INFO-F-514 - Course Report
Secure computation

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1 Introduction

In this paper we will propose formal definitions of secure computation, also refered as secure multiparty computation. We will also gives known example in the litterature that are defined following the logical of secure computation. We will then recall state of the art techniques and protocols that allow to resolve secure computations.

2 Secure computation

A secure multi-party computation problem, is a problem where a computation, or a result, must be computed but the input that each party must used is confidential and not share between all parties. Such problem can be defined as a function $f(\cdot)$, that takes n parameters. The idea is to be able to compute the function $f(x_0, .., x_n)$ where the input x_i can only be accessed by the party i. The final results is accessible to everyone.

The first secure computation problem was first introduced by Yao in 1985 [Yao82], with the millionnaires problem. This problem is a secure two-party computation, a subproblem of multi-party computation problem. Unlike other cryptographic protocols, the malicious behaviour come from the participant in the exchange. Indeed in secure computation we can define two party behaviour. A *semi-honest adversary* is a party in the protocol that will always follow the steps that must be performed as stated in the protocol. That is, a semi-honest adversary will always send well-formed messages. It is not fully honest because such adversaries will try to learn other participants secrets by analyzing their protocol messages. A *malicious adversary*

can behave in the worst way possible. That is it can deviate from the protocol, send mal-formed message, and can use any other possible way to find the other parties secrets.

3 Oblivious Transfer

3.1 Definition and variants

The Oblivious Transfer introduced by Rabin during in 1982 [Rab]. The Oblivious Transfer has many application and has been first introduced has a protocol to resolve the Exchange Of Secret problem. The oblivious transfer protocol is defined as follow: a sender want to send a message to a receiver, but it must not be able to tell if the receiver got the message, that is there is a probability of $\frac{1}{2}$, that the message has been sent to the receiver.

In the context of secure computation, The 1-out-of-2 Oblivious Transfer (OT_2^1), an another approach to the original Oblivious Transfer, is used. It is the problem that for a sender and a receiver, one of two message must be sent from the receiver to the sender. The message receive can be chosen by the receiver. Two constraints are that the sender must never know which message has been chosen, and the receiver must not know the content of the other message.

There exists also 1-out-of-n Oblivious Transfer (OT_n^1) is an extension of OT_2^1, where the sender has n messages to send and the receiver must choose one of them. Those two protocols are theorically equivalent has proven in [Cre88, Cac98].

3.2 1-out-of-2 Oblivious Transfer protocol

On possible protocol for the OT_2^1 problem is by using a pair of key using the RSA protocol, first proposed in [EGL85]. Let Alice be the sender and Bob the receiver. Alice has two messages m_0, m_1, and in addition to that a public RSA key (e, d, n). The protocol is a multi-step communication between the two parties using the RSA public key of Alice.

The first step is for Alice to send the public key and two random values, that is the public key (e, n) and two random values x_0, x_1 contains in the domain $[1, n-1]$. Now that Bob has those inputs, he will generates on his side two other random values. The first one is the bit b which value is either 0 or 1, and is used to choose which random inputs received from Alice, that is x_b would be either x_0 or x_1. The second generated random value of Bob is a value k in the domain $[1, n-1]$.

The second step is for Bob to return his response. Since we don't want Alice to know which value has been chosen, Bob will encrypt the value x_b by blinding it using the random value k that he generated. That is Bob will send to Alice the value $v = (x_b + k^e) \mod n$. Upon receival of v, Alice will decrypt v two times, by removing the random values. That is, Alice will have two values k_0, k_1 where $k_i = (v - x_i)^d \mod n$.

Finally, the last step if for Alice to send back the real message. Since v was blinded by Bob with the value k, Alice doesn't know which random x_i has been chosen. By computing the two k_i based on both values, one of them will be identical to the k value of Bob. The last inputs that Alice will send to Bob are the two messages m'_0, m'_1 where $m'_i = m_i + k_i$. Upon receival, Bob will have to decrypt the message with k, that is $m_b = m'_b - k$. Since Bob only has the k_i value associate to his message, he will not be able to decrypt the other message.

4 An application: Yao's millionaires problem

Oblivious Transfer provide a protocol to share inputs without giving to much informations to the other parties. It is still necessary to provide a protocol that will compute the function for the secure computation, and share the results among parties. With this objective, we will focus on a well known problem in secure computation, that is Yao's millionaires problem.

The millionaires problem introduced by Yao in [Yao82], is the problem that for two millionaires they both want to know which one of them is the richer, but they don't want to know the difference. In this problem, the computation function is the usual comparison $<$, and the inputs are the incomes of the individuals. In [LT05], proposed an efficient solution to Yao's problem, using 1-out-of-2 Oblivious Transfer.

4.1 Ioannidis and Ananth solution

Ioannidis and Ananth protocol to resolve the millionaires problem [IG03] is divided in five major steps, and make used of the 1-out-of-2 oblivious transfer protocol.

Alice and Bob wants to know which one is richer that the other. Let a be the number of millions she posses and b the amount of Bob. Before the computation, they first both agree that $a, b < d$, for $d \in \mathbb{N}$.

The goal of this protocol is for Alice to first create a matrix where all elements are values of k bits, where k is the size of the RSA key of Alice, used in the Oblivous Transfer. The first step for alice is to set the matrix to specific values depending on the key size k, and bits of her value a. The second major steps is to generate $d-1$ k bits random numbers S_i. The create another value S_d so that all the bits are random except for the the last, that is $k-1$ and $k-2$. Those two last bits are set using the bitwise XOR operation, based on the S_i random values and the generated matrix. Then a rotation of the elements in the two first column is made, by rotating the the cell with the second random value generated by Alice at the beginning. The resulting matrix is send to Bob, which will resend using the oblivious transfer protocol for each line i, the value at the column $b_i + 1$, where b_i is the i^{th} bit of b. At reception, Alice will rotate the S_i values using her generated random numbers, and send back the result to Bob. This will allow Bob to scan the value from the matrix at indices $b_i + 1$ (as explained above) and the S_i rotated values received from Alice. The scan should reveal a large sequence of zero bits. If the bit to the right of the sequence is equals to 1 then $a \geq b$, otherwise $a < b$.

Proof of OT12 et OT equivalement Crepeau page 351 et Event page 640

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