Edge Detection

General 2D Image Operations

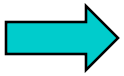
- Point operation – some function of the pixel value
  - $I'(x,y) = \boldsymbol{f}\,(I(x,y))$ or $I'_{ij} = \boldsymbol{f}\,(I_{ij})$
  - Examples: log, sqrt, threshold

- Local area operation – some function of the pixel values in the area surrounding the pixel
  - $I'_{ij} = \boldsymbol{f}\,(\{I_{(i+u)(j+v)}\})$ where $-m<u<m$ and $-n<v<n$
  - Examples: blur, low-pass, high-pass, gradient, center-surround

- Global operation – some function of the whole image
  - Examples: histogram, mean value, median value, 2^{nd} moment

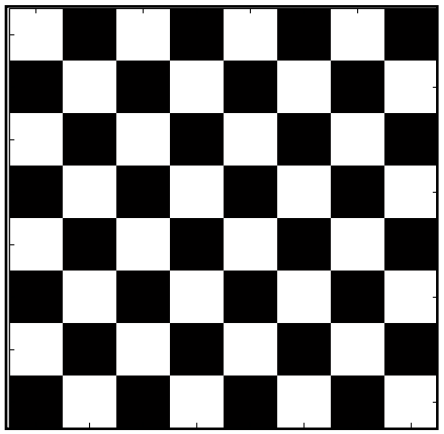
Point Operations

Point Operations



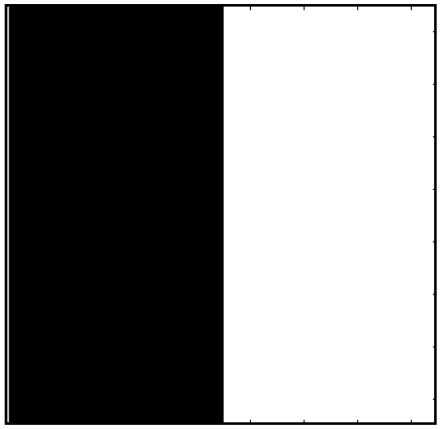
Global examples

- Histogram: Distribution (count) of possible image values
- Average pixel value



Same histogram

Same average value



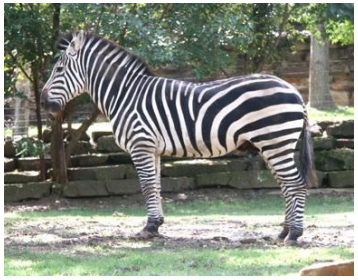
Global examples

Indoors

Outdoors

Image neighborhoods

- Q: What happens if we reshuffle all pixels within the images?




- A: Its histogram won't change.
  Point-wise processing unaffected.

- Need to measure properties relative to small *neighborhoods* of pixels

Comparison

- Point operations and global operations don't tell much about the object and scene
  - Objects/surfaces have finite area
  - Most surfaces have texture (defined by a local area)
  - This is referred to as "aperture problem"
- Point operations – very small aperture
  - Only information of a *single* pixel is used, no other spatial or temporal information
- Global operations – very large aperture
  - How do you focus on what is relevant?
  - How do you deal with computational complexity?
  - Again, human vision system can solve focusing and complexity issues, but not current machine vision systems

Area operations: Linear filtering

- Much of computer vision analysis starts with local area operations and then builds from there
  - Texture, edges, contours, shape, etc.
  - Perhaps at multiple scales (because how do you know how large should your aperture be?)
- Linear filtering is an important class of local operators
  - Convolution
  - Correlation
  - Fourier (and other) transforms
  - Sampling and aliasing issues

2D Convolution

- The response of a linear shift-invariant system can be described by the ***convolution*** operation

$$R_{ij} = \sum_{u,v} H_{i-u,j-v} F_{uv}$$

Output image

Convolution filter kernel

Input image

$$R = H * F = H \otimes F$$

Convolution notations

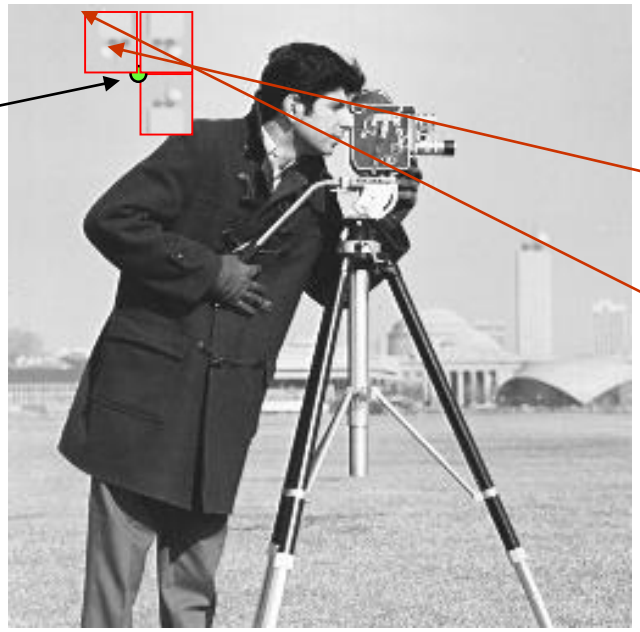
Textbook

My preference →

$$R_{ij} = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} H_{mn} F_{i-m,\, j-n}$$

Convolution

- Think of 2D convolution as the following procedure
- For every pixel (i,j):
  - Line up the image at (i,j) with the filter kernel
  - Flip the kernel in both directions (vertical and horizontal)
  - Multiply and sum (dot product) to get output value $R(i,j)$



Sometimes defined so that this value will go here (kernel and image line up at center)

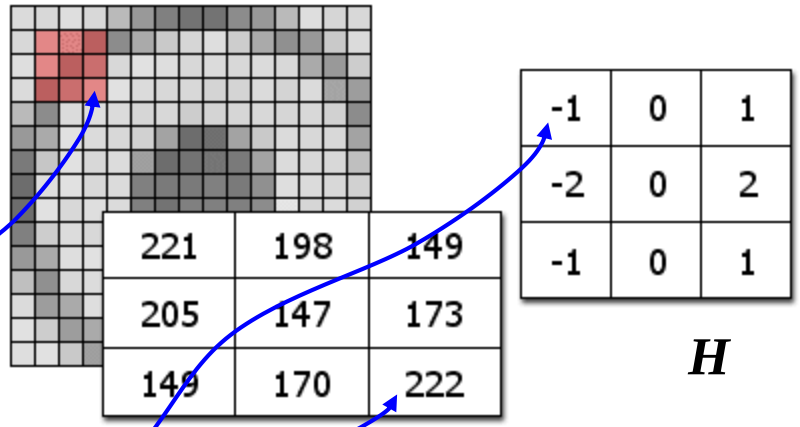
Or here (opposite corner)

(Minor detail)

Convolution

$$R_{ij} = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} H_{mn} F_{i-m, j-n}$$

- For every (i,j) location in the output image R, there is a summation over the local area



$$R_{4,4} = H_{0,0}F_{4,4} + H_{0,1}F_{4,3} + H_{0,2}F_{4,2} + H_{1,0}F_{3,4} + H_{1,1}F_{3,3} + H_{1,2}F_{3,2} + H_{2,0}F_{2,4} + H_{2,1}F_{2,3} + H_{2,2}F_{2,2}$$

$$= -1*222+0*170+1*149+ -2*173+0*147+2*205+ -1*149+0*198+1*221$$

$$= 63$$

Convolution

- So, what do you get if you convolve an image with

0	0	0
0	1	0
0	0	0

0.5	0	0
0	0	0
0	0	0.5

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

-1	1

-1
1

1	0	1
2	0	2
1	0	-1

- Convolution is a linear filter (a linear shift-invariant operation)

$$F * G = G * F \qquad F * (G * H) = (F * G) * H$$

Symmetric **Associative**

Convolution

- What do you get if you convolve an image with

-1	1

and then

-1
1

?

Differentiation

- Because of associative property, (I*A)*B = I*(A*B)
  - So it is the same as convolving with this

1	-1
-1	1

Convolution

- What about

1	1	1
1	1	1
1	1	1

and then

1	1	1
1	1	1
1	1	1

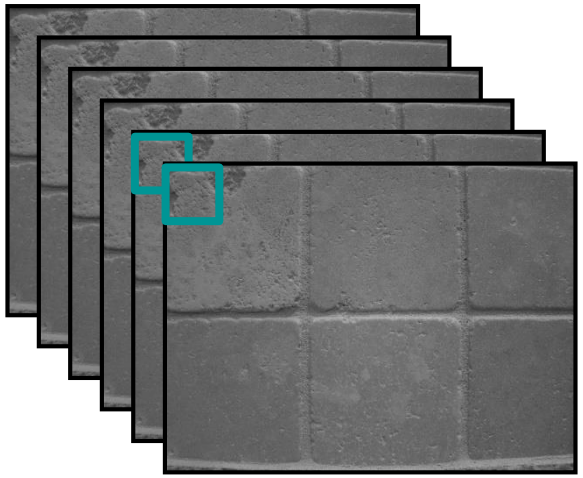
?

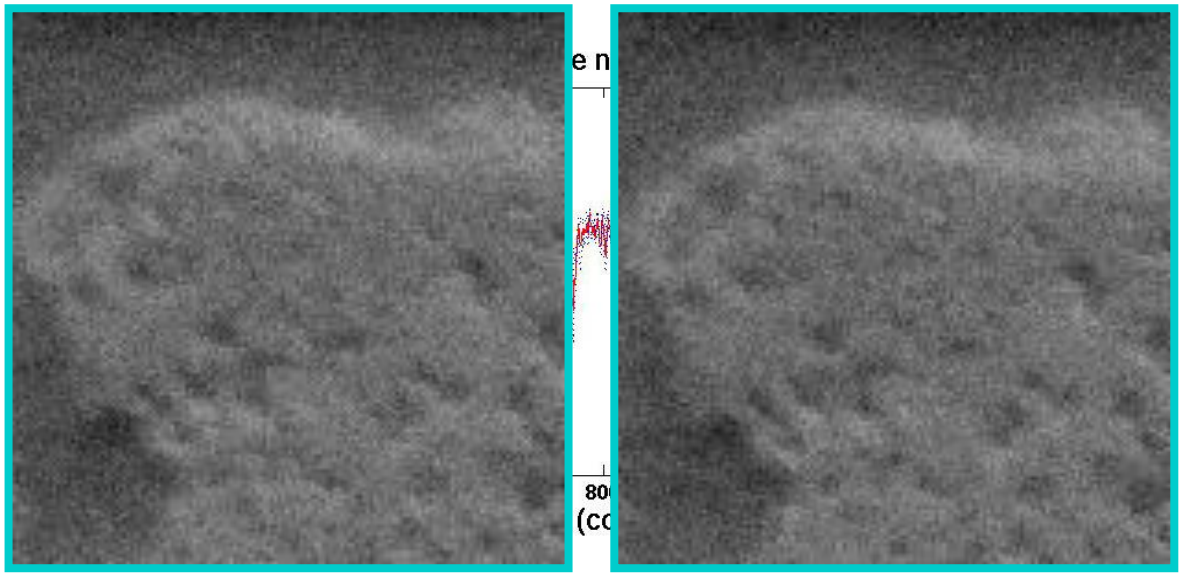
It is the same as convolving with this

1	2	3	2	1
2	4	6	4	2
3	6	9	6	3
2	4	6	4	2
1	2	3	2	1

Border effects can be a bit tricky

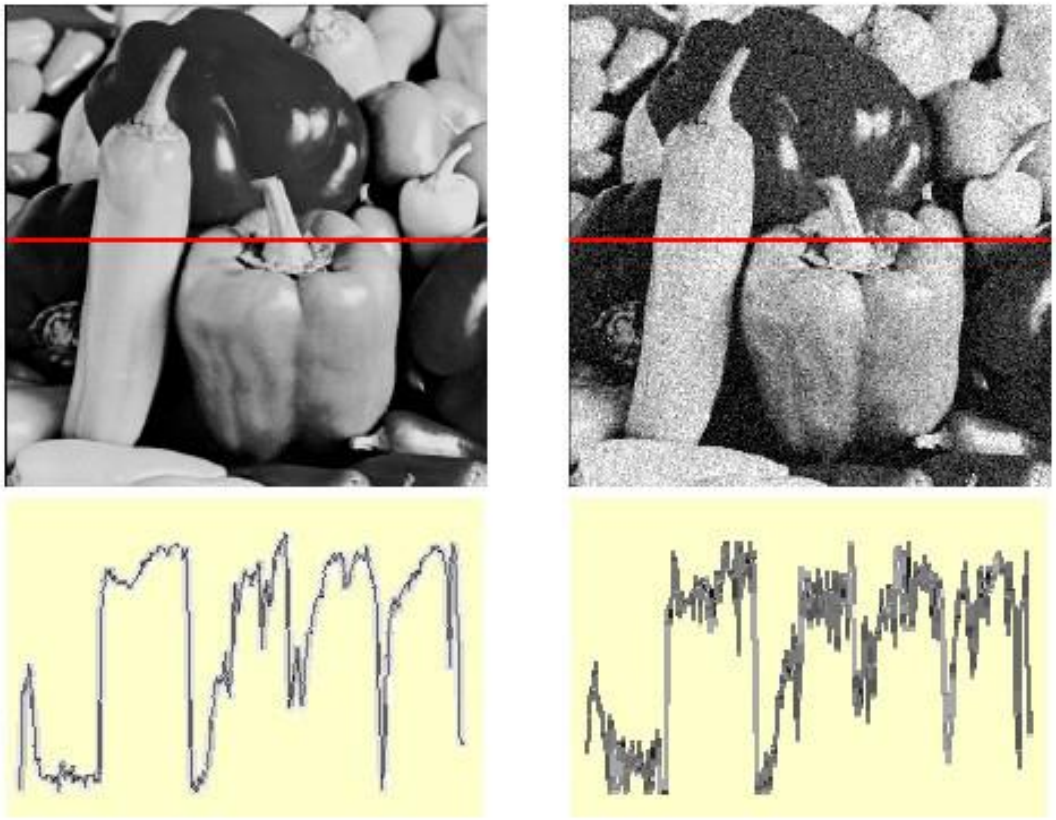
Linear Filter: noise reduction





- We can measure **noise** in multiple images of the same static scene.
- How could we reduce the noise, i.e., give an estimate of the true intensities?

Gaussian noise



$$f(x,y) = \overbrace{\bar{f}(x,y)}^{\text{Ideal Image}} + \overbrace{\eta(x,y)}^{\text{Noise process}}$$

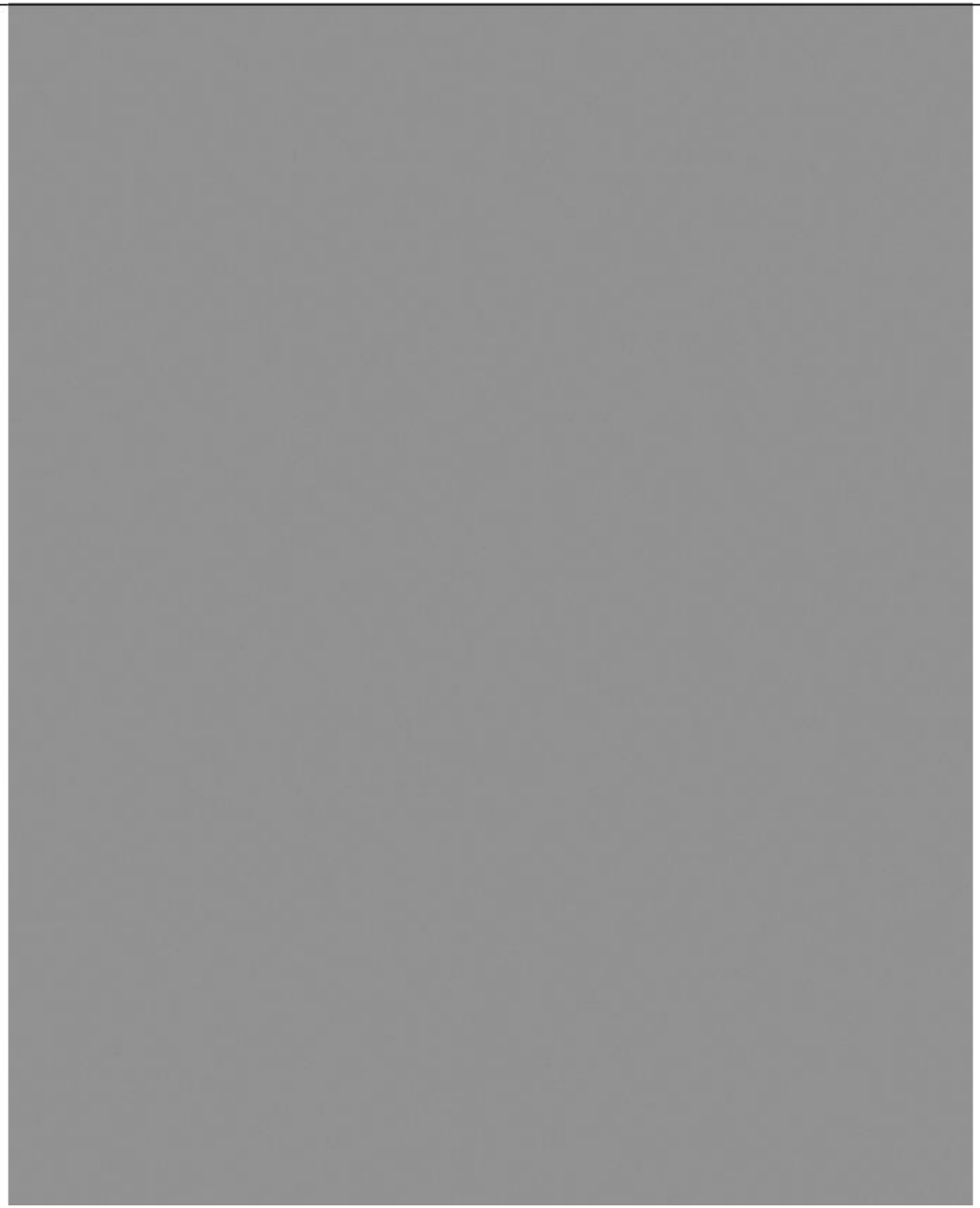
Gaussian i.i.d. ("white") noise:
$\eta(x,y) \sim \mathcal{N}(\mu,\sigma)$

```
>> noise = randn(size(im)).*sigma;

>> output = im + noise;
```

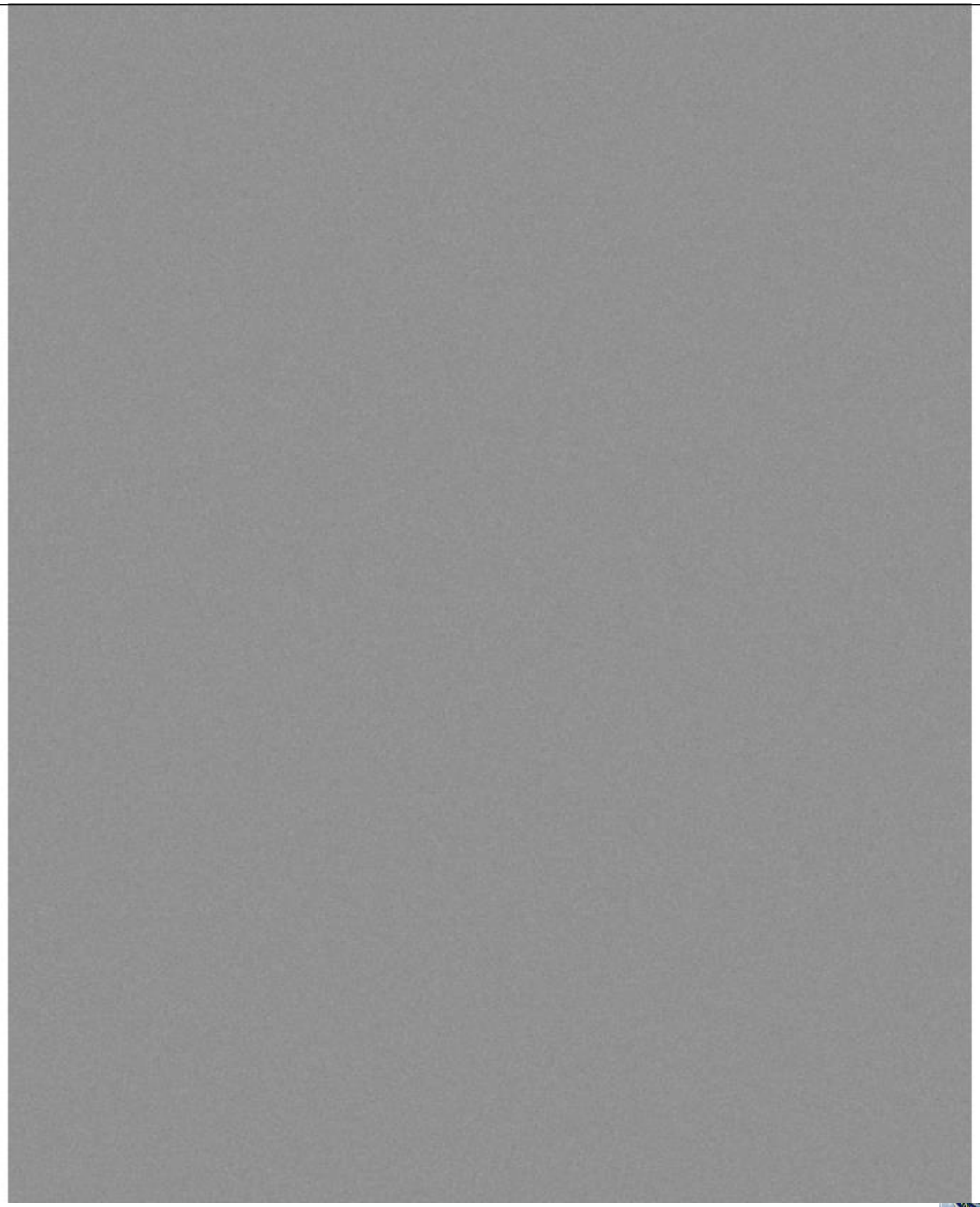
Fig: M. Hebert

sigma=1



Effect of sigma on Gaussian noise:

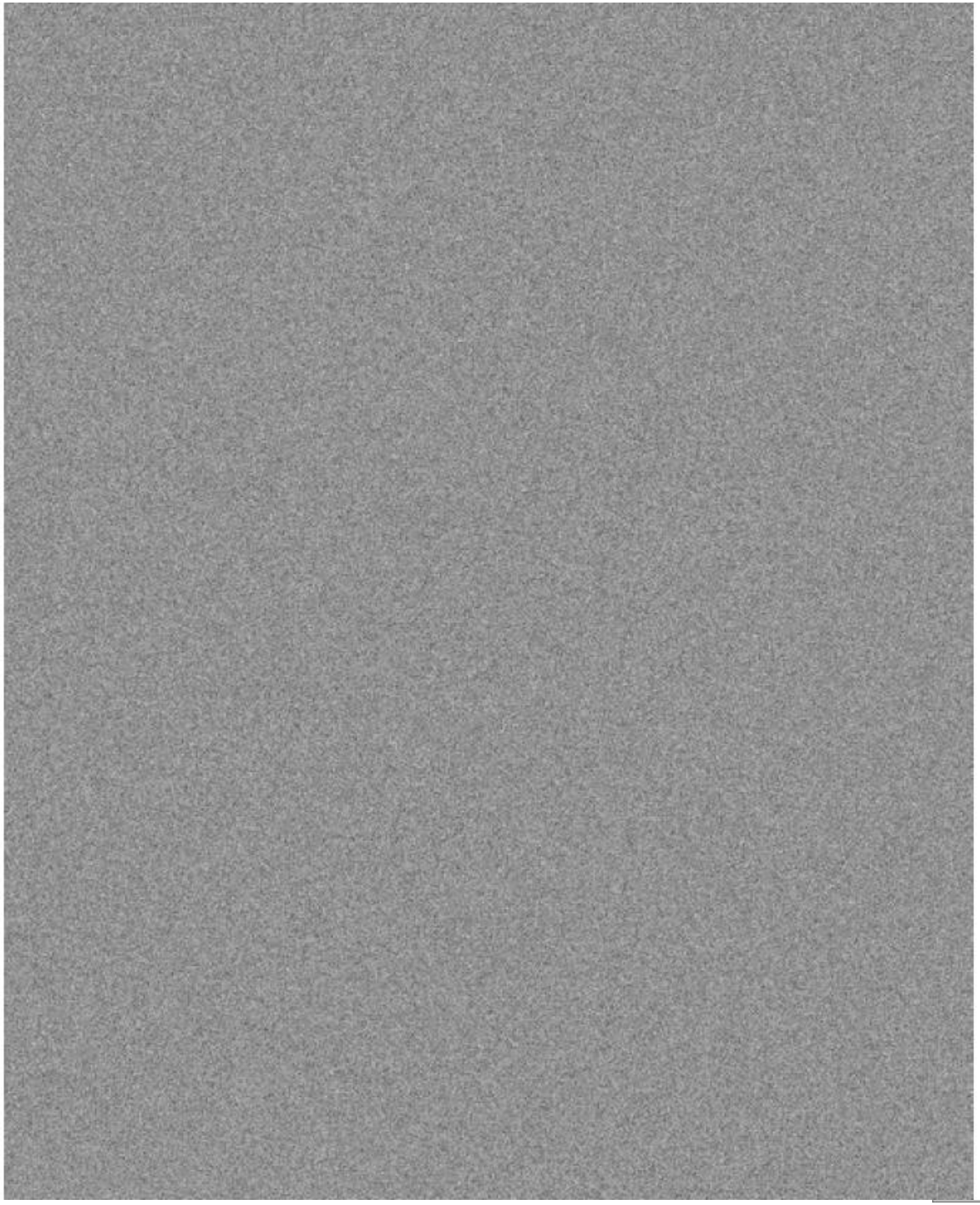
Image shows the noise values themselves.



sigma=4

Effect of sigma on Gaussian noise:

Image shows the noise values themselves.



sigma=
16

Effect of sigma on Gaussian noise:

Image shows the noise values themselves.

sigma=1

Effect of sigma on Gaussian noise:

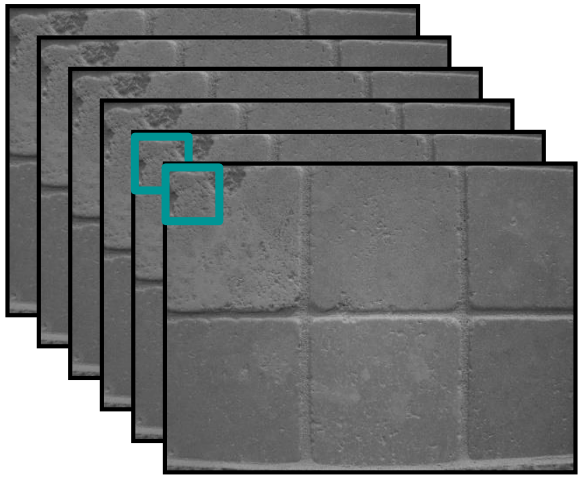
This shows the noise values added to the raw intensities of an image.

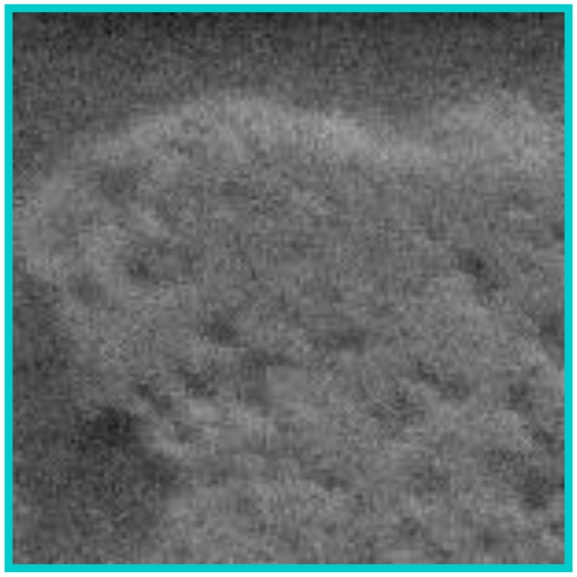
sigma=16

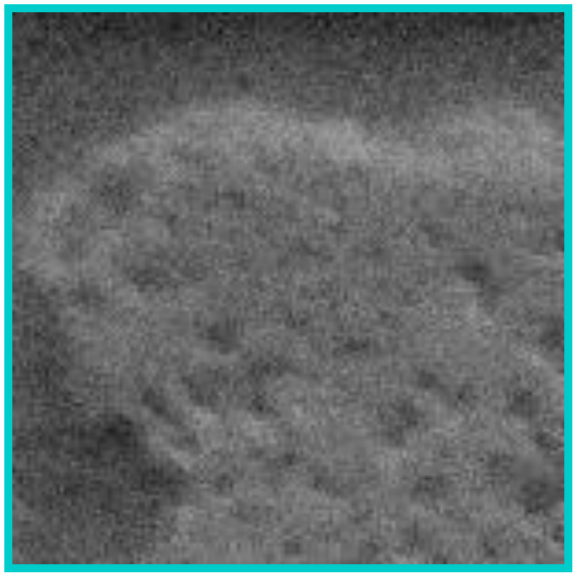
Effect of sigma on Gaussian noise

This shows the noise values added to the raw intensities of an image.

Motivation: noise reduction







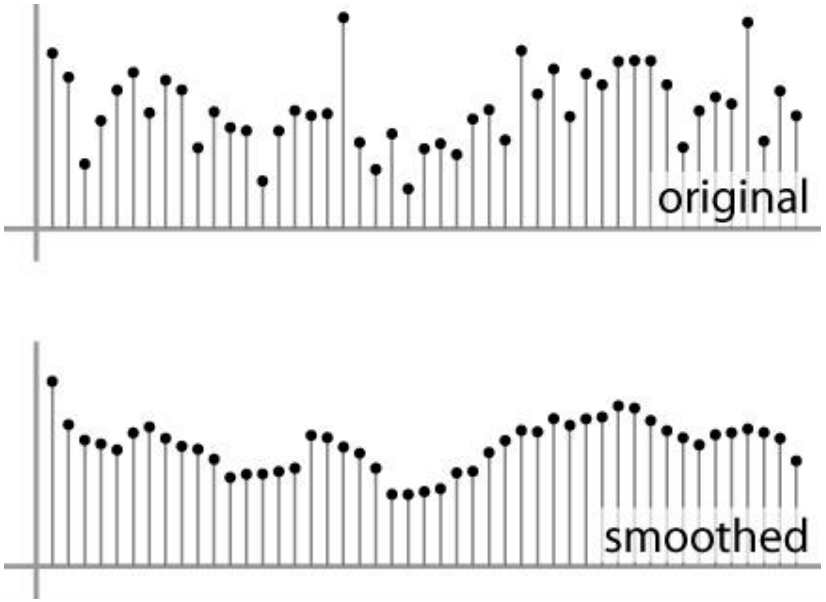
- How could we reduce the noise, i.e., give an estimate of the true intensities?
- **What if there's only one image?**

First attempt at a solution

- Let's replace each pixel with an average of all the values in its neighborhood
- Assumptions:
  - Expect pixels to be like their neighbors
  - Expect noise processes to be independent from pixel to pixel

First attempt at a solution

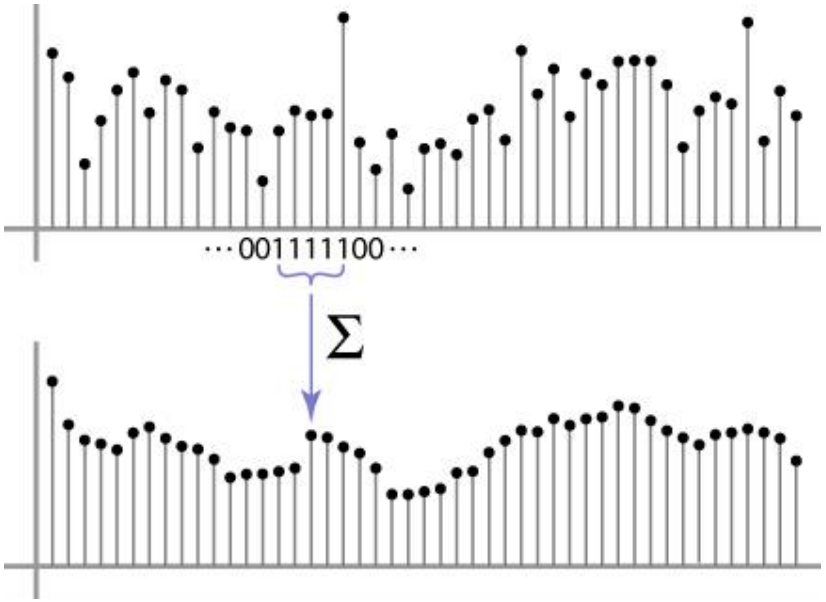
- Let's replace each pixel with an average of all the values in its neighborhood
- Moving average in 1D:



Source: S. Marschner

Weighted Moving Average

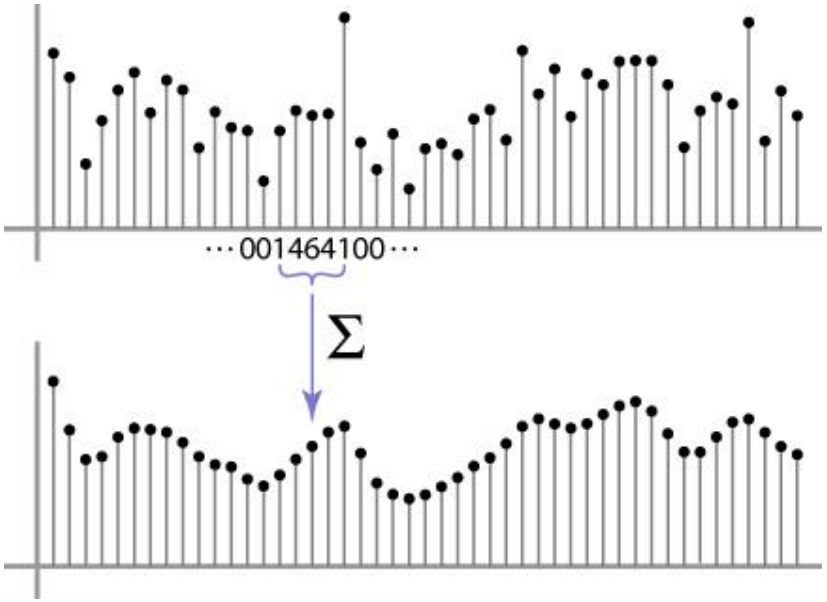
- Can add weights to our moving average
- *Weights* [1, 1, 1, 1, 1] / 5



Source: S. Marschner

Weighted Moving Average

- Non-uniform weights [1, 4, 6, 4, 1] / 16



Source: S. Marschner

Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0								

Source: S. Seitz

Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10							

Source: S. Seitz

Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10	20						

Source: S. Seitz

Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10	20	30					

Source: S. Seitz

Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

	0	10	20	30	30				

Source: S. Seitz

Moving Average In 2D

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G[x, y]$

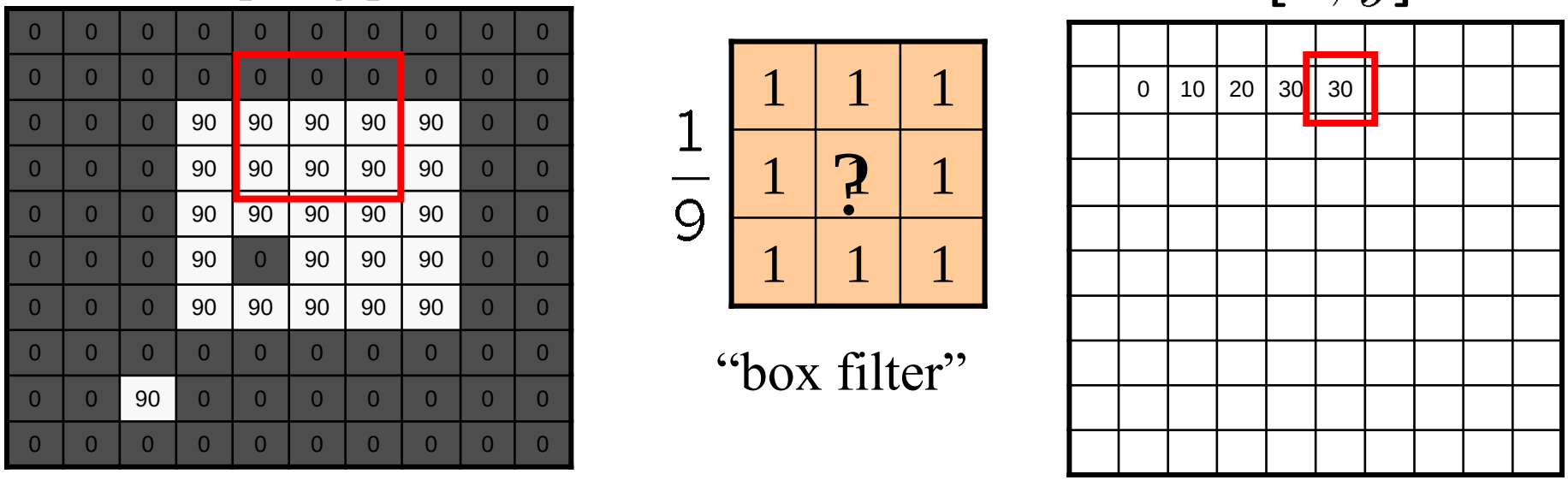
	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

Source: S. Seitz

Averaging filter

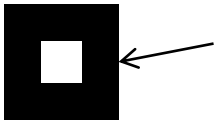
- What values belong in the kernel *H* for the moving average example?

$F[x, y]$ $\otimes$ $H[u, v]$ $G[x, y]$



$$G = H \otimes F$$

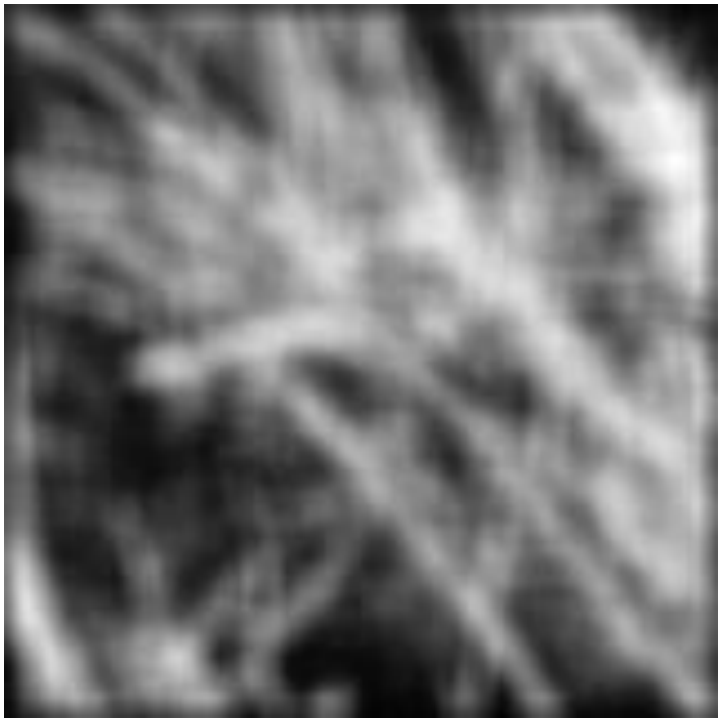
Smoothing by averaging



depicts box filter:

white = high value, black = low value

original



filtered

Gaussian filter

- What if we want nearest neighboring pixels to have the most influence on the output?

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$F[x, y]$

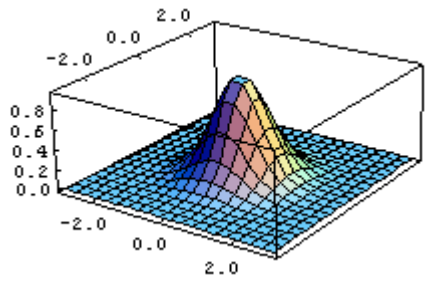
$$\frac{1}{16}$$

1	2	1
2	4	2
1	2	1

$H[u, v]$

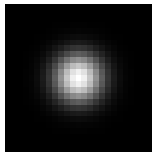
This kernel is an approximation of a Gaussian function:

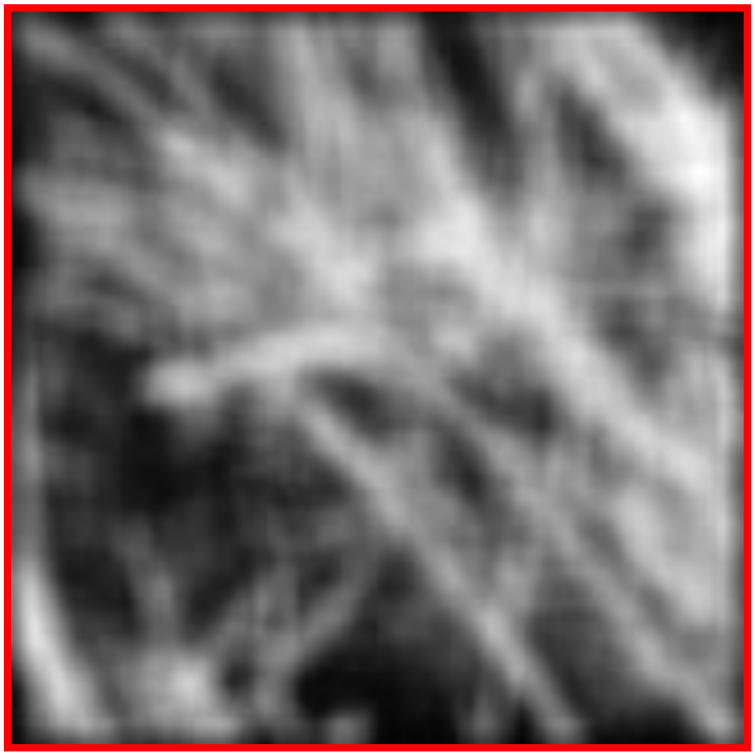
$$h(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$

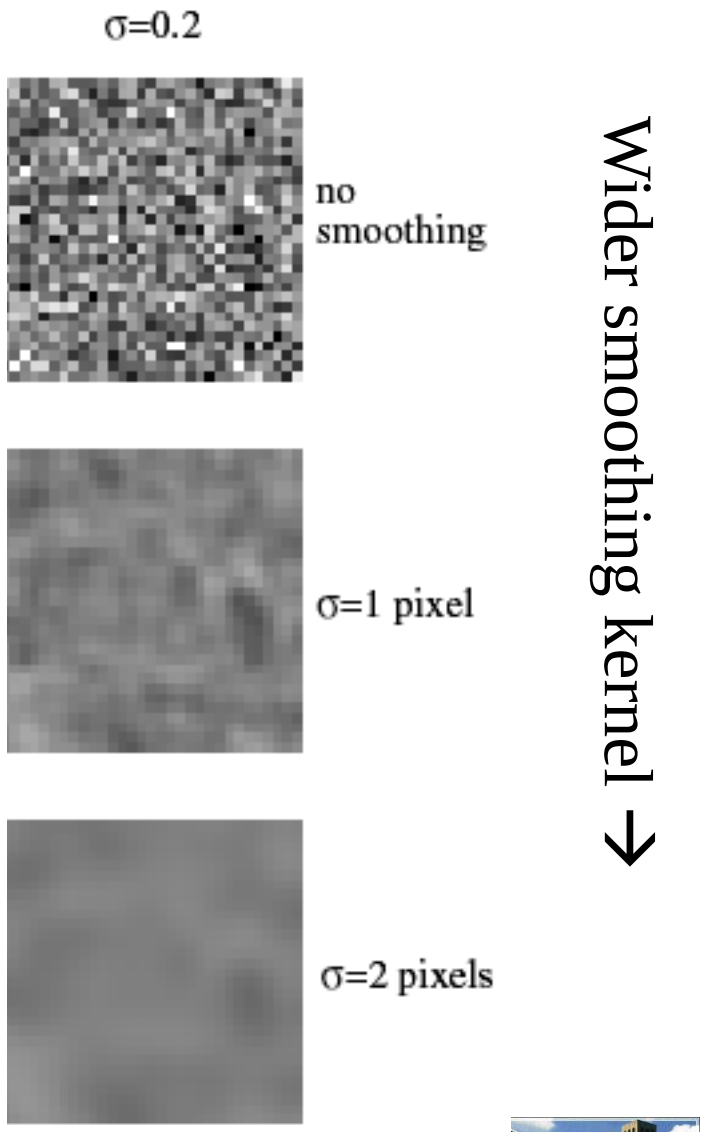


Source: S. Seitz

Smoothing with a Gaussian

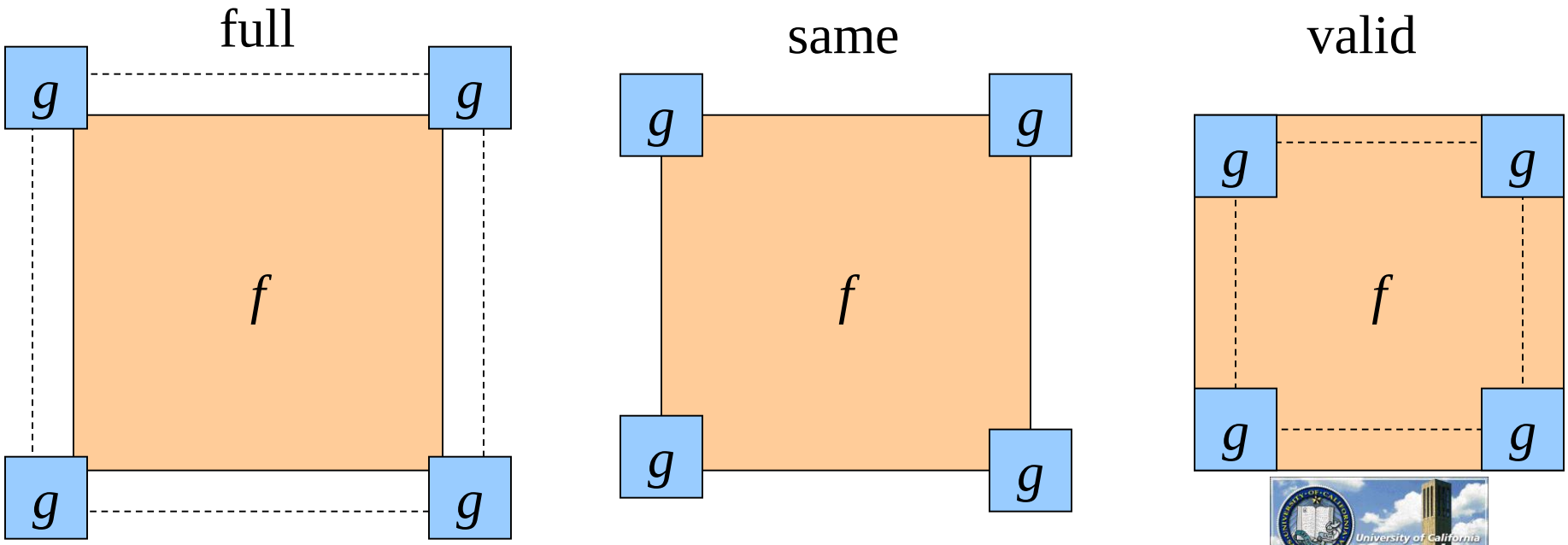





σ=0.2
no smoothing
σ=1 pixel
σ=2 pixels
Wider smoothing kernel →

Boundary issues

- What is the size of the output?
- MATLAB: filter2(g, f, *shape*)
  - *shape* = 'full': output size is sum of sizes of f and g
  - *shape* = 'same': output size is same as f
  - *shape* = 'valid': output size is difference of sizes of f and g



Source: S. Lazebnik

Boundary issues

- What about near the edge?
  - the filter window falls off the edge of the image
  - need to extrapolate
  - methods:
    - clip filter (black)
    - wrap around
    - copy edge
    - reflect across edge

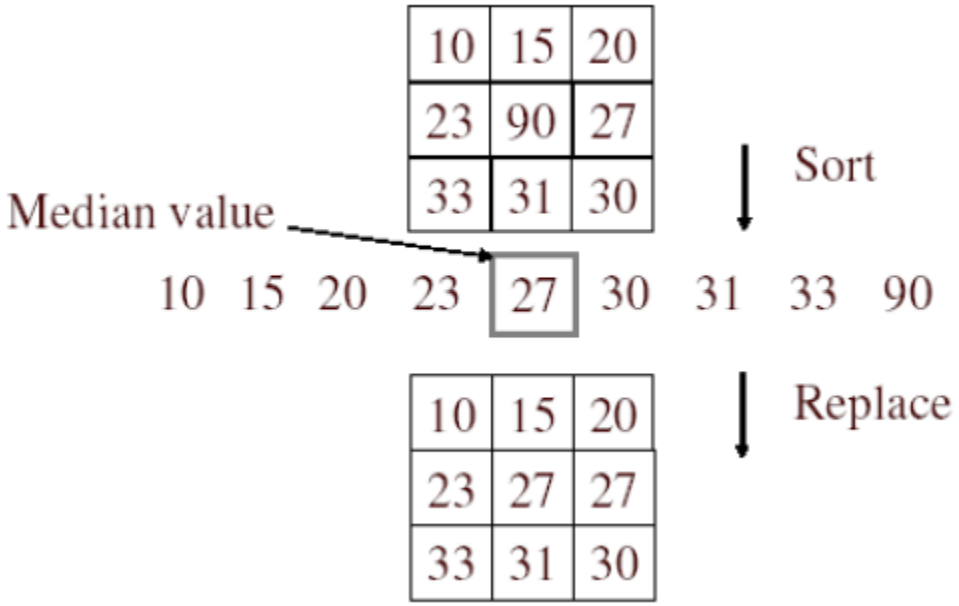
Source: S. Marschner

Boundary issues

- What about near the edge?
  - the filter window falls off the edge of the image
  - need to extrapolate
  - methods (MATLAB):
    - clip filter (black): imfilter(f, g, 0)
    - wrap around: imfilter(f, g, ‘circular’)
    - copy edge: imfilter(f, g, ‘replicate’)
    - reflect across edge: imfilter(f, g, ‘symmetric’)

Source: S. Marschner

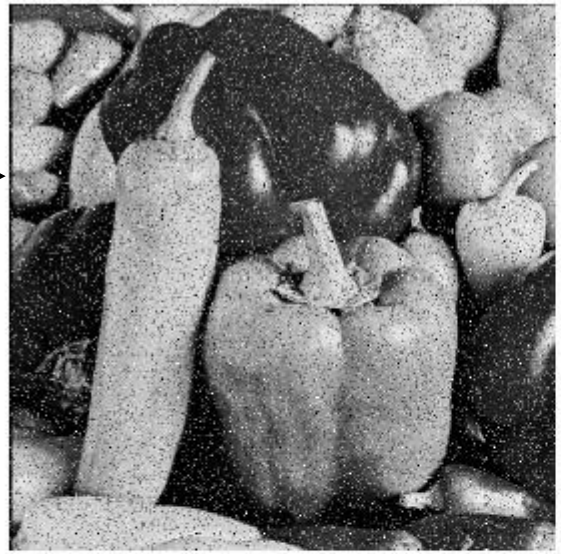
Median filter



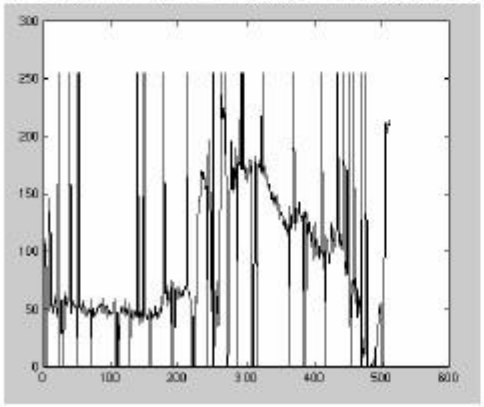
- No new pixel values introduced
- Removes spikes: good for impulse, salt & pepper noise

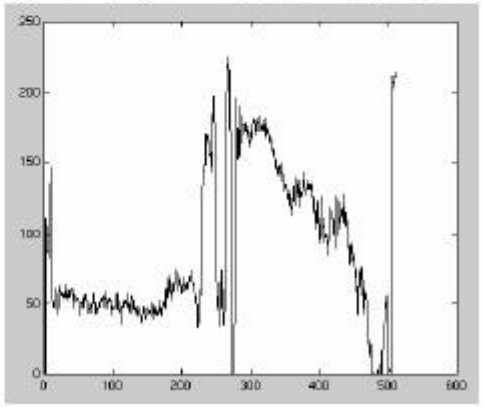
Median filter

Salt and pepper noise



Median filtered



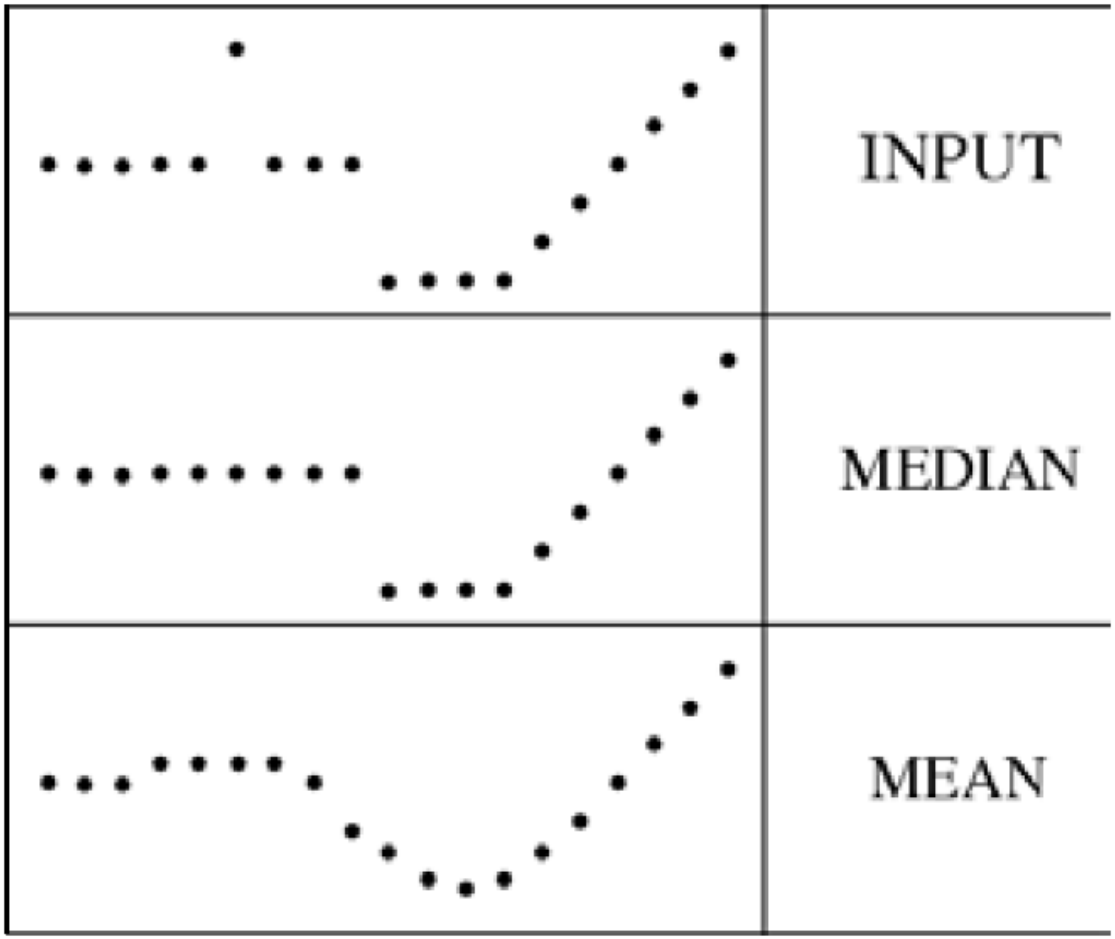


Plots of a row of the image

Source: M. Hebert

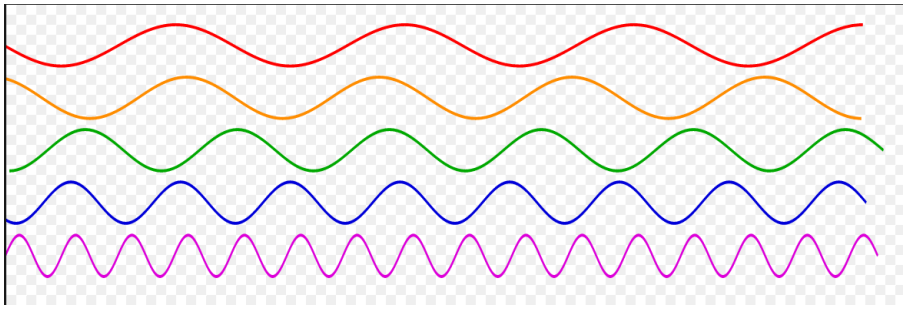
Median filter

- Median filter is edge preserving



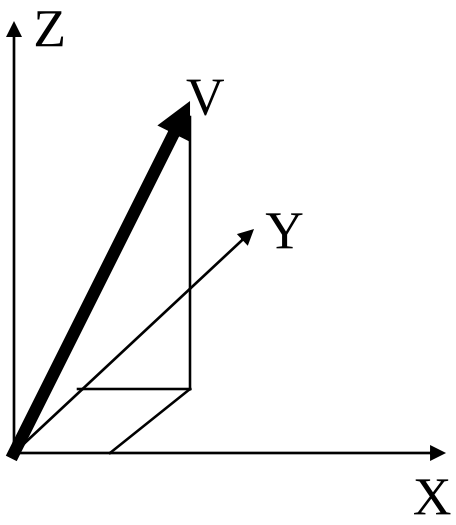
University of California
Santa Barbara

Frequency Decomposition

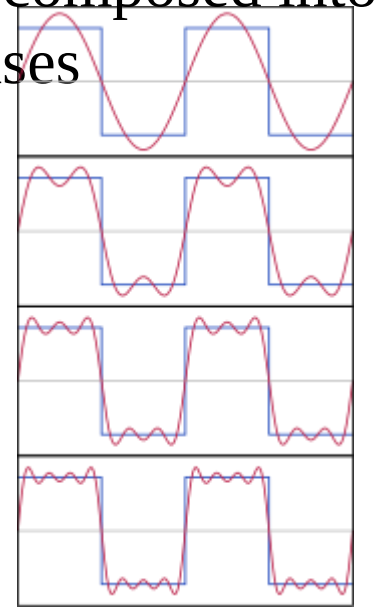


Basis

- X, Y, Z
- V = a X + b Y + c Z
- Any vector can be decomposed into the bases
- a = < V. X>
- b = <V . Y>
- c = <V . Z>



- Any waveform can be decomposed into the Fourier bases



High vs. Low Frequencies

- Slow varying components
- Smooth regions
- Accentuated by suppressing the high-frequency component
- E.g., using box or Gaussian smoothing

- Fast varying components
- Edges and noise
- Accentuated by suppressing the low-frequency component
- E.g. using derivative operators

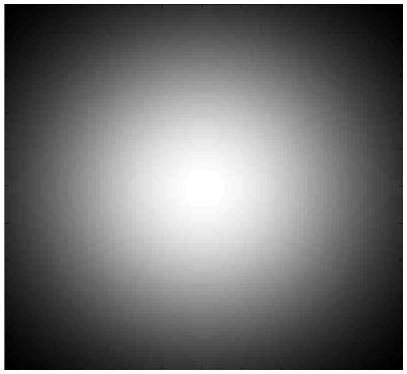
Gaussian filter

Symmetric Gaussian kernel:

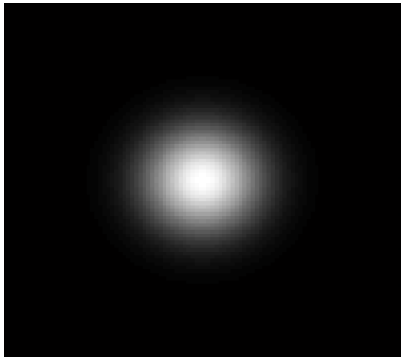
$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2} e^{\frac{-(x^2+y^2)}{2\sigma^2}}$$

Mean: (0,0)

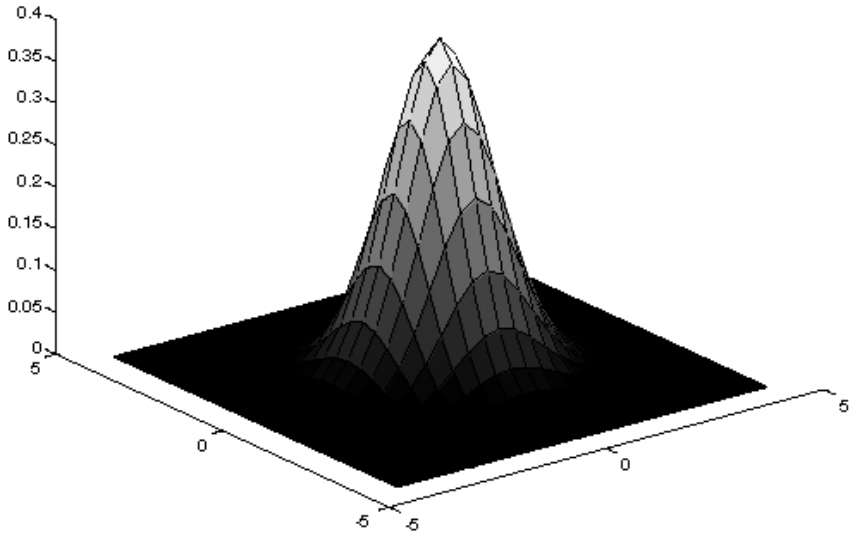
Standard deviation: σ



Large σ



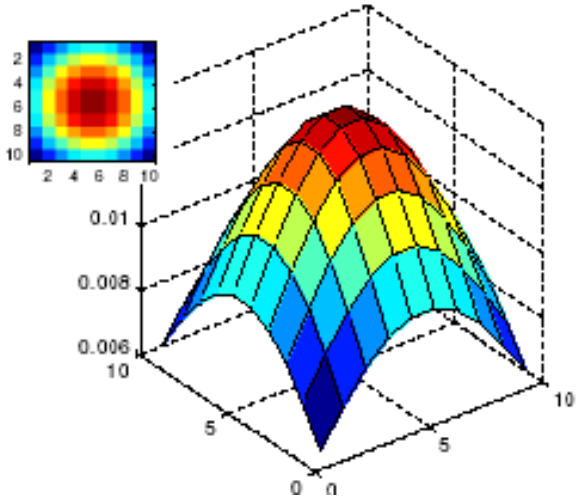
Small σ



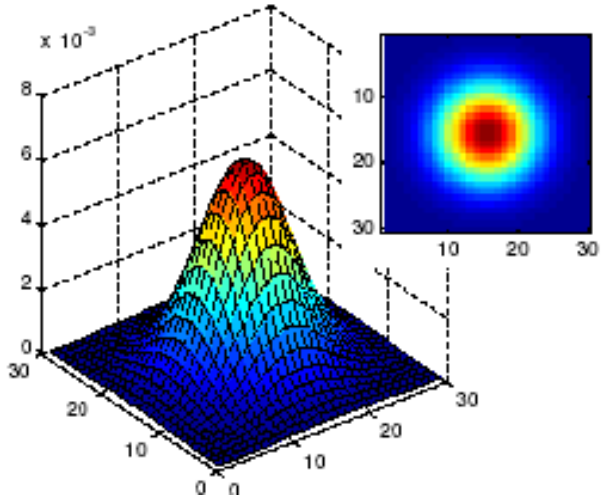
Gaussian filtering (smoothing, blurring) is a good model of camera optics

Gaussian filters

- What parameters matter here?
- **Size** of kernel or mask
  - Note, Gaussian function has infinite support, but discrete filters use finite kernels



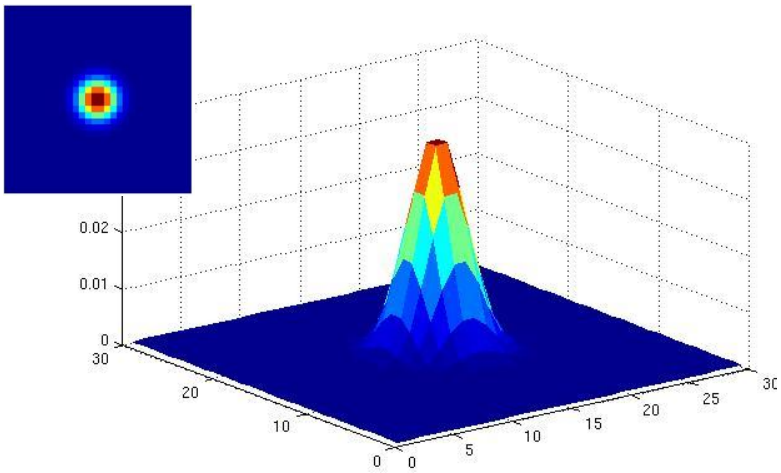
σ = 5 with 10 x 10 kernel



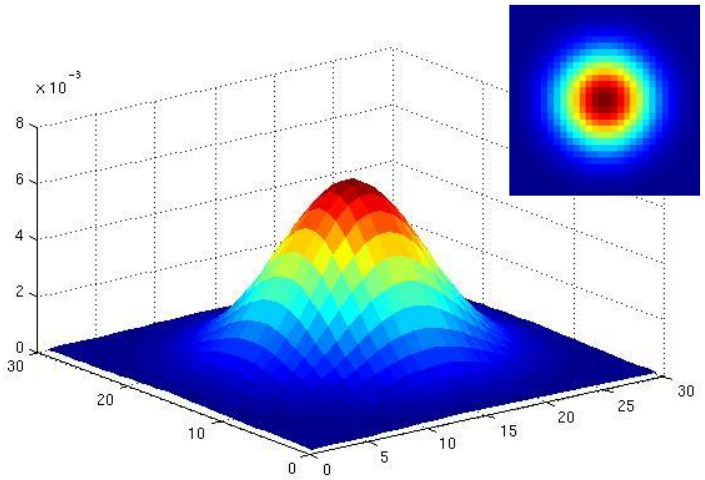
σ = 5 with 30 x 30 kernel

Gaussian filters

- What parameters matter here?
- **Variance** of Gaussian: determines extent of smoothing



σ = 2 with 30 x 30 kernel



σ = 5 with 30 x 30 kernel

Convolution

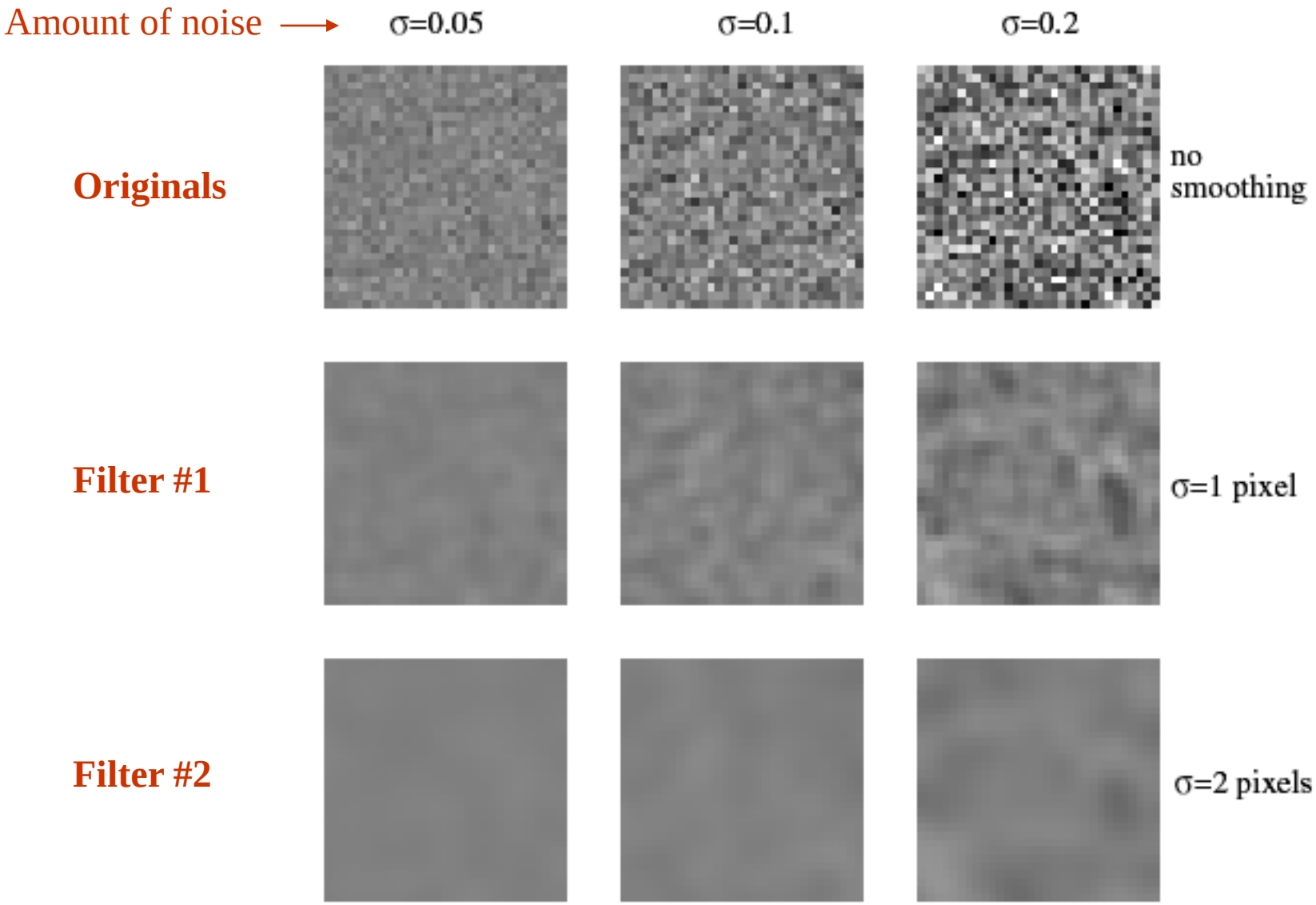
- For a high-pass filter, the kernel values should add to zero
  - Blocks out low spatial frequencies (gradual changes), accentuates high spatial frequencies (rapid changes)
  - Accentuates image noise!
- For a low-pass filter, the kernel values should add to one
  - Blocks out high spatial frequencies
  - Smoothing, averaging, blurring
  - Reduces image noise!

High pass

Low pass

High-pass filter example

Low-pass Gaussian filter example



Correlation

- Correlation is *almost* the same thing as convolution
  - Minus the "flip the kernel in both directions" step
  - So it's somewhat more intuitive than convolution

- ***Normalized correlation*** is a frequently used operation in computer vision
  - More on that later…

Edge Detection

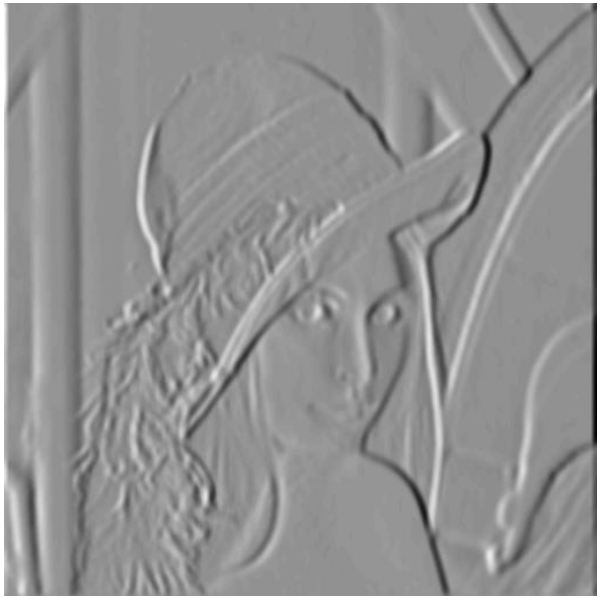
- All the discussions so far
  - Filtering
  - Convolution, correlation, etc.
  - Average, mean, Gaussian
  - High-pass, low-pass
- Lead us to edge detection which needs all the terminologies and techiques

Edge Detection

- Edge detection is a local area operator that seeks to find significant, meaningful changes in image intensity (color?) that correspond to
  - Boundaries of objects and patterns
  - Texture
  - Changes in object color or brightness
  - Highlights
  - Occlusions
  - Etc.

Example

Original

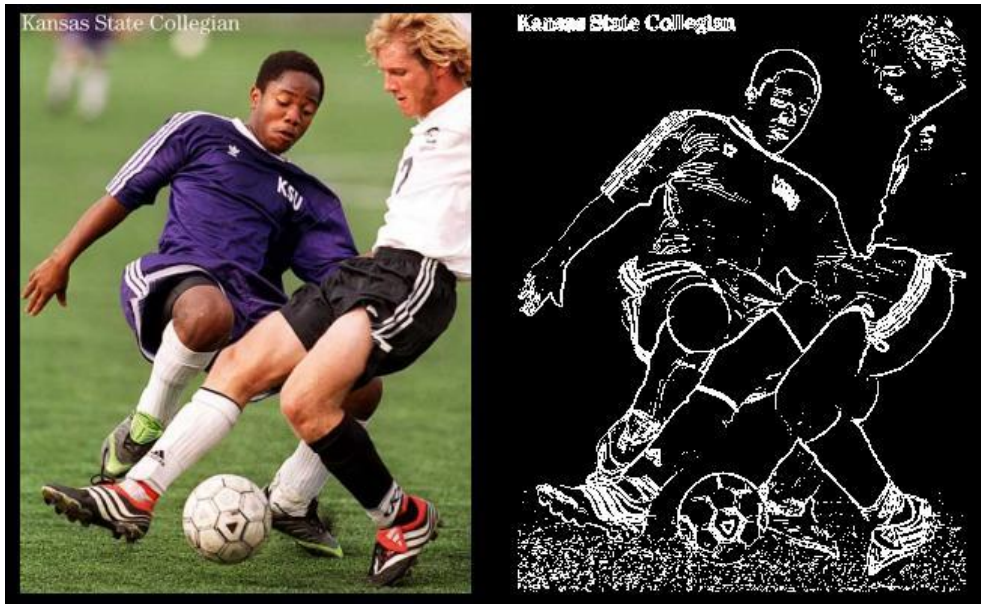


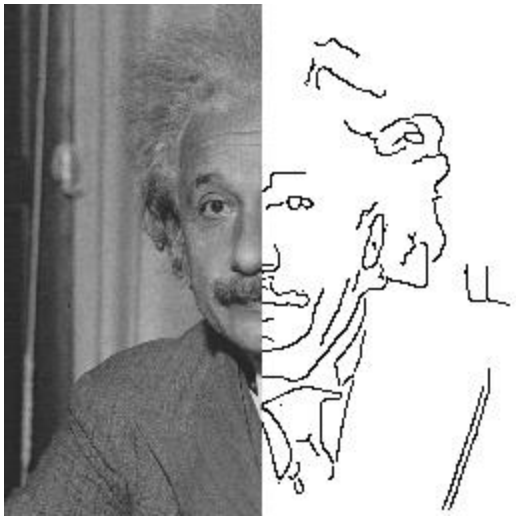
Vertical edges

Edge magnitude

Horizontal edges

Edge detection examples





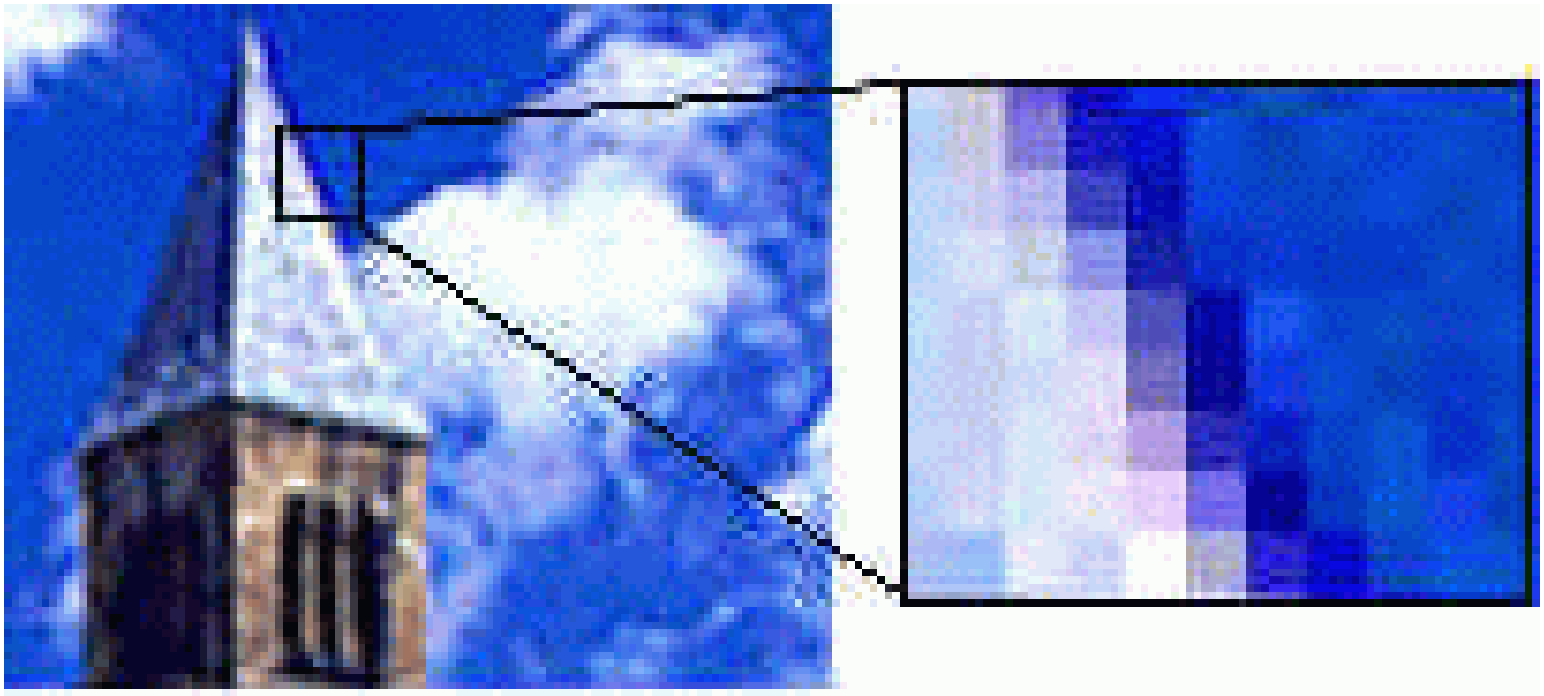
The bad news

- Unfortunately, it's very hard to tell *significant* edges from *bogus* edges!
  - Noise is a big problem!

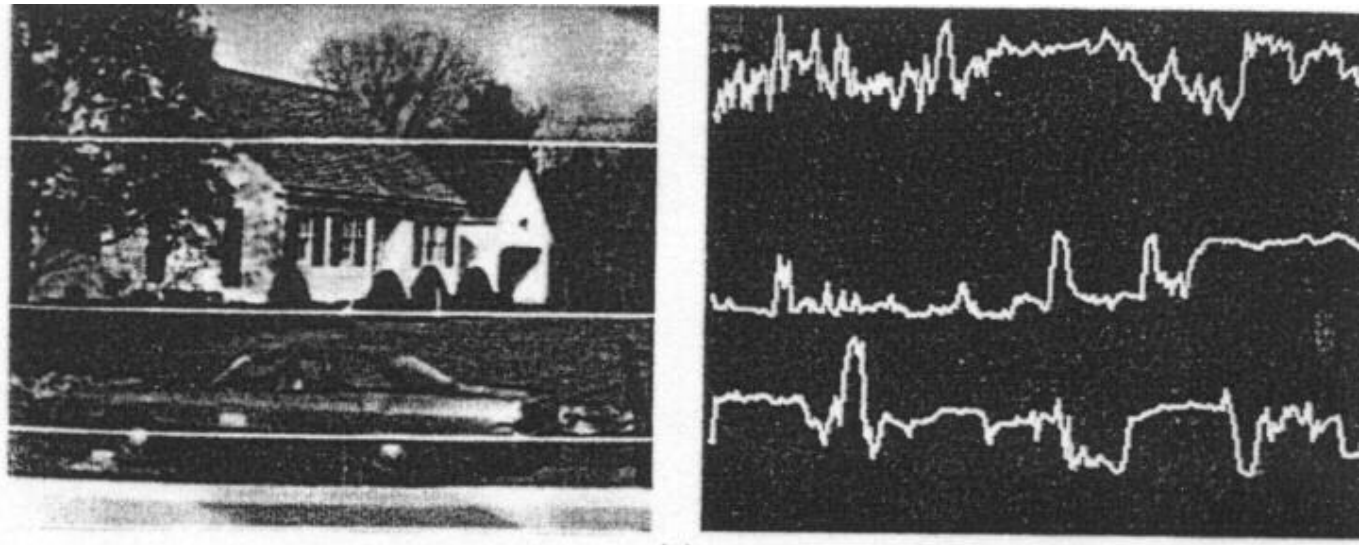
- An edge detector is basically a high-frequency filter, since sharp intensity changes are high-frequency events
- But image noise is also high-frequency, so edge detectors tend to accentuate noise!
- Some things to do:
  - Smooth before edge detection (hoping to get rid of noise but not edges!)
  - Look for edges at multiple scales (pyramids!)
  - Use an adaptive edge threshold

Caveats

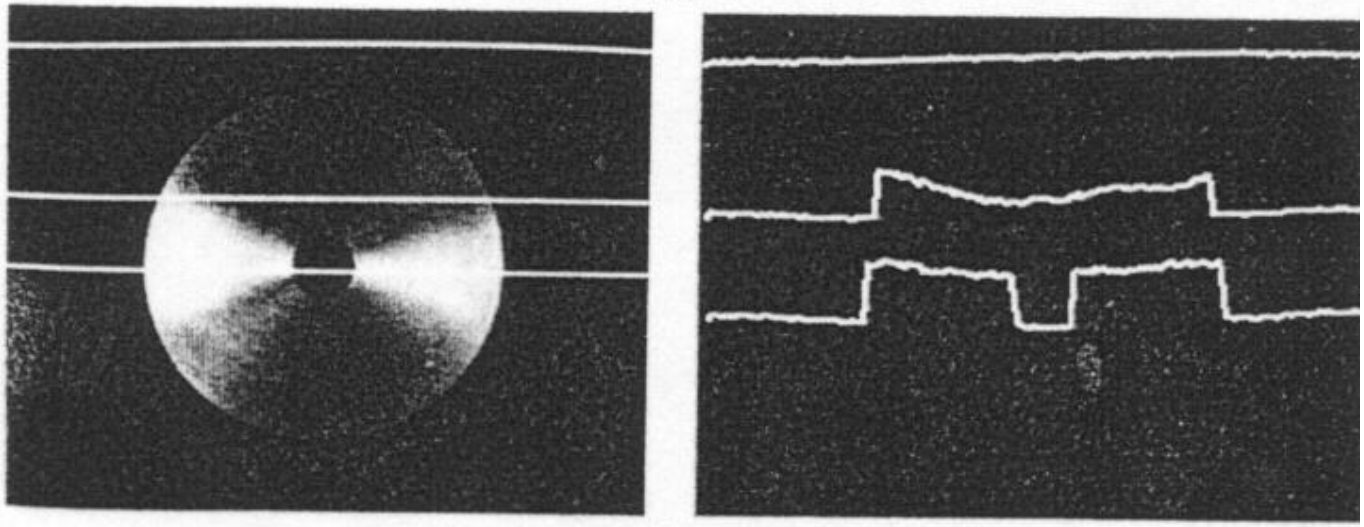
- In reality, low light levels and random noise lead to high fluctuations in individual pixel values, leading to bad estimations.



Graphically



(*a*)



Edge detection history

- Edge detection has a long history and a huge literature
  - Edge modeling: Step edges, roof edges, impulse edges…
  - Biological modeling: How does human vision do it?
  - Elegant and complex mathematical models
  - Simple and computationally cheap edge detectors
  - Etc., etc., etc…..

- Typical usage:
  - Detect "edge points" in the image (filter then threshold)
    - Edges may have magnitude **and** orientation
  - Throw away "bad" ones (isolated points)
  - Link edge points together to make edge segments
  - Merge segments into lines, corners, junctions, etc.
  - Interpret these higher-level features in the context of the problem

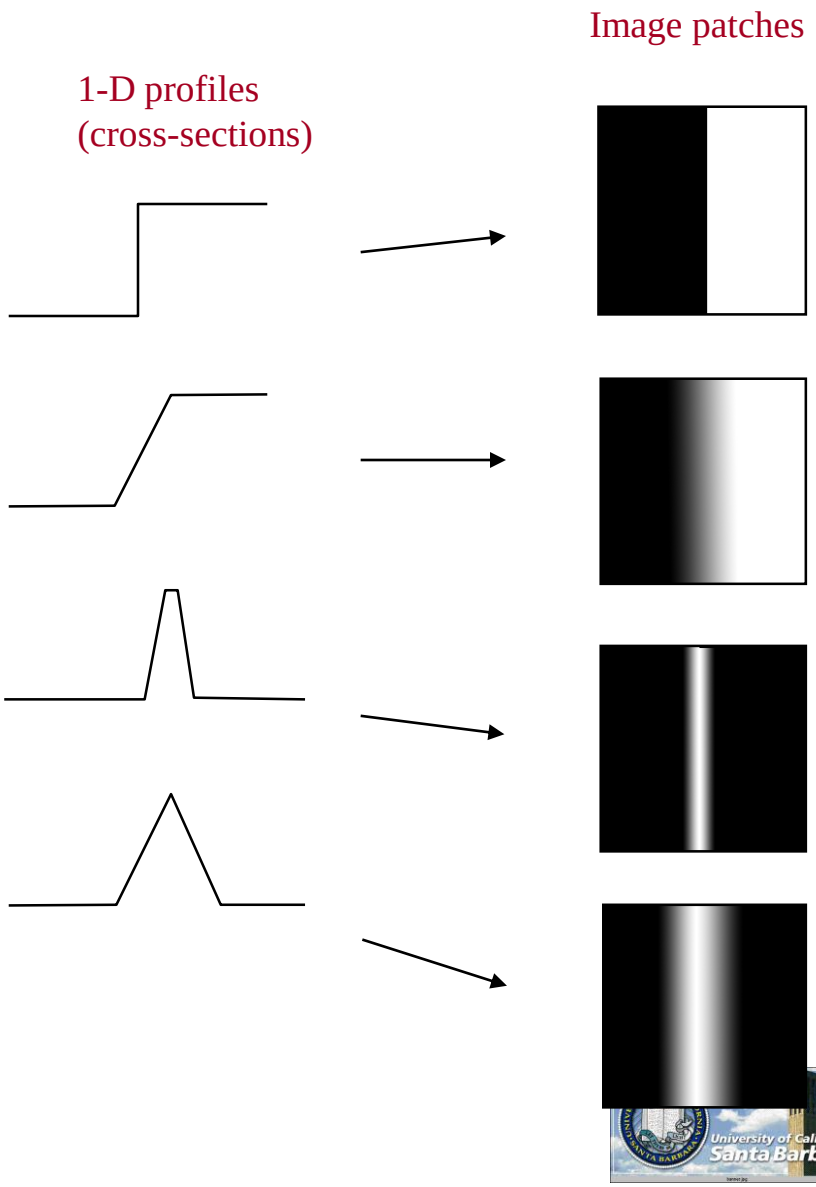
Edge detection

- The bottom line:
  - It doesn't work!
  - At least, now how we'd like it to:
    - Too many false positives (noise)
    - Too many omissions (little or no local signal)

- Still, edge detection is often the first step in a computer vision program
  - We have to learn to live with imperfection

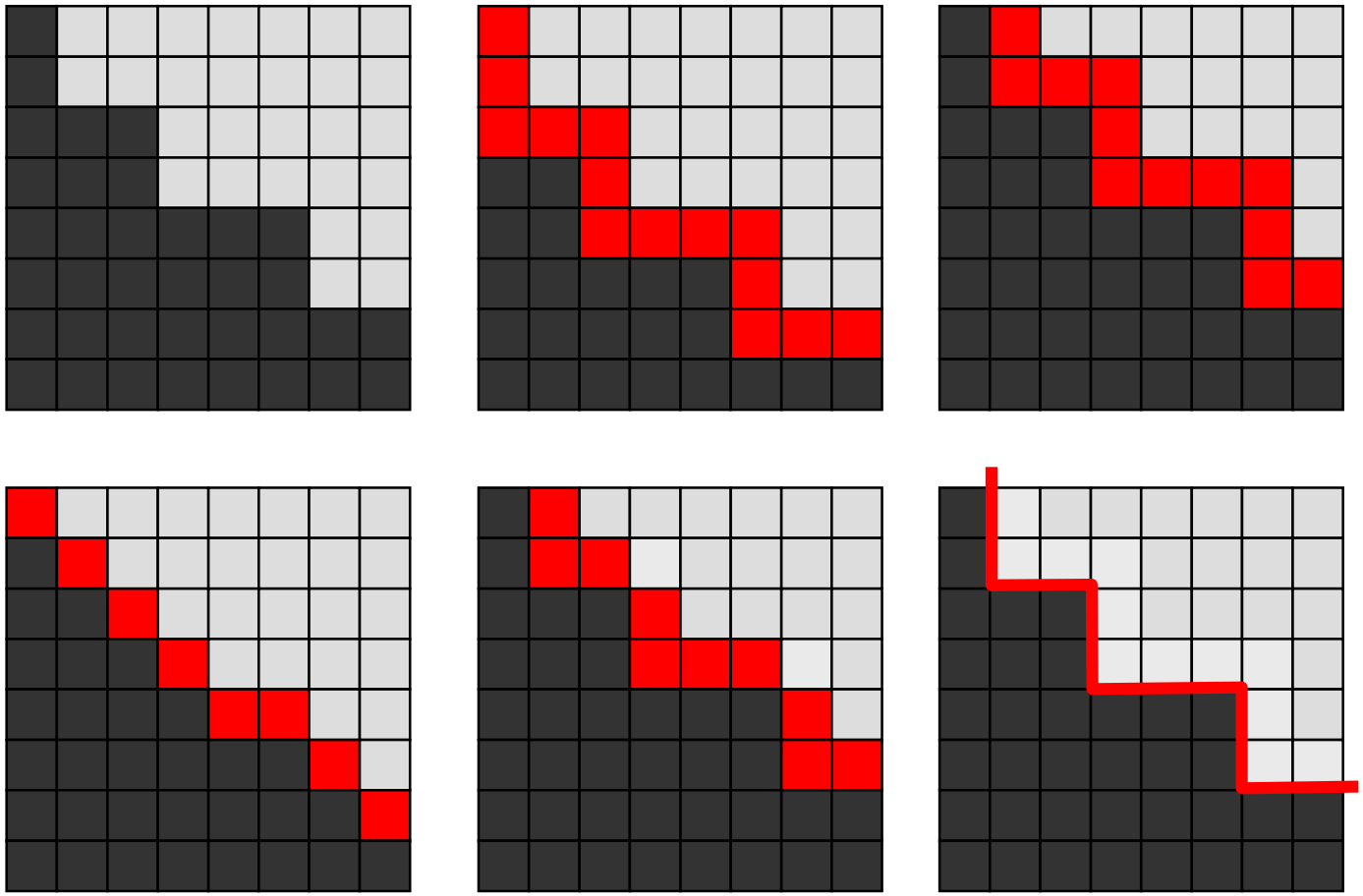
How to design an edge detector?

- What specifically are we trying to detect?
  - Step edges
  - Ramp edges
  - Ridge edges
  - Roof edges
  - Texture edges
  - Color edges
  - etc.
- Do the math
  - Build a filter to detect the edge type
- What about noise?



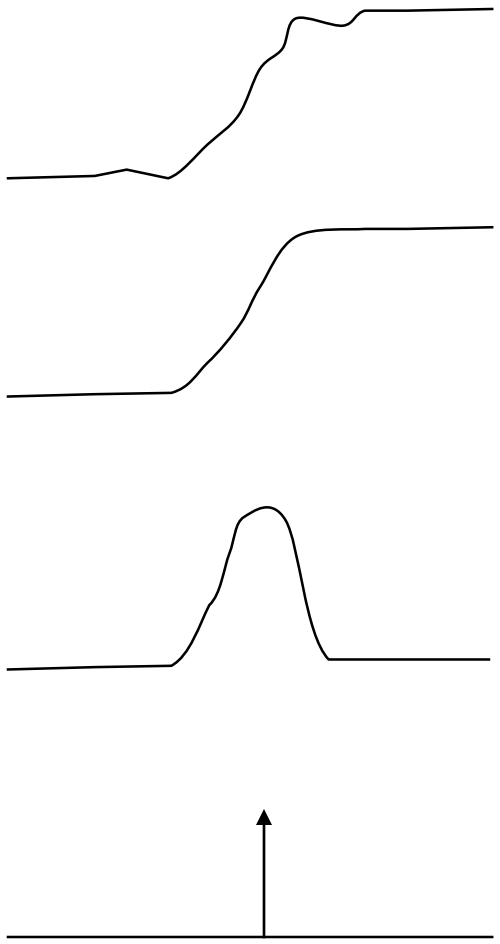
What should an edge detector output?

- Where does the edge exist?



Edge detection: Three primary steps

- Smoothing
  - Reduce noise
  - Reduce frequency content not of interest
    - Are we looking for low, medium, or high frequency edges?
- Edge enhancement
  - Filter (usually linear) to produce high values where there are strong edges, low values elsewhere
- Edge localization
  - Where specifically are the edge points?



Edge detection criteria

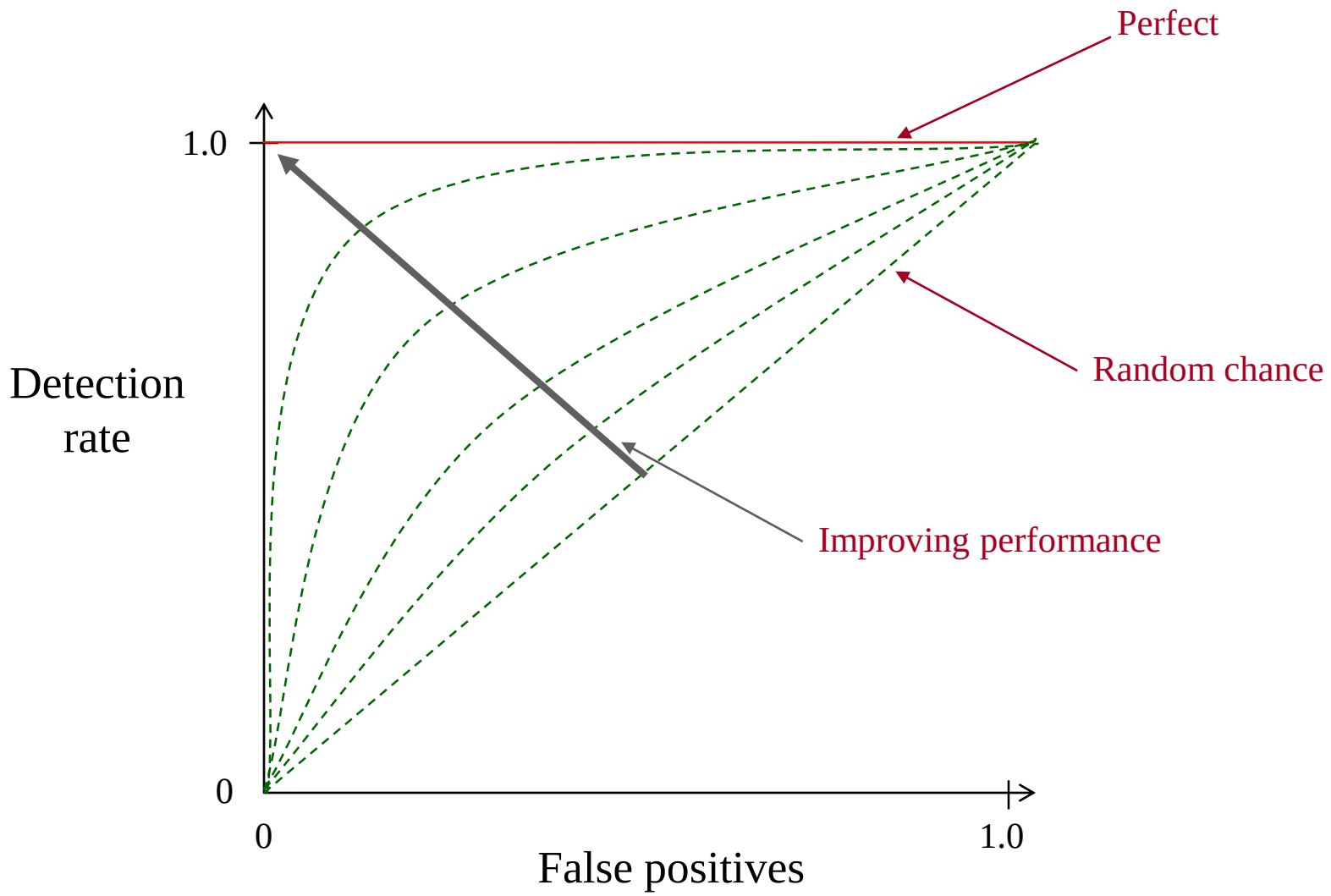
- Optimal edge detector for competing criteria:
  - Detection
    - Maximize detection (minimize misses)
    - Minimize false positives
  - Localization
    - Location of detected edge should be as close as possible to true location

- There is a tradeoff between these two criteria
  - Compromise between good detection and good localization for a given approach

How can you assure no misses?

How can you assure no false positives?

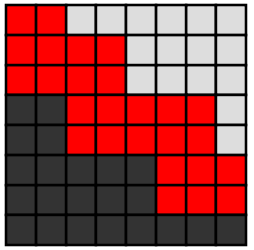
Evaluating performance – the ROC curve



Canny edge detector

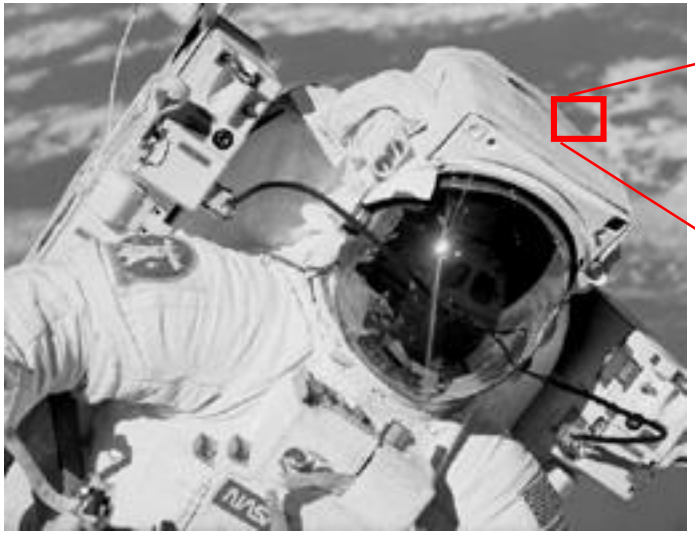
- Properties of the Canny edge detector
  - Designed for step edges (but generalizes relatively well)
  - Good detection
  - Good localization
  - Single response constraint
    - Uses *nonmaximum supression* to return just one edge point per "true" edge point

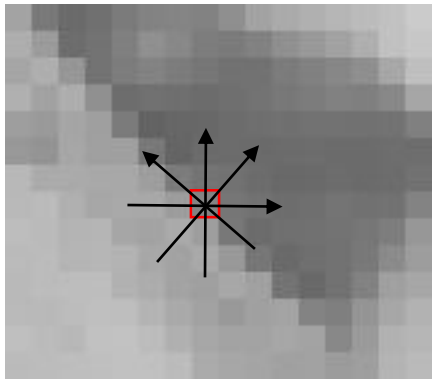
Avoids this:



- Basic detection filter:
  - 1D first derivative of a Gaussian
  - Apply this at multiple orientations to detect 2D edges and their orientations

Example of measuring at multiple orientations





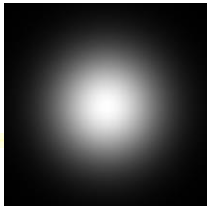
θ_1

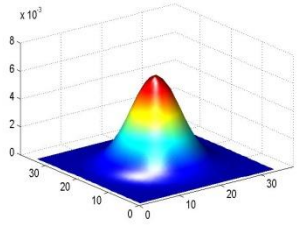
θ_2

θ_3

θ_4

Canny steps





- Apply Gaussian smoothing
  - 2D Gaussian filter over the whole image
  - Can actually be done with H and V 1D filters (much faster)
- Run 1D detectors at multiple orientations
  - Produce *edge strength* and *edge orientation* maps
- Run *nonmaximum suppression* to eliminate extra edge points, producing only one per true edge point (ideally)
- Threshold to produce a binary (*edge* or *no edge*) edge image
  - Alter the threshold value depending on local edge information

The Canny edge detector

original image (Lena)

The Canny edge detector

Combine the 1D detectors

→ Edge strength map

The Canny edge detector

Thresholding

The Canny edge detector

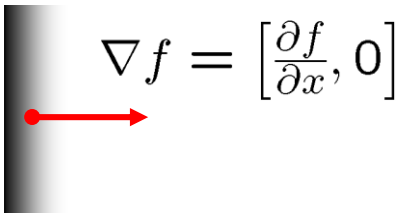
Thinning

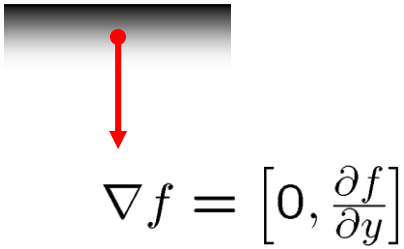
(non-maximum suppression)

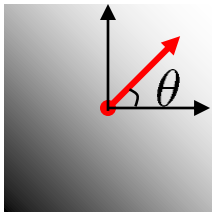
Image gradient

- The gradient of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

- The gradient points in the direction of most rapid change in intensity







The gradient direction is given by:

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$$

The *edge strength* is given by the gradient magnitude

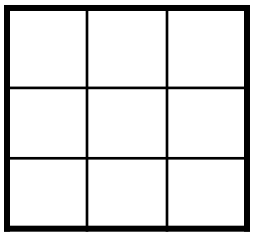
$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

The discrete gradient

- How can we differentiate a *digital* image f[x,y]?
  - Option 1: reconstruct a continuous image, then take gradient
  - Option 2: take discrete derivative (finite difference)

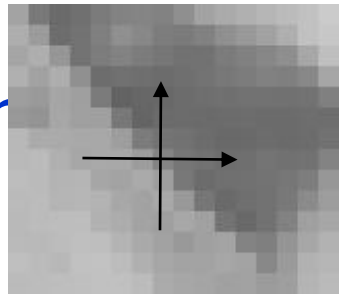
$$\frac{\partial f}{\partial x}[x, y] \approx f[x+1, y] - f[x, y]$$

How would you implement this as a correlation?



H

Other (simpler) edge detector



- Sobel detector

G_x

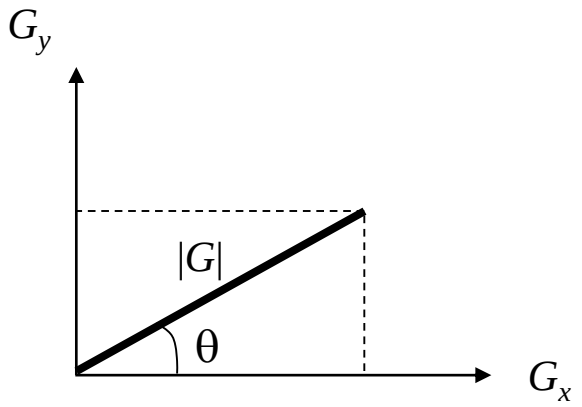
-1	0	1
-2	0	2
-1	0	1

G_y

-1	-2	-1
0	0	0
1	2	1

$$|G| = G_x^2 + G_y^2$$

$$\theta = atan\ G_y/G_x$$



- Prewitt detector

G_x

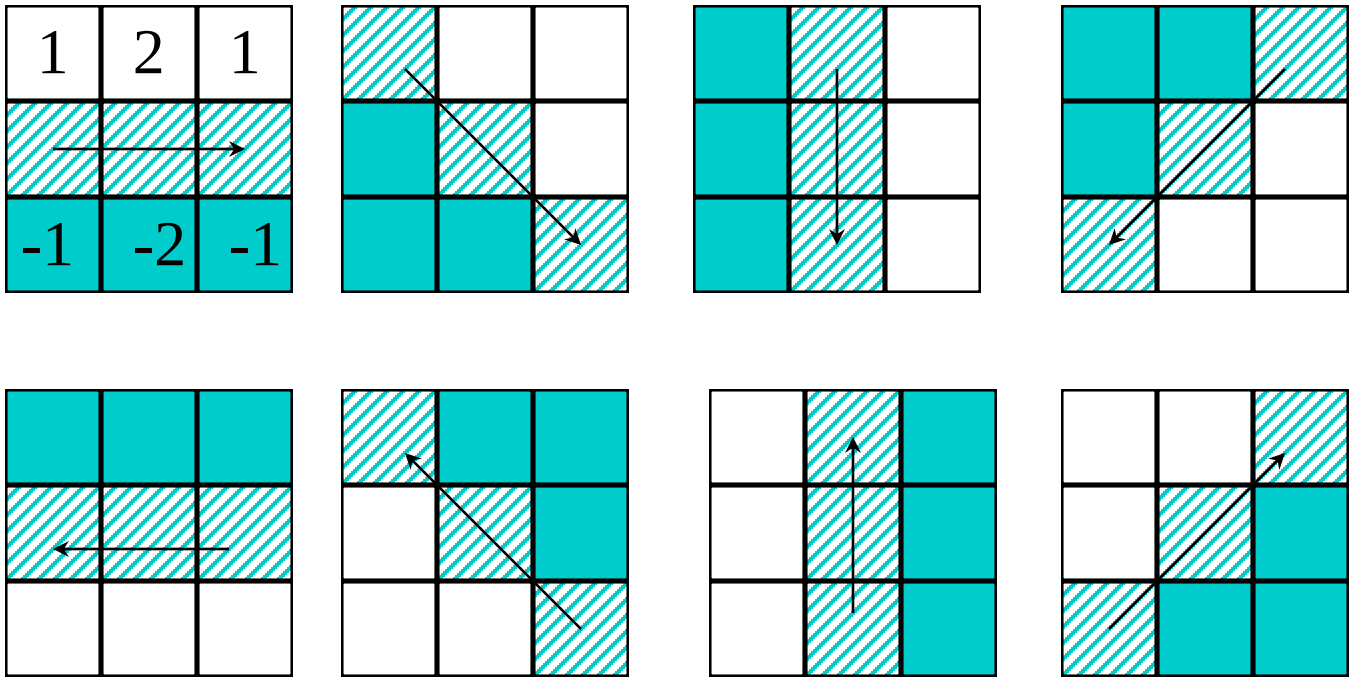
-1	0	1
-1	0	1
-1	0	1

G_y

-1	-1	-1
0	0	0
1	1	1

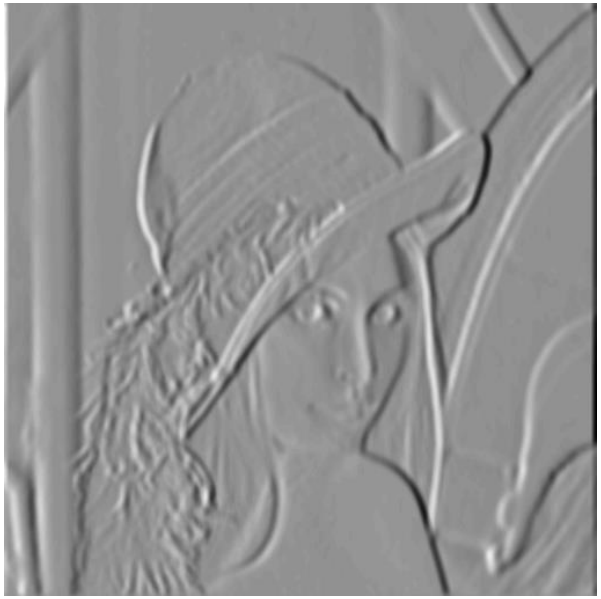
Other Possibilities

- 8-directional masks



Sobel example

Original

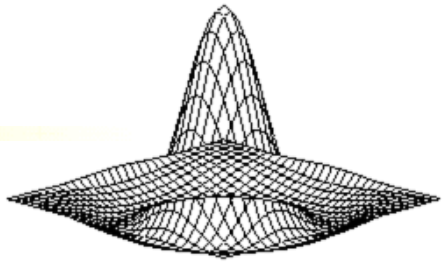


Vertical edges

Horizontal edges

Edge magnitude

More edge detectors



- Laplacian detectors
  - Single kernel

0	-1	0
-1	4	-1
0	-1	0

-1	-1	-1
-1	8	-1
-1	-1	-1

1	-2	1
-2	4	-2
1	-2	1

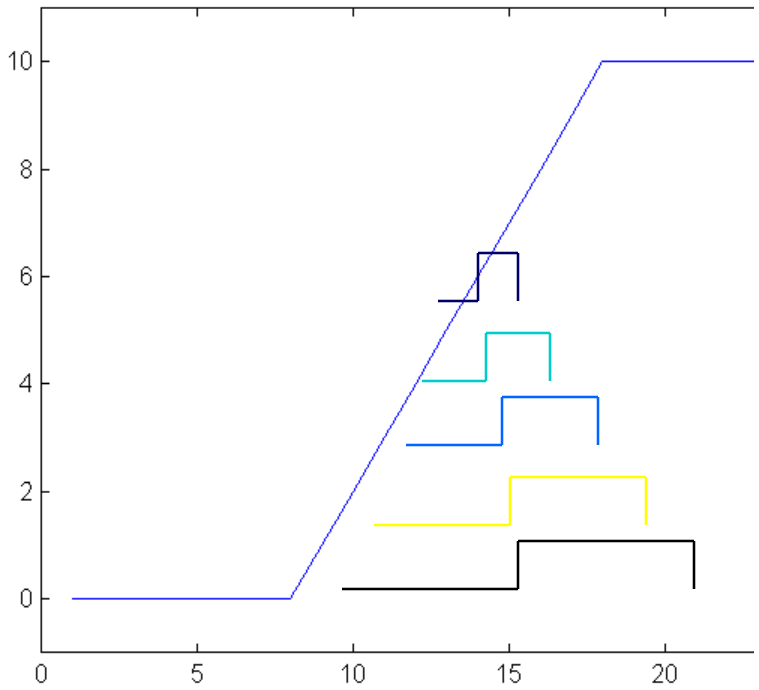
Etc.

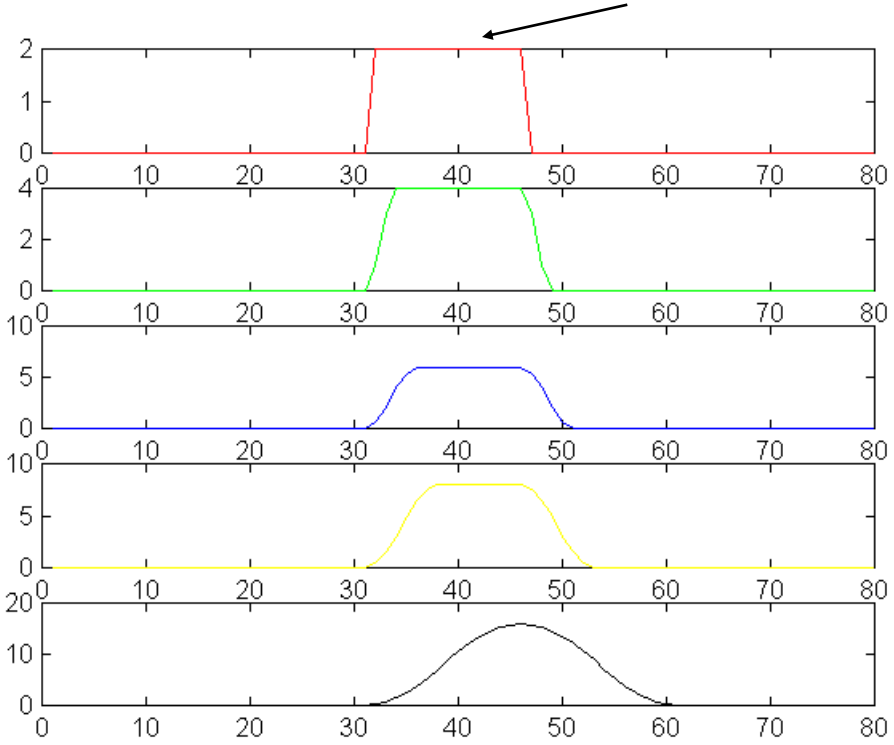
Edge detectors are not limited to 3x3 kernels

Multiple Scales

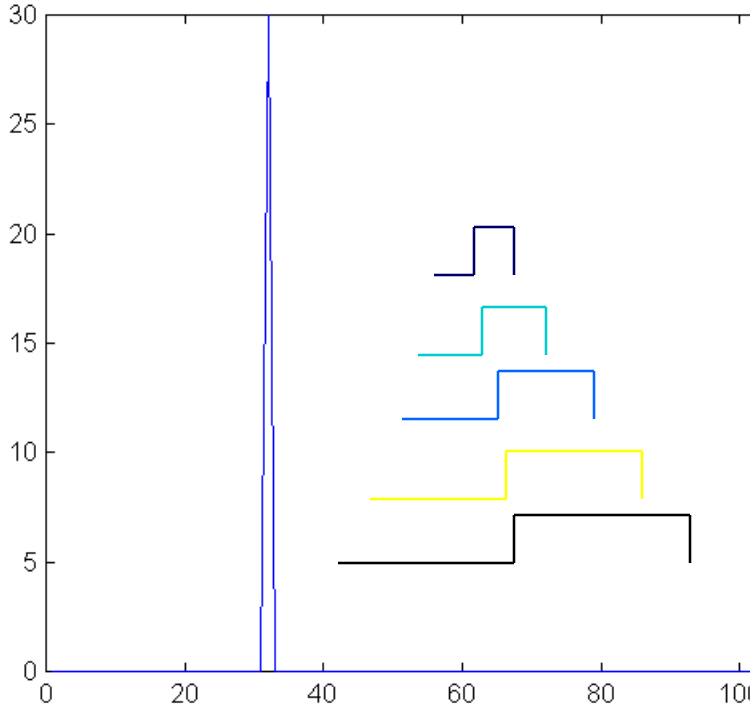
Bad localization

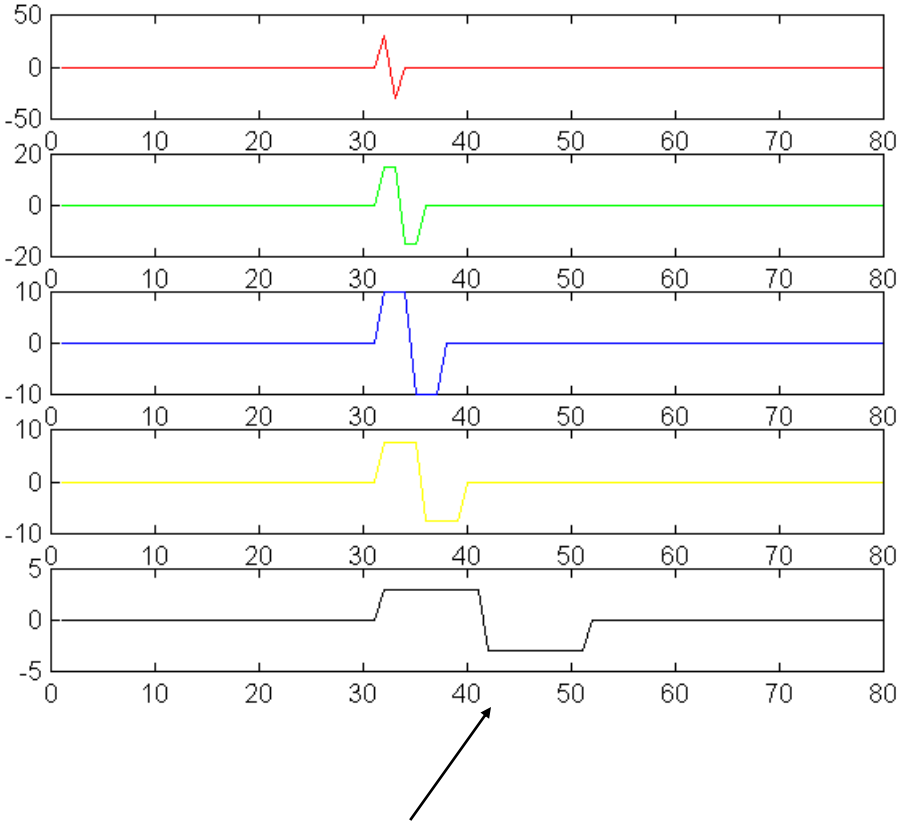
Where is the edge?





Multiple Scales (cont.)

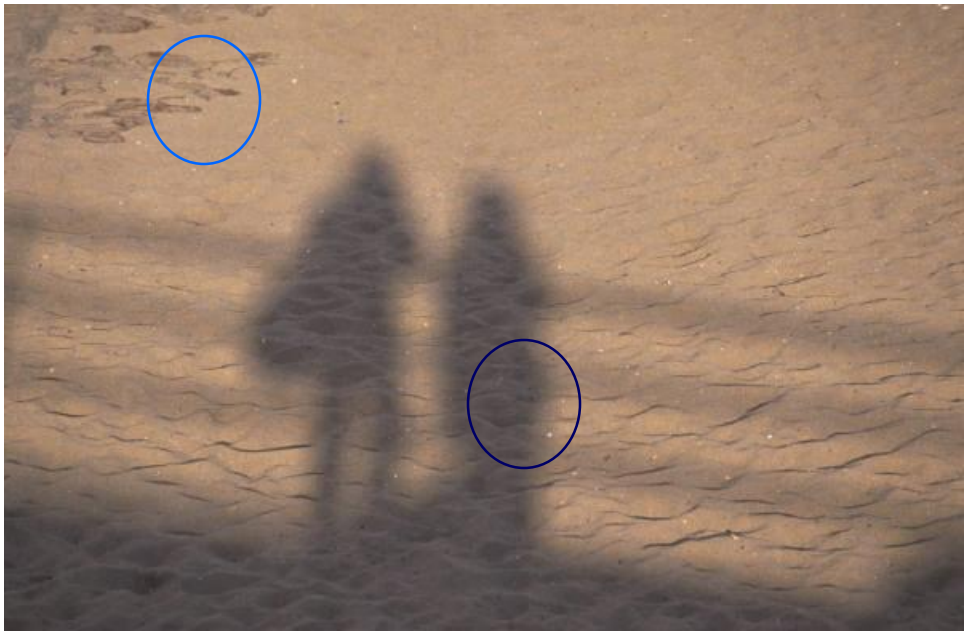




Bad localization

Where is the edge?

Sharp vs. Fuzzy



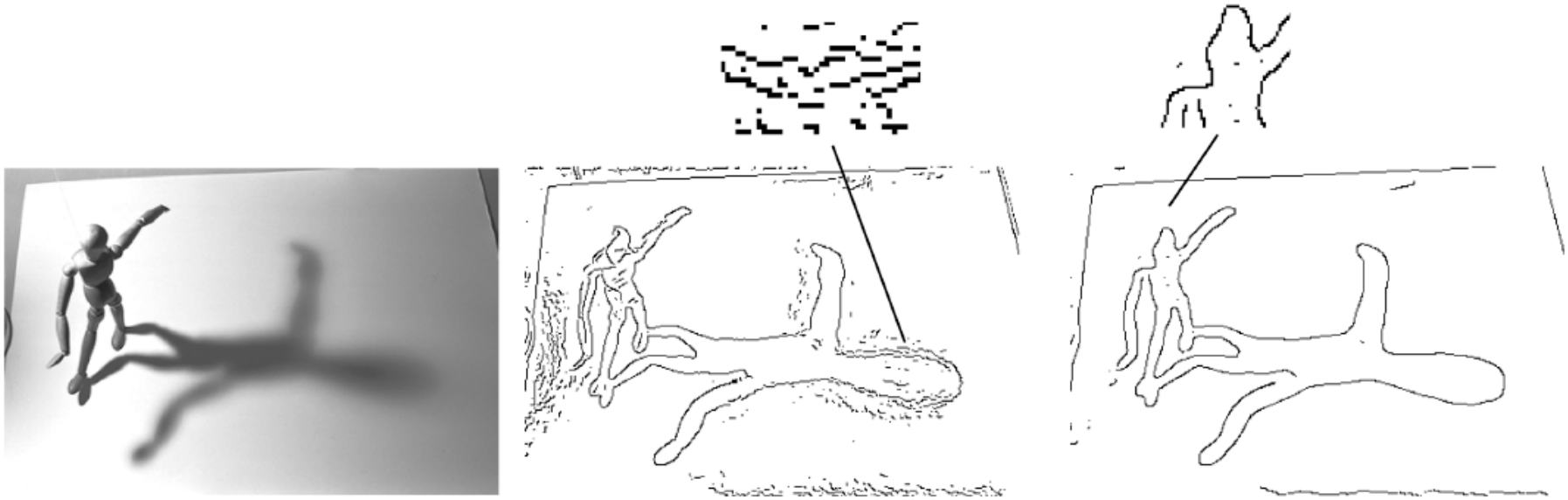
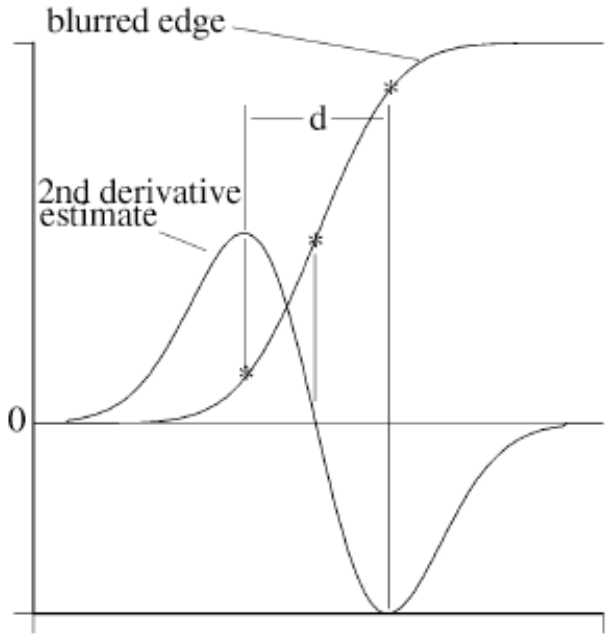
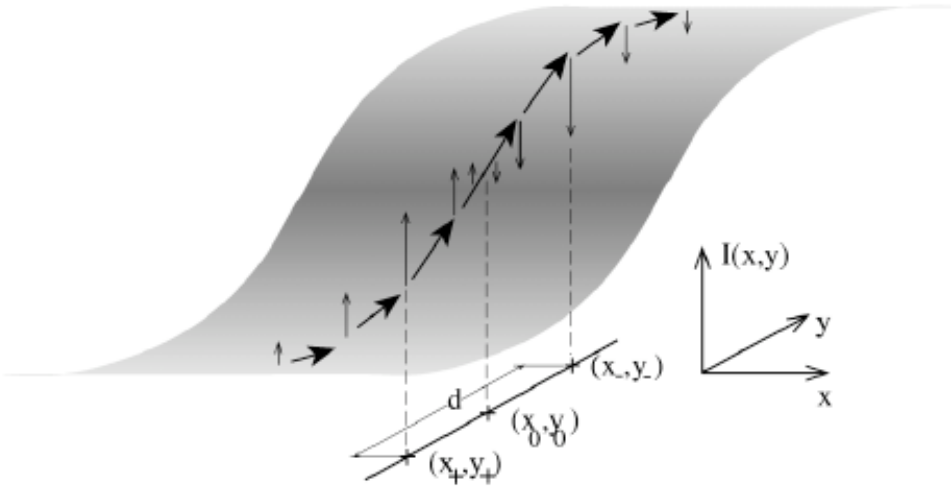


Fig. 2. The problem of local estimation scale. Different structures in a natural image require different spatial scales for local estimation. (a) The original image contains edges over a broad range of contrasts and blur scales. (b) The edges detected with a Canny/Deriche operator tuned to detect structure in the mannequin. (c) The edges detected with a Canny/Deriche operator tuned to detect the smooth contour of the shadow. Parameters are ($\alpha = 1.25$, $\omega = 0.02$) and ($\alpha = 0.5$, $\omega = 0.02$), respectively. See [2] for details of the Deriche detector.

Response to a blurred step edge





$$\sigma_b = \sqrt{\left(d/2^2\right) - \sigma_2^2}$$

Thus, the blur due to defocus can be estimated from the measured thickness of the contours, after compensation for the blur induced by the estimation itself.

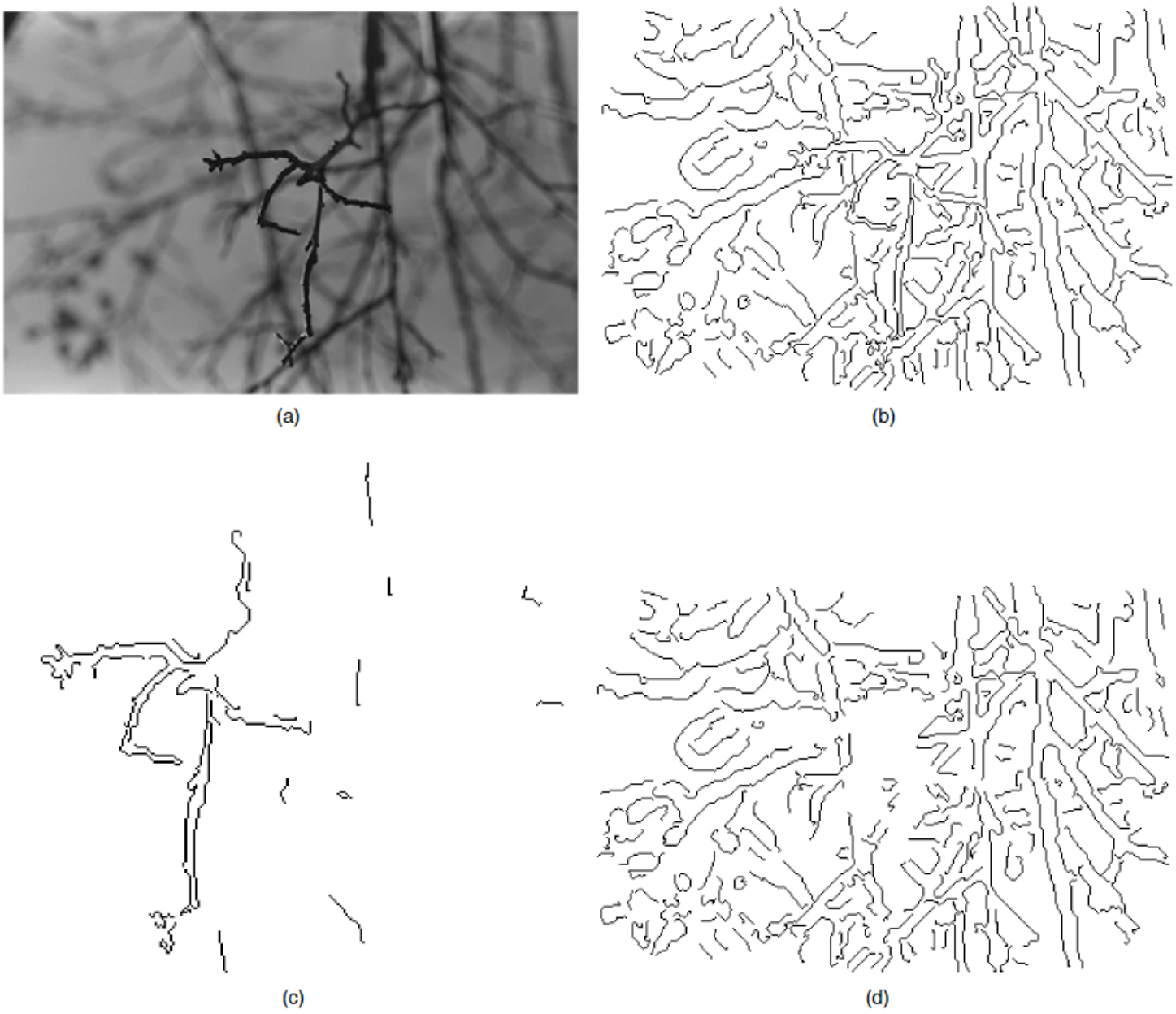
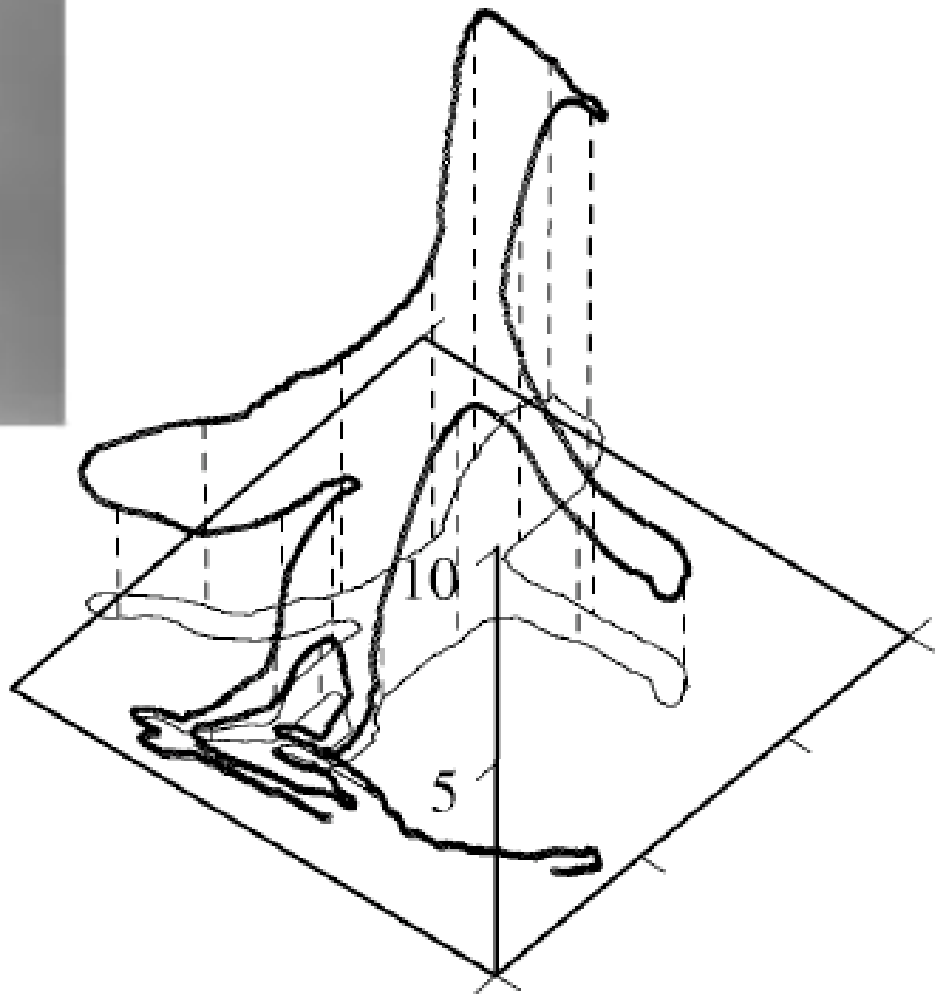


Fig. 10. Depth segmentation based on focal blur. (a) A photograph of tree branches with shallow depth of field (*f*/3.5) and near focus. (b) Edge map. (c) Foreground structure (focused contours). (d) Background structure (blurred contours).


10
5

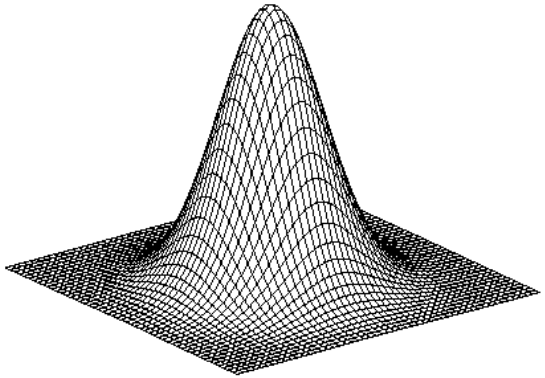
Multiple Scales (cont.)

- Basically, considering your operator myopic – that it uses only information in a small neighborhood of a pixel
- Why?
  - Saving in computation effort
  - Local coherency
- But how large should the neighborhood be?
  - Too small – not enough information
  - Too large – Too noisy, with multiple edges

Multiple Scales (cont.)

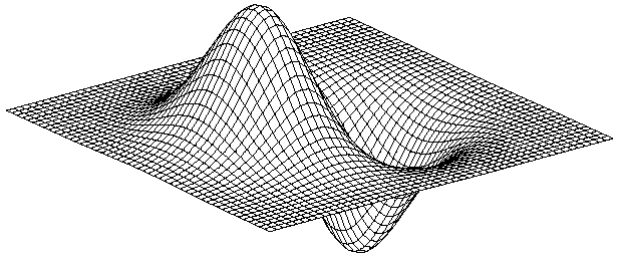
- No single scale will suffice
  - E.g. sharply focused objects have fast transition edges, out-of-focus objects have slow transition edges
- Need operators that are tuned to edges of different scales
- For an operator with a large processing window, the content can be noisy (multiple edge presence)
  - Need a mechanism to smooth out small edges

2D edge detection filters



Gaussian

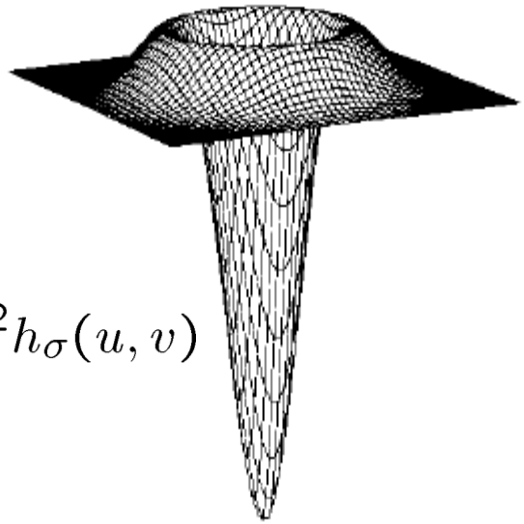
$$h_\sigma(u,v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$



derivative of Gaussian

$$\frac{\partial}{\partial x} h_\sigma(u,v)$$

Laplacian of Gaussian

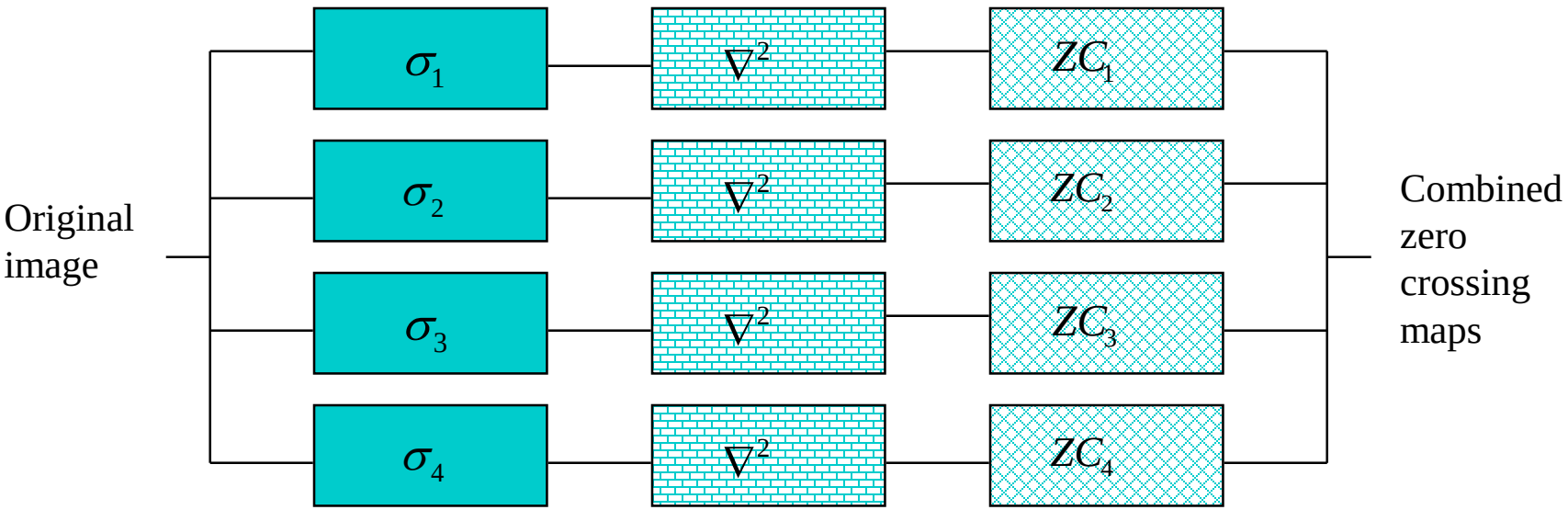


$$\nabla^2 h_\sigma(u,v)$$

∇^2 is the **Laplacian** operator:

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

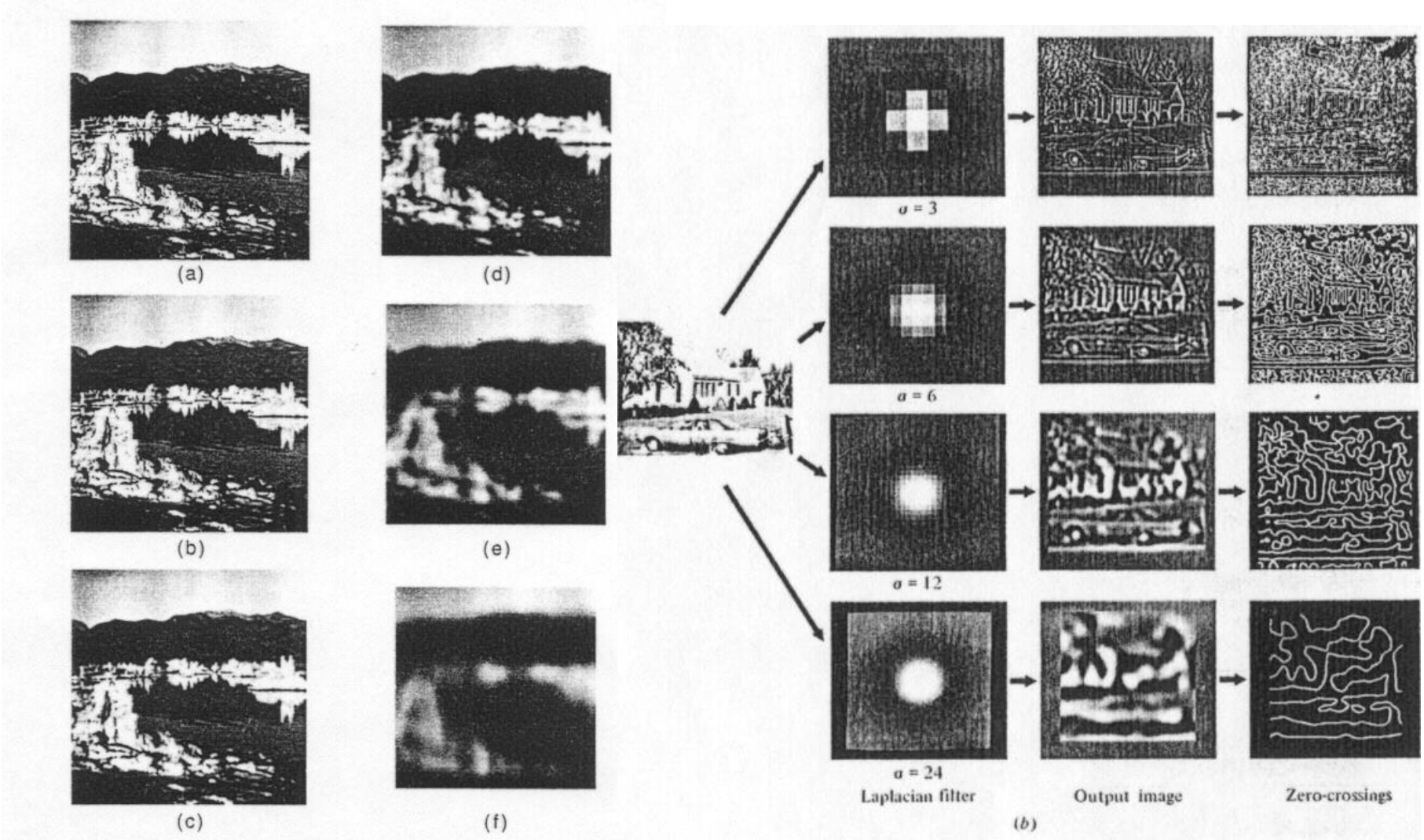
Multiple Scales

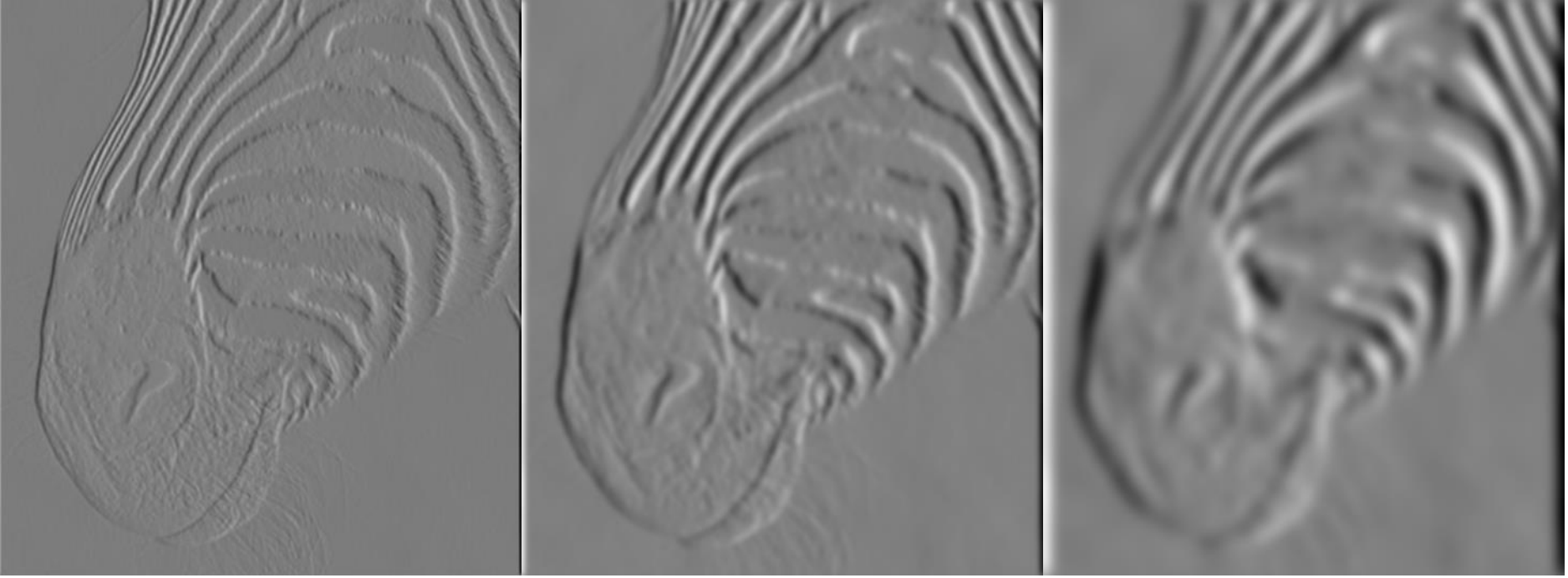


$$G = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} \qquad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

$$\nabla^2 G = \frac{1}{\pi\sigma^4}\left(1 - \frac{x^2+y^2}{2\sigma^2}\right) e^{-\frac{x^2+y^2}{2\sigma^2}}$$

Multiple Scales



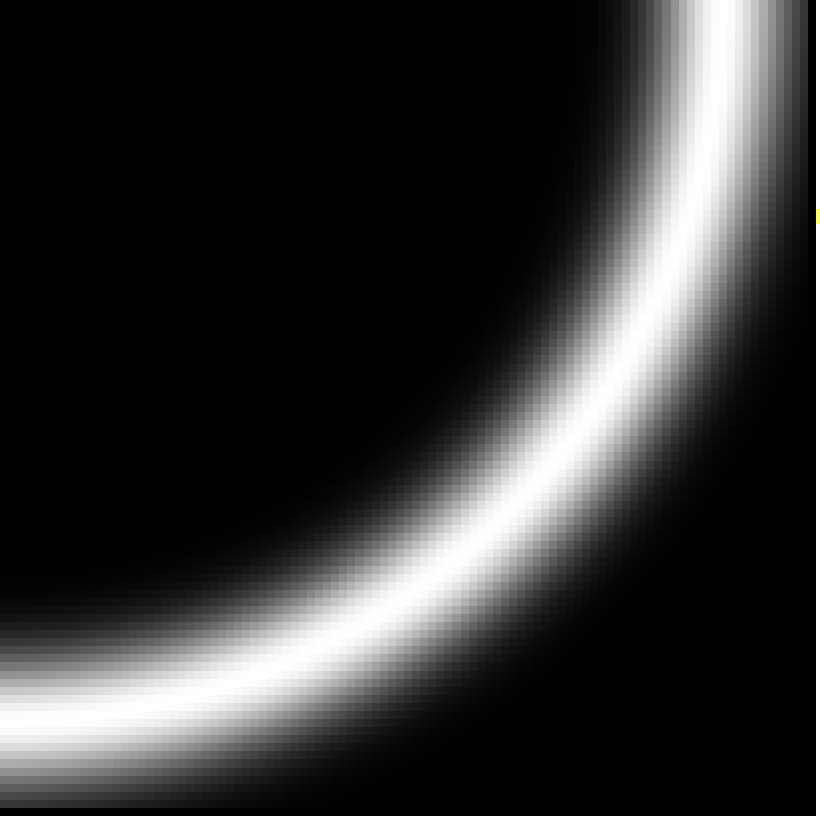


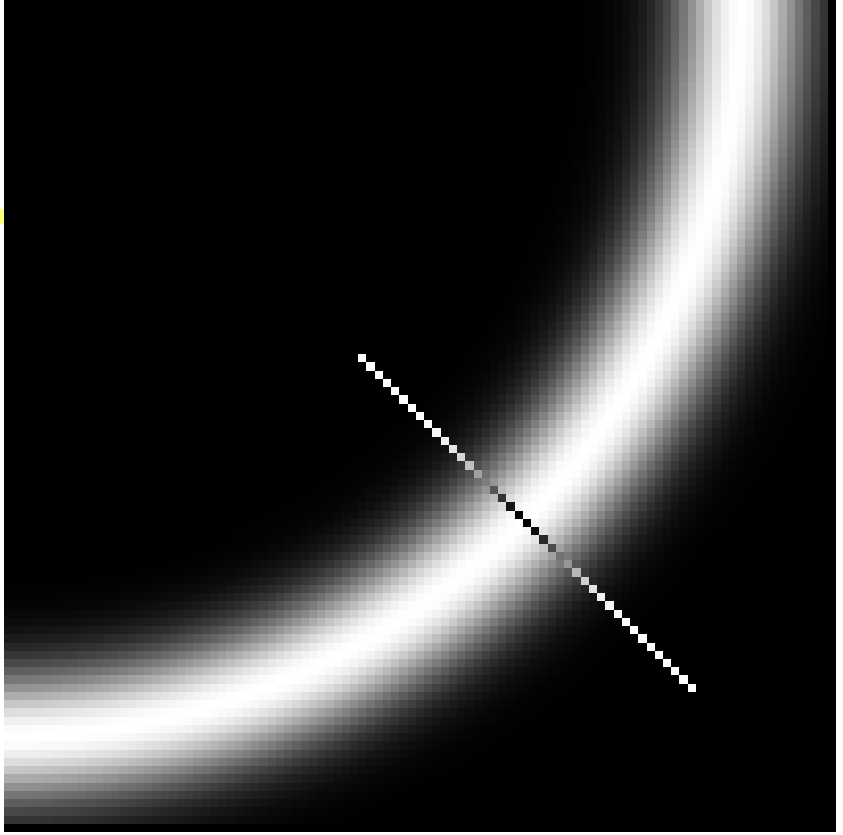
1 pixel

3 pixels

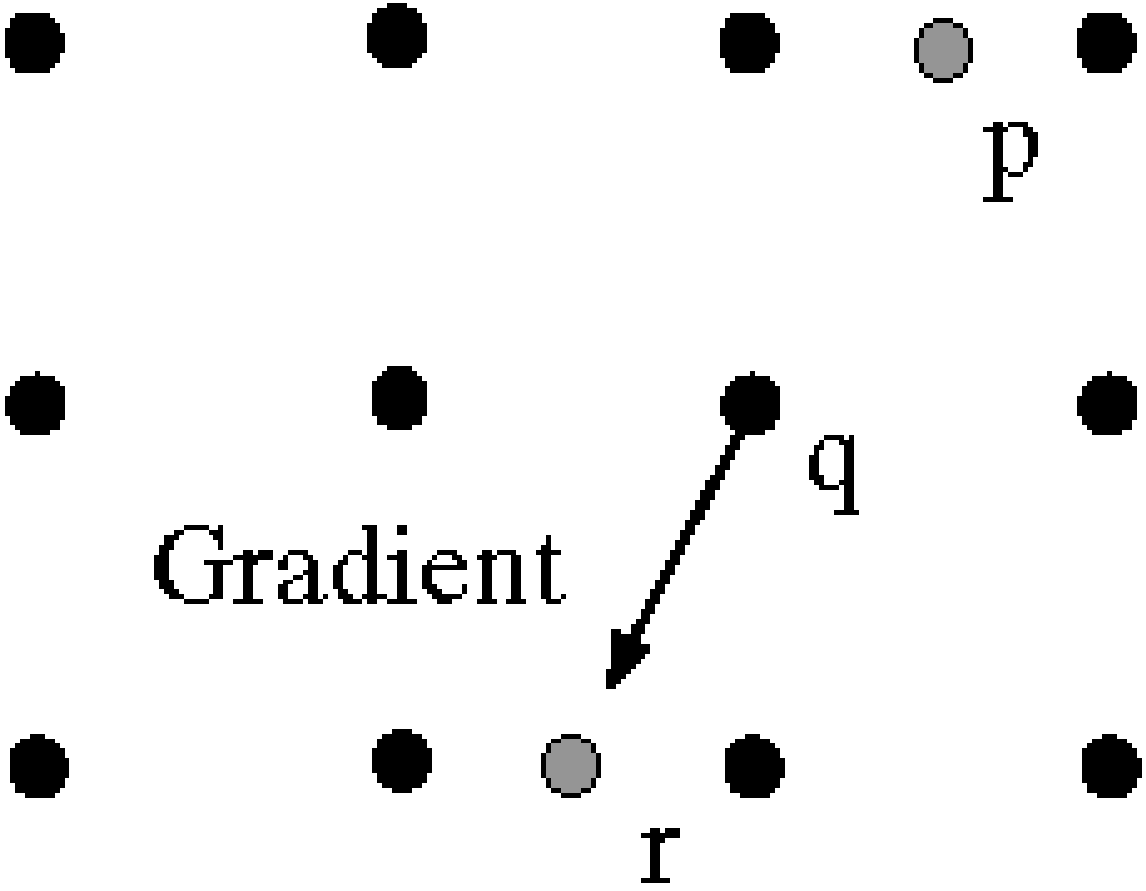
7 pixels

The scale of the smoothing filter affects derivative estimates, and also the semantics of the edges recovered.



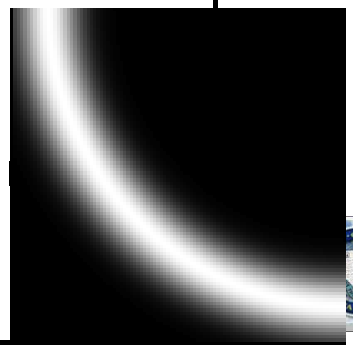


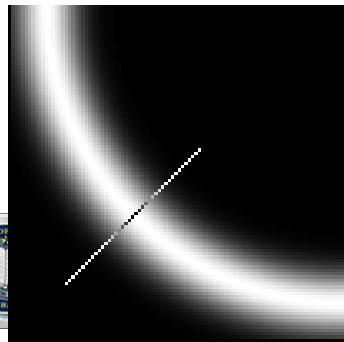
We wish to mark points along the curve where the magnitude is biggest. We can do this by looking for a maximum along a slice normal to the curve (non-maximum suppression). These points should form a curve. There are then two algorithmic issues: at which point is the maximum, and where is the next one?

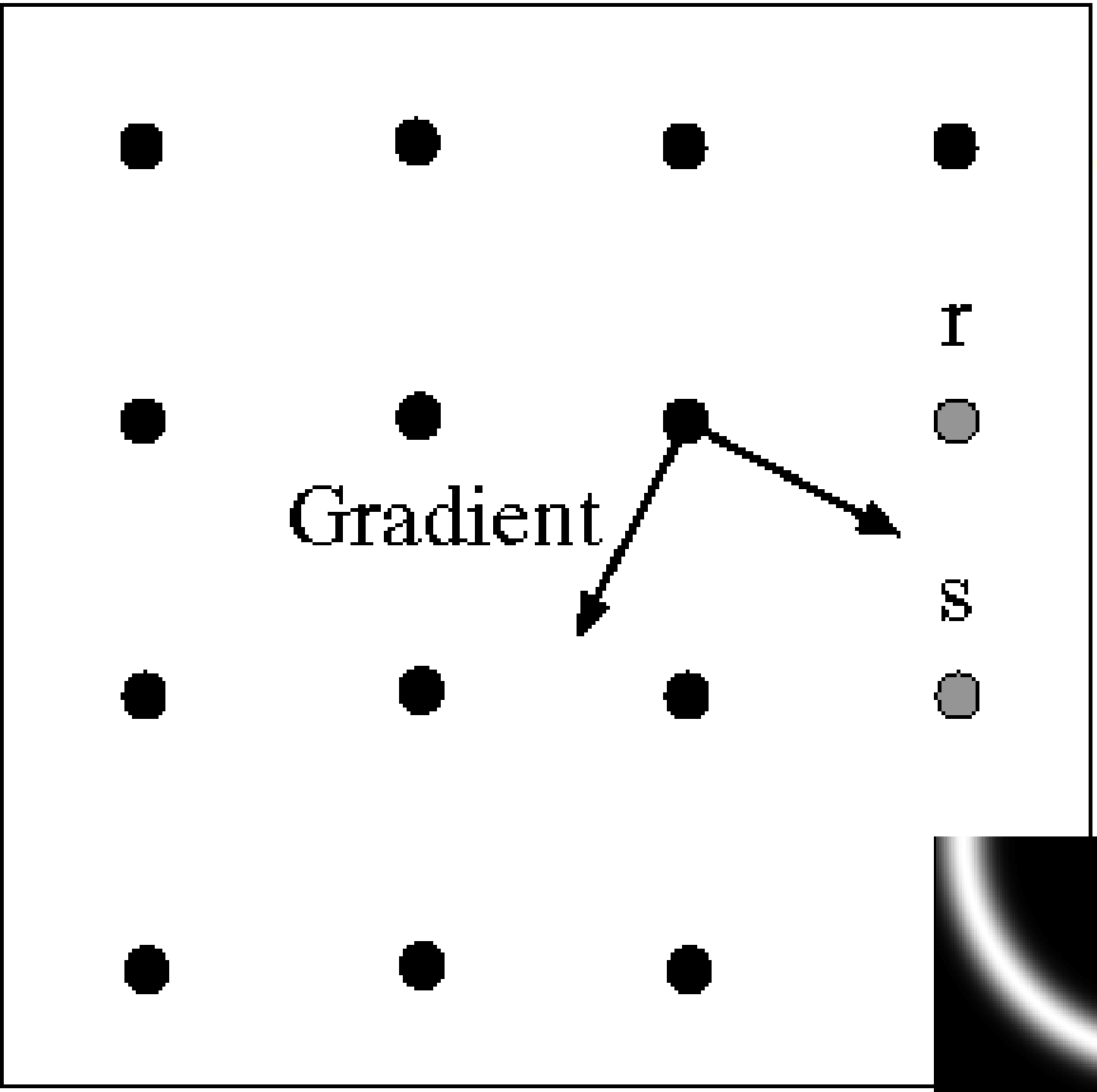


Non-maximum suppression

At q, we have a maximum if the value is larger than those at both p and at r. Interpolate to get these values.

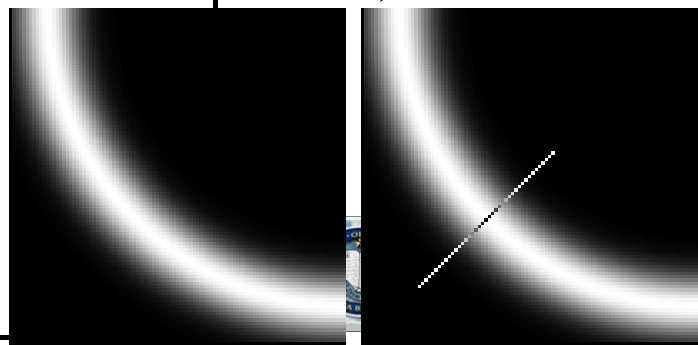


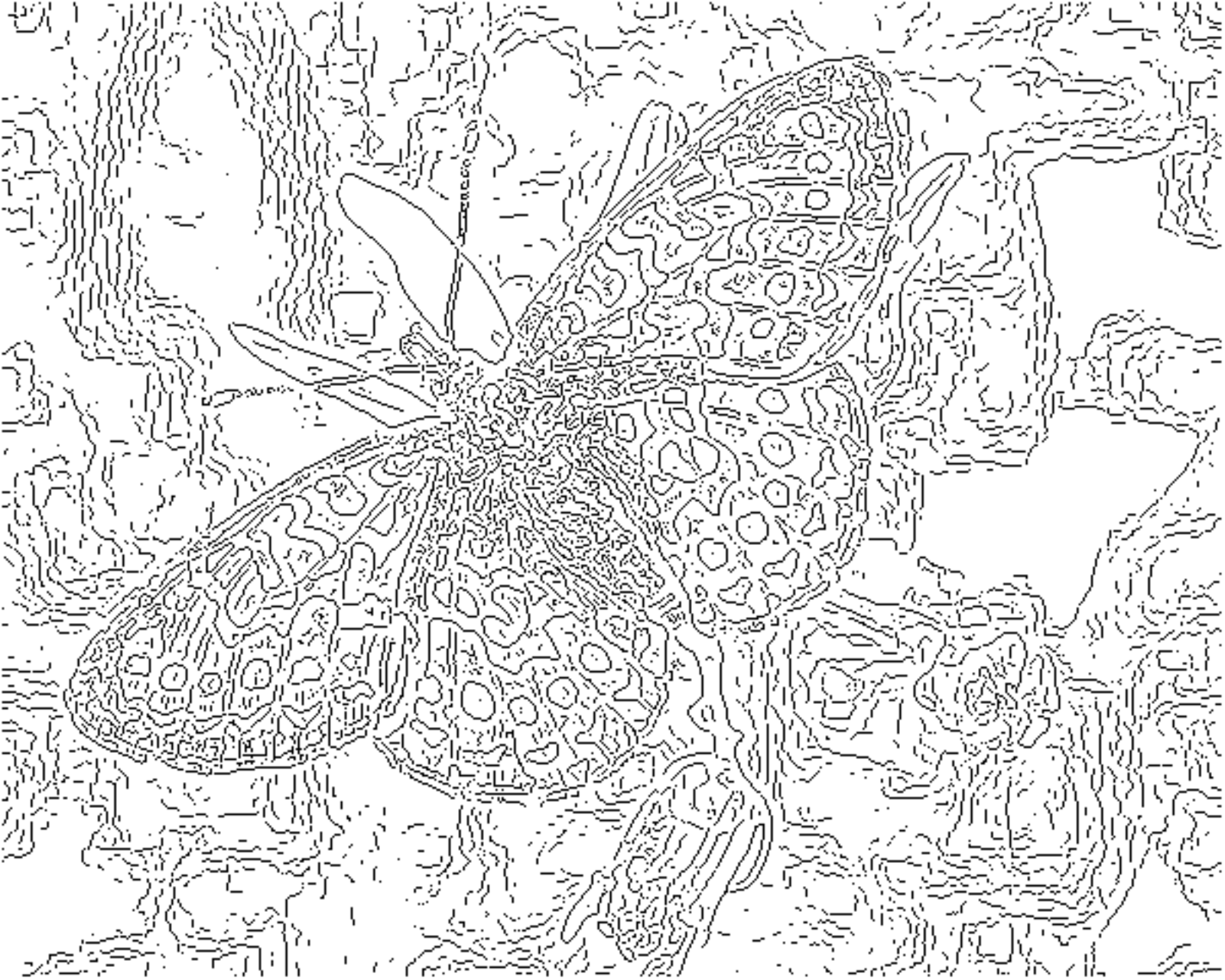




Predicting the next edge point

Assume the marked point is an edge point. Then we construct the tangent to the edge curve (which is normal to the gradient at that point) and use this to predict the next points (here either r or s).





fine scale
high
threshold

coarse scale, high threshold

coarse
scale
low
threshold

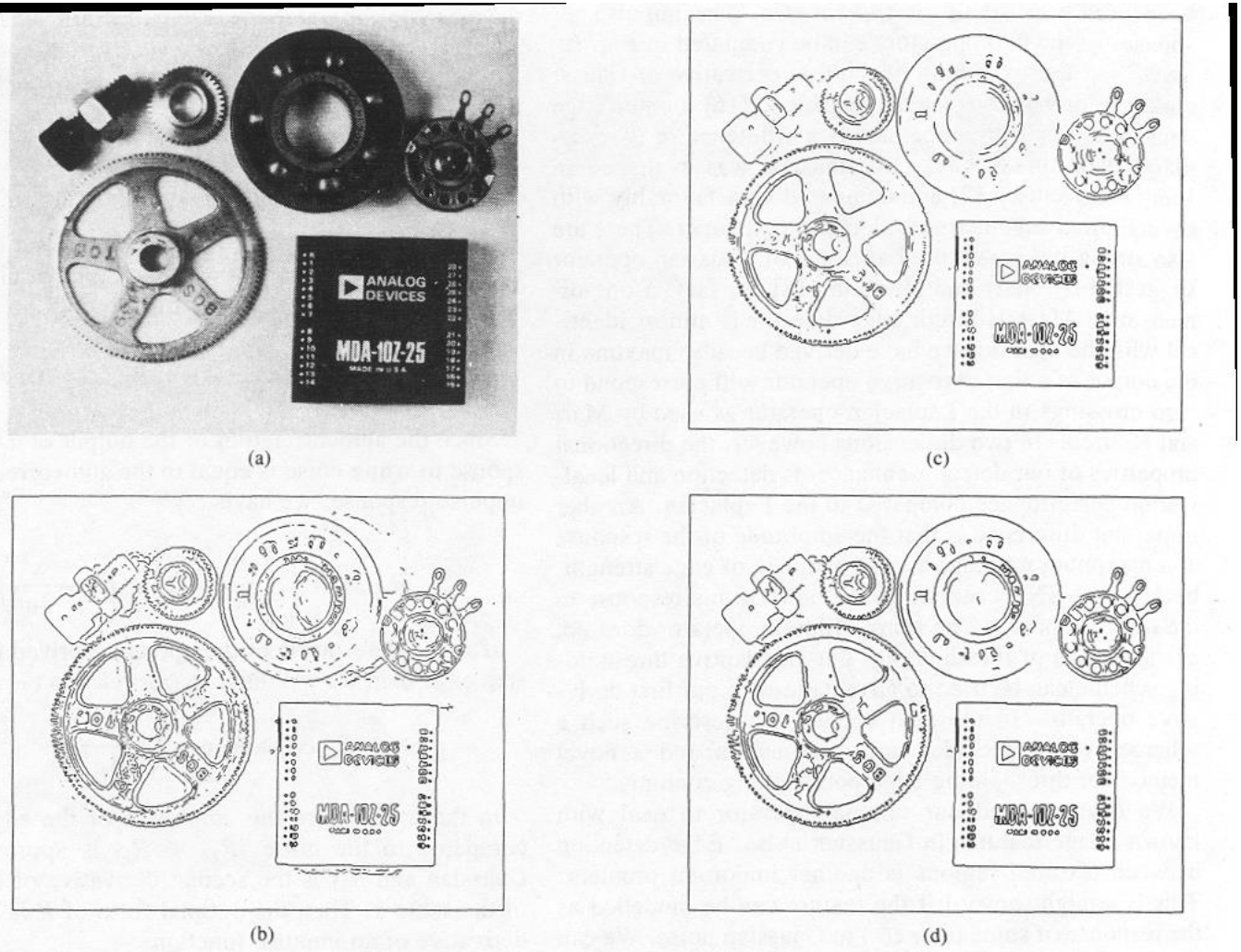


Fig. 7. (a) Parts image, 576 by 454 pixels. (b) Image thesholded at T_1. (c) Image thresholded at 2 T_1. (d) Image thresholded with hysteresis using both the thresholds in (a) and (b).

Don't Use Afterward

Linear filters

- Three important properties of *linear*, *shift-invariant systems:*
  - Superposition
    - $R(f + g) = R(f) + R(g)$
  - Scaling
    - $R(kf) = k\,R(f)$
    - $R(af + bg) = aR(f) + bR(g)$

Linear system

  - Shift invariance
    - Translation of stimulus → translation of response
    - $h(t) = R(g(t)) \rightarrow h(t+k) = R(g(t+k))$

System Theory

- What is a system?
- How to study a system?

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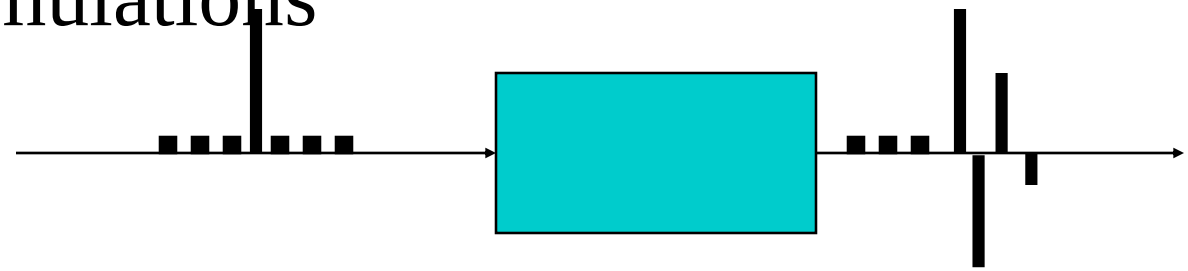
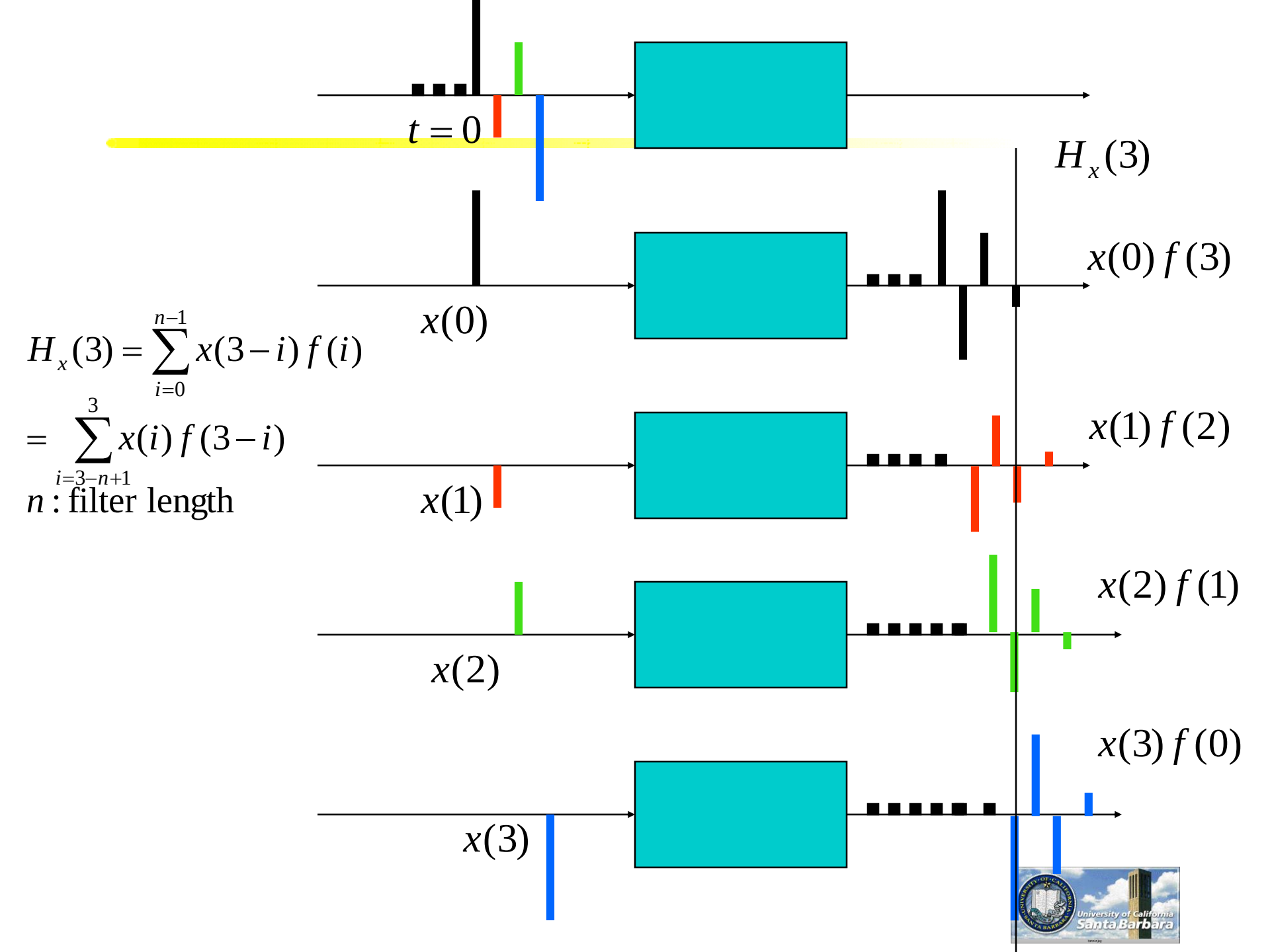
Impulse Response

- The response to the simplest stimulation (an impulse)
- Using impulse response and linear, time invariant behavior, one can predict exactly what the system will do to any *arbitrary* stimulations





$$H_x(3) = \sum_{i=0}^{n-1} x(3-i)\,f\,(i)$$

$$= \sum_{i=3-n+1}^{3} x(i)\,f\,(3-i)$$

n : filter length

In General

- The filter response is a "convolution" of input and filter's impulse response function

$$H_x(j) = \sum_{i=0}^{n-1} x(j-i) f(i) = \sum_{i=j-n+1}^{j} x(i) f(j-i)$$

- In continuous domain, summation becomes integration

$$H_x(t) = \int_{u=0}^{u=w} x(t-u) f(u) du = \int_{u=t-w}^{u=t} x(u) f(t-u) du$$

Edge Detection

- In 2D:
  - Edges separate regions
- In 3D:
  - Occluding edges (jump boundaries) separate objects from background
  - Internal edges separate surface patches of different orientation
  - Texture edges separate surface areas of different patterns

Terminology

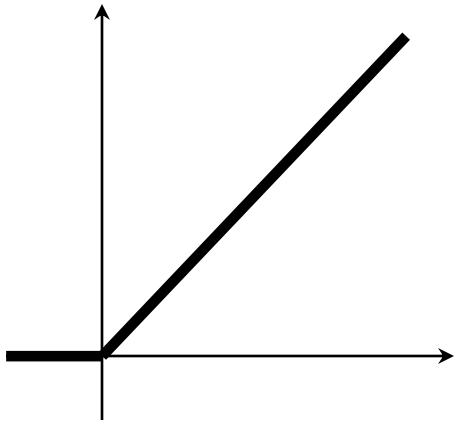
- Edges:
  - Represent abrupt jumps or discontinuities of image brightness
- Edge detection:
  - To accentuate the above image features
- Edge maps:
  - Edge pixel marked as 1
  - Nonedge pixel marked as 0
- Strictly speaking, this is an “image to image” transform topic and should belong in ECE 178 (but the goal is usually to prepare images for further analysis)

Approaches

- How do you model an edge profile?
- How do you transform an edge profile to a 0, 1 edgemap?
- How good is the approach in the presence of
  - Noise
  - Multiple scales
  - Etc.

Useful Mathematics Funcs.

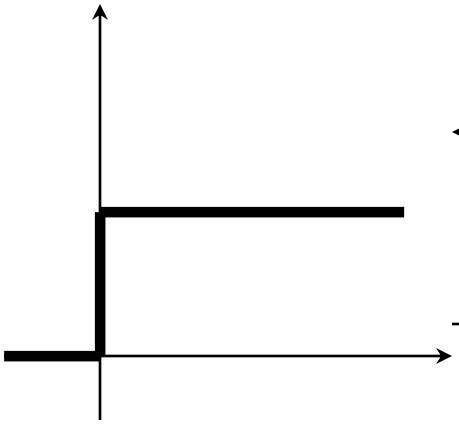
Ramp



$\int dx$

$\frac{d}{dx}$

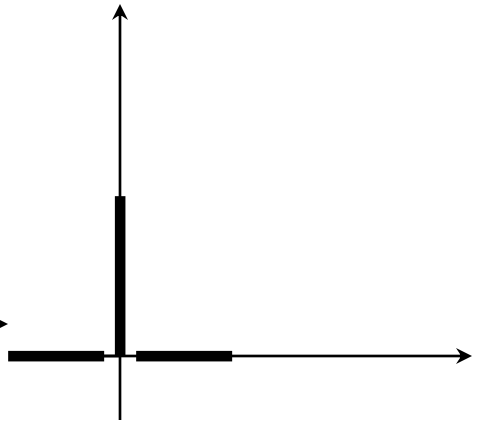
Step



$\int dx$

$\frac{d}{dx}$

Impulse



$$R(x) = \begin{cases} 0 & x < 0 \\ 0 & x = 0 \\ x & x > 0 \end{cases}$$

$$U(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2} & x = 0 \\ 1 & x > 0 \end{cases}$$

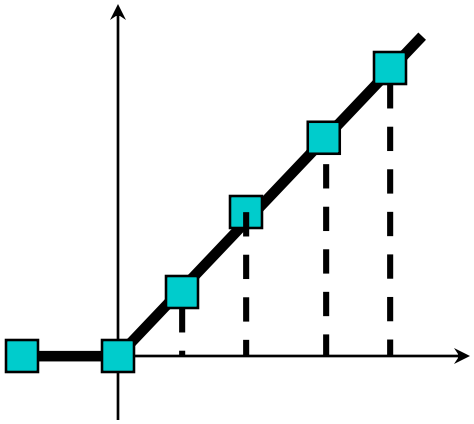
$$\delta(x) = \begin{cases} 0 & x < 0 \\ 1 & x = 0 \\ 0 & x > 0 \end{cases}$$

Useful Math Functions

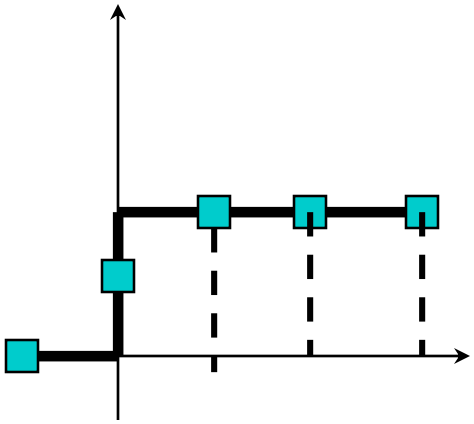
- Strictly speaking, those functions do not have derivatives at zero (the functions are not even continuous at that point!)
- Might be easier to think in terms of sampled (discrete) versions

$$\frac{df}{dx} \cong \frac{f(x+\Delta x)-f(x)}{\Delta x} \cong f(n+0.5)-f(n-0.5)$$

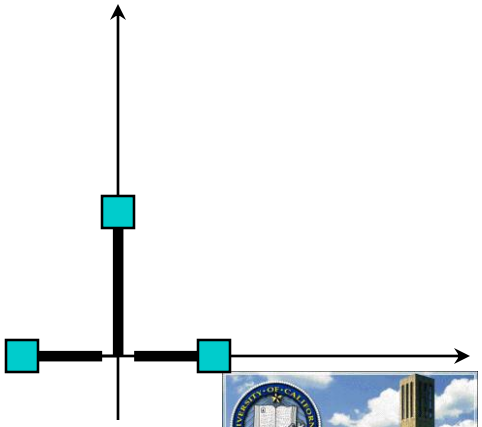
Ramp



Step



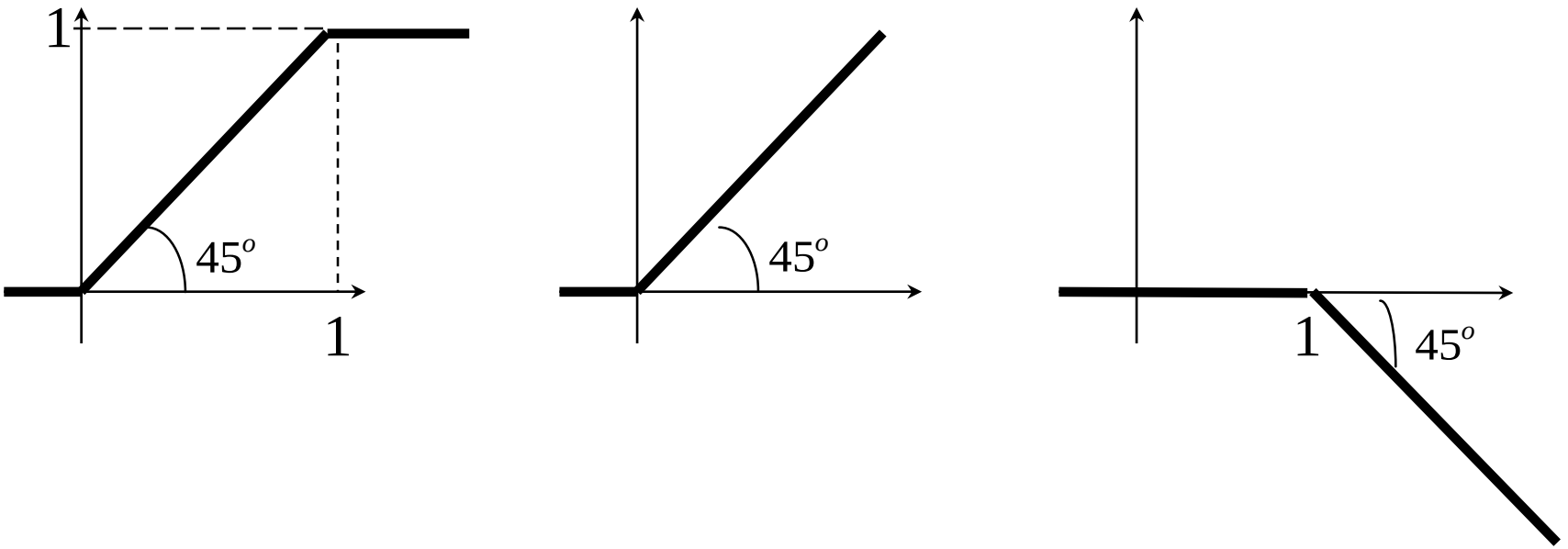
Impulse



Modeling

- Use these functions to
  - Create 1-D edge profile
  - Study method for accentuation (transformation)

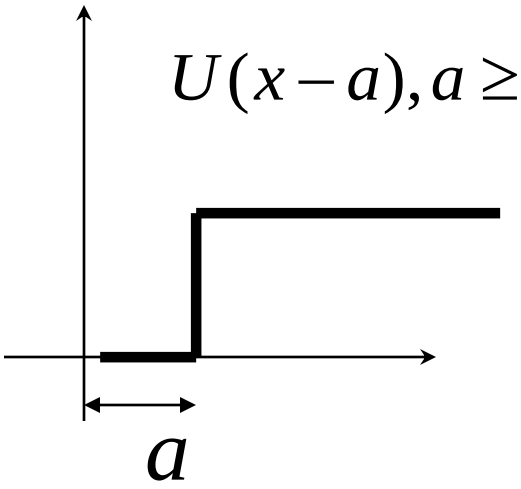
simple ramp edge = *R(x)* + *-R(x-1)*

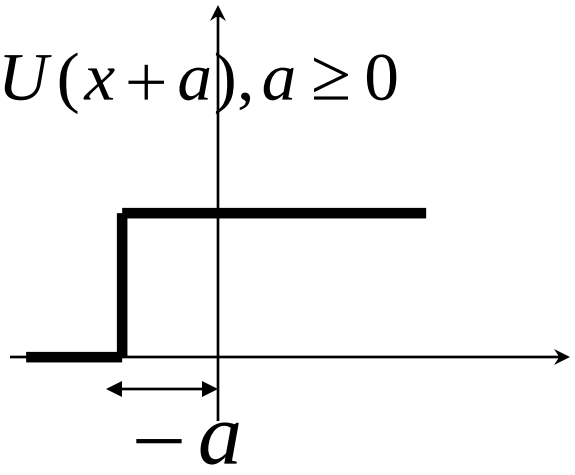


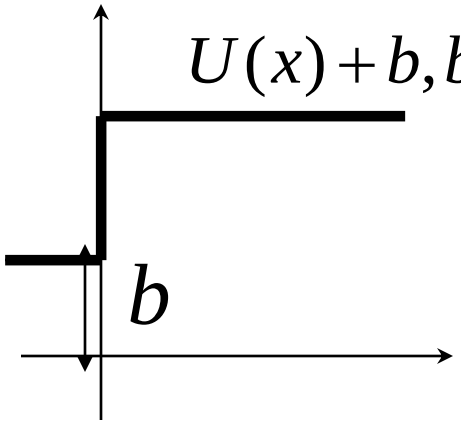
Useful Tricks

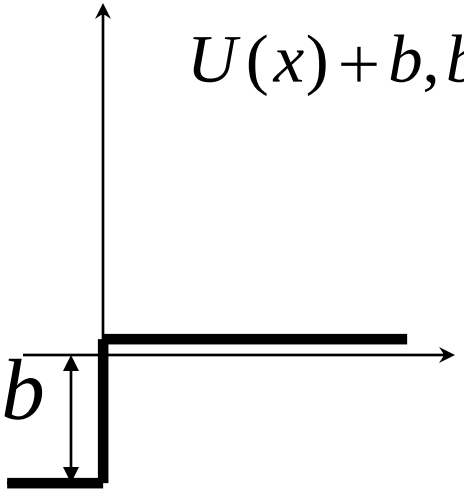
- To shift a function left and right $U(x+a)$
- To move a function up and down $U(x) + b$
- To scale a function $c \cdot U(x)$
- A combination of above $c \cdot U(x+a) + b$

Useful Tricks (cont.)



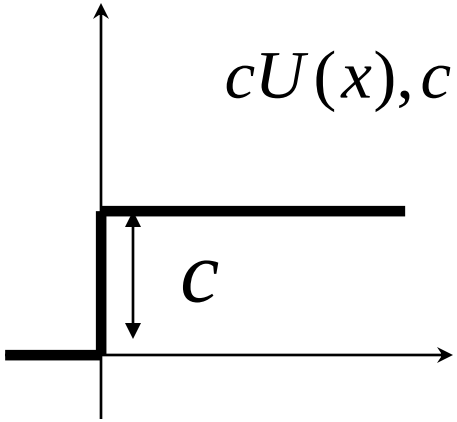




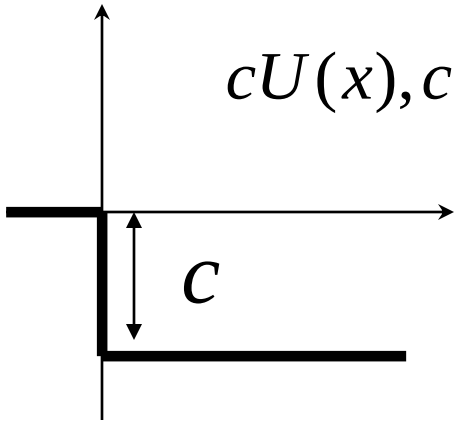


Useful Tricks (cont.)

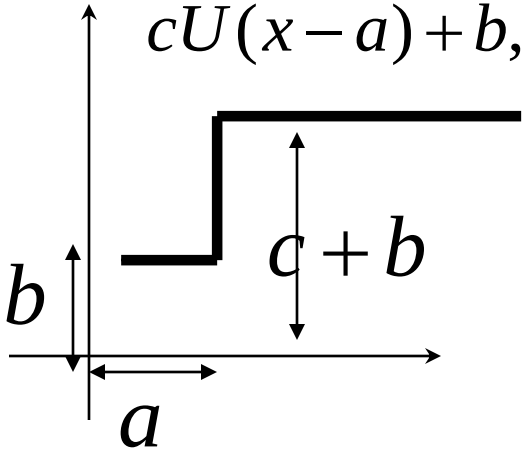
$cU(x), c \geq 0$



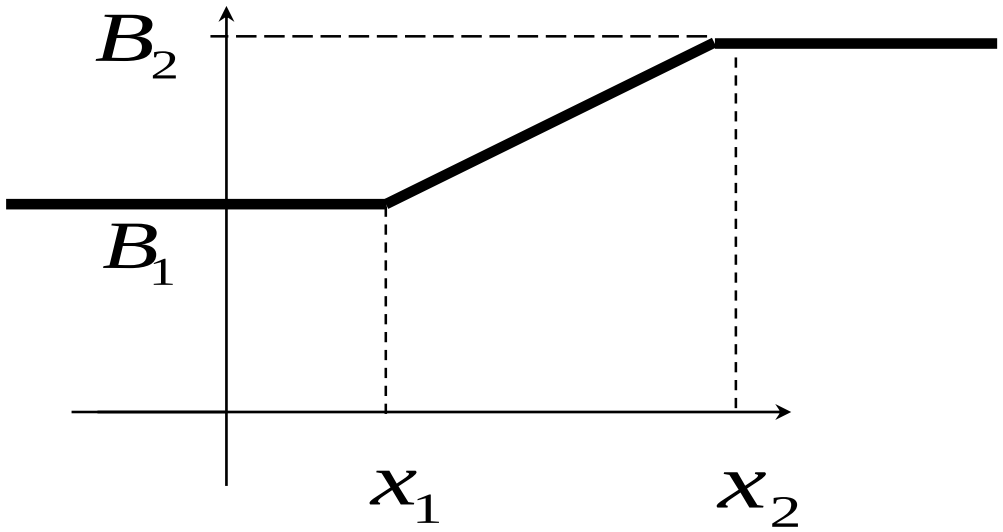
$cU(x), c < 0$



$cU(x-a)+b, a, b, c \geq 0$



Building a General Ramp Edge

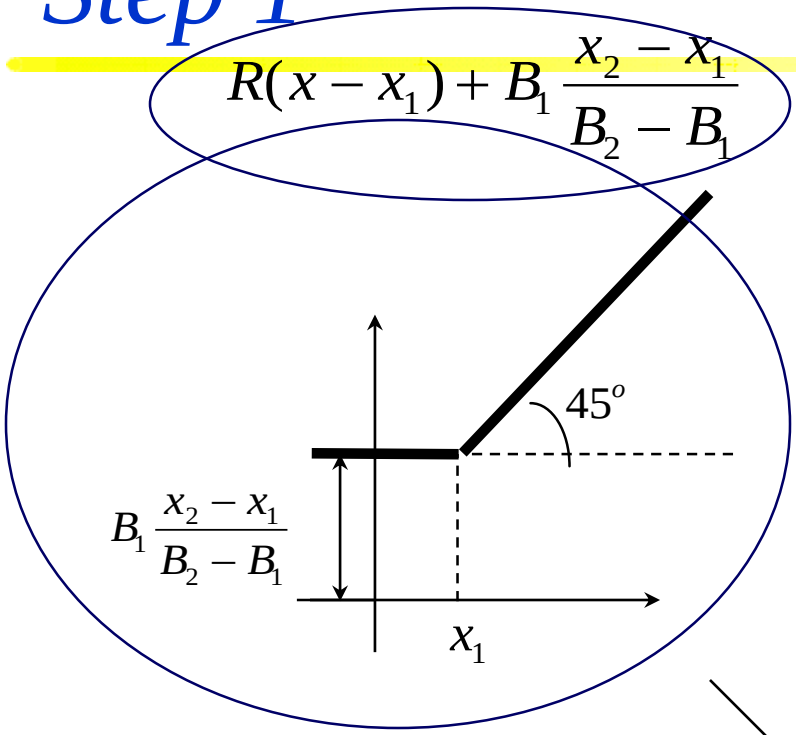


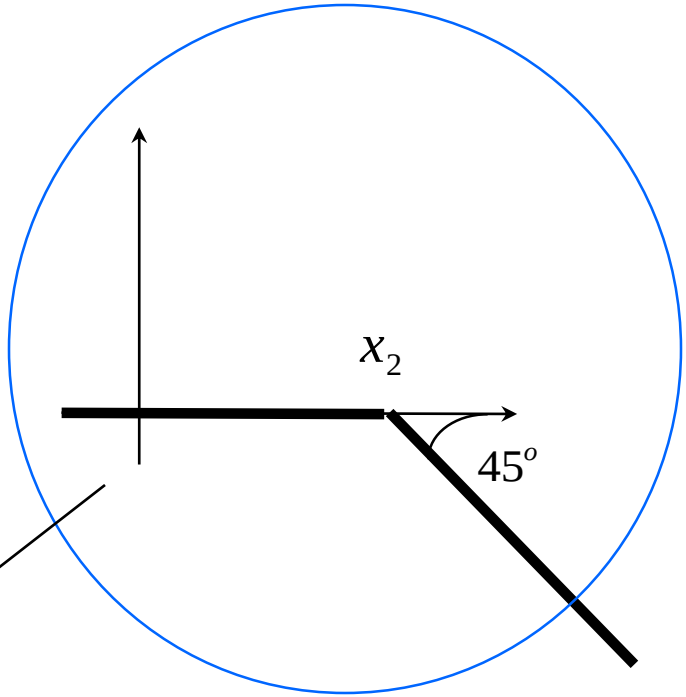
Step 1

$$R(x-x_1)+B_1\frac{x_2-x_1}{B_2-B_1}$$

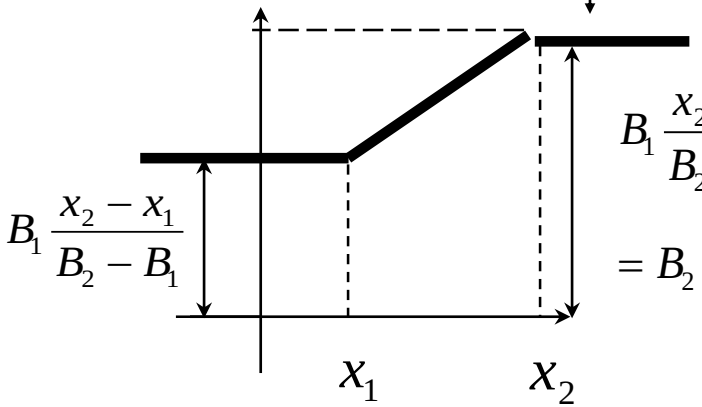
$+$

$$-R(x-x_2)$$





$=$



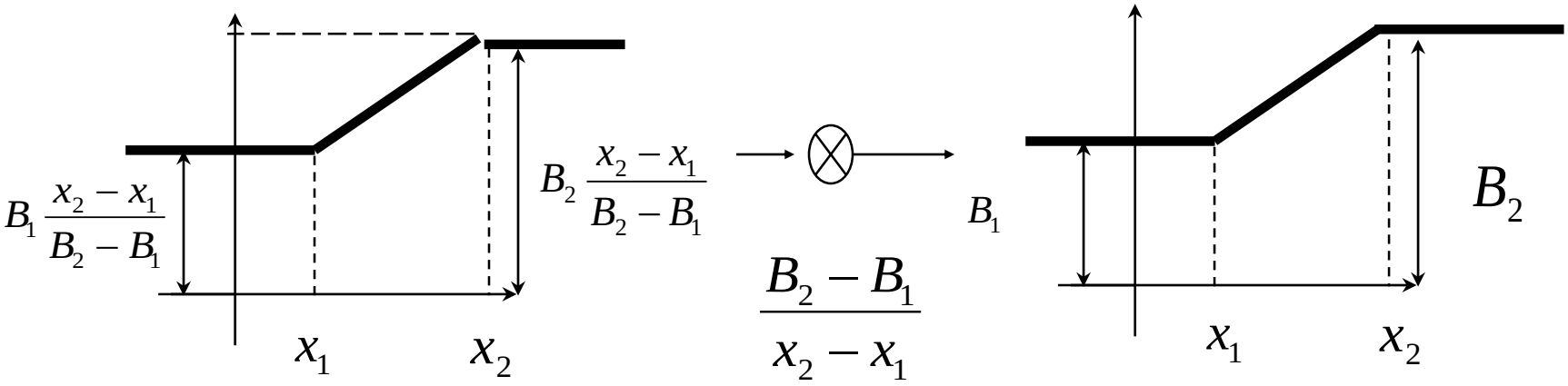
$$B_1\frac{x_2-x_1}{B_2-B_1}+(x_2-x_1)$$

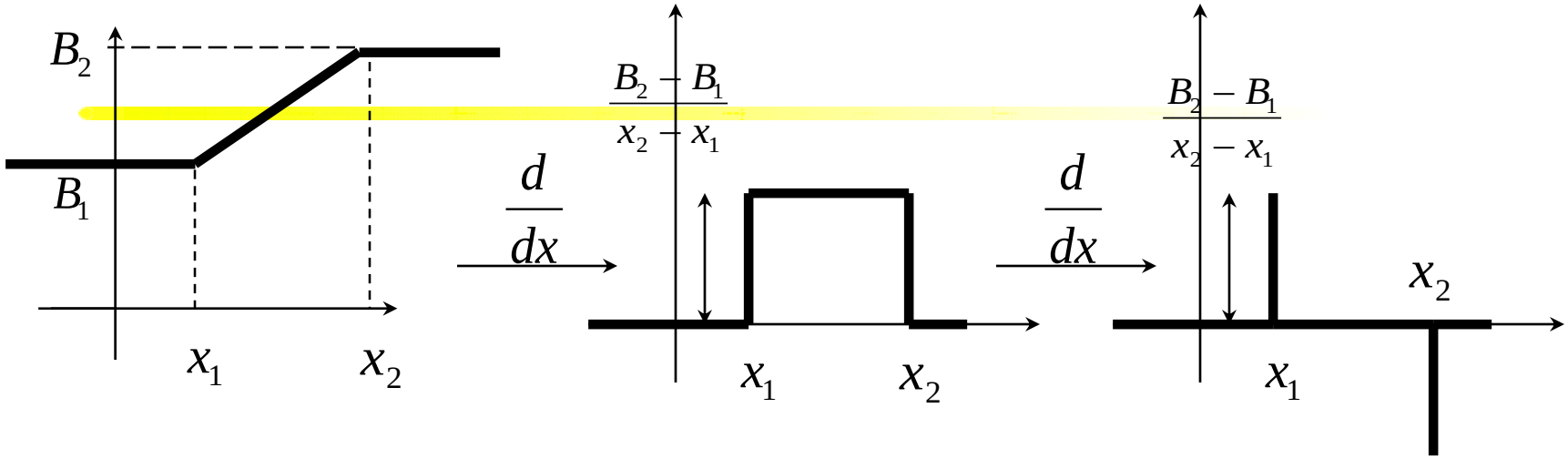
$$=B_2\frac{x_2-x_1}{B_2-B_1}$$

Step 2

$$\frac{B_2 - B_1}{x_2 - x_1}\{R(x - x_1) + B_1 + \frac{x_2 - x_1}{B_2 - B_1} - R(x - x_2)\}$$

$$= B_1 + \frac{B_2 - B_1}{x_2 - x_1}\{R(x - x_1) - R(x - x_2)\}$$

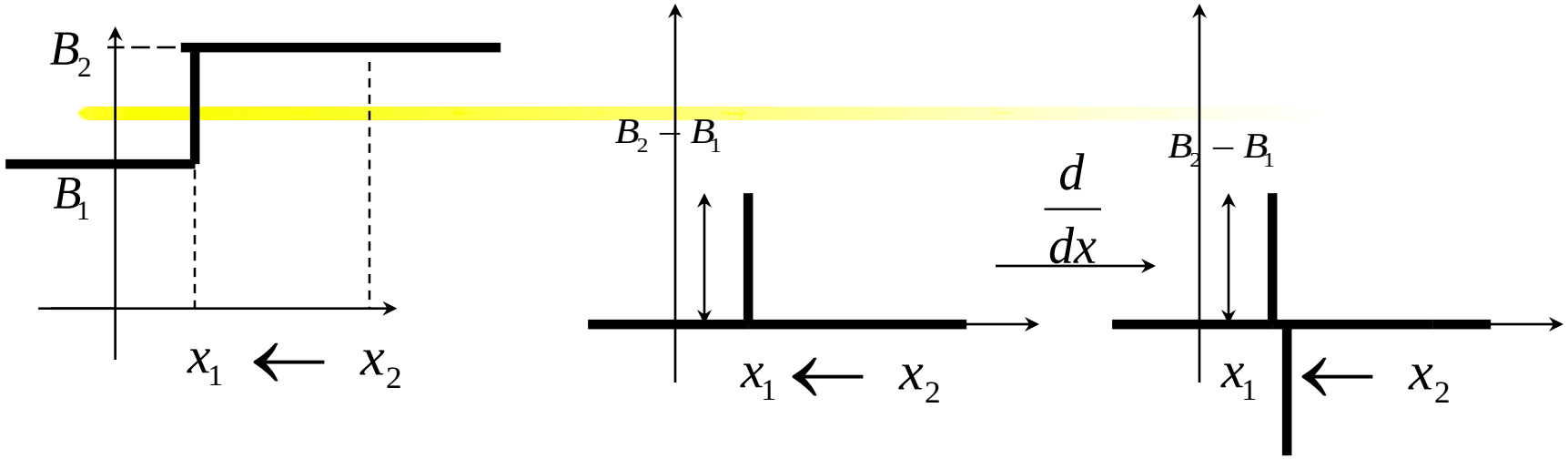




$$E(x) = B_1 + \frac{B_2 - B_1}{x_2 - x_1}\left(R(x - x_1) - R(x - x_2)\right)$$

$$\frac{dE(x)}{dx} = \frac{B_2 - B_1}{x_2 - x_1}\left(u(x - x_1) - u(x - x_2)\right)$$

$$\frac{d^2E(x)}{dx^2} = \frac{B_2 - B_1}{x_2 - x_1}\left(\delta(x - x_1) - \delta(x - x_2)\right)$$



$$E(x) = \lim_{x_2 \to x_1} B_1 + \frac{B_2 - B_1}{x_2 - x_1}(R(x - x_1) - R(x - x_2))$$

$$= \lim_{x_2 \to x_1} B_1 + (B_2 - B_1)\frac{R(x - x_1) - R(x - x_2)}{x_2 - x_1}$$

$$= B_1 + (B_2 - B_1)u(x - x_2)$$

$$\frac{dE(x)}{dx} = (B_2 - B_1)\delta(x - x_2)$$

$$\frac{d^2E(x)}{dx^2} = (B_2 - B_1)\{\delta((x - x_2)^-) - \delta((x - x_2)^+)\}$$

Observation in 1D

- Accentuate edges
  - with 1st derivative: look for peaks
  - with 2nd derivative: look for zero crossings (not zeros!)
- 1D 1st order edge operator $\frac{dE(x)}{dx} = \frac{B_2 - B_1}{x_2 - x_1}(u(x - x_1) - u(x - x_2))$
  - a magnitude
  - a "+" or "-" direction
  - magnitude
    - proportional to intensity difference
    - inversely proportional to transition distance
  - direction
    - positive if $B_2 > B_1$
    - negative if $B_2 < B_1$

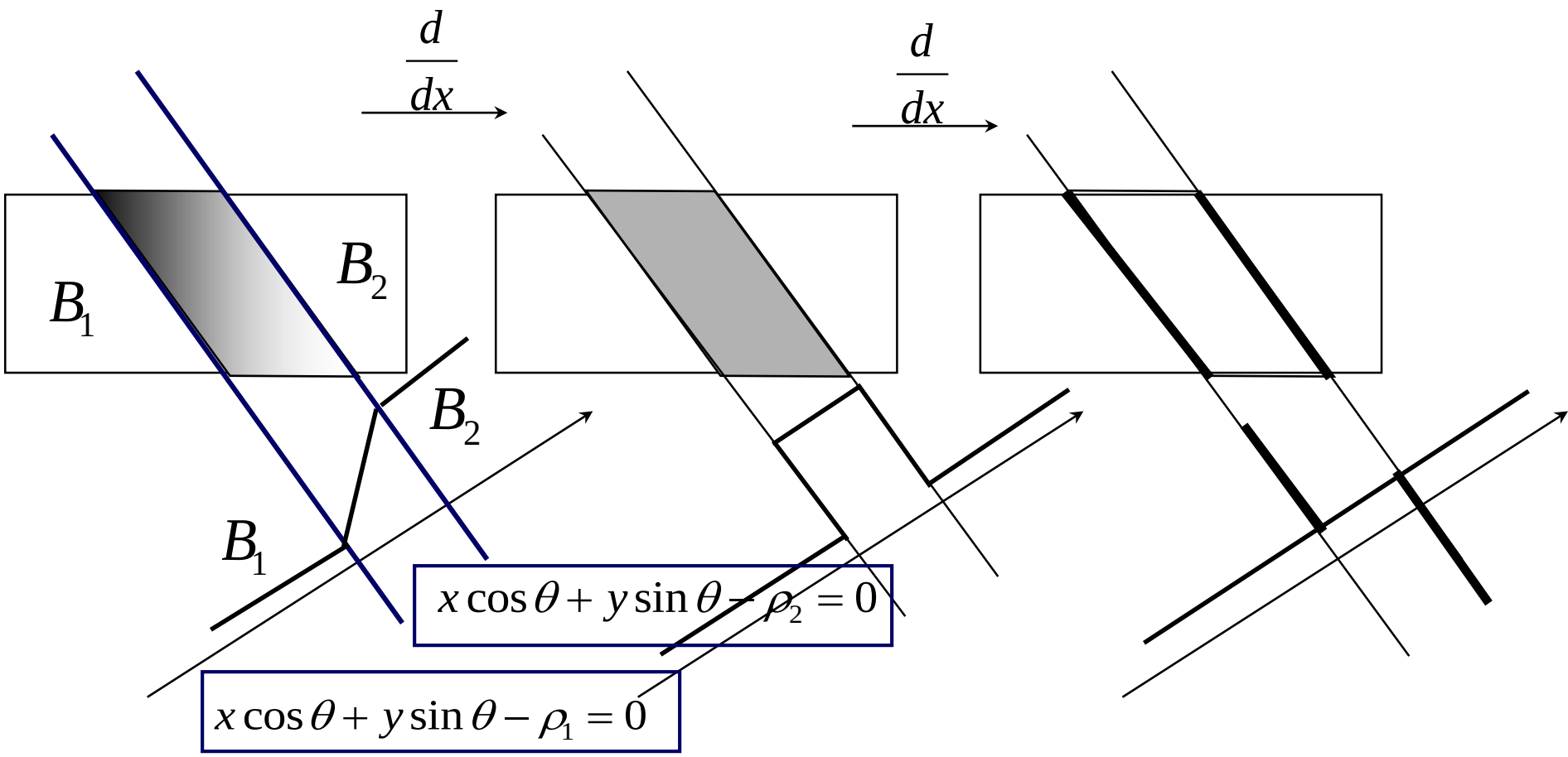
Observation in 1D (cont.)

- 1D 2nd order edge operator
  - a magnitude
  - a "+" or "-" direction

$$\frac{d^2E(x)}{dx^2} = \frac{B_2 - B_1}{x_2 - x_1}(\delta(x - x_1) - \delta(x - x_2))$$

  - magnitude of spikes around zero
    - proportional to intensity difference
    - inversely proportional to transition distance
  - zero crossing direction
    - positive if $B_2 > B_1$
    - negative if $B_2 < B_1$

Generalization to 2D



Reminder: Partial Derivatives

- $y = f(x)$
- Derivative is change rate
- One degree-of-freedom
- dy/dx

- $z=f(x,y)$
- Derivative is change rate
- Two degrees-of-freedom
- $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$

Change rate along any direction can be written as the combination of that along two independent directions (e.g., x and y)

$$E(x,y) = B_1 + \frac{B_2 - B_1}{\rho_2 - \rho_1}[R(x\cos\theta + y\sin\theta - \rho_1) - R(x\cos\theta + y\sin\theta - \rho_2)]$$

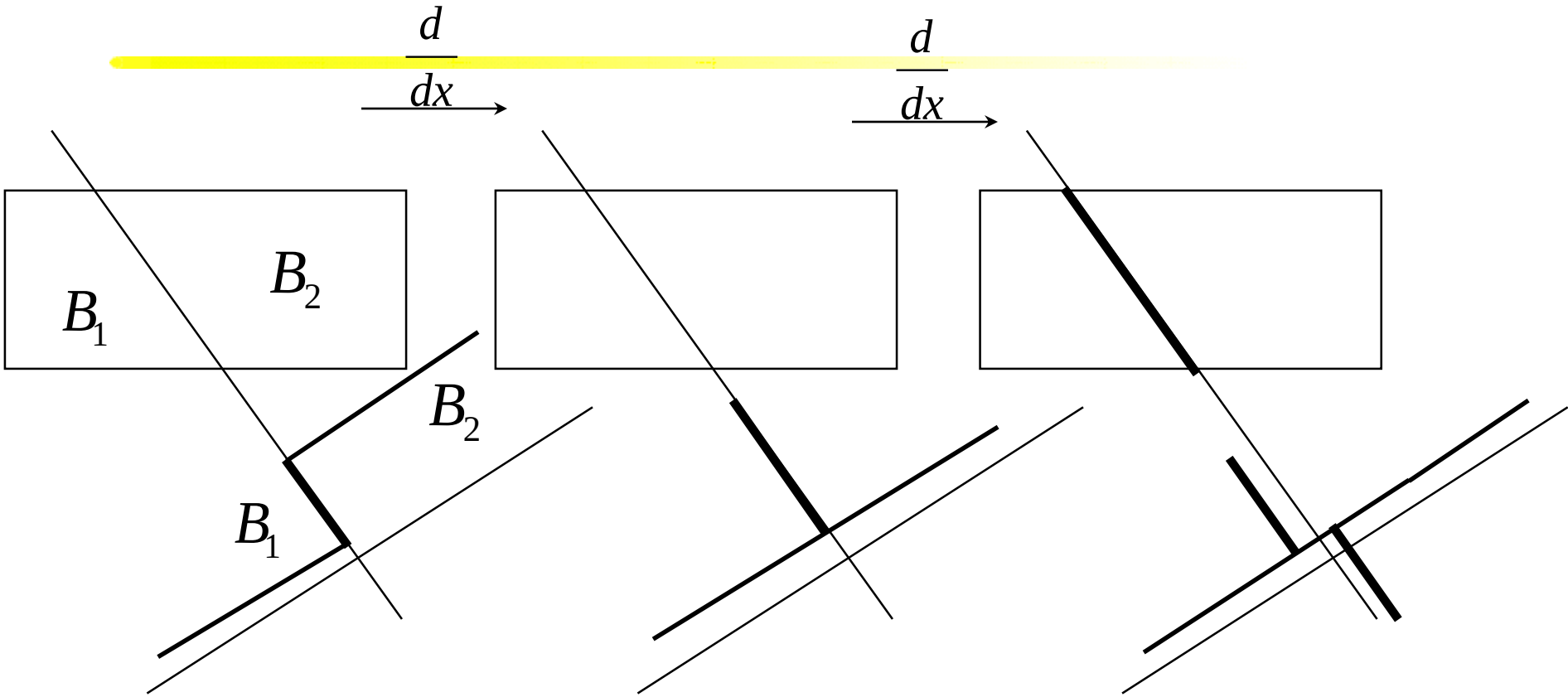
$$\frac{\partial E}{\partial x} = \cos\theta \frac{B_2 - B_1}{\rho_2 - \rho_1}[u(x\cos\theta + y\sin\theta - \rho_1) - u(x\cos\theta + y\sin\theta - \rho_2)]$$

$$\frac{\partial E}{\partial y} = \sin\theta \frac{B_2 - B_1}{\rho_2 - \rho_1}[u(x\cos\theta + y\sin\theta - \rho_1) - u(x\cos\theta + y\sin\theta - \rho_2)]$$

$$\frac{\partial^2 E}{\partial x^2} = \cos^2\theta \frac{B_2 - B_1}{\rho_2 - \rho_1}[\delta(x\cos\theta + y\sin\theta - \rho_1) - \delta(x\cos\theta + y\sin\theta - \rho_2)]$$

$$\frac{\partial^2 E}{\partial x \partial y} = \sin\theta\cos\theta \frac{B_2 - B_1}{\rho_2 - \rho_1}[\delta(x\cos\theta + y\sin\theta - \rho_1) - \delta(x\cos\theta + y\sin\theta - \rho_2)]$$

$$\frac{\partial^2 E}{\partial y^2} = \sin^2\theta \frac{B_2 - B_1}{\rho_2 - \rho_1}[\delta(x\cos\theta + y\sin\theta - \rho_1) - \delta(x\cos\theta + y\sin\theta - \rho_2)]$$



$$\rho_1 \rightarrow \rho_2$$

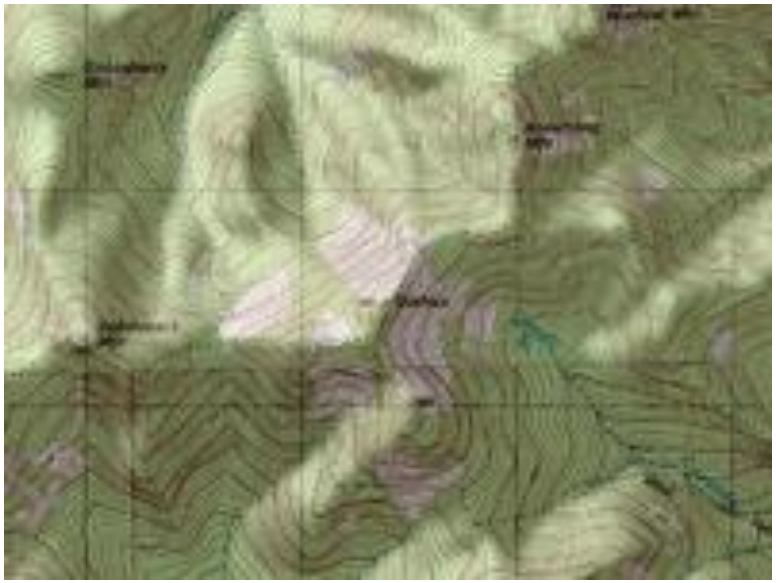
Observation in 2D

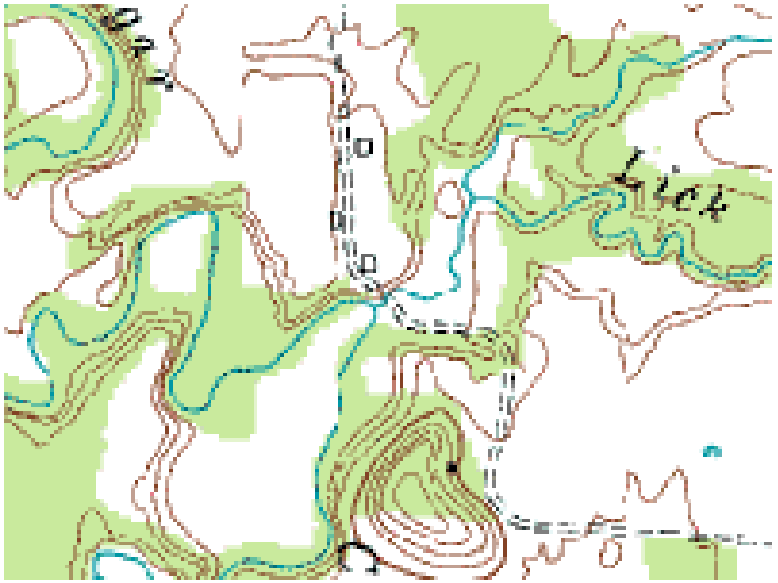
- 2D 1st order edge operator
  - A magnitude
  - A direction, but ...
    - Edge direction: iso-brightness direction
    - Gradient direction: largest brightness change direction

$$magnitude = \sqrt{(\frac{\partial E}{\partial x})^2 + (\frac{\partial E}{\partial y})^2}$$

$$direction = \tan^{-1}(\frac{\frac{\partial E}{\partial y}}{\frac{\partial E}{\partial x}})$$

Analogy: Topo Maps

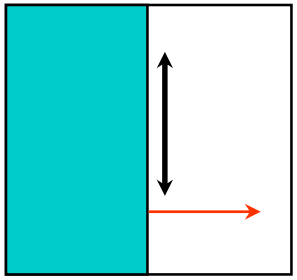


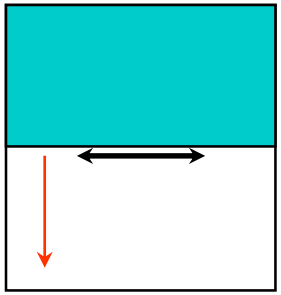


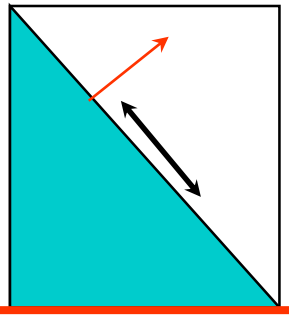
Comparison

- Topo maps
  - Constant elevation regions
  - Separated by iso-elevation contours
  - Move in the direction perpendicular to the contours to change elevation
- Regular images
  - Constant brightness regions
  - Separated by edge
  - Move in the direction perpendicular to the edges to change brightness

Gradient vs. Iso-brightness dirs







$$\left(\frac{\partial E}{\partial x} > 0, \frac{\partial E}{\partial y} = 0\right) \qquad \left(\frac{\partial E}{\partial x} = 0, \frac{\partial E}{\partial y} < 0\right) \qquad \left(\frac{\partial E}{\partial x} > 0, \frac{\partial E}{\partial y} > 0\right)$$

$$\tan^{-1}\left(\frac{0}{>0}\right) = 0^o \qquad \tan^{-1}\left(\frac{<0}{0}\right) = -90^o \qquad \tan^{-1}\left(\frac{>0}{>0}\right) = 0^o \rightarrow 90^o$$

Observation in 2D

- 2D 2nd order edge operator
  - A magnitude
  - A direction
  - Can be made *isotropic* (non-directional)

$$\frac{\partial^2 E}{\partial^2 x} + \frac{\partial^2 E}{\partial^2 y} \quad (Laplacian\ operator)$$

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} = (\cos^2\theta + \sin^2\theta)\frac{B_2 - B_1}{\rho_2 - \rho_1}[\delta(x\cos\theta + y\sin\theta - \rho_1) - \delta(x\cos\theta + y\sin\theta - \rho_2)]$$

$$= \frac{B_2 - B_1}{\rho_2 - \rho_1}[\delta(x\cos\theta + y\sin\theta - \rho_1) - \delta(x\cos\theta + y\sin\theta - \rho_2)]$$

Comparison

- 1st order operator

$$|\nabla E|^2 = \left(\frac{\partial E}{\partial x}\right)^2 + \left(\frac{\partial E}{\partial y}\right)^2$$

  - less sensitive to noise
  - directional
  - threshold on edge magnitude

- 2nd order operator

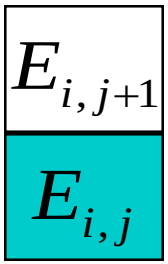
$$div(\nabla E) = \frac{\partial^2 E}{\partial^2 x} + \frac{\partial^2 E}{\partial^2 y}$$

  - more sensitive to noise
  - can be made isotropic
  - marked by zero crossing
  - threshold on transition magnitude around zero

Digital Implementations

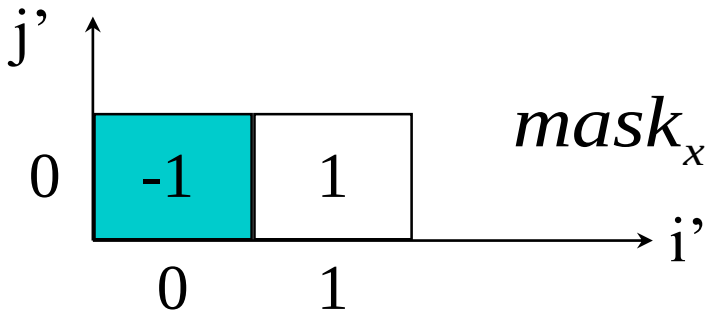
- 1st order operator - 1x2 or 2x1 mask
  - simple
  - unbalanced (forward differencing)
  - sensitive to noise

$$\frac{\partial E}{\partial x} \approx E_{i+1,j} - E_{i,j}$$

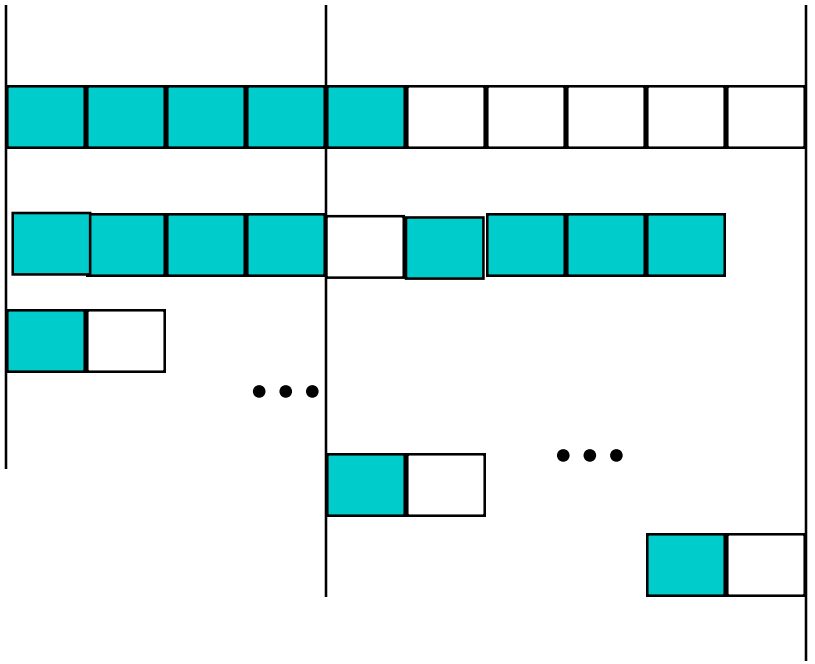


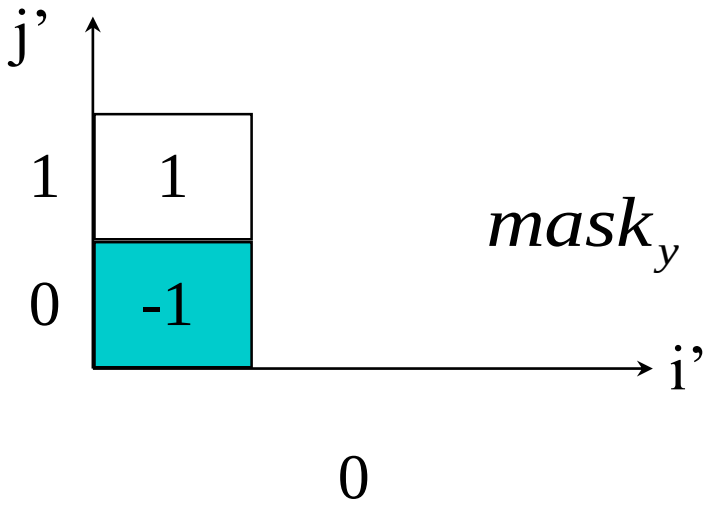
$$\frac{\partial E}{\partial y} \approx E_{i,j+1} - E_{i,j}$$

$$Edge\_map(i,j) = \sqrt{\left(\sum_{i'=0}^{1}\sum_{j'=0}^{1} E(i+i', j+j') \cdot mask_x(i', j')\right)^2 + \left(\sum_{i'=0}^{1}\sum_{j'=0}^{1} E(i+i', j+j') \cdot mask_y(i', j')\right)^2}$$



$mask_x$





$mask_y$

Another Implementation

- 1st order operator - 2x2 mask
  - simple
  - unbalanced (forward differencing)
  - more resistive to noise

$E_{i,j+1}$	$E_{i+1,j+1}$
$E_{i,j}$	$E_{i+1,j}$

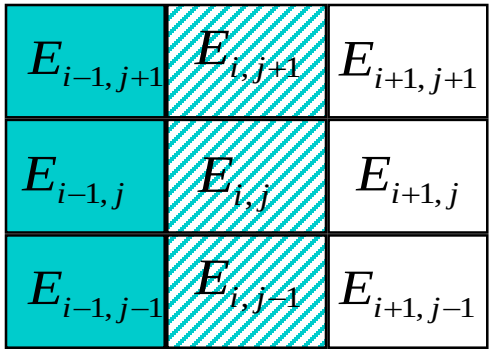
$$\frac{\partial E}{\partial x} \approx \frac{1}{2}((E_{i+1,j+1} - E_{i,j+1}) + (E_{i+1,j} - E_{i,j}))$$

$E_{i,j+1}$	$E_{i+1,j+1}$
$E_{i,j}$	$E_{i+1,j}$

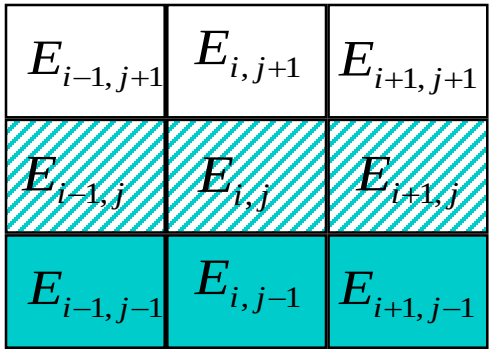
$$\frac{\partial E}{\partial y} \approx \frac{1}{2}((E_{i+1,j+1} - E_{i+1,j}) + (E_{i,j+1} - E_{i,j}))$$

Yet Another Implementation

- 1st order operator - 3x3 mask
  - simple
  - balanced
  - more resistive to noise



$$\frac{\partial E}{\partial x} \approx \frac{1}{6}((E_{i+1,j+1} - E_{i-1,j+1}) + (E_{i+1,j} - E_{i-1,j}) + (E_{i+1,j-1} - E_{i-1,j-1}))$$



$$\frac{\partial E}{\partial y} \approx \frac{1}{6}((E_{i+1,j+1} - E_{i+1,j-1}) + (E_{i,j+1} - E_{i,j-1}) + (E_{i-1,j+1} - E_{i-1,j-1}))$$

2nd order operator

- 3x3 mask

$$\frac{\partial^2 E}{\partial x^2} \approx \frac{\partial E}{\partial x}_{i,j} - \frac{\partial E}{\partial x}_{i-1,j}$$

$$\approx (E_{i+1,j} - E_{i,j}) - (E_{i,j} - E_{i-1,j})$$

$$= E_{i+1,j} - 2E_{i,j} + E_{i-1,j}$$

$$\frac{\partial^2 E}{\partial y^2} \approx E_{i,j+1} - 2E_{i,j} + E_{i,j-1}$$

$$Laplacian: \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} \approx E_{i+1,j} + E_{i-1,j} + E_{i,j+1} + E_{i,j-1} - 4E_{i,j}$$

$E_{i-1,j+1}$	$E_{i,j+1}$	$E_{i+1,j+1}$
$E_{i-1,j}$	$E_{i,j}$	$E_{i+1,j}$
$E_{i-1,j-1}$	$E_{i,j-1}$	$E_{i+1,j-1}$

	1	
1	-4	1
	1	

2nd order operator (cont.)

- Zero-crossing
  - if pixel > t and one of its neighbor < -t, or
  - if pixel<-t and one of its neighbor > t