Learning Dynamics: Assignment 2
Evolutionary dynamics in a spatial context

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Contents

1 Part I

Specifications Plots in Figure 1 shows the average cooperation level of 100 simulations with unconditional imitation as the update mechanism. The game played si the weak prisoner's dilemma. The first rounds were played randomly, where a player would choose cooperate with a probabilty of $\frac{1}{2}$.

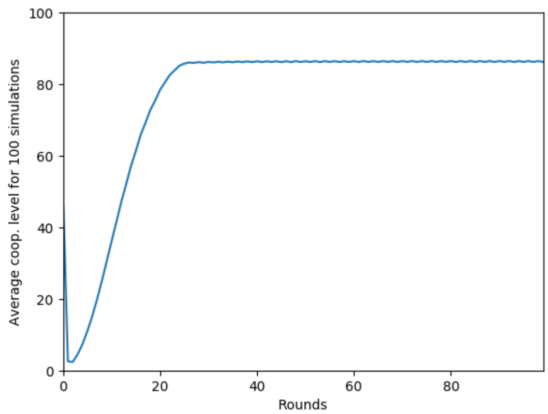
1.1 Neighborhood analysis

Remark The analysis is based on results from 50x50 lattice simulations.

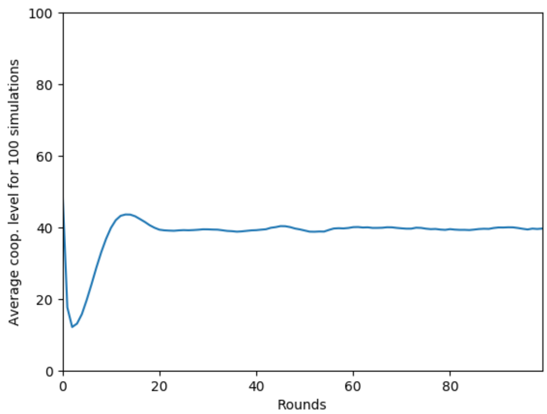
Moore Figure 1a shows the average cooperation level using a Moore neighborhood for each player. We can observe that the level after the first randomly played round, the cooperation dropped at around 2%. Then grows to stabilize at around 87%.

Von Neumann Figure 1b shows the average cooperation level using a Von Neumann neighborhood for each player. We can observe that the level after the first randomly played round, the cooperation dropped at around 15%. Then grows to stabilize at around 40%.

We can see that the cooperation level follows the same pattern but on a different scale. With Moore we have more neighbors, which can explain why the behaviour of the players are more *extreme*.



(a) Moore neighborhood

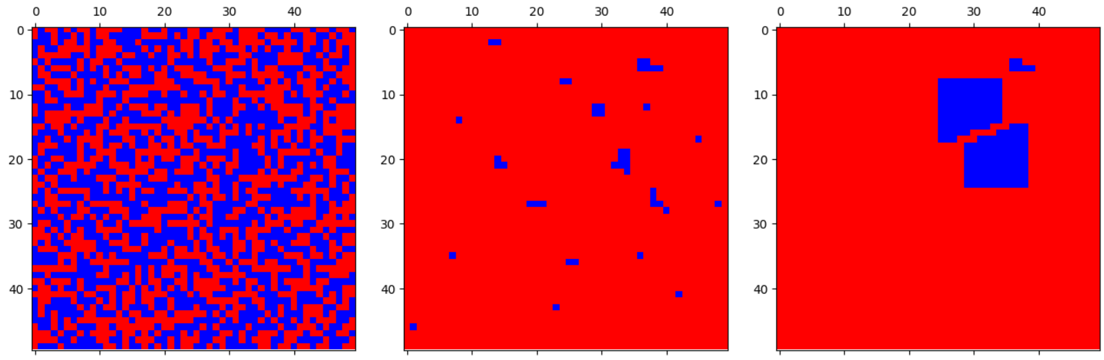


(b) Von Neumann neigborhood

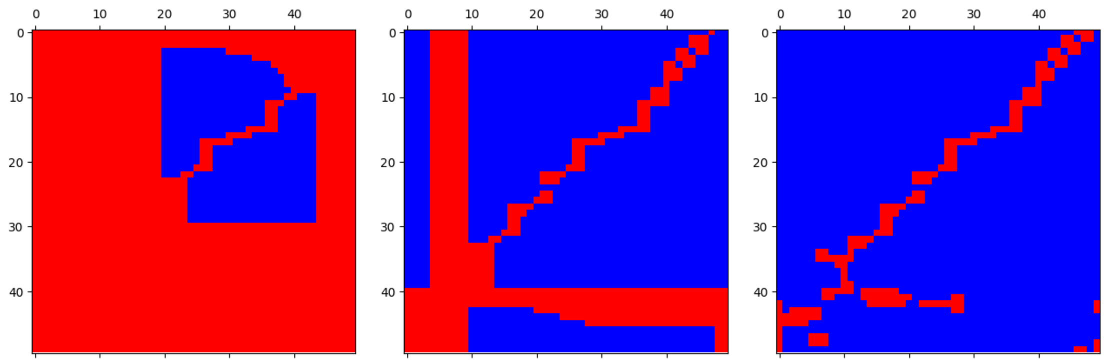
Figure 1: Cooperation level using unconditional imitation and weak prisoner's dilemma on a 50x50 lattice.

1.2 Lattice observation

Figure 2 shows the full matrix of cooperation for the rounds $t_0, t_1, t_5, t_{10}, t_{20}, t_{50}$. We can observe that in the first round there is more or less the same number of players cooperating and defecting. But in the second round, a large percentage of players will choose to defect, leaving only smalls zone of cooperation. This is due to the fact that a defecting player will usually have a better score around a mixed neighborhood of player than a cooperating player. But when a co-operating player has a cooperating neighborhood he will have high enough score to influence defecting players in his neighborhood. We can observe that in the following rounds, a sort of *cluster* of cooperation will be formed and influence the full lattice until reaching the cooperation level explained in section 1.1.



(a) t_0 (b) t_1 (c) t_5



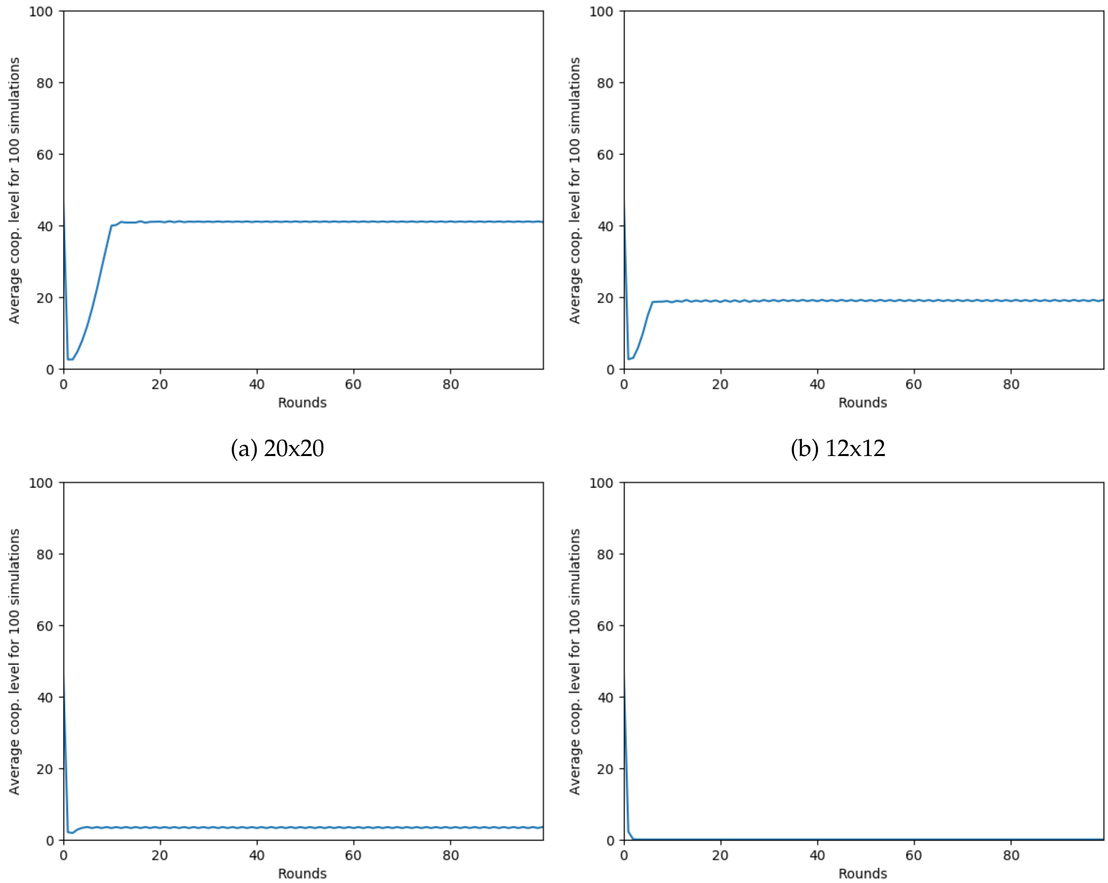
(d) t_{10} (e) t_{20} (f) t_{50}

Figure 2: Visualization of a the lattice with unconditional imitation, Moore neighborhood and weak prisoner's, dilemma.

1.3 Lattice size analysis

Figure 3 shows the average cooperation level of lattices of size 20, 12, 8 and 4. The behavior seems to the same as in the analysis made against lattice of size 50 in section 1.1, it crashes to a small level of cooperation to grows and stabilize after a number of rounds. The plots show that when the lattice size is small, the cooperation stabilize to a smaller cooperation level than bigger lattices. We can observe that for a size 4 lattice, the cooperation level is even 0 after the first round.

The reason could be that there is less players so less possibilities to form some cooperation neighborhood, that will form the *clusters* and grews as explained in previous section.



(a) 20x20

(b) 12x12

(c) 8x8

(d) 4x4

Figure 3: Cooperation level using unconditional imitation, Moore neighborhood and weak prisoner's dilemma.

2 Part II

2.1 Update mechanism

$$P_{ij} = (1 + [W_j - W_i]/[N \cdot (max\{P, R, T, S\} - min\{P, R, T, S\}])/2 \tag{1}$$

Intuitive observation This probabilty is interesting to be used as an update mechanism because the probability to change the action to the neighbor action is proportional to the difference between the players payoffs.

Probability variables $[W_j - W_i]$ is the difference between the two payoffs. N represent the number of neighbor that the payoff calculation are based on. The difference between the maximum and minimum multiply by N is the maximum payoff of a player. Since the payoffs cannot be bigger than the maximum score, it is clear that P_{ij} is a probability.

Analysis We can highlight different results from the fration $[W_j - W_i]/[N \cdot (max\{P, R, T, S\} - min\{P, R, T, S\}]$ between the difference of payoffs and the maximum score:

1. Fraction is positive when $W_j > W_i$. A special case is when W_j is maximum and W_i is null, the fraction is equal to 1.

2. Fraction is negative when $W_i > W_j$. A special case is when W_i is maximum and W_j is null, the fraction is equal to -1.

3. Fraction is equal to 0 when $W_i = W_j$

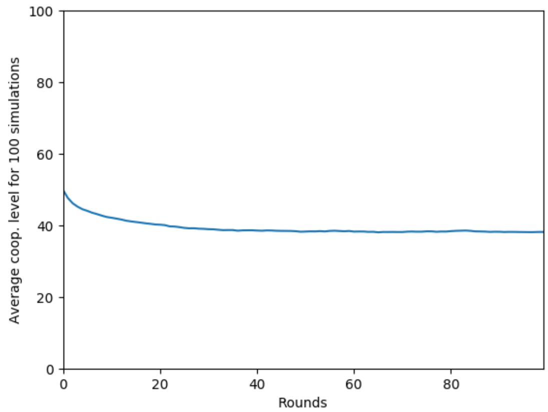
By using the full definition of the probablity we can see that when in case (1), it is more probable that the player will change is action to the neighbor action, and sure if the fraction is equal to 1 because $P_{ij} = (1 + 1)/2 = 1$. When in (2), it is more probable that the player will keep is action, and sure that he will not change it when the fraction is equal to -1 because $P_{ij} = (1 - 1)/2 = 0$. When in (3), $P_{ij} = (1 - 0)/2 = \frac{1}{2}$, the payoff of the player and his neighbor are the same, which means that both of their actions lead to the same payoff, so the probability to change or to keep is the same.

2.2 Neighborhood analysis

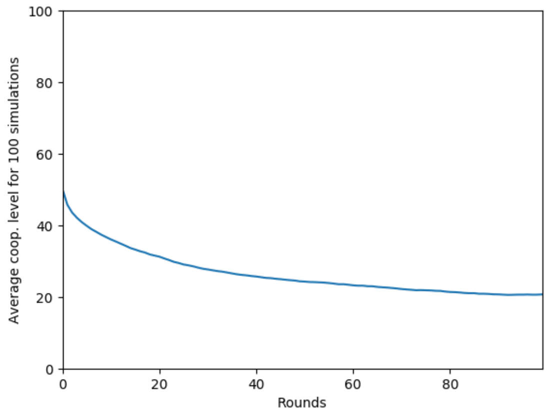
Specifications Plots in Figure 4 shows the average cooperation level of 100 simulations with replicator rule as the update mechanism. The game played is the snowdrift game. The first rounds were played randomly, where a player would choose cooperate with a probabilty of $\frac{1}{2}$. The analysis is based on results from 50x50 lattice simulations.

Moore Figure 4a shows the average cooperation level using the specifications above and a Moore neighborhood system. In those simulations we can see that the cooperation level drops at each round, but with a smaller factor over time. Here we see that around round 50, the cooperation level is *stable* at around 40%.

Von Neumann Figure 4b shows the average cooperation level using the specifications above and a Von Neumann neighborhood system. Even with less neighbors per player, the pattern seems to be the same than a Moore neighborhood over time. It drops to be *stable*, but with a far less cooperation level, around 20%. The difference between the Moore neighborhood is that the drop factor, the lost of cooperation level over time, is bigger for a Von Neumann neighborhood, and it takes more time to have a more *stabilized* cooperation level.



(a) Moore neighborhood



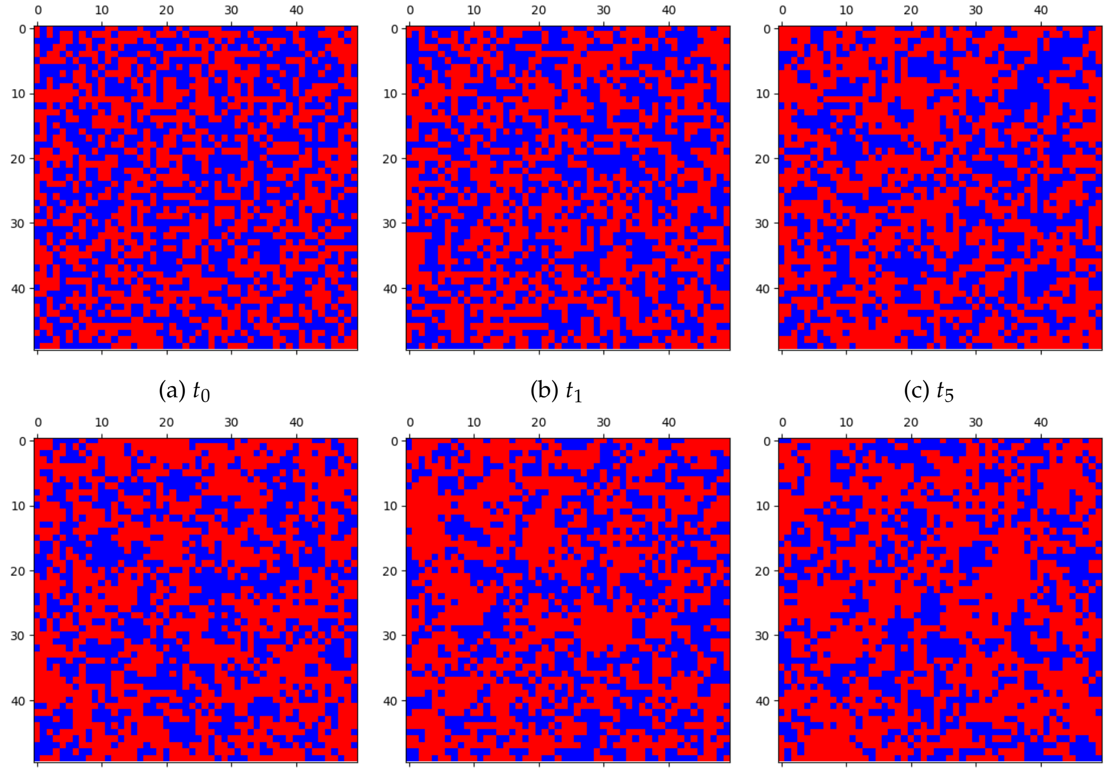
(b) Von Neumann neigborhood

Figure 4: Cooperation level using replicator rule and snowdrift game on a 50x50 lattice.

Comparaison with Part I In comparaison to Part I specifications and simulations, we see that here the players seems to be less *influenced* by their neighborhood. This is due to the update mechanism. First the decision of the player to change his action is base only on a random player action from his neighborhood. The replicator rule provides also to the players a way to respond to their neighborhood in a more *intelligent* manner. Indeed, when using the unconditional imitation, we saw that after the first round the reactions of the players is *extreme*, a big majority of the players, change their actions to defect, because in a equally distributed population, defect usually have a better score. With the replicator rule each player, based on his probability, will likely change his move only if it is *probably* better than his previous one.

2.3 Lattice observavion

Figure 5 shows the evolution of the lattice over time. In opposite the the lattice in Part I, her we see that the player are less influenced by their neighborhood. Indeed, there is no form of *zone of cooperation*, that takes the advantage overtime. We can see that overtime players tend to prefer to defect.



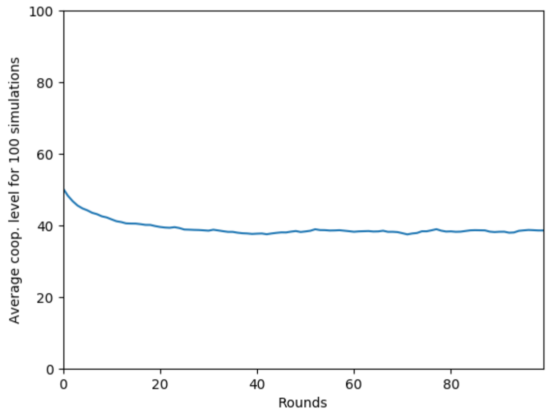
(a) t_0 (b) t_1 (c) t_5

(d) t_{10} (e) t_{20} (f) t_{50}

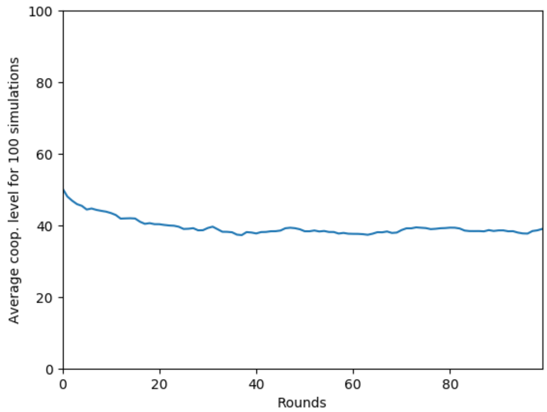
Figure 5: Visualization of a the lattice with replicator rule, Moore neighborhood and snowdrift game.

2.4 Lattice size analysis

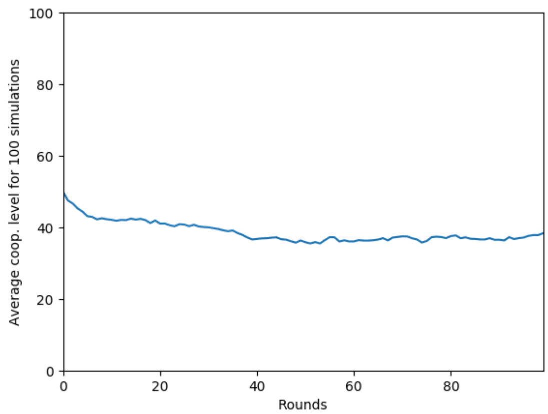
Figure 6 shows the cooperation level for matrix of different size. The pattern here is the same for the lattice 50x50. The cooperation level drops over time to be *steady* at around 40%. Except for the lattice of size 4 where the cooperation level drops more and it is smaller than the other size. We can suppose that here when the lattice is too small, and the probability of cooperate in a smaller lattice is smaller than with bigger lattice. In conclusion, we can see that for this specifications, size does not matter, up to a point where the lattice is too small, in opposite of specifications of Part I, where the cooperation level is different with lattice of different size.



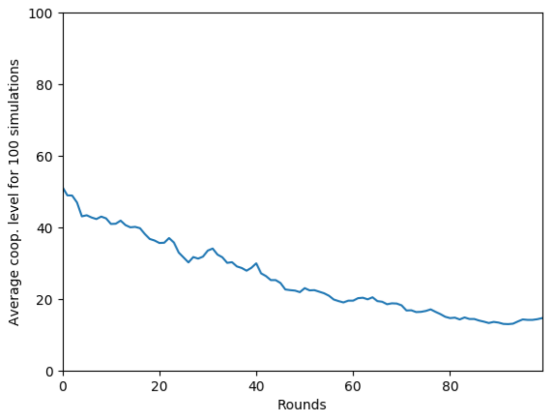
(a) 20x20



(b) 12x12



(c) 8x8



(d) 4x4

Figure 6: Cooperation level using replicator rule, Moore neighborhood and snowdrift game

3 Part III

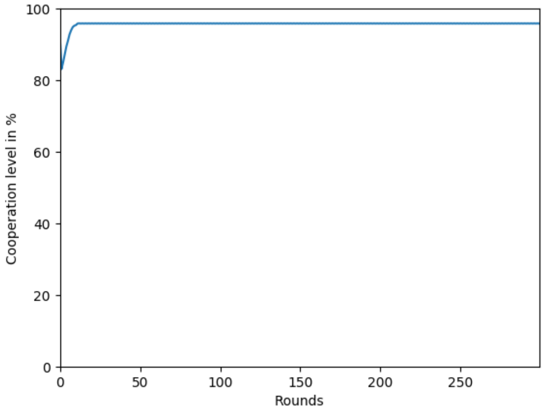
3.1 Specifications

In this part we will discuss the behaviour of the same mechanisms used in sections above but by changing the start round population. The idea is the following: a zone in the center that is equally distributed between cooperation and defection, i.e. a random start round for the players in the center, and this population be surrounded by full cooperation or full defection players. We will ask two questions: How the cooperation level changes over time, and is the final results identical as in Part I and Part II.

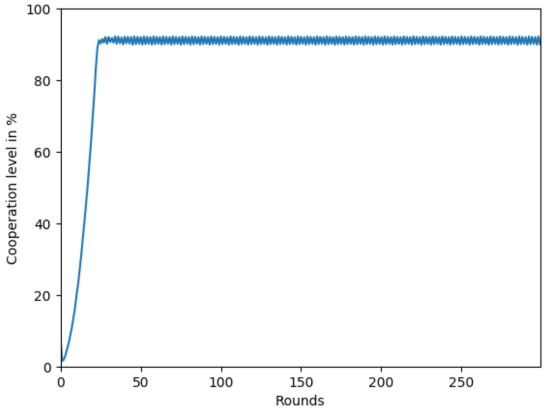
3.2 Results

Remark The simulations were run on a 50x50 lattice, with a 10x10 *center zone* and 300 rounds played.

3.2.1 Part I mechanisms



(a) Cooperation surrounding



(b) Defection surrounding

Figure 7: Cooperation level using unconditional imitation and weak prisoner's dilemma on a 50x50 lattice and 10x10 center zone.

Cooperation surrounding Figure 7a shows the cooperation level when using the same configuration as Part I with a Moore neighborhood, with the center zone surrounded by cooperation players. Figure 8 shows the lattice of this simulation over time. We can see that in this simulation the surrounded players are not affected by the center zone (except for the closest players). Indeed the behaviour is as explained in Part I, the cooperation level drops to a very low rate, and form a cluster that will take advantage over time and *stabilize* around 95%. This percentage is higher than in Part I because the cooperation level changes only in the center zone.

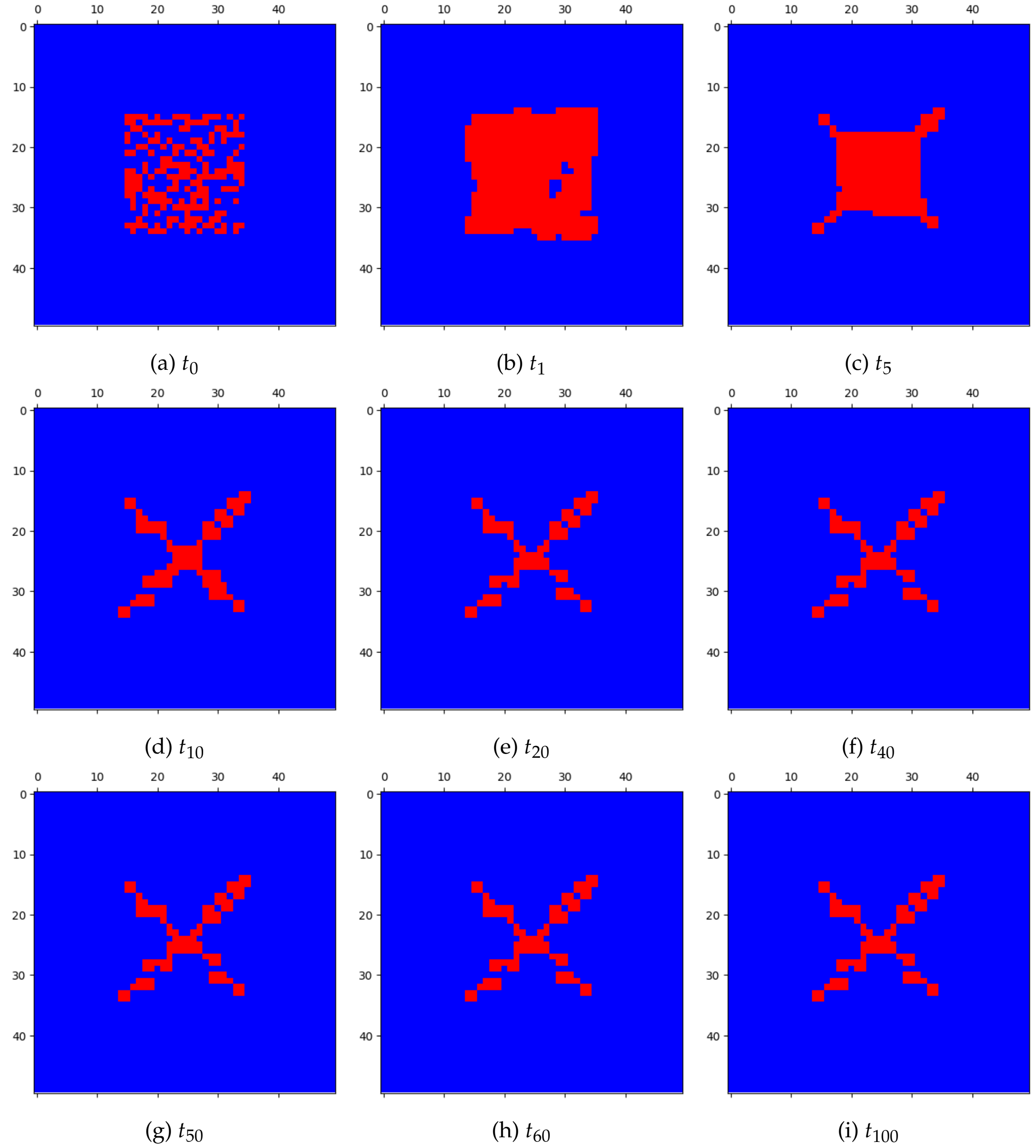
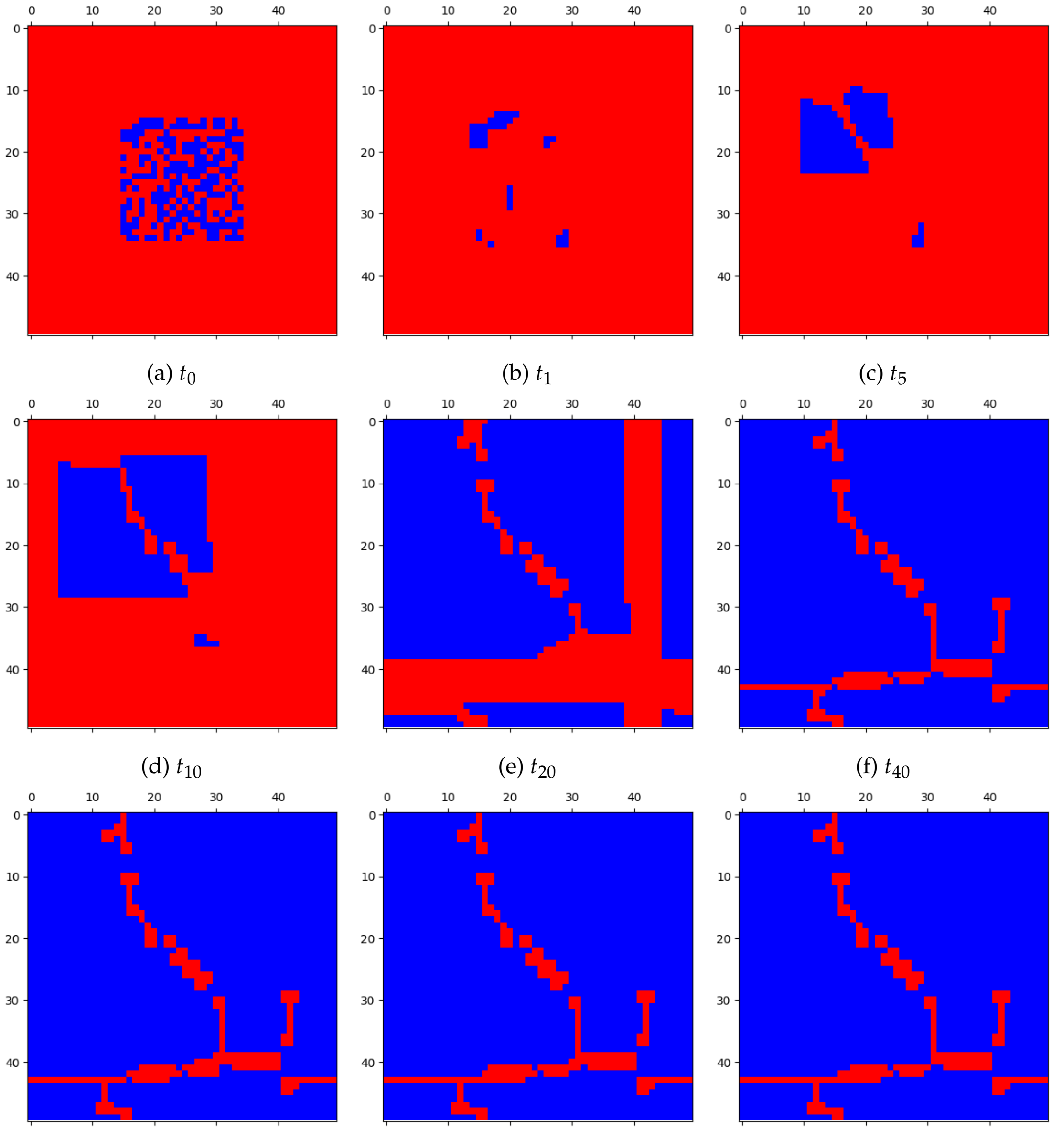


Figure 8: Visualization of the lattice with unconditional imitation, Moore neighborhood and weak prisoner's, dilemma. The start round is a 10x10 center zone and surrounded by cooperation players.

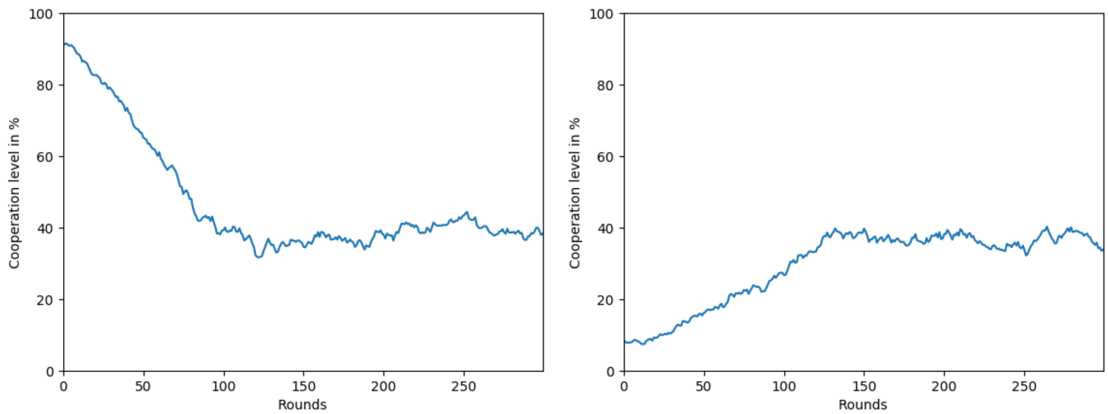
Defection surrounding Figure 7b shows the cooperation level when using the same configuration as Part I with a Moore neighborhood, with the center zone surrounded by defecting players. Figure 9 shows the lattice of this simulation over time. Here we can see that the center zone influenced the surrounding population. As soon as the cluster are formed in the center zone, the cooperation level grows up to *stabilize* at around 90%. Here the cooperation level is more or less the same as exposed in Part I, because in t_1, the defecting players are a majority, as opposed to cooperation surrounding where only the center zone has a majority of defecting players.



(a) t_0 (b) t_1 (c) t_5 (d) t_{10} (e) t_{20} (f) t_{40} (g) t_{50} (h) t_{60} (i) t_{100}

Figure 9: Visualization of a the lattice with unconditional imitation, Moore neighborhood and weak prisoner's, dilemma. The start round is a 10x10 center zone and surrounded by defection players.

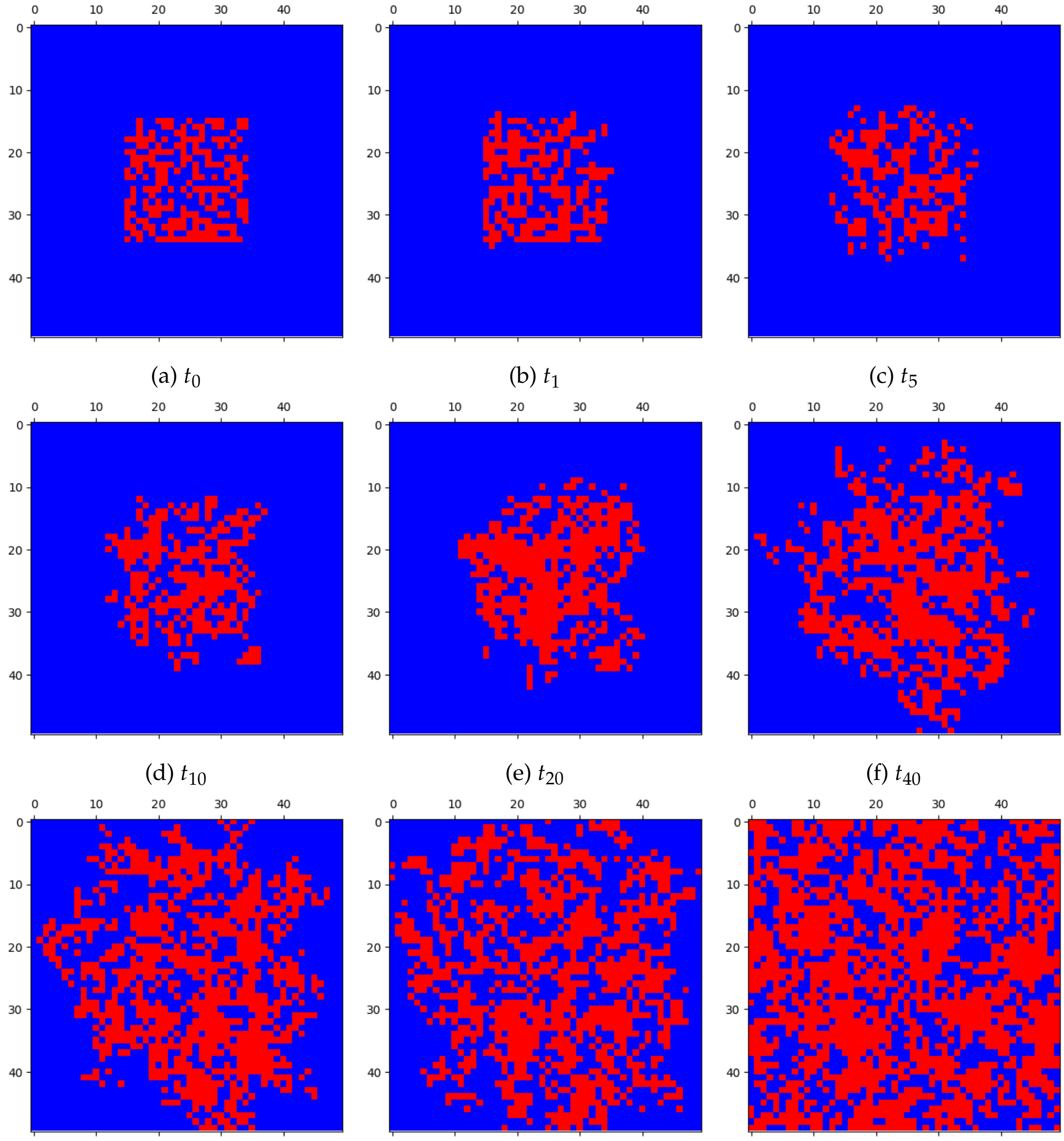
3.2.2 Part II mechanisms



(a) Cooperation surrounding (b) Defection surrounding

Figure 10: Cooperation level using replicator rule and snowdrift game on a 50x50 lattice and 10x10 center zone.

Cooperation surrounding Figure 10a shows the cooperation level when using the same configuration as Part II with a Moore neighborhood, with the center zone surrounded by cooperation players. Figure 11 shows the lattice of this simulation over time. The cooperation level over time is decreasing until reaching a level around 40%. As shown in the visualization we see that the surrounding cooperating players are influenced by the center zone players. Indeed, players close to the center zone are changing their actions based on the results of the center players score. We can see that at round t_{100}, the full population has been influenced by the results of the center zone games, and the distribution of players seems identical to the results exposed in Part II, the cooperation is *stabilize* at around 40%. As opposed to cooperation surrounding in section 3.2.1, here the full population is influenced and change their actions, where in the cooperation surrounding using Part I mechanisms the players stick to their cooperating action.



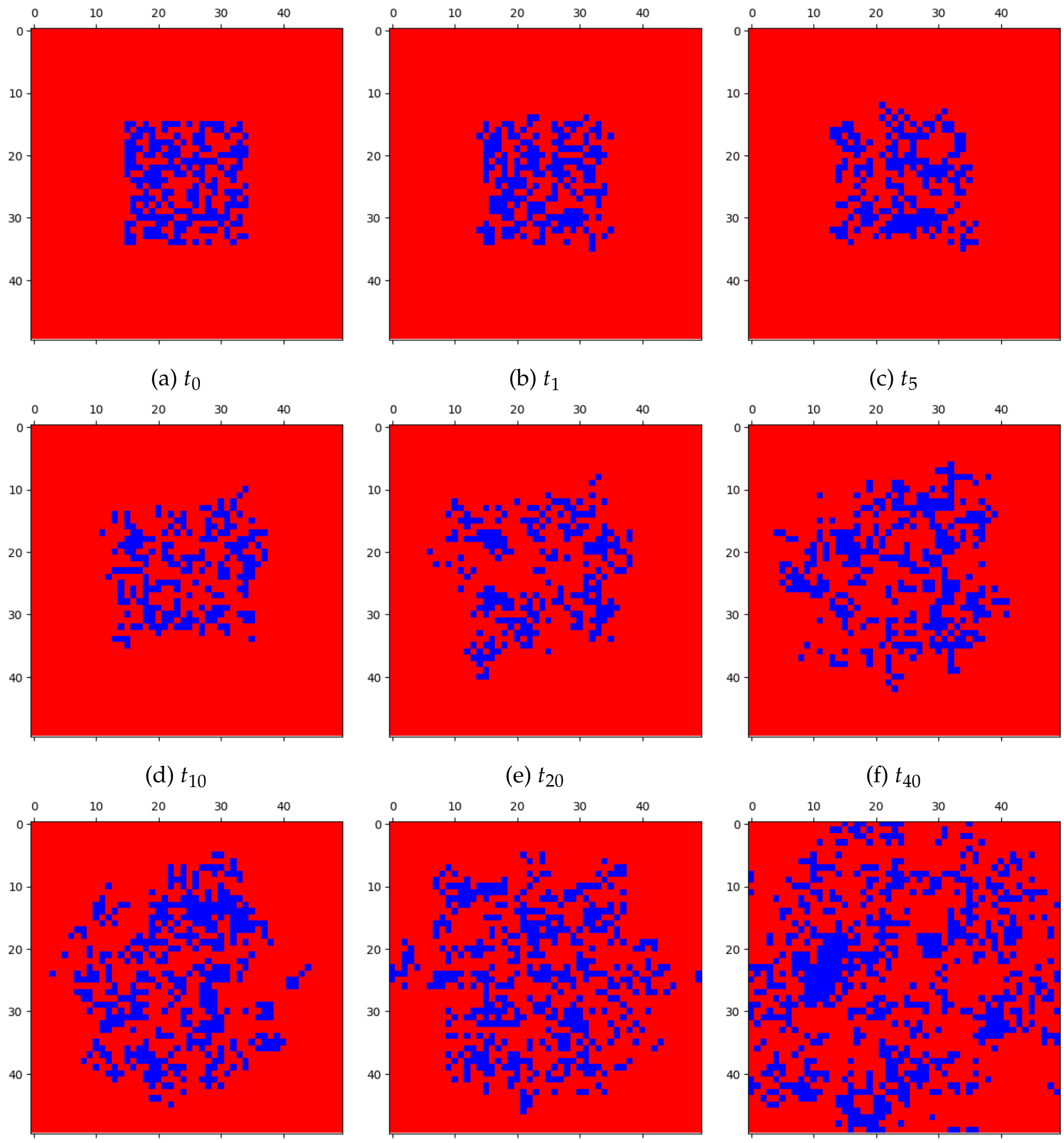
(a) t_0 (b) t_1 (c) t_5

(d) t_{10} (e) t_{20} (f) t_{40}

(g) t_{50} (h) t_{60} (i) t_{100}

Figure 11: Visualization of a the lattice with replicator rule, Moore neighborhood and weak snowdrift game. The start round is a 10x10 center zone and surrounded by cooperation players.

Defection surrounding Figure 10b shows the cooperation level when using the same configuration as Part II with a Moore neighborhood, with the center zone surrounded by defecting players. Figure 12 shows the lattice of this simulation over time. The pattern seems to be the same as exposed in cooperation surrounding. The center zone, influenced the players close to the them, until it reaches the full population. Here the cooperation level is low, since cooperating players can only be found in the center zone in the first round. It is growing over time to reach a *stabilize* state at around 40% which.



(a) t_0 (b) t_1 (c) t_5

(d) t_{10} (e) t_{20} (f) t_{40}

(g) t_{50} (h) t_{60} (i) t_{100}

Figure 12: Visualization of a the lattice with replicator rule, Moore neighborhood and weak snowdrift game. The start round is a 10x10 center zone and surrounded by defecting players.