

(Pre)Thesis draft

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# Contents

<b>Contents</b>	<b>1</b>
<b>1 Introduction</b>	<b>5</b>
1.1 Motivation . . . . .	5
1.2 Objective . . . . .	5
1.3 Related work . . . . .	5
<b>2 Data Structures</b>	<b>7</b>
2.1 Binary relations . . . . .	7
2.2 Partially ordered set . . . . .	8
2.3 Antichains and pseudo-antichains . . . . .	8
<b>3 Implementation</b>	<b>9</b>
3.1 Summary of objectives . . . . .	9
3.2 Existing implementation . . . . .	9
3.3 Motivation and objective . . . . .	9
3.4 Notions . . . . .	10
3.5 Questions . . . . .	10
<b>Bibliography</b>	<b>11</b>



## Todo list

cite the original papers of those problems . . . . .	5
Ref for complexity of univers. pbl . . . . .	5
Talk about the problems, complexity and alternative (Safra vs antichain)	5
This small paragraph is an open discussion . . . . .	5
Includes limitation of Java built-in and different possible solution for antichains found on stack overflow . . . . .	5
is this Total def correct ? . . . . .	7
Meaning of — vs . vs : in set definition ? . . . . .	8
Is this total order affirmation correct ? . . . . .	8
Everything below is a fast/draft notes . . . . .	9



# Chapter 1

## Introduction

### 1.1 Motivation

Automata theory is used in various field in Computer Science, especially in Computer-Aided Verification.

Problems such as synthesis [FJR09] or the universality problem [DWRLH06] has shown to be PSPACE-complete [?, DWRLH06].

cite the original papers of those problems

More efficient algorithms to resolve those problems have been implemented using antichain based-algorithms. Antichains are data structures that allow to represent a partial order sets, in a more compact way.

Ref for complexity of univers. pbl

### 1.2 Objective

The goal of this thesis is to provide an efficient implementation of data structures used in those algorithms, especially antichain. We will mainly focus on implementing antichain-related data structure and provide a library to be used in different tool such as Acacia+, Owl.

Talk about the problems, complexity and alternative (Safra vs antichain)

The first, and main, objective is to implement an Antichain object in Java to be used in Owl. Then if possible used this implementation for other tool such as Acacia+, by either providing bindings or other.

As we mainly focus on efficiency, it could be interesting to use a C implementation and provide binding to a Java class.

This small paragraph is an open discussion

### 1.3 Related work

AaPAL library is a that was implemented in the context of Bohy's PhD thesis to provide an antichain library and be able to implement the antichains based algorithm for the synthesis problem.

Java already provide built-in implementation for Set.

Includes limitation of Java built-in and different possible solution for antichains found on stack overflow



## Chapter 2

# Data Structures

In this section, we will provide formal definitions of the data structures that we will implement. We recall the notion of binary relations and important properties of such relations. We then define partially ordered set, totally order set and closed set. Finally we give a formal definition for antichains.

The definitions and examples for this section are based on [Boh14b].

### 2.1 Binary relations

A binary relation for an arbitrary set  $S$  is a set of pair  $R \subseteq S \times S$ . There are five important properties: reflexivity, transitivity, symmetry, antisymmetry and total.

A relation  $R$  on  $S$  is said to be:

- Reflexive: iff  $\forall s \in S$  it holds that  $(s, s) \in R$
- Transitive: iff  $\forall s_1, s_2, s_3 \in S$ , if  $(s_1, s_2) \in R$  and  $(s_2, s_3) \in R$  then it holds that  $(s_1, s_3) \in R$
- Symmetric: iff  $(s_1, s_2) \in R$  then  $(s_2, s_1) \in R$ .
- Antisymmetric: iff  $(s_1, s_2) \in R$  and  $(s_2, s_1) \in R$  then  $s_1 = s_2$
- Total: iff  $\forall s_1, s_2 \in S$  then  $(s_1, s_2) \in R$  or  $(s_2, s_1) \in R$  is this Total def correct ?

**Orders** A *partial order* is a binary relation that is *reflexive*, *transitive* and *antisymmetric*. We note a partial order relation by  $R$ . We note  $s_1 R s_2$  to show the belonging of a binary relation to a partial order, which is equivalent to  $(s_1, s_2) \in R$ . A *total order* is a partial order that is *total*.

## 2.2 Partially ordered set

An arbitrary set  $S$  associated with a partial order  $\preceq$  is called a *partially ordered set* or *poset*. It is denoted by the pair  $\langle S, \preceq \rangle$ .

**Comparable** Let  $s_1, s_2 \in S$  and  $\langle S, \preceq \rangle$  a poset. The two elements  $s_1$  and  $s_2$  are called *comparable* if either  $s_1 \preceq s_2$  or  $s_2 \preceq s_1$ . If neither of those two comparisons are correct, then  $s_1$  and  $s_2$  are called *uncomparable*.

### Bounds

**Lattices** A poset where every elements has a LUB and GLB is called a lattice.  
 lower semilattice: all elements have LUB. upper semilattice: all elements have GLB.

## 2.3 Antichains and pseudo-antichains

### Closed sets

A closed set is a set  $L \subseteq S$  of a lower semilattice  $\langle S, \preceq \rangle$  where  $\forall l \in L$  we have that  $\forall s \in S$  such that  $s \preceq l$ , then  $s \in L$ .

Note that for two closed sets  $L_1, L_2 \subseteq S$ , we have that  $L_1 \cup L_2$  and  $L_1 \cap L_2$  are also closed sets, but  $L_1 \setminus L_2$  does not result necessarily to a closed set.

**Maximal/minimal elements** We denote by  $\lceil L \rceil$  (resp.  $\lfloor L \rfloor$ ) the set of maximal (resp. minimal) elements of a closed set  $L$  which correspond to  $\lceil L \rceil = \{l \in L \mid \forall l' \in L : l \preceq l' \Rightarrow l = l'\}$  and  $\lfloor L \rfloor = \{l \in L \mid \forall l' \in L : l' \preceq l \Rightarrow l = l'\}$ .

Meaning of — vs . vs : in set definition ?

Is this total order affirmation correct ?

**Closure** In this thesis we are more interested in partial ordered sets as total order sets can be easily implemented as lists.

**Ordered sets** Partial/Order sets

Lattice (semi upper and lower)

Closed sets

Antichain

Includes properties to implement



## Chapter 3

# Implementation

### 3.1 Summary of objectives

The main focus of the thesis is to be able to provide an efficient implementation of antichains and pseudo-antichains in **Java**. The first step is to provide an interface for the different operations that can be applied to antichains. We then give a description of the implementation. Antichains provide a way to represent in a compact way partially ordered set that are closed. Pseudo-antichains are an extension of antichains and provide a compact way to represent partially ordered sets. Pseudo-antichains does not specifically require closed set.

### 3.2 Existing implementation

Everything below is a fast/draft notes

### 3.3 Motivation and objective

For the moment, the implementation to represent the data structures in Acacia+ is specifically designed for a specific set. The idea is to propose a new library implementation to provide an API that will implement important data structures that are used in synthesis algorithms.

The objective of the thesis is to provide an efficient library to represent antichain data structures and implement different operation. An final goal is to be able to use this library within Aaron/Acacia.

\* Impl. datastructure antichain in Java \* Theoretical context: synthesis, universality and automata theory known problem \* Practical context: Owl, Acacia+ and AaPAL

\* Why interesting: More efficient than CTL symbolic with BDD

## Related Work

\* Java Built-in IMPL \* AaPAL \* Stackoverflow \* Acacia ?

## 3.4 Notions

### Model checking and synthesis

Model checking is the process to verify a software model with specifications.

Synthesis is the process to derive the system from specifications.

[?]

[FJR09]

### Data structures

#### Binary Decision Diagram

**Antichain** BDD vs antichain

Antichain vs pseudo-antichain

## 3.5 Questions

\* Maastricht library ? [BBFR] [Boh14a] \* How to references ? \* What to impl (about operations) ?

\* Is implementing LTL Rea. or Universality a final goal of the thesis ?

# Bibliography

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