Data structures for Partially Ordered Sets

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Hakim BOULAHYA

supervised by
Emmanuel Filiot and Guillermo A. Pérez

Département d’Informatique
Université Libre de Bruxelles
Belgium

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Introduction

Subject

- Implement data structures to represent partially ordered sets
- Main focus on antichain-based algorithms in automata theory

Motivations

- There exists new algorithms that uses antichains, for known pratical and theoritical problems:
  - Model checking [3]
  - Synthesis problem [4]
  - Language universality [2]
- There is a need for an efficient implementation of antichains

Partial order

A partial order $\preceq$ on a set S is a binary relation $\preceq \subseteq S \times S$ that is reflexive, transitive and antisymmetric.

Partially ordered set (or poset)

A partially ordered set is a pair $\langle S, \preceq \rangle$ where S is a set, and $\preceq$ a partial order on S.

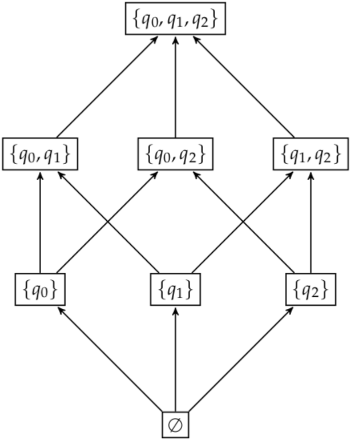
Antichains

An antichain α is a set of incomparable elements of poset $\langle S, \preceq \rangle$ w.r.t. to $\preceq$.

Interestring operations

- **Lower closure** of an antichain α on S, denoted $\downarrow\alpha$, is the set of all elements of S that are smaller or equal to an element of α
- **Appartenance** $s \in \downarrow\alpha_1$
- **Inclusion** $\downarrow\alpha_1 \subseteq \downarrow\alpha_2$
- **Union** $\downarrow\alpha_1 \cup \downarrow\alpha_2 = \downarrow\lceil\alpha_1 \cup \alpha_2\rceil$
- **Intersection** $\downarrow\alpha_1 \cap \downarrow\alpha_2$

Example



- Poset is $\langle 2^Q, \subseteq \rangle$, where $Q = \{q_0, q_1, q_2\}$
- $\alpha = \{\{q_0, q_2\}, \{q_1, q_2\}\}$ is an antichain
- Can retrieve all subset of cardinality 1 using the closure on the antichain: $\downarrow\alpha$
- $\downarrow\alpha = \{\{q_0\}, \{q_1\}, \{q_2\}, \{q_0, q_2\}, \{q_1, q_2\}\}$

Existing implementations

Do implementation already exists ?

Yes, 2 for antichains:

1. **AaPAL**[1]: Antichains implementation in *C*
   - More of an API: user must implement comparison and intersection
2. Antichains for the Dedekind number problem [5]
   - Bitarray represention (a bit is a subset)
   - Can only be applied to natural numbers

Requirements

- `Owl` library: an automata library
- But symbolic antichain-based algorithm are missing (including antichain data structures)
- Implementation in `Java 10`

Objective

What to do ?

- Provide an API and implement it against `Owl`
- Provide some implementation for antichains depending on the universe of the sets
- Define the algorithms to test our antichains implementation against
- Study the performance of those implementations

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Thank you!

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