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# A Survey of Recent Progress in the Study of Distributed High-Order Linear Multi-Agent Coordination

Jie Huang<sup>\*,\*\*</sup>, Hao Fang<sup>\*</sup>, Jie Chen<sup>\*</sup>, Lihua Dou<sup>\*</sup>, and Jie Zeng<sup>\*</sup>

<sup>\*</sup>School of Automation, Beijing Institute of Technology  
Key Laboratory of Intelligent Control and Decision of Complex Systems  
Beijing, 100081, China

<sup>\*\*</sup>Fujian Institute of Education  
Fuzhou, 350001, China

E-mail: autohuangjie@gmail.com

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**Most previous studies of multi-agent systems only considered the first- and second-order dynamics. In this review, we present the major results and progress related to distributed high-order linear multi-agent coordination. We also discuss the current challenges and propose several promising research directions, as well as open problems that demand further investigation.**

**Keywords:** high-order, multi-agent systems, linear system, coordination, overview

## 1. Introduction

Research into multi-agent systems has attracted much attention in recent years and numerous results have been obtained. Multi-agent coordination was motivated initially by the study of the group behavior in nature [1], statistical physics [2], and distributed computing [3]. The cooperative control of multi-agent systems has received increasing attention because many benefits can be obtained if a single complex agent is replaced by multiple simple agents. In early research before 2000, most studies assumed the availability of global team information, the ability to plan group actions in a centralized manner, and perfect and unlimited communication among the agents [4–9]. However, centralized coordination schemes do not work well, especially when the groups of agents are large, because real-world communication topologies are generally not fully connected. Thus, distributed control was investigated because of the disadvantages of centralized coordination schemes. During the distributed control of a group of autonomous agents, the main objective is to ensure that the group of agents works in a cooperative manner via a distributed protocol. In this case, cooperative refers to a close relationship among all agents in the group where information sharing plays a crucial role. Thus, graph theory must be applied in this research area. The distributed approach has many advantages for achieving cooperative group performance, especially its low operational costs, low system requirements, high robustness,

strong adaptivity, and flexible scalability. The control theory for multi-agent systems can be applied to many practical engineering applications, such as cooperative control of unmanned ground/air/underwater vehicles [10–12], distributed sensor networks [13, 14], aggregation and rendezvous control [15], and attitude alignment of spacecraft [16, 17]. Therefore, the cooperative control of multi-agent systems is recognized and appreciated widely. Indeed, important reviews and books have been published about the control of multi-agent systems [18–32].

However, most previous studies have only considered first- and second-order multi-agent systems. Recently, some researchers have begun to focus on the coordination of high-order multi-agent systems. This is because it is not acceptable to model many practical applications using only single- or double-integrator dynamics and many systems must be modeled using high-order dynamics. For example, the jerk systems are described using third-order differential equations and they are of particular interest in mechanical engineering. A single link flexible joint manipulator has also been modeled as a fourth-order nonlinear system [33]. Another motivation for the study of high-order coordination comes from observations of the behavior of flocks of birds [34]. Flocks of birds often fly in formation, where they maintain a specific separation distance from each other while traveling at the same velocity. Second-order coordination can reproduce this separation and common velocity based on information exchange, but bird flocks sometimes change direction abruptly, possibly because one of them suddenly perceives a source of danger or food. Clearly, birds in this setting need to build a consensus on their relative position and velocity, but also their acceleration. Thus, it is necessary to extend the coordination problem from lower-order dynamics to high-order dynamics. However, because of the coupled states of each agent, it is more difficult to analyze the dynamics of high-order multi-agent systems than first-order and second-order systems. High-order multi-agent systems include more details about the interactions between the system dynamics, the control design problem, and the communication graph, which is reflected in the Laplacian matrix. Therefore, high-order coordination must address



the interface between control systems and the communication graph structure more directly. Chained systems with a strict-feedback form are too simple to describe practical systems, but they are used widely as a basic form during research into distributed coordination.

In this review, we provide a classification of distributed high-order linear multi-agent coordination and discuss recent studies in this area. The remainder of this paper is organized as follows. In Section 2, we provide the necessary preliminaries related to the cooperative control problem. In Section 3, we summarize the main models of high-order linear multi-agent systems and discuss them briefly. The final section contains a short summary and we propose several promising research directions, as well as open problems that require further investigation.

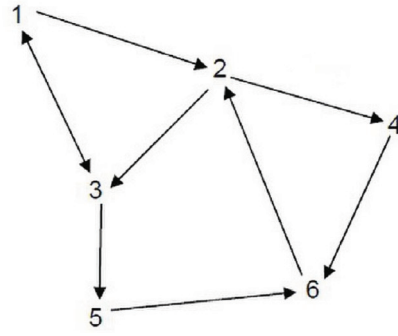
## 2. Preliminaries: Graph Theory for Multi-Agent Systems

Graph theory is a very useful mathematical tool for studying the coordination of multi-agent systems. If a team of  $N$  agents labeled as systems 1 to  $N$  are considered, the topology of a communication network can be expressed using a graph, either directed or undirected, which depends on whether the information flow is unidirectional or bidirectional.

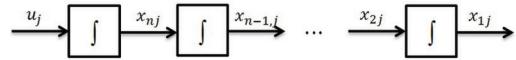
A weighted graph (undirected graph or digraph)  $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$  comprises a nonempty finite set of  $N$  nodes  $\mathcal{V} = \{v_1, v_2, \dots, v_N\}$ , a set of edges  $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ , and a weighted adjacency matrix  $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ , where  $\mathcal{V}$  is the set of the indices in the systems and  $\mathcal{E}$  is a set of edges, which describe the communication between the agents. In a directed graph, if  $(v_i, v_j) \in \mathcal{E}$ , then  $i$  is a neighbor of  $j$ , which means that system  $j$  can obtain information from system  $i$ . Thus, it can be said that  $v_i$  is the parent node and  $v_j$  is the child node. The set of neighbors of node  $i$  is denoted as  $N_i = \{j | (v_j, v_i) \in \mathcal{E}\}$ . In most previous studies, it is assumed that self-edges  $(v_i, v_i)$  are not permitted. Each entry  $a_{ij}$  of adjacency matrix  $\mathcal{A}$  is the weight associated with the edge  $(v_j, v_i)$  and  $a_{ij} > 0$  if  $(v_j, v_i) \in \mathcal{E}$ . Otherwise,  $a_{ij} = 0$ . For an undirected graph  $\mathcal{G}$ , the weighted adjacency matrix  $\mathcal{A} = [a_{ij}]_{N \times N}$  satisfies  $a_{ij} = a_{ji} > 0$ , where its Laplacian matrix  $L = [L_{ij}]$  is defined as follows.

$$L_{ij} = \begin{cases} -a_{ij} & \text{if } i \neq j \text{ and } i \in \mathcal{N}_j \\ \sum_{l \in \mathcal{N}_j} a_{jl} & \text{if } i = j \\ 0 & \text{otherwise} \end{cases} \quad . \quad (1)$$

the in-degree of node  $i$  as  $d_i = \sum_{j=1}^N a_{ij}$  and the in-degree matrix as  $D = \text{diag}\{d_i\} \in \mathbb{R}^{N \times N}$ . Therefore, the graph Laplacian matrix is  $L = D - A$ . If we let  $1_N = [1, 1, \dots, 1]^T \in \mathbb{R}^N$ , then  $L1_N = 0$ . Accordingly, define the out-degree of node  $i$  as  $d_i^o = \sum_{j=1}^N a_{ji}$  and the out-degree matrix as  $D^o = \text{diag}\{d_i^o\} \in \mathbb{R}^{N \times N}$ . Then, the graph column Laplacian matrix can be defined as  $L^o = D^o - A^T$ . A node is balanced if its in-degree equals its out-degree.



**Fig. 1.** A directed graph models the local information flow between agents.



**Fig. 2.** The fundamental model of high-order linear multi-agent systems (the  $j$ -th agent).

A directed graph is balanced if all its nodes are balanced. For an undirected graph,  $\mathcal{A}^T = \mathcal{A}$ , thus all undirected graphs are balanced.

In a directed graph (Fig. 1), a sequence of successive edges in the form  $\{(v_i, v_k), (v_k, v_l), \dots, (v_m, v_j)\}$  is a direct path from node  $i$  to node  $j$ . An undirected path is defined in a similar manner. A digraph is said to have a spanning tree if there is a root node  $i_r$ , such that there is a directed path from the root to any other node in the graph. A digraph is said to be strongly connected if there is a direct path from node  $i$  to node  $j$  for all distinct nodes  $v_i, v_j \in \mathcal{V}$ . A digraph has a spanning tree if it is strongly connected, but not vice versa. A digraph (or undirected graph) is said to be connected if, for any orderless pair of nodes, a directed (or undirected) path connects them. For a digraph, its underlying graph is the graph obtained by replacing all of the directed edges with undirected edges. A digraph is weakly connected if its underlying graph is connected. More details can be found in the book *Graph Theoretic Methods in Multiagent Networks* [35].

## 3. Coordination of High-Order Linear Systems

In this section, we review the main research progress in distributed high-order linear multi-agent coordination. The main high-order linear multi-agent systems models, control problems, and approaches are described. Each subsection is followed by a short summary or discussion.

### 3.1. Coordination of High-Order Integrator Linear Systems

Basic high-order linear multi-agent systems can be represented with integrator chained systems in a strict-

feedback form (**Fig. 2**), as follows:

$$\begin{aligned} \dot{\xi}_i^{(0)} &= \xi_i^{(1)} \\ &\vdots \\ \dot{\xi}_i^{(l-2)} &= \xi_i^{(l-1)} \dots\dots\dots (2) \\ \dot{\xi}_i^{(l-1)} &= u_i \end{aligned}$$

where  $\xi_i^{(k)} \in \mathbb{R}^m$ ,  $k = 0, 1, \dots, l-1$  and  $l \geq 3$ .  $\xi_i^{(k)}$  denotes the states of the  $i$ -th system,  $u_i \in \mathbb{R}^m$  is the control input of the  $i$ -th system, and  $\xi_i^{(k)}$  denotes the  $k$ -th derivative of  $\xi_i$  with  $\xi_i^{(0)} = \xi_i$ . Model (2) generalizes the single-integrator and double-integrator (first-order and second-order) dynamics as special cases.

The basic concept of a coordination algorithm involves obtaining a consensus, which aims to impose similar dynamics on the information states of each high-order system. If the communication network among systems allows continuous communication or if the communication bandwidth is sufficiently large, then the information state update of each system is modeled using a differential equation. However, if the communication data arrive in discrete packets, the information state update is modeled using a difference equation. The fundamental consensus algorithms for high-order linear systems (2) are as follows:

$$u_i = - \sum_{j=1}^n a_{ij} k_{ij} \left[ \sum_{k=0}^{l-1} \gamma_k (\xi_i^k - \xi_j^k) \right] \quad . \quad . \quad . \quad . \quad (3)$$

where  $i = 1, \dots, n$ ,  $k_{ij} > 0$ ,  $\gamma_k > 0$ ,  $a_{ii} \triangleq 0$ , and  $a_{ij} = 1$  if information flows from system  $j$  to system  $i$ , whereas  $a_{ij} = 0$ . Consensus is reached among the  $m$  systems if  $\xi_i^{(k)} \rightarrow \xi_j^{(k)}$ ,  $k = 0, 1, \dots, l-1$ ,  $\forall i \neq j$ . The characteristics of multi-agent systems mean that the closed-loop system can be expressed in a vector form. Therefore, matrix theory has been used frequently in stability analyses of the distributed coordination of linear system. By applying the consensus control protocol (3), multi-agent systems (2) can be written in matrix form as follows.

$$\begin{bmatrix} \xi^{(0)} \\ \xi^{(1)} \\ \vdots \\ \xi^{(l-1)} \end{bmatrix} = (\Gamma \otimes I_m) \begin{bmatrix} \xi^{(0)} \\ \xi^{(1)} \\ \vdots \\ \xi^{(l-1)} \end{bmatrix} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

As a fundamental cooperative control problem, leaderless consensus was considered initially in previous studies. In [34, 36], Ren et al. proposed a class of  $l$ -order ( $l \geq 3$ ) consensus algorithms and determined the necessary and sufficient conditions where each information variable and their high-order derivatives converged to common values. They also introduced the concept of high-order consensus with a leader and the concept of an  $l$ -order model-reference consensus problem, where each information variable and their high-order derivatives not only reach consensus but also converge to the solution of a prescribed dynamic model. However, this approach

cannot solve the rendezvous problem because of the non-zero velocity when consensus is achieved. In [37], Jiang et al. investigated the consensus problem where individual agents were modeled by high-order integrators under a fixed/switching topology and with zero/nonzero communication time-delays. In [38], Yang et al. studied the consensus of high-order integrator multi-agent systems with time-delays and switching topologies, which showed that consensus could be reached with arbitrarily bounded time-delays even though the communication topology might not include spanning trees. In [39], Zhang et al. also investigated the consensus of high-order multi-agent systems. In contrast to existing protocols [34, 36], a dynamic neighbor-based protocol was proposed, which only used the relative information of the first states of agents by extending second-order protocols from previous studies. Sufficient conditions were derived where all of the agents reached consensus asymptotically. In [40], He et al. proposed a linear consensus protocol for solving this type of consensus problem, which had two components: a feedback controller and interactions with neighbors. A sufficient and necessary condition for consensus in high-order systems was obtained. They proposed the control protocol as  $u_i = u_{i1} + u_{i2}$ , where  $u_{i1} = b \sum_{k=1}^{l-1} \xi_i^k$  is the feedback controller and  $b$  is a non-zero constant that needs to be determined. In addition,  $u_{i2} = \sum_{j=1}^n c_{ij} \left( \sum_{k=1}^{l-2} \eta_k (\xi_j^k - \xi_i^k) \right)$ . As special cases, the criteria for second- and third-order systems were given, which established the exact relationship between feedback gain and the system parameters. The parameters introduced in the consensus protocol make the design of feedback gain more flexible. In [41], the leader-following consensus problems of high-order multi-agent linear systems with noise and time delays were discussed under the condition of undirected topologies.

These previous studies based on matrix theory employed this basic framework, where the models of multi-agent systems were linear with a chained form. In addition, other methods have considered high-order linear system cooperative control problems, such as the well-known backstepping technique. In the backstepping technique combined with graphic theory, the high-rank state of each differential equation is used for virtual control because of the characteristic structure of the lower-triangular strict-feedback formed system. The consensus control problem for multiple high-order systems can then be broken down into a sequence of design problems for multiple lower order subsystems in a distributed manner. In addition, the backstepping technique can be combined with other methods, such as adaptive control methods, sliding mode control, neural networks, and fuzzy control methods, to solve linear system control problems and nonlinear control problems [33, 42].

### 3.2. Coordination of High-Order Linear Systems with Disturbance

In practical applications, systems are often subjected to various disturbances such as actuator bias, measurement



or calculation errors, and variations in the communication topology. Thus, multi-agent systems (2) with disturbances have been studied. A class of high-order linear systems with external disturbances can be described as follows:

$$\begin{aligned} \xi_i^{(0)} &= \xi_i^{(1)} \\ &\vdots \\ \xi_i^{(l-2)} &= \xi_i^{(l-1)} \dots \dots \dots (5) \\ \xi_i^{(l-1)} &= u_i + w_i \end{aligned}$$

where  $i (1 \leq i \leq n)$  denotes the index number of the systems,  $\xi_i = [\xi_i^{(0)}, \xi_i^{(1)}, \dots, \xi_i^{(l-1)}]^T \in \mathbb{R}^l$  denotes the states of the  $i$ -th system,  $u_i \in \mathbb{R}$  is the control input of the  $i$ -th system, and  $w_i$  is the external disturbance.

For multi-agent systems (5), Mo et al. presented a convergence analysis of the consensus problem with time delays [43]. A dynamic neighbor-based protocol was proposed as follows:

$$u_i(t) = - \sum_{k=0}^{l-1} g_k \sum_{j \in N_i(t)} a_{ij} (\xi_i^{(k)}(t - \tau) - \xi_j^{(k)}(t - \tau)), \quad (6)$$

where for any  $i \in \{1, 2, \dots, n\}$ ,  $a_{ij} > 0$  denotes the edge weight,  $g_k > 0$  for  $k = 0, 1, \dots, l-1$ , and  $\tau > 0$  denotes the communication delay. Output functions were defined to combine relative information by computing the average relative displacements of all agents as follows:

$$\begin{aligned} z_{il}(t) &= \frac{1}{n} \sum_{j=1}^n \left[ \xi_i^{(l-1)}(t) - \xi_j^{(l-1)}(t) \right] \\ &= \xi_i^{(l-1)}(t) - \frac{1}{n} \sum_{j=1}^n \xi_j^{(l-1)}(t), \quad . \quad . \quad . \quad (7) \end{aligned}$$

and the  $H_\infty$  performance index was defined as follows:

$$J = \int_0^\infty [z^T(t)z(t) - \gamma^2 w^T(t)w(t)] dt, \quad (8)$$

where  $\gamma$  is a given positive constant. The approach used in [43] does not require any model transformation, which differs from the results for first-order [44] and second-order multi-agent systems [45].

Liu et al. [46] studied the output consensus problem for  $l$ -order multi-agent systems with external disturbances in networks with fixed and switching directed topologies. A controlled output was defined to measure the disagreement between each agent's measured output relative to the average for all agents. In [43], the controlled output functions were similar to those described by Eq. (7). The consensus problem can be transformed into an  $H_\infty$  control problem. A distributed protocol was proposed for each agent, which used its own information and its neighbors' measured outputs, and a closed-loop system was derived with a singular state matrix, thereby invalidating the traditional  $H_\infty$  theory. Next, they conducted a model transformation in two steps, and obtained an equivalent nonsingular reduced-order system with respect to the  $H_\infty$  performance, which depended on the consensus

of the multi-agent system under investigation. In particular, for directed networks with a fixed topology, a necessary and sufficient condition was obtained that ensured the output consensus with a given  $H_\infty$  performance index. This consensus controller differed from those mentioned earlier because only the  $i$ -th agent and its neighbors' measured outputs  $y_j(t) = \xi_j^{(1)}(j \in \mathcal{N}_i)$  were directly obtainable, so the other variables related to the neighbors had to be estimated. Thus, it can use the available information  $\xi_j^{(1)}(j \in \mathcal{N}_i)$ , instead of all-order derivatives of  $\xi_j^{(1)}(t)$ , to design

$$u_i(t) = \sum_{k=2}^l -K_k \xi_i^{(k)}(t) + K_1 \sum_{j \in \mathcal{N}_i} a_{ij} (\xi_j^{(1)} - \xi_i^{(1)}(t)), \quad (9)$$

for  $i = 1, 2, \dots, n$ , where  $K_k \in \mathbb{R} (k = 1, 2, \dots, l)$  are consensus gains. Local negative feedback  $\sum_{k=2}^l -K_k \xi_i^{(k)}(t)$  also has a role in decreasing the modulus of high-order variables  $\xi_i^{(k)}(t)$ , ( $k = 2, 3, \dots, l$ ).

Compared with multi-agent systems (2) and (5), the main difference is that a nonlinear control technique must be introduced to deal with disturbances, especially robust control, adaptive control, and neural network techniques. The absence of global information means that a distributed estimation scheme is often needed to achieve group coordination. Without distributed estimators, it would be very difficult or even impossible to design the controller. Thus, joint estimation and the control problem are now research hotspots, where the problem was considered subject to disturbances in [47]. However, dealing with stochastic disturbances in high-order multi-agent systems, or even those with stochastic communication topologies, still remains a challenging problem at present.

### 3.3. Coordination of Discrete-Time High-Order Linear Systems

Many practical systems are discrete-time models and high-order discrete-time multiple systems can be described as follows.

$$\begin{aligned}\xi_i^{(0)}(k+1) &= \xi_i^{(0)}(k) + \xi_i^{(1)}(k)T \\ &\vdots \\ \xi_i^{(l-2)}(k+1) &= \xi_i^{(l-2)}(k) + \xi_i^{(l-1)}(k)T \quad . \quad . \quad . \quad (10) \\ \xi_i^{(l-1)}(k+1) &= \xi_i^{(l-1)}(k) + u_i(k)T\end{aligned}$$

Suppose that the multi-agent systems (10) under consideration comprise  $N$  discrete-time agents. Each agent at instant  $kT$  is regarded as a node in a directed graph  $\mathcal{G}(kT) = (\mathcal{V}, \mathcal{E}(kT), \mathcal{A}(kT))$ , where  $T > 0$  is the sample time,  $\mathcal{V} = \{v_1, \dots, v_N\}$  is the set of nodes,  $\mathcal{E}(kT) \subseteq \mathcal{V} \times \mathcal{V}$  is the set of edges, and  $\mathcal{A}(kT) = [a_{ij}(kT)]$  is a weighted adjacency matrix. Each edge  $(v_j, v_i) \in \mathcal{E}(kT)$  denotes an information link from agent  $v_i$  to agent  $v_j$  at instant  $kT$ . The set of the neighbors of the  $i$ -th agent and the Laplacian of the graph  $\mathcal{G}(kT)$  at time  $kT$  are represented by  $N_i(kT)$  and  $L(kT)$ , respectively.

In contrast to [14], the rule for the discrete-time multi-

agent high-order systems was given as

$$u_i(k) = - \sum_{j=1}^{l-1} p_j \xi_i^{(j)}(k) - \sum_{s_j \in N_i(k)} a_{ij}(k) (\xi_i^{(0)}(k) - \xi_j^{(0)}(k - \tau_{ij})), \quad (11)$$

where for any  $i \in \{1, 2, \dots, n\}$ ,  $p_j > 0$  for  $j = 1, 2, \dots, l-1$ ,  $a_{ij} > 0$  denotes the edge weight chosen from a finite set  $\bar{\alpha}$ , and  $\tau_{ij}(t) \in \mathbb{Z}_+$ ,  $\tau_{ij}(t) \leq \tau_{\max}$  ( $i \neq j$ ) is the communication time-delay from  $v_j$  to  $v_i$  for a given positive constant  $\tau_{\max}$ . In [48], sufficient conditions are derived for the introduced rule to make all agents reach consensus in dynamically changing topologies. It was shown that consensus could be reached, even when the communication topologies changed dynamically and the corresponding communication graphs had no spanning trees. It was also shown that the communication time-delays did not affect the stability of the multi-agent systems. In [49], the leader-following consensus of discrete-time multi-agent systems was considered and an observer-based protocol was proposed.

### 3.4. Coordination of General High-Order LTI Systems

Recently, a class of general high-order linear time-invariant (LTI) systems has been proposed [50–57]. In this type of multi-agent system, all of the agents have identical multi-input multi-output (MIMO) linear dynamics, which can be of any order. The dynamics of the  $i$ -th agent are described by

$$\begin{aligned} \dot{x}_i &= Ax_i + Bu_i \\ y_i &= Cx_i, \quad i = 1, \dots, N, \end{aligned} \quad (12)$$

where  $x_i \in \mathbb{R}^n$ ,  $u_i \in \mathbb{R}^p$ , and  $y_i \in \mathbb{R}^q$  are the state, control input, and measured output, respectively.  $A$ ,  $B$ , and  $C$ , are constant matrices with compatible dimensions, where  $C$  is assumed to have full row rank.

In Section 3.1–3.3, the protocols described were based on all of the states of neighboring agents. However, in general high-order LTI systems, the agent should have inputs that match the order of the system [47]. The output feedback consensus problem has been considered by Scardovi and Sepulchre [51], where all the states of the observer for each agent need to be transmitted to the neighbors, so the quantity of transmitted information is the same as the state feedback case. Tuna [52] discussed various conditions for achieving consensus based on output feedback, although this study was limited to static output feedback cases. Based on these previous studies, Seo et al. [53] studied the consensus problem in a generalized environment from the perspective that each agent is a MIMO linear dynamic system, which is stabilizable and detectable, and a dynamic consensus algorithm was proposed that used only the output information (rather than the full state) from the neighboring agents. The output feedback consensus problem for  $N$  identical linear dynamics was solved in a unified manner. They also showed

that the consensus problem can be solved if there is a stable dynamic filter that simultaneously stabilizes  $N-1$  linear dynamic systems in a special form. Next, they proved that such a filter exists under a very general condition and they showed how to construct the filter. In [52], it was assumed that agent  $i$  collected the output information from its neighboring agents according to the following rule:

$$z_i(t) = \sum_{j \in \mathcal{N}_i} \alpha_{ij}(y_j(t) - y_i(t)) = - \sum_{j \in \mathcal{N}} l_{ij} y_j(t), \quad (13)$$

where  $\mathcal{N}_i = \{j \in \mathcal{N} : \alpha_{ij} \neq 0\}$ . They also assumed that the collected information  $z_i(t)$  was filtered by a stable filter  $\kappa(s)$  and fed back to agent  $i$  as follows.

$$\begin{aligned} u_i &= \kappa(s) z_i = \kappa(s) \sum_{j \in \mathcal{N}_i} \alpha_{ij}(y_j - y_i) \\ &= -\kappa(s) \sum_{j \in \mathcal{N}} l_{ij} y_j. \end{aligned} \quad (14)$$

Li [54] considered consensus problems for continuous- and discrete-time linear multi-agent systems with directed communication topologies. Distributed reduced-order observer-based consensus protocols were proposed, which were based on the relative outputs of neighboring agents. A multi-step algorithm was introduced for the construction of a reduced-order protocol, where a continuous-time multi-agent system with communication topology that contained a directed spanning tree could reach consensus. The algorithm described in [54] was modified further to achieve consensus with a prescribed convergence rate. These two algorithms have a favorable decoupling property. Based on the modified algebraic Riccati equation, an algorithm was then proposed for constructing a reduced-order protocol for the discrete-time case. Wieland et al. [58] considered the problem of reaching static or dynamic consensus over fixed interconnection topologies for multi-agent systems where the agents were modeled as general LTI systems. The consensus condition of the overall multi-agent system was derived based on the stability of some matrices, which comprised the agent dynamics and the spectrum of the Laplacian of the corresponding graph. In addition, a meaningful interpretation of the role of the adjoint null space of the Laplacian matrix was given. Systematic methods were proposed for selecting the gains in a consensus algorithm so the multi-agent systems reached consensus asymptotically with a prespecified convergence rate. Jiang et al. [59] reported results for a consensus-seeking problem with high-order LTI multi-agent systems with fixed and switching topologies. A necessary and sufficient condition was derived for the consensus of the multi-agent systems with fixed topology and zero time-delay. The protocol designs were also discussed for directed and undirected topologies. A sufficient condition for the consensus of the multi-agent system was established with a switching topology and zero time-delay. Zeng et al. [60] discussed the consensus problem for a group of general linear agents in an undirected topology. The input and communication delays were both considered. The factorization of the characteristic equation of the system into

decreased-order factors relied only on the set of eigenvalues of a matrix, which described the structure of the network topology and simplified the stability analysis considerably. Tang et al. [61] investigated hierarchical distributed control for multi-agent systems using an approximate simulation, where the proposed form of distributed control had two parts: local control for coordination and a virtual leader. In [62], the consensus of high-order linear multi-agent systems using output error feedback was considered by Wang et al., who investigated an observer-based approach for the design of dynamic output error feedback consensus control laws.

A recent new direction is formation swarm control for multiple LTI systems. Xi et al. [55–57] considered consensus analysis and design problems for high-order linear time-invariant swarm systems. In [55], consensus problems for high-order continuous-time linear time-invariant swarm systems with directed interaction topologies were investigated, where a method was proposed to deal with consensus problems based on state space decomposition. Two subspaces of a complex space were introduced, i.e., a consensus subspace and a complement consensus subspace. Based on this decomposition, necessary and sufficient conditions for consensus and consensusability were presented. An approach for determining and designing a consensus function that may be time-variant was also described. The applications of theoretical results to multi-agent support systems were also studied. In [56], the swarm stability problem of high-order linear time-invariant swarm systems with a directed graph topology was considered. Consensus problems can be regarded as a specific type of swarm stability problem. The necessary and sufficient conditions for both swarm stability and consensus were presented. In [57], consensus analysis and design problems for high-order linear time-invariant swarm systems with time-varying delays were addressed. First, a consensus subspace and a complement consensus subspace were introduced. Next, based on state projection onto the two subspaces, the consensus problems were converted into simultaneous stabilization problems with multiple time-delayed subsystems and low dimensions, and a method was proposed for analyzing and designing the consensus function. The sufficient conditions for consensus and consensusability were presented, which include only four linear matrix inequality constraints. At present, high-order swarm coordination control problems remain a challenging problem.

#### 4. Discussion

This review summarized the main research progress in distributed high-order linear multi-agent coordination, where we described the main high-order linear multi-agent systems models, control problems, and approaches. The use of chained systems in strict-feedback form is too simple to describe practical systems, but they are used widely as a basic model in studies of distributed high-order multi-agent systems coordination.

The main challenge of distributed high-order multi-agent coordination is the complexity of the control problem, which includes the communication topology and complex dynamics. First, the communication restrictions imposed by graph topologies can severely limit the performance of any control laws used by agents. In contrast to single-agent and centralized control systems, cooperative control studies the dynamics of multi-agent dynamical systems, which are linked to each other via a communication topology. The communication graph represents the local information flow between agents. Thus, the controllers are distributed in the sense that the controller design for each agent only requires relative state information between itself and its neighbors. Clearly, this complicates the design of distributed controllers. Second, studies of high-order multi-agent systems control are required to implement coordination in many real-world applications, such as formation control among multi-vehicles where the position, velocity, and even the acceleration must be controlled. Compared with the first-order and second-order dynamics, high-order dynamics involve more detailed considerations of the interactions between the system dynamics, as well as the control design problem and the communication graph, which is reflected in the Laplacian matrix.

Thus, high-order coordination must consider the interface between control systems and the communication graph structure more directly, which leads directly to the design of complex Lyapunov functions in a distributed manner by combining the model and the graph structure simultaneously. Using a single-integrator multi-agent system  $\dot{\xi}_j = u_j, j = 1, \dots, n$  as an example of the first-order case, the fundamental consensus algorithm is  $u_j = -\sum_{i=1}^n a_{ij}(\xi_j - \xi_i), j = 1, \dots, n$ . These two equations can be written in matrix form as  $\dot{\xi} = -[L_n(t) \otimes I_m]$ , where  $\xi = [\xi_1^T, \dots, \xi_n^T]^T, L_n(t) \in \mathbb{R}^{n \times n}$  is the nonsymmetrical Laplacian matrix at time  $t$ . In this case, the consensus equilibrium is a constant. In contrast to the constant consensus equilibrium, it might be appropriate to derive consensus algorithms for double-integrator dynamics ( $\dot{\xi} = \zeta_j, \dot{\zeta}_j = u_j, j = 1, \dots, n$ ) so some information states (e.g., position) can converge to a consistent value, while others (e.g., velocity) converge to another consistent value. The extension of consensus algorithms from single-integrator dynamics to double-integrator dynamics is nontrivial. In contrast to the single-integrator case, it has been shown that a directed spanning tree is a necessary rather than a sufficient condition for consensus-seeking with double-integrator dynamics. The fundamental consensus algorithm for double-integrator dynamics is  $u_j = -\sum_{i=1}^n a_{ij}[(\xi_j - \xi_i) + \gamma(t)(\zeta_j - \zeta_i)], j = 1, \dots, n$  with the matrix form

$$\begin{bmatrix} \dot{\xi} \\ \dot{\zeta} \end{bmatrix} = (\Theta \otimes I_m) \begin{bmatrix} \xi \\ \zeta \end{bmatrix}$$

where

$$\Theta = \begin{bmatrix} 0_n & I_n \\ -L_n(t) & -\gamma(t)L_n(t) \end{bmatrix}$$



and  $L_n(t) \in \mathbb{R}^{n \times n}$  is the nonsymmetrical Laplacian matrix associated with the communication graph at time  $t$  [26]. Because of the challenges involved in the design of cooperative controls for systems distributed on communication networks, it is not straightforward to extend the results obtained for first- and second-order systems to high-order dynamics. For the  $l$ -order case (4) [63],

$$\Gamma = \begin{bmatrix} 0_n & I_n & 0_n & \dots & 0_n \\ 0_n & 0_n & I_n & \dots & 0_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0_n & 0_n & 0_n & \dots & I_n \\ -\gamma_0 L & -\gamma_1 L & -\gamma_2 L & \dots & -\gamma_{l-1} L \end{bmatrix}$$

where  $\Gamma$  has at least  $l$  zero eigenvalues, there are exactly  $l$  zero eigenvalues if and only if  $-L$  has a simple zero eigenvalue. Moreover, if  $-L$  has a simple zero eigenvalue, the geometricity of the zero eigenvalue of  $\Gamma$  is equal to one.

In addition to these approaches, other approaches have been used to achieve consensus. For example, backstepping is a recursive design procedure where the complexity increases drastically with the order of systems. Compared with the first-order and second-order dynamics, the high-order dynamics involve more details related to the interactions between the system dynamics (states and their derivatives) and the communication graph, which are reflected in the virtual controls and the Lyapunov functions [42, 64].

For high-order multi-agent systems, most previous studies were concerned only with integrators that used chained systems with strict-feedback form. Due to the inherent characteristics of multiple linear systems, matrix theory approaches are used frequently in stability analysis, but other techniques include dissipativity theory, non-smooth analysis, the backstepping control technique, and Lyapunov functions in particular. Previous theoretical research and experiments have solved a number of the technical problems that affect distributed multi-agent coordination, but many important and challenging research problems merit further investigation. First, most physical systems are inherently nonlinear, which means that the cooperative control of high-order nonlinear systems is very challenging. Furthermore, in many practical applications, the dynamics of the systems are generally nonlinear but there are also uncertainties. Thus, solving consensus problems for multiple high-order uncertain nonlinear systems would be beneficial for practical applications. The matrix theory-based approaches mentioned above are not applicable in many scenarios, especially for nonlinear systems. The extension of adaptive control to high-order dynamics is also not straightforward because of the growth in the order. The control law needs to be distributed in cooperative adaptive controllers for multi-agent systems on graphs, but the adaptive tuning laws also need to be distributed. High-order systems contain more states and their derivatives, so the design of adaptive control becomes more complicated. This requires the careful crafting of a suitable Lyapunov function, which automat-

ically yields a distributed adaptive controller that depends only on local information [65]. For undirected graphs or balanced graphs, it is not difficult to extend the Lyapunov techniques to the design of adaptive controllers. However, if the underlying graph is directed, the Lyapunov function must be constructed carefully and it must generally contain information about the graph topology. Thus, the cooperative control of high-order nonlinear systems with uncertainties is more challenging than that of certain high-order linear systems. Most previous studies have proved the convergence asymptotically, such as [34, 37–44, 46]. However, achieving finite-time convergence is also a challenging direction for high-order multi-agent systems.

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**Name:**  
Jie Huang

**Affiliation:**  
Beijing Institute of Technology

**Address:**

206 Lab, No.5 Yard, Zhong Guan Cun South Street, Haidian District, Beijing 100081, China

**Brief Biographical History:**

2005 Received the B.E. degree in electrical engineering and automation from Fuzhou University

2010 Received the M.E. degree in control science and engineering from Fuzhou University

2011- Ph.D. candidate at School of Automation, Beijing Institute of Technology

**Main Works:**

- Control of mobile robots and multi-agent systems

**Membership in Academic Societies:**

- Student Member, The Institute of Electrical and Electronics Engineers (IEEE)



**Name:**  
Jie Chen

**Affiliation:**  
Professor, Beijing Institute of Technology

**Address:**

206 Lab, No.5 Yard, Zhong Guan Cun South Street, Haidian District, Beijing 100081, China

**Brief Biographical History:**

1986 Received the B.E. degree in control theory and control engineering from Beijing Institute of Technology

1989-1990 Visiting scholar in California State University

1996 Received the M.E. degree in control theory and control engineering from Beijing Institute of Technology

1996-1997 Research fellow in school of E&E, the University of Birmingham

1998- Professor of control science and engineering, Beijing Institute of Technology

2000 Received the Ph.D. degree in control theory and control engineering from Beijing Institute of Technology

**Main Works:**

- Complex system multi-objective optimization and decision, intelligent control, constrained nonlinear control, and optimization methods

**Membership in Academic Societies:**

- Senior Member, The Institute of Electrical and Electronics Engineers (IEEE)
- Member, The 6th Review Panel on Control Science and Engineering
- Chinese State Council Academic Degrees Committee
- Executive Member and Deputy Director, The Technical Committee on Control Theory, Chinese Association of Automation
- Executive Member, The Chinese Association for Artificial Intelligence
- Member, The Academic Committee for the State Key Laboratory of Industrial Control Technology



**Name:**  
Hao Fang

**Affiliation:**  
Professor, Beijing Institute of Technology

**Address:**

206 Lab, No.5 Yard, Zhong Guan Cun South Street, Haidian District, Beijing 100081, China

**Brief Biographical History:**

1995 Received the B.E. degree in automatic control from the Xian University of Technology

1998 Received the M.E. degree in automatic control from the Xian Jiaotong University

2002 Received the Ph.D. degree in automatic control from the Xian Jiaotong University

2011- Professor of control science and engineering, Beijing Institute of Technology

**Main Works:**

- Control of mobile robots, parallel manipulators, and multi-agent systems



**Name:**  
Lihua Dou

**Affiliation:**  
Professor, Beijing Institute of Technology

**Address:**

206 Lab, No.5 Yard, Zhong Guan Cun South Street, Haidian District, Beijing 100081, China

**Brief Biographical History:**

1979 Received the B.E. degree in Control Theory and Control Engineering from Beijing Institute of Technology

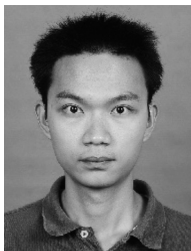
1987 Received the M.E. degree in Control Theory and Control Engineering from Beijing Institute of Technology

2000- Director of Ordinary University Key Laboratory of Beijing (Automatic Control System), and Professor of Control Science and Engineering at Key Laboratory of Complex System Intelligent Control and Decision, School of Automation, Beijing Institute of Technology

2001 Received the Ph.D. degree in Control Theory and Control Engineering from Beijing Institute of Technology

**Main Works:**

- Intelligent control, pattern recognition, and image processing



**Name:**

Jie Zeng

**Affiliation:**

Beijing Institute of Technology

**Address:**

206 Lab, No.5 Yard, Zhong Guan Cun South Street, Haidian District,  
Beijing 100081, China

**Brief Biographical History:**

2009 Received the B.E. degree in Automation from Beijing Institute of  
Technology

2012- Ph.D. candidate in the Pattern recognition and Intelligent System  
Lab, Beijing Institute of Technology

**Main Works:**

- Cooperative missile guidance and control
-