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Linearization Exercises **Email:** hqdhftw@uchicago.edu

Exercise 1. We know that for the Brock-Mirman model, the steady state value of capital is $\bar{K} = (\alpha\beta)^{\frac{1}{1-\alpha}}$. We have

$$
\begin{aligned}
F &= \frac{\alpha \bar{K}^{\alpha-1}}{\bar{K}^{\alpha} - \bar{K}} \\
&= \frac{1}{\beta(\alpha\beta)^{\frac{\alpha}{1-\alpha}}(1-\alpha\beta)} \\
&= \frac{1}{\alpha^{\frac{\alpha}{1-\alpha}}\beta^{\frac{1}{1-\alpha}}(1-\alpha\beta)} \\
G &= -\frac{\alpha \bar{K}^{\alpha-1}(\alpha + \bar{K}^{\alpha-1})}{\bar{K}^{\alpha} - \bar{K}} \\
&= -F(\alpha + (\alpha\beta)^{-1}) \\
&= \frac{-\alpha - (\alpha\beta)^{-1}}{\alpha^{\frac{\alpha}{1-\alpha}}\beta^{\frac{1}{1-\alpha}}(1-\alpha\beta)} \\
&= \frac{-\alpha^2 - \beta^{-1}}{(\alpha\beta)^{\frac{1}{1-\alpha}}(1-\alpha\beta)} \\
H &= F(\alpha \bar{K}^{\alpha-1}) \\
&= \frac{1}{\alpha^{\frac{\alpha}{1-\alpha}}\beta^{\frac{2-\alpha}{1-\alpha}}(1-\alpha\beta)} \\
L &= F(-\bar{K}^{\alpha}) \\
&= \frac{-(\alpha\beta)^{\frac{\alpha}{1-\alpha}}}{\alpha^{\frac{\alpha}{1-\alpha}}\beta^{\frac{1}{1-\alpha}}(1-\alpha\beta)} \\
&= \frac{-1}{\beta(1-\alpha\beta)} \\
M &= H = \frac{1}{\alpha^{\frac{\alpha}{1-\alpha}}\beta^{\frac{2-\alpha}{1-\alpha}}(1-\alpha\beta)} \\
P &= \frac{-G \pm \sqrt{G^2 - 4FH}}{2F} \\
&= \frac{-F(\alpha + (\alpha\beta)^{-1}) \pm \sqrt{[-F(\alpha + (\alpha\beta)^{-1})]^2 - 4F^2\beta^{-1}}}{2F} \\
&= \frac{-\alpha - \alpha\beta^{-1} \pm [\alpha - (\alpha\beta)^{-1}]}{2} = -\alpha \text{ or } -(\alpha\beta)^{-1} \\
Q &= -\frac{LN + M}{FN + FP + G} \\
&= \frac{NF(-\bar{K}^{\alpha}) + F(\beta^{-1})}{FN + FP - F(\alpha + (\alpha\beta)^{-1})} \\
&= \frac{-N(\alpha\beta)^{\frac{\alpha}{1-\alpha}} + \beta^{-1}}{N + P - (\alpha + (\alpha\beta)^{-1})} \\
&= \frac{-\rho(\alpha\beta)^{\frac{\alpha}{1-\alpha}} + \beta^{-1}}{\rho - 2\alpha - (\alpha\beta)^{-1}} \text{ or } \frac{-\rho(\alpha\beta)^{\frac{\alpha}{1-\alpha}} + \beta^{-1}}{\rho - \alpha - 2(\alpha\beta)^{-1}}
\end{aligned}
$$

Exercises 3. The equations are

$$
\begin{aligned}
E_t\left\{F\tilde{X}_{t+1}+G\tilde{X}_t+H\tilde{X}_{t-1}+L\tilde{Z}_{t+1}+M\tilde{Z}_t\right\}&=0\\
E_t\left\{FP^2\tilde{X}_{t-1}+FPQ\tilde{Z}_t+FQ\tilde{Z}_{t+1}+GP\tilde{X}_{t-1}+GQ\tilde{Z}_t+H\tilde{X}_{t-1}+L\tilde{Z}_{t+1}+M\tilde{Z}_t\right\}&=0\\
E_t\left\{(FP^2+GP+H)\tilde{X}_{t-1}+(FPQ+GQ+M)\tilde{Z}_t+(FQ+L)\tilde{Z}_{t+1}\right\}&=0\\
E_t\left\{[(FP+G)P+H]\tilde{X}_{t-1}+(FPQ+GQ+M)\tilde{Z}_t+(FQ+L)(N\tilde{Z}_t+\varepsilon_t)\right\}&=0\\
E_t\left\{[(FP+G)P+H]\tilde{X}_{t-1}+[(FP+G)Q+M+(FQ+L)N]\tilde{Z}_t+(FQ+L)\varepsilon_t\right\}&=0\\
[(FP+G)P+H]\tilde{X}_{t-1}+[(FP+G)Q+M+(FQ+L)N]\tilde{Z}_t&=0
\end{aligned}
$$