Learning Dynamics: Assignement 1

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1 Hawk-Dove game

	Hawk	Dove
Hawk	$\frac{1}{2}(V-D), \quad \frac{1}{2}(V-D)$	$V, \quad 0$
Dove	$0, \quad V$	$\frac{V}{2}-T, \quad \frac{V}{2}-T$

Figure 1: Hawk-Dove game

1.1 Expected payoff for player 1

Given player 2's mixed strategy α_2, player 1 expected payoff for the pure strategy $Hawk(p)$ is:

$$E_1(H,\alpha_2) = q(\frac{1}{2}(V-D)) + (1-q)V = V - \frac{q}{2}(V-D) \tag{1}$$

Given player 2's mixed strategy α_2, player 1 expected payoff for the pure strategy $Dove(1-p)$ is:

$$E_1(D,\alpha_2) = q \cdot 0 + (1-q)(\frac{V}{2}-T) = (1-q)(\frac{V}{2}-T) \tag{2}$$

1.2 Expected payoff for player 2

Given player 1's mixed strategy α_1, player 2 expected payoff for the pure strategy $Hawk(q)$ is:

$$E_2(H,\alpha_1) = p(\frac{1}{2}(V-D)) + (1-p)V = V - \frac{p}{2}(V-D) \tag{3}$$

Given player 1's mixed strategy α_1, player 2 expected payoff for the pure strategy $Dove(1-q)$ is:

$$E_2(D, \alpha_1) = p \cdot 0 + (1-p)(\frac{V}{2} - T) = (1-p)(\frac{V}{2} - T) \tag{4}$$

1.3 Player 1's mixed strategies

Since this is a game with two-players two actions *The linearity implies 3 possible outcomes*:

player 1's unique best response is the pure strategy $Hawk(p)$ when:

$$\begin{aligned}
\iff & \quad E_1(H, \alpha_2) > E_1(D, \alpha_2) \\
\iff & V - \frac{q}{2}(V-D) > (1-q)(\frac{V}{2} - T) \\
\iff & V - \frac{V}{2}q + \frac{D}{2}q > \frac{V}{2} - T - \frac{V}{2}q + T \\
\iff & \quad q > -\frac{V}{D}
\end{aligned}$$

player 1's unique best response is the pure strategy $Dove(1-p)$ when:

$$\begin{aligned}
\iff & E_1(H, \alpha_2) < E_1(D, \alpha_2) \\
\iff & \quad q < -\frac{V}{D}
\end{aligned}$$

all player 1's mixed strategies are all best responses when:

$$\begin{aligned}
\iff & E_1(H, \alpha_2) = E_1(D, \alpha_2) \\
\iff & \quad q = -\frac{V}{D}
\end{aligned}$$

1.4 Player 2's mixed strategies

This game is symmetric, the payoff matrix of player 1 and 2 are the same ($A = B^T$). The strategies of player 2 are the same than the player 1, except that it depends on p and not on q.

Player 2's unique best response is pure strategy $Hawk(q)$ when $q > -\frac{V}{D}$, is pure strategy $Dove(1-q)$ when $q < -\frac{V}{D}$ and all mixed strategies are all best response when $q = -\frac{V}{D}$.

1.5 Discussions

2 Social Dilemma

Figure 2 represents the expected payoff of A based on her belief. For example the first column indicates the payoff of A when she believes that B will defect in all games type.

Since A is sure that each game is equally likely, there is a probability of $1/3$ that she is in Prisonner's dilemma, probability of $1/3$ in Stag-Hunt game and probability of $1/3$ in Snowdrift game.

Figure 3 represents the best response of player B in all three games, she doesn't need to form any beliefs since she knows in which game she is playing.

	DDD	CCC	CCD	CDC	CDD	DCC	DCD	DDC
C	1/3	3	**8/3**	4/3	1	7/3	**2**	2/3
D	**2/3**	**4**	7/3	**11/3**	**2**	**8/3**	1	**7/3**

Figure 2: Expected payoff of player A for all possible combinations of player B's types

	C	D	C	D	C	D
C	2	**5**	**5**	2	2	**5**
D	0	**1**	0	**1**	**1**	0

Figure 3: Best response of player B against player A in all games (in the following order): Prisonners, Stag-hunt, Snowdrift

2.1 Payoff function

Let's associate Prisonner's dilemma, Stag-hunt game and Snowdrift game with the indexes $I = \{1, 2, 3\}$, respectively. The actions cooperate and defect be the set $A = \{C, D\}$. Let $b_i \ \forall i \in I$, the beliefs of A, with $b_i \in A$ an action. Let $o_i(x, y) \ \forall i \in I \wedge \forall x, y \in A$ the outcomes of A for game i and action x for player A and action y for B. The payoff function is $p(a, b_1, b_2, b_3) \ \forall a \in A$:

$$p(a, b_1, b_2, b_3) = \frac{1}{3}(o_1(a, b_1) + o_2(a, b_2) + o_3(a, b_3)) \tag{5}$$

For example the payoff for A action cooperate (C) and beliefs $b_1 = D, b_2 = D, b_3 = D$, the fist row and column of Figure 2, is:

$$p(C, D, D, D) = \frac{1}{3}(0 + 0 + 1) = 1/3 \tag{6}$$

Other examples:

$$p(C,C,C,D) = \frac{1}{3}(2+5+1) = 8/3 \tag{7}$$

$$p(D,C,C,C) = \frac{1}{3}(5+2+5) = 4 \tag{8}$$

2.2 Best response and equilibrium

The best response given the action of all game type for player A are the bolded values in Figure 2. The best response for all games for player B are the bolded values in Figure 3. To find the pure Nash Equilibria, we need to match the best response of A with the best response of B for each beliefs. There are two pure Nash Equilibria, shows in red in Figure 2, $\{C,(DCD)\}$ and $\{D,(DDC)\}$. Indeed, for $\{C,(DCD)\}$, when A plays action C, the best response of B in Prisonner's dilemma is D, in Stag-hunt game it is C and in Snowdrift game it is D, so it is a pure Nash Equilibria. The same goes for $\{D,(DDC)\}$.

3 Sequential truel

The diagram representing the subgames are drawn as trees in Figure 4, 5 and 6. When we will refer to T_1, T_2 and T_3 it means that we are refering to, respectively, Figure 4, 5 and 6. In the subgames, the action $t(i)$ where $i \in \{A,B,C\}$, means that the current player is targeting player i. The current player can of course not targeting himself.

The main game is represented by T_1. As mentioned in the assignement specifications, the subgames when A fails to hits in intended target are the same, it is T_2. When in subgame T_2, we also found that when B misses is intended target, the subgames are the same, it is T_3.

3.1 Preferences

- Players prefer outcomes with fewer people
- Players prefer to stay alive

3.2 Subgame perfect equilibria

C equilibrium in T_3 To find the SPE, we have to use the backward induction, so we need to find the SPE of T_3 first. Player C always stay alive whatever target she is choosing. The SPE is $t(A)$ if $p_a > p_b$, and $t(B)$ if $p_b > p_a$.

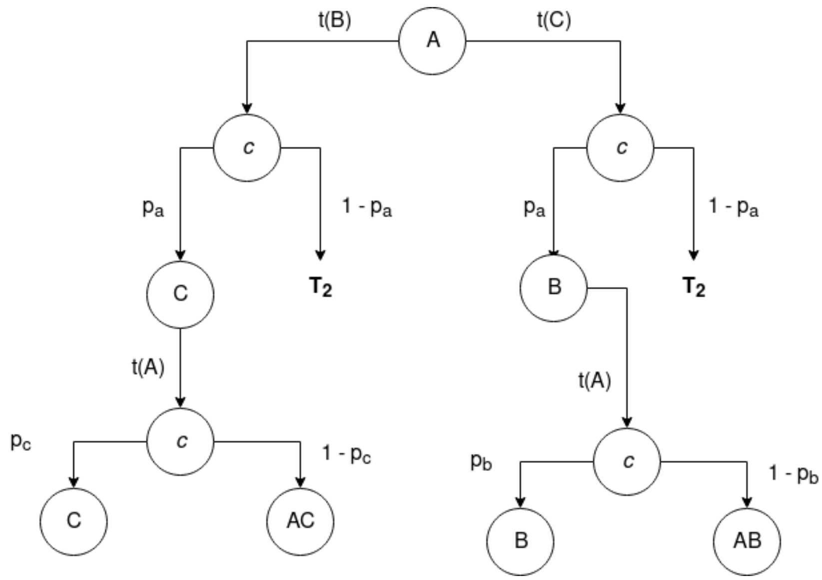

Figure 4: T_1, main subgame

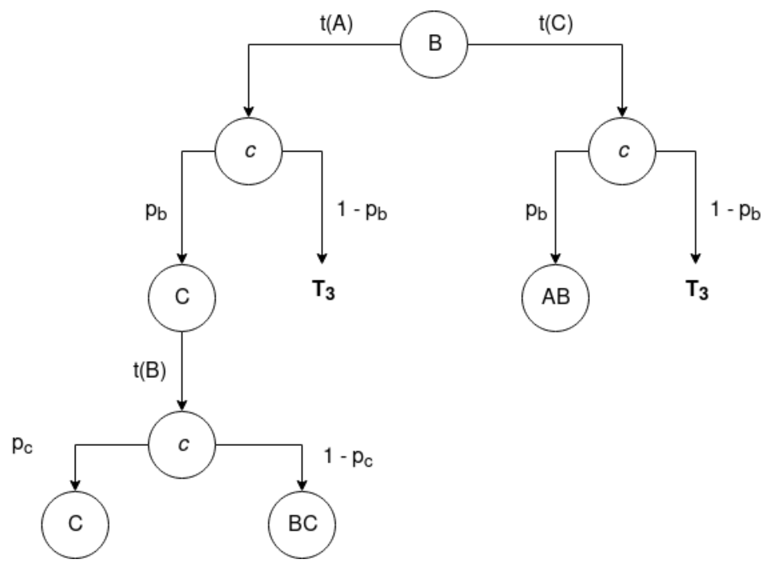

Figure 5: T_2, subgame when A misses her intended target

B equilibrium in T_2 Let's consider subgame T_2. Player B needs either to target A or target C. If B targets A, she has less chance to survive because, since next turn C will play and will still be alive whatever the result is of this shoot, so it B will always target C in T_3. Formally, if B misses her outcome

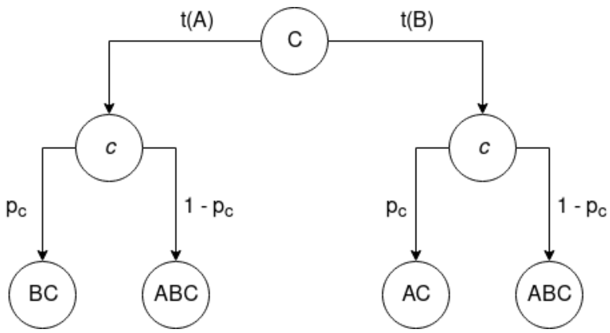

Figure 6: T_3, subgame when A and B miss their intended targets

will be the same, because it will have the SPE of subgame T_3, which is unique. If B targets A and hits her, she has a probabilty of $1 - p_c$ to stay alive. If B targets C and hits her, she has a probablity of 1 to stay alive. So B will always choose to target C.

A equilibrium in T_1 Let's consider the full game, subgame T_1. Intuitively, it is best for A to target the player with the biggest probability, because she will have more chance to stay alive if she manage to eliminate the strongest opponent. Formally, if A misses her intended target, the outcome will be the same since T_2 has an unique SPE. If A targets B and hits her, she has a probability of $1 - p_c$ to stay alive, because C is the remaining shooter and has a probability of $1 - p_c$ to fail. If A targets C and hits her, she has a probability of $1 - p_b$ to stay alive, because B is the remaining shooter and has a probability of $1 - p_b$ to fail. So A targets B if $(1 - p_c) > (1 - p_b)$, which can be simplify as $p_c < p_b$, and A targets C if $(1 - p_b) > (1 - p_c)$, which can be simplify as $p_b < p_c$.

Weakness is strength In the previous paragraph, we explained that if $p_c > p_b$ A will target C. If C is the target, the her probablity of survival is the probability that A misses and B misses, formally:

$$(1 - p_a)(1 - p_b) = 1 - p_b - p_a + p_a p_b = 1 - p_a - p_b(1 - p_a) \tag{9}$$

If $p_b > p_c$ A will target B, then C chance of survival is A hits her target and C will be the last player with a bullet or A misses her target and B misses also her target. Formally:

$$p_a + (1 - p_a)(1 - p_b) = p_a + 1 - p_b - p_a + p_a p_b = 1 - p_b(1 - p_a) \tag{10}$$

The difference between the probablity (9) and (10), is that (9) is decreasing (10) with p_a, so (9) will always be smaller that (10). The probablity of survival of C when $p_c > p_b$ will always be smaller than the probability of survival of C when $p_b > p_c$. C is always better off when $p_b > p_c$.