1 Cook-Levin Theorem: SAT is NP-complete

To proof that we have to:

1. SAT $\in$ NP
2. any laguage in NP is $\leq_p$ to SAT

Let A a language in NP and N be the NTM (Nondeterministic Turing Machine) that decides A. The machine N decides A in n^k time for some constant k.
We will construct a tableau for N on w of size $n^k \times n^k$. The tableau is constructed following these propreties:

- Each row represent the configurations of a branch of computation of N on w
- Each row starts and ends with the symbol #
- The first row is the starting configuration and each row follows the previous according to N's transition function
- A tableau is **accepting** if any row of the tableau is an accepting configuraton

The reduction build multiple tableau based on the computation tree, and every accepting tableau corresponds to an accepting computation branch of N on w.

1.1 Reduction

On input w the reduction produces a formula ϕ.

$$\phi = \phi_{cell} \wedge \phi_{start} \wedge \phi_{move} \wedge \phi_{accept}$$

This formula is composed of a set of literals $x_{i,j,s}$ where $1 \leq i, j \leq n^k$ and $s \in C$ with $C = Q \cup \Gamma \cup \{\#\}$ (i.e. a symbol that can be in the tableau).

1.1.1 Cell formula

Ensure that each cell contains one and only one character:

$$\phi_{cell} = \bigwedge_{1 \leq i,j \leq n^k} [\bigvee_{s \in C} x_{i,j,s}]$$

The idea is the following: for each cell, at least one symbol must be true and two symbols cannot be true together.

1.1.2 Start formula

Ensure that the first row is the start configuration of N on w:

1.1.3 Accept formula

Guarantees that an accepting configurations occurs in the tableau

1.1.4 Move formula

Guarantees that each row of the tableau corresponds to a configuration that legally follows the configuration of the preceding row according to N's rules. It does so by ensuring that each 2×3 window of the tableau is **legal**.

A windows is legal when it respects the following transitions: $\delta(q_1, a) = \{(q_1, b, R)\}$ and $\delta(q_1, b) = \{(q_2, c, L), (q_2, A, R)\}$