Neurocomputing 199 (2016) 204–218

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Multi-agent distributed coordination control: Developments and directions via graph viewpoint ☆

Xiangke Wang [a,*], Zhiwen Zeng [a], Yirui Cong [a,b]

[a] College of Mechatronic Engineering and Automation, National University of Defense Technology, Changsha 410073, China
[b] Research School of Engineering, The Australian National University, Canberra ACT 2601, Australia

ARTICLE INFO

Article history:
Received 4 July 2015
Received in revised form
22 February 2016
Accepted 7 March 2016
Available online 26 March 2016

Keywords:
Multi-agent system
Distributed coordination control
Graph theory
Consensus
Formation control

ABSTRACT

In this paper, the recent developments of distributed coordination control problems are summarized in a graph-theory-based framework. Distributed coordination has attracted tremendous attention in control and robot communities because of its potential applications in the past decade. The graph is used to describe the interconnections among agents, and different distributed coordination control problems, such as consensus, formation control, rendezvous, alignment, swarming, flocking, containment control and circumnavigation control, are adopted to this description by considering different cooperative objects. Therefore it is natural to study the distributed coordination control problems via graph theory, and the graph-theory-based results on consensus, formation control, and some closely related issues, i.e., rendezvous/alignment, swarming/flocking, containment control and circumnavigation control, are reviewed, and provide a cohesive overview in the coordination control problems, in system modeling, control law designs and analysis, and structure transformation. Finally, towards the practical applications, some potential directions possibly deserving investigation in distributed coordination control are discussed.

1. Introduction

Researchers have long noticed and carried on detailed analysis on many coordinated behaviors, for example, the forage for food or defense against predators of insects, birds and fishes in nature, and self-organization or self-excitation of particles in physics [1–5]. These behaviors attracted researchers to consider seriously why the creature and particles take initiative to coordinate, and motivated the theoretical and applied studies on multi-agent coordination. With these inspirations, the coordination control in a network of mobile autonomous agents, such as unmanned aerial vehicles (UAVs), unmanned ground vehicles (UGVs) and unmanned underwater vehicles (UUVs), was of interest in control and robotics in the past decade. Though it appears to be more complicated than single-agent systems, there are indeed many significant advantages in the coordinations of MAS (multi-agent system) over the single-agent system, for example, [6–13]:

- Distributed sensors and actuators, as well as inherent parallelism.
- Larger redundancy, higher robustness and greater fault tolerance. If one agent is destroyed, its task can be re-allocated and completed by others.
- Performing tasks that single-agent systems cannot do, such as in cargo transportation multiple vehicles can cooperate to put up big goods.
- Completing missions usually with higher performance and lower cost than single-agent systems, such as multiple mini-satellite formation can greatly reduce the cost in economy and enhance the accuracy in celestial observations that a single costly satellite.

No doubt that there are more advantages as well.

In general, two approaches are commonly adopted for coordination of multiple agents, i.e., a centralized control and a distributed control. In the centralized control, all computations and controls are completed in a global central station, which may result in high communication burden, high computational load and high memory requirement. On the contrary, the distributed approach requires no central controller, and all measures and controls are done in many local centers. Although both approaches are considered practical depending on the conditions of the real applications, the distributed approach is believed more promising due to many inevitable physical constraints such as narrow communication bandwidths, limited computing/memory resources,

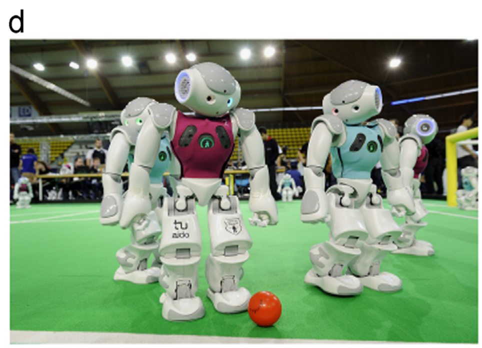
☆The work is supported by the National Natural Science Foundation of China under No. 61403406.
* Corresponding author.
E-mail address: xkwang@nudt.edu.cn (X. Wang).

http://dx.doi.org/10.1016/j.neucom.2016.03.021
0925-2312/

Multiple mobile robot systems in military

Multiple satellite formation

BionicAnt robots transport large object (from https://www.festo.com/group/en/cms/10157.htm)



Soccer robots of standard platform league in RoboCup (from http://www.robocup2014.org/?p=893)

Fig. 1. Some typical applications of multi-agent distributed coordinations.

and large sizes of vehicles to manage and control [12]. In addition, with the improvements of the embedded computing and communicating technologies, the distributed coordinations of MAS have become easy to materialize.

Generally speaking, distributed coordination uses *local* interactions between agents to achieve *collective* behaviors of multiple agents, and therefore to accomplish *global* tasks. It has a broad range of potential applications, such as in military, aerospace, industry, and entertainment. Fig. 1 shows some typical applications of multi-agent distributed coordinations. In the military field, multiple mobile robot systems shown in Fig. 1(a) can adopt a proper geometric pattern to perform military tasks for taking the place of human soldiers, such as reconnaissance, searching, mine clearance, and patrol under adverse/hazardous circumstances. Taking the reconnaissance mission as an example, a single robot has limited ability to gain environmental information. However, if multiple robots keep proper formation to cooperatively apperceive the surrounding, they are likely to rapidly and accurately obtain the environmental information. In the aerospace field, satellite formation shown in Fig. 1(b) is the leading technique in the space application in 21st century, which opens up a brand-new direction for the application of satellites, especially for mini-satellites. Satellite formation can not only greatly reduce the cost and enhance the reliability and survivability, but also broaden and override the function of individual satellites and achieve the tasks that multiple single spacecrafts cannot finish. In the industrial field, multiple mobile robots can deal with the dull, dirty and dangerous work in coordination. For example, when multiple robots cooperatively carry large scale goods in a poisonous environment, their positions and orientations are strictly restricted in order to meet the requirements of load balance shown in Fig. 1(c). In the entertainment field, for example, in Fig. 1(d), multiple soccer robots, in order to keep neat or meet tactical needs, must keep some special patterns, and may dynamically switch the patterns for avoiding obstacles.

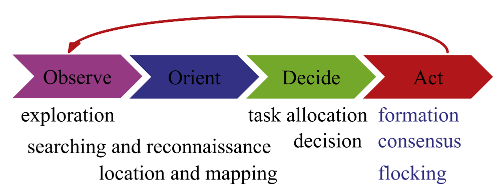


Fig. 2. Typical researches on multi-agent coordination in the view of the OODA (observe–orient–decide–act) loop.

Because of huge advantages and wide applications, all phrases in Boyd's OODA (observe–orient–decide–act) loop [14], which is clearly the dominant model of control and command today, are apt to be accomplished by distributed coordination. Fig. 2 provides some typical researches on multi-agent coordinations in the view of the OODA loop, such as cooperative exploration, searching and reconnaissance, location and mapping, decision, task assignment, formation, consensus, flocking, and to name a few.

In this survey, we mainly pay attentions to coordination control oriented problems in the last cycle, i.e., "act", in the OODA loop. Only the control problems of multi-agent distributed coordination are reviewed, in which the control inputs are designed by utilizing local information to drive the agents moving, and achieving collective objects. Coordination control is the foundation for the multiple agents to realize coordination, and much work has been done on different coordination control problems, such as consensus, formation, rendezvous, alignment, synchronization, swarming, flocking, containment control and circumnavigation control. A number of control approaches have been proposed for achieving the distributed coordination, and in general the conventional approaches include the leader–follower method [15–19], behavior-based method [20–23], virtual structure method [24–27], graph-based method [6,7,28–30], etc. Even though these methods are with different considerations, they can be concluded in a graph-based framework, because the interactions in MAS can be modeled by a graph (more details will be provided in the next section), and hence almost all the coordination control problems can be contained into a unified framework and studied by utilizing the graph theory.

The remainder of this survey is organized as follows. In Section 2, the multi-agent distributed coordination control problems are described with graphs drawing the interconnections of the agents, and different typical distributed coordination control problems are categorized by different control objectives accordingly. Afterwards, with the graph-based framework, the recent developments on typical distributed coordination control problems, including consensus (Section 3), formation control (Section 4), and some closely related issues (Section 5), i.e., rendezvous/alignment, swarming/flocking, containment control and circumnavigation control, are provided, in order to draw a cohesive overview of multi-agent distributed coordination controls. In Section 6, some topics which might be interesting for future investigation are discussed.

2. System descriptions of multi-agent coordinations

2.1. Preliminaries on graph

A *graph* is a representation of a set of objects where some pairs of objects are connected by links. In mathematics, a *graph* $\mathcal{G}=(\mathcal{V},\mathcal{E})$ comprises a finite nonempty set of vertices $\mathcal{V}=\{1,2,\ldots,n\}$; with a set $\mathcal{E}\in\mathcal{V}\times\mathcal{V}$ of edges [31]. If the edges are ordered, then the graph is called *directed graph* or *digraph* for short; otherwise, it is an *undirected graph*, or directly *graph* for short. Clearly, if treating each edge of an undirected graph as two directed edges, the undirected graph becomes a digraph. In addition, if considering the weights of the edges, the graph can be extended to a weighted one as $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{W})$, where the set of the weights $\mathcal{W}$ is often referred to be as the "cost" of the edges such as measure of the length of a route, the capacity of a line, and the energy required to move between locations in applications. A *directed path* is a sequence of directed edges of the form $(i_1,i_2),(i_2,i_3),\ldots,(i_{l-1},i_l)$. Further, if i_1 and i_l are coincided, i.e., $i_1=i_l$, then it is a *cycle*. We say that a digraph has a *spanning tree* if there exists at least one vertex, called the *root* node, having a directed path to all the other vertices.

The *algebraic graph theory* studies the relationships between the structure of graphs and different matrix representations of graphs [31]. The most important concepts of the algebraic graph theory used in MAS are the *adjacent matrix* and the *Laplacian matrix*. For a graph $\mathcal{G}=(\mathcal{V},\mathcal{E})$, the *adjacency matrix* $\mathcal{A}_G=[a_{ij}]$ is an $n\times n$ matrix given by $a_{ij}>0$ if and only if $(j,i)\in\mathcal{E}$ and $a_{ij}=0$. The *indegree* of vertex i is given by $d_i=\sum_j a_{ij}$, and the *Laplacian matrix* of $\mathcal{G}$ is $\mathcal{L}_{\mathcal{G}}=\mathrm{diag}(d_1,\ldots,d_N)-\mathcal{A}_{\mathcal{G}}$.

2.2. System descriptions

A MAS is a set of isolated agents in physics, while connected in topology such as perception or communication. It is in general distinguished with large scale (often decided by the numbers of agents) and decentralized perception and communication structures, and forms inter-connected network between agents consequently. Thus, the interconnections of the MAS can be naturally modeled by a graph with vertices being used to describe agents and the edges being used to represent the topological relationships between agents such as perception and communication links. Both directed and undirected graphs can be used to describe the MAS. For example, when the agent i is required to keep the predetermined distance to the agent j, while agent j is not required to keep the distance to agent i, the edge $\overrightarrow{ij}$ is directed from vertex i to j, and at this time a directed graph is used to model the MAS. Conversely, when the interconnection between agents i and j is bi-directional, for example, the agent i could perceive the agent j if and only if agent j could perceive agent i, the edge ij is undirected, and at this time the graph is also undirected.

The dynamics of the isolated agent i are represented by

$$\dot{x}_i=f_i(x_i,\mu_i,w_i),\quad \text{for } i=1,\ldots n, \tag{1}$$

where $x_i\in\mathbb{R}^m$ and $\mu_i\in\mathbb{R}^m$ are the state and the control input, respectively, and $w_i\in\mathbb{R}^m$ is the disturbance if exists. In addition, the input $\mu_i(i=1,\ldots,n)$ is designed to achieve the coordinated *objectives* with the form of

$$\mu_i=h_i(x_i,x_j\,|\,j\in\mathcal{N}_i), \tag{2}$$

where $\mathcal{N}_i$ is the set of agents who have relationships with agent i. Therefore, with the aid of graph, the distributed coordination control problems of the overall multi-agent systems can generally be described as shown in Fig. 3, where digraph $\mathcal{G}$ represents the perception or communication topology of multi-agent systems. The feedback connection between the agents and the integrator represents the dynamics with control laws, whereas the feedback connection between the integrator and $\mathcal{G}$ represents a relation as the Laplacian dynamics $\dot{x}=-\mathcal{L}_{\mathcal{G}}x$ in [32].

Then, different coordination control problems can be all contained in a unified framework that Fig. 3 described. The objectives of multi-agent distributed coordination control problems are to design μ_i for each agent i, using local information $x_j\,|\,j\in\mathcal{N}_i$, in order to accomplish different global *tasks*.

Considering the differences of the tasks, the multi-agent distributed coordination control problems can be categorized. For example, some typical coordination control problems and their tasks in short are listed in Table 1. More detailed explanations about these typical coordination control problems can be found in the rest of this paper.

The relationships among the typical coordination control problems are described in Fig. 4, with graph playing the center roles. As preceding description, the interconnections of the multi-agent distributed coordination control systems can be modeled by graphs, therefore, the mature graph theory can be employed to study different coordination control problems. The consensus is

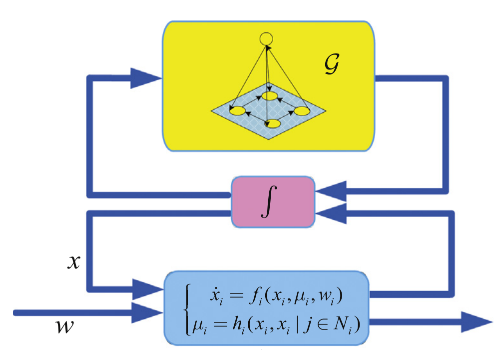


Fig. 3. Systematic description for multi-agent distributed coordination control problems with aid of graph.

Table 1
Some typical coordination control problems and their tasks in short.

Issues	Tasks
Consensus	All states reach an agreement
Formation	States achieve and preserve a geometric pattern
Rendezvous	States reach the same position
Alignment	States reach the same attitude
Swarming	Imitating collective behaviors and self-organization phenomenons
Containment control	Driving the followers' states to the convex hull spanned by the states of multiple leaders
Circumnavigation	Circling around targets

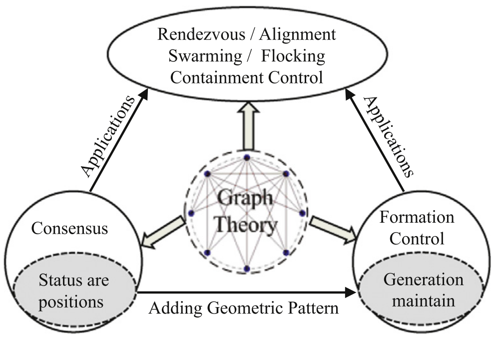


Fig. 4. Relationships among consensus, formation control and some closely related issues.

the bedrock of these problems, just like the status that taking zeros as the equilibrium in control theory. Also, it has many practical applications, e.g., the velocities of UAVs should achieve consensus in the multiple UAV formation flights. The formation control is to keep a predetermined geometric pattern and adapt to the environmental constraints for multi-agent systems. In many literatures, by adding a predefined geometric pattern, the consensus problem can be used to solve the generation and preserving of formations. The other problems, such as rendezvous, alignment, swarming, flocking, containment control and circumnavigation control, can be taken as the applications of consensus and formation control problems.

In the sequel, the developments of consensus and formation control are given in Sections 3 and 4, respectively. As their applications, some closely related issues such as rendezvous/alignment, swarming/flocking, containment control and circumnavigation control, are presented in Section 5.

3. Recent developments on consensus

In the networks of agents, the *consensus* means to reach an agreement regarding a certain quantity of interest that depends on the states of all agents. A consensus *protocol* is an interaction rule that specifies the information exchange between an agent and all of its neighbors in the network [33,34]. Mathematically, recalling dynamics (1), the consensus problem is to design a protocol μ_i by using the local information $x_j | j \in \mathcal{N}_i$ for each agent i, to make $x_1 = \cdots = x_n$.

By considering the predominance of agents, the consensus problems can be divided into two categories. If an agent has priority so that the other agents must follow its trajectory, then such an agent is called the "leader"; accordingly, if the MAS has at least one leader, then the consensus is *leader–follower consensus*; otherwise, it is *leaderless consensus*. Even though it seems that there is an obvious difference between leader–follower and leaderless consensus, they have the same foundations, because the

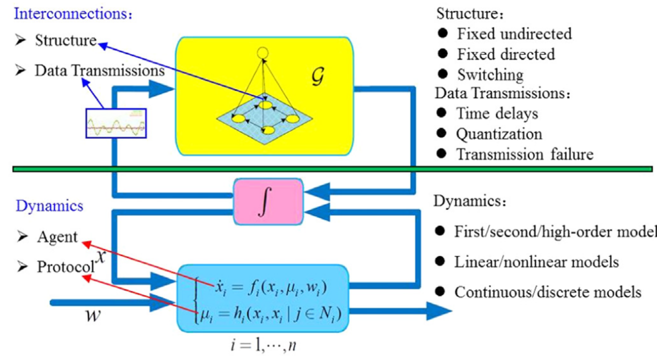


Fig. 5. Key elements considered in consensus problems.

topology and consensus protocol can make their common behavior act as a leader. Afterwards, we will mainly focus on the developments of leaderless consensus and then point out some important results about leader–follower consensus.

The beginning of researches on consensus is [35], in which Vicsek et al. propose a simple discrete-time model of particles all moving in the plane at the same speed but with different headings, and the system behavior is analyzed only through simulations, since at that time the researchers did not realize the importance of the algebraic graph theory. The seminar work on consensus analysis using the algebraic graph theory is given by Jadbabaie et al. [21], in which a theoretical explanation for the observed behavior in [35] is provided. After that a large number of literatures use the algebraic graph theory to study the consensus problem, for example, literature [34,36–44], to name a few.

Recall that in Fig. 3, two key elements for the multi-agent coordinations are considered in general, namely, the dynamics of agents and the interconnections among agents. For consensus, we further develop these two key elements in Fig. 5. To be more specific, the dynamics are normally integrated by the dynamics of the agent (1) and the protocol μ_i used among agents. The former is the inherent model of the agents, such as the first/second/high-order models, linear/nonlinear models, and continuous/discrete models; the latter is used to modify the agent inherent model in order to make the MAS achieve a consensus. The interconnections among agents are normally considered the topology structures and data transmissions. The topology structure is mainly the interconnection of agents, represented by $\mathcal{G}$ in Fig. 3, which can be directed, undirected, or even switching; while data transmissions often considered the time delays, quantization, noise and transmission failure between the communication/perception of agents. This completes the description of key elements for consensus problems.

In the sequel, following the way in Fig. 5, much attention is drawn to the dynamics and the interconnections. In the view of the dynamics and topology structure, the consensus for first-order, high-order and nonlinear dynamics under undirected/directed topologies with/without switching are considered, in Section 3.1–3.3, respectively; in the special view of the data transmission of interconnections, the general results for time delay and quantization will be explored, in Sections 3.4 and 3.5, respectively. Note that in all aspects, the algebraic graph theory plays a central role in the protocol designs, stability proofs and convergence rate analysis. Therefore, the convergence rate analysis, reviewed in Section 3.6, are concluded to end this section.

3.1. First-order dynamics

The most basic case of consensus is that the considered agents are governed by first-order dynamics, especially by single-integrator dynamics. The theoretical framework for reaching a consensus for networked agents with single-integrator dynamics under fixed/switching communicating topology was introduced by Olfati-Saber and Murray in literature [36], building on their earlier work of [28]. The subsequent studies mainly followed up on the way of [36].

For single-integrator dynamics, the dynamics (1) is simply represented by $\dot{x}_i = \mu_i$, and the commonly used continuous-time consensus protocol is [34,36–38,43]

$$\mu_i = \dot{x}_i(t) = -\sum_{j=1}^{n} a_{ij}(t)(x_j(t) - x_i(t)), \quad i = 1, \ldots, n; \tag{3}$$

in which parameters $x_i(t)$ and $a_{ij}(t)$ represent the state of agent i and the (i,j) entry in the adjacent matrix $\mathcal{A}_{\mathcal{G}}(t)$ of a given graph $\mathcal{G}(t)$ at time t, respectively.

Remark 1. Similar to the continuous-time case, for first-order discrete systems, the commonly used consensus protocol is given in [21,34,38,44]

$$x_i[k+1] = \sum_{j=1}^{n} a_{ij}[k]x_j[k], \quad i = 1, \ldots, n, \tag{4}$$

in which parameters $x_i[k]$ and $a_{ij}[k]$ represent the states of agent i and the (i,j) entry in the adjacent matrix $\mathcal{A}_{\mathcal{G}}[k]$ of a given graph $\mathcal{G}[k]$ at time k, respectively. The only difference for the continuous-time case and the discrete case is that time t in continuous-time domain is replaced with time k in discrete-time domain.

Note that protocols (3) and (4) only use the relative state information of the adjacent agent, so it is local and distributed. Then, by using the tools of the algebraic graph theory, different conclusions for different cases can be made. When the topology is fixed, protocols (3) and (4) can guarantee a consensus if $\mathcal{G}$ is connected for an undirected graph or has a rooted spanning tree for a digraph. When the topology is switching, protocols (3) and (4) result in a consensus if there exists an infinite sequence of contiguous, uniformly bounded time intervals, with the property that across each interval, the *union* of the $\mathcal{G}$ graphs is connected for an undirected case or has a rooted spanning tree for a directed case.

3.2. High-order dynamics

The results of the first-order system have been extended to the consensus of the second-order and high-order linear dynamical MAS, and similar consensus protocols and graph conditions are obtained. Consensus for multiple agents governed by the second-order linear dynamics has been considered by the similar framework of using the algebra graph theory [45–48]. For example, in the representative work [45], a linear distributed consensus protocol for the second-order MAS is designed with aid of the Laplacian matrix without requiring velocity information of neighbors; a variable structure control law is used to design the consensus protocol by taking the second-order system as two cascade fist-order systems in [48]. It is worth noting that many consensus protocols for linear systems requires the knowledge of the eigenvalues of the Laplacian matrix, which actually cannot be computed and implemented by each agent in a fully distributed fashion. To overcome this limitation, fully distributed consensus protocols using only local information of its own and neighbors without requiring the eigenvalues of the Laplacian matrix are proposed in [49–52].

In terms of high-order consensus problems, the first discussion is completed by Ren [53], where the second-order MAS is generalized to the lth-order integrator MAS. Afterwards, the consensus protocol is modified in [54] and χ-consensus is investigated under undirected communication topology. It should be noted that both the aforementioned literatures have no self-feedback information in the consensus protocols. With adding the self-feedback items, Ref. [55] investigates the constant-value consensus problem for high-order MASs under fixed and switching directed topology. Based on those meaningful explorations, the robust analysis and the convergence results with time delays are further provided, for example, the work in [56–58].

The consensus governed by more complex high-order linear dynamics, such as the SISO (single input single output) and the MIMO (multiple inputs multiple outputs) linear dynamics are also investigated. At the very beginning, researcher did not notice the relationship between the consensus in MAS and the controllability in individual agents; therefore, many studies are done with the assumption that the isolated agent is controllable, e.g., the work in [59]. It is proven that this technical assumption is not necessary in [60].

3.3. Nonlinear dynamics

Considered that most physical systems are inherently nonlinear in nature, the consensus where the agents are governed by nonlinear dynamics has also aroused the attention of some researchers recently. According to the nonlinear item $f_i(x_i, \mu_i, w_i)$ in (1), the problems can be categorized by consensus with continuously differentiable, global Lipschitz, and local Lipschitz nonlinear dynamics, respectively.

In the early phase, the agents with continuously differentiable nonlinear dynamics are considered in [61–63]. For instant, a backstepping based distributed adaptive consensus tracking control scheme for a class of smooth nonlinear systems with mismatched uncertainties is proposed in [63], and successfully applied to solve a formation control problem for multiple nonholonomic mobile robots. And further, the agents with nonlinear dynamics satisfying the global Lipschitz condition are also investigated. For example, Refs. [64–66] solved the consensus problem under a directed communicating graph, respectively, without and with a leader agent; Ref. [67] showed that the exponential consensus of the nonlinear multi-agent system can be achieved by transforming the original leader–follower consensus problem into the design problem of an appropriate time-varying parameter; and Ref. [68] proposed a novel nonlinear consensus protocol with an improved closed-loop performance for multi-agent networks with unknown inherently nonlinear dynamics to guarantee the finite-time convergence. However, it is worth pointing out that in most of this case, the nonlinear dynamics can be approximated by linear dynamics. As a result, the techniques performed in such work are in fact for linear systems, and hard to be extended to deal with the consensus problems with the locally Lipschitz continuous dynamics, which is a rather relaxed condition used in a wide-range of practical nonlinear systems.

Fortunately, for local Lipschitiz continuous nonlinear dynamics, the modern nonlinear control law design methods can be utilized. The robust H_∞ control is used to deal with the uncertain nonlinear networked systems with sufficient conditions in terms of LMIs under directed networks [69]. Adaptive control is employed to achieve the synchronization of uncertain nonlinear networked systems [70], and to consensus a group of nonlinear mechanical systems with parametric uncertainties within finite time [71], respectively. For local Lipschitiz continuous nonlinear second-order dynamic, the ISS (input-to-state stability) concept, together with the small gain theory, especially the recently developed cyclic-small-gain theorem, are employed to design new distributed control strategies, in order to tackle the challenge caused by the locally Lipschitz [72–76]. When the absolute position measurements and local communication are available and only relative position measurements are available, distributed control algorithms are developed for leader-following tracking problem with external disturbances [77]. Further, for partly unmeasurable states, a output feedback controller with a dynamic observer is proposed to achieve consensus [78].

3.4. Time delays

Time delays associated with both message transmission and processing, for instance caused by the communication congestion, are inevitable and also taken into account in consensus problems. Recall Fig. 5. In this case, the data transmissions are effected by delay functions.

Let δ_{ij} denote the time delay communicated from the jth agent to the ith agent, protocol (3) is modified as

$$\dot{x}_i(t) = -\sum_{j=1}^{n} a_{ij}(t)(x_j(t-\delta_{ij}) - x_i(t-\delta_{ij})). \tag{5}$$

Ref. [36] investigated the simplest case where $\delta_{ij} = \delta$ and the communication topology was fixed, undirected, and connected, and concluded that the consensus was achieved if and only if $0 \le \delta < \pi/(2\lambda_{\max}(\mathcal{L}_\mathcal{G}))$, where $\mathcal{L}_\mathcal{G}$ is the Laplacian of $\mathcal{G}$. Along the way developed by [36], different time delays and communication topologies are considered in the consensus problem. If the time delay affects only the state being transmitted, protocol (5) is modified as

$$\dot{x}_i(t) = -\sum_{j=1}^{n} a_{ij}(t)(x_j(t-\delta_{ij}) - x_i(t)), \tag{6}$$

and the consensus for switching directed topologies remains valid for an arbitrarily uniformly distributed time delay when $\delta_{ij} = \delta$ [79]. The constant or time-varying, uniformly or non-uniformly distributed time delays are considered in [80], and sufficient conditions for the existence of average consensus under bounded communication delays are given correspondingly. The consensus problems with noise-perturbed under fixed and switching topologies as well as time-varying communication delays are investigated in [81]. It is shown that the consensus is reached for arbitrarily large constant, time-varying, or distributed delays if consensus is reached without delays [82]. For the discrete consensus protocol (4), conclusions similar to that of the continuous case can be obtained. It is shown in [83] that if a consensus is reached under a time-invariant undirected communication topology, then the presence of communication delays does not affect the consensus. In addition, sufficient conditions for the consensus under dynamically changing communication topologies and bounded time-varying communication delays are shown in Ref. [84].

3.5. Quantization

It is known that constraints on communication have considerable impacts on the performance of multi-agent systems. To cope with the limitations of the finite bandwidth channels, the quantized information is generally encoded by the transmitter before transmitting and dynamically decoded at the receiver. Recall Fig. 5. In this case, the data transmissions are effected by quantization functions.

Mathematically, the quantization function, or quantizer can be modeled by a discontinuous mapping from a continuous region to a discrete set of numbers. According to the normally used communication scheme, two typical quantizers are considered, i.e., uniform and logarithmic quantizers. For a given $\delta_u > 0$, the uniform quantizer $q_u : \mathbb{R} \to \mathbb{R}$ satisfies $|q_u(a) - a| \le \delta_u, \forall a \in \mathbb{R}$; for the given $\delta_l > 0$, a logarithmic quantizer $q_l : \mathbb{R} \to \mathbb{R}$ satisfies $|q_l(a) - a| \le \delta_l |a|, \forall a \in \mathbb{R}$, where positive δ_u and δ_l are the predefined quantization intervals, which may be constant or time-varying.

After considering the quantization, protocol (3) can be rewritten by

$$\dot{x}_i(t) = -\sum_{j \in \mathcal{N}_i} a_{ij}(t) q(x_j(t) - x_i(t)), \quad i = 1, \dots, n. \tag{7}$$

It is clear that, compared with protocol (3), the quantization, which results in strong nonlinear characteristics such as discontinuity and saturation, plays a key role on the coordinated behaviors. Hereby, we divided the consensus problems into the *static quantized consensus* and the *dynamic quantized consensus*, according to the quantization.

The *static quantized consensus* implies that the used quantizer, no matter uniform and logarithmic, is not changed during the consensus evolution. The spectral properties of the incidence matrix are usually employed to carry out the convergence analysis of multi-agent system in this problem [85,86]. The performances for the second-order consensus problem with different quantizers are compared in detailed in [87]. The collective coordination of second-order passive nonlinear systems under quantized measurements are considered in [88]; and that via sampled data under directed topology is investigated in [89]. However, though it is simple, the static quantizer always leads to marginally stability rather than asymptotic stability, since it requires infinite quantization levels which cannot be achieved by the realistic digital channels, and therefore the dynamic quantizer with finite quantization levels is more of practice.

Conversely, in the *dynamic quantized consensus*, causal-coding technique is employed, which always results in asymptotic stability, such as a coding/decoding strategies based on room in-room out scheme is proposed in [90] to maintain the average consensus. Based on dynamic encoding and decoding scheme with embedding a scaling function, Ref. [91] provided an explicit relationship of the asymptotic convergence and the network parameters especially the quantization level under undirected communication. Follow this way, the quantized consensus under directed networks are further discussed [92–95].

3.6. Convergence rate analysis

As discussed above, an important performance in consensus problem is its convergence speed, which characterizes how fast the agreement can be achieved. The results show that this problem is heavily depended on the interconnected structure of agents, i.e., the properties of graph $\mathcal{G}$.

Recall the commonly used continuous-time consensus protocol (3). Denoting $\boldsymbol{x} = [x_1, \dots, x_n]$, the dynamics of the overall multi-agent system can be rewritten in a compact form,

$$\dot{\boldsymbol{x}} = -\mathcal{L}_\mathcal{G}\boldsymbol{x}, \tag{8}$$

where $\mathcal{L}_\mathcal{G}$ is the Laplacian matrix of the $\mathcal{G}$.

The convergence rate of the above consensus protocol can be naturally measured by the eigenvalues of the Laplacian matrix $\mathcal{L}_\mathcal{G}$ [36]. For a connected undirected graph, the worse-case convergence speed was shown to be the Laplacian spectral gap $\min_{\boldsymbol{x} \ne 0, 1^T\boldsymbol{x} = 0} \frac{\boldsymbol{x}^T L \boldsymbol{x}}{\|\boldsymbol{x}\|^2} = \lambda_2$, where λ_2 is the second smallest eigenvalue of $\mathcal{L}_\mathcal{G}$. Note that the smallest eigenvalue of a Laplacian for a connected undirected graph is zero and all the other eigenvalues are positive, therefore λ_2 is the smallest nonzero eigenvalue, called by the *algebraic connectivity*, of the graph. For a digraph, the convergence speed is depended on the Fiedler eigenvalue of the mirror graph of a digraph, that is the second smallest eigenvalue of a symmetric matrix $(\mathcal{L}_\mathcal{G} + \mathcal{L}_\mathcal{G}^T)/2$. In order to speed up the convergence, it is necessary to maximize the second smallest eigenvalue of $\mathcal{L}$, such as the work in [96], in which an iterative algorithm with a semidefinite programming solver is employed to fix this problem.

Another performance index to measure the convergence rate of protocol (8) is the asymptotic convergence factor, defined by

$$\rho = \sup_{x(0)\neq \bar{x}} \lim_{t\to\infty} \left(\frac{\| x(t) - x^* \|_2}{\| x(0) - x^* \|_2} \right)^{1/t}, \tag{9}$$

where x^* stands for $\lim_{t\to\infty} x(t)$, i.e., the final equilibrium of the systems [97,98]. To finding the fastest convergence speed, the problem was casted into a semidefinite programming problem in [97]. With this convergence factor, more considerations about the lower bounds on the worst-case convergence time for various classes of linear, time-invariant, distributed consensus methods are investigated in [98].

4. Recent developments on formation control

The *formation control* is that a team comprised of multiple agents keeps a predetermined geometric pattern and adapts to the environmental constraints (e.g., obstacle avoidance) at the same time during the movement towards a specific goal. In general, the main control problems in formation are summarized as follows [99,100]:

- Formation generation: how to design a formation pattern for MASs and then achieve it?
- Formation maintaining: how to maintain the formation pattern for MASs during the movement?
- Formation transformation: how to transit the formation pattern, which means transiting one formation pattern into another?
- Obstacle avoidance with formation: how to change a motion plan or a formation pattern for MASs in order to avoid obstacles during movement?
- Self-adaptation: how to automatically change the formation in order to best adapt to the dynamical unknown environment?

It is worth pointing out that the graph theory is a powerful tool to study formation control involving all the above five aspects. Similar to the study on consensus problem, the algebraic graph is employed. While except for the algebraic graph, another graph tool, rigid graph is also developed for the distanced based formation in order to minimize the required information. Therefore, in the following, after reviewing the main results by using algebraic graph in formation control, the rigid graph based results will be presented.

4.1. Algebraic graph based formation control

In formation control problems, the state x_i of agent i is often described by its position $p_i \in \mathbb{R}^m (m = 2, 3)$, and assumed that each agent i has only access to the relative position $\hat{p}_i^j = p_j - p_i, j \in \mathcal{N}_i$. Then the dynamics with control laws are re-presented by

$$\begin{cases} \dot{p}_i = f_i(p_i, \mu_i, w_i) \\ \mu_i = \ell\left(\hat{p}_i^j | j \in \mathcal{N}_i\right) \end{cases} \tag{10}$$

where $i = 1, \ldots, n$, and $\dot{p}_i \in \mathbb{R}^m$ and $\mu_i \in \mathbb{R}^m$ are the position and the control input, respectively.

By adding a pre-specified geometric pattern $\delta_{ij}^* = p_j^* - p_i^*$, where i and j are neighbored agents, the formation generation and maintaining can be achieved in the corresponding consensus problem, which is usually known as *consensus-based formation control*, e.g., in [101,102]. Generally, the control objective is to drive the relative position of neighbors to satisfy the geometric constraint, that is $\hat{p}_i^j = \delta_{ij}^*$. In this setting, graphs are naturally used to generate and maintain a formation, and consequently to achieve

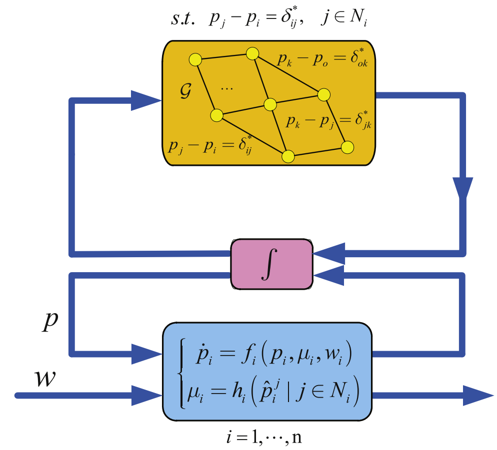


Fig. 6. System description for formation control based on algebraic graph.

transformation between different formation patterns. Recall that the neighbors of vertex i in $\mathcal{G}$ is exactly the collection of agents that have a topological relationship such as perception with the agent i. Therefore, the scheme of the consensus-based formation control can be illustrated with Fig. 6.

For the single-integrator case $\dot{p}_i = \mu_i$ with $e_\ell := p^* - p$, the feedback connection between the integrator and $\mathcal{G}$ represents a relation as

$$\dot{e}_\ell = -(L \otimes I_n) e_\ell \tag{11}$$

with the control law μ_i evolving according to $\hat{p}_i^j$. Therefore, by using the properties of the Laplacian matrix, local, distributed and scalable formation controllers can be designed, and their stabilities can also be verified by virtue of the eigenvalue of the Laplacian matrix, for a typical example, the work in the classic literature [29]. And then, Fax and Murray set up the relation between the formation controller and the topological structure of the communicating network with the Laplacian matrix, and proved that if the local controller was stable, then the stability of formation with linear dynamics depended on the stability of the information flow [28]; Lin et al. proved that if and only if there is a global accessible vertex in the perception graph, the formation is stable by using tools of the algebraic graph theory [103]. Since many phenomena in nature, such as macromolecule fluids and porous media, cannot be explained in integer-order dynamics, the formation control of fractional-order multi-agent systems under undirected/directed graph were investigated [104–106], with the help of the Laplacian matrix, as well as the developments in fractional calculus.

Consider more practical issues such as the connectivity preserving and obstacle avoidance. The desired formation shape can be maintained and all the agents are kept within limited sensing range to preserve the connectivity, by assigning each edge of $\mathcal{G}$ with positive and bounded weight function $w_{ij}(t), (i,j) \in \mathcal{E}$ in [107]. By adding appropriate edge tension function to the edges in the graph, the formation translation with the collision avoidance between robots and obstacles is obtained in [108].

In addition, the formation stability concepts are defined with the aid of the graph theory. Tanner et al. defined the *leader-to-formation stability* notion based on the graph and ISS concept, which includes both transient and steady-state errors [109]. In contrast, the *pairwise asymptotic stability* is defined based on directed graphs by involving a single pair of neighboring agents in [110], which implies that any two agents can asymptotically achieve and maintain a desired relative attitude and position though the pair of agents which may not be neighbors and do not interact with each other directly.

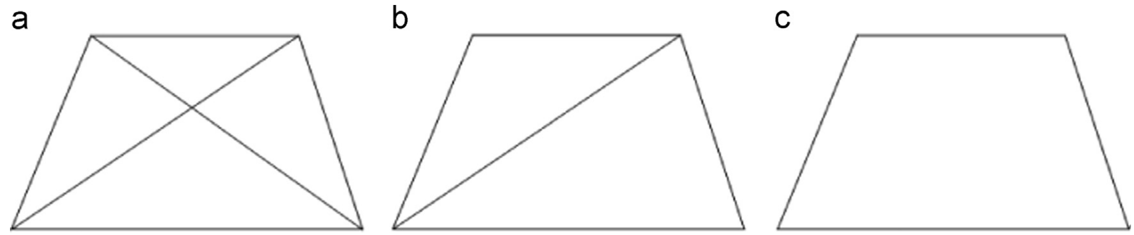


Fig. 7. (a) Rigid graph. (b) Minimally rigid graph. (c) Non-rigid graph.

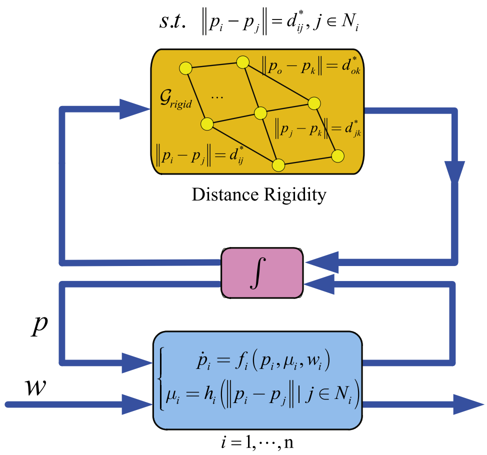


Fig. 8. Distance rigid graph based formation control.

4.2. Rigid graph based formation control

Another important tool of the graph theory employed in multi-agent formation control is the rigid graph theory. An undirected graph is *rigid* if for almost every structure, the only possible continuous moves are those which preserve every inter-agent distance; further, an undirected graph is called *minimally rigid* if it is rigid and if there exists no rigid graphs with the same number of vertices and a smaller number of edges (see Fig. 7 for more illustrations of *rigid* and *minimally rigid* graphs).

Given an arbitrary ordering of edges in $\mathcal{E}$, an edge function $r_{\mathcal{G}} : \mathbb{R}^{nN} \to \mathbb{R}^M$ associated with the framework $(\mathcal{G}, p)$ is defined as

$$r_{\mathcal{G}}(p) = \frac{1}{2}\left[\cdots, \|p_i - p_j\|^2, \cdots\right]^T, \quad (i,j) \in \mathcal{E}, \tag{12}$$

where the kth component $\|p_i - p_j\|^2$ corresponds to the length of the edge $e_k = p_j - p_i, (i,j) \in \mathcal{E}$. To characterize the rigidity property of a framework $(\mathcal{G}, p)$, the *rigidity matrix* $R(p) \in \mathbb{R}^{m \times 2n}$ is introduced, defined by

$$R(p) = \frac{\partial r_{\mathcal{G}}(p)}{\partial p}. \tag{13}$$

Therefore, the row of $R(p)$ looks like $\left[\cdots \left(p_i - p_j\right)^T \cdots \left(p_j - p_i\right)^T \cdots\right]$. Consider an arbitrary orientation of the undirected graph, we can indicate a simple expression for rigidity matrix which involves both the network topology and position configuration as

$$R(p) = diag\left(e_k^T\right)\left(E_{\mathcal{G}}^T \otimes I_n\right), \tag{14}$$

where $E_{\mathcal{G}}^T$ is the incidence matrix of $\mathcal{G}$, which is a $\{0, \pm 1\}$-matrix with rows and columns indexed by vertices and edges of $\mathcal{G}$, respectively. For edge $e_k = (j,i) \in \mathcal{E}$, $[E_{\mathcal{G}}]_{jk} = +1$, $[E_{\mathcal{G}}]_{ik} = -1$ and $[E_{\mathcal{G}}]_{lk} = 0$ if $l \neq i,j$.

The rigidity matrix can be used to determine the *infinitesimal rigidity* of the framework. A framework is infinitesimally rigid in d-dimensional space if $rank(R(p)) = dn - d(d+1)/2$. The framework $(\mathcal{G}, p)$ is *minimally infinitesimally rigid* if it is both infinitesimally and minimally rigid. An useful observation is that the matrix $R(p)R(p)^T$ is positive definite if the framework $(\mathcal{G}, p)$ is minimally infinitesimally rigid [111].

In rigid graph based formation control problem, the pre-specified geometric pattern is defined by a set of distance constraints d_{ij}^* between agent i and j, rather than the desired relative positions. Accordingly, the control laws are designed by using the distance terms $\|p_i - p_j\|$, instead of the relative positions. Therefore, the dynamics with control laws are re-presented by

$$\begin{cases} \dot{p}_i = f_i(p_i, \mu_i, w_i) \\ \mu_i = h_i\left(\|p_i - p_j\| \mid j \in \mathcal{N}_i\right) \end{cases} \tag{15}$$

where μ_i is the control input evolving according to the distance term $\|p_i - p_j\|$. The control objective is to drive each pair of the adjacent agents to satisfy the distance constraint $\|p_i - p_j\| = d_{ij}^*$ for $(i,j) \in \mathcal{E}$. As pointed out, the connection topology $\mathcal{G}$ can be explicitly incorporated into the dynamical system, so that the overall system based on distance rigidity can be illustrated as in Fig. 8.

Gradient control laws were widely employed to design control laws for rigid formation. Define potential function $\Psi(p) = \frac{1}{4}\sum_{k=1}^{m} \sigma_k^2$ with $\sigma_k = \left(p_i - p_j\right)^T\left(p_i - p_j\right) - (d_{ij}^*)^2$. By taking derivative of $\Psi(p)$, then obtain $\dot{\Psi}(p) = \sigma^T R(p)\dot{p}$ and the control law can be designed as

$$\psi = -\nabla\Psi(p) = -R^T(p)\sigma \tag{16}$$

such that

$$\dot{\Psi}(p) = \sigma^T R(p) R^T(p)\sigma \leq 0. \tag{17}$$

Under the gradient control law (16), Krick et al. show that infinitesimal rigidity is a sufficient condition for local asymptotical stability of the equilibrium manifold [112]; Dorfler and Francis [113] presented a Lyapunov stability analysis based on (16) and pointed out that the cooperative graphs admit a local stability result of the formation together with a guaranteed region of attraction. However, while the neighbored agents suffer to mismatched distance understanding under the gradient control law, the rigid formation will be distorted [114–116]. Particularly, the mismatched distance drives the rigid formation to converge exponentially fast to a closed circular orbit in $\mathbb{R}^2$, and the orbit becomes helical in $\mathbb{R}^3$. Then, a robust control law is proposed to eliminate the reported inconsistency-induced orbits in rigid formation [117,118]. Consider the situation that all the agents are located at a common point or a straight line, as the existence of trivial undesired equilibrium points, the gradient control law can only lead to local asymptotical stability. The pioneering work on the global stability properties investigated the undirected triangular/rectangular formations in [119–122]; and recently, Tian and Wang [123] proposed a perturbation method combined with the gradient control law to stabilize the desired rigid formation in a global sense. However, generally speaking, the global stability properties of the rigid formation control remains open.

Another main tools of rigid graphs are Henneberg sequence and Laman's theorems [124]. The former was raised by Henneberg to construct two-dimensional minimally rigid graphs, and the latter were raised by Laman in 1970 to verify if a two-dimensional

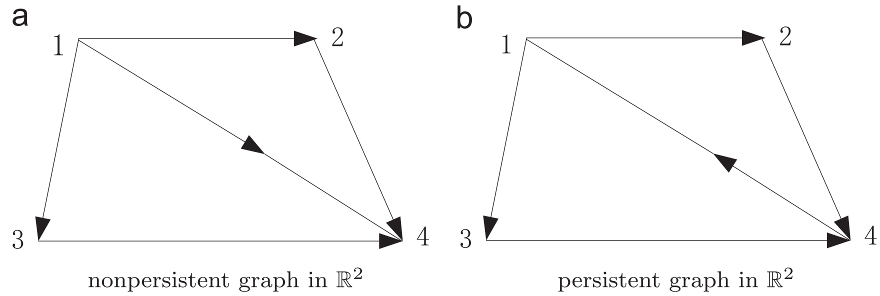


Fig. 9. (a) Nonpersistent graph in $\mathbb{R}^2$. (b) Persistent graph in $\mathbb{R}^2$.

graph was rigid [125]. With the aid of Henneberg sequence and Laman's theorems, some primitive operators are defined to deal with the transformations such as splitting and restructuring of rigid formation [126,127].

Note that the concept of rigidity is mainly for the undirected graph. Based on the concept of rigidity, a "directed rigidity" concept, named *persistence* is developed. A directed graph is *persistent* if and only if its underlying graph is rigid, and itself is constraint consistent [30], as illustrated in Fig. 9.[1] Similar to a minimally rigid one, a graph is *minimally persistent* if it is persistent and no single edge can be removed without losing persistence. The main difference between rigidity and persistence is that rigidity assumes all the constraints are satisfied, as if they were enforced by an external agency or through some mechanical properties, while persistence considers each constraint to be the responsibility of a single agent.

Based on the concept of persistence, a series of innovation work has been done by the group led by Prof. Brian D.O. Anderson, such as developing the persistent/minimum persistent notions from the rigid/minimum rigid concepts [30]; fixing the transformation, splitting and reconstruction of formation in a two-dimensional space through defining three primitive graph operators by virtue of Henneberg sequence [128,129]; designing a formation control law to keep the persistence for formation in two-dimensional space [130]; proposing distributed formation control laws in order to keep the minimum persistence for two kinds (leader–remote-rfollower and coleader) of formation with small perturbations [131].

On another way, Zelazo et al. extend the rigidity theory to the weighted framework and accordingly propose the rigidity eigenvalue, which can be used as the algebraic characterization of the infinitesimal rigidity. By estimating a common relative position reference frame amongst a team of robots with only range measurements, a fully decentralized strategy for maintaining the formation rigidity is studied in [132]. As known, the rigid/persistent formation control is heavily depended on the gradients of the potential functions closely related to the distance constraints between the neighbored agents; whereas, bearing-based formation control problem has also attracted a considerable attention as the bearing measurements are often cheaper and more accessible than the distance measurements [133,134]. Therefore, by using bearing-only measurements, a SE(2) framework combining the rigidity theorem to estimate the unscaled relative positions of robots is investigated in [135].

[1] In $\mathbb{R}^2$, the graph represented in (a) is not persistent. For almost all uncoordinated displacements of 2, 3 and 4 (even if they satisfy their constraints), 4 is indeed unable to satisfy its three constraints. This problem cannot happen for the graph represented in (b), which is persistent however.

5. Recent developments on some closely related issues

5.1. Rendezvous/alignment

The *rendezvous* problem refers to designing a distributed local control strategy to make multiple agents reach the same specified position at the same time. To some extent, rendezvous can be taken as the application of consensus in actual systems, such as robots and spacecrafts [136]. This problem was firstly proposed in literature [137], in which a distributed simple memoryless point rendezvous algorithm was proposed with proven convergence. And then, literature [138,139] extended this algorithm to a "stop-and-go" strategy under synchronous and asynchronous situations, respectively, and analyzed the convergence of the strategies. Similar to other multi-agent coordinations, the graph theory also played key roles in the research of the rendezvous problem. Cortes et al. presented a robust rendezvous algorithm in an arbitrarily dimensional space with the aid of the proximity graph [140]; and a connectivity-preserving protocol consisting of a set of distributed control rules was proposed in the case of the link in communication graph failure in literature [141]. In addition, the rendezvous problems with dynamical constraints for different practical systems are considered. For multiple omni-directional mobile robots equipped with line-of-sight limited-range sensors moving in a connected, nonconvex, unknown environment, literature [142] presented a perimeter minimizing rendezvous algorithm by using the notions of connectivity-preserving constraint sets and proximity graphs. For multiple nonholonomic unicycles, a discontinuous and time-invariant rendezvous algorithm was designed with tools from the nonsmooth Lyapunov theory and the graph theory [143]. The discontinuous rendezvous policies were further investigated in literature [144], in which the new proposed sufficient conditions for characterizing the control policies were less restrictive than those presented in the above-mentioned literature. For a multiple agent system moving like a Dubins car, a simple quantized control law was designed with three values to make agents achieve rendezvous given a connected initial assignment with minimalism in sensing and control [145].

The *attitude alignment* problem, sometimes called attitude consensus or attitude synchronization, also received many attentions in multi-agent fields, especially for multiple spacecrafts [146], multiple satellites [147] and multiple marine robots [148]. Similar to the rendezvous problem, the attitude alignment is required to design a distributed control strategy to make the attitude of multiple agents tend to be consistent simultaneously. Lawton and Beard [149] adopted the behavior-based method to design two kinds of control strategies that made a group of aircrafts achieve attitude alignment; and further, this work was extended to more general scenes which did not need bi-directional communication [146]. For the attitude alignment with limited communication and no reference signal, Sarlette et al. proposed attitude alignment strategies based on artificial potential methods [150].

5.2. Swarming/flocking

The *swarming* is often inspired by biological and physical systems, which refers to the prevalent collective behavior and the self-organization phenomenon, such as bacterial chemotaxis, ant colonies, bee colonies, flocks of birds and schools of fishes [4,2,3,151]. And recently it has emerged in MAS with focuses on physical embodiment and realistic interactions among the individuals themselves and also between the individuals and the environment [152]. Generally speaking, swarming is characterized by the following: (1) flexibility: adaptability to the environment; (2) robustness: anti-jamming to the internal and external disturbances; (3) dispersion: dynamical behavior based on individuals; and (4) self-organization: obvious overall system properties in evolution, namely emerging [153].

As a special case of swarming, the *flocking* refers to the phenomena that the MAS (usually with the second-order integrator dynamics) presents certain coordinated behaviors by decentralized control with the aid of local perception and reaction between individuals. The classical flocking model is proposed by Reynolds in [154], in which the animation of flocking behaviors, such as bird flock and fish school, are realized through three rules, i.e., collision avoidance, velocity matching and flock centering. These three rules can make the distance between agents converge to an expected value, the speed of agents tends to be consistent, and agents cannot collide with each other. After then, for the linear systems with the second-order integrator dynamics, flocking algorithms are usually designed by local artificial potential fields integrated with the graph theory [155–157]. In [155], a theoretical framework based on the algebraic graph theory and the spatially induced graphs is provided for the design and analysis of scalable flocking algorithms under a connected topology. Further, this work was extended in [156] from two directions: one is the case where only a fraction of agents are informed and the other is that where the velocity of the virtual leader is varying. In [157], the stability properties of a group of agents governed by decentralized, nearest-neighbor interaction rules are analyzed. It is worthy pointing out that the flocking problem sometimes is without considerations of collision between multi-agents, such as the work in [158,159]. And little work has been done on flocking with nonlinear dynamics, except that Dong proposed a backstepping based control law design method for the flocking of multiple nonholonomic wheeled mobile robots with directed topology in [160].

5.3. Containment control

The *containment control* can be taken as an expansion of consensus or flocking with pinning control, which refer to designing a distributed control law to drive the states of the followers to the convex hull spanned by the states of multiple leaders. In particular, when the multiple leaders are moving, some literature call it as *set tracking* problem [161]. In early 2005, the simplest containment control, which took two agents as leaders and drove the rest converge to the line decided by the two leaders, was discussed in [29].

For the containment control with the first-order dynamics under fixed undirected graphs, a simple "stop-and-go" strategy is proposed and analyzed in literature [162]. Further, the containment control with single- or double-integrator dynamics was studied in [162–166], of which, literature [165] proposed two containment control algorithms via only position measurements where the leaders were neighbors of only a subset of the followers, and literature [166] examined the containment control for a second-order multi-agent system with random switching topologies. Moreover, the containment control strategy for multiple Lagrangian system in directed topological structure was proposed in [167]; and the containment control of multiple rigid-body systems with uncertainty was studied in [168]. For the more general nonlinear dynamics, Shi et al. [161] investigated the distributed set tracking problem with unmeasurable velocities under switching directed topologies, and provided the necessary and sufficient conditions for set input-to-state stability and set integral input-to-state stability.

5.4. Circumnavigation control

Inspired by the behaviors of biological organisms, such as the encirclement techniques of bottlenose dolphins to entrap the fishes [169] and the mysterious death-vortex phenomenon of ants [170], a simple coordination strategy which is called as *circumnavigation* or *cyclic pursuit* aroused great interesting. The *circumnavigation* is a group of agents moving around the targets, which can be employed to several interesting missions such as coverage, patrolling, escorting and entrapment [171]. The 3-dimensional circular formation behavior was studied in [172] via set stabilization and a reduction principle for asymptotic stability of closed sets. Ref. [173] proposed a distributed control scheme combining attraction/repulsion from the neighbored agents and the target on different obits for a group of unicycles. Ref. [171] discussed 3-types of the encirclement control schemes for 3-dimensional space with the collision-free motion and decentralized estimation. Distributed cyclic pursuit approach for target capture task is proposed by using local distance and bearing information [174]; and in [175,176], the bearing-only circular formation scheme was discussed. Furthermore, to improve the performances with different constraints, distributed optimal methods are always used. For example, PSO (particle swarm optimization) approach is employed for target searching in unknown environments in [177].

Remark 2. The studies on distributed coordination control are still ongoing, both in depth and width. On the one hand, the studied agents are found with more and more complicated dynamics and limited perceptions, for example, from nonlinear dynamics to complex environmental constraints [178], again to communication bandwidth limitation [179–181]. On the other hand, more and more attention has been paid to various practical multi-robot systems, for example, multiple micro-satellites [182], multiple spacecrafts [183,184], multiple marine robots [185], multiple wheeled mobile robots [186,187], and multiple Lagrangian systems [188].

6. Potential directions

Theoretical work on multi-agent distributed coordination control has made significant achievements in the past decade, which have however rare practical applications. In fact, the behaviors of actual robots are much more complicated than those considered in the existing work. Therefore, how to promote the theoretical work for serving in practice should be considered seriously. Along the way presented in Fig. 5, the two key elements, i.e., the dynamics and interconnections, are both deserving further investigations in the study of multi-agent distributed coordination control.

(1) Coordination control with practical dynamics

- *Strong nonlinear dynamics*. The dynamics of UAVs, UGVs or UUVs in practice are generally found with strong nonlinearities, which are hard to be described simply by using single/double-integrator or continuously differentiable/global Lipschitz continuous function.

Therefore, it is necessary to study the multi-agent coordination control in the presence of the strong nonlinearities by means of the nonlinear control theory.

- *Heterogenous dynamics*. In existing work on coordination control, the agents are usually considered to be homogenous, which means all agents are with the same dynamics. However, in many practical applications, for example, in the cooperative reconnaissance carried out by aerial and ground vehicles, the dynamics between agents are quite different. So, the coordination control for heterogeneous robots is also a direction worth to pay attentions to in the future.
- *Intelligent Coordination.* The nonlinear dynamics of the robots are hard to be obtained accurately in practice. For single robot, many intelligent control approaches, such as learning control, neural network control, fuzzy control, or even data-driven control have been developed, to deal with the in-accurate dynamics or modeless cases. For multi-agent systems, there is less work on intelligent coordination, which is worthy of further exploration.
- *Optimization of Coordination Control.* Most of existing work in the coordination control problems only consider the stability of the control laws or the achievement of the protocols because of the complexity of the networked systems. In the practical, more performances except for the stability are asked. For example, in the formation flight problems, the additive least energy is required to prolong the sail; in some special cases, the formation is required to be formed as quick as possible. Therefore, it is necessary and valuable to consider the parameter optimization in the coordination control problems.
- *Manned/unmanned coordination.* In the forseeable future, the wide usage of multiple unmanned vehicles should be directed and administrated by vehicles with man. How to control a coordinated system blending manned and unmanned vehicles is an attractive and challenging problem.

(2) Coordination control with interconnecting constraints.

- *Rigid graph theory in three-dimensional space*. The graph theory acts as an important tool used in multi-agent distributed coordination control. However, the existing results on the graph theory are mainly considers agents in a plane, and the work cannot be directly extended to the three-dimensional space. For example, Laman's theorem and Henneberg sequence in three-dimensional space are still open problems [130]. Improving and expanding the existing foundations of the graph theory to three-dimensional space have not been effectively solved.
- *Communication constrained coordinations*. Communications for actual systems are usually not ideal. For example, time-delays, unstable signals, and limited bandwidth are inevitable appearing in the actual systems. Therefore, multi-agent distributed coordination control with non-ideal communication is a challenging problem, and much work can be further considered, such as the design of fault tolerance topological structure which is robustness against the loss of communicating nodes or links, coordination algorithms with limited communicating bandwidth or quantitative communication, or even without communications, and underlying communication protocols with reliable robustness.
- *Perception limited coordinations.* It is usually the ideal perceptions considered by the existing MAS, which means agents can obtain all the required information in real-time. However in practice, sensors equipped by agents certainly have some perceptional limitations. For example, the commonly used cameras have conical perceptional fields, and the laser ranging sensor (typically Hokuyo's series products) can only perceive the information in a sector. Meanwhile, the perceived information is usually accompanied by noise and time-delay, and further more information, such as other agents' relative velocities and accelerations, is generally difficult to obtain precisely. Therefore, the distributed coordination control with perceptional constraints is also a problem that requires to be fixed, such as coordination control with perceptional directions, coordination control without velocity measurement, and coordination control with time-delay and noisy information.

(3) Experiments on coordination control.

Many experiments have been design to validate the theoretical analysis. For instance, multiple marine robots cooperative exploration [185], flight array [189], and multiple flying vehicles cooperative handling objects [183] or playing tennis [184]. However, we have to admit that most of the existing theoretical results are only verified by simulations, rather than by actual systems, partly due to the high cost and various restrictions of the experiments. Therefore, to verify and apply the theoretical results to actual multi-robot systems is a most pressing issue too.

(4) Combination with other collective behaviors.

For the last 10 years, the work on coordination control in fact has led to the research of the distributed networked system and provided some necessary supports to different types of collective behaviors, e.g., the wireless sensor network. Therefore, how to expand and fuse the existing results of coordination control to other collective behaviors such as wireless sensor network is also a notable direction.

7. Conclusions

This paper has reviewed the recent developments on multi-agent distributed coordination control problems with focusing on the consensus, formation control, and some closely related topics including rendezvous/alignment, swarming/flocking, containment control and circumnavigation control, where the graph theory plays a central role. Furthermore, some potential directions in the coordination control have been provided. The work on coordination control problem involves many practical respects, and the authors believe that in the coming decade, more problems related to coordination control will be fixed along with the developments of other related disciplines and technologies, and will also get more practical applications in more fields.

Acknowledgements

The authors would like to thank the fruitful suggestions and comments provided by the associate editor and the anonymous reviewers, which are very helpful to improve the quality of this paper.

References

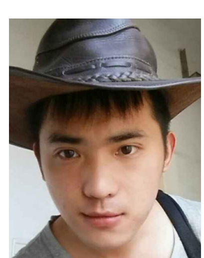
[1] S. Hubbard, B. Babak, S.T. Sigurdsson, K.G. Magnusson, A model of the formation of fish schools and migrations of fish, Ecol. Model. 174 (4) (2004) 359–374.
[2] J. Toner, Y. Tu, Hydrodynamics and phases of flocks, Ann. Phys. 318 (1) (2005) 170–244.
[3] S. Janson, M. Middendorf, M. Beekman, Honey bee swarms: how do scouts guide a swarm of uninformed bees?, Anim. Behav. 70 (2) (2005) 349–358.
[4] I.D. Couzin, J. Krause, N.R. Franks, S.A. Levin, Effective leadership and decision-making in animal groups on the move, Nature 433 (3) (2005) 513–516.
[5] H. Levine, W.J. Rappel, Self organization in systems of self-propelled particles, Phys. Rev. E 63 (1) (2001) 208–211.

[6] W. Ren, R.W. Beard, E.M. Atkins, Information consensus in multivehicle cooperative control, IEEE Control Syst. Mag. 27 (2) (2007) 71–82.
[7] B.D.O. Anderson, C. Yu, B. Fidan, J.M. Hendrickx, Rigid graph control architecture for autonomous formation, IEEE Control Syst. Mag. 28 (6) (2008) 48–63.
[8] Z. Lin, B. Francis, M. Maggiore, Distributed Control and Analysis of Coupled Cell Systems, VDM Verlag, Germany, 2008.
[9] W. Ren, R. Beard, Distributed Consensus in Multi-vehicle Cooperative Control, Spring, New York, 2008.
[10] F. Bullo, J. Cortes, S. Martinez, Distributed Control of Robotic Networks: A Mathematical Approach to Motion Coordination Algorithms, Princeton University Press, Princeton and Oxford, 2009.
[11] M. Mesbahi, M. Egerstedt, Graph Theoretic Methods in Multiagent Networks, Princeton University Press, Princeton and Oxford, 2010.
[12] Y. Cao, W. Yu, W. Ren, G. Chen, An overview of recent progress in the study of distributed multi-agent coordination, IEEE Trans. Ind. Inf. 9 (1) (2013) 427–438.
[13] K.-K. Oh, M.-C. Park, H.-S. Ahn, A survey of multi-agent formation control, Automatica 53 (2015) 424–440.
[14] R. Boyd John, A Discourse on Winning and Losing, Air University document MU43947, Briefing 1, 1987.
[15] J.P. Desai, J.P. Ostrowski, V. Kumar, Modeling and control of formations of nonholonomic mobile robots, IEEE Trans. Robot. Autom. 17 (6) (2001) 905–908.
[16] J.P. Desai, A graph theoretic approach for modeling mobile robot team formations, J. Robot. Syst. 19 (11) (2002) 511–525.
[17] L. Consolini, F. Morbidi, D. Prattichizzo, M. Tosques, Leader–follower formation control of nonholonomic mobile robots with input constraints, Automatica 44 (5) (2008) 1343–1349.
[18] D.V. Dimarogonasa, P. Tsiotrasb, K.J. Kyriakopoulosc, Leader–follower cooperative attitude control of multiple rigid bodies, Syst. Control Lett. 58 (6) (2009) 429–435.
[19] J. Chen, D. Sun, J. Yang, H. Chen, Leader–follower formation control of multiple non-holonomic mobile robots incorporating a receding-horizon scheme, Int. J. Robot. Res. 29 (6) (2010) 727–747.
[20] T. Balch, R.C. Arkin, Behavior-based formation control for multirobot teams, IEEE Trans. Robot. Autom. 14 (6) (1998) 926–939.
[21] A. Jadbabaie, J. Lin, A.S. Morse, Coordination of groups of mobile autonomous agents using nearest neighbor rules, IEEE Trans. Autom. Control 48 (6) (2003) 988–1001.
[22] G. Antonelli, F. Arrichiello, S. Chiaverini, Experiments of formation control with multirobot systems using the null-space-based behavioral control, IEEE Trans. Control Syst. Technol. 17 (5) (2009) 1173–1182.
[23] F. Arrichiello, S. Chiaverini, G. Indiveri, P. Pedone, The null-space based behavioral control for mobile robots with velocity actuator saturations, Int. J. Robot. Res. 29 (10) (2010) 1317–1337.
[24] M.A. Lewis, K.-H. Tan, High precision formation control of mobile robots using virtual structures, Auton. Robot. 4 (4) (1997) 387–403.
[25] W. Ren, R.W. Beard, Decentralized scheme for spacecraft formation flying via the virtual structure approach, J. Guid. Control Dyn. 27 (1) (2004) 73–82.
[26] N.H.M. Li, H.H.T. Liu, Formation UAV flight control using virtual structure and motion synchronization, in: Proceedings of the 2008 American Control Conference, Washington, USA, 2008, pp. 1782–1787.
[27] T.H.A. van den Broek, N. van de Wouw, H. Nijmeijer, Formation control of unicycle mobile robots: a virtual structure approach, in: Proceedings of the Joint 48th IEEE Conference on Decision and Control and 28th Chinese Control Conference, Shanghai, China, 2009, pp. 8328–8333.
[28] J.A. Fax, R.M. Murray, Information flow and cooperative control of vehicle formations, IEEE Trans. Autom. Control 49 (9) (2004) 1465–1476.
[29] Z. Lin, B. Francis, M. Maggiore, Necessary and sufficient graphical conditions for formation control of unicycles, IEEE Trans. Autom. Control 50 (1) (2005) 121–127.
[30] C. Yu, J.M. Hendrickx, B. Fidan, B.D.O. Anderson, V.D. Blondel, Three and higher dimensional autonomous formations: rigidity, persistence and structure persistence, Automatica 43 (3) (2007) 387–402.
[31] C. Godsil, G. Royle, Algebraic Graph Theory, Springer-Verlag, New York, 2004.
[32] D. Zelazo, Graph-theoretic methods for the analysis and synthesis of networked dynamic systems (Ph.D. thesis), University of Washington, 2009.
[33] R. Olfati-saber, J.A. Fax, R.M. Murray, Consensus and cooperation in networked multi-agent systems, Proc. IEEE 95 (1) (2007) 215–233.
[34] W. Ren, W. Beard, E.M. Atkins, Information consensus in multivehicle cooperative control, IEEE Control Syst. Mag. 27 (2) (2007) 71–82.
[35] T. Vicsek, A. Czirook, E. Ben-Jacob, I. Cohen, O. Shochet, Novel type of phase transition in a system of self-deriven particles, Phys. Rev. Lett. 75 (6) (1995) 1226–1229.
[36] R. Olfati-Saber, R.M. Murray, Consensus problems in networks of agents with switching topology and time-delays, IEEE Trans. Autom. Control 49 (9) (2004) 1520–1553.
[37] Z. Lin, M. Broucke, B. Francis, Local control strategies for groups of mobile autonomous agents, IEEE Trans. Autom. Control 49 (4) (2004) 622–629.
[38] W. Ren, R.W. Beard, Consensus seeking in multiagent systems under dynamically changing interaction topologies, IEEE Trans. Autom. Control 50 (5) (2005) 655–661.
[39] D. Zelazo, M. Mesbahi, Edge agreement: graph-theoretic performance bounds and passivity analysis, IEEE Trans. Autom. Control 56 (3) (2011) 544–555.
[40] Z. Lin, B. Francis, M. Maggiore, State agreement for continuous-time coupled nonlinear system, SIAM J. Control Optim. 46 (1) (2007) 288–307.
[41] M. Cao, A.S. Morse, B.D.O. Anderson, Reaching a consensus in a dynamically changing environment: a graphical approach, SIAM J. Control Optim. 47 (2) (2008) 575–600.
[42] A. Rahmani, M. Ji, M. Mesbahi, M. Egerstedt, Controllability of multi-agent systems from a graph-theoretic perspective, SIAM J. Control Optim. 48 (1) (2009) 162–186.
[43] Y. Su, J. Huang, Stability of a class of linear switching systems with applications to two consensus problems, IEEE Trans. Autom. Control 57 (6) (2012) 1420–1430.
[44] L. Moreau, Stability of multiagent systems with time-dependent communication links, IEEE Trans. Autom. Control 50 (2) (2005) 169–182.
[45] G. Xie, L. Wang, Consensus control for a class of networks of dynamic agents, Int. J. Robust Nonlinear Control 17 (10–11) (2007) 941–959.
[46] H. Su, X. Wang, Z. Lin, Synchronization of coupled harmonic oscillators in a dynamic proximity network, Automatica 45 (10) (2009) 2286–2291.
[47] J. Qin, W.X. Zheng, H. Gao, Consensus of multiple second-order vehicles with a time-varying reference signal under directed topology, Automatica 47 (9) (2011) 1983–1991.
[48] Y. Cao, W. Ren, Distributed coordinated tracking with reduced interaction via a variable structure approach, IEEE Trans. Autom. Control 57 (1) (2012) 33–48.
[49] Z. Li, X. Liu, W. Ren, L. Xie, Distributed tracking control for linear multiagent systems with a leader of bounded unknown input, IEEE Trans. Autom. Control 58 (2) (2013) 518–523.
[50] Z. Li, W. Ren, X. Liu, M. Fu, Consensus of multi-agent systems with general linear and Lipschitz nonlinear dynamics using distributed adaptive protocols, IEEE Trans. Autom. Control 58 (7) (2013) 1786–1791.
[51] Z. Li, W. Ren, X. Liu, L. Xie, Distributed consensus of linear multi-agent systems with adaptive dynamic protocols, Automatica 49 (7) (2013) 1986–1995.
[52] Z. Li, Z. Duan, F.L. Lewis, Distributed robust consensus control of multi-agent systems with heterogeneous matching uncertainties, Automatica 50 (3) (2014) 883–889.
[53] W. Ren, K. Moore, Y.Q. Chen, High-order consensus algorithms in cooperative vehicle systems, in: Proceedings of the 2006 IEEE International Conference on Networking, Sensing and Control, 2006, pp. 457–462.
[54] F.C. Jiang, G.M. Xie, W. Long, X.J. Chen, The chi-consensus problem of high-order multi-agent systems with fixed and switching topologies, Asian J. Control 10 (2) (2008) 246–253.
[55] F.C. Jiang, L. Wang, Consensus seeking of high-order dynamic multi-agent systems with fixed and switching topologies, Int. J. Control 83 (2) (2010) 404–420.
[56] Y. Liu, Y.M. Jia, Consensus problem of high-order multi-agent systems with external disturbances: an H-infinity analysis approach, Int. J. Robust Nonlinear Control 20 (14) (2010) 1579–1593.
[57] L. Mo, Y. Jia, H-infinity consensus control of a class of high-order multi-agent systems, IET Control Theory Appl. 5 (1) (2011) 247–253.
[58] P. Lin, Z. Li, Y. Jia, M. Sun, High-order multi-agent consensus with dynamically changing topologies and time-delays, IET Control Theory Appl. 5 (8) (2011) 976–981.
[59] J.H. Seo, H. Shim, J. Back, Consensus of high-order linear systems using dynamic output feedback compensator: low gain approach, Automatica 45 (11) (2009) 2659–2664.
[60] J. Xi, N. Cai, Y. Zhong, Consensus problems for high-order linear time-invariant swarm systems, Physica A—Stat. Mech. Appl. 389 (24) (2010) 5619–5627.
[61] Y. Zhao, B. Li, J. Qin, H. Gao, H.R. Karimi, Consensus and synchronization of nonlinear systems based on a novel fuzzy model, IEEE Trans. Cybern. 43 (6) (2013) 2157–2169.
[62] H. Li, X. Liao, T. Huang, Second-order locally dynamical consensus of multiagent systems with arbitrarily fast switching directed topologies, IEEE Trans. Syst. Man Cybern.: Syst. 43 (6) (2013) 1343–1353.
[63] W. Wang, J. Huang, C. Wen, H. Fan, Distributed adaptive control for consensus tracking with application to formation control of nonholonomic mobile robots, Automatica 50 (4) (2014) 1254–1263.
[64] W. Yu, G. Chen, M. Cao, J. Kurths, Second-order consensus for multiagent systems with directed topologies and nonlinear dynamics, IEEE Trans. Syst. Man Cybern. Part B 40 (3) (2010) 881–891.
[65] Q. Song, J. Cao, W. Yu, Second-order leader–following consensus of nonlinear multi-agent systems via pinning control, Syst. Control Lett. 59 (9) (2010) 553–562.
[66] Q. Song, F. Liu, J. Cao, W. Yu, M-matrix strategies for pinning-controlled leader-following consensus in multiagent systems with nonlinear dynamics, IEEE Trans. Cybern. 43 (6) (2013) 1688–1697.
[67] X. Zhang, L. Liu, G. Feng, Leader–follower consensus of time-varying nonlinear multi-agent systems, Automatica 52 (6) (2015) 8–14.
[68] Y. Cao, W. Ren, Finite-time consensus for multi-agent networks with unknown inherent nonlinear dynamics, Automatica 50 (10) (2014) 2648–2656.
[69] P. Wang, Y. Jia, Robust h_∞ containment control for second-order multi-agent systems with nonlinear dynamics in directed networks, Neurocomputing 153 (2015) 235–241.
[70] A. Das, F.L. Lewis, Cooperative adaptive control for synchronization of second-order systems with unknown nonlinearities, Int. J. Robust Nonlinear Control 21 (13) (2011) 1509–1524.

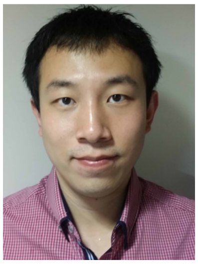
[71] J. Huang, C. Wen, W. Wang, Y.-D. Song, Adaptive finite-time consensus control of a group of uncertain nonlinear mechanical systems, Automatica 51 (2015) 292–301.
[72] X. Wang, T. Liu, J. Qin, Second-order consensus with unknown dynamics via cyclic-small-gain method, IET Control Theory Appl. 6 (18) (2012) 2748–2756.
[73] T. Liu, Z.-P. Jiang, Distributed formation control of nonholonomic mobile robots without global position measurements, Automatica 49 (2) (2013) 592–600.
[74] T. Liu, Z.-P. Jiang, Distributed output-feedback control of nonlinear multi-agent systems, IEEE Trans. Autom. Control 58 (11) (2013) 2912–2917.
[75] X. Wang, J. Qin, C. Yu, ISS method for coordination control of nonlinear dynamical agents under directed topology, IEEE Trans. Cybern. 44 (10) (2014) 1832–1845.
[76] Z. Zeng, X. Wang, Z. Zheng, Convergence analysis using the edge Laplacian: robust consensus of nonlinear multi-agent systems via ISS method, Int. J. Robust Nonlinear Control, Online (doi:0.1002/rnc.3351), 26 (5) (2016) 1051–1072.
[77] Z. Meng, Z. Lin, W. Ren, Robust cooperative tracking for multiple non-identical second-order nonlinear systems, Automatica 49 (8) (2013) 2363–2372.
[78] M.-C. Fan, Z. Chen, H.-T. Zhang, Semi-global consensus of nonlinear second-order multi-agent systems with measurement output feedback, IEEE Trans. Autom. Control 59 (8) (2014) 2222–2227.
[79] L. Moreau, Stability of continuous-time distributed consensus algorithms, in: Proceedings of 43rd IEEE Conference on Decision and Control, Paradise Island, Bahamas, USA, 2004, pp. 3998–4003.
[80] P.-A. Blimana, G. Ferrari-Trecateb, Average consensus problems in networks of agents with delayed communications, Automatica 44 (3) (2008) 1985–1995.
[81] Y. Sun, D. Zhao, J. Ruan, Consensus in noisy environments with switching topology and time-varying delays, Physica A: Stat. Mech. Appl. 389 (19) (2010) 4149–4161.
[82] U. Munz, A. Papachristodoulou, F. Allgower, Consensus in multi-agent systems with coupling delays and switching topology, IEEE Trans. Autom. Control 56 (12) (2011) 2976–2982.
[83] H.G. Tanner, D.K. Christodoulakis, State synchronization in local-interaction networks is robust with respect to time delays, in: Proceedings of the 44th IEEE Conference on Decision and Control, and the European Control Conference, Seville, Spain, 2005, pp. 4945–4950.
[84] F. Xiao, L. Wang, State consensus for multi-agent systems with switching topologies and time-varying delays, Int. J. Control 79 (10) (2006) 1277–1284.
[85] D.V. Dimarogonas, K.H. Johansson, Stability analysis for multi-agent systems using the incidence matrix: quantized communication and formation control, Automatica 46 (4) (2010) 695–700.
[86] M. Guo, D.V. Dimarogonas, Consensus with quantized relative state measurements, Automatica 49 (8) (2013) 2531–2537.
[87] H. Liu, M. Cao, C. De Persis, Quantization effects on synchronized motion of teams of mobile agents with second-order dynamics, Syst. Control Lett. 61 (12) (2012) 1157–1167.
[88] C.D. Persis, B. Jayawardhana, Coordination of passive systems under quantized measurements, SIAM J. Control Optim. 50 (6) (2012) 3155–3177.
[89] W. Chen, X. Li, L. Jiao, Quantized consensus of second-order continuous-time multi-agent systems with a directed topology via sampled data, Automatica 49 (7) (2013) 2236–2242.
[90] R. Carli, F. Bullo, S. Zampieri, Quantized average consensus via dynamic coding/decoding schemes, Int. J. Robust Nonlinear Control 20 (2) (2010) 156–175.
[91] T. Li, M. Fu, L. Xie, J.-F. Zhang, Distributed consensus with limited communication data rate, IEEE Trans. Autom. Control 56 (2) (2011) 279–292.
[92] K. You, L. Xie, Network topology and communication data rate for consensusability of discrete-time multi-agent systems, IEEE Trans. Autom. Control 56 (10) (2011) 226–2275.
[93] D. Li, Q. Liu, X. Wang, Z. Lin, Consensus seeking over directed networks with limited information communication, Automatica 49 (2) (2013) 610–618.
[94] D. Li, Q. Liu, X. Wang, Z. Yin, Quantized consensus over directed networks with switching topologies, Syst. Control Lett. 65 (2014) 13–22.
[95] Q. Zhang, B.-C. Wang, J.-F. Zhang, Distributed dynamic consensus under quantized communication data, Int. J. Robust Nonlinear Control 25 (11) (2015) 1704–1720.
[96] Y. Kim, M. Mesbahi, On maximizing the second smallest eigenvalue of a state-dependent graph Laplacian, IEEE Trans. Autom. Control 51 (1) (2006) 116–120.
[97] L. Xiao, S. Boyd, Fast linear iterations for distributed averaging, Syst. Control Lett. 53 (1) (2004) 65–78.
[98] A. Olshevsky, J.N. Tsitsiklis, Convergence speed in distributed consensus and averaging, SIAM J. Control Optim. 48 (1) (2009) 33–55.
[99] M. Lemay, F. Michaud, D. Letourneau, J. Marc Valin, Autonomous initialization of robot formations, in: Proceedings of the IEEE International Conference on Robotics and Automation, New Orleans, LA, 2005, pp. 3018–3023.
[100] D. Ren, G. Lu, Thinking in formation control, Control Decis. 20 (6) (2005) 601–606.
[101] W. Ren, Consensus based formation control strategies for multi-vehicle systems, in: Proceedings of the 2006 American Control Conference, Minneapolis, Minnesota, USA, 2006, pp. 4237–4242.
[102] Z. Wu, Z. Guan, X. Wu, T. Li, Consensus based formation control and trajectory tracing of multi-agent robot systems, J. Intel. Robot Syst. 48 (3) (2007) 397–410.
[103] Z. Lin, M. Broucke, B. Francis, Local control strategies for groups of mobile autonomous agents, IEEE Trans. Autom. Control 49 (4) (2004) 622–629.
[104] Y. Cao, Y. Li, W. Ren, Y. Chen, Distributed coordination of networked fractional-order systems, IEEE Trans. Syst. Man Cybern. Part B: Cybern. 40 (2) (2010) 362–370.
[105] Y. Cao, W. Ren, Distributed formation control for fractional-order systems: dynamic interaction and absolute/relative damping, Syst. Control Lett. 59 (3) (2010) 233–240.
[106] J. Bai, G. Wen, A. Rahmani, Y. Yu, Distributed formation control of fractional-order multi-agent systems with absolute damping and communication delay, Int. J. Syst. Sci. 46 (13) (2015) 2380–2392.
[107] M. Ji, M. Egerstedt, Distributed coordination control of multiagent systems while preserving connectedness, IEEE Trans. Robot. 23 (4) (2007) 693–703.
[108] R. Falconi, L. Sabattini, C. Secchi, C. Fantuzzi, C. Melchiorri, Edge-weighted consensus-based formation control strategy with collision avoidance, Robotica 33 (02) (2015) 332–347.
[109] H.G. Tanner, G.J. Pappas, V. Kumar, Leader-to-formation stability, IEEE Trans. Robot. Autom. 20 (3) (2004) 443–455.
[110] X. Wang, C. Yu, Z. Lin, A dual quaternion solution to attitude and position control for rigid-body coordination, IEEE Trans. Robot. 28 (5) (2012) 1162–1170.
[111] Z. Sun, S. Mou, M. Deghat, B. Anderson, A.S. Morse, Finite time distance-based rigid formation stabilization and flocking, in: Proceedings of the 19th IFAC Congress, 2014, pp. 9183–9189.
[112] L. Krick, M.E. Broucke, B.A. Francis, Stabilisation of infinitesimally rigid formations of multi-robot networks, Int. J. Control 82 (3) (2009) 423–439.
[113] F. Dorfler, B. Francis, Formation control of autonomous robots based on cooperative behavior, in: 2009 European Control Conference (ECC), IEEE, New York, 2009, pp. 2432–2437.
[114] A. Belabbas, S. Mou, A.S. Morse, B.D. Anderson, Robustness issues with undirected formations, in: 2012 IEEE 51st Annual Conference on Decision and Control (CDC), IEEE, 2012, pp. 1445–1450.
[115] Z. Sun, S. Mou, B. Anderson, A.S. Morse, Non-robustness of gradient control for 3-d undirected formations with distance mismatch, in: Proceedings of the 3rd Australian Control Conference (AUCC 2013), IEEE, 2013, pp. 369–374.
[116] S. Mou, A.S. Morse, M.A. Belabbas, B. Anderson, Undirected rigid formations are problematic, in: 2014 IEEE 53rd Annual Conference on Decision and Control (CDC), IEEE, 2014, pp. 637–642.
[117] S. Mou, A. Morse, B. Anderson, Toward robust control of minimally rigid undirected formations, in: 2014 IEEE 53rd Annual Conference on Decision and Control (CDC), IEEE, 2014, pp. 643–647.
[118] Z. Sun, S. Mou, B.D. Anderson, A. Morse, Formation movements in minimally rigid formation control with mismatched mutual distances, in: 2014 IEEE 53rd Annual Conference on Decision and Control (CDC), IEEE, 2014, pp. 6161–6166.
[119] S.L. Smith, M.E. Broucke, B.A. Francis, Stabilizing a multi-agent system to an equilateral polygon formation, in: Proceedings of the 17th International Symposium on Mathematical Theory of Networks and Systems (MTNS2006), Kyoto Japan, 2006, pp. 2415–2424.
[120] K.-K. Oh, H.-S. Ahn, Formation control of mobile agents based on inter-agent distance dynamics, Automatica 47 (10) (2011) 2306–2312.
[121] B.D. Anderson, C. Yu, S. Dasgupta, T.H. Summers, Controlling four agent formations, in: Proceedings of the 2nd IFAC Workshop on Distributed Estimation and Control in Networked Systems, 2010, pp. 139–144.
[122] S. Dasgupta, B. Anderson, C. Yu, T.H. Summers, Controlling rectangular formations, in: Australian Control Conference (AUCC), 2011, IEEE, 2011, pp. 44–49.
[123] Y.-P. Tian, Q. Wang, Global stabilization of rigid formations in the plane, Automatica 49 (5) (2013) 1436–1441.
[124] J. Graver, B. Servatius, H. Servatius, Combinatorial Rigidity (Graduate Studies in Mathematics), American Mathematical Society, New York, USA, 1994.
[125] G. Laman, On graphs and rigidity of plane skeletal structures, J. Eng. Math. 4 (4) (1970) 331–340.
[126] R. Olfati-Saber, R.M. Murray, Graph rigidity and distributed formation stabilization of multi-vehicle systems, in: Proceedings of the IEEE International Conference on Decision and Control, Las Vegas, Nevada, USA, 2002, pp. 2965–2971.
[127] T. Eren, B. Anderson, A. Morse, W. Whiteley, P. Belhumeur, Operations on formations of autonomous agents, Commun. Inf. Syst. 3 (4) (2004) 223–258.
[128] J.M. Hendrickx, B. Fidan, C. Yu, B.D.O. Anderson, V.D. Blondel, Formation reorganization by primitive operations on directed graphs, IEEE Trans. Autom. control 53 (4) (2008) 968–979.
[129] J.M. Hendrickx, B.D.O. Anderson, J.-C. Delvenne, V.D. Blondel, Directed graphs for the analysis of rigidity and persistence in autonomous agent system, Int. J. Robust Nonlinear Control 17 (10–11) (2007) 960–981.
[130] C. Yu, B.D.O. Anderson, S. Dasgupta, B. Fidan, Control of minimally persistent formations in the plane, SIAM J. Control Optim. 48 (1) (2009) 206–233.
[131] T.H. Summers, C. Yu, S. Dasgupta, B.D.O. Anderson, Control of minimally persistent leader–remote–follower and coleader formations in the plane, IEEE Trans. Autom. Control 56 (12) (2011) 2778–2792.
[132] D. Zelazo, A. Franchi, H.H. Bülthoff, P.R. Giordano, Decentralized rigidity maintenance control with range measurements for multi-robot systems, Int. J. Robot. Res. 34 (1) (2015) 105–128.

[133] S. Zhao, D. Zelazo, Bearing Rigidity and Almost Global Bearing-Only Formation Stabilization, Arxiv preprint arXiv:1408.6552.
[134] S. Zhao, F. Lin, K. Peng, B.M. Chen, T.H. Lee, Distributed control of angle-constrained cyclic formations using bearing-only measurements, Syst. Control Lett. 63 (2014) 12–24.
[135] D. Zelazo, A. Franchi, P.R. Giordano, Rigidity theory in SE (2) for unscaled relative position estimation using only bearing measurements, in: 2014 European Control Conference (ECC), IEEE, 2014, pp. 2703–2708.
[136] E. Kranakis, D. Krizanc, S. Rajsbaum, Mobile agent rendezvous: A survey, in: Structural Information and Communication Complexity, vol. 4056/2006, Springer, Berlin, Heidelberg, New York, 2006, pp. 1–9.
[137] H. Ando, Y. Oasa, I. Suzuki, M. Yamashita, Distributed memoryless point convergence algorithm for mobile robots with limited visibility, IEEE Trans. Robot. Autom. 15 (5) (1999) 818–828.
[138] J. Lin, A.S. Morse, B.D.O. Anderson, The multi-agent rendezvous problem. Part 1: the synchronous case, SIAM J. Control Optim. 46 (6) (2007) 2096–2119.
[139] J. Lin, A.S. Morse, B.D.O. Anderson, The multi-agent rendezvous problem. Part 2: the asynchronous case, SIAM J. Control Optim. 46 (6) (2007) 2120–2147.
[140] J. Cortes, S. Martinez, F. Bullo, Robust rendezvous for mobile autonomous agents via proximity graphs in arbitrary dimensions, IEEE Trans. Autom. Control 51 (8) (2006) 1289–1298.
[141] F. Xiao, L. Wang, T. Chen, Connectivity preservation for multi-agent rendezvous with link failure, Automatica 48 (1) (2012) 25–35.
[142] A. Ganguli, J. Cortes, F. Bullo, Multirobot rendezvous with visibility sensors in nonconvex environments, IEEE Trans. Robot. 25 (2) (2009) 340–352.
[143] D.V. Dimarogonas, K.J. Kyriakopoulos, On the rendezvous problem for multiple nonholonomic agents, IEEE Trans. Autom. Control 52 (5) (2007) 916–922.
[144] G. Conte, P. Pennesi, The rendezvous problem with discontinuous control policies, IEEE Trans. Autom. Control 55 (1) (2010) 279–283.
[145] J. Yu, S.M. LaValle, D. Liberzon, Rendezvous without coordinates, IEEE Trans. Autom. Control 57 (2) (2012) 421–434.
[146] W. Ren, Formation keeping and attitude alignment for multiple spacecraft through local interactions, J. Guid. Control Dyn. 30 (2) (2007) 633–638.
[147] A.K. Bondhus, K.Y. Pettersen, J.T. Gravdahl, Leader/follower synchronization of satellite attitude without angular velocity measurements, in: Proceedings of the 44th IEEE Conference on Decision and Control, and the European Control Conference 2005, Seville, Spain, 2005, pp. 7270–7277.
[148] T.R. Smith, H. Hanssmann, N.E. Leonard, Orientation control of multiple underwater vehicle with symmetry-breaking potentials, in: Proceedings of the 40th IEEE Conference on Decision and Control, Orlando, Florida, USA, 2001, pp. 4598–4603.
[149] J.R. Lawton, R.W. Beard, Synchronized multiple spacecraft rotations, Automatica 38 (8) (2002) 1359–1364.
[150] A. Sarlette, R. Sepulchre, N.E. Leonard, Autonomous rigid body attitude synchronization, Automatica 45 (2) (2009) 572–577.
[151] B. Yang, Y. Ding, Y. Jin, K. Hao, Self-organized swarm robot for target search and trapping inspired by bacterial chemotaxis, Robot. Auton. Syst. 72 (2015) 83–92.
[152] J.C. Barca, Y.A. Sekercioglus, Swarm robotics reviewed, Robotica 31 (3) (2013) 349–359.
[153] D. Cheng, H. Chen, From swarm to social behavior control, Sci. Technol. Rev. 22 (8) (2004) 4–7.
[154] C.W. Reynolds, Flocks, herds, and schools: a distributed behavioral model, Comput. Graph. 21 (4) (1987) 25–34.
[155] R. Olfati-Saber, Flocking for multi-agent dynamic systems: algorithms and theory, IEEE Trans. Autom. Control 51 (3) (2006) 401–420.
[156] H. Su, X. Wang, Z. Lin, Flocking of multi-agents with a virtual leader, IEEE Trans. Autom. Control 54 (2) (2009) 293–307.
[157] H.G. Tanner, A. Jadbabaie, G.J. Pappas, Flocking in fixed and switching networks, IEEE Trans. Autom. Control 52 (5) (2007) 863–868.
[158] D. Lee, M.W. Spong, Stable flocking of multiple inertial agents on balanced graphs, IEEE Trans. Autom. Control 52 (8) (2007) 1469–1475.
[159] N. Moshtagh, A. Jadbabaie, Distributed geodesic control laws for flocking of nonholonomic agents, IEEE Trans. Autom. Control 52 (4) (2007) 681–686.
[160] W. Dong, Flocking of multiple mobile robots based on backstepping, IEEE Trans. Syst. Man Cybern.: Part B 41 (2) (2011) 414–424.
[161] G. Shi, Y. Hong, K.H. Johansson, Connectivity and set tracking of multi-agent systems guided by multiple moving leaders, IEEE Trans. Autom. Control 57 (3) (2012) 663–676.
[162] M. Ji, G. Ferrari-Trecate, M. Egerstedt, A. Buffa, Containment control in mobile networks, IEEE Trans. Autom. Control 53 (5) (2008) 1972–1975.
[163] Y. Cao, D. Stuart, W. Ren, Z. Meng, Distributed containment control for multiple autonomous vehicles with doubleintegrator dynamics: algorithms and experiments, IEEE Trans. Control Syst. Technol. 19 (4) (2011) 929–938.
[164] Y. Cao, W. Ren, Containment control with multiple stationary or dynamic leaders under a directed interaction graph, in: Proceedings of the IEEE Conference on Decision Control, Shanghai, China, 2009, pp. 3014–3019.
[165] J. Li, W. Ren, S. Xu, Distributed containment control with multiple dynamic leaders for double-integrator dynamics using only position measurements, IEEE Trans. Autom. Control 57 (6) (2012) 1553–1559.
[166] Y. Lou, Y. Hong, Target containment control of multi-agent systems with random switching interconnection topologies, Automatica 48 (5) (2012) 879–885.
[167] J. Mei, W. Ren, G. Ma, Distributed containment control for Lagrangian networks with parametric uncertainties under a directed graph, Automatica 48 (4) (2012) 653–659.
[168] Z. Meng, W. Ren, Z. You, Distributed finite-time attitude containment control for multiple rigid bodies, Automatica 46 (12) (2010) 2092–2099.
[169] M. Haque, A. Rahmani, M. Egerstedt, Geometric foraging strategies in multi-agent systems based on biological models, in: 2010 49th IEEE Conference on Decision and Control (CDC), IEEE, 2010, pp. 6040–6045.
[170] J. Marshall, M.E. Broucke, B. Francis, et al., Formations of vehicles in cyclic pursuit, IEEE Trans. Autom. Control 49 (11) (2004) 1963–1974.
[171] A. Franchi, P. Stegagno, G. Oriolo, Decentralized multi-robot encirclement of a 3d target with guaranteed collision avoidance, Auton. Robot. (2015) 1–21.
[172] M.I. El-Hawwary, Three-dimensional circular formations via set stabilization, Automatica 54 (2015) 374–381.
[173] R. Zheng, Z. Lin, M. Fu, D. Sun, Distributed control for uniform circumnavigation of ring-coupled unicycles, Automatica 53 (2015) 23–29.
[174] T.-H. Kim, T. Sugie, Cooperative control for target-capturing task based on a cyclic pursuit strategy, Automatica 43 (8) (2007) 1426–1431.
[175] R. Zheng, Y. Liu, D. Sun, Enclosing a target by nonholonomic mobile robots with bearing-only measurements, Automatica 53 (2015) 400–407.
[176] S. Daingade, A. Sinha, Target centric cyclic pursuit using bearing angle measurements only, Adv. Control Optim. Dyn. Syst. 3 (1) (2014) 491–496.
[177] M. Dadgar, S. Jafari, A. Hamzeh, A pso-based multi-robot cooperation method for target searching in unknown environments, Neurocomputing 177 (2) (2016) 62–74.
[178] A. Ganguli, J. Cortses, F. Bullo, Multirobot rendezvous with visibility sensors in nonconvex environments, IEEE Trans. Robot. 25 (2) (2009) 340–352.
[179] T. Li, L. Xie, Distributed consensus over digital networks with limited bandwidth and time-varying topologies, Automatica 47 (9) (2011) 2006–2015.
[180] T. Li, L. Xie, J. Zhang, Distributed consensus with limited communication data rate, IEEE Trans. Autom. Control 56 (2) (2011) 279–292.
[181] J. Lavaei, R.M. Murray, Quantized consensus by means of gossip algorithm, IEEE Trans. Autom. Control 57 (1) (2012) 19–32.
[182] R. Schlanbusch, P.J. Nicklasson, Synchronization of target tracking cascadeded leader–follower spacecraft formation, in: J. Hall (Ed.), Advances in Spacecraft Technologies, INTECH Open Access Publisher, 2011, pp. 563–584.
[183] N. Michael, J. Fink, V. Kumar, Cooperative manipulation and transportation with aerial robots, Auton. Robot. 30 (1) (2011) 73–86.
[184] M. Muller, S. Lupashin, R. D'Andrea, Quadrocopter ball juggling, in: Proceedings of the 2011 IEEE/RSJ International Conference on Intelligent Robots and Systems, San Francisco, CA, USA, 2011, pp. 5113–5120.
[185] Y. Girdhar, A. Xu, B.B. Dey, M. Meghjani, F. Shkurti, I. Rekleitis, G. Dudek, Mare: Marine autonomous robotic explorer, in: Proceedings of the 2011 IEEE/RSJ International Conference on Intelligent Robots and Systems, San Francisco, CA, USA, 2011, pp. 5048–5053.
[186] W. Dong, Tracking control of multiple-wheeled mobile robots with limited information of a desired trajectory, IEEE Trans. Robot. 28 (1) (2012) 262–268.
[187] L. Consolini, F. Morbidi, D. Prattichizzo, M. Tosques, On a class of hierarchical formations of unicycles and their internal, IEEE Trans. Autom. Control 57 (4) (2012) 845–859.
[188] S. Mastellone, J.S. Mejia, D.M. Stipanovi, M.W. Spong, Formation control and coordinated tracking via asymptotic decoupling for Lagrangian multi-agent systems, Automatica 47 (12) (2011) 2355–2363.
[189] R. Oung, R. D'Andrea, The distribute flight array, Mechatronics 21 (5) (2011) 908–917.

Xiangke Wang received the B.S., M.S., and Ph.D. degrees from National University of Defense Technology, China, in 2004, 2006 and 2012, respectively. From 2012, he is with the College of Mechatronic Engineering and Automation, National University of Defense Technology, China, and currently he is an associate professor. He was a visiting Ph. D. student at the Research School of Engineering, Australian National University, supported by the China Scholarship Council from September 2009 to September 2011. His current research interests include coordination control of multiple UAVs, nonlinear control and robot soccer.



Zhiwen Zeng is now a Ph.D. candidate in the College of Mechatronics and Automation at National University of Defense Technology. He received his B.S. degree from the University of Electronic Science and Technology of China in 2009 and M.S. degree from National University of Defense Technology in 2011, respectively. His research interests include distributed control and estimation of networked dynamical systems and its application to intelligent and robotic systems.



Yirui Cong is currently pursuing his Ph.D. degree in the Research School of Engineering, Australian National University, Canberra, Australia. He received his B.S. degree (outstanding graduates) in automation from Northeastern University, Shenyang, China, in 2011. In 2013, he got the M.S. degree (graduated in advance) in control science and engineering from National University of Defense Technology, Changsha, China. His research interests are in the fields of control theory and communication theory.