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## An overview of consensus problems in constrained multi-agent coordination

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In recent years, multi-agent coordination has gained significant development in theoretical research in parallel with the increasing attention in the practical applications of such systems in various areas including consensus. This paper presents a survey of recent research in consensus problems for constrained multi-agent coordination. We first focus on theoretical directions of multi-agent systems paying particular attention from the system dynamics and control algorithms perspective. Then we present several problems on constrained multi-agent coordination via consensus schemes which have gained significant attention recently. Relevant to these problems, we summarize some of the recent results on constrained multi-agent coordination via consensus schemes which appeared in the literature. Finally, this paper is concluded with existing results and some remarks on the practical direction of consensus problems developed for constrained multi-agent coordination.

**Keywords:** multi-agent coordination; consensus problems; constrained control; saturation constraints

### 1. Introduction

During the past decades, distributed coordination for networks of dynamical agents has attracted many researchers from various disciplines of engineering and science due to the broad applications of multi-agent systems in areas such as sensor networks, unmanned aerial vehicles (UAVs), formation control, congestion control in communication networks, swarming and flocking models. The first study of distributed coordination for multi-agent systems might be motivated by the efforts on distributed decision-making (Tsitsiklis, Bertsekas, & Athans, 1986), statistical physics (Vicsek, Czirók, Ben-Jacob, Cohen, & Shochet, 1995) and distributed computing (Lynch, 1996).

In the study of distributed coordination for multi-agent systems, consensus problems arise naturally under various information flow constraints (Fax & Murray, 2004; Jadbabaie, Lin, & Morse, 2003; Moreau, 2005; Olfati-Saber & Murray, 2004; Ren & Beard, 2005). Consensus means that a group of agents reach an agreement on a common value by negotiating with their neighbors (Ren, Beard, & Atkins, 2005). It is the most basic behavior of multi-agent coordination, and many of coordinated behaviors are also directly related to consensus. The study of consensus problems not only reveal how the behaviors of multi-agent coordination emerge, but also provide the result of applying the method of lines to other new technologies for the coordinated control of multi-agent systems. For more details

and progress reports, see the survey papers (Cao, Yu, Ren, & Chen, 2013; Chebotarev & Agaev, 2009; Leonard et al., 2007; Olfati-Saber, Fax, & Murray, 2007; Ren, Beard, & Atkins, 2007; Zhang, Gao, & Kaynak, 2013), the books (Bai, Arcak, & Wen, 2011; Gazi & Fidan, 2007; Mesbahi & Egerstedt, 2010; Qu, 2009; Ren & Beard, 2008; Ren & Cao, 2011; Shamma, 2007; Wu, 2007) and references therein.

The main idea behind consensus is to design distributed and coordinated algorithms for multi-agent systems. As is known, consensus always focuses on the behaviors of multiple agents. Therefore, it is natural to understand that system dynamics of practical models should be considered when studying the consensus problem in multi-agent systems. To fully fill the gap between studying consensus algorithms and some inherent properties or constraints in practical models, it is necessary to investigate consensus problems with some practical factors, such as communication constraints (Gao, Chen, & Lam, 2008) and actuator saturations, which can be regarded as key-constrained features in practical models. Taking into account the fact that saturation constraint has received considerable attention owing to the physical limitations in real control systems and their universal existence in various application domains, distributed coordination for constrained multi-agent systems is not only theoretically challenging but also practically important. When considering the saturation constraint in multi-agent coordination, can the distributed algorithms reported before still

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be effective with the saturation constraint. This problem is pretty significant since the answer to it determines whether we should design new coordinated algorithms when the saturation constraint exists in multi-agent systems (Li, Xiang, & Wei, 2011). Motivated by these observations, the consensus problems in constrained multi-agent coordination were widely studied in recent years.

This paper reviews some main results and recent progress of consensus problems in constrained multi-agent coordination. Here, the term *constrained multi-agent coordination* is used for the study of consensus problems when the system dynamics are constrained (e.g. saturated/bounded) or with constrained consensus algorithms. Relevant to the consensus problem, some applications via consensus schemes for constrained multi-agent coordination shall be investigated.

The paper is organized as follows. Section 2 formulates some preliminaries. In Section 3, results of global consensus, semi-global consensus and other constrained problems with undirected, directed, switching topologies and complex network topologies are firstly studied. Then, constrained asymptotical, constrained finite-time and constrained complex consensus algorithms are summarized. Section 4 reviews some constrained consensus schemes, such as constrained formation control, attitude coordination via constrained control and flocking via constrained control. Finally, remarks and future research directions are concluded in Section 5.

## 2. Preliminaries

*Notation* (Wang, Gao, & Yu, 2013; Wang, Yu, Gao, & Liu, 2013): Some notations will be stated here. The notation  $\text{diag}\{\dots\}$  denotes a block-diagonal matrix. Use  $\text{col}\{x_1, x_2, \dots, x_n\}$  and  $1_N$  to denote a column vector and an  $N$ -dimensional column vector with all elements of 1, respectively. Matrices, if their dimensions are not explicitly stated are assumed to be compatible for algebraic operations. The notation  $P > 0$  ( $\geq 0$ ) means that  $P$  is a real symmetric positive- (semi-positive) definite matrix.  $I$  and  $0$  represent, respectively, the identity matrix and zero matrix. The set of real numbers is denoted by  $\mathbb{R}$ . The set of real-valued vectors of length  $m$  is given by  $\mathbb{R}^m$ . The set of arbitrary real-valued  $m \times n$  matrices is given by  $\mathbb{R}^{m \times n}$ .

*Graph theory* (Godsil & Royle, 2011): Let  $G = (\mathcal{V}, \varepsilon, E)$  be a weighted directed graph, where  $N$  is a finite nonempty set of nodes  $\mathcal{V} = \{1, \dots, N\}$  and edges  $\varepsilon = \mathcal{V} \times \mathcal{V}$ .  $E = [e_{ij}] \in \mathbb{R}^{N \times N}$  is the weighted adjacency matrix, where  $e_{ij}$  is the coupling strength of the directed edge  $(j, i)$  satisfying  $e_{ij} \neq 0$  if  $(j, i)$  is an edge of  $G$  and  $e_{ij} = 0$  otherwise. Let  $\mathcal{N}_i = \{j \in \mathcal{V} : (j, i) \in \varepsilon\}$  be the set of neighbors of node  $i$  in  $G$ . For any pair of vertices  $(i, j)$ , if  $e_{ij} = e_{ji}$ , the graph is called an undirected graph. Let  $d_i = \sum_{j=1}^N e_{ij}$  be the in-degree of vertex  $i$ , and  $D = \text{diag}\{d_1, \dots, d_N\}$  the in-degree matrix of  $G$ . The Laplacian matrix  $L = [l_{ij}]$  of

weighted digraph  $G$  is defined by  $L = D - E$ . A directed path of length  $l$  is defined as a sequence of edges in a directed graph of the form  $((i_1, i_2), (i_2, i_3), \dots, (i_l, i_{l+1}))$  in which  $(i_j, i_{j+1}) \in \varepsilon$  for  $j = 1, \dots, l$  and  $i_j \neq i_k$  for  $j, k = 1, \dots, l$  and  $j \neq k$ . The graph  $G$  contains a directed spanning tree if there is a node which can reach all the other nodes through a directed path. In this paper, the interaction topologies of multi-agent systems are modeled by undirected graphs or directed graphs without multiple edges.

*Saturation function* (Abdessameud & Tayebi, 2013): A strictly increasing continuously differentiable function  $\delta : \mathbb{R} \rightarrow \mathbb{R}$  is said to be a saturation function if it satisfies the three properties:

- (1)  $\delta(0) = 0$  and  $x\delta(x) > 0$  for  $x \neq 0$ .
- (2)  $|\delta(x)| \leq \delta_b$  for  $\delta_b > 0$ .
- (3) The diagonal matrix  $h(x) = \text{diag}[d\delta(x_k)/dx_k]$  satisfies  $\|h(x)\|_\infty \leq \delta_h$ , where we denote  $x = \text{col}\{x_1, x_2, \dots, x_n\}$  and  $\delta_h > 0$ .

It is clearly seen that  $\tanh(x)$  and  $x/\sqrt{1+x^2}$  can serve as two examples of saturation functions. Also, the definition of the above saturation function can be applied to the standard saturation functions introduced in Lin (1998).

## 3. Consensus problems in constrained multi-agent coordination

Early works on consensus mainly focus on first-order leaderless consensus problem, where the reached consensus value depends on the initial states of the agents, cannot be controlled and no dynamic exchange among agents. The first-order leaderless consensus problem can be described as follows: consider a group of  $N$  agents, and the dynamic of each agent is given by

$$\dot{x}_i(t) = u_i(t), \quad i = 1, 2, \dots, N, \quad (1)$$

where  $x_i(t)$  and  $u_i(t)$  are the state and the control input of the  $i$ th agent. To achieve consensus, the following algorithm is designed as

$$u_i(t) = \sum_{j=1}^N e_{ij}(t)[x_j(t) - x_i(t)], \quad (2)$$

where  $e_{ij}(t)$  is the  $(i, j)$ th entry of the corresponding adjacency matrix at time  $t$ . By adding a (virtual) leader agent whose information is only available to a subset of networked agents, the cooperative tracking control problem was investigated in Vicsek et al. (1995), Jadbabaie et al. (2003) and Spanos, Olfati-Saber, and Murray (2005). It is shown in Moreau (2005) and Ren and Beard (2005) that first-order consensus can be achieved if the underlying directed graph has a directed spanning tree.

As extensions of studying first-order consensus, second-order or high-order consensus problems were widely studied in Ren (2008), Xie and Wang (2007), Yu, Chen, Cao,

and Kurths (2010), Li, Duan, Chen, and Huang (2010) and You and Xie (2011). Note that the two main factors, the network topology and the agent dynamics, greatly influence the behaviors of the high-order multi-agent systems. As a result, some new problems should be investigated when considering consensus problems for high-order agents with some practical factors, such as saturation constraints. Therefore, we will review recent theoretical progress of consensus problems for multi-agent systems with saturation constraints in this section.

### 3.1. Consensus problems via constrained dynamics

In this section, we review recent theoretical progress of consensus problems when the system dynamics are constrained.

For the continuous-time multi-agent systems, we consider  $N$  agents and every agent has a state vector  $x_i(t) \in \mathbb{R}^n$  and a control input  $u_i(t) \in \mathbb{R}^m$ . Let the agents, labeled from 1 to  $N$ , be modeled by

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + B\delta(u_i(t)), \\ y_i(t) &= Cx_i(t), \end{aligned} \quad i = 1, \dots, N, \quad (3)$$

where  $A, B$  and  $C$  are constant matrices with compatible sizes,  $\delta(\cdot)$  is the saturation function and  $u_i(t)$  is decided by the neighborhood of agent  $i$ . The matrix pair  $(A, B)$  is stabilizable. Note that the first-order and high-order integrator dynamics with saturation functions can be included as the special cases of system (3) for properly choosing  $A, B$  and  $C$ . For the discrete-time multi-agent systems, the dynamics can be similarly described by

$$\begin{aligned} x_i(k+1) &= Ax_i(k) + B\delta(u_i(k)), \\ y_i(k) &= Cx_i(k), \end{aligned} \quad i = 1, \dots, N, \quad (4)$$

where  $u_i(k)$  is the algorithm designed based on the information of its neighborhood, received at or before step  $k$ .

#### 3.1.1. Global consensus problems

We firstly introduce the global consensus problem. For the multi-agent system (3) (or (4)) under an algorithm  $u_i(t)$  (or  $u_i(k)$ ), we say that system (3) (or (4)) solves the global consensus problem if for *any initial states*, there exists a common equilibrium point for all agents. Generally speaking, the global consensus means that, for any initial states, a number of coupled individual systems with saturation constraints converge with each other under a graph  $G$ .

On the one hand, it is well known (Eduardo, 1984) that, for the continuous-time linear individual system subject to saturation constraints, global stabilization is possible if and only if the linear system is asymptotically null controllable with bounded controls (ANCBC), that is, the linear system is stabilizable and all its open-loop poles are located in the closed left-half plane. On the other hand, for an undirected

graph or a directed graph, the sufficient condition for achieving consensus of multi-agent systems is that the undirected graph is connected or the directed graph contains a directed spanning tree (Olfati-Saber et al., 2007). Therefore, from the work of saturation (Teel, 1992), in general the conditions that the linear system is ANCBC and the undirected graph is connected or the directed graph contains a directed spanning tree are necessary for multi-agent system (3) to achieve global consensus. For the discrete-time linear multi-agent systems (4), similar to the continuous-time case, all the eigenvalues of matrix  $A$  are on or within the unit disk and the undirected graph is connected or the directed graph contains a directed spanning tree are sufficient conditions to achieve global consensus.

To achieve global consensus in constrained dynamics, leader-following and leaderless linear algorithms are standard and widely studied by the researchers. In the literature, the global leaderless consensus was studied in Wang, Peng, Gao, and Basin (2013) for the continuous- and discrete-time single-integrator agents under undirected graphs. In Meng, Zhao, and Lin (2013), the global leader-following consensus of continuous-time double-integrator agents and marginally stable agents was investigated with undirected or detailed balance-directed graphs. Accordingly, the literature (Yang, Meng, Dimarogonas, & Johansson, 2013) focused on the global leaderless consensus for discrete-time double-integrator agents and marginally stable agents under undirected graphs. The dynamic in Wang et al. (2013) for the global leader-following consensus was the discrete-time marginally stable agents, where the agents were subject to saturation constraints and the graph topology was undirected or detailed balance directed. Although the global consensus problem is to be assumed that the topologies among agents are time-invariant in the beginning, topologies may change dynamically in reality. For example, the global consensus of the continuous-time single-integrator agents with saturation constraints and time-varying topology was studied in Shi and Hong (2009) by using the bang-bang control scheme.

Looking into the issues discussed above, it can be seen that for single-integrator, double-integrator and marginally stable agents, continuous or discontinuous linear control algorithms are capable of achieving global consensus for constrained dynamic agents. It is also observed that the above literature is based on special cases of ANCBC agents, such as single-integrator and double-integrator agents. For the general linear ANCBC agents, nonlinear algorithms are needed to achieve the global consensus problem (Yang, Sontag, & Sussmann, 1997). For example, by using the nested saturation function, the global consensus for a group of multi-integrator agents subject to saturation constraints was investigated in Zhao, Lin, and Meng (2012) and Wang and Gao (2013), where there exists dynamics exchange between agents. However, if the researchers still concentrate on linear algorithms, it will lead to the semi-global consensus problem.

### 3.1.2. Semi-global consensus problems

In what follows, we will introduce the semi-global consensus problem, which is defined as follows: for the multi-agent systems (3) (or (4)) and under an algorithm  $u_i(t)$  (or  $u_i(k)$ ), we say that system (3) (or (4)) solves the semi-global consensus problem if for any a priori given bounded set  $\chi \subset \mathbb{R}^n$  and the initial states  $x_i(0) \in \chi, i = 1, \dots, N$ , there exists a common stable equilibrium point for all agents.

Using linear control algorithms, the main technique to achieve semi-global consensus is the small gain feedback. More specifically, by using low-gain state feedback (Lin, 1998), the works of Wei, Hu, and Wang (2013) and Wang, Yu, and Gao (2013) actually studied the semi-global consensus of linear multi-agent systems subject to saturation constraints with undirected and directed topologies, respectively. In Su, Chen, Lam, and Lin (2013) and Su, Chen, Lam, and Lin (2014), the problems of semi-global consensus and observer-based semi-global consensus for a linear multi-agent system with saturation constraints were presented on a switching network. The leader-following low-gain feedback design for the multi-agent system (3) is carried out in the following two steps.

*Step 1* Solve the following parametric algebraic Riccati equation (ARE)

$$A^T P(\gamma) + P(\gamma) A - 2\mu P(\gamma) B B^T P(\gamma) + \gamma I = 0, \\ \gamma \in (0, 1],$$

where  $\mu > 0$  is a positive constant. Let  $\gamma I = \gamma P(\gamma)$ , the parametric ARE becomes a parametric Lyapunov equation.

*Step 2* Construct a linear low-gain feedback for agent  $i$  as

$$K = -B^T P(\gamma).$$

The fact that  $P(\gamma) \rightarrow 0$  as  $\gamma \rightarrow 0$  motivates the term of low-gain feedback. As for the leaderless low-gain feedback, the design procedure is the same. By taking the semi-global framework, the distributed output regulation problem of linear multi-agent systems subject to saturation constraints was considered in Wang, Ni, and Yang (2013) under a switching topology.

Note that two main factors, the network topology and the agent dynamics, greatly influence the motion of the multi-agent systems. For systems described by first-order dynamics, the consensus problem depends on the network topology. For high-order or linear systems, both the network topology and the agent dynamics can influence the consensus convergence. For example, first-order consensus often converges to a constant value, while second-order consensus usually converges to a dynamic value. These different behaviors motivate the focus on the consensus problem with different dynamics. It is also observed that the technique of semi-global consensus is based on the small gain feedback, which means that saturation constraints can be avoided, such as in the domain of attraction. Therefore, it is natural to assume that the actuator will not be saturated any more,

and consider absolute the magnitude of the coupling of the network. In this sense, the semi-global consensus problem can achieve global motions.

### 3.1.3. Other constrained problems

All the above analysis is under the assumption that the agents are identical. It also may happen that the network of multi-agent systems subject to actuator saturation is heterogeneous. For example, the global consensus problem of multi-agent systems with linear follower agents and a leader agent whose control input is bounded was considered in Li, Liu, Ren, and Xie (2013) under undirected topologies. The work of Yang, Stoorvogel, Grip, and Saberi (2014) actually investigated the semi-global output regulation of consensus problem for heterogeneous networks of invertible linear agents subject to saturation constraints. For a heterogeneous network of discrete-time introspective right-invertible agents, the global output consensus problem was studied in Wang, Saberi, and Yang (2013) under directed graphs. The finite-time consensus problem of heterogeneous multi-agent systems composed of first-order and second-order integrator agents was studied in Zheng and Wang (2012a) with and without velocity measurements. Some other results related with bounded consensus have been obtained, interested readers may refer to Zhong, Liu, and Thomas (2012), Xie (2013), Li and Fang (2012), Wang, Yan, and Wang (2013).

Some related consensus problems with constrained complex dynamics were widely studied. In Chopra and Spong (2009), the problem of exponential synchronization was studied for nonlinear oscillators, where the dynamics are assumed to be Kuramoto equations. The actual consensus algorithms design problem for underactuated mobile robots was investigated in Dimarogonas and Kyriakopoulos (2007). The consensus problem for inherent nonlinear dynamics was investigated in Cao, Ren, Chen, and Zong (2011) under an undirected interaction graph. As an application, distributed multi-hop reactive power compensation was proposed in Bolognani et al. (2012) for smart micro-grids subject to saturation constraints.

It is worth pointing out that, compared with existing consensus methods, the techniques to solve semi-global consensus problems in constrained multi-agent coordination are mainly based on the low-gain approach, which in turn leads to weak couplings. In fact, the strong coupling may destroy semi-global consensus for higher order dynamics and, thus, the low-gain approach is a safe way to reach semi-global consensus.

It is noting that nonlinear algorithms are needed to achieve the global consensus problem for general linear ANCBC agents. Furthermore, saturation constraints exist commonly in practical control inputs. Therefore, it is important to develop appropriately constrained nonlinear consensus algorithms for different kinds of ANCBC agents.



### 3.2. Consensus problems via constrained algorithms

In this section, we will study some recent theoretical progress of consensus problems when consensus algorithms are constrained.

For the continuous-time multi-agent systems, we consider  $N$  agents and every agent has a state vector  $x_i(t) \in \mathbb{R}^n$  and a control input  $u_i(t) \in \mathbb{R}^m$ . Let the agents, labeled from 1 to  $N$ , be modeled by

$$\begin{aligned}\dot{x}_i(t) &= Ax_i(t) + Bu_i(t), \\ y_i(t) &= Cx_i(t),\end{aligned}\quad i = 1, \dots, N, \quad (5)$$

where  $A, B$  and  $C$  are constant matrices with compatible sizes and  $u_i(t)$  is decided by the neighborhood of agent  $i$  and is affected by control input saturation. The matrix pair  $(A, B)$  is stabilizable. Note that the first-order and high-order integrator dynamics can be included as the special cases of system (5) for properly choosing  $A, B$  and  $C$ . For the discrete-time multi-agent systems, the dynamics can be similarly described by

$$\begin{aligned}x_i(k+1) &= Ax_i(k) + Bu_i(k), \\ y_i(k) &= Cx_i(k),\end{aligned}\quad i = 1, \dots, N, \quad (6)$$

where  $u_i(k)$  is the algorithm designed based on the information of its neighborhood, received at or before step  $k$ , and it is affected by control input saturation.

#### 3.2.1. Constrained asymptotical consensus algorithms

We firstly present the asymptotical consensus problem. Consider system (5) (or (6)), and design algorithm  $u_i(t)$  (or  $u_i(k)$ ),  $i = 1, 2, \dots, N$ , which is affected by control input saturation, the algorithm or the system is said to *solve an asymptotical consensus problem*, if for any given initial states, there exists a common asymptotically stable equilibrium  $x^* \in \mathbb{R}^N$  for all agents, such that for any  $i$ ,  $x_i(t) \rightarrow x^*$  as time  $t \rightarrow \infty$  (Olfati-Saber & Murray, 2004).

Some works have taken saturation constraints to design asymptotical consensus algorithms in multi-agent systems with single-integrator dynamics (Li et al., 2011; Zhang & Duan, 2011) or double-integrator dynamics (Ren, 2008). Without using velocity measurement, the asymptotical consensus algorithm for double-integrator dynamics with control input saturation was proposed in Abdessameud and Tayebi (2010). The saturated consensus algorithms design for double-integrator dynamics with bounded inputs was considered in Abdessameud and Tayebi (2013). However, these works on asymptotical consensus algorithms with input saturation are based on undirected graphs and are conducted under information consensus, which means there are no dynamics except information exchange between agents. Accounting for dynamics exchange between agents, the work of Ren (2009) proposed a distributed, leaderless and model-independent asymptotical consensus algorithm

for networked Euler–Lagrange systems with saturation constraints. Detailed analysis of asymptotical consensus algorithms under bounded inputs was investigated in Ni, Xiong, and Yang (2013) for both fixed and switching network topologies.

Different from these works, the asymptotical consensus problem of heterogeneous multi-agent system composed of first-order and second-order integrator agents was discussed in Zheng, Zhu, and Wang (2011) with saturated consensus algorithms. Without using velocity measurement, the asymptotical consensus algorithm accounting for saturation constraints was proposed in Zheng and Wang (2012b) by studying the same asymptotical consensus problem of the heterogeneous multi-agent system.

To represent the performance of distributed algorithms in consensus problems, convergence rate is an important factor. Note that the above literature overlook this factor. How to design effective distributed algorithms with finite settling times is more interesting. Furthermore, finite settling-time systems may have better disturbance rejection and robustness against uncertainties.

#### 3.2.2. Constrained finite-time consensus algorithms

In what follows, we shall study the finite-time consensus problem. Consider system (5) (or (6)), and design algorithm  $u_i(t)$  (or  $u_i(k)$ ),  $i = 1, 2, \dots, N$ , which is affected by control input saturation, it is said to *solve a finite-time consensus problem* if it solves an asymptotical consensus problem and, for any initial states, there exist a finite time  $t^*$  and a vector  $x^* \in \mathbb{R}^N$ , such that for all  $t \geq t^*$  and for all  $i$ ,  $x_i(t) = x^*$ . Note that in the context of switching topologies, the final state  $x^*$  is usually unpredictable, and it depends on the initial states and the switching sequence of the interaction topology (Ren & Beard, 2005).

In practice, a finite-time control technique was adopted in Chen, Wang, Zhang, and Zhang (2013) to solve the saturated tracking control problem of nonholonomic mobile robots. The work of Hui, Haddad, and Bhat (2008) developed a general framework for designing nonlinear algorithms in dynamical networks for achieving coordination tasks in finite time. By using nonsmooth stability analysis, the nonsmooth gradient flows was proposed in Cortés (2006) to achieve consensus in finite time. In Lu, Chen, and Lü (2013), the bounded finite-time tracking algorithm for double-integrator multi-agent systems was studied under the conditions of fixed and switching jointly reachable digraphes. It also may happen that the information exchange is not transmitted or processed in time due to the limited bandwidth of information links or the limited processing capacities of agents. This will introduce time delays into the overall systems. Based on a kind of novel nonlinear saturation functions, bounded finite-time consensus algorithms were further developed in Du, Li, and Ding (2013) for the single-integrator agents with communication delay.

In the study of the above consensus algorithms, asymptotical consensus algorithms and finite-time consensus algorithms are classified by the convergence rate, which is an important factor for the performance of multi-agent systems. According to the definition of finite-time consensus, the final state  $x^*$  is usually a constant vector. Compared with this point, we think that the finite-time boundedness concept introduced by Amato, Ariola, and Dorato (2001) is more general. Because the authors in Amato et al. (2001) integrate the asymptotical concept and finite-time concept from the definition of the finite-time boundedness.

### 3.2.3. Constrained complex algorithms

It should be pointed out that the above-reported control algorithms for multi-agent systems are mainly based on linear agents. However, there are also some related results in constrained complex algorithms research directions. For example, consensus problems for nonlinear multi-agent dynamical systems were addressed in Hui and Haddad (2008) by using constrained complex algorithms and with the fixed and switching topologies. By using a novel Takagi–Sukeno fuzzy modeling method, Zhao, Li, Qin, and Gao (2013) investigated the  $H^\infty$  consensus control problem of multi-agent systems by using constrained complex algorithms and with an arbitrary topology. The problem of connectivity preservation in multi-agent systems was widely studied by bounded control algorithms, interested readers may refer to Dimarogonas and Johansson (2010), Fan, Feng, and Gao (2012), Ajorlou, Momeni, and Aghdam (2010) and Ajorlou and Aghdam (2013).

Although the aforementioned consensus algorithms are different from constrained asymptotical, constrained finite-time and constrained complex, the main target is the same, that is, to make all agents to some common states via coupling among agents. The main task can be regraded as designing proper constrained control algorithms and providing some conditions such that consensus can be achieved.

## 4. Constrained multi-agent coordination via consensus schemes

In this section, we investigate some applications of the consensus schemes to constrained multi-agent coordination, such as constrained formation control, attitude coordination via constrained control and flocking via constrained control.

### 4.1. Constrained formation control

Formation control means the final states of all agents are diversified via formation, which is different from the consensus problem where the final states of all agents typically reach a common state. Many practical applications can be regarded as formation control, such as formation flying,

distributed sensor networks and surveillance. In general, formation control can be divided by its group reference. *Constrained formation producing* means formation control without a group reference, and it refers to the algorithm design *with constraints* (for example, bounded/saturation) for a group of agents to reach some control objective in advance, while *constrained formation tracking* means the same task but following a group reference. Recent research results and progress in formation control without constraints were reviewed in Cao et al. (2013). The following two parts will review constrained formation control including constrained formation producing and constrained formation tracking.

#### 4.1.1. Constrained formation producing

The aim of formation control is to study formation behaviors with some control algorithms. In addition, the work of stability for formation control is included.

By using a model-independent coordination strategy, formation-constrained multi-agent control was proposed in Egerstedt and Hu (2001) by studying the stability of a formation error via a bounded tracking error assumption. In Yang, Liu, Chen, and Pei (2006), distributed robust control algorithms were presented for formation stabilization of uncertain nonholonomic wheeled mobile robots with actuator saturations. A bounded cooperative controller was designed in Do (2007) to stabilize a desired formation behavior for mobile robots with bounded sensing ranges. An operator–vehicle adversarial network was considered in Zhu and Martínez (2013) to make the vehicles to the desired formation with the given state and input constraints. The problem of formation control with collision avoidance and bounded velocity was presented in Fukushima, Kon, and Matsuno (2013) by a model predictive control method. Due to the nonlinearity of constrained dynamics, it is more challenging than considering the formation control with time delay, disturbances and quantization in the future.

#### 4.1.2. Constrained formation tracking

Since control objectives or common values for a group of agents are more interesting in reality, formation control with a group reference has received wide attentions. This section is to review constrained formation tracking.

Under saturation constraints on the control inputs, a consensus scheme was proposed in Kostic, Adinandra, Caarls, Van de Wouw, and Nijmeijer (2010) to solve the formation tracking problem where multiple unicycle agents are required to follow individual reference trajectories while maintaining a time-varying formation. By introducing a formation control strategy, the leader–follower formation problem of nonholonomic mobile robots with input constraints was studied in Consolini, Morbidi, Prattichizzo, and Tosques (2008). Without using velocity measurement and accounting for actuator saturations, the formation tracking

problem with a general vehicle model was developed in Li (2009) for both leaderless and leader–follower formation scenarios. In Do (2014), a bounded formation controller is proposed to make a group of mobile agents with second-order dynamics to track reference trajectories and to avoid collision. By using a fuzzy-logic-based potential function in a constrained environment, a virtual leader was introduced in Dong, Kang, Yang, and Sun (2013) to direct the group mobile agents to maintain a desired geometric formation and to track a desired path. The formation tracking problem can be converted to a traditional stability problem while the formation producing problem, in general, cannot be solved by redefining the variables as the formation errors. As a result, the constrained formation tracking problem is generally much easier than the constrained formation producing problem with switching topologies.

It is observed that both constrained formation producing and constrained formation tracking are based on fixed formation in which the distances between agents are fixed. Also, the constraints mainly exist in the formation controller design procedure. Considering practical applications there exists constrained dynamics among agents, it may require more performance to manage formation control behaviors. Meanwhile, considerable attention in complex dynamics with noise and disturbances could be more challenging.

#### 4.2. Attitude coordination via constrained control

For attitude alignment among a group of spacecraft, consensus algorithms are effective solutions when appropriate information states on which consensus is reached through local information exchange are chosen. By using consensus algorithms combined with behavior-based control, the decentralized coordinated control problem for multiple spacecraft was investigated in Zhang and Song (2012) and Zhang and Song (2011) by coordinated formation controllers with input saturation and undirected graphs. The attitude alignment problem for a team of UAVs was studied in Bauso, Giarré, and Pesenti (2003) using nonlinear centralized and decentralized information consensus algorithms. By using the Chebyshev neural network, a distributed attitude coordinated control method was proposed for spacecraft with control input saturation when the time-varying reference attitude is available to a subset of spacecraft (Zou & Kumar, 2012). The work of Septanto, Trilaksono, Syaichu-Rohman, and Poetro (2012) developed a piecewise-continuous consensus scheme of quaternion-based control law with saturation constraints such that it would avoid the unwinding phenomenon.

Due to the nonlinear dynamics of the spacecraft, the problem of attitude coordination for a group of spacecraft is more challenging, especially when the control input saturation is considered. It is also observed that the results are mainly based on fixed topologies, and there are some open problems. For example, whether the time-varying reference

attitude must be available to a subset of spacecraft or to all spacecraft is still open even without input saturation.

#### 4.3. Flocking via constrained control

Flocking behavior can be described as a group of agents with individual control laws eventually reaching an equidistant spacing among themselves, and can be found very common in nature (Shaw, 1975; Vicsek et al., 1995). The flocking problem in multi-agent coordination was considered in Liu and Yu (2009) with a bounded control input. By using the ideas of collective potential functions and velocity consensus, a connectivity-preserving flocking algorithm with bounded potential function was proposed in Wen, Duan, Su, Chen, and Yu (2012) to investigate the flocking problem of multi-agent systems with second-order nonlinear dynamics. In Chen and Sun (2012), the flocking problem of micro-scaled particles was solved to achieve the flocking manipulation via saturated velocities. The controller with actuator saturation was proposed in He, Feng, and Ren (2012) to study the flocking problem of multi-agent coordination by consensus schemes and pinning control. In Chen, Wang, and Lou (2013), it presented a velocity saturation method to achieve micro-particles flocking with a group of robotics. By using algebraic graph theory and Barbalat's Lemma, the velocity-free flocking control problem with input saturation was studied in Fan, Chen, and Zhang (2013) to guarantee that the multi-agent system converges to a rigid flock behavior. Under a switching topology, Li, Jia, and Yuan (2008) considered the flocking problem to a class of multi-agent systems in a bounded gradient environment.

Note that flocking control aims at stabilizing multiple agents toward desired formation, and the method of flocking control is mainly described by some nonnegative gradient functions. Constrained controls are usually used to achieve the flocking manipulation because of constrained dynamics or environment.

### 5. Conclusions and future directions

In this paper, we have reviewed the consensus problems of constrained multi-agent coordination in the current literature, and have shown some consensus results on both constrained multi-agent dynamics and constrained consensus algorithms. The consensus results on constrained multi-agent dynamics consist of the results of global consensus, semi-global consensus and other consensus with the topologies of both undirected graphs and directed graphs, and also with switching topologies. The constrained consensus algorithms in multi-agent systems are also considered in this paper, and the results of constrained consensus algorithms contain asymptotical, finite-time and constrained complex algorithms. The main results are based on the assumption of single-integrator, double-integrator and linear dynamic agents. Relevant to these problems, we summarized some of



the recent results on constrained multi-agent coordination via consensus schemes which appeared in the literature.

Despite these results, there are still some points that should be further considered in the future. We present some of them as follows.

**Complex dynamics:** Most research in constrained multi-agent systems assumes that the agent system is stabilizable and all its open-loop poles are located in the closed left-half plane. For some special cases, the dynamics of agents might not be strictly stable. For example, the marginally stable agents are easier to be unstable when they are subject to disturbances. Therefore, it will be interesting to study consensus problems where agents are heterogeneous (one or some unstable) in constrained multi-agent systems.

**Complex network issues:** In the current literature, most research activities focus on certain fixed and switching networks. Further investigations on the direction of uncertain or partially unknown network will be practical. In addition, issues such as network-induced disturbances, network-induced delay and communication/sensor noise should also be taken into account.

Note that it is not necessary to take all the network-related constraints into account for constrained multi-agent systems. Also, it should be noted that the comprehensive studies combining all the related constraints are not sufficient. For example, if one simultaneously considers the network-induced delay, the uncertain dynamics and the saturation constraints, the analysis and synthesis of constrained multi-agent systems will be more complex and challenging.

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