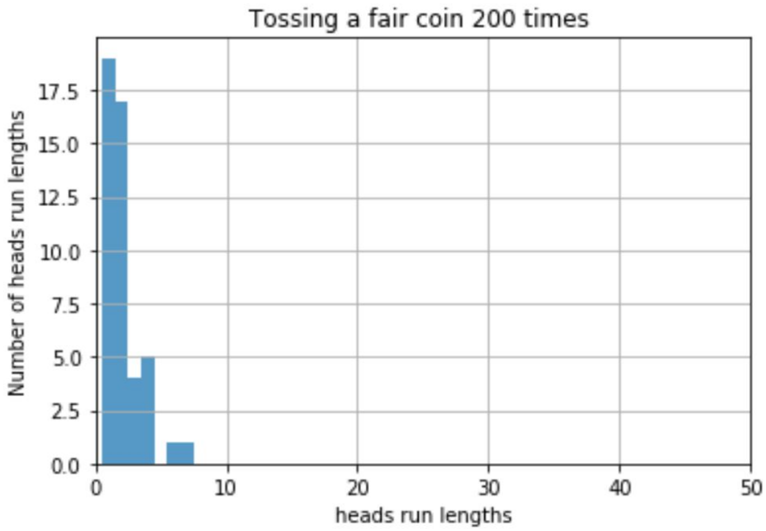


Figure 7: The heads run lengths for Tossing a fair coin 200 times.

```
import numpy as np
import matplotlib.pyplot as plt

number=200
nums = np.random.randint(2, size=number)
# Longest sequence of 1's:
idx_pairs = np.where(np.diff(np.hstack(([False],nums==1,[False]))))[0].reshape(-1,2)
head_number=np.diff(idx_pairs,axis=1)
head_number=np.concatenate((head_number), axis=None)
consH_res = np.max(np.diff(idx_pairs,axis=1))
print('pair number: ',str(head_number))
print('Original array is: ',str(nums))
print('Longest sequence of 1s :',str(consH_res))
bins=np.arange(52)-0.5
N, bins, patches = plt.hist(head_number, bins, alpha=0.75)
plt.ylabel('Number of heads run lengths')
plt.xlabel('heads run lengths')
plt.title('Tossing a fair coin 200 times '   )
plt.xlim(0, 50)

plt.grid(True)
plt.show()
```

```
pair number:  [1 1 2 4 1 3 2 1 1 1 1 2 6 3 2 2 7 4 1 1 1 3 2 1 1 1 1 2 2 2 2 4 2 2 1 2 1
 2 3 2 2 4 1 2 4 1 1]
Original array is:  [0 1 0 0 0 1 0 1 1 0 1 1 1 1 0 0 0 1 0 0 1 1 1 0 1 1 0 1 0 0 1 0 1 0 0 1 0
 1 1 0 1 1 1 1 1 1 0 0 1 1 1 0 0 1 1 0 1 1 0 1 1 1 1 1 1 1 0 1 1 1 1 0 1 0
 0 0 0 0 0 0 0 0 0 1 0 1 0 0 0 0 1 1 1 0 1 1 0 1 0 1 0 1 0 1 0 1 1 0 0 1 1
 0 1 1 0 1 1 0 1 1 1 1 0 0 0 1 1 0 0 0 0 0 1 1 0 0 0 0 0 1 0 0 0 1 1 0 0 0
 1 0 0 1 1 0 0 0 0 0 0 0 1 1 1 0 1 1 0 0 0 0 0 1 1 0 1 1 1 1 0 1 0 0 0 1 1
 0 0 0 0 0 0 1 1 1 1 0 1 0 1 0]
Longest sequence of 1s : 7
```

Figure 8: Code and Result for the heads run lengths for Tossing a fair coin 200 times.

Conclusion: The majority of heads run lengths are 1, because the probability of multiple heads is 0.5^n, n represent the number of heads run lengths. As the n become larger, the probability will become smaller and smaller. As the graph shows the same result. That means the experiment is successful.

Question 4

Statement This experiment will have user input. When the user input a number that represent the number of heads, the python code will run the code to find the number of tosses until reaching the this amount of heads. Figure 9 shows the code and result for this experiment.

```
# input from user
import numpy as np
import matplotlib.pyplot as plt
from random import randint


a = input("please input a number that you want to reach")
a = int(a)
number_head=0
number_filp=0

while number_head != a:
    number_filp +=1
    if randint(0, 1)==1:
        number_head += 1

print('user input number: ',str(a))
print('number of filp: ',str(number_filp))
```

```
please input a number that you want to reach 20
user input number:  20
number of filp:  62
```

Figure 9: Code and Result for finding the number of tosses until reaching the number of heads from user input.

Conclusion: From our many testing, the number of tosses is almost double of the number of heads from the user input. The reasoning is because the experiment is tossing a fair coin, the probability of heads of fair coin is 0.5, Therefore, the tossing number will be double of the user input.

References

Wikipedia:
https://en.wikipedia.org/wiki/Binomial_distribution.html

Appendix

project1.tex

```latex
%This is my super simple Real Analysis Homework template

\documentclass{article}
\usepackage[utf8]{inputenc}
\usepackage[english]{babel}
\usepackage[]{amsthm} %lets us use \begin{proof}
\usepackage[]{amssymb} %gives us the character \varnothing
\usepackage{graphicx}
\usepackage{subcaption}

\title{Project 1- Coin  Flips \\\hfill \break\hfill \break\hfill \break \hfill \break\hfill \break EE 511:Spring 2020 \\\hfill \break\hfill \break}
\author{\\Qidi Han\\\\\hfill \break\hfill \break}
\date\today
%This information doesn't actually show up on your document unless you use the maketitle command below

\begin{document}
\maketitle %This command prints the title based on information entered above



\section*{Question 1}

    Statement:
    Tossing a fair coin 50 times, each flips can be a Bernoulli trial. The Bernoulli trial has two outcomes: head and tail, which means a Bernoulli trial means each head and tail has 50 and 50 percent. Figure 1 shows the number of heads and tails of those 50 flip. 0 represent tail, and 1 represent head. There are 27 heads of 50 flips. The longest sequence of heads is 5. The longest sequence of heads is the max of the heads run lengths. The code and result shows in Figure2. \\
\hfill \break


\begin{figure}[th!]
  \includegraphics[width=\linewidth]{figure1_graphheads.jpg}
  \caption{The number of heads and tails of those 50 flip.}
  \label{fig:figure1}
  \centering
\end{figure}

\begin{figure}[th!]
  \includegraphics[width=\linewidth]{question1_code.jpg}
```

project1.tex

```latex
  \includegraphics[width=\linewidth]{question1—code.jpg}
  \caption{Code and Result for Berniulli trial(Tossing a fair coin).}
  \label{fig:figure2}
  \centering
\end{figure}

Conclusion:
    The number of tails and heads are almost equal. This is also the meaning of the Bernoulli trial. As the number of flips increasing, the percentage of head and tail will close to 50 \%.\\


\section*{Question 1a}

    In this question, the 50 flips will repeated 20,100,200, and 1000 times. The Figure 3 shows the histogram for each showing the number of heads of 50 flips. The Figure 4 shows the python code for each experiments.

\begin{figure}[th!]

\centering
  \begin{subfigure}[b]{0.4\linewidth}
    \includegraphics[width=\linewidth]{figure3_20.jpg}
    \caption{Repeated 20 times.}
  \end{subfigure}
  \begin{subfigure}[b]{0.4\linewidth}
    \includegraphics[width=\linewidth]{figure3_100.jpg}
    \caption{Repeated 100 times.}
  \end{subfigure}
  \begin{subfigure}[b]{0.4\linewidth}
    \includegraphics[width=\linewidth]{figure3_200.jpg}
    \caption{Repeated 200 times.}
  \end{subfigure}
  \begin{subfigure}[b]{0.4\linewidth}
    \includegraphics[width=\linewidth]{figure3_1000.jpg}
    \caption{Repeated 1000 times.}
  \end{subfigure}
  \caption{The histogram for each showing the number of heads of 50 flips.}
  \label{fig:figure3}


\end{figure}

\begin{figure}[th!]
  \includegraphics[width=\linewidth]{figure4.jpg}
```

```latex
\begin{figure}[th!]
  \includegraphics[width=\linewidth]{figure4.jpg}
  \caption{The python code for each experiments.}
  \label{fig:figure4}
  \centering
\end{figure}

Conclusion:
    Comment on the limit of the histogram: Repeat the experiment many times, the distribution become binomial distribution. Binomial distribution is the sum of each i.i.d Bernoulli trials. The probability of each head is 0.5, and there are 50 flips from last experiment. The expectation of binomial distribution is E[x]=np=50*0.5=25. When the number of repeated increasing, the graph will look like a binomial distribution with expectation of 25.


\section*{Question 2}

Statement:
    This experiment is looks like the first experiment, but it is not a fair coin for this experiment. The probability of head is 0.8. The experiment will toss a biased coin 200 times. Figure 5 shows a histogram for this outcomes. Figure 6 shows the code and result for this experiment. The code is generating a random number between 0 and 1. When the number greater than 0.8, the trial will become tail(0) as the result. When the number smaller than 0.8, this trial will become head(1) as the result.



\begin{figure}[th!]
  \includegraphics[width=\linewidth]{figure5.jpg}
  \caption{The number of heads and tails of those 200 flips with biased coin.}
  \label{fig:figure5}
  \centering
\end{figure}

\begin{figure}[th!]
  \includegraphics[width=\linewidth]{figure6.jpg}
  \caption{Code and Result for a biased coin.}
  \label{fig:figure6}
  \centering
\end{figure} \newpage
\newpage
```

```latex
\newpage


Conclusion:
    The result shows that there are 162 heads of 200 flips. The longest sequence of heads are 23. From the graph, the blue represent tail, and the red represent head. The expectation of head are E[x]=np=200*0.8=160. The result is close to 160, that means that the experiment is successful.

\section*{Question 3}
Statement:
    This experiment will toss a fair coin 200 times. The python code will record the heads run lengths for each. Figure 7 shows the heads run lengths for Tossing a fair coin 200 times. Figure 8 shows the code and result for those 200 times.


\begin{figure}[th!]
  \includegraphics[width=\linewidth]{figure7.jpg}
  \caption{The heads run lengths for Tossing a fair coin 200 times.}
  \label{fig:figure7}
  \centering
\end{figure}

\begin{figure}[th!]
  \includegraphics[width=110 MM]{figure8.jpg}
  \caption{Code and Result for the heads run lengths for Tossing a fair coin 200 times.}
  \label{fig:figure8}
  \centering
\end{figure}



Conclusion:
    The majority of heads run lengths are 1, because the probability of multiple heads is $0.5^n$, n represent the number of heads run lengths. As the n become larger, the probability will become smaller and smaller. As the graph shows the same result. That means the experiment is successful.

\section*{Question 4}

Statement
    This experiment will have user input. When the user input a number that represent the number of heads, the python code will run the code to
```

project1.tex

```latex



Conclusion:
    The majority of heads run lengths are 1, because the probability of multiple heads is $0.5^n$, n represent the number of heads run lengths. As the n become larger, the probability will become smaller and smaller. As the graph shows the same result. That means the experiment is successful.

\section*{Question 4}

Statement
    This experiment will have user input. When the user input a number that represent the number of heads, the python code will run the code to find the number of tosses until reaching the this amount of heads. Figure 9 shows the code and result for this experiment.

\begin{figure}[th!]
  \includegraphics[width=\linewidth]{figure9.jpg}
  \caption{Code and Result for finding the number of tosses until reaching the number of heads from user input.}
  \label{fig:figure9}
  \centering
\end{figure}


Conclusion:
    From our many testing, the number of tosses is almost double of the number of heads from the user input. The reasoning is because the experiment is tossing a fair coin, the probability of heads of fair coin is 0.5, Therefore, the tossing number will be double of the user input.

\pagebreak
\newpage

\begin{thebibliography}{9}

Wikipedia:
\\\texttt{https://en.wikipedia.org/wiki/Binomial\_distribution.html}


\end{thebibliography}

\end{document}

```