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Exercise 2. Let $S_\lambda = \sum_{t=1}^{T}(y_t - g_t)^2 + \lambda \sum_{t=2}^{T-1}[(g_{t+1} - g_t) - (g_t - g_{t-1})]^2$ and let S be the value obtained when $\{g_t\}$ is a linear series. We know that

$$S_\lambda \geq \sum_{t=1}^{T}(y_t - g_t)^2 + \lambda \max_t \{[(g_{t+1} - g_t) - (g_t - g_{t-1})]^2\}$$

Also note that we can always obtain S from S_λ by setting $\{g_t\}$ to be a linear series. Hence $\min_{\{g_t\}} S_\lambda \leq S$. Thus, we deduce that

$$\sum_{t=1}^{T}(y_t - g_t)^2 + \lambda \max_t \{[(g_{t+1} - g_t) - (g_t - g_{t-1})]^2 \leq S.$$

Hence, $\lim_{\lambda \to \infty} [(g_{t+1} - g_t) - (g_t - g_{t-1})]^2 = 0$. Thus, when $\lambda \to \infty$, $(g_{t+1} - g_t) - (g_t - g_{t-1}) = 0$ for all t. This means that for all t, $g_{t+1} - g_t = \beta$, a constant. From this we deduce that $g_t = g_0 + \beta t$ for all t.