An Overview of Recent Progress in the Study of Distributed Multi-Agent Coordination

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***Abstract*—This paper reviews some main results and progress in distributed multi-agent coordination, focusing on papers published in major control systems and robotics journals since 2006. Distributed coordination of multiple vehicles, including unmanned aerial vehicles, unmanned ground vehicles, and unmanned underwater vehicles, has been a very active research subject studied extensively by the systems and control community. The recent results in this area are categorized into several directions, such as consensus, formation control, optimization, and estimation. After the review, a short discussion section is included to summarize the existing research and to propose several promising research directions along with some open problems that are deemed important for further investigations.**

***Index Terms*—Distributed coordination, formation control, multi-agent system, sensor network.**

I. Introduction

CONTROL theory and practice may date back to the beginning of the last century when the Wright Brothers attempted their first test flight in 1903. Since then, control theory has gradually gained popularity, receiving more and wider attention especially during the World War II when it was developed and applied to fire-control systems, missile navigation and guidance, as well as various electronic automation devices. In the past several decades, modern control theory was further advanced due to the booming of aerospace technology based on large-scale engineering systems.

During the rapid and sustained development of the modern control theory, technology for controlling a single vehicle, albeit higher dimensional and complex, has become relatively mature and has produced many effective tools such as PID control, adaptive control, nonlinear control, intelligent control, and robust control methodologies. In the past two decades in particular, control of multiple vehicles has received increasing demands spurred by the fact that many benefits can be obtained when a single complicated vehicle is equivalently replaced by multiple yet simpler vehicles. In this endeavor, two approaches are commonly adopted for controlling multiple vehicles: a centralized approach and a distributed approach. The centralized approach is based on the assumption that a central station is available and sufficiently powerful to control a whole group of vehicles. Essentially, the centralized approach is a direct extension of the traditional single-vehicle-based control philosophy and strategy. On the contrary, the distributed approach does not require a central station for control, at the cost of becoming far more complex in structure and organization. Although both approaches are considered to be practical depending on the situations and conditions of the real applications, the distributed approach is believed more promising due to many inevitable physical constraints such as limited resources and energy, short wireless communication ranges, narrow bandwidths, and large sizes of vehicles to manage and control. Therefore, the focus of this overview is placed on the distributed approach.

In distributed control of a group of autonomous vehicles, the main objective typically is to have the whole group of vehicles working in a cooperative fashion throughout a distributed protocol. Here, *cooperative* refers to a close relationship among all vehicles in the group where *information sharing* plays a central role. The distributed approach has many advantages in achieving cooperative group performances, especially with low operational costs, less system requirements, high robustness, strong adaptivity, and flexible scalability, therefore has been widely recognized and appreciated.

The study of distributed control of multiple vehicles was perhaps first motivated by the work in distributed computing [1], management science [2], and statistical physics [3]. In the control systems society, some pioneering works are generally referred to [4], [5], where an asynchronous agreement problem was studied for distributed decision-making problems. Thereafter, some consensus algorithms were studied under various information-flow constraints [6]–[10]. There are several journal special issues on the related topics published after 2006, including [11]–[15]. In addition, there are some recent reviews and progress reports given in the surveys [16]–[20] and the books [21]–[28], among others.

This paper reviews some main results and recent progress in distributed multi-agent coordination, published in major control systems and robotics journals since 2006. Due to space limitations, we refer the readers to [29] for a more complete version

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of the same overview. For results before 2006, the readers are referred to [16]–[19].

Specifically, this paper reviews the recent research results in the following directions, which are not independent but actually may have overlapping to some extent.

1) Consensus and the like (synchronization, rendezvous): consensus refers to the group behavior that all of the agents asymptotically reach a certain common agreement through a local distributed protocol, with or without predefined common speed and orientation.
2) Distributed formation and the like (flocking): distributed formation refers to the group behavior that all of the agents form a predesigned geometrical configuration through local interactions with or without a common reference.
3) Distributed optimization: this refers to algorithmic developments for the analysis and optimization of large-scale distributed systems.
4) Distributed estimation and control: this refers to distributed control design based on local estimation about the needed global information.

The remainder of this paper is organized as follows. In Section II, basic notations of graph theory and stochastic matrices are introduced. Sections III–VI describe the recent research results and progress in consensus, formation control, optimization, and estimation. Finally, we conclude the paper with a short discussion on future perspectives.

II. Preliminaries

A. Graph Theory

For a system of n connected agents, its network topology can be modeled as a directed graph denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{W})$, where $\mathcal{V} = \{v_1, v_2, \cdots, v_n\}$ and $\mathcal{W} \subseteq \mathcal{V} \times \mathcal{V}$ are, respectively, the set of agents and the set of edges which directionally connect the agents together. Specifically, the directed edge denoted by an ordered pair (v_i, v_j) means that agent j can access the state information of agent i. Accordingly, agent i is a neighbor of agent j. A directed path is a sequence of directed edges in the form of $(v_1, v_2), (v_2, v_3), \cdots$, with all $v_i \in \mathcal{V}$. A directed graph has a directed spanning tree if there exists at least one agent that has a directed path to every other agent. The union of a set of directed graphs with the same set of agents, $\{\mathcal{G}_{i_1}, \cdots, \mathcal{G}_{i_m}\}$, is a directed graph with the same set of agents and its set of edges is given by the union of the edge sets of all the directed graphs $\mathcal{G}_{i_j}$, $j = 1, \cdots, m$. A complete directed graph is a directed graph in which each pair of distinct agents is bidirectionally connected by an edge, thus there is a directed path from any agent to any other agent in the network.

Two matrices are used to represent the network topology: the adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ with $a_{ij} > 0$ if $(v_j, v_i) \in \mathcal{W}$ and $a_{ij} = 0$ otherwise, and the Laplacian matrix $\mathcal{L} = [\ell_{ij}] \in \mathbb{R}^{n\times n}$ with $\ell_{ii} = \sum_{j=1}^{n} a_{ij}$ and $\ell_{ij} = -a_{ij}, i \neq j$, which is generally asymmetric for directed graphs.

B. Stochastic Matrices

A nonnegative square matrix is called (row) stochastic matrix if its every row is summed up to one. The product of two stochastic matrices is still a stochastic matrix. A row stochastic matrix $P \in \mathbb{R}^{n\times n}$ is called indecomposable and aperiodic if $\lim_{k\to\infty} P^k = \mathbf{1}y^T$ for some $y \in \mathbb{R}^n$ [30], where $\mathbf{1}$ is a vector with all elements being 1.

III. Consensus

Consider a group of n agents, each with single-integrator kinematics described by

$$\dot{x}_i(t) = u_i(t), \quad i = 1, \cdots, n \tag{1}$$

where $x_i(t)$ and $u_i(t)$ are, respectively, the state and the control input of the ith agent. A typical consensus control algorithm is designed as

$$u_i(t) = \sum_{j=1}^{n} a_{ij}(t)\,[x_j(t) - x_i(t)] \tag{2}$$

where $a_{ij}(t)$ is the (i, j)th entry of the corresponding adjacency matrix at time t. The main idea behind (2) is that each agent moves towards the weighted average of the states of its neighbors. Given the switching network pattern due to the continuous motions of the dynamic agents, coupling coefficients $a_{ij}(t)$ in (2), hence the graph topologies, are generally time varying. It is shown in [9], [10] that consensus is achieved if the underlying directed graph has a directed spanning tree in some jointly fashion in terms of a union of its time-varying graph topologies.

The idea behind consensus serves as a fundamental principle for the design of distributed multi-agent coordination algorithms. Therefore, investigating consensus has been a main research direction in the study of distributed multi-agent coordination. To bridge the gap between the study of consensus algorithms and many physical properties inherited in practical systems, it is necessary and meaningful to study consensus by considering many practical factors, such as actuation, control, communication, computation, and vehicle dynamics, which characterize some important features of practical systems. This is the main motivation to study consensus. In Section III-A, an overview of the research progress in the study of consensus is given, regarding stochastic network topologies and dynamics, complex dynamical systems, delay effects, and quantization, mainly after 2006. Several milestone results prior to 2006 can be found in [2], [4]–[6], [8]–[10], and [31].

A. Stochastic Network Topologies and Dynamics

In multi-agent systems, the network topology among all vehicles plays a crucial role in determining consensus. The objective here is to explicitly identify necessary and/or sufficient conditions on the network topology such that consensus can be achieved under properly designed algorithms.

It is often reasonable to consider the case when the network topology is deterministic under ideal communication channels. Accordingly, main research on the consensus problem was conducted under a deterministic fixed/switching network topology. That is, the adjacency matrix $\mathcal{A}(t)$ is deterministic. Some other times, when considering random communication failures, random packet drops, and communication channel instabilities inherited in physical communication channels, it is necessary and important to study consensus problem in the stochastic setting where a network topology evolves according to some random distributions, that is, the adjacency matrix $\mathcal{A}(t)$ is stochastically evolving.

In the deterministic setting, consensus is said to be achieved if all agents eventually reach agreement on a common state. In the stochastic setting, consensus is said to be achieved *almost surely* (respectively, *in mean-square* or *in probability*) if all agents reach agreement on a common state almost surely (respectively, in mean-square or with probability one). Note that the problem studied in the stochastic setting is slightly different from that studied in the deterministic setting due to the different assumptions in terms of the network topology. Consensus over a stochastic network topology was perhaps first studied in [32], where some sufficient conditions on the network topology were given to guarantee consensus with probability one for systems with single-integrator kinematics equation (1), where the rate of convergence was also studied. Further results for consensus under a stochastic network topology were reported in [33]–[35], where research effort was conducted for systems with single-integrator kinematics [33], [34] or double-integrator dynamics [35]. Consensus for single-integrator kinematics under stochastic network topology has been extensively studied in particular, where some general conditions for almost-surely consensus was derived [34]. Loosely speaking, an almost sure consensus for single-integrator kinematics can be achieved, i.e., $x_i(t) - x_j(t) \to 0$ almost surely, if and only if the expectation of the network topology, namely, the network topology associated with expectation $E[\mathcal{A}(t)]$, has a directed spanning tree. It is worth noting that the conditions are analogous to that in [9] and [10], but in the stochastic setting. In view of the special structure of the closed-loop systems concerning consensus for single-integrator kinematics, basic properties of the stochastic matrices play a crucial role in the convergence analysis of the associated control algorithms. Consensus for double-integrator dynamics was studied in [35], where the switching network topology is assumed to be driven by a Bernoulli process, and it was shown that consensus can be achieved if the union of all the graphs has a directed spanning tree. Apparently, the requirement on the network topology for double-integrator dynamics is a special case of that for single-integrator kinematics due to the difference nature of the final states (constant final states for single-integrator kinematics and possible dynamic final states for double-integrator dynamics) caused by the substantial dynamical difference. It is still an open question as if some general conditions (corresponding to some specific algorithms) can be found for consensus with double-integrator dynamics.

In addition to analyzing the conditions on the network topology such that consensus can be achieved, a special type of consensus algorithm, the so-called gossip algorithm [36], [37], has been used to achieve consensus in the stochastic setting. The gossip algorithm can always guarantee consensus almost surely if the available pairwise communication channels satisfy certain conditions (such as a connected graph). The way of network topology switching does not play any role in the consideration of consensus.

The current study on consensus over stochastic network topologies has shown some interesting results regarding: 1) consensus algorithm design for various multi-agent systems; 2) conditions of the network topologies on consensus; and 3) effects of the stochastic network topologies on the convergence rate. Future research on this topic includes, but not limited to, the following two directions. 1) When the network topology itself is stochastic, how to determine the probability of reaching consensus almost surely? (2) Compared with the deterministic network topology, what are the advantages and disadvantages of the stochastic network topology, regarding such as robustness and convergence rate?

As is well known, disturbances and uncertainties often exist in networked systems, for example, channel noise, communication noise, and uncertainties in network parameters. In addition to the stochastic network topologies discussed above, the effect of stochastic disturbances [38], [39] and uncertainties [40] on the consensus problem also needs investigation. Study has been mainly devoted to analyzing the performance of consensus algorithms subject to disturbances and to presenting conditions on the uncertainties such that consensus can be achieved. In addition, another interesting direction in dealing with disturbances and uncertainties is to design distributed local filtering algorithms so as to save energy and improve computational efficiency. Distributed local filtering algorithms play an important role and are more effective than traditional centralized filtering algorithms for multi-agent systems. For example, in [41]–[43], some distributed Kalman filters are designed to implement data fusion. In [44], by analyzing consensus and pinning control in synchronization of complex networks, distributed consensus filtering in sensor networks is addressed. Recently, Kalman filtering over a packet-dropping network is designed through a probabilistic approach [45]. Today, it remains a challenging problem to incorporate both dynamics of consensus and probabilistic (Kalman) filtering into a unified framework.

B. Complex Dynamical Systems

Since consensus is concerned with the behavior of a group of vehicles, it is natural to consider the system dynamics for practical vehicles in the study of the consensus problem. Although the study of consensus under various system dynamics is due to the existence of complex dynamics in practical systems, it is also interesting to observe that system dynamics play an important role in determining the final consensus state. For instance, the well-studied consensus of multi-agent systems with single-integrator kinematics often converges to a constant final value instead. However, consensus for double-integrator dynamics might admit a dynamic final value (i.e., a time function). These important issues motivate the study of consensus under various system dynamics.

As a direct extension of the study of the consensus problem for systems with simple dynamics, for example, with single-integrator kinematics or double-integrator dynamics, consensus with general linear dynamics was also studied recently [46]–[48], where research is mainly devoted to finding feedback control laws such that consensus (in terms of the output states) can be achieved for general linear systems

$$\dot{x}_i = Ax_i + Bu_i, \quad y_i = Cx_i \tag{3}$$

where A, B, and C are constant matrices with compatible sizes. Apparently, the well-studied single-integrator kinematics and double-integrator dynamics are special cases of (3) for properly choosing A, B, and C.

As a further extension, consensus for complex systems has also been extensively studied. Here, the term *consensus for complex systems* is used for the study of consensus problem when the system dynamics are nonlinear [49]–[53] or with nonlinear consensus algorithms [54], [55]. Examples of the nonlinear system dynamics include the following.

- Nonlinear oscillators [50]: the dynamics are often assumed to be governed by the Kuramoto equation $\dot{\theta}_i = \omega_i + (K/N)\sum_{j=1}^{N}\sin(\theta_j - \theta_i)$, $i = 1, 2, \cdots, N$, where θ_i and ω_i are, respectively, the phase and natural frequency of the ith oscillator, N is the number of oscillators, and K is the control gain. Generally, the control gain K plays a crucial role in determining the synchronizability of the network.
- Complex networks [53]: the dynamics are typically represented as

$$\dot{x}_i(t) = f(x_i(t)) + c\sum_{j=1, j\neq i}^{N} a_{ij}(t)\Gamma(x_j(t) - x_i(t)), \quad i = 1, 2, \cdots, N \quad (4)$$

where $x_i = (x_{i1}, x_{i2}, \cdots, x_{in})^T \in \mathbb{R}^n$ is the state vector of the ith node, $f : \mathbb{R}^n \mapsto \mathbb{R}^n$ is a nonlinear vector function, c is the overall coupling strength, $A(t) = [a_{ij}(t)]$ is the outer coupling matrix with $a_{ij}(t) = 1$ if node i and node j are connected at time t but otherwise $a_{ij}(t) = 0$, with $a_{ii}(t) = k_i$ (degree of node i), and Γ is a general inner coupling matrix describing the inner interactions between different state components of agents. It is easy to see that model (1) with control input (2) is a special case of (4) with $f = 0$.
- Nonholonomic mobile robots [56]: the dynamics are described by

$$\dot{x}_i = u_i\cos\theta_i, \quad \dot{y}_i = u_i\sin\theta_i \quad \dot{\theta}_i = \omega_i, \qquad i = 1, \cdots, N \quad (5)$$

where $[x_i, y_i]$ denotes the location of the ith agent, and u_i and ω_i denote, respectively, its translational and rotational velocity. Note that there are three states and two control inputs. Therefore, the dynamics for nonholonomic mobile robots are underactuated. This poses substantial difficulties in designing proper consensus algorithms with corresponding stability analysis.
- Rigid bodies and the like [51], [52]. One typical (but not unique) description of the dynamics is

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + g_i(q_i) = \tau_i, \qquad i = 1, \cdots, N \quad (6)$$

where $q_i \in \mathbb{R}^p$ is the vector of generalized coordinates, $M_i(q_i) \in \mathbb{R}^{p\times p}$ is the symmetric positive-definite inertia matrix, $C_i(q_i, \dot{q}_i)\dot{q}_i \in \mathbb{R}^p$ is the vector of Coriolis and centrifugal torques, $g_i(q_i)$ is the vector of gravitational torques, and $\tau_i \in \mathbb{R}^p$ is the vector of torques produced by the actuators associated with the ith agent. In practice, the dynamics of many mechanical systems are similar to (6). A notable property regarding the dynamics of rigid bodies is that $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$ is skew-symmetric (i.e., $z^T[\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)]z = 0$ for all $z \in \mathbb{R}^p$), which plays a crucial role in finding Lyapunov functions and the subsequent stability analysis.

Although the aforementioned system dynamics are different from the well-studied single-integrator kinematics and double-integrator dynamics, the main research problem is the same, namely, to drive all agents to some common states through local interactions among agents. Similarly to the consensus algorithms proposed for systems with simple dynamics, the consensus algorithms used for these complex models are also based on a weighted average of the state differences, with some additional terms if necessary. Main research work has been conducted to design proper control algorithms and derive necessary and/or sufficient conditions such that consensus can be achieved ultimately.

Note that although the objective is same, i.e., to guarantee reaching agreement on some final states, the problem is more complicated due to the nonlinearity of the closed-loop systems. In addition, most properties of stochastic matrices cannot be directly applied to their convergence analysis. The main techniques used in their stability analysis include dissipativity theory [49], nonsmooth analysis [55], [56], and especially Lyapunov functions [50]–[52], [56].

The current research on consensus with complex systems focuses on fully actuated systems although consensus for nonholonomic mobile robots [56], which are typical underactuated systems. Note that many mechanical devices are described by systems with underactuation. Therefore, it is important to develop appropriate consensus algorithms for underactuated systems.

C. Delay Effects

Time delay appears in almost all practical systems due to several reasons: 1) limited communication speed when information transmission exists; 2) extra time required by the sensor to get the measurement information; 3) computation time required for generating the control inputs; and 4) execution time required for the inputs being acted. In general, time delay reflects an important property inherited in practical systems due to actuation, control, communication, and computation.

Knowing that time delay might degrade the system performance or even destroy the system stability, studies have been conducted to investigate its effect on system performance and stability. A well-studied consensus algorithm for (1) is given in (2), where it is now assumed that time delay exists. Two types of time delays, *communication delay* and *input delay*, have been considered in the literature. Communication delay accounts for the time for transmitting information from origin to destination. More precisely, if it takes time T_{ij} for agent i to receive information from agent j, the closed-loop system of (1) using (2) under a fixed network topology becomes

$$\dot{x}_i(t) = \sum_{j=1}^{n} a_{ij}(t)\left[x_j(t - T_{ij}) - x_i(t)\right]. \quad (7)$$

An interpretation of (7) is that at time t, agent i receives information from agent j and uses data $x_j(t - T_{ij})$ instead of $x_j(t)$ due to the time delay. Note that agent i can get its own information instantly, therefore, input delay can be considered as the summation of computation time and execution time. More precisely, if the input delay for agent i is given by T_i^p, then the closed-loop system of (1) using (2) becomes

$$\dot{x}_i(t) = \sum_{j=1}^{n} a_{ij}(t)\left[x_j\left(t - T_i^p\right) - x_i\left(t - T_i^p\right)\right]. \quad (8)$$

Clearly, (7) refers to the case when only communication delay is considered while (8) refers to the case when only input delay is considered. It should be emphasized that both communication delay and input delay might be time-varying and they might coexist at the same time.

In addition to time delay, it is also important to consider packet drops in exchanging state information. Fortunately, consensus with packet drops can be considered as a special case of consensus with time delay, because re-sending packets after they were dropped can be easily done but just having time delay in the data transmission channels.

Thus, the main problem involved in consensus with time delay is to study the effects of time delay on the convergence and performance of consensus, referred to as *consensusability* [57].

Because time delay might affect the system stability, it is important to study under what conditions consensus can still be guaranteed even if time delay exists. In other words, can one find conditions on the time delay such that consensus can be achieved? For this purpose, the effect of time delay on the consensusability of (1) using (2) was investigated. When there exists only (constant) input delay, a sufficient condition on the time delay to guarantee consensus under a fixed undirected interaction graph is presented in [8]. Specifically, an upper bound for the time delay is derived under which consensus can be achieved. This is a well-expected result because time delay normally degrades the system performance gradually but will not destroy the system stability unless the time delay is above a certain threshold. Further studies can be found in, e.g., [58], [59], which demonstrate that for (1) using (2), the communication delay does not affect the consensusability but the input delay does. In a similar manner, consensus with time delay was studied for systems with different dynamics, where the dynamics (1) are replaced by other more complex ones, such as double-integrator dynamics [60], [61], complex networks [62], [63], rigid bodies [64], [65], and general nonlinear dynamics [66].

In summary, the existing study of consensus with time delay mainly focuses on analyzing the stability of consensus algorithms with time delay for various types of system dynamics, including linear and nonlinear dynamics. Generally speaking, consensus with time delay for systems with nonlinear dynamics is more challenging. For most consensus algorithms with time delays, the main research question is to determine an upper bound of the time delay under which time delay does not affect the consensusability. For communication delay, it is possible to achieve consensus under a relatively large time-delay threshold. A notable phenomenon in this case is that the final consensus state is constant. Considering both linear and nonlinear system dynamics in consensus, the main tools for stability analysis of the closed-loop systems include matrix theory [58], Lyapunov functions [62], frequency-domain approach [59], passivity [63], and the contraction principle [67].

Although consensus with time delay has been studied extensively, it is often assumed that time delay is either constant or random. However, time delay itself might obey its own dynamics, which possibly depend on the communication distance, total computation load, and computation capability. Therefore, it is more suitable to represent the time delay as another system variable to be considered in the study of the consensus problem. In addition, it is also important to consider time delay and other physical constraints simultaneously in the study of the consensus problem.

D. *Quantization*

Quantized consensus has been studied recently with motivation from digital signal processing. Here, quantized consensus refers to consensus when the measurements are digital rather than analog therefore the information received by each agent is not continuous and might have been truncated due to digital finite precision constraints. Roughly speaking, for an analog signal s, a typical quantizer with an accuracy parameter δ, also referred to as quantization step size, is described by $Q(s) = q(s, \delta)$, where $Q(s)$ is the quantized signal and $q(\cdot, \cdot)$ is the associated quantization function. For instance [68], a quantizer rounding a signal s to its nearest integer can be expressed as $Q(s) = n$, if $s \in [(n - 1/2)\delta, (n + 1/2)\delta]$, $n \in \mathcal{Z}$, where $\mathcal{Z}$ denotes the integer set. Note that the types of quantizers might be different for different systems, hence $Q(s)$ may differ for different systems. Due to the truncation of the signals received, consensus is now considered achieved if the maximal state difference is not larger than the accuracy level associated with the whole system. A notable feature for consensus with quantization is that the time to reach consensus is usually finite, that is, it often takes a finite period of time for all agents' states to converge to an accuracy interval. Accordingly, the main research is to investigate the convergence time associated with the proposed consensus algorithm.

Quantized consensus was probably first studied in [68], where a quantized gossip algorithm was proposed and its convergence was analyzed. In particular, the bound of the convergence time for a complete graph was shown to be polynomial in the network size. In [69], coding/decoding strategies were introduced to the quantized consensus algorithms, where it was shown that the convergence rate depends on the accuracy of the quantization but not the coding/decoding schemes. In [70], quantized consensus was studied via the gossip algorithm, with both lower and upper bounds of the expected convergence time in the worst case derived in terms of the principle submatrices of the Laplacian matrix. Further results regarding quantized consensus were reported in [71]–[73], where the main research was also on the convergence time for various proposed quantized consensus algorithms as well as the quantization effects on the convergence time. It is intuitively reasonable that the convergence time depends on both the quantization level and the network topology. It is then natural to ask if and how the quantization methods affect the convergence time. This is an important measure of the robustness of a quantized consensus algorithm (with respect to the quantization method).

Note that it is interesting but also more challenging to study consensus for general linear/nonlinear systems with quantization. Because the difference between the truncated signal and the original signal is bounded, consensus with quantization can be considered as a special case of one without quantization when there exist bounded disturbances. Therefore, if consensus can be achieved for a group of vehicles in the absence of quantization, it might be intuitively correct to say that the differences among the states of all vehicles will be bounded if the quantization precision is small enough. However, it is still an open question to rigorously describe the quantization effects on consensus with general linear/nonlinear systems.

E. Remarks

In summary, the existing research on the consensus problem has covered a number of physical properties for practical systems and control performance analysis. However, the study of the consensus problem covering multiple physical properties and/or control performance analysis has been largely ignored. In other words, two or more problems discussed in the above subsections might need to be taken into consideration simultaneously when studying the consensus problem. In addition, consensus algorithms normally guarantee the agreement of a team of agents on some common states without taking group formation into consideration. To reflect many practical applications where a group of agents are normally required to form some preferred geometric structure, it is desirable to consider a task-oriented formation control problem for a group of mobile agents, which motivates the study of formation control presented in Section IV.

IV. Formation Control

Compared with the consensus problem where the final states of all agents typically reach a singleton, the final states of all agents can be more diversified under the formation control scenario. Indeed, formation control is more desirable in many practical applications such as formation flying, cooperative transportation, sensor networks, as well as combat intelligence, surveillance, and reconnaissance. In addition, the performance of a team of agents working cooperatively often exceeds the simple integration of the performances of all individual agents. For its broad applications and advantages, formation control has been a very active research subject in the control systems community, where a certain geometric pattern is aimed to form with or without a group reference. More precisely, the main objective of formation control is to coordinate a group of agents such that they can achieve some desired formation so that some tasks can be finished by the collaboration of the agents. Generally speaking, formation control can be categorized according to the group reference. Formation control without a group reference, called *formation producing*, refers to the algorithm design for a group of agents to reach some pre-desired geometric pattern in the absence of a group reference, which can also be considered as the control objective. Formation control with a group reference, called *formation tracking*, refers to the same task but following the predesignated group reference. Due to the existence of the group reference, formation tracking is usually much more challenging than formation producing and control algorithms for the latter might not be useful for the former. As of today, there are still many open questions in solving the formation tracking problem.

The following part of the section reviews and discusses recent research results and progress in formation control, including formation producing and formation tracking, mainly accomplished after 2006. Several milestone results prior to 2006 can be found in [74]–[76].

A. Formation Producing

The existing work in formation control aims at analyzing the formation behavior under certain control laws, along with stability analysis.

1) Matrix Theory Approach: Due to the nature of multi-agent systems, matrix theory has been frequently used in the stability analysis of their distributed coordination.

Note that consensus input to each agent [see, e.g., (2)] is essentially a weighted average of the differences between the states of the agent's neighbors and its own. As an extension of the consensus algorithms, some coupling matrices were introduced here to offset the corresponding control inputs by some angles [77], [78]. For example, given (1), the control input (2) is revised as $u_i(t) = \sum_{j=1}^{n} a_{ij}(t)C[x_j(t) - x_i(t)]$, where C is a coupling matrix with compatible size. If $x_i \in \mathbb{R}^3$, then C can be viewed as the 3-D rotational matrix. The main idea behind the revised algorithm is that the original control input for reaching consensus is now rotated by some angles. The closed-loop system can be expressed in a vector form, whose stability can be determined by studying the distribution of the eigenvalues of a certain transfer matrix. Main research work was conducted in [77], [78] to analyze the collective motions for systems with single-integrator kinematics and double-integrator dynamics, where the network topology, the damping gain, and C were shown to affect the collective motions. Analogously, the collective motions for a team of nonlinear self-propelling agents were shown to be affected by the coupling strength among the agents, the time delay, the noise, and the initial states [79].

Note that the collective motions for nonholonomic mobile robots were also studied recently in, e.g., [80], [81]. Although the study in [77], [78] is different from that in [80] and [81], similarities exist in the sense that all agents will not move to the weighted average of the states of neighboring agents, but to some offsetted state. Noticeably, the offsetted state in [77], [78] is properly designed, yet the one in [80], [81] is induced by some special nonlinear system dynamics.

In the study of formation producing with linear closed-loop systems, the associated system matrix has two interesting properties: 1) the existence of at least one zero eigenvalue and (2) the existence of at least one pair of eigenvalues on the imaginary axis. The two properties play an important role in the formation producing problem under a fixed network topology. However, the two properties might not be able to solve the formation producing problem under a switching network topology, which remains a challenging problem due to the complexity in the analysis of switching systems.

2) Lyapunov Function Approach: Although matrix theory is a relatively simple approach for stability analysis of the formation producing problem, it is not applicable in many formation producing scenarios, especially with nonlinear systems. It is then natural to consider the Lyapunov function approach.

By using the Lyapunov function approach, several typical formation-producing scenarios have been studied, including the inverse agreement problem [82], leaderless flocking and stabilization [83]–[86], and circular formation alike [80], [81], [87]–[89]. In the inverse agreement problem [82], the objective is to force a team of agents to disperse in space. Roughly speaking, for the single-integrator kinematics (1), the corresponding control input takes the form of

$u_i(t) = \sum_{j=1}^{n} b_{ij}(\|x_i - x_j\|)[x_i(t) - x_j(t)]$, where $b_{ij}(\cdot)$ is a nonnegative function. Assuming that each agent can communicate with all other agents within a radius R, the agents will disperse in space with the relative distance between any two agents being larger than R.

For the case of leaderless flocking, research has been conducted to stabilize a group of agents towards some desired geometric formation, where the inter-agent interaction is described directly or indirectly by some nonnegative potential function $V_{ij}(\|x_i - x_j\|)$ regardless of the final group velocity. Some notable properties of $V_{ij}(\|x_i - x_j\|)$ includes: 1) $V_{ij}(\|x_i - x_j\|)$ achieves its minimum when $\|x_i - x_j\|$ is equal to the desired inter-agent distance between agents i and j; 2) $V_{ij}(\|x_i - x_j\|)$ increases as $\|x_i - x_j\|$ decreases from the desired distance to zero and $V_{ij}(\|x_i - x_j\|)$ could approach infinity as $\|x_i - x_j\|$ approaches zero; 3) $V_{ij}(\|x_i - x_j\|)$ increases as $\|x_i - x_j\|$ increases from the desired distance to the maximum communication range R. The basic idea behind the potential function $V_{ij}(\|x_i - x_j\|)$ is to drive the inter-agent distance to the desired value while avoiding possible inter-agent collision. The corresponding control law for each agent is usually chosen to be the same as the corresponding consensus algorithm except that the $x_i - x_j$ term is replaced by $\nabla_{x_i} V(\|x_i - x_j\|)$ here. A fundamental limitation is that all agents will normally converge to some (constant) inter-agent configuration locally in the sense that some nonnegative potential function achieves its local minimum. Accordingly, the inter-agent distance might not converge to the desired value globally. It is an interesting future research topic to study how to ensure the desired inter-agent distance be achieved globally under a properly designed control algorithm. In addition, the network topology associated with a team of agents is usually assumed to be undirected, which is not applicable to many practical systems which are described by directed networks.

For the case of circular formation and the like, the main research in [80], [81], [87], and [90] was devoted to the collective motion for nonhonolomic mobile robots with dynamics given in (5). Denote $r_i = x_i + \iota y_i$, where $\iota = \sqrt{-1}$. Then, (5) becomes $\dot{r}_i = u_i e^{\iota\theta_i}$, $\dot{\theta}_i = \omega_i$, $i = 1, \cdots, N$. Due to the nature of the nonlinear dynamics, a consensus-like algorithm often renders a circular-like ultimate formation where the trajectories of all agents are circular and the relative phase difference (namely, $\theta_i - \theta_j$) is constant. The current work mainly focuses on the case when all agents share a common unit speed. Similar circular-like formation was analyzed in [88] and [89], where the system dynamics are different from (5) but share a similar nonlinearity. Due to the nonlinearity of the system dynamics, it is a challenging task to incorporate time delay, disturbances, and quantization into the existing research.

B. Formation Tracking

Although formation control without a group reference is interesting in theory, it is more realistic to study formation control in the presence of a group reference because it may represent a control objective or a common interest of the whole group. This scenario is now reviewed in this subsection.

1) Matrix Theory Approach: Similarly to the case of formation producing, matrix theory is often used in the study of the formation tracking problem.

An interesting problem in formation tracking is to design a distributed control algorithm to drive a team of agents to track some desired state. For example, given the single-integrator kinematics, control algorithms were designed in [91], [92], where the algorithms are similar to those consensus algorithms except for that an extra term is introduced here due to the existence of the group reference. If properly designed, all agents can track the group reference accurately as reported in [91], with bounded tracking errors analyzed in [92], where a discretized version in [91] was considered. It is worth mentioning that the group reference can be arbitrarily chosen as long as its derivative is bounded. In [93], [94], the synchronization of a group of linear systems to the output of another linear exosystem was investigated with or without parameter uncertainties. In [91] and [92], a general group reference was discussed while, in [93] and [94], a general system model was considered. How to solve formation tracking for general linear systems with a general group reference is still an open problem.

The formation tracking problem can be converted to a traditional stability problem by redefining the variables as the errors between each agent's state and the group reference. Then, the formation tracking problem is solved if the corresponding errors can be driven to zero. However, the formation producing problem, in general, cannot be solved in this way. Therefore, under a switching network topology, the formation tracking problem is generally easier to manage than the formation producing problem.

2) Lyapunov Function Approach: Due to the broad applications of the Lyapunov function approach in stability analysis, it has become an important tool in the study of the formation tracking problem as well.

Flocking with a dynamic group reference has been studied recently [95]–[97], where the objective is to design distributed control algorithms such that a team of agents move cohesively along the group reference. Compared with leaderless flocking where no specific final group velocity is required, the study of flocking with a dynamic group reference is much more challenging both theoretically and technically due to the existence of the dynamic group reference and the requirement on the cohesive movement of the agents along the dynamic group reference. In other words, the agents not only have to maintain some desired geometric formation but also need to follow the group reference as a whole. The combination of the two objectives makes the problem much more difficult than the leaderless flocking problem where only the first objective is involved. If enough information of the group reference is known, such as the acceleration and/or velocity information of the group reference, flocking with a dynamic group reference can be solved by employing a gradient-based control law [95], [96]. Another approach was proposed in [97], where a variable structure-based control law was used to solve the problem with less information required. Similarly to the study of the leaderless flocking problem, the existing approaches on flocking with a dynamic group reference can only reach a local minimization of certain potential functions because the potential function is generally unspecified but satisfies the conditions stated in Subsection IV-A. Accordingly, the inter-agent distance is not identical to the desired one. However, the potential-based control can be easily designed to guarantee collision avoidance and maintain the initial inter-agent communication patterns. Nevertheless, it is still an open problem to accomplish the task with global inter-agent distance stabilization, collision avoidance, and initial communication pattern maintenance.

Formation control with a group reference was studied in both linear systems [98], [99] and nonlinear systems [100]–[103] when the potential function $V(\|x_i - x_j\|)$ is replaced by some known functions, generally in the form of $\|x_i - x_j - d_{ij}\|^2$, where d_{ij} denotes the desired distance between agents i and j. Briefly, the nonlinear systems studied in this case include nonholonomic mobile robots [see (5)] [101], [102], rigid bodies [see (6)] [103], and linear systems with other nonlinear terms [100]. Compared with the flocking problem, the problem studied here is relatively easier due to the known $V(\|x_i - x_j\|)$. In general, the inter-agent distance can be driven to the desired one. As a tradeoff, the collision avoidance and initial communication pattern maintenance need to be considered separately.

C. Remarks

Current research on formation control mainly focuses on a fixed formation where the inter-agent distance is fixed. Considering practical applications, however, it might require the formation be adaptive with respect to the events performed by the team of agents. In addition, it is important to consider constraints, such as input saturation, quantization, and power limitation, in the formation control problem. Meanwhile, the robustness is another important factor that deserves considerable attention in real applications where noise and disturbances exist.

V. Optimization

Optimization is an important subject in the studies of control systems. The main objective of optimization is to find an optimal strategy subject to some given constraints such as cost functions. Recently optimization in distributed multi-agent coordination has been studied concerning convergence speed and some specific cost functions, which are respectively reviewed below.

A. Convergence Speed

As discussed above, one important problem in consensus is the convergence speed, which characterizes how fast consensus can be achieved therefore is desirable to optimize. In this regard, the convergence speed is the cost function to be maximized.

Consider a group of n agents with dynamics described by the single-integrator kinematics (1). Equipped with (2), the dynamical system (1) can be written in a matrix form as

$$\dot{X}(t) = -\mathcal{L}X(t) \tag{9}$$

where $X(t) = [x_1(t), \cdots, x_n(t)]^T$ and $\mathcal{L}$ is the Laplacian matrix. For a network with a fixed topology, $\mathcal{L}$ is constant.

Motivated by the observation that the smallest nonzero eigenvalue of the Laplacian matrix, $\lambda_2(\mathcal{L})$, determines the worst-case convergence speed [8], research has been conducted to maximize the convergence speed [104], [105] by choosing optimal weights associated with edges. In contrast to [104], [105], where the systems are assumed to have single-integrator kinematics, optimization of the convergence speed for double-integrator dynamics was considered in [106], where the convergence speed is defined in a similar way to the $\lambda_2(\mathcal{L})$ for the case with single-integrator kinematics. It is worth mentioning that optimal convergence for general linear and nonlinear systems is still an open problem.

Other than $\lambda_2(\mathcal{L})$, another commonly used measure for the convergence speed is given by

$$\rho = \lim_{t\to\infty, X(t)\neq X^\star} \left(\frac{\|X(t) - X^\star\|}{\|X(0) - X^\star\|} \right)^{\frac{1}{t}} \tag{10}$$

where $X^\star$ represents the final equilibrium given by $\sigma\mathbf{1}$, where σ is a constant. The corresponding optimization problem is

$$\max_{u_i \in U} \rho \tag{11}$$

where U is a set of admissible controllers. Existing research in [107], [108] focuses on the case when all agents converge to the average of the initial states, i.e., $X^\star = [(1/n)\sum_{i=1}^n x_i(0)]\mathbf{1}$. For an arbitrary fixed or switching network topology, the optimization problem (11) is challenging if $X^\star$ is unknown. But if $X^\star$ is chosen as $[(1/n)\sum_{i=1}^n x_i(t)]\mathbf{1}$, then the problem becomes much easier.

B. Specific Cost Functions

In addition to the fastest convergence speed requirement, various cost functions are also subject to minimization.

One interesting problem studied in this setting is distributed multi-agent optimization, which is motivated by solving one challenge in wireless sensor networks, namely, to estimate the environment parameters and/or some functions of interest, such as temperature and source location [109]. As a simple strategy, each sensor node can send its data to an existing central location which can then process the data if it is sufficiently powered. However, due to the limited power resources and communication capabilities, this strategy is often not applicable. An alternative approach to achieving a similar objective is to estimate the environment parameters and/or some functions of interest locally, which requires much less communication bandwidth and power. In wireless sensor networks [109], the estimation problem is usually modeled as a distributed multi-agent optimization problem. Roughly speaking, the objective of distributed multi-agent optimization is to cooperatively minimize the cost function $\sum_{i=1}^n f_i(x)$, where the function $f_i : \mathbb{R}^n \mapsto \mathbb{R}$ represents the cost of agent i, known by this agent only, and $x \in \mathbb{R}^m$ is a decision vector. In [109], an incremental subgradient approach was used to solve the optimization problem for a ring type of network. It should be noted that [109] does not provide much discussion on the optimization problem under other types of network topologies.

Ref. [110] was probably the first paper studying the distributed multi-agent optimization problem under a consensus-based framework. The problem considered therein is formulated so as to minimize $\sum_{i=1}^n f_i(x)$, where $x \in \mathbb{R}^n$ and each $f_i : \mathbb{R}^n \to \mathbb{R}$ is assumed to be a convex function. Inspired by the average consensus algorithm and the standard subgradient method, a consensus-like algorithm was proposed as

$$x_i(k+1) = \sum_{j=1}^n a_{ij}(k)x_j(k) - \alpha g_i(x_i(k)) \tag{12}$$

where α is the step size and $g_i(x_i(k))$ is the subgradient of $f_i(x)$ at $x = x_i(k)$. In [109], $\sum_{j=1}^{n} a_{ij}(k)x_j(k)$ in (12) was replaced by $x_{i-1}(k)$ with $x_0(k) = x_n(k-1)$. Note that the algorithm (12) can only find suboptimal solutions, determined by the constant step size α. Further results on this topic can be found in [111]–[113], where a similar distributed multi-agent optimization problem was studied within various scenarios, under constraints [111], over random networks [112], and with broadcast-based communications in an asynchronous setting [113]. In the existing literature, time delay and disturbances have not been taken into consideration. Therefore, it is important to consider time delay and disturbances in the distributed multi-agent optimization problem due to their wide existence in wireless sensor networks.

In addition to the distributed multi-agent optimization problem, where the cost function is a sum of a series of convex functions, distributed optimization has also been considered for both infinite-horizon cost functions [114], [115] given by $J_i = \int_0^\infty [X^T(t)QX(t) + U^T(t)RU(t)]\mathrm{d}t$ and finite-horizon cost functions [116], [117] given by $J_f = \int_0^{t_f} [X^T(t)QX(t) + U^T(t)RU(t)]\mathrm{d}t$, where $X \in \mathbb{R}^n$ is the state, $U \in \mathbb{R}^n$ is the control input, and t_f is a positive constant. It is worth mentioning that the receding horizon control, known also as model predictive control, typically has finite-horizon cost functions. Differing from the research reported in [110]–[113], which is to find the optimal estimated state, the objective here is to find the optimal control laws subject to the minimization of certain cost functions. Due to requirements of optimizing the cost functions when designing the control laws, the computational complexity becomes an important problem to study. Meanwhile, the network topology plays a significant role in the optimization problem with certain cost functions, therefore it is also important to optimize the network topology subject to certain cost functions.

VI. Estimation

Due to the absence of global information, used for achieving group coordination in many cases, a distributed estimation scheme is often needed for estimation.

The first problem is to design local distributed estimators such that some global information can be estimated asymptotically or in finite time. The second problem is to design local controllers based on the local estimators such that the closed-loop system is stable. The estimation-based distributed control is essentially a combination of both centralized control and distributed control in such a way that distributed control is used in the estimation of some global information and the centralized control idea is used for local controllers design. The estimation-based distributed control strategy often inherits the merits of both centralized control and distributed control. However, it is worth emphasizing that a closed-loop system with distributed estimators is much more complicated to design than one without distributed estimators.

Main research on this topic has been reported in [118]–[121], where the joint estimation and control problem was considered subject to disturbances [118], [119] or without disturbances [120], [121]. In [118]–[121], a joint estimation and control problem is solved in the sense that the distributed estimator is used in the design of proper control algorithms such that certain global objective can be achieved. Without the aid of distributed estimators, the control design is very hard and even impossible. As can be noticed from [122]–[124], the distributed estimation problem has been considered without much discussion on specific control problems. In general, the joint estimation and control idea has provided an important approach in the study of distributed multi-agent coordination where only neighbor-based information is not sufficient for the controllers design. On the other hand, in real applications properly designed distributed estimators might be used to replace some expensive sensors.

In general, it remains a challenging problem to study task-oriented coordination control systems where the use of distributed estimation is either necessary or an appropriate replacement of certain expensive measurement devices, at the costs of difficult control system design and complex system stability analysis. Moreover, physical limitations such as bounded control input, asynchronous communication, and information quantization, could potentially reduce the applicability of the joint estimation and control scheme in various distributed multi-agent coordination systems.

VII. Discussions

In this paper, we have reviewed some recent research and development in distributed multi-agent coordination. In addition to the theoretical results reviewed above, many experiments have also been conducted to validate the theoretical designs and analysis, as can be found in [125], [126], to name just a couple of representative reports. Although the existing theoretical research and experiments have solved a number of technical problems in distributed multi-agent coordination, there are still many interesting, important and yet challenging research problems deserving further investigation. Some of them are briefly summarized here.

- *Intelligent coordination*. Intelligent coordination refers to the distributed coordination of a team of agents in the presence of artificial intelligence, namely, each agent is intelligent in the sense that they can choose the best possible responses based on its own objective. Intelligent coordination has potential applications not only in engineering and technology but also in economics and social studies. Although research problems, such as pursuer-invader problem [127], [128] and the game theory in distributed coordination [129], [130], have been studied recently, there are still many open questions especially the understanding of group behaviors in the presence of agent intelligence. One interesting problem is how to interpret the underlying complex networks as well as to stabilize and optimize the network in the presence of agent intelligence.
- *Competition and cooperation*. Today, most research is conducted based on local cooperation but not competition. This posts an obvious limitation because competition not only exists but also plays an positive role in group coordination. For example, due to the lack of competition, the final states of the traditional consensus algorithms are always limited to be within some region in the state space determined by the initial agent states. One interesting question is how to introduce competition into distributed co-

ordination so as to arrive at different desired regions and to improve the system performance by rewarding different agents with different benefits.

- *Centralization and decentralization*. Although decentralization shows obvious advantages over centralization, such as scalability and robustness, decentralization also has its own drawbacks. One shortcoming is that, under decentralized protocols, some agents cannot predict the group behavior based only on the available local information. Consequently, some group behavior cannot be controlled. As a sensible example, current economic crisis actually illustrates some disadvantages of behavioral decentralization. One interesting question, therefore, is how to balance decentralization and centralization so as to further improve the overall systems performance.

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