Hammad Qureshi <u>qureshi434@csu.fullerton.edu</u> Brian Tan <u>brian388@csu.fullerton.edu</u>

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Project 1

Pseudocode for Alternate Algorithm

```
sort alternate(before)
after = before
                        // 1tu
swap_count = 0
                        //1tu
run_count = 0
                        //1tu
if after.issorted() == true then // 8n + 16 tu
        return sequence unchanged
else
                                               // \frac{5n}{2} + 7 \text{ tu}
        if after.is_initialized() == true then
               for i = 0 to n-1 do
                                               //outer for loop
                                                                       //2 tu
                       if run_count % 2 != 0 then
                               for j = 0 to 2n-1, step 2, do
                                                                      //inner loop 1
                                       if color[j] != color[j+1] then //2tu
                                               swap()
                                                                       //5tu
                                                                      //1tu
                                               swap count++
                                       endif
                               endfor
                       else
                               for j = 1 to 2n-2, step 2, do
                                                                       //inner loop 2
                                       if color[j] != color[j+1] then //2tu
                                                                      //5tu
                                               swap()
                                                                      //1tu
                                               swap_count++
                                       endif
                               endfor
                       endif
                                                                      //1tu
                       run_count++
               endfor
        else
               return sequence unchanged
        endif
endif
// is_initialized() function step count calculated to be \frac{5n}{2} + 7 tu
// is_sorted() function step count calculated to be 8n + 16 tu
//swap() function determined to be 5tu
```

Proof and Time Complexity for Alternate Algorithm

(Independent for loops)

outer for loop:

using
$$\frac{end-start}{step} + 1$$
:

$$\frac{n-1-0}{1}$$
 + 1 = n

inner for loop 1:

$$\frac{2n-1}{2}$$
 + 1 = n + $\frac{1}{2}$

multiply with the if statement time units: $8(n + \frac{1}{2}) = 8n + 4$ //2tu + 5tu + 1tu = 8tu

inner for loop 2:

$$\frac{2n-2-1}{2} + 1 = n - \frac{1}{2}$$

multiply with the if statement time units: 8($n - \frac{1}{2}$) = 8n - 4

Combined:

outer for loop * (2 + max(inner loop 1, inner loop 2) + 1) //+1 because of the run_count increment

$$= n * (2 + (8n + 4) + 1)$$

$$= n * (8n + 7)$$

$$= 8n^2 + 7n$$

Add the time units outside the outer loop:

$$8n^2 + 7n + 3 + \frac{5n}{2} + 7 + 8n + 16$$

$$=8n^2+\frac{35n}{2}+26$$

highest power is 2. Therefore, algorithm efficiency class is O(n²).

Hammad Qureshi <u>qureshi434@csu.fullerton.edu</u> Brian Tan <u>brian388@csu.fullerton.edu</u>

Proof by definition

Prove
$$8n^2 + \frac{35n}{2} + 26 \in O(n^2)$$

choose C > 0 and a value $n_0 >= 0$ such that:

$$8n^2 + \frac{35n}{2} + 26 \le C * n^2 \forall n \ge n_0$$

choose C =
$$8 + \frac{35}{2} + 26 \cong 52$$

$$8n^2 + \frac{35n}{2} + 26 \le 52n^2$$
, true for n >= 1

choose
$$n_0 = 1$$

$$\therefore 8n^2 + \frac{35}{2} + 26 \in O(n^2)$$

Pseudocode for Lawnmower Algorithm

```
sort lawnmower(before)
                              //1tu
after = before
                              //1tu
swap count = 0
temp_n = total_n - 1
                              //2tu
if after.issorted() == true then
                                     //8n + 16 tu
       return sequence unchanged
else
                                                                           //\frac{5n}{2} + 7 tu
       if after.is_initialized() == true then
               for i = 0 to n-1 do
                                                                           //outer loop
                                                                           //inner loop 1
                      for j = i to 2n-1, step 2, do
                              if color[j] != color[j+1] then
                                                                           //2tu
                                     swap()
                                                                           //5tu
                                     swap_count++
                                                                           //1tu
                              endif
                      endfor
                                                                           //1tu
                      temp_n--
                                                                           //1tu
                      if temp n < i then
                              for k = 2n-1 down to i, step 2, do
                                                                           //inner loop 2
                                     if color[k] != color[k-1] then
                                                                           //2tu
                                             swap()
                                                                           //5tu
                                                                           //1tu
                                             swap_count++
                                     endif
                              endfor
                      endif
               endfor
       else
               return sequence unchanged
       endif
endif
//swap() function determined to be 5tu
```

Proof and Time Complexity for Lawnmower Algorithm

Proof Time Complexity

(Dependent for loops)

outer for loop:

part of this loop has 1 decrement, and 1 if statement. So multiply by 2tu

$$\sum_{i=0}^{n-1} 2^{i}$$

inner loop 1:

1 if statement with 2tu operations, and carries out 5tu function and 1tu increment. So, 2+5+1=8tu

since step of 2, use $\frac{end-star}{step}$ + 1.

8 (
$$\frac{2n-1-i}{2}$$
 + 1) = 8 (n + $\frac{1-i}{2}$)

inner loop 2:

1 if statement with 2tu operations, and carries out 5tu function and 1tu increment. So, 2+5+1= 8tu

since step of -2 use $\frac{end-start}{step}$ + 1.

8 (
$$\frac{i-2n-1}{-2}$$
 + 1) = 8 (n + $\frac{3-i}{2}$)

Combined:

(outer for loop) * (inner loop 1 + inner loop 2)

$$\sum_{i=0}^{n-1} 2 \left[8 \left(n + \frac{1-i}{2} \right) + 8 \left(n + \frac{3-i}{2} \right) \right]$$

=
$$\sum_{i=0}^{n-1} 2 * 8[(n + \frac{1-i}{2}) + (n + \frac{3-i}{2})]$$

$$=\sum_{i=0}^{n-1} 16 (2n+2-i) = \sum_{i=0}^{n-1} 16 (2n-i+2)$$

$$=\sum_{i=0}^{n-1}(32n-16i+32)$$

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$$= \sum_{i=0}^{n-1} 16n - \sum_{i=0}^{n-1} 16i + \sum_{i=0}^{n-1} 32 \qquad // \sum_{i=0}^{n-1} 16i = 16 * 0 + \sum_{i=1}^{n-1} 16i = 16 \left(\frac{n-1(n-1+1)}{2}\right)$$

= 16n(n-1 + 1) - 16 (
$$\frac{n-1(n-1+1)}{2}$$
) + 32(n-1 + 1)

$$= 16n^2 - 8n^2 - 8n + 32n$$

$$=8n^2-8n+32$$

Add time units outside of outer loop:

$$8n^2 - 8n + 32 + 4 + \frac{5n}{2} + 7 + 8n + 16$$

$$=8n^2+\frac{5n}{2}+59$$

highest power is 2. Therefore, algorithm efficiency class is $O(n^2)$.

Proof by definition

Prove
$$8n^2 + \frac{5n}{2} + 59 \in O(n^2)$$

choose C > 0 and a value $n_0 >= 0$ such that:

$$8n^2 + \frac{5n}{2} + 59 \le C * n^2 \forall n \ge n_0$$

choose C =
$$8 + \frac{5}{2} + 59 \cong 70$$

$$8n^2 + \frac{5n}{2} + 59 \le 70n^2$$
, true for $n \ge 1$

choose
$$n_0 = 1$$

$$\therefore 8n^2 + \frac{5n}{2} + 59 \in O(n^2)$$