Special Issue

Contents

Endogenously Determined Subjective Discount Rate

Before moving forward, let's have another look at the FOC (13). For compactness, we define $\delta = -\log\beta$. Essentially, FOC says

$$\log(1+\mathsf{a}) - \delta + \log\tilde{\mathbb{E}}\left[\left(\frac{V_{t+1}}{R_t}\right)^{\rho-1}\left(\frac{U_{t+1}}{U_t}\right)^{-\rho}\left(\frac{MC_{t+1}/MU_{t+1}}{MC_t/MU_t}\right)\Big|\mathfrak{F}_t\right] = 0 \qquad (14)$$

where $\tilde{\mathbb{E}}[\cdot|\mathfrak{F}_t]$ is the conditional expectation operator under the **altered probability measure**, i.e. taking into account the $\left(\frac{V_{t+1}}{R_t}\right)^{1-\gamma}$ term. Here a is an exogenous one period risk-free capital growth rate, thus the subjective discount rate δ is not exogenously given, but rather endogenously pinned down by (14). Our small noise expansion method gives the law of motion of model variables $X_{t+1}(\mathsf{q}) = \psi[X_t(\mathsf{q}), \mathsf{q}W_{t+1}, \mathsf{q}]$, and if we plug it into (14), there will surely be a bunch of terms that involve W_{t+1}. Since the structure of W_{t+1} under the altered probability measure differs as the order of expansion differs, it's clear that β pinned down by (14) in order 0, 1, 2 differs. Let's write them as δ^0, δ^1 and δ^2.

In the deterministic steady state (order 0), δ^0 can be easily solved from (14) because change of measure and expectation operator don't matter (no W_{t+1} terms).

In order 1, an **additional constant term** needs to be added to (14), which equals $\delta^0 - \delta^1$. Without this constant term, the LHS of (14) is obviously non-zero, and the approximated law of motion coming from expansion that can make the wrong FOC "hold" will be wrong (not the equilibria that we want).

Similarly, in order 2, an **additional constant term** needs to be added to (14), which equals $\delta^1 - \delta^2$.

Technical details: When introducing the additional free constant term, we also impose an additional restriction, to set the constant component that applies to $\frac{K_t}{Y_t}$ to zero. We make this latter adjustment because we want to set $\frac{K_0}{Y_0}$ as an initial condition and don't want to adjust it later when we include higher order terms in an expansion.

As in LPH's *PI Model Notes*, we provide a simplified example where there's no habit/durable goods. It can be viewed as a special case of the preference described in section 1.1, by setting $\alpha = 0$, $\rho = 1$. In this special case, LPH showed that

In order 0 approximation,

$$\delta^0 = \mathsf{a} - \mathsf{g}$$

In order 1 approximation,

$$\delta^1 = \mathsf{a} - \mathsf{g} - (1-\gamma)\sigma_c^1 \cdot \sigma_c^1$$

The last term is an adjustment for precaution, which is exactly the "additional constant term" that should be included when we move from order 0 to order 1 expansion in this special case.

```python
# discount rate adjustment graph
from demonstration import plot_figure_2
plot_figure_2()
```