CSCE 633 - Machine Learning HW3

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I. ABSTRACT

In this homework, I mess around with Neural Networks and cool stuff.

QUESTION I

Activation Function in Neural NetworksDecision Tree

a) The scheme is in figure 1:

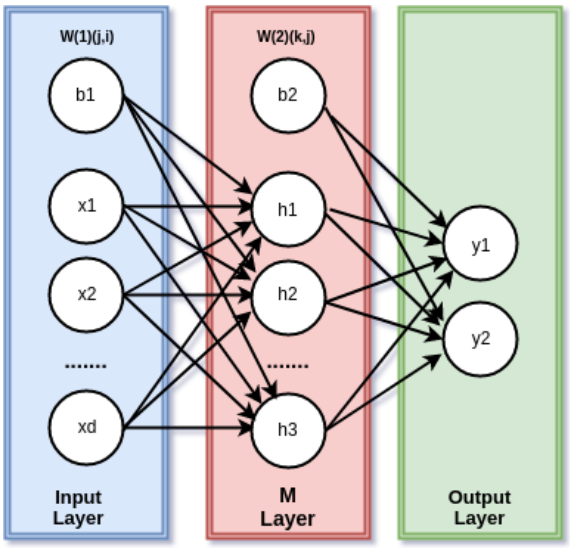


FIG. 1: Decision tree for problem 1 a

For convenience, it was easier to write out the wights in the matrices in the matrices below. The explanation is that for the first layer, (From input layer to the hidden layer), the weight $W^{(1)}(j,i)$ signifies the weight to the j hidden Perceptron from the i input Perceptron.

I am assuming that the input dimensions are (D=3) and the output are only from two classes (K=2). The biases are from Perceptron 0. The weights for the layers are the following:

$$W^{(1)}(j,i) = \begin{bmatrix} W^{(1)}_{1,0} & W^{(1)}_{2,0} & W^{(1)}_{3,0} \\ W^{(1)}_{1,1} & W^{(1)}_{2,1} & W^{(1)}_{3,1} \\ W^{(1)}_{1,2} & W^{(1)}_{2,2} & W^{(1)}_{3,2} \\ W^{(1)}_{3,1} & W^{(1)}_{3,2} & W^{(1)}_{3,3} \end{bmatrix} \tag{1}$$

and for the hidden layer to the output layer:

$$W^{(2)}(j,i) = \begin{bmatrix} W^{(2)}_{1,0} & W^{(2)}_{2,0} \\ W^{(2)}_{1,1} & W^{(2)}_{2,1} \\ W^{(2)}_{1,2} & W^{(2)}_{2,2} \\ W^{(2)}_{3,1} & W^{(2)}_{3,2} \end{bmatrix} \tag{2}$$

b) To express the output as a function of the input I am assuming that:

$$s(\alpha) = \frac{1}{1 + exp(-\alpha)} \tag{3}$$

lets first define out first layer after the activation function:

$$h^{(2)}_1(x) = s(W^{(1)}_{1,1}x_1 + W^{(1)}_{1,2}x_2 + W^{(1)}_{1,3}x_3 + W^{(1)}_{1,0}b_1) \tag{4a}$$
$$h^{(2)}_2(x) = s(W^{(1)}_{2,1}x_1 + W^{(1)}_{2,2}x_2 + W^{(1)}_{2,3}x_3 + W^{(1)}_{2,0}b_1) \tag{4b}$$
$$h^{(2)}_3(x) = s(W^{(1)}_{3,1}x_1 + W^{(1)}_{3,2}x_2 + W^{(1)}_{3,3}x_3 + W^{(1)}_{3,0}b_1) \tag{4c}$$

Now we can calculate the output:

$$y^{(3)}_1(x) = s(W^{(2)}_{1,1}h_1 + W^{(2)}_{1,2}h_2 + W^{(2)}_{1,3}h_3 + W^{(2)}_{1,0}b_2) \tag{5a}$$
$$y^{(2)}_2(x) = s(W^{(2)}_{2,1}h_1 + W^{(2)}_{2,2}h_2 + W^{(2)}_{2,3}h_3 + W^{(2)}_{2,0}b_2) \tag{5b}$$

c) The number of parameters needed to be inferred are (neurons and biases):

$$\boxed{(3*3) + (3*2) + 3 + 2 = 20} \tag{6}$$

d) The relationship between the sigmoid function and the hyperbolic tan is as follows. If we rewrite the tanh(x) as:

$$tanh(x) + 1 = \frac{e^x - e^{-x}}{e^x + e^{-x}} + \frac{e^x + e^{-x}}{e^x + e^{-x}} \tag{7a}$$
$$= \frac{2e^x}{e^x + e^{-x}} \tag{7b}$$
$$= \frac{2}{1 + e^{-2x}} \tag{7c}$$
$$tanh(x) + 1 = 2\sigma(2x) \tag{7d}$$

therefore we have:

$$\boxed{tahn(x) = 2\sigma(2x) - 1} \tag{8}$$

HW3_PCA

November 27, 2017

1 Hw 3 Principal Component Analysis

```
In [280]: import os
          from time import time
          import pandas as pd
          import numpy as np
          from skimage import io
          from matplotlib import pyplot as plt
          from skimage.transform import rescale
          import matplotlib.cm as cm
```

Lets define a function that can import and image as grey, and then rescale the image to downsize it by 3.0 times the orignal size. This will help us with our calculation of the covariant matrix to find the eigenvectors later.

```
In [230]: def import_image(filename):
              image = io.imread('yalefaces/' +filename, as_grey = True)
              image_resize = rescale(image,1.0/4.0)
              size = image_resize.shape
              image_flat = [item for sub in image_resize for item in sub]
              return image_flat, image_resize, size
```

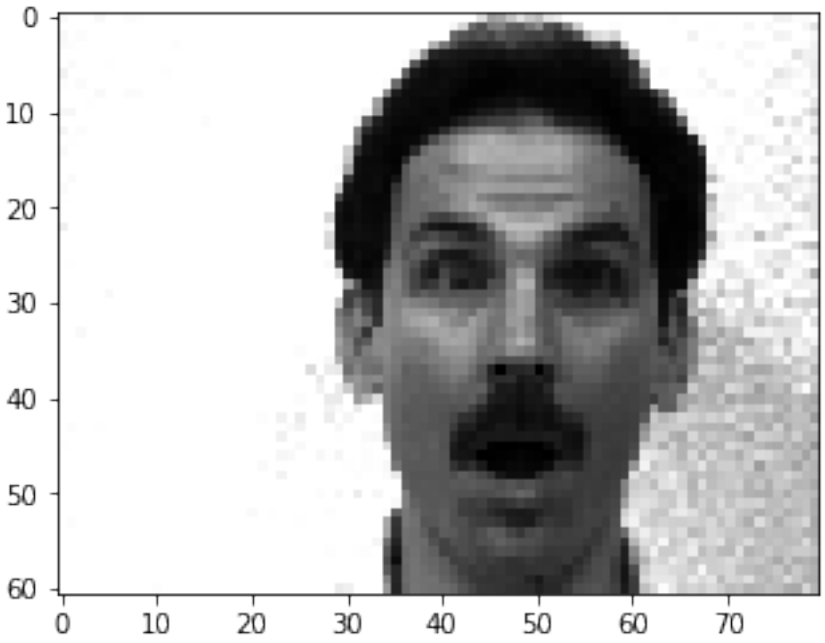
Import the images by using the list directory from the yaleface folder.

```
In [231]: # list_of_images = os.listdir('yalefaces')
          list_of_images = os.listdir('yalefaces')  #import the filenames as a list
          data = pd.DataFrame(list_of_images)
```

Create a Pandas Dataframe which contains the subject, the expression, the filename and the flattened image data.

```
In [232]: data = []
          for title in list_of_images:
              row = {}
              row['filename'] = title
              row['subject'] = title.split('.')[0]
              row['expression'] = title.split('.')[1]
              row['flat'], row['resized'], shape = import_image(title)
```

```
        data.append(row)
    data = pd.DataFrame.from_records(data)
    plt.imshow(data['resized'][2], cmap = cm.gray)
    plt.show()
    print "size of image: ",shape
```




```
size of image:  (61, 80)
```

Now we will normalize the Data by calculating the mean of the row and then divinding each value in the row by the mean.

```
In [233]: for row in range(data['flat'].count()):
              entries = len(data['flat'][0])
              numerator = 0
              for entry in data['flat'][row]:
                  numerator += entry
              mean = numerator/entries
              data['resized'][row] = data['resized'][row] / mean
              data['flat'][row] = data['flat'][row] / mean
```

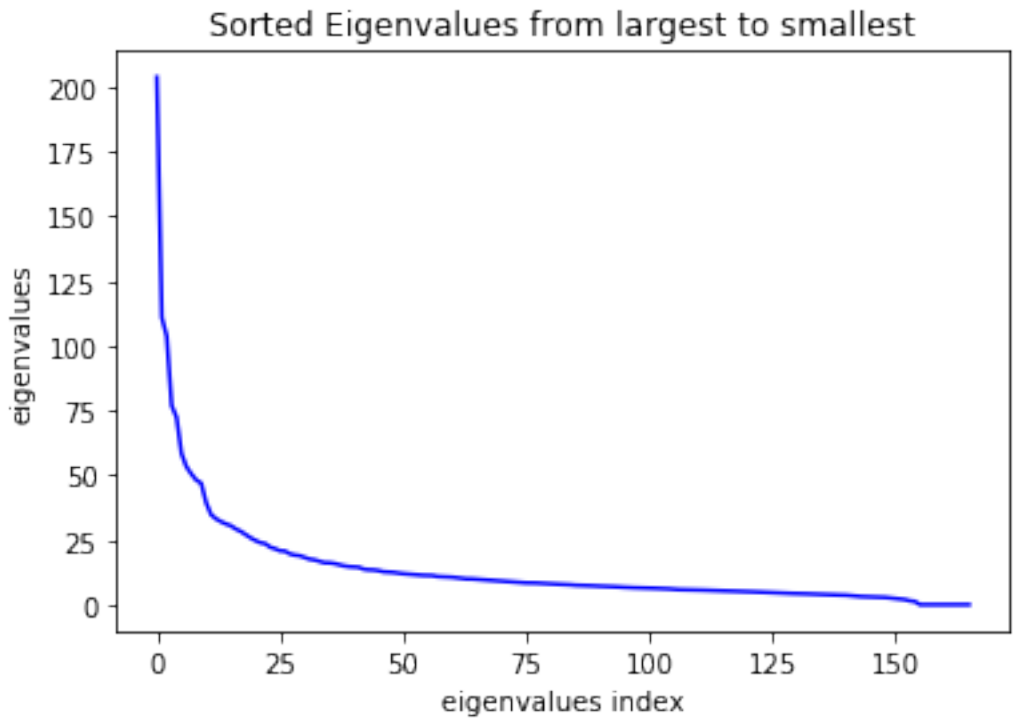
We now normalize the data by dividing the entries by the mean

1.0.1 Lets compute the covariance matrix. I used the slided from class which say

$$S = \frac{1}{N} X^T X$$

which gives us a dimensionality of: **{DxD}**

```
In [234]: X = []
          for row in range(data['flat'].count()):
              img_mean =  np.array([data['flat'][row]])
              img_mean =  img_mean[0]
              X.append(img_mean)
          X = np.array(X)
          S = np.dot(X.T,X)/len(X)

In [235]: eigenvalues,eigenvectors =  np.linalg.eig(S)  #lets find the eigenvalues

In [236]: idx = eigenvalues.argsort()[::-1] #large to small
          eigenvalues = eigenvalues[idx]   #orders the eignevalues
          eigenvectors = eigenvectors[:,idx]  #orders the eignevtors as columns

In [263]: print "We have obtained", len(eigenvalues), "eigenvalues"
          a = range(len(eigenvalues))
          plt.plot(range(len(S)), S, 'b')
          plt.xlabel("eigenvalues index")
          plt.ylabel("eigenvalues")
          plt.title("Sorted Eigenvalues from largest to smallest")
          plt.show()
          Uvector =  eigenvectors[:,0:10]   #take the first ten
          Ueigen = eigenvalues[0:10]     #take the corresponding eigenvectors

We have obtained 4880 eigenvalues
```



```
In [270]: half_energy =  sum(S)/2
          runningsum = 0
          k = 0
          for value in S:
              if runningsum < half_energy:
                  runningsum += value
                  k +=1
          print "the number of eigenvalues to reach half energy is: ", k
          print k, " <- are the number of eigenvalues needed to reach half energy o

the number of eigenvalues to reach half energy is:  22
22  <- are the number of eigenvalues needed to reach half energy of ->  1165.650770
```

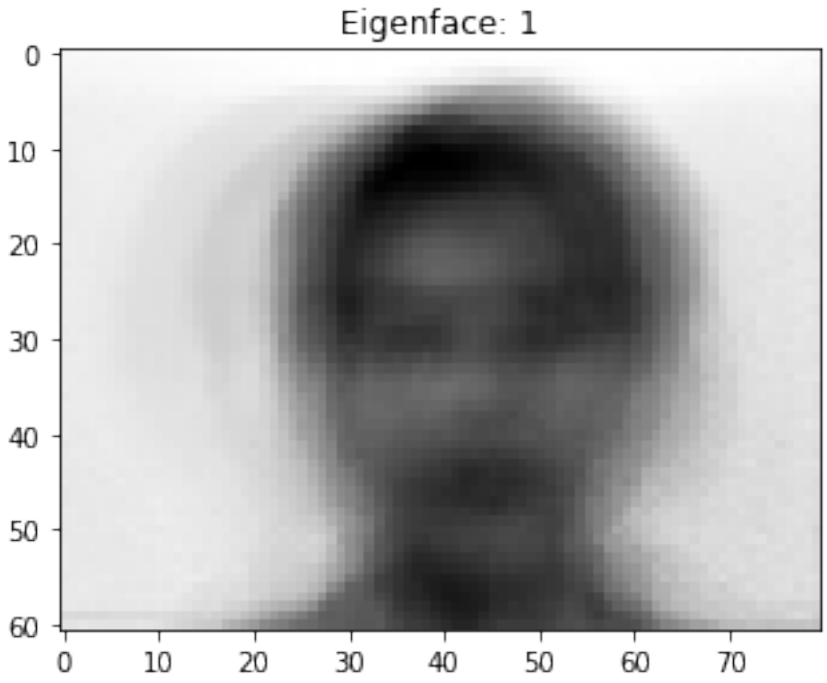
Then number of eigenvalues necessary to reach 50% of the energy of the system is above Now I will print out the dimensionalities of our matrices for clear computations

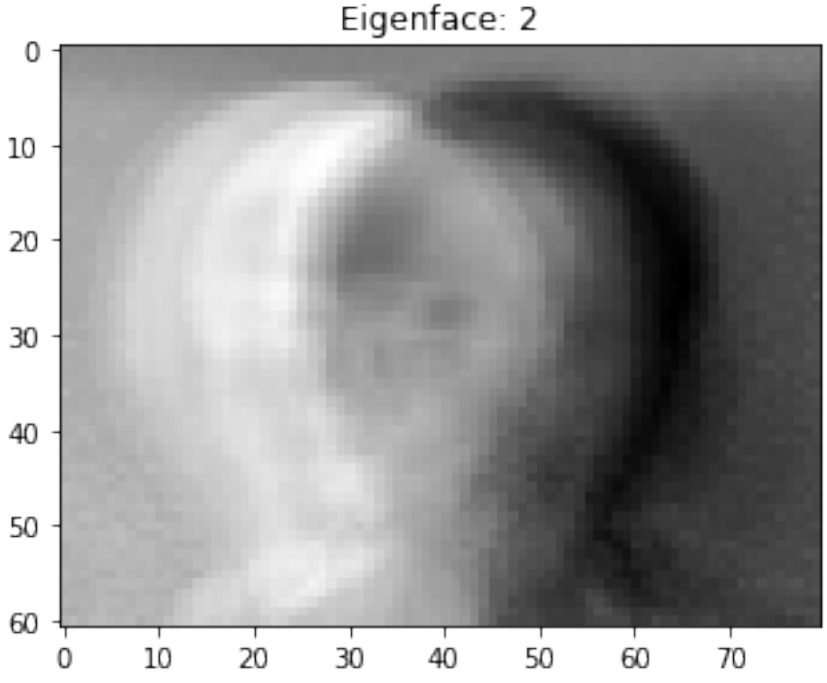
```
In [239]: print X.shape, "X original NxD"
          print S.shape,  "S that is DxD"
          print Uvector.shape , "U which is DxM"
          print Uvector.T.shape , "UT which is MxD"
          print np.dot(X,Uvector).shape, "X dot Ut which is NxM Dimensionality redu

(166, 4880) X original NxD
(4880, 4880) S that is DxD
(4880, 10) U which is DxM
(10, 4880) UT which is MxD
(166, 10) X dot Ut which is NxM Dimensionality reduced!
```

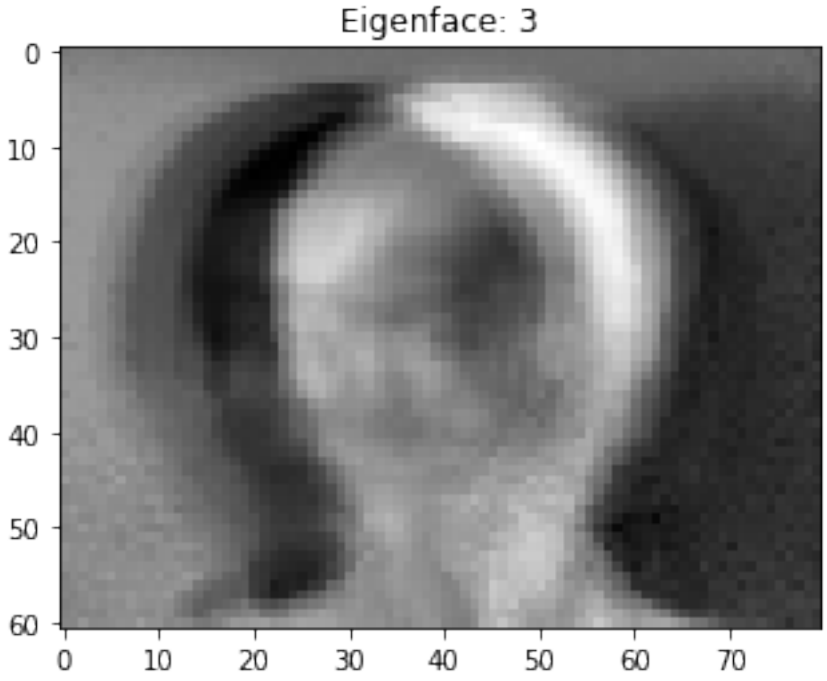
2 Top 10 Eigenfaces

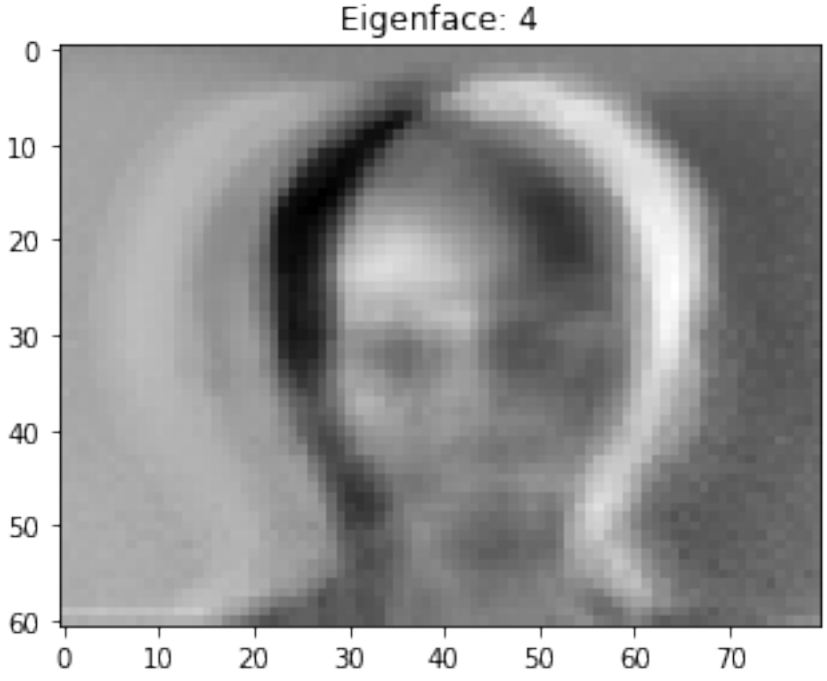
2.0.1 From the computed eignevectors, we can extract the eigenfaces.

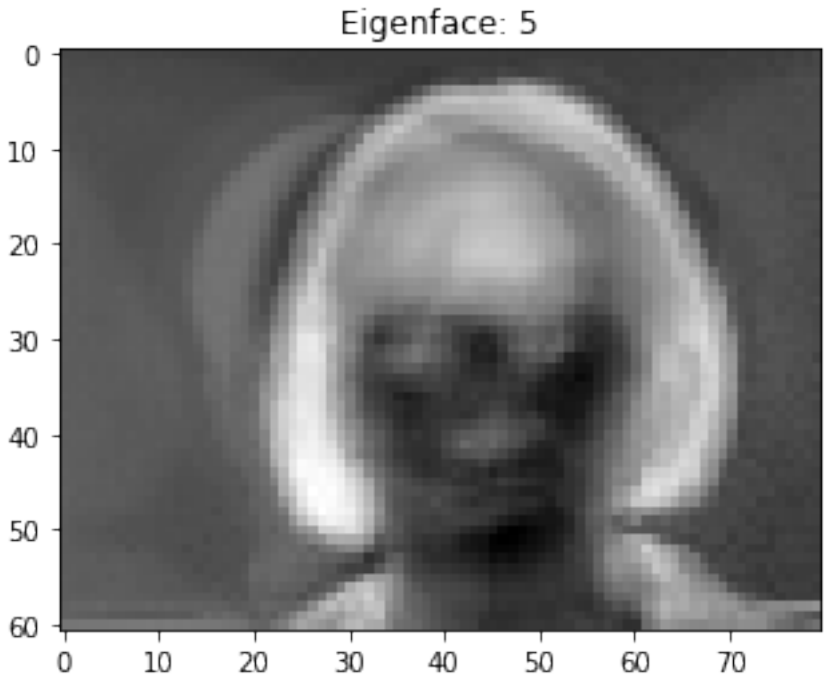
```
In [244]: for i in range(10):
              UT =  Uvector.T[i]
              temp = UT.real
              temp = temp.reshape( shape )
              plt.imshow(temp, cmap = cm.gray)
              plt.title("Eigenface: "+str(i+1))
              plt.show()
```

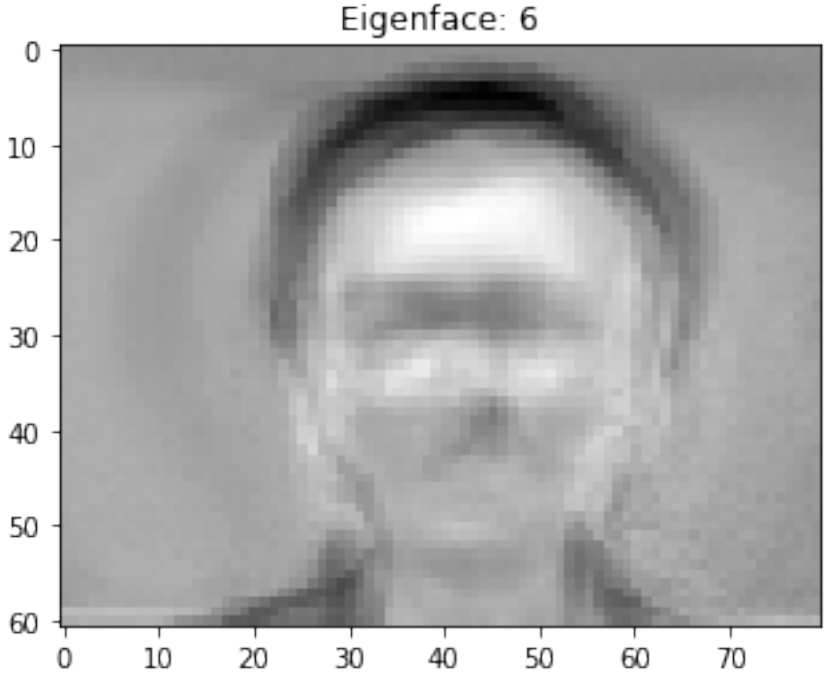

Eigenface: 1
0
10
20
30
40
50
60
0
10
20
30
40
50
60
70

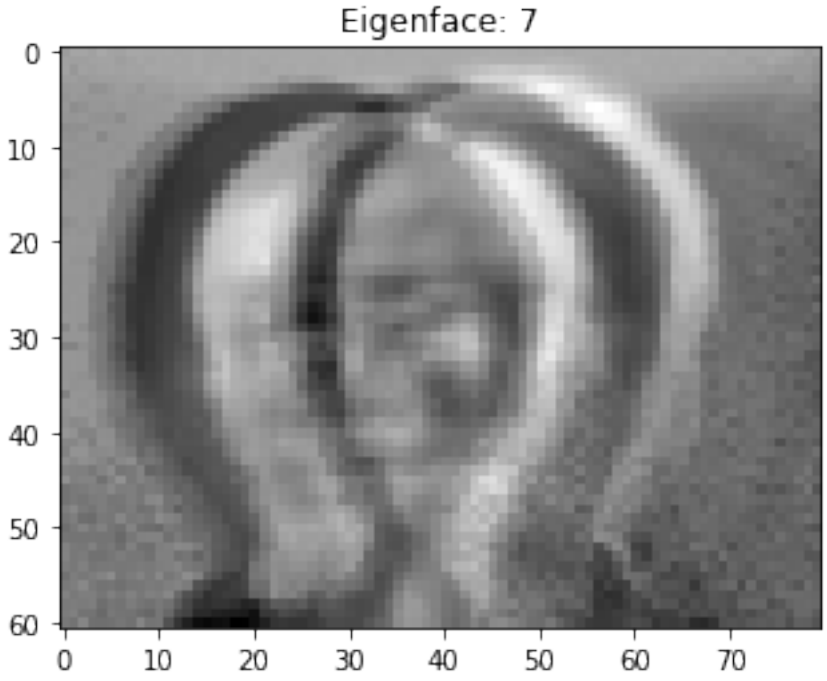

Eigenface: 2
0
10
20
30
40
50
60
0
10
20
30
40
50
60
70

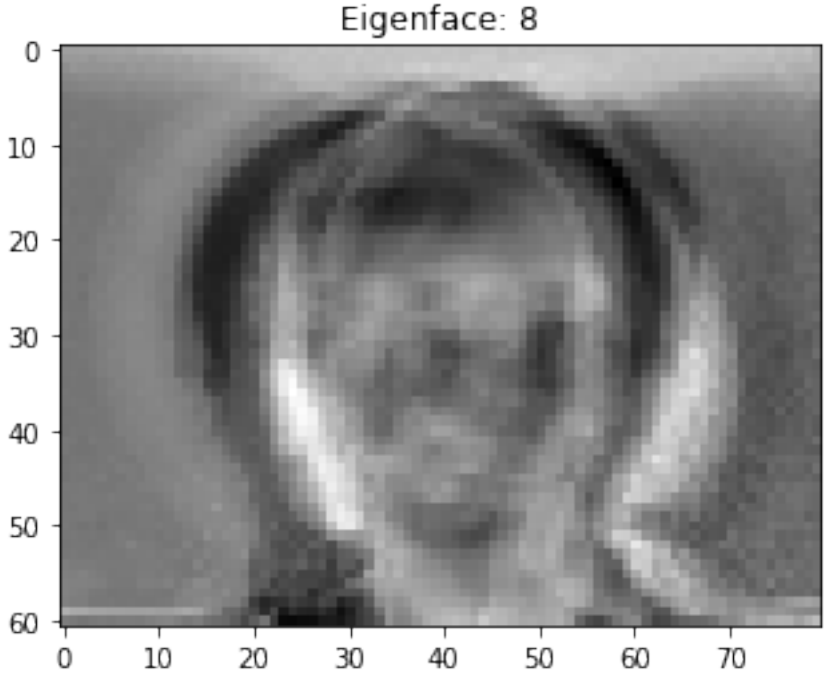

Eigenface: 3
0
10
20
30
40
50
60
0
10
20
30
40
50
60
70

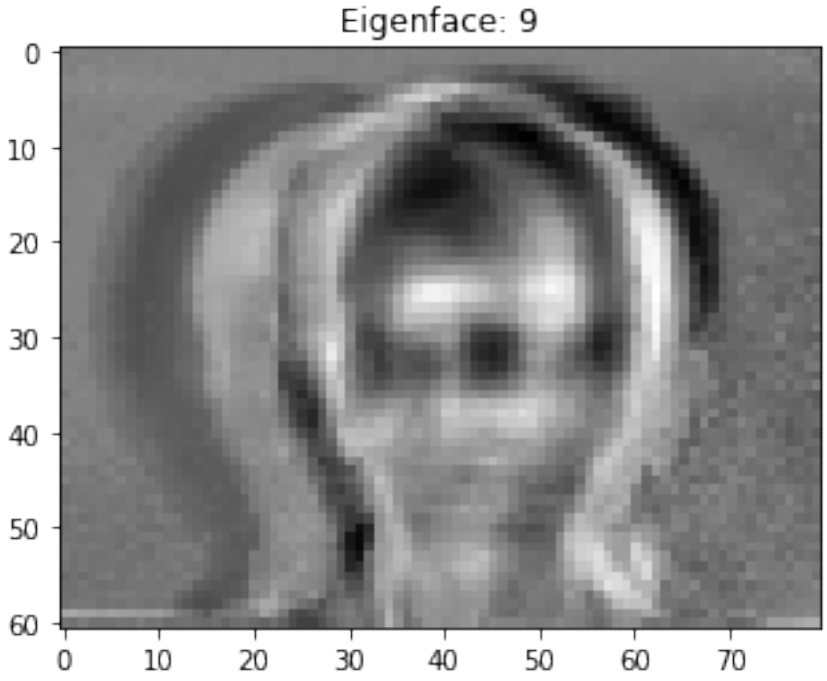

Eigenface: 4
0
10
20
30
40
50
60
0
10
20
30
40
50
60
70

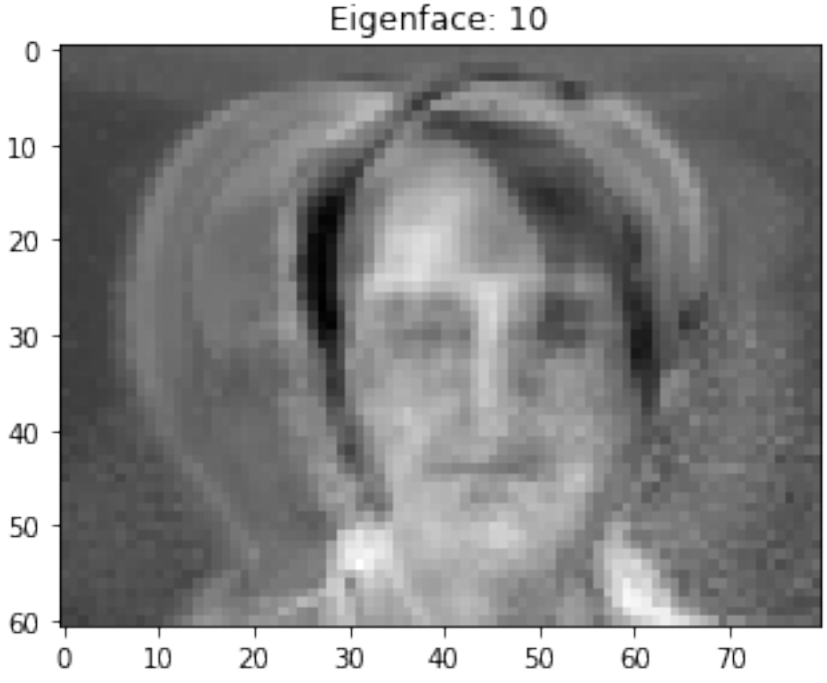

Eigenface: 5
0
10
20
30
40
50
60
0
10
20
30
40
50
60
70


Eigenface: 6
0
10
20
30
40
50
60
0
10
20
30
40
50
60
70


Eigenface: 7
0
10
20
30
40
50
60
0
10
20
30
40
50
60
70


Eigenface: 8
0
10
20
30
40
50
60
0
10
20
30
40
50
60
70


Eigenface: 9
0
10
20
30
40
50
60
0
10
20
30
40
50
60
70


Eigenface: 10
0
10
20
30
40
50
60
0
10
20
30
40
50
60
70

2.0.2 Reconstructing different images from the eigenfaces.

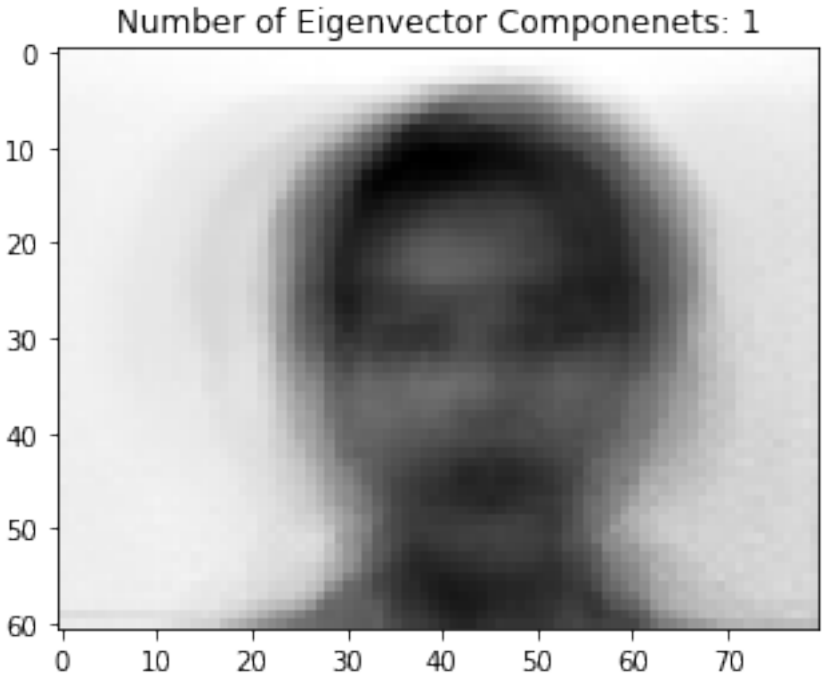
In this section, I will try to reconstruct the different input images by using different number of eigenfaces. I will use the following equation:

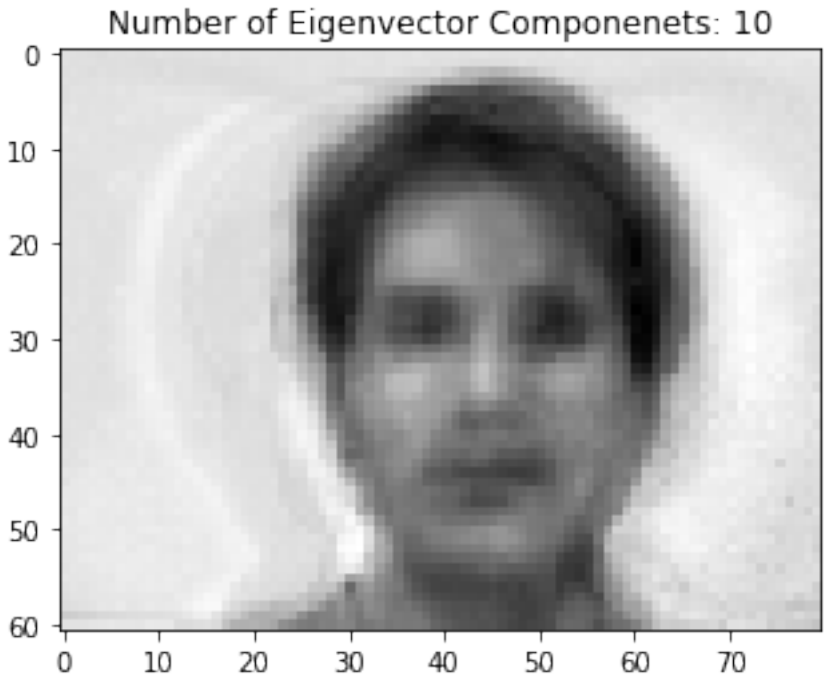
$$\mathbf{x} \approx \mathbf{x0} + \sum_{k=1}^{K} \boldsymbol{c_k u_k}$$

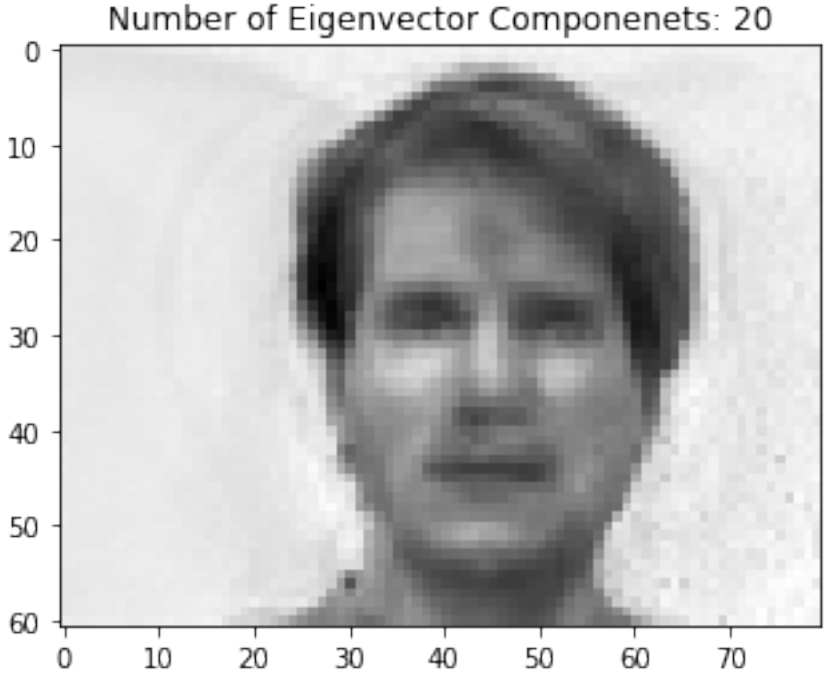
where

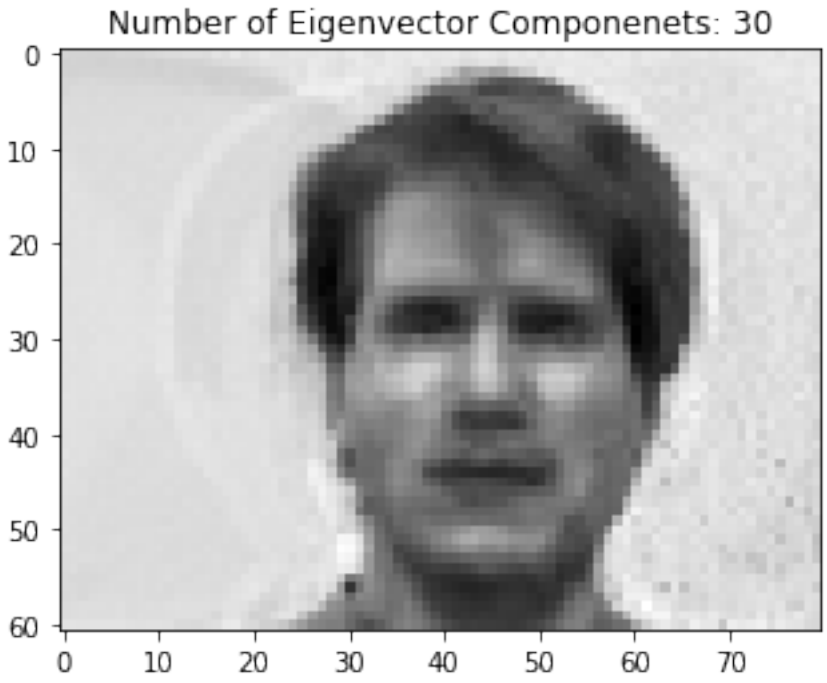
$$c_k = \boldsymbol{u_k^T x}$$

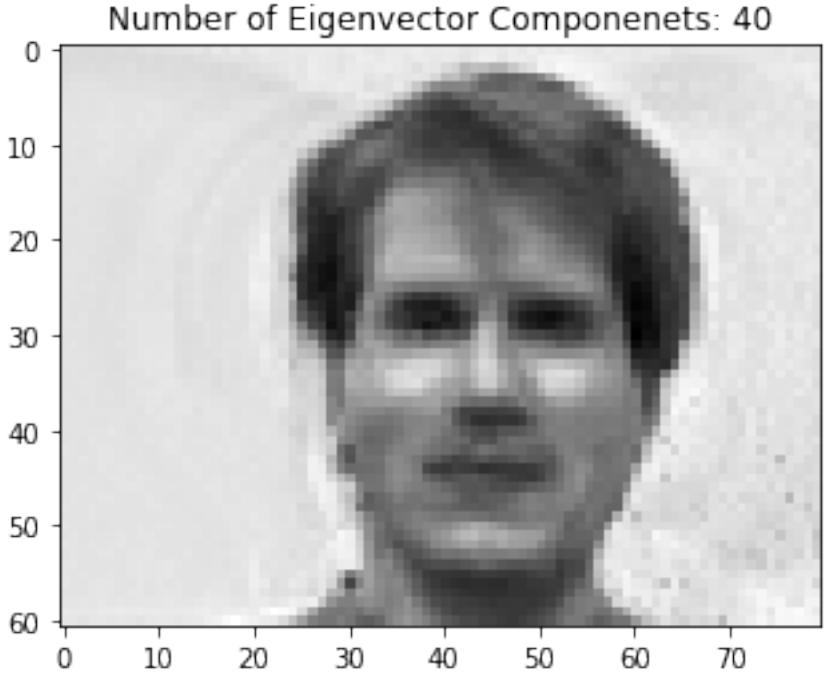
```
In [241]: mu = np.mean(X,0)    #find the mean face of our input images
          ma_data = X - mu     #mean adjust the data
          U, S, V = np.linalg.svd(ma_data.transpose(), full_matrices=False)  #use S
          weights = np.dot(ma_data, U)  #our U matrix is the eigenvectors, now we c

In [297]: k = [1,10,20,30,40,50]  #number of eigenvectors used to reconstruct the i
          for eigen in k:
              recon = mu + np.dot(weights[6, 0:eigen], U[:, 0:eigen].T)
              plt.imshow(recon.reshape(shape), cmap = cm.gray)
              plt.title("Number of Eigenvector Componenets: "+str(eigen))
              plt.show()
```

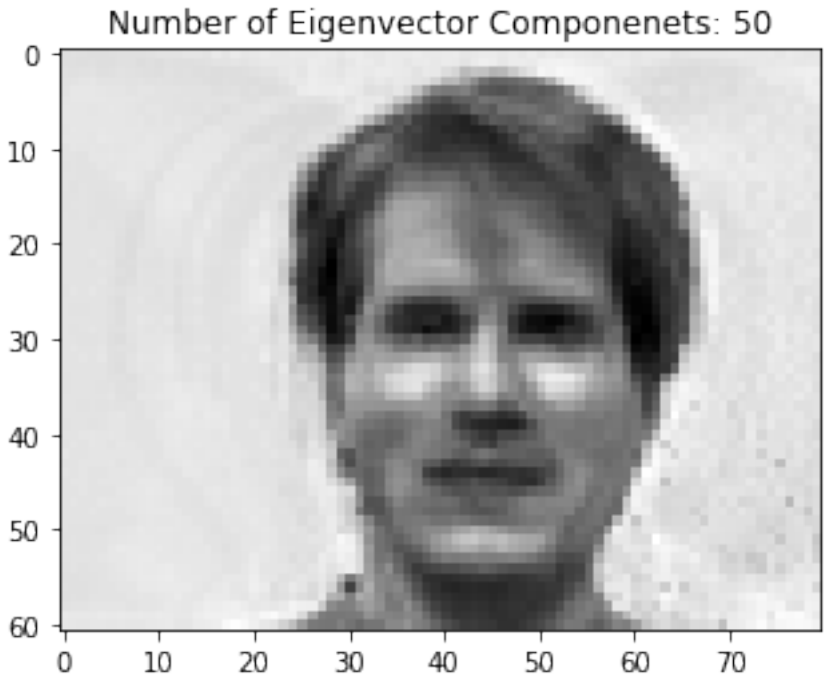



Number of Eigenvector Componenets: 10
0
10
20
30
40
50
60
0
10
20
30
40
50
60
70


Number of Eigenvector Componenets: 20
0
10
20
30
40
50
60
0
10
20
30
40
50
60
70


Number of Eigenvector Componenets: 30
0
10
20
30
40
50
60
0
10
20
30
40
50
60
70


Number of Eigenvector Componenets: 40
0
10
20
30
40
50
60
0
10
20
30
40
50
60
70



As we can see, It is usually best to use more than 1 eigenvector to reconstruct our image. In my case, since the resize function compresses our data by a significant ammount, by the **50th eigenvector, we have reconstructed a very nice looking image compared to our original resized input.** More eigenfaces will be a better reconstruction of our image!

2.1 Face Recognition

I will try to identify the subject based on the recation. By splitting the data into training and testing I can then train a Support Vector Machine to identify the correct subject number from the training set images.

```
In [277]: from sklearn.model_selection import train_test_split

          train, test = train_test_split(data, test_size=0.33)

In [377]: y_train, y_test = [], []
          for ids in train['subject']:
              y_train.append(ids[7:9])
          for ids in test['subject']:
              y_test.append(ids[7:9])

In [416]: from sklearn import svm
          t0 = time()
          A = train['flat'].values.tolist()
          B = test['flat'].values.tolist()
          clf = svm.SVC(kernel="linear",C=100000)
```

```
          clf.fit(A,y_train)
          print ("Training Time:"),(round(time()-t0,3)),("s")

Training Time: 0.105 s


In [419]: tp = time()
          pred = clf.predict(B)

          from sklearn.metrics import accuracy_score
          count = 0
          for i in range(len(pred)):
              if pred[i] == y_test[i]:
                  count += 1

          print ("Time Predict:"),(round(time()-tp,3)),("s")
          print("Accuracy: "), (accuracy_score(pred, y_test))*100,"%"
          print ("final count: "),(count)

Time Predict: 0.03 s
Accuracy:  90.9090909091 %
final count:  50
```

By using a **Linear Kernel** with a **C = 100,000** . This gives an Accuracy of classifying correctly:

2.2 Accuracy: 90.9%!