

Learning Dynamics: Assignement 1

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1 Sequential truel

The diagram representing the subgames are drawn as trees in Figure 1, 2 and 3. When we will refer to T_1 , T_2 and T_3 it means that we are referring to, respectively, Figure 1, 2 and 3. In the subgames, the action $t(i)$ where $i \in \{A, B, C\}$, means that the current player is targeting player i . The current player can of course not targeting himself.

The main game is represented by T_1 . As mentioned in the assignement specifications, the subgames when A fails to hits in intended target are the same, it is T_2 . When in subgame T_2 , we also found that when B misses is intended target, the subgames are the same, it is T_3 .

1.1 Preferences

- Players prefer outcomes with fewer people
- Players prefer to stay alive

1.2 Subgame perfect equilibria

C equilibrium in T_3 To find the SPE, we have to use the backward induction, so we need to find the SPE of T_3 first. Player C always stay alive whatever target she is choosing. The SPE is $t(A)$ if $p_a > p_b$, and $t(B)$ if $p_b > p_a$.

B equilibrium in T_2 Let's consider subgame T_2 . Player B needs either to target A or target C . If B targets A , she has less chance to survive because, since next turn C will play and will still be alive whatever the result is of this shoot, so it B will always target C in T_3 . Formally, if B misses her outcome will be the same, because it will have the SPE of subgame T_3 , which is unique. If B targets A and hits her, she has a probability of $1 - p_c$ to stay alive. If B targets C and hits her, she has a probability of 1 to stay alive. So B will always choose to target C .

A equilibrium in T_1 Let's consider the full game, subgame T_1 . Intuitively, it is best for A to target the player with the biggest probability, because she will have more chance to stay alive if she manage to eliminate the strongest opponent. Formally, if A misses her intended target, the outcome will be the same since T_2 has an unique SPE. If A targets B and hits her, she has a probability of $1 - p_c$ to stay alive, because C is the remaining shooter and has a probability of $1 - p_c$ to fail. If A targets C and hits her, she has a probability of $1 - p_b$ to stay alive, because B is the remaining shooter and has a probability of $1 - p_b$ to fail. So A targets B if $(1 - p_c) > (1 - p_b)$, which can be simplify as $p_c < p_b$, and A targets C if $(1 - p_b) > (1 - p_c)$, which can be simplify as $p_b < p_c$.

Weakness is strength In the previous paragraph, we explained that if $p_c > p_b$ A will target C . If C is the target, the her probability of survival is the probability that A misses and B misses, formally

$$(1 - p_a)(1 - p_b) = 1 - p_b - p_a + p_a p_b = 1 - p_a - p_b(1 - p_a) \quad (1)$$

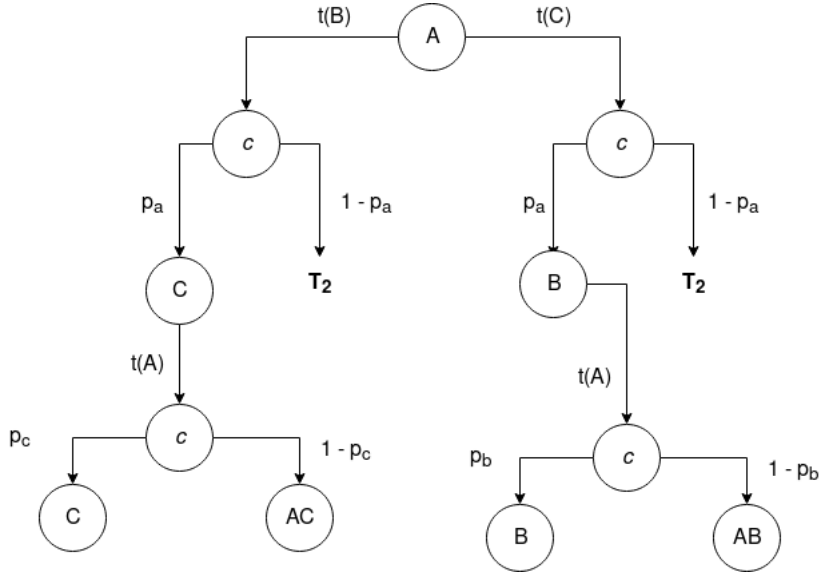


Figure 1: T_1 , main subgame

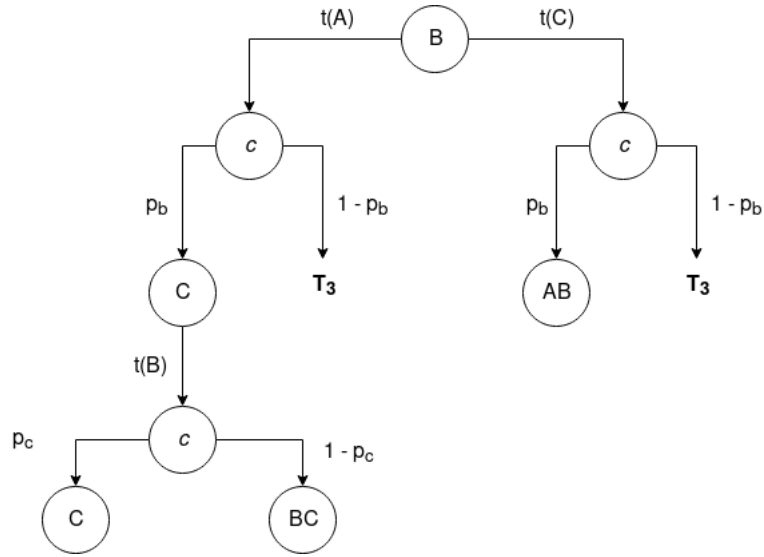


Figure 2: T_2 , subgame when A misses her intended target

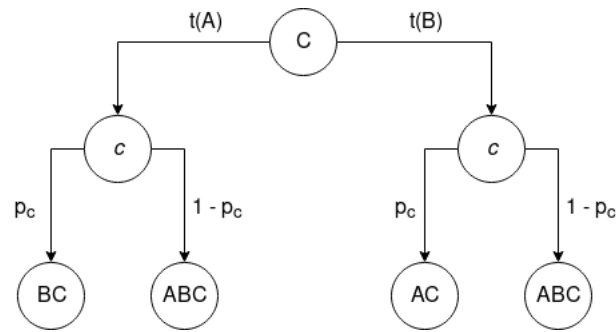


Figure 3: T_3 , subgame when A and B miss their intended targets