Learning Dynamics: Assignment 2
Evolutionary dynamics in a spatial context

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Contents

1 Part I

Specifications Plots in this part shows the average cooperation level of 100 simulations with unconditional imitation as the update mechanism. The first rounds were played randomly, where a player would choose cooperate with a probabilty of $\frac{1}{2}$.

1.1 Neighborhood analysis

Remark The analysis is based on results from 50x50 lattice simulations.

Moore Figure 1a shows the average cooperation level using a Moore neighborhood for each player. We can observe that the level after the first randomly played round, the cooperation dropped at around 2%. Then grows to stabilize at around 87%.

Von Neumann Figure 1b shows the average cooperation level using a Von Neumann neighborhood for each player. We can observe that the level after the first randomly played round, the cooperation dropped at around 15%. Then grows to stabilize at around 40%.

We can see that the cooperation level follows the same pattern but on a different scale. With Moore we have more neighbors, which can explain why the behaviour of the players are more *extreme*.

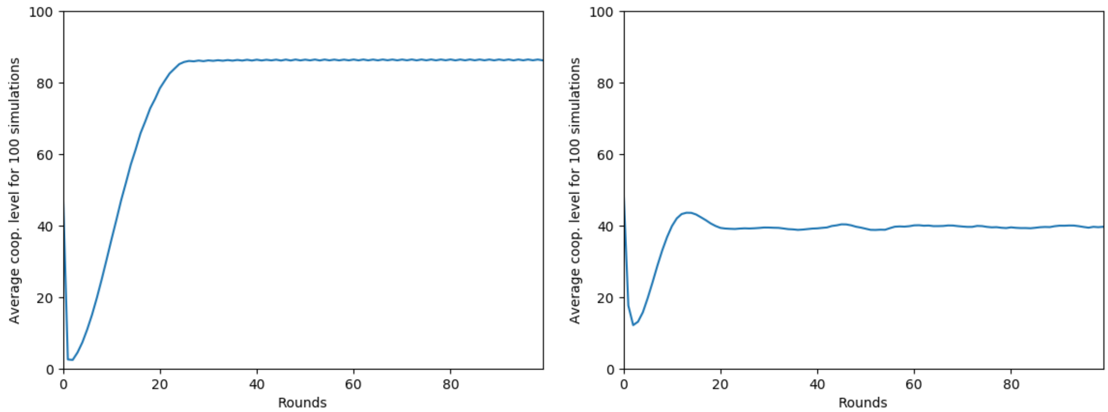
1.2 Lattice observation

Figure 2 shows the full matrix of cooperation for the rounds $t_0, t_1, t_5, t_{10}, t_{20}, t_{50}$. We can observe that in the first round there is more or less the same number of players cooperating and defecting. But in the second round, a large percentage of players will choose to defect, leaving only smalls zone of cooperation. This is due to the fact that a defecting player will usually have a better score around a mixed neighborhood of player than a cooperating player. But when a cooperating player has a cooperating neighborhood he will have high enough score to influence defecting players in his neighborhood. We can observe that in the following rounds, a sort of *cluster* of cooperation will be formed and influence the full lattice until reaching the cooperation level explained in section 1.1.

1.3 Lattice size analysis

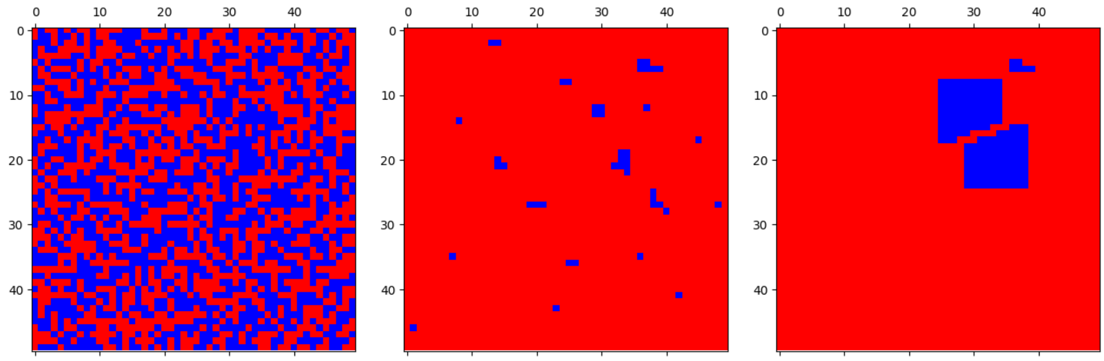
Figure 3 shows the average cooperation level of lattices of size 20, 12, 8 and 4. The behavior seems to the same as in the analysis made against lattice of size 50 in section 1.1, it crashes to a small level of cooperation to grows and stabilize after a number of rounds. The plots show that when the lattice size is small, the cooperation stabilize to a smaller cooperation level than bigger lattices. We can observe that for a size 4 lattice, the cooperation level is even 0 after the first round.

The reason could be that there is less players so less possibilities to form some cooperation neighborhood, that will form the `clusters` and grews as explained in previous section.

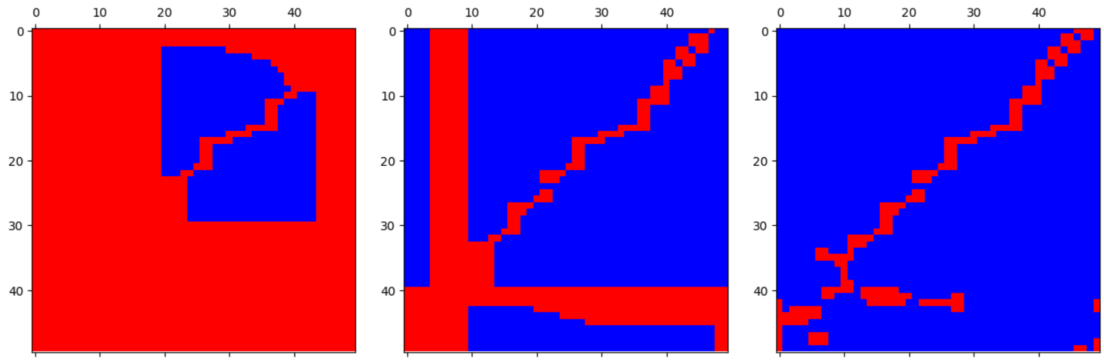


(a) Moore neighborhood (b) Von Neumann neigborhood

Figure 1: Cooperation level using unconditional imitation and weak prisoner's dilemma on a 50x50 lattice

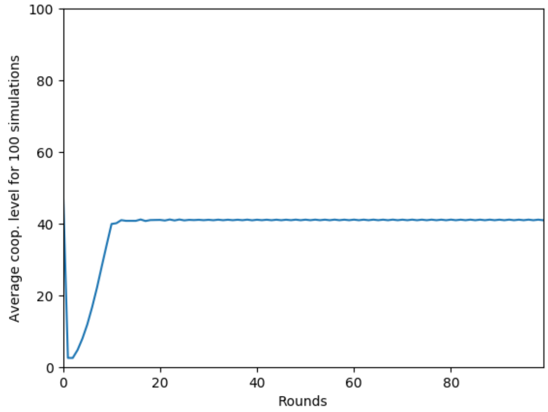


(a) t_0 (b) t_1 (c) t_5

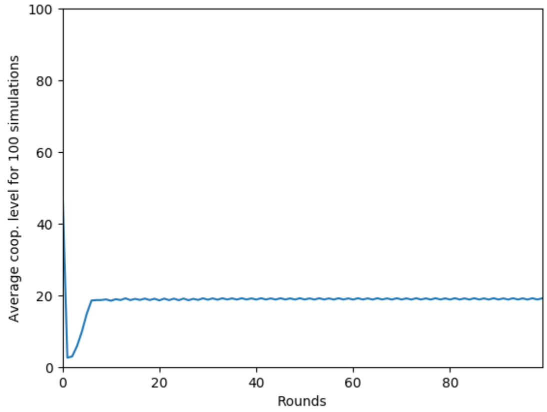


(d) t_{10} (e) t_{20} (f) t_{50}

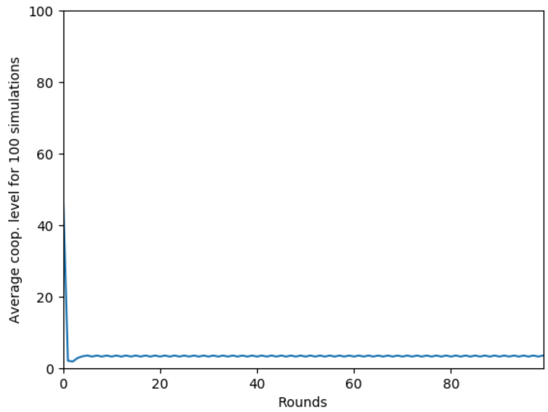
Figure 2: Visualization of a the lattice with unconditional imitation, Moore neighborhood and weak prisoner's, dilemma



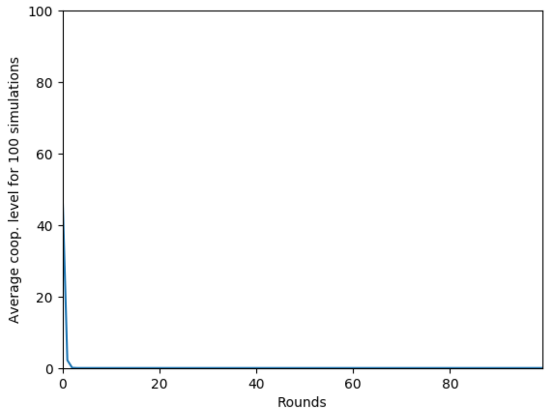
(a) 20x20



(b) 12x12



(c) 8x8



(d) 4x4

Figure 3: Cooperation level using unconditional imitation, Moore neighborhood and weak prisoner’s dilemma

2 Part II

3 Update mechanism

$$P_{ij} = (1 + [W_j - W_i]/[N \cdot (max\{P,R,T,S\} - min\{P,R,T,S\}])/2 \quad (1)$$

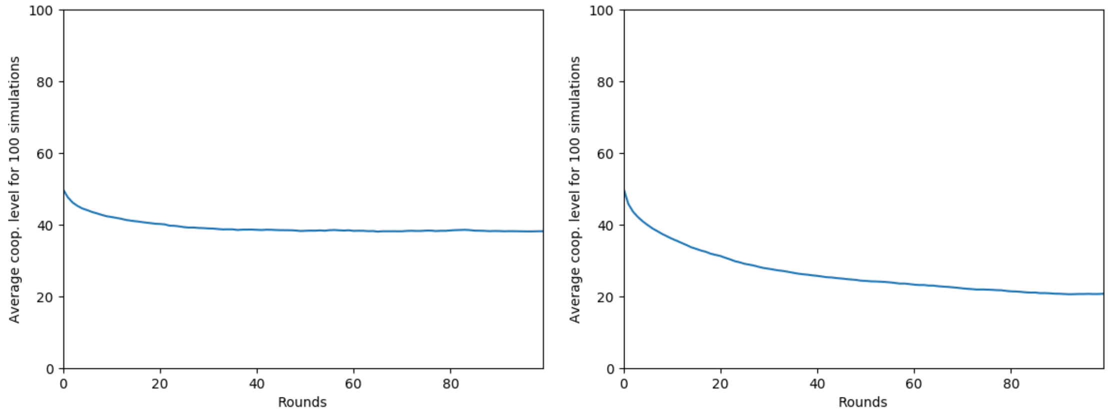
Intuitive observation This probabilty is interesting to be used as an update mechanism because the probability to change the action to the neighbor action is proportional to the difference between the players payoffs.

Probability variables $[W_j - W_i]$ is the difference between the two payoffs. N represent the number of neighbor that the payoff calculation are based on. The difference between the maximum and minimum multiply by N is the maximum payoff of a player. Since the payoffs cannot be bigger than the maximum score, it is clear that P_{ij} is a probability.

Analysis We can highlight different result from the fration $[W_j - W_i]/[N \cdot (max\{P,R,T,S\} - min\{P,R,T,S\}]$ between the difference of payoffs and the maximum score:

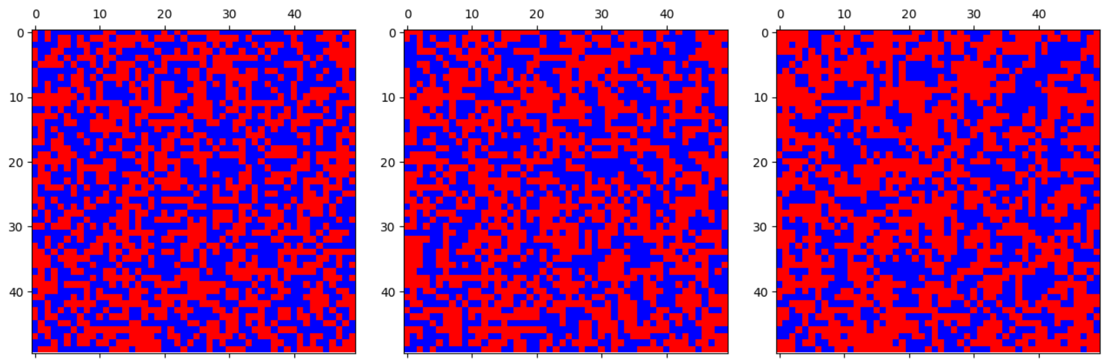
1. Fraction is positive when $W_j > W_i$. A special case is when W_j is maximum and W_i is null, the fraction is equal to 1.
2. Fraction is negative when $W_i > W_j$. A special case is when W_i is maximum and W_j is null, the fraction is equal to -1.
3. Fraction is equal to 0 when $W_i = W_j$

By using the full definition of the probablity we can see that when in case (1), it is more probable that the player will change is action to the neighbor action, and sure if the fraction is equal 1 because $P_{ij} = 1$. When in (2), it is more probable that the player will keep is action, and sure that he will not change it when the fraction is equal to -1 because $P_{ij} = 0$. When in (3), $P_{ij} = \frac{1}{2}$, the payoff of the player and his neighbor are the same, which means that both of their actions lead to the same payoff, so the probability to change or to keep is the same.

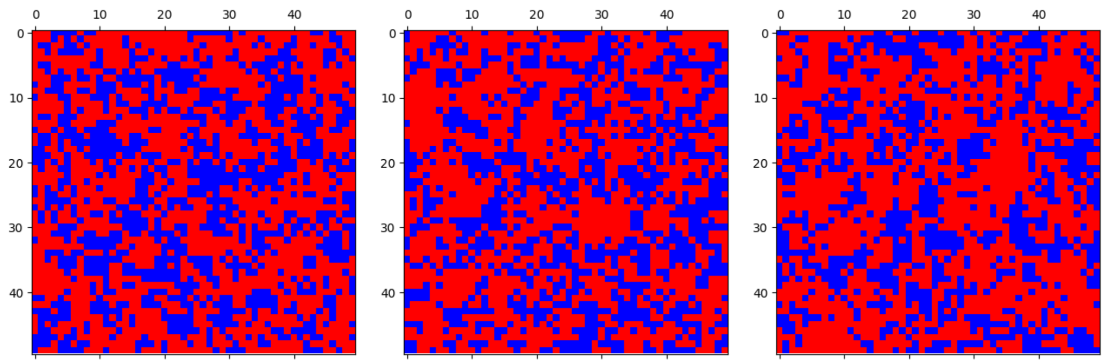


(a) Moore neighborhood (b) Von Neumann neigborhood

Figure 4: Cooperation level using replicator rule and snowdrift game on a 50x50 lattice

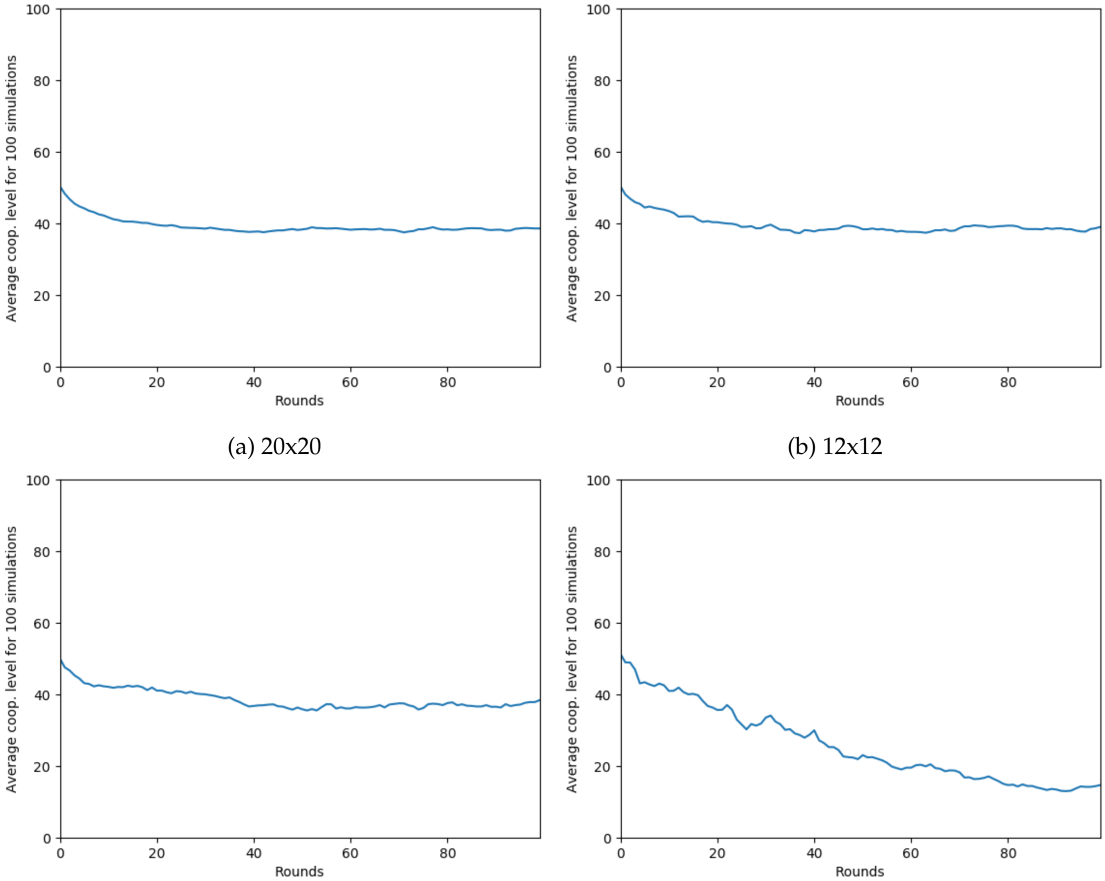


(a) t_0 (b) t_1 (c) t_5



(d) t_{10} (e) t_{20} (f) t_{50}

Figure 5: Visualization of a the lattice with replicator rule, Moore neighborhood and snowdrift game



(a) 20x20

(b) 12x12

(c) 8x8

(d) 4x4

Figure 6: Cooperation level using replicator rule, Moore neighborhood and snowdrift game