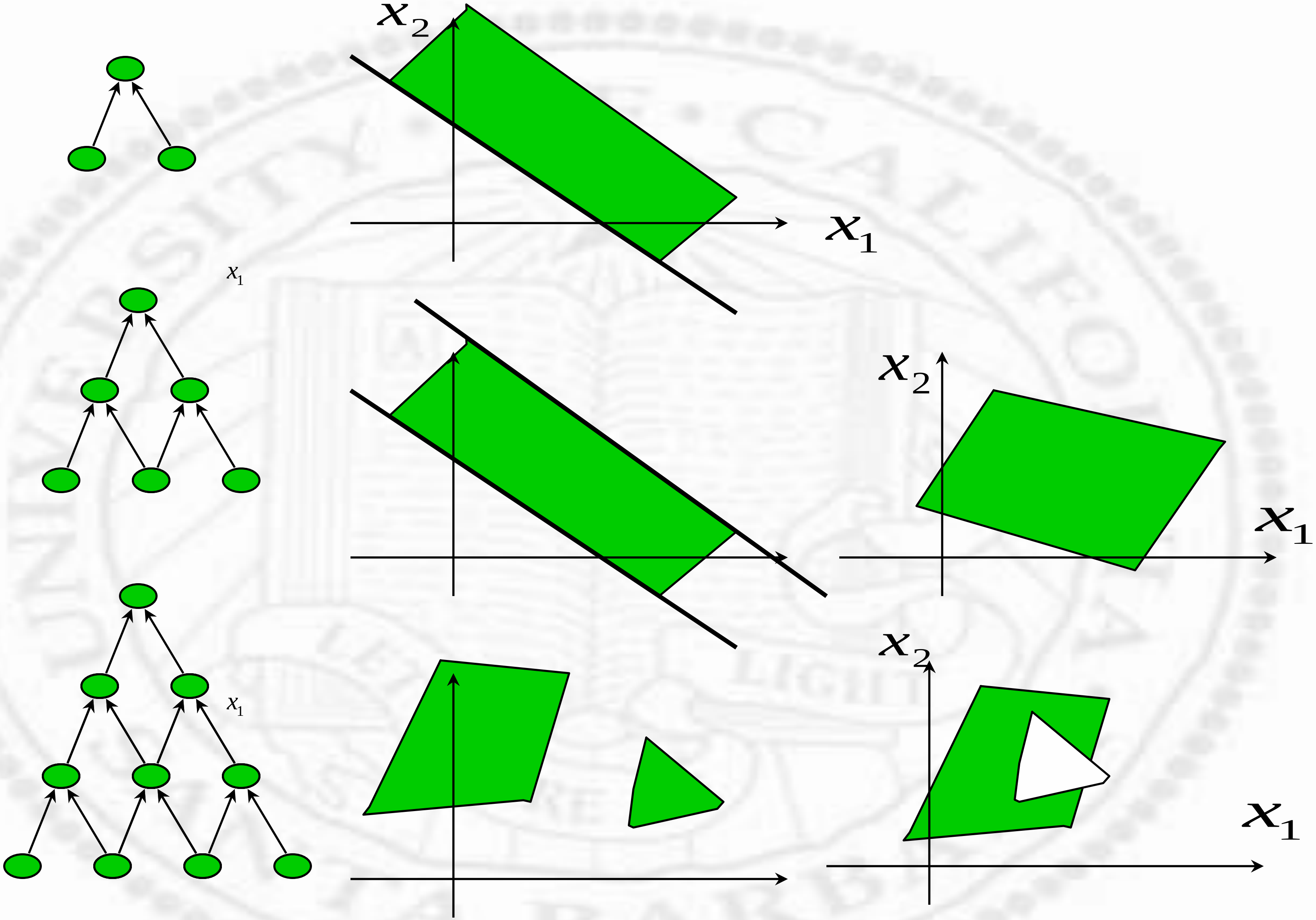
Multi-Layer Perceptrons

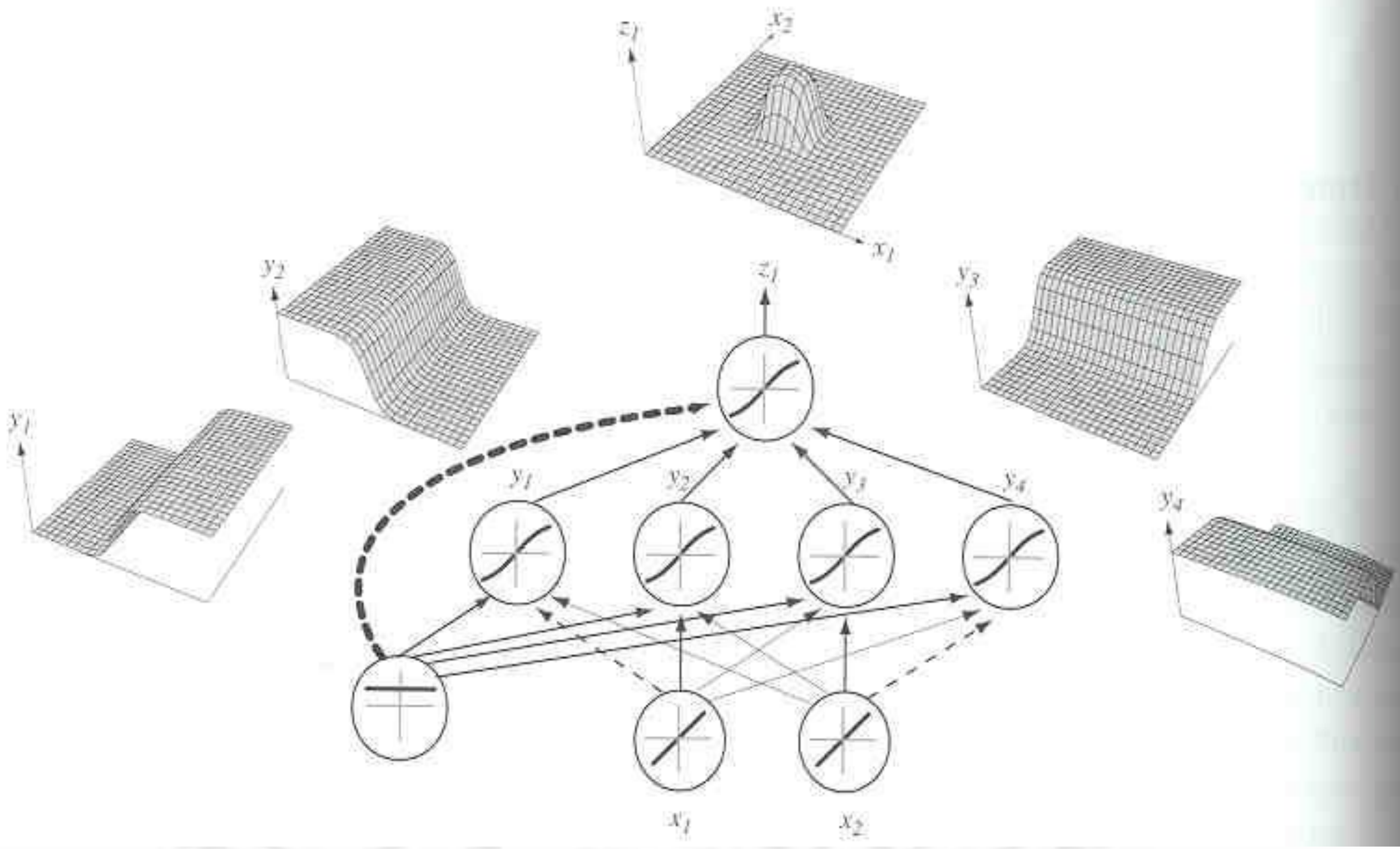
Multi-Layer Perceptrons

- With “hidden” layers
- One hidden layer - any Boolean function or convex decision regions
- Two hidden layers - arbitrary decision regions

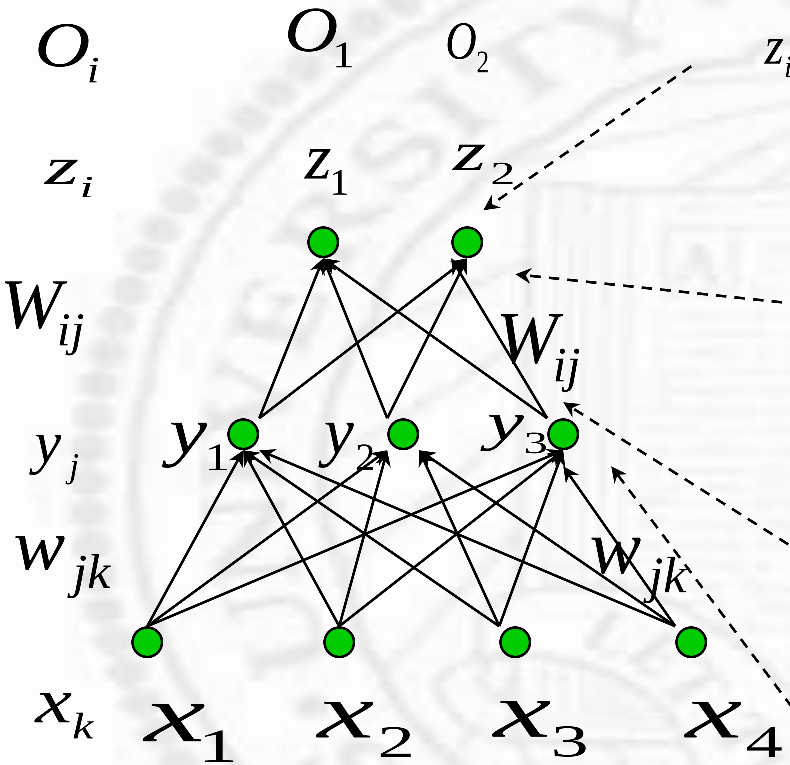
Decision boundaries



Decision Boundaries



Backpropagation Learning rule



$$z_i^u = g(NET_i^u) = g(\sum_j W_{ij} y_j^u) = g(\sum_j W_{ij} g(\sum_k w_{jk} x_k^u))$$

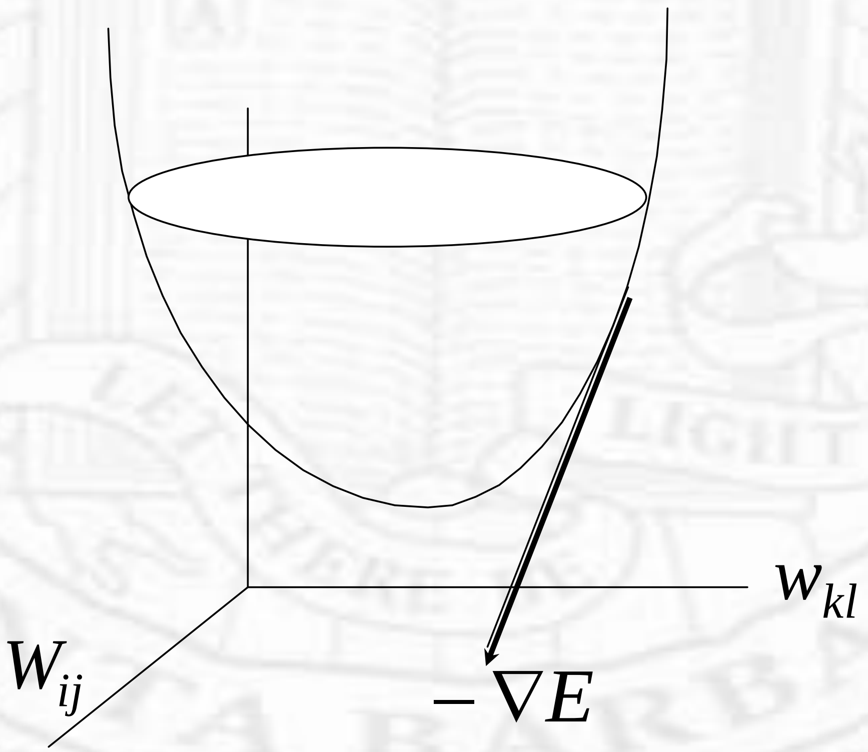
$$NET_i^u = \sum_j W_{ij} y_j^u = \sum_j W_{ij} g(\sum_k w_{jk} x_k^u)$$

$$y_j^u = g(net_j^u) = g(\sum_k w_{jk} x_k^u)$$

$$net_j^u = \sum_k w_{jk} x_k^u$$

Cost function

$$E(\mathbf{W},\mathbf{w}) = \frac{1}{2}\sum_{u,i}(O_i^{\ u} - z_i^{\ u})^2 = \frac{1}{2}\sum_{u,i}(O_i^{\ u} - g(\sum_j W_{ij}\, g(\sum_k w_{jk} x_k^{\ u}))^2$$



Change w.r.t. W_{ij}

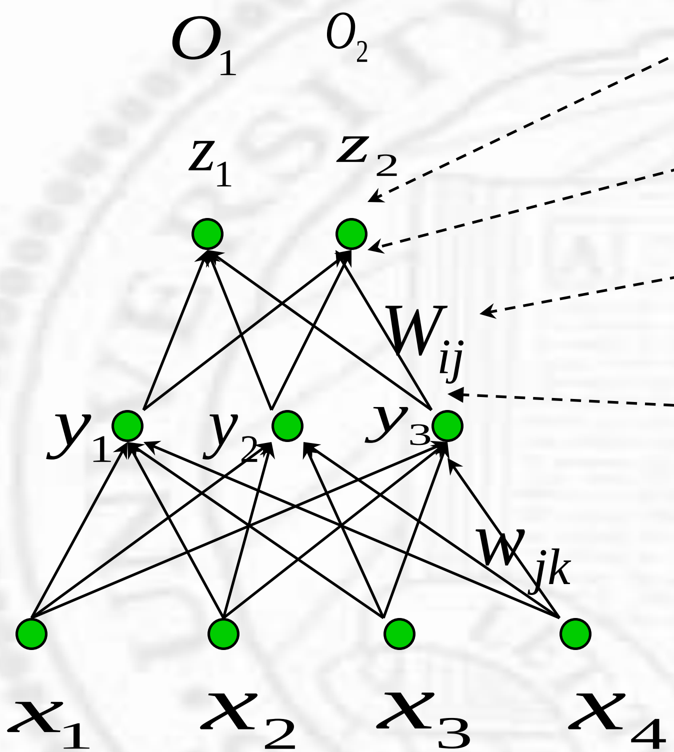
$$\Delta W_{ij} = -\eta \frac{\partial E}{\partial W_{ij}} = -\eta \frac{\partial (O_i^u - g(\sum_j W_{ij} y_j^u))^2}{\partial W_{ij}}$$

$$= -\eta \frac{\partial (O_i^u - z_i^u)^2}{\partial (O_i^u - z_i^u)} \frac{\partial (O_i^u - g(NET_i^u))}{\partial NET_i^u} \frac{\sum_j W_{ij} y_j^u}{\partial W_{ij}}$$

$$= \eta \sum_u \underline{(O_i^u - z_i^u) g'(NET_i^u)} y_j^u$$

$$= \eta \sum_u \underline{\delta_i^u} y_j^u \qquad \delta_i^u = (O_i^u - z_i^u) g'(NET_i^u)$$

Interpretation



$$(O_i^u - z_i^u)$$

$$g'(NET_i^u)$$

$$\delta_i^u = (O_i^u - z_i^u) g'(NET_i^u)$$

$$\eta \sum_u \delta_i^u y_j^u$$

$$y_j^u$$

- A form of Hebb’s rule
  - If error, reward mutual excitation (large feedback output and large input)

Change w.r.t. w_{ij}

$$\Delta w_{jk} = -\eta \frac{\partial E}{\partial w_{jk}} = -\eta \frac{\partial \sum_{u,i} (O_i^u - g(\sum_j W_{ij} g(\sum_k w_{jk} x_k^u))^2}{\partial w_{jk}}$$

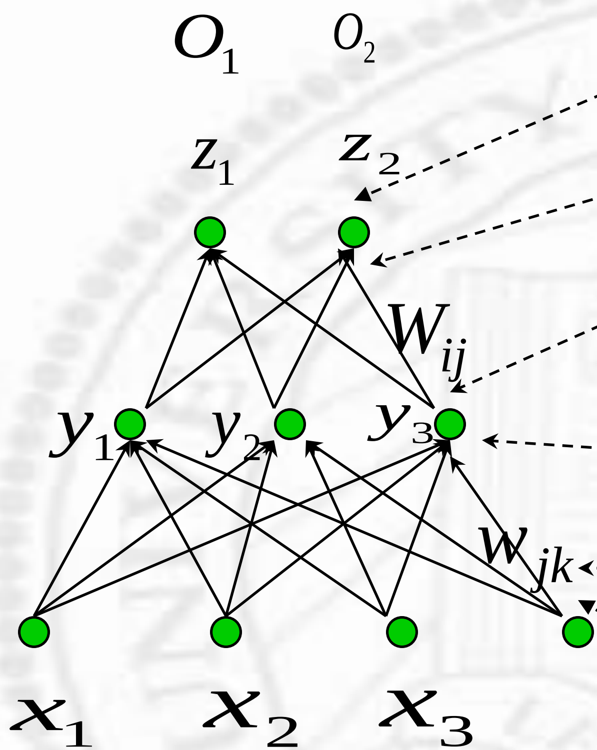
$$= -\eta \frac{\partial E}{\partial y_j^u} \frac{\partial y_j^u}{\partial w_{jk}}$$

$$= \eta \sum_{u,i} \underline{(O_i^u - z_i^u) g'(NET_i^u)} W_{ij} g'(net_j^u) x_k^u$$

$$= \underline{\eta \sum_{u,i} \underline{\delta_i^u} W_{ij}^u g'(net_j^u)} x_k^u$$

$$= \eta \sum_u \underline{\delta_j^u} x_k^u \qquad \delta_j^u = g'(net_j^u) \sum_i \delta_i^u W_{ij}$$

Interpretation (cont.)



$(O_i^u - z_i^u)$ Error

$\delta_i^u = (O_i^u - z_i^u) g'(NET_i^u)$ backpropagated error

$\sum_i \delta_i^u W_{ij}$ Weighted backpropagated error

$\delta_j^u = g'(net_j^u) \sum_i \delta_i^u W_{ij}$

$g'(net_j^u)$ Weighted backpropagated error backpropagated again

$\eta \sum_u \delta_j^u x_k^u$ Weighted backpropagated error backpropagated again and weighted again

x_k^u

- A two-level Hebb's rule
  - If error, reward mutual excitation (large feedback output and large input)

Interpretation (cont.)

$$\Delta w_{pq} = \eta \sum_{patterns} \delta_{output} \times V_{input}$$

- Hebb's learning
- Error at the output end
- Activation at the input end
- Learning rate

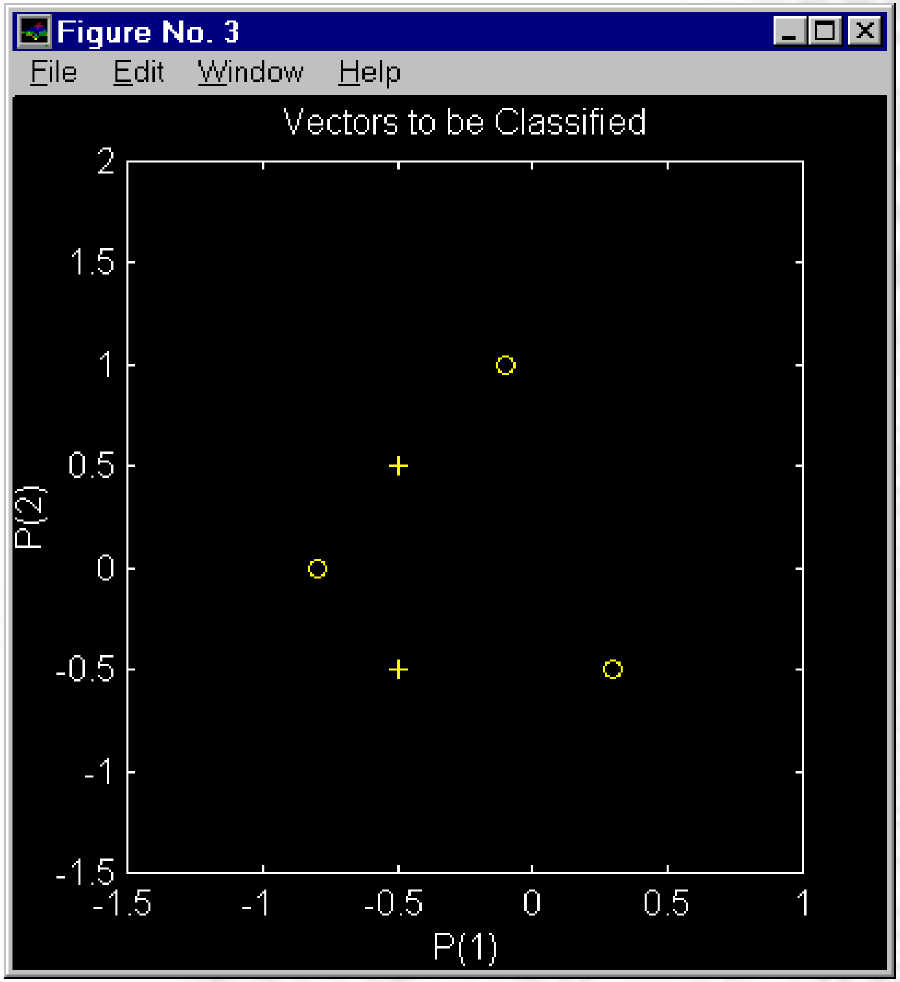


Figure No. 3
File Edit Window Help
Vectors to be Classified
2
1.5
1
0.5
0
-0.5
-1
-1.5
-1.5
-1
-0.5
0
0.5
1
P(1)
P(2)

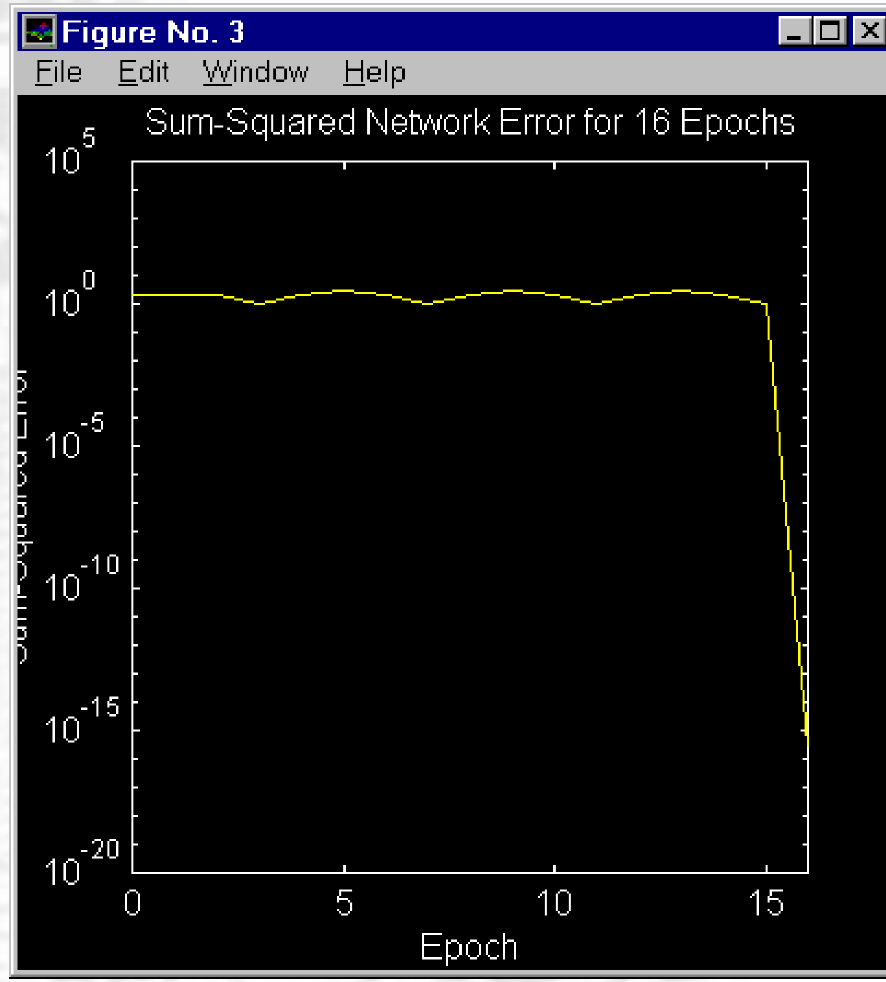
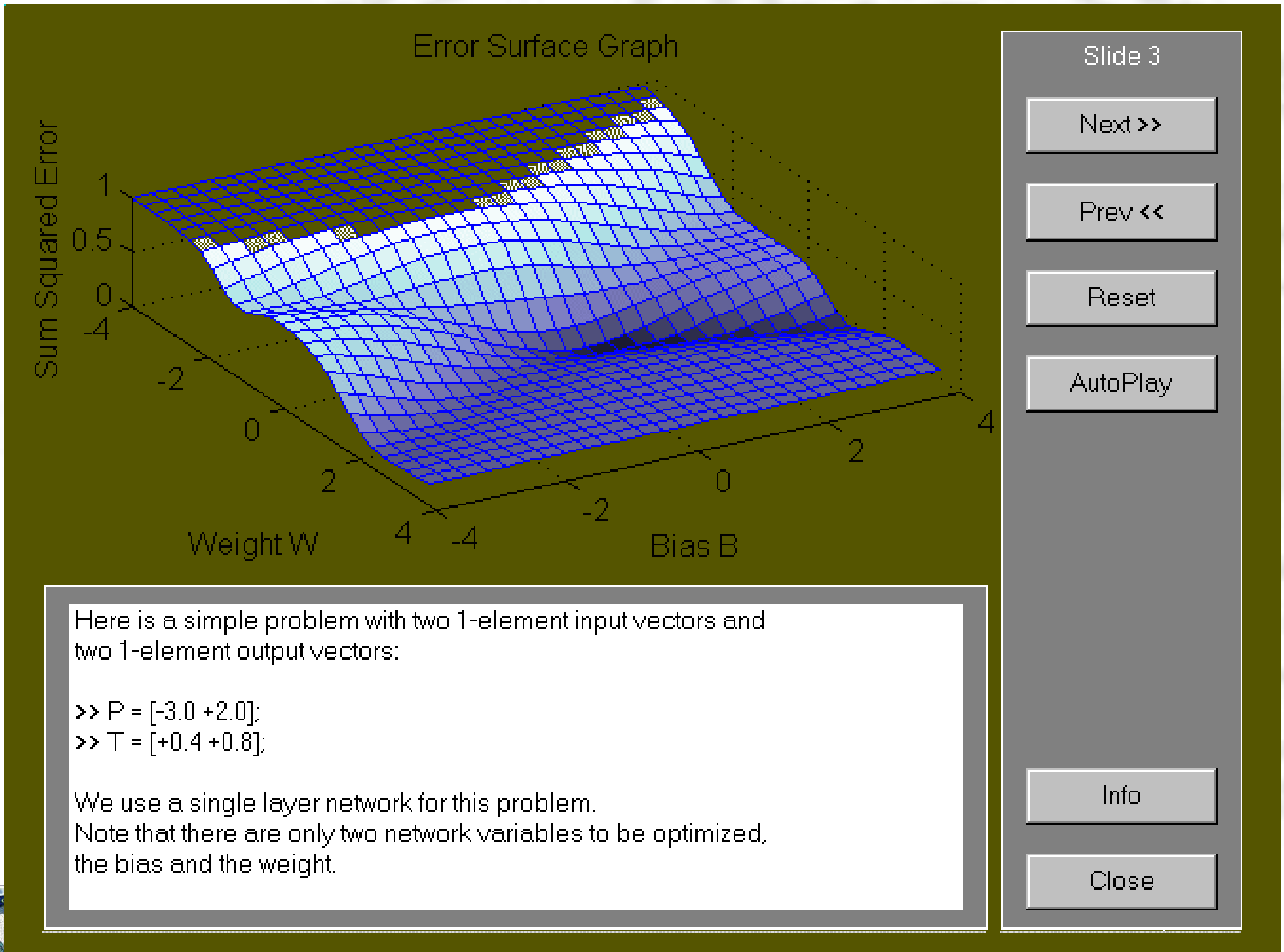
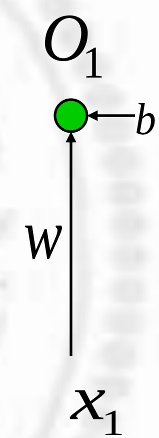
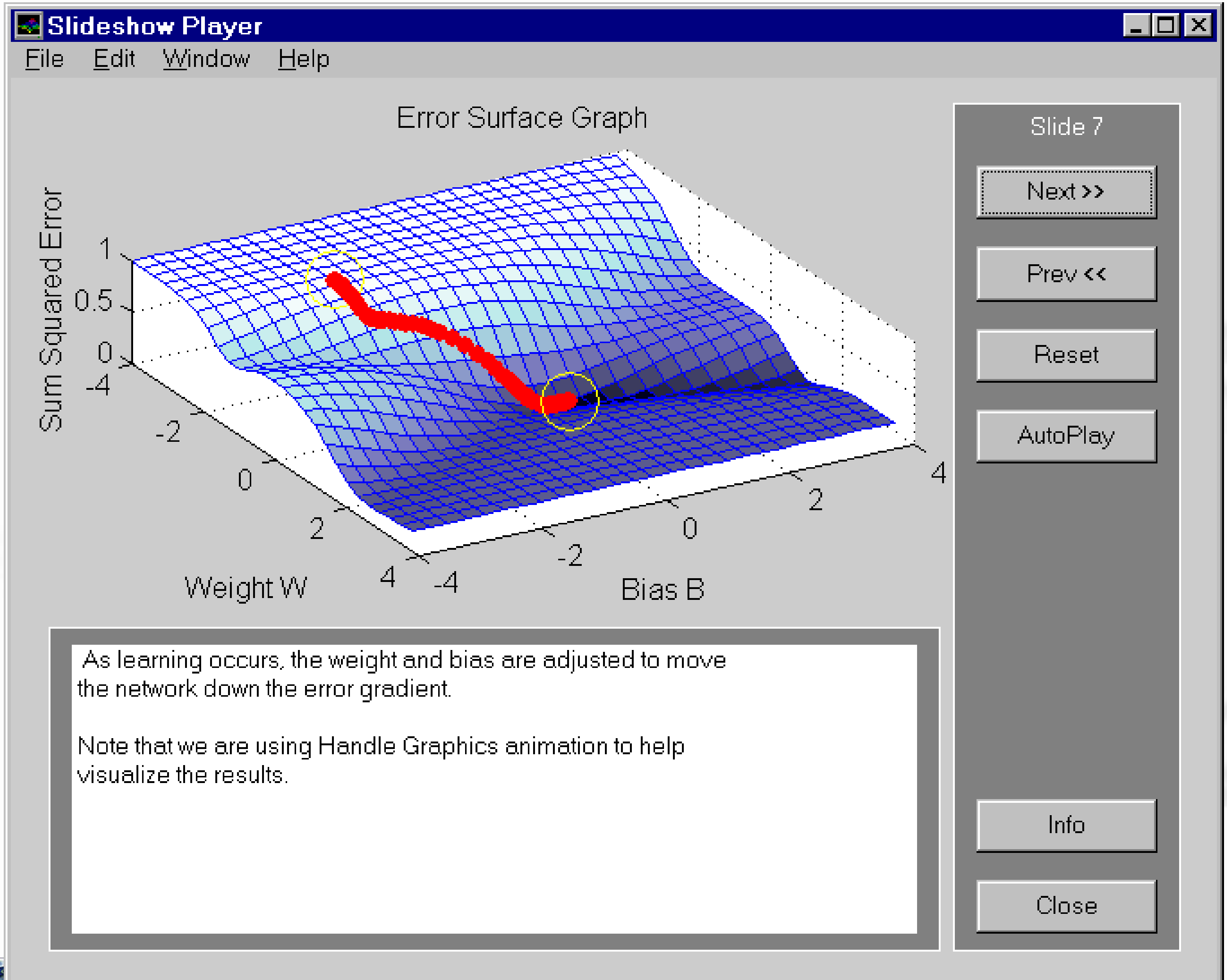


Figure No. 3
File Edit Window Help
Sum-Squared Network Error for 16 Epochs
10^{5}
10^{0}
10^{-5}
10^{-10}
10^{-15}
10^{-20}
0
5
10
15
Epoch

Graphics Illustration of Backpropagation






Slideshow Player
File Edit Window Help
Error Surface Graph
Sum Squared Error
1
0.5
0
-4
-2
0
2
4
Weight W
-4
-2
0
2
4
Bias B
Slide 7
Next >>
Prev <<
Reset
AutoPlay
Info
Close
As learning occurs, the weight and bias are adjusted to move the network down the error gradient.
Note that we are using Handle Graphics animation to help visualize the results.

Caveats on Backpropagation

- Slow
- Network Paralysis
  - if weights become large
  - operates at limits of squash (transfer) functions
  - derivatives of squash function (feedback) small
- Step size
  - too large may lead to saturation
  - too small cause slow convergence

Caveats on Backpropagation

- Local minima
  - many different initial guesses
  - momentum
  - varying step size (large initially, getting small as training goes on)
  - simulated annealing
- Temporal instability
  - learn B and forgot about A

Other than BackPropagation

- In reality, gradient descent is slow and highly dependent on initial guess
- More sophisticated numerical methods exist Trust region methods, combination of
  - Gradient descent
  - Newton's methods

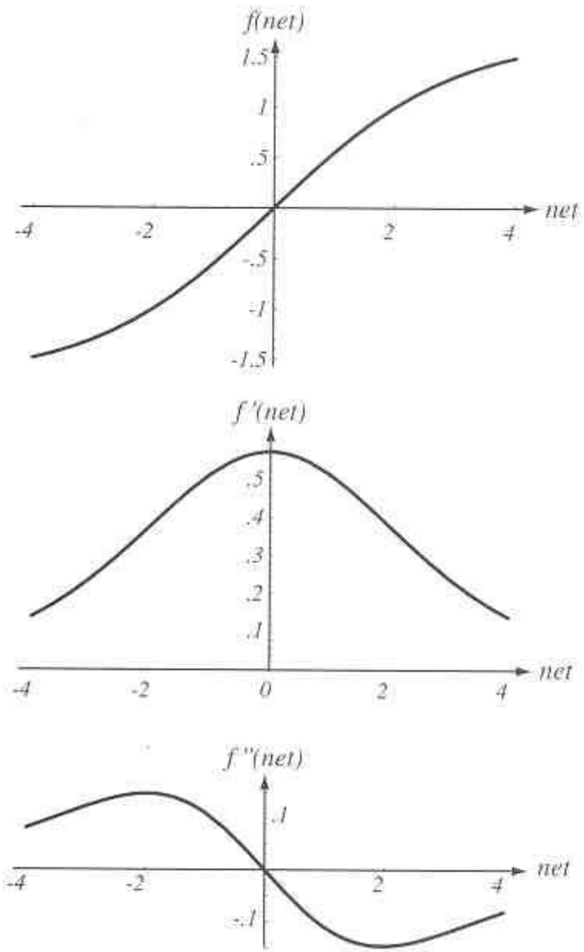
Other Practical Issues

- Which transfer function (g)?
  - g must be nonlinear

$$net_j^u = \sum_k w_{jk} x_k^u \Rightarrow \mathbf{H} = \mathbf{WX}$$

  - g should be continuous and smooth
    - So g and g' are defined
  - g should saturate
    - Limit training time
    - Biologically (electronically) plausible

Sigmoid Function



$$g = a\tanh(b \cdot net) = a\frac{1-e^{-b \cdot net}}{1+e^{-b \cdot net}} = \frac{2a}{1+e^{-b \cdot net}} - a$$

$$a = 1.716$$
$$b = 2/3$$

Input Scaling

- Inputs (weight, size, etc.) have different units and dynamic range and may be learned at different rates
- Small input ranges make small contribution to the error and are often ignored
- Normalization to same range and same variance (similar to Whitening transform)

Output Scaling

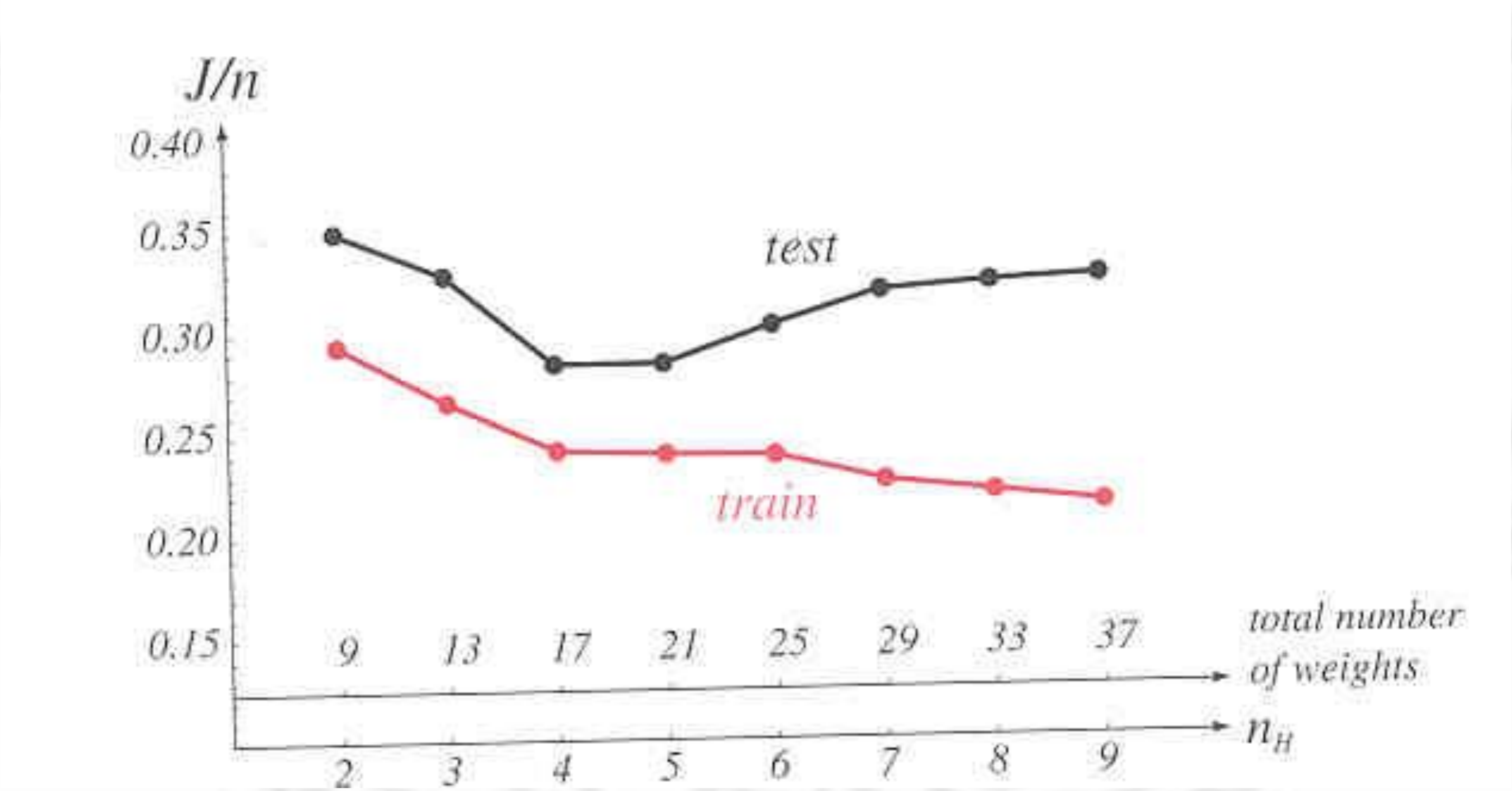
- Rule of thumb: Avoid operating neurons in the saturation (tail) regions
  - Tendency for weight saturation
  - g' is small, learning is very slow
  - For sigmoid function as shown before, use range (-1, 1) instead of (-1.716, 1.716)

Weight initialization

- Don’t set the initial weights to zero, the network is not going to learn at all
- Don’t set the initial weights too high, that leads to paralysis and slow learning
- Both positive and negative random weights to insure uniform learning

Number of Hidden Layers

- Too few – poor fitting
- Too many – over fitting, poor generalization



Numerical Stability – step size

❖ Adaptive

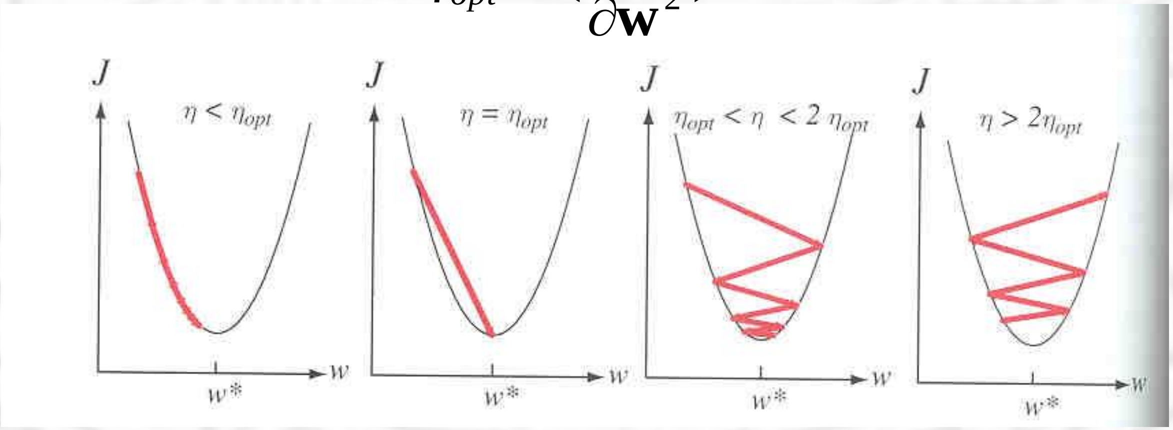
$$J(\mathbf{w}+\Delta\mathbf{w}) = J(\mathbf{w}) + \frac{\partial J}{\partial \mathbf{w}}\Delta\mathbf{w} + \frac{1}{2}\frac{\partial^2 J}{\partial \mathbf{w}^2}\Delta\mathbf{w}^2$$

$$\frac{J(\mathbf{w}+\Delta\mathbf{w}) - J(\mathbf{w})}{\Delta\mathbf{w}} \approx 0 = \frac{\partial J}{\partial \mathbf{w}} + \frac{\partial^2 J}{\partial \mathbf{w}^2}\Delta\mathbf{w}$$

$$\eta_{opt} < \eta < 2\eta_{opt}$$

$$\frac{\partial J}{\partial \mathbf{w}} = \frac{\partial^2 J}{\partial \mathbf{w}^2}\Delta\mathbf{w}$$

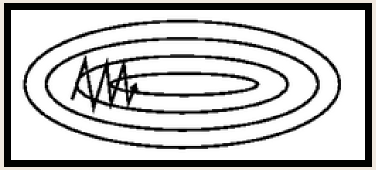
$$\eta_{opt} = \left(\frac{\partial^2 J}{\partial \mathbf{w}^2}\right)^{-1}$$



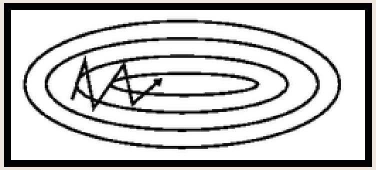
Numerical Stability - momentum

$$\mathbf{w}^{new} = \mathbf{w}^{curr} + \underline{(1-\alpha)\Delta\mathbf{w}_{bp}^{curr}} + \underline{\alpha\Delta\mathbf{w}^{prev}}$$

$\alpha \approx 0.9$

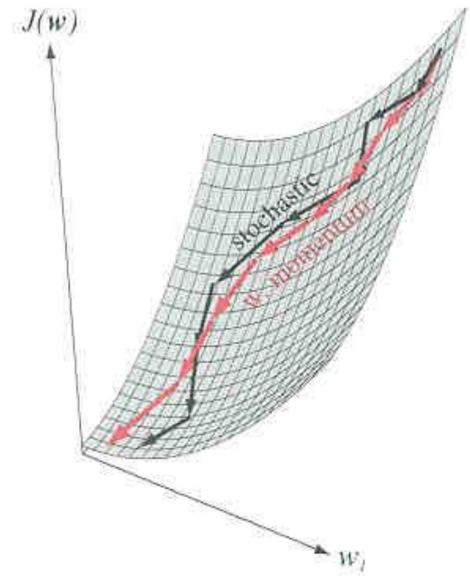


(Fig. 2a)



(Fig. 2b)

Without w. momentum



- Red: as computed from current back propagation
- Blue: as computed from previous back propagation

Numerical Stability

- Weight decay
  - To ensure no single large weight dominates the training process

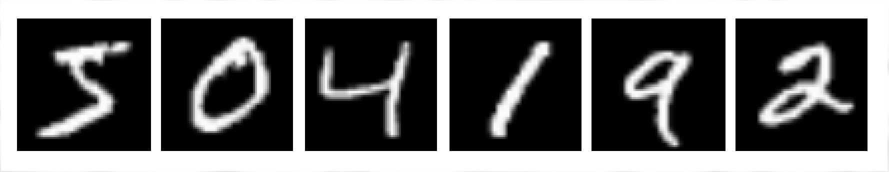
$$\mathbf{w}^{new} = \mathbf{w}^{old}(1-\xi)$$

Essentially

- Yes, multi-layer perceptrons can distinguish classes even when they are not linearly separable?
- Questions: How many layers? How many neurons per layers?
- Can # layers/# neurons per layer be *learned* too? (in addition to weights)

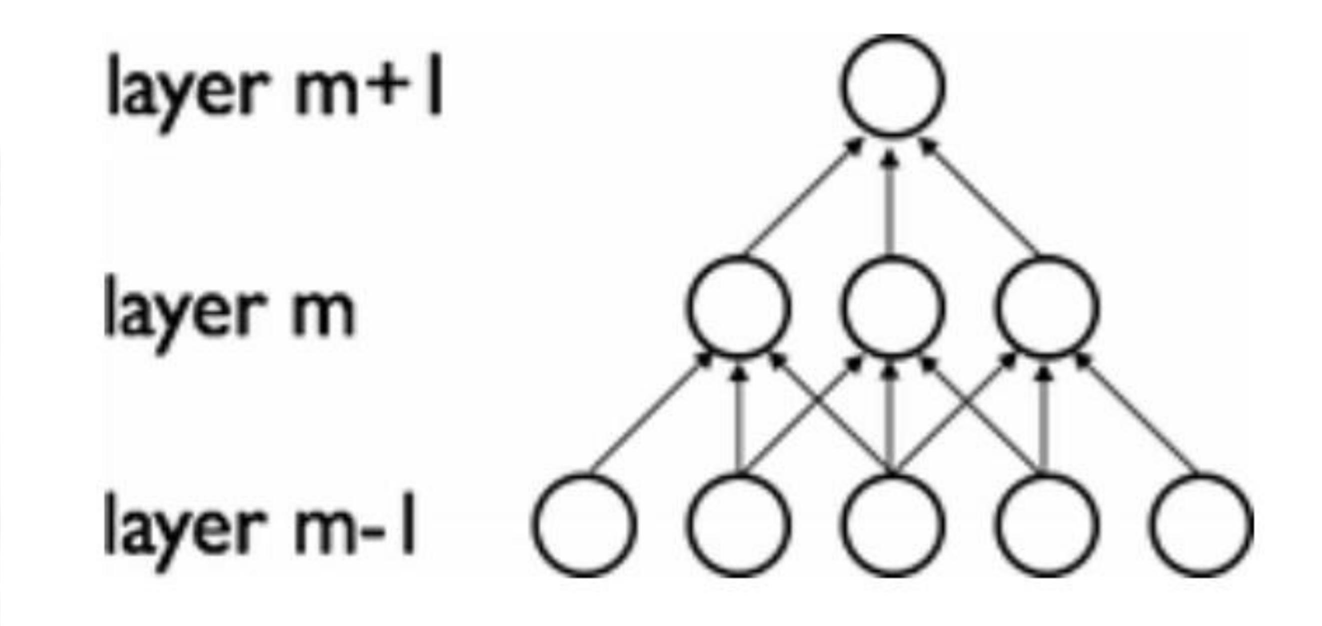
Easier Said than Done

- Blind learning with large number of parameters is numerically impossible
- Major recent advance
  - Reduced number of parameters
  - Layered learning



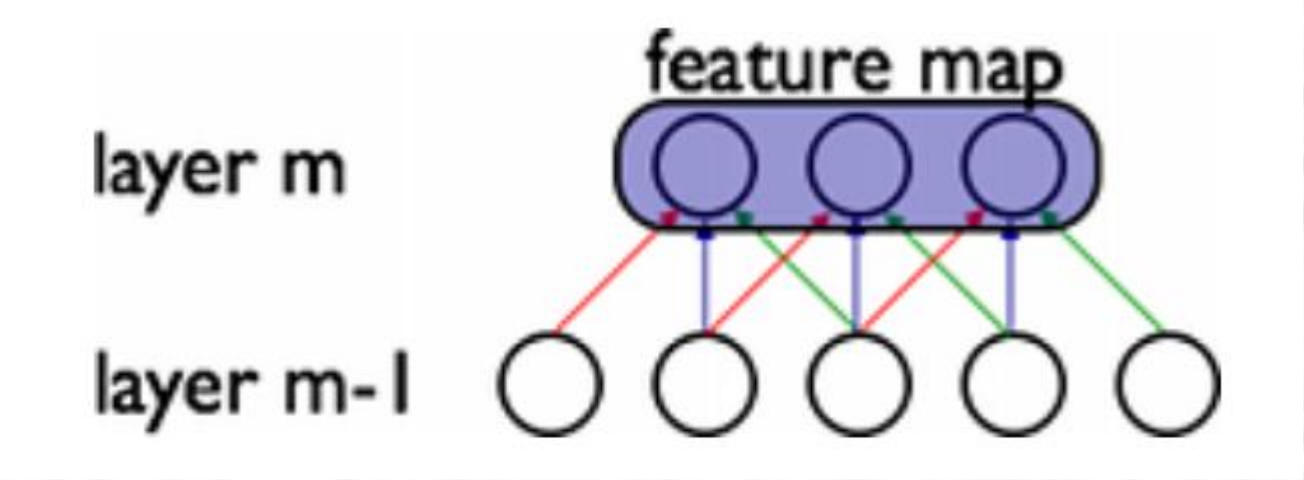
Emulation of Human Vision

- Sparsity of connection



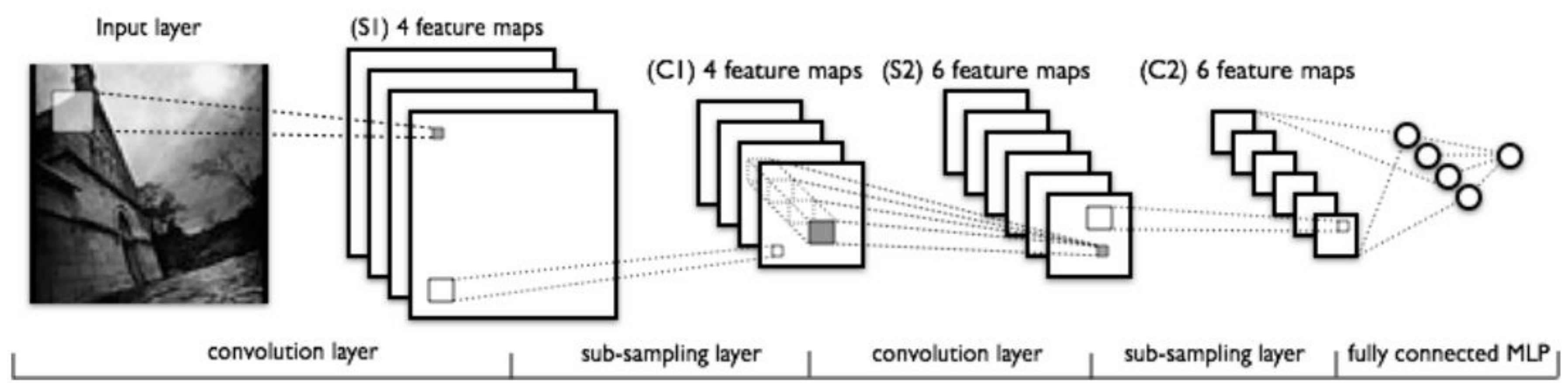
Emulation of Human Vision

- Shared weight



Layered Learning

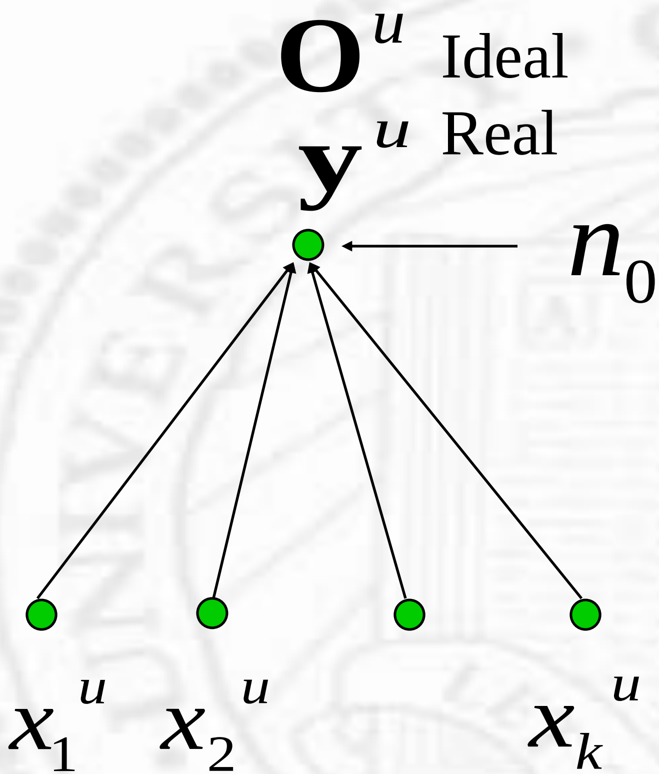
- A hierarchical "feature descriptor"
- Learning automatically from input data
- Layer-by-layer learning with auto encoder
- Partition:
  - CNN: feature detection
  - Fully-connected network: recognition


Input layer
(S1) 4 feature maps
(C1) 4 feature maps
(S2) 6 feature maps
(C2) 6 feature maps
convolution layer
sub-sampling layer
convolution layer
sub-sampling layer
fully connected MLP

Adaptive Networks

- Network size/layer is not fixed initially
- Layer/size are added when necessary (or when a large number of epochs progress without finding suitable weights)
- Assumptions:
  - two classes (1,0)
  - may not be linearly separable (e.g., multiple concave regions)

Initially one neuron

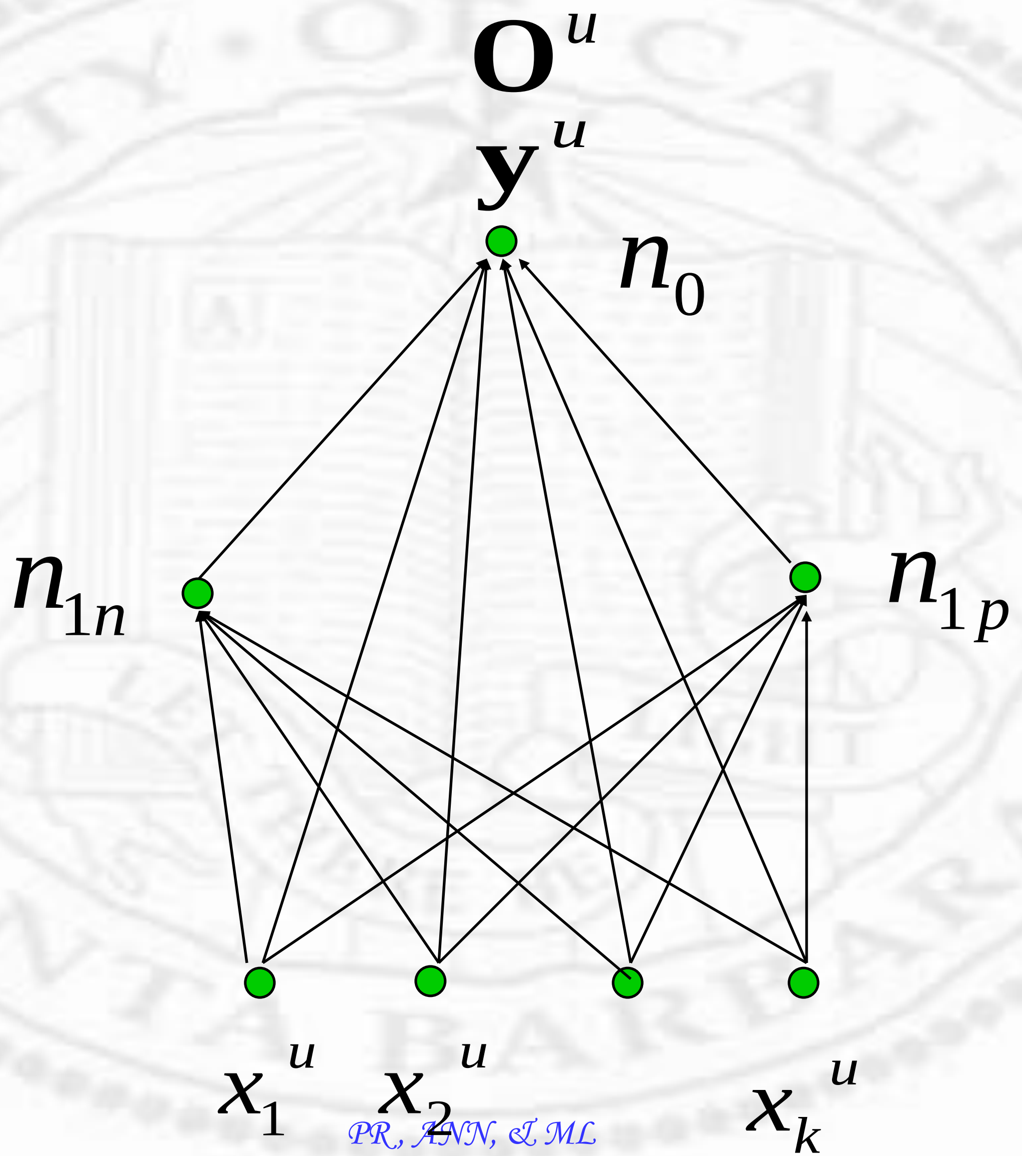


$$\text{wrongly on} \quad O^u = 0 \quad y^u = 1$$
$$\text{wrongly off} \quad O^u = 1 \quad y^u = 0$$

Refinement with more neurons

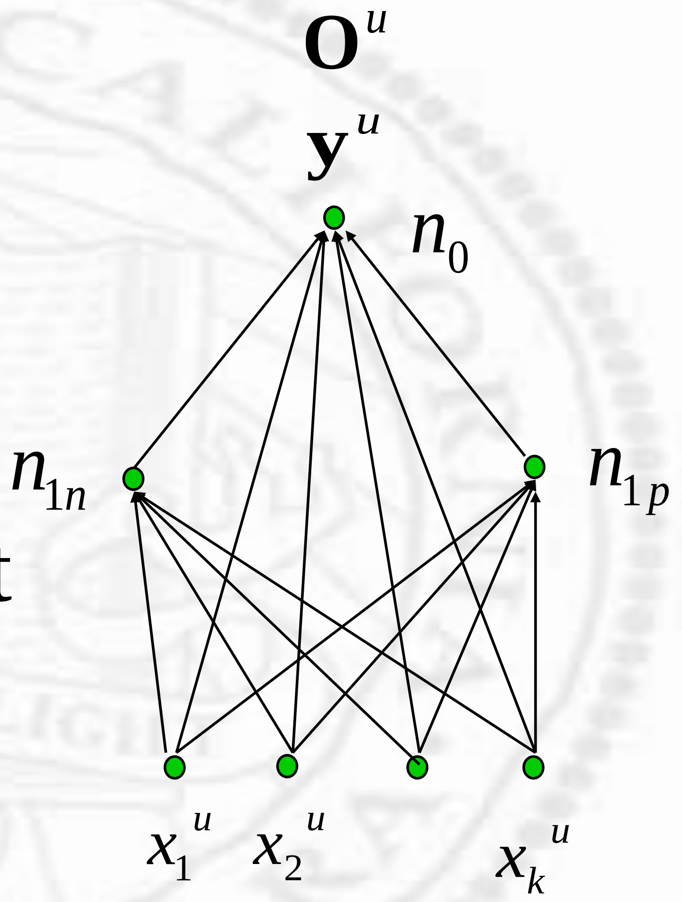
- Train through a number of epochs
- if no wrongly on/off cases, the two classes are linearly separable, stop
- if there are wrongly on/off cases, the two classes are not linearly separable, then
  - remember the best weights (the weights that cause the less number of misclassification)
  - introduce more units (instead of throwing away everything and restarting from scratch with a larger network)

Increase Network Complexity



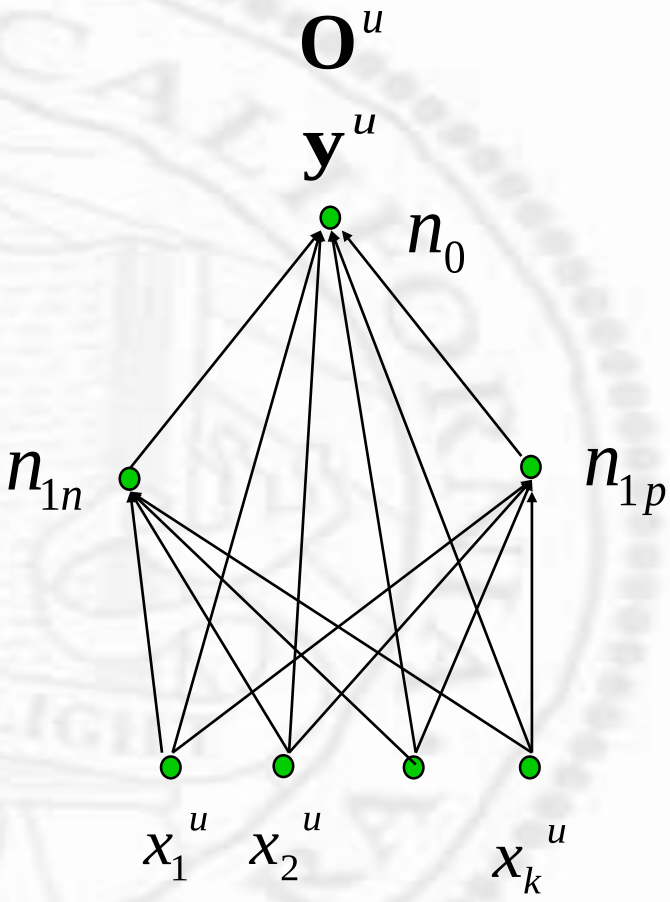
N_{1n}: correct wrongly-on error fire negative feedback only

O	y	*action*
0	0	don't care
1	1	*off*
0	1	large negative output
1	0	*off*

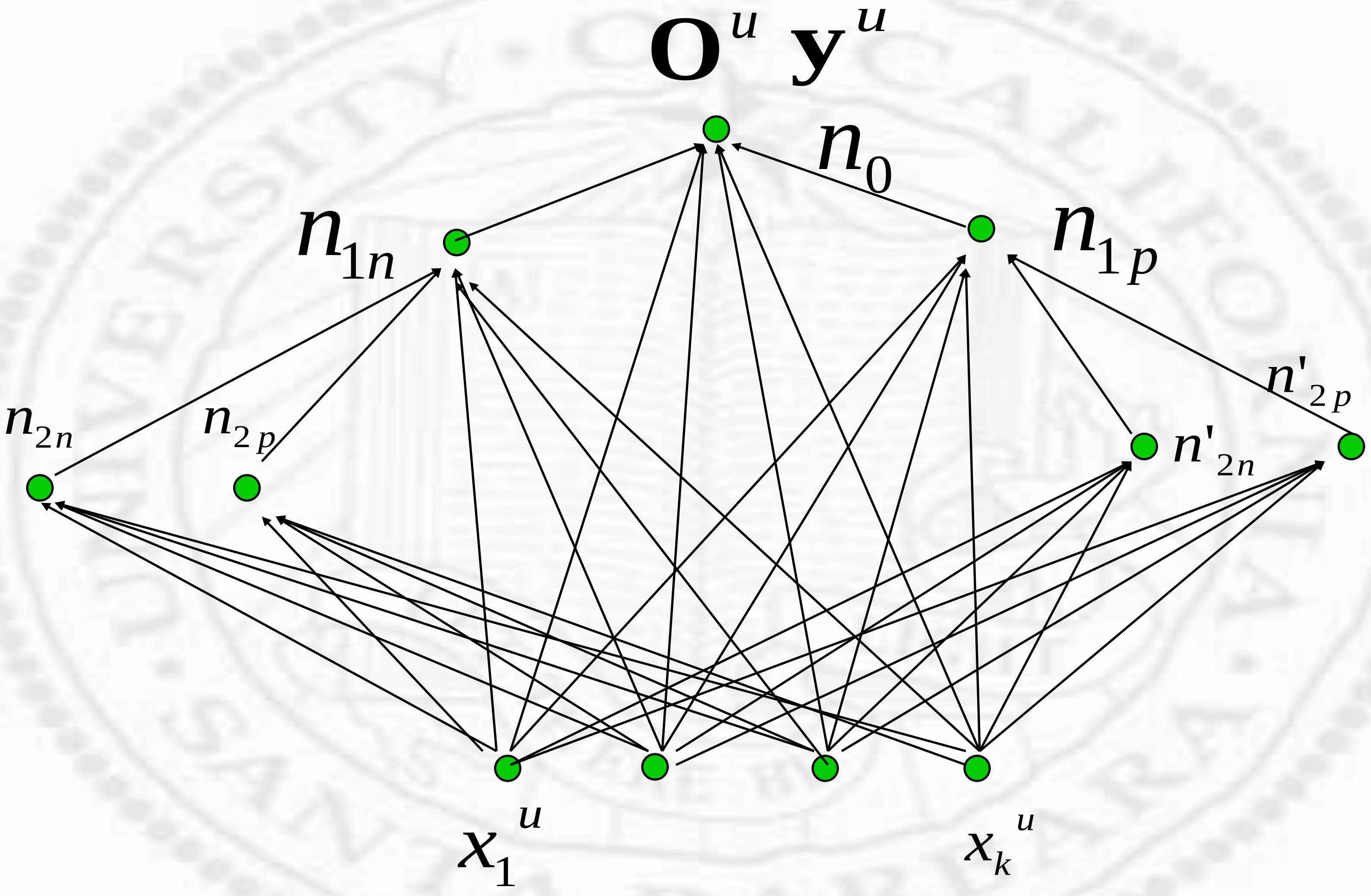


N_{1p}: correct wrongly-off error fire positive feedback only

O	y	*action*
0	0	off
1	1	don't care
0	1	*off*
1	0	large positive output



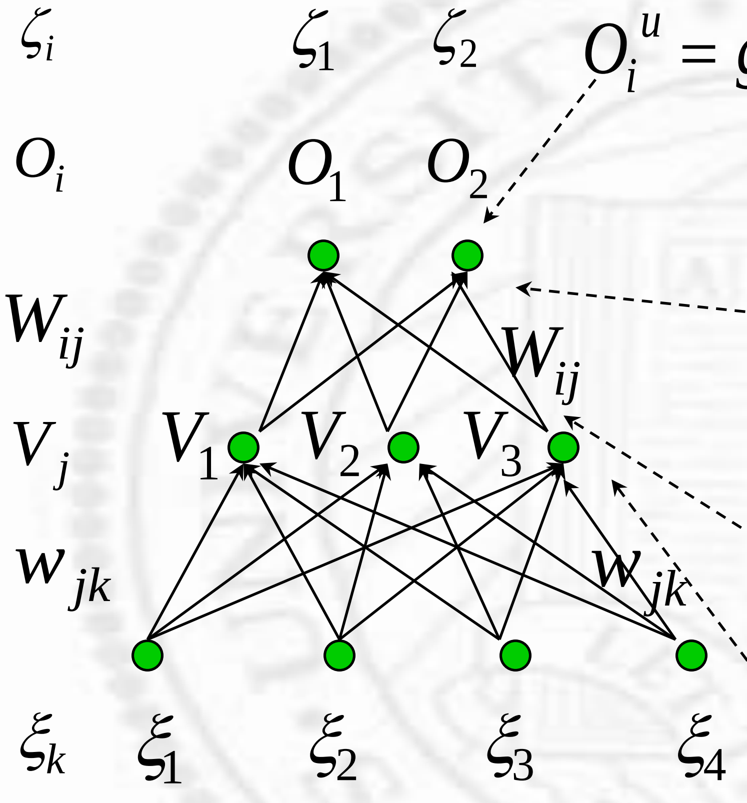
Further Refinement



General Learning Rule

- N_{xn}
  - Fire negative impulse
  - Correct wrongly on cases
  - Turn off if O=1 (no matter what y is)
  - Don't care if O=0 and y=0
- N_{xp}
  - Fire positive impulse
  - Correct wrongly off cases
  - Turn off if O=0 (no matter what y is)
  - Don't care if O=1 and y=1

Backpropagation Learning rule



$$O_i^u = g(h_i^u) = g(\sum_j W_{ij} V_j^u) = g(\sum_j W_{ij} g(\sum_k w_{jk} \xi_k^u))$$

$$h_i^u = \sum_j W_{ij} V_j^u = \sum_j W_{ij} g(\sum_k w_{jk} \xi_k^u)$$

$$V_j^u = g(h_j^u) = g(\sum_k w_{jk} \xi_k^u)$$

$$h_j^u = \sum_k w_{jk} \xi_k^u$$

Change w.r.t. w_ij

$$\Delta W_{ij} = -\eta \frac{\partial E}{\partial W_{ij}} = -\eta \frac{\partial (\zeta_i^u - g(\sum_j W_{ij} V_j^u))^2}{\partial W_{ij}}$$

$$= \eta \sum_u (\zeta_i^u - O_i^u) g'(h_i^u) V_j^u$$

$$= \eta \sum_u \delta_i^u V_j^u \qquad \delta_i^u = (\zeta_i^u - O_i^u) g'(h_i^u)$$

Change w.r.t. w_ij

$$\Delta w_{jk} = -\eta \frac{\partial E}{\partial w_{jk}} = -\eta \frac{\partial \sum_{u,i} (\zeta_i^u - g(\sum_j W_{ij} g(\sum_k w_{jk} \xi_k^u))^2}{\partial w_{jk}}$$

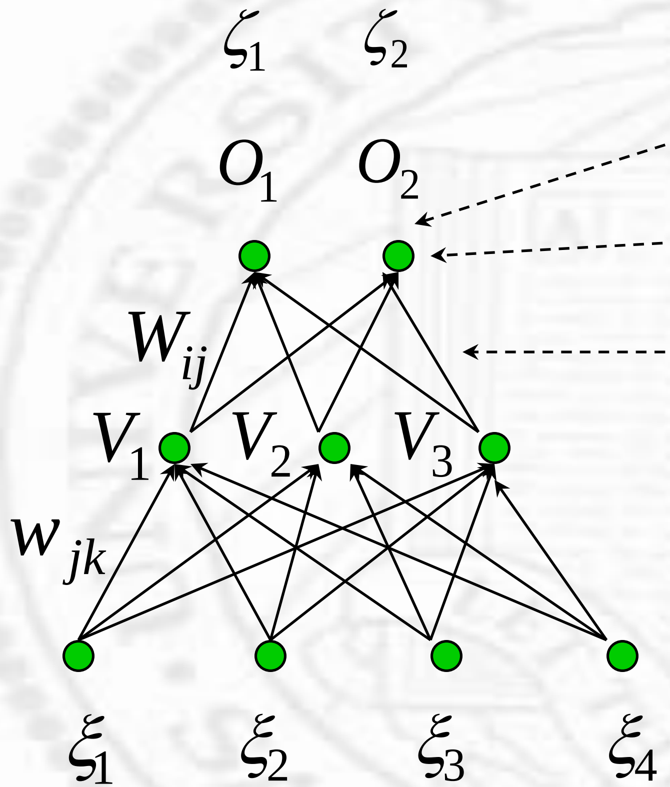
$$= -\eta \frac{\partial E}{\partial V_j^u} \frac{\partial V_j^u}{\partial w_{jk}}$$

$$= \eta \sum_{u,i} (\zeta_i^u - O_i^u) g'(h_i^u) W_{ij} g'(h_j^u) \xi_k^u$$

$$= \eta \sum_{u,i} \delta_i^u W_{ij}^u g'(h_j^u) \xi_k^u$$

$$= \eta \sum_u \delta_j^u \xi_k^u \qquad \delta_j^u = g'(h_j^u) \sum_i \delta_i^u W_{ij}$$

Interpretation



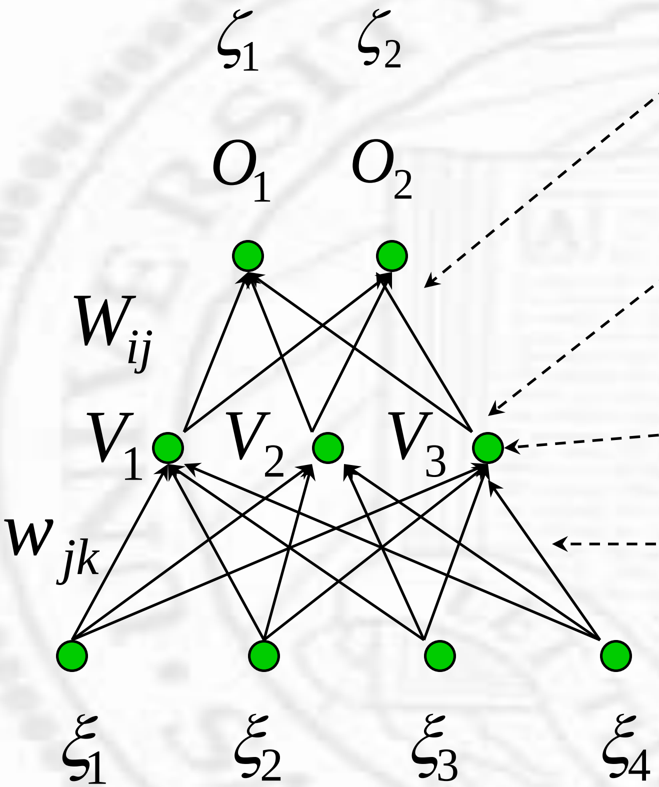
$(\zeta_i^u - O_i^u)$

$g'(h_i^u)$

$$\delta_i^u = (\zeta_i^u - O_i^u) g'(h_i^u)$$

$$\eta \sum_u \delta_i^u V_j^u$$

Interpretation (cont.)



$$\delta_i^u = (\zeta_i^u - O_i^u) g'(h_i^u)$$

$$\sum_i \delta_i^u W_{ij}$$

$$\delta_j^u = g'(h_j^u) \sum_i \delta_i^u W_{ij}$$

$$g'(h_j^u)$$

$$\eta \sum_u \delta_j^u \xi_k^u$$