Learning Dynamics: Assignement 1

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1 Sequential truel

The diagram representing the subgames are drawn as trees in Figure 1, 2 and 3. When we will refer to T_1, T_2 and T_3 it means that we are refering to, respectively, Figure 1, 2 and 3. In the subgames, the action $t(i)$ where $i \in \{A, B, C\}$, means that the current player is targeting player i. The current player can of course not targeting himself.

The main game is represented by T_1. As mentioned in the assignement specifications, the subgames when A fails to hits in intended target are the same, it is T_2. When in subgame T_2, we also found that when B misses is intended target, the subgames are the same, it is T_3.

1.1 Preferences

- Players prefer outcomes with fewer people
- Players prefer to stay alive

1.2 Subgame perfect equilibria

C equilibrium in T_3 To find the SPE, we have to use the backward induction, so we need to find the SPE of T_3 first. Player C always stay alive whatever target she is choosing. The SPE is $t(A)$ if $p_a > p_b$, and $t(B)$ if $p_b > p_a$.

B equilibrium in T_2 Let's consider subgame T_2. Player B needs either to target A or target C. If B targets A, she has less chance to survive because, since next turn C will play and will still be alive whatever the result is of this shoot, so it B will always target C in T_3. Formally, if B misses her outcome will be the same, because it will have the SPE of subgame T_3, which is unique. If B targets A and hits her, she has a probabilty of $1 - p_c$ to stay alive. If B targets C and hits her, she has a probablity of 1 to stay alive. So B will always choose to target C.

A equilibrium in T_1 Let's consider the full game, subgame T_1. Intuitively, it is best for A to target the player with the biggest probability, because she will have more chance to stay alive if she manage to eliminate the strongest opponent. Formally, if A misses her intended target, the outcome will be the same since T_2 has an unique SPE. If A targets B and hits her, she has a probability of $1-p_c$ to stay alive, because C is the remaining shooter and has a probability of $1-p_c$ to fail. If A targets C and hits her, she has a probability of $1-p_b$ to stay alive, because B is the remaining shooter and has a probability of $1-p_b$ to fail. So A targets B if $(1-p_c) > (1-p_b)$, which can be simplify as $p_c < p_b$, and A targets C if $(1-p_b) > (1-p_c)$, which can be simplify as $p_b < p_c$.

Weakness is strength In the previous paragraph, we explained that if $p_c > p_b$ A will target C. If C is the target, the her probablity of survival is the probability that A misses and B misses, formally:

$$(1-p_a)(1-p_b) = 1-p_b-p_a+p_a p_b = 1-p_a-p_b(1-p_a) \tag{1}$$

If $p_b > p_c$ A will target B, then C chance of survival is A hits her target and C will be the last player with a bullet or A misses her target and B misses also her target. Formally:

$$p_a + (1-p_a)(1-p_b) = p_a + 1 - p_b - p_a + p_a p_b = 1 - p_b(1-p_a) \tag{2}$$

The difference between the probablity (1) and (2), is that (1) is decreasing (2) with p_a, so (1) will always be smaller that (2). The probablity of survival of C when $p_c > p_b$ will always be smaller than the probability of survival of C when $p_b > p_c$. So it will always be better for C to have $p_b > p_c$.

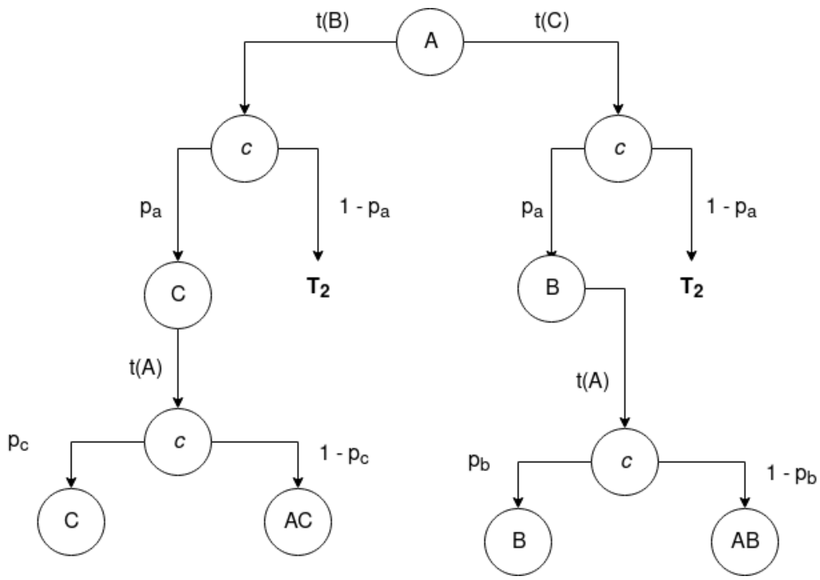

Figure 1: T_1, main subgame

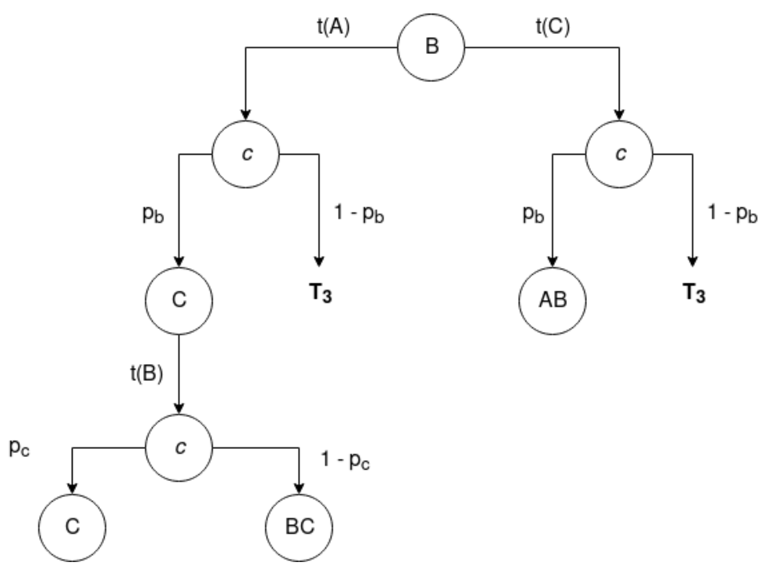

Figure 2: T_2, subgame when A misses her intended target

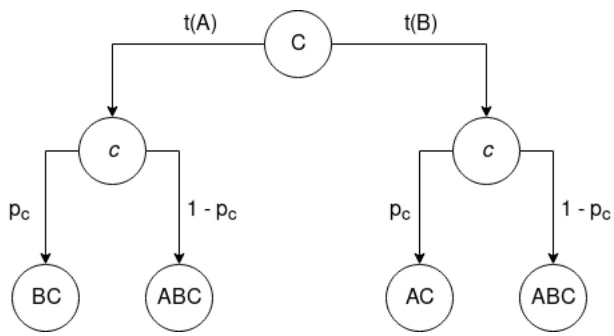

Figure 3: T_3, subgame when A and B miss their intended targets