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DSGE Exercises **Email:** hqdhftw@uchicago.edu

Exercise 1. The Euler equation of the Brock-Mirman model is given by

$$\frac{1}{e^{z_t}K_t^{\alpha}-K_{t+1}} = \beta E_t\left\{\frac{\alpha e^{z_{t+1}}K_{t+1}^{\alpha-1}}{e^{z_{t+1}}K_{t+1}^{\alpha}-K_{t+2}}\right\}. \quad (1)$$

Substitute $K_{t+1}=Ae^{z_t}K_t^{\alpha}$ and $K_{t+2}=Ae^{z_{t+1}}K_{t+1}^{\alpha}$ into (1), we obtain

$$\begin{aligned}
\frac{1}{e^{z_t}K_t^{\alpha}-K_{t+1}} &= \beta E_t\left\{\frac{\alpha e^{z_{t+1}}K_{t+1}^{\alpha-1}}{e^{z_{t+1}}K_{t+1}^{\alpha}-Ae^{z_{t+1}}K_{t+1}^{\alpha}}\right\}\\
\Leftrightarrow \frac{1}{e^{z_t}K_t^{\alpha}(1-A)} &= \beta E_t\left\{\frac{\alpha}{K_{t+1}(1-A)}\right\}\\
\Leftrightarrow K_{t+1} &= \alpha\beta e^{z_t}K_t^{\alpha}.
\end{aligned}$$

From this we deduce that $A=\alpha\beta$.

Exercise 2. The equations are

$$\begin{aligned}
c_t &= (1-\tau)[w_t\ell_t+(r_t-\delta)K_t]+K_t+T_t-K_{t+1}\\
\frac{1}{c_t} &= \beta E_t\left\{\frac{1}{c_{t+1}}[(r_{t+1}-\delta)(1-\tau)+1]\right\}\\
\frac{a}{1-\ell_t} &= \frac{w_t(1-\tau)}{c_t}\\
r_t &= \alpha e^{z_t}K_t^{1-\alpha}L_t^{1-\alpha}\\
w_t &= (1-\alpha)e^{z_t}K_t^{\alpha}L_t^{-\alpha}\\
T_t &= \tau[w_t\ell_t+(r_t-\delta)K_t]\\
z_t &= (1-\rho_z)\bar{z}+\rho_z z_{t-1}+\varepsilon_t^z.
\end{aligned}$$

Exercises 3. The equations are

$$\begin{aligned}
c_t &= (1-\tau)[w_t\ell_t+(r_t-\delta)K_t]+K_t+T_t-K_{t+1}\\
c_t^{-\gamma} &= \beta E_t\left\{c_{t+1}^{-\gamma}[(r_{t+1}-\delta)(1-\tau)+1]\right\}\\
\frac{a}{1-\ell_t} &= w_t(1-\tau)c_t^{-\gamma}\\
r_t &= \alpha e^{z_t}K_t^{1-\alpha}L_t^{1-\alpha}\\
w_t &= (1-\alpha)e^{z_t}K_t^{\alpha}L_t^{-\alpha}\\
T_t &= \tau[w_t\ell_t+(r_t-\delta)K_t]\\
z_t &= (1-\rho_z)\bar{z}+\rho_z z_{t-1}+\varepsilon_t^z.
\end{aligned}$$

Exercises 4. The equations are

$$
\begin{aligned}
c_t &= (1-\tau)[w_t \ell_t + (r_t - \delta)K_t] + K_t + T_t - K_{t+1} \\
c_t^{-\gamma} &= \beta E_t \left\{ c_{t+1}^{-\gamma} [(r_{t+1} - \delta)(1-\tau) + 1] \right\} \\
\frac{a}{(1-\ell_t)^{\xi}} &= w_t (1-\tau) c_t^{-\gamma} \\
r_t &= \alpha K_t^{\eta-1} e^{z_t} [\alpha K_t^{\eta} + (1-\alpha) L_t^{\eta}]^{\frac{1-\eta}{\eta}} \\
w_t &= (1-\alpha) L_t^{\eta-1} e^{z_t} [\alpha K_t^{\eta} + (1-\alpha) L_t^{\eta}]^{\frac{1-\eta}{\eta}} \\
T_t &= \tau [w_t \ell_t + (r_t - \delta) K_t] \\
z_t &= (1-\rho_z)\bar{z} + \rho_z z_{t-1} + \varepsilon_t^z.
\end{aligned}
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