

Assignment 01 Solution

Download and install the latest version of R and R Studio.

Problem 1

Create a vector V with 8 elements (7,2,1,0,3,-1,-3,4).

- Transform that vector into a rectangular matrix A of dimensions 4X2 (4- rows, 2- columns).
- Create a matrix transpose to the above matrix A. Call that matrix AT.
- Calculate matrix products: $A \cdot AT$ and $AT \cdot A$. Present the results. What are the dimensions of those two product matrices.
- Square matrixes sometimes have an inverse matrix. Try calculating inverse matrices (or matrixes, if you prefer) of above matrices (matrixes) $A \cdot AT$ and $AT \cdot A$.
- Extend the above vector V with the ninth number of value -2. Do it elegantly by concatenating two vectors (☺).
- Transform that extended vector into a 3X3 matrix B.
- Calculate the inverse matrix of matrix B. Call it Binv. Demonstrate that the product of B and Binv is the same as the product of Binv and B and is equal to what?
- Determine the eigenvectors of matrixes B.
- Construct a new matrix C which is made by using each eigenvector of matrix B as a column. Calculate the product of matrix C and matrix B and the product of matrix B and C. Is there any significance to the elements of the product matrixes.
- Transform matrix B into a matrix with names columns and named rows.
- Transformed that fully “named” matrix into a data.frame.
- Ask the object you just created what is its class().

Solution:

1. First we create a vector V with (7, 2, 1, 0, 3, -1, -3, 4).

```
> V <- c(7,2,1,0,3,-1,-3,4)
> V
[1] 7 2 1 0 3 -1 -3 4
```

2. Transform the vector into a 4x2 matrix. “byrow = True” means that the matrix is filled by rows.

```
> A <- matrix(V, nrow = 4, ncol = 2, byrow = TRUE)
> A
      [,1] [,2]
[1,]    7    2
[2,]    1    0
[3,]    3   -1
[4,]   -3    4
```

3. Create a matrix transpose to matrix A.

```
> AT <- t(A)
> AT
      [,1] [,2] [,3] [,4]
[1,]    7    1    3   -3
[2,]    2    0   -1    4
```

4. Calculate the product of $A \cdot AT$ and $AT \cdot A$. The dimension of $A \cdot AT$ is 4×4 . The dimension of $AT \cdot A$ is 2×2 .

```
> A %*% AT
      [,1] [,2] [,3] [,4]
[1,]   53    7   19  -13
[2,]    7    1    3   -3
[3,]   19    3   10  -13
[4,]  -13   -3  -13   25
> dim(A %*% AT)
[1] 4 4
> AT %*% A
      [,1] [,2]
[1,]   68   -1
[2,]   -1   21
> dim(AT %*% A)
[1] 2 2
```

5. Calculate the inverse matrix of $A \cdot AT$ and $AT \cdot A$. The product of $A \cdot AT$ doesn't have inverses. It's a singular matrix. The product of $AT \cdot A$ is invertible.

```
> solve(A %*% AT)
Error in solve.default(A %*% AT) :
  system is computationally singular: reciprocal condition number = 3.4479e-19
> solve(AT %*% A)
      [,1]      [,2]
[1,] 0.0147161878 0.0007007708
[2,] 0.0007007708 0.0476524177
```

6. Extend V with the ninth number of -2.

```
> V <- c(V, -2)
> V
[1] 7 2 1 0 3 -1 -3 4 -2
```

7. Transform the extended matrix into a 3×3 matrix B.

```
> B <- matrix(V, nrow = 3, ncol = 3, byrow = TRUE)
> B
      [,1] [,2] [,3]
[1,]    7    2    1
[2,]    0    3   -1
[3,]   -3    4   -2
```

8. Calculate the inverse matrix of B. Call is Binv. The product of $B \cdot Binv$ is equal to the product of $Binv \cdot B$. The product is an identity matrix.

```

> Binv <- solve(B)
> Binv
      [,1] [,2] [,3]
[1,]  -2    8  -5
[2,]   3  -11   7
[3,]   9  -34  21
> B %%% Binv
      [,1]      [,2]      [,3]
[1,] 1.000000e+00 -7.105427e-15 3.552714e-15
[2,] 1.776357e-15 1.000000e+00 0.000000e+00
[3,] 3.552714e-15 0.000000e+00 1.000000e+00
> Binv %%% B
      [,1]      [,2] [,3]
[1,] 1.000000e+00 -3.552714e-15 0
[2,] 3.552714e-15 1.000000e+00 0
[3,] 0.000000e+00 1.421085e-14 1
> zapsmall(B %%% Binv) == zapsmall(Binv %%% B)
      [,1] [,2] [,3]
[1,] TRUE TRUE TRUE
[2,] TRUE TRUE TRUE
[3,] TRUE TRUE TRUE

```

9. Calculate the eigenvectors of matrix B. The `eigen()` will return the eigenvectors and eigenvalues at the same time.

```

> Beigen <- eigen(B)
> Beigen
$values
[1] 6.854102 1.000000 0.145898

$vectors
      [,1]      [,2]      [,3]
[1,] 0.95425723 -0.2857143 -0.2280090
[2,] 0.07508986 0.4285714 0.3219539
[3,] -0.28940397 0.8571429 0.9188893

> Beigen$vectors = Beigen$vectors
> Beigen$vectors
      [,1]      [,2]      [,3]
[1,] 0.95425723 -0.2857143 -0.2280090
[2,] 0.07508986 0.4285714 0.3219539
[3,] -0.28940397 0.8571429 0.9188893
> Beigen$values = Beigen$values

```

10. The eigenvector of B is already made by using each eigenvector of matrix B as a column. So `Beigen$vectors` is matrix C. The results below show that the product of the inverse matrix of C^*B^*C is a diagonal matrix. It has the eigenvalues on the diagonal.

```

> Beigen$vectors %%% B
      [,1]      [,2]      [,3]
[1,] 7.3638277 0.1393355 1.6959896
[2,] -0.4402327 2.7237097 -0.9973894
[3,] -4.7824957 5.6681779 -2.9843254

```

```

> B %%% Beigenvectors
      [,1]      [,2]      [,3]
[1,]  6.5405763 -0.2857143 -0.03326607
[2,]  0.5146735  0.4285714  0.04697244
[3,] -1.9836043  0.8571429  0.13406414

> solve(Beigenvectors) %%% B %%% Beigenvectors
      [,1]      [,2]      [,3]
[1,]  6.854102e+00  2.775558e-16  1.110223e-16
[2,]  1.221245e-15  1.000000e+00  4.440892e-16
[3,] -3.608225e-15  2.664535e-15  1.458980e-01

```

11. Transform matrix B into a matrix with names columns and named rows.

```

> B
      [,1] [,2] [,3]
[1,]    7    2    1
[2,]    0    3   -1
[3,]   -3    4   -2
> dimnames(B) <- list(c("R1", "R2", "R3"), c("C1", "C2", "C3"))
> B
      C1 C2 C3
R1    7  2  1
R2    0  3 -1
R3   -3  4 -2
> class(B)
[1] "matrix"

```

12. Transformed that fully “named” matrix into a data.frame. Ask the object you just created what is its class().

```

> dataframe = data.frame(B)
> class(dataframe)
[1] "data.frame"
> is.data.frame(dataframe)
[1] TRUE
> labels(dataframe)
[[1]]
[1] "R1" "R2" "R3"

[[2]]
[1] "C1" "C2" "C3"

> names(dataframe)
[1] "C1" "C2" "C3"
> row.names(dataframe)
[1] "R1" "R2" "R3"

```

Problem 2

Consider file 2006Data.csv upload to the class site in Assignment 01 folder. File represents actual measurement of power consumption in a country somewhere in a California. Import data contained in that file into a data frame. You are expected to Google and find a function that will let you perform that import. Create a scatter plot of power consumption vs. temperature and power consumption vs. hour of the day.

Subsequently create a boxplot with power on the vertical axis and hour of the day on the horizontal axis. The objective is to present the distribution (variation) of power consumption for every hour of the day.

Solution:

1. Import the data from the csv file into a data frame. Use colClasses here to remove the redundant columns.

```
> mydata <- read.csv("~/Downloads/2006Data.csv", header = TRUE, sep = ",", colClasses = c(NA, NA, NA, NA, NA, NA, "NULL"))
> summary(mydata)
```

Month	Day	Hour	DayOfWeek	Holiday
Min. : 1.000	Min. : 1.00	Min. : 1.00	Min. : 1.000	Min. : 0.0000
1st Qu.: 4.000	1st Qu.: 8.00	1st Qu.: 6.75	1st Qu.: 2.000	1st Qu.: 0.0000
Median : 7.000	Median : 16.00	Median : 12.50	Median : 4.000	Median : 0.0000
Mean : 6.526	Mean : 15.72	Mean : 12.50	Mean : 4.008	Mean : 0.0274
3rd Qu.: 10.000	3rd Qu.: 23.00	3rd Qu.: 18.25	3rd Qu.: 6.000	3rd Qu.: 0.0000
Max. : 12.000	Max. : 31.00	Max. : 24.00	Max. : 7.000	Max. : 1.0000

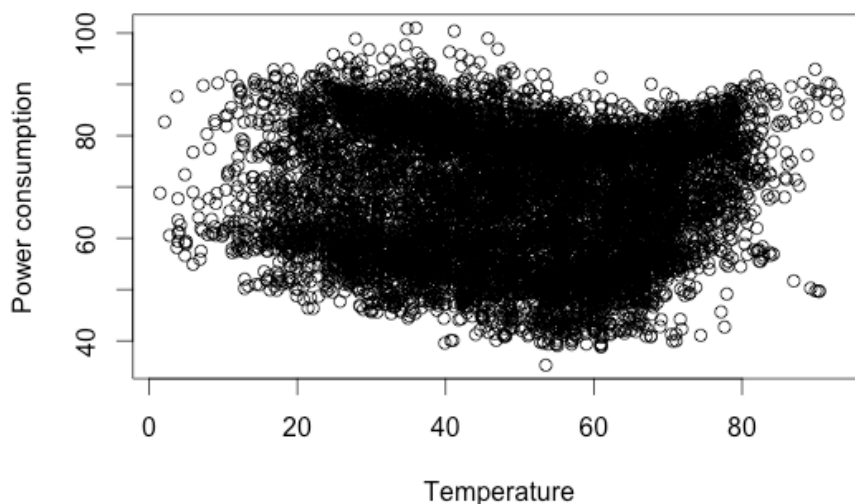
Power	Temperature
Min. : 35.26	Min. : 1.475
1st Qu.: 57.19	1st Qu.: 35.865
Median : 67.85	Median : 49.835
Mean : 67.62	Mean : 49.106
3rd Qu.: 78.31	3rd Qu.: 62.676
Max. : 100.99	Max. : 93.000

```
> class(mydata)
[1] "data.frame"
```

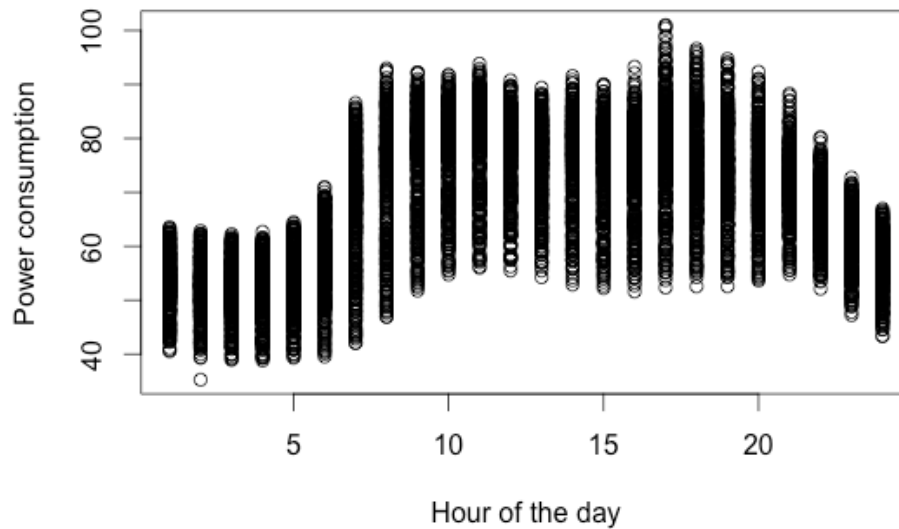
2. Create a scatter plot of power consumptions vs. temperature and power consumptions vs. hour of the day.

```
> power = mydata$Power
> temperature = mydata$Temperature
> hour = mydata$Hour
> plot(temperature, power, xlab = "Temperature", ylab = "Power consumption")
> plot(hour, power, xlab = "Hour of the day", ylab = "Power consumption")
```

The scatter plot of power consumptions vs. temperature.

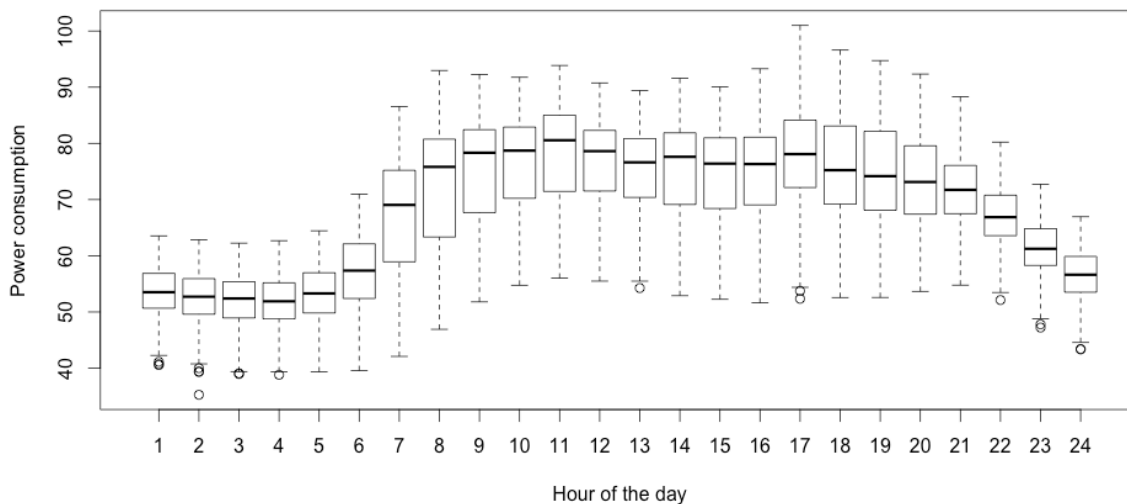


The scatter plot of power consumptions vs. hour of the day.



3. Create a boxplot of power consumptions vs. hour of the day to present the distribution of power for every hour of the day. The dots outside the min value are outliers, which are less than $3/2$ times of the lower quartile.

The distribution of power consumption for every hour of the day



Problem 3

Separate temperature scale in a reasonable number of intervals: 50 or 100. Calculate average power consumption, minimum power consumption and maximum power consumptions for every interval. Present those three sets of values on a single scatter graph (perhaps in different colours). Calculate three covariance matrixes between temperature and each of those power indicators (min, average, max).

Solution:

1. Separate the temperature scale in 50 intervals. Assign values from temperature to the intervals delimited by sequence breaks. “right = FALSE” means the intervals are close on the left and open on the right.

```
> breaks = seq(1.0, 94.0, by = 1.86);
> breaks
[1] 1.00 2.86 4.72 6.58 8.44 10.30 12.16 14.02 15.88 17.74 19.60 21.46 23.32 25.18
[15] 27.04 28.90 30.76 32.62 34.48 36.34 38.20 40.06 41.92 43.78 45.64 47.50 49.36 51.22
[29] 53.08 54.94 56.80 58.66 60.52 62.38 64.24 66.10 67.96 69.82 71.68 73.54 75.40 77.26
[43] 79.12 80.98 82.84 84.70 86.56 88.42 90.28 92.14 94.00
> temperature.cut = cut(temperature, breaks, right=FALSE);
```

2. Calculate average, minimum and maximum power consumptions for every interval.

```
> maxpower = tapply(power, temperature.cut, max)
> minpower = tapply(power, temperature.cut, min)
> meanpower = tapply(power, temperature.cut, mean)
> maxpower
 [1,2.86) [2.86,4.72) [4.72,6.58) [6.58,8.44) [8.44,10.3) [10.3,12.2) [12.2,14)
 82.6960 87.6270 76.7704 89.7771 90.2336 91.6283 88.9911
 [14,15.9) [15.9,17.7) [17.7,19.6) [19.6,21.5) [21.5,23.3) [23.3,25.2) [25.2,27)
 90.9417 92.8717 92.9389 90.4748 92.9723 95.8191 95.1873
 [27,28.9) [28.9,30.8) [30.8,32.6) [32.6,34.5) [34.5,36.3) [36.3,38.2) [38.2,40.1)
 98.8383 96.7866 96.6031 94.3725 100.9896 95.0553 90.8336
 [40.1,41.9) [41.9,43.8) [43.8,45.6) [45.6,47.5) [47.5,49.4) [49.4,51.2) [51.2,53.1)
 100.3930 95.7247 94.2719 98.9550 93.0825 88.6777 91.7737
 [53.1,54.9) [54.9,56.8) [56.8,58.7) [58.7,60.5) [60.5,62.4) [62.4,64.2) [64.2,66.1)
 91.9065 86.3998 87.1483 87.9378 91.3635 86.1453 85.5433
 [66.1,68) [68,69.8) [69.8,71.7) [71.7,73.5) [73.5,75.4) [75.4,77.3) [77.3,79.1)
 90.0549 88.1541 87.3791 85.8301 87.3303 88.4328 89.0667
 [79.1,81) [81,82.8) [82.8,84.7) [84.7,86.6) [86.6,88.4) [88.4,90.3) [90.3,92.1)
 89.7736 91.5709 89.0597 82.7765 90.3311 92.9252 90.2133
 [92.1,94)
 88.1772
> minpower
 [1,2.86) [2.86,4.72) [4.72,6.58) [6.58,8.44) [8.44,10.3) [10.3,12.2) [12.2,14)
 60.5326 58.1601 54.9205 55.9044 60.4221 57.2195 50.2237
 [14,15.9) [15.9,17.7) [17.7,19.6) [19.6,21.5) [21.5,23.3) [23.3,25.2) [25.2,27)
 50.9279 49.8641 50.9160 46.3978 46.3474 49.5164 47.6825
 [27,28.9) [28.9,30.8) [30.8,32.6) [32.6,34.5) [34.5,36.3) [36.3,38.2) [38.2,40.1)
 46.0882 45.6591 45.6151 45.3238 44.4238 46.4085 39.5643
 [40.1,41.9) [41.9,43.8) [43.8,45.6) [45.6,47.5) [47.5,49.4) [49.4,51.2) [51.2,53.1)
 40.0671 44.2881 41.2311 40.1007 40.8894 40.9541 39.7150
 [53.1,54.9) [54.9,56.8) [56.8,58.7) [58.7,60.5) [60.5,62.4) [62.4,64.2) [64.2,66.1)
 35.2605 39.0662 39.3408 39.3739 38.8142 41.4718 41.8363
 [66.1,68) [68,69.8) [69.8,71.7) [71.7,73.5) [73.5,75.4) [75.4,77.3) [77.3,79.1)
 40.8431 42.7762 39.9362 42.2921 41.0651 45.6383 42.7008
 [79.1,81) [81,82.8) [82.8,84.7) [84.7,86.6) [86.6,88.4) [88.4,90.3) [90.3,92.1)
 54.4724 55.4768 56.6712 68.0002 51.6770 49.7290 49.6333
 [92.1,94)
 84.2437
```

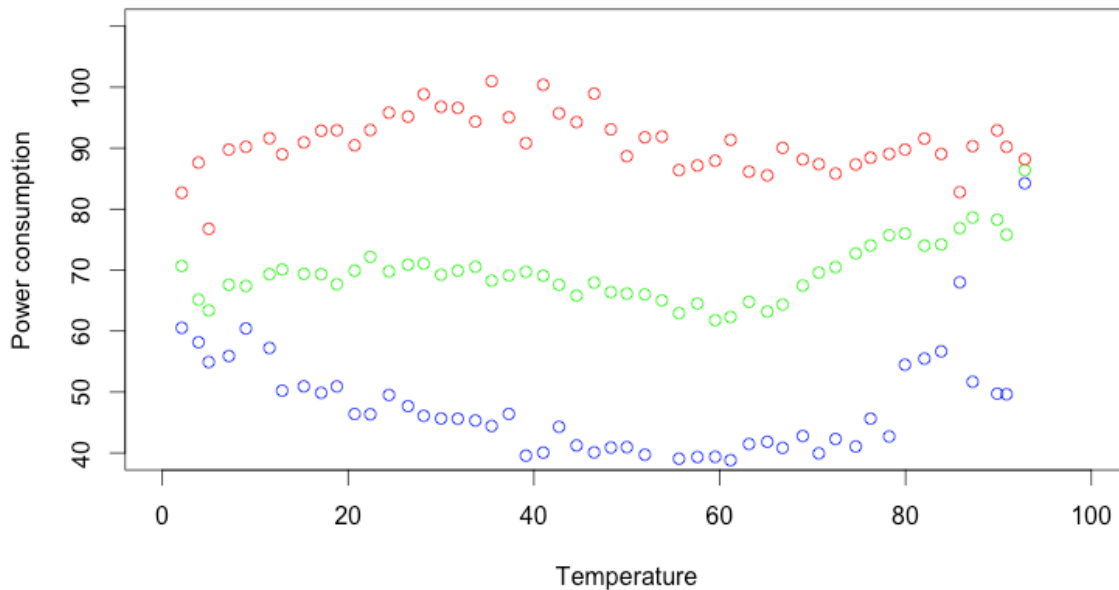


```
> meanpower
[1,2.86) [2.86,4.72) [4.72,6.58) [6.58,8.44) [8.44,10.3) [10.3,12.2) [12.2,14)
70.67890 65.16306 63.39036 67.59992 67.39706 69.32481 70.09464
[14,15.9) [15.9,17.7) [17.7,19.6) [19.6,21.5) [21.5,23.3) [23.3,25.2) [25.2,27)
69.38817 69.32528 67.67428 69.90888 72.16975 69.78337 70.89329
[27,28.9) [28.9,30.8) [30.8,32.6) [32.6,34.5) [34.5,36.3) [36.3,38.2) [38.2,40.1)
71.06004 69.23769 69.89605 70.54779 68.25706 69.10285 69.71936
[40.1,41.9) [41.9,43.8) [43.8,45.6) [45.6,47.5) [47.5,49.4) [49.4,51.2) [51.2,53.1)
69.08667 67.59824 65.81948 67.94535 66.38124 66.14448 66.01584
[53.1,54.9) [54.9,56.8) [56.8,58.7) [58.7,60.5) [60.5,62.4) [62.4,64.2) [64.2,66.1)
65.03037 62.93600 64.51590 61.77305 62.30138 64.79231 63.19053
[66.1,68) [68,69.8) [69.8,71.7) [71.7,73.5) [73.5,75.4) [75.4,77.3) [77.3,79.1)
64.35930 67.46678 69.57974 70.49110 72.74420 74.04072 75.73548
[79.1,81) [81,82.8) [82.8,84.7) [84.7,86.6) [86.6,88.4) [88.4,90.3) [90.3,92.1)
75.99739 74.02740 74.20681 76.86791 78.63333 78.26831 75.82372
[92.1,94)
86.39373
```

3. Present the above three sets of values on a single scatter graph. For the temperature, take the midpoints of every interval. Use points() to plot the values on the same graph.

```
> midtemperature = tapply(temperature, temperature.cut, median)
> midtemperature
[1,2.86) [2.86,4.72) [4.72,6.58) [6.58,8.44) [8.44,10.3) [10.3,12.2) [12.2,14)
2.09500 3.90000 5.00000 7.16500 9.00000 11.53000 12.90000
[14,15.9) [15.9,17.7) [17.7,19.6) [19.6,21.5) [21.5,23.3) [23.3,25.2) [25.2,27)
15.23500 17.10000 18.78250 20.70000 22.38375 24.40000 26.44620
[27,28.9) [28.9,30.8) [30.8,32.6) [32.6,34.5) [34.5,36.3) [36.3,38.2) [38.2,40.1)
28.13500 30.00000 31.80450 33.70000 35.44500 37.30000 39.13500
[40.1,41.9) [41.9,43.8) [43.8,45.6) [45.6,47.5) [47.5,49.4) [49.4,51.2) [51.2,53.1)
41.00000 42.70000 44.59055 46.48000 48.28500 50.00000 51.93790
[53.1,54.9) [54.9,56.8) [56.8,58.7) [58.7,60.5) [60.5,62.4) [62.4,64.2) [64.2,66.1)
53.77430 55.61500 57.60000 59.50500 61.16500 63.15000 65.11905
[66.1,68) [68,69.8) [69.8,71.7) [71.7,73.5) [73.5,75.4) [75.4,77.3) [77.3,79.1)
66.76500 68.93500 70.66560 72.46500 74.63500 76.23000 78.23500
[79.1,81) [81,82.8) [82.8,84.7) [84.7,86.6) [86.6,88.4) [88.4,90.3) [90.3,92.1)
79.93500 82.03000 83.83500 85.83500 87.22000 89.86500 90.87500
[92.1,94)
92.83500
> plot(midtemperature, meanpower, main = "Distribution of power consumption for different temperature", xlab = "Temperature", ylab = "Power consumption", xlim = c(0, 100), ylim = c(40, 110), col = "green")
> points(midtemperature, maxpower, col = "red")
> points(midtemperature, minpower, col = "blue")
```


Distribution of power consumption for different temperature



4. Calculate three covariance matrix between temperature and each of those power indicators (min, average, max).

```
> cov(midtemperature, maxpower)
[1] -21.50168
> cov(midtemperature, minpower)
[1] -11.55951
> cov(midtemperature, meanpower)
[1] 57.88333
> M <- cbind(midtemperature, maxpower, minpower, meanpower)
> cov(M)
```

	midtemperature	maxpower	minpower	meanpower
midtemperature	733.81772	-21.5016801	-11.55951	57.8833333
maxpower	-21.50168	21.3789126	-10.87648	0.8847414
minpower	-11.55951	-10.8764830	77.90368	25.8822440
meanpower	57.88333	0.8847414	25.88224	22.9500587

The covariance matrix is a matrix that only concerns the relationship between variables. We can create a matrix from the vectors (temperature, min, average, max). Each value in the covariance matrix represents the covariance (or variance) between two of the vectors. The first column shows the covariance between temperatures and power indicators.