

CS769 Advanced NLP

Latent Variable Models

Junjie Hu



Slides adapted from Graham
<https://junjiehu.github.io/cs769-spring23/>

Goal for Today

- **Variational Auto-encoder (VAE)**
 1. Maximize Evidence Lower Bound (ELBO)
 2. Reparamertizction trick
 3. Stabilizing Training
- **Discrete Latent Variable Models (LVM)**
 1. Enumerate
 2. Sampling
 3. Gumbel-softmax
- **LVM Applications in NLP**

Discriminative vs. Generative Models

- **Discriminative model:** calculate the probability of output given input $P(Y|X)$
- **Generative model:** calculate the probability of a variable $P(X)$, or multiple variables $P(X,Y)$
- Which of the following models are discriminative vs. generative?
 - Standard BiLSTM POS tagger
 - Language model

Types of Variables

- Observed vs. Latent:
 - **Observed:** something that we can see from our data, e.g. X or Y
 - **Latent:** a variable that we assume exists, but we aren't given the value
- Deterministic vs. Random:
 - **Deterministic:** variables that are calculated directly according to some deterministic function
 - **Random (stochastic):** variables that obey a probability distribution, and may take any of several (or infinite) values

Quiz: What Types of Variables?

- In the an attentional sequence-to-sequence model using MLE/teacher forcing, are the following variables observed or latent? deterministic or random?
 - The input word ids **f**
 - The encoder hidden states **h**
 - The attention values **a**
 - The output word ids **e**

Latent Variable Models

- A vector of latent variables \mathbf{z} in a high-dim space \mathcal{Z} which we can sample from some PDF $p(\mathbf{z})$ over \mathcal{Z}
- For every observed \mathbf{x} in our dataset, there is a \mathbf{z} that causes the model to generate \mathbf{x}
- A latent variable model (LVM) is a probability distribution over two sets of variables \mathbf{x}, \mathbf{z} :

$$p(\mathbf{x}|\mathbf{z}; \theta)$$

Maximum Likelihood Framework

- Maximize the probability of each x in the training set under the generative process according to:

$$p(\mathbf{x}) = \int p(\mathbf{x}|\mathbf{z}; \theta)p(\mathbf{z})d\mathbf{z}$$

Why LVM?

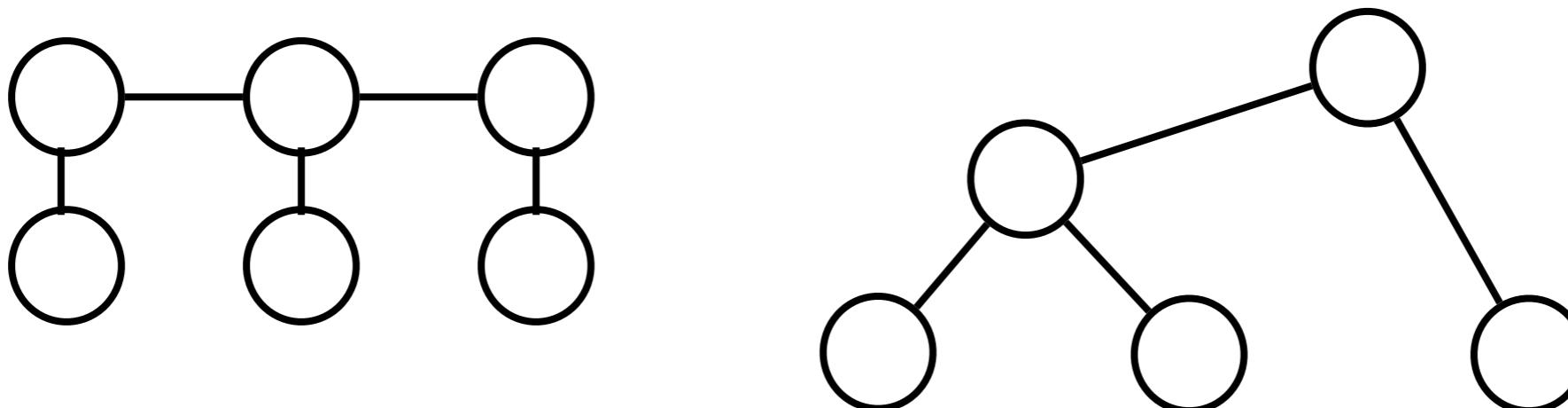
- Intuitively, latent variable z enables the model to first decide which property to generate before it assigns values to the output x
- Example:
 - Sample a LV z from the set $[0, \dots, 9]$ before generating an image for the digit
- Sample a sentiment from $\{\text{positive}, \text{negative}\}$ before generating a positive/negative movie review.

$z=2 \rightarrow x=$ 

$z=\text{"negative"} \rightarrow x= \text{"This is a bad movie."}$

What Types of Latent Variables?

- Latent continuous vector (e.g. variational auto-encoder)
- Latent discrete vector (e.g. topic model)
- Latent structure (e.g. HMM or tree-structured model)

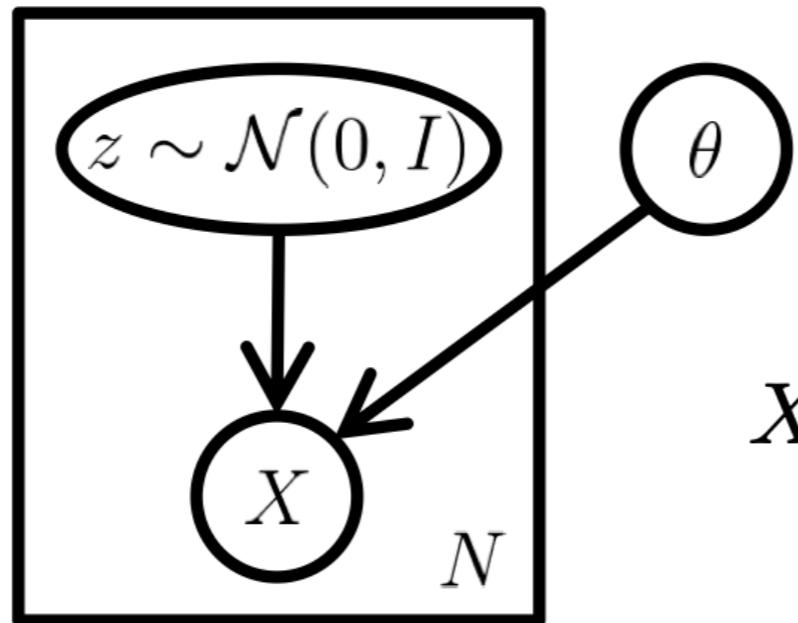


Variational Auto-encoders

(Kingma and Welling 2014)

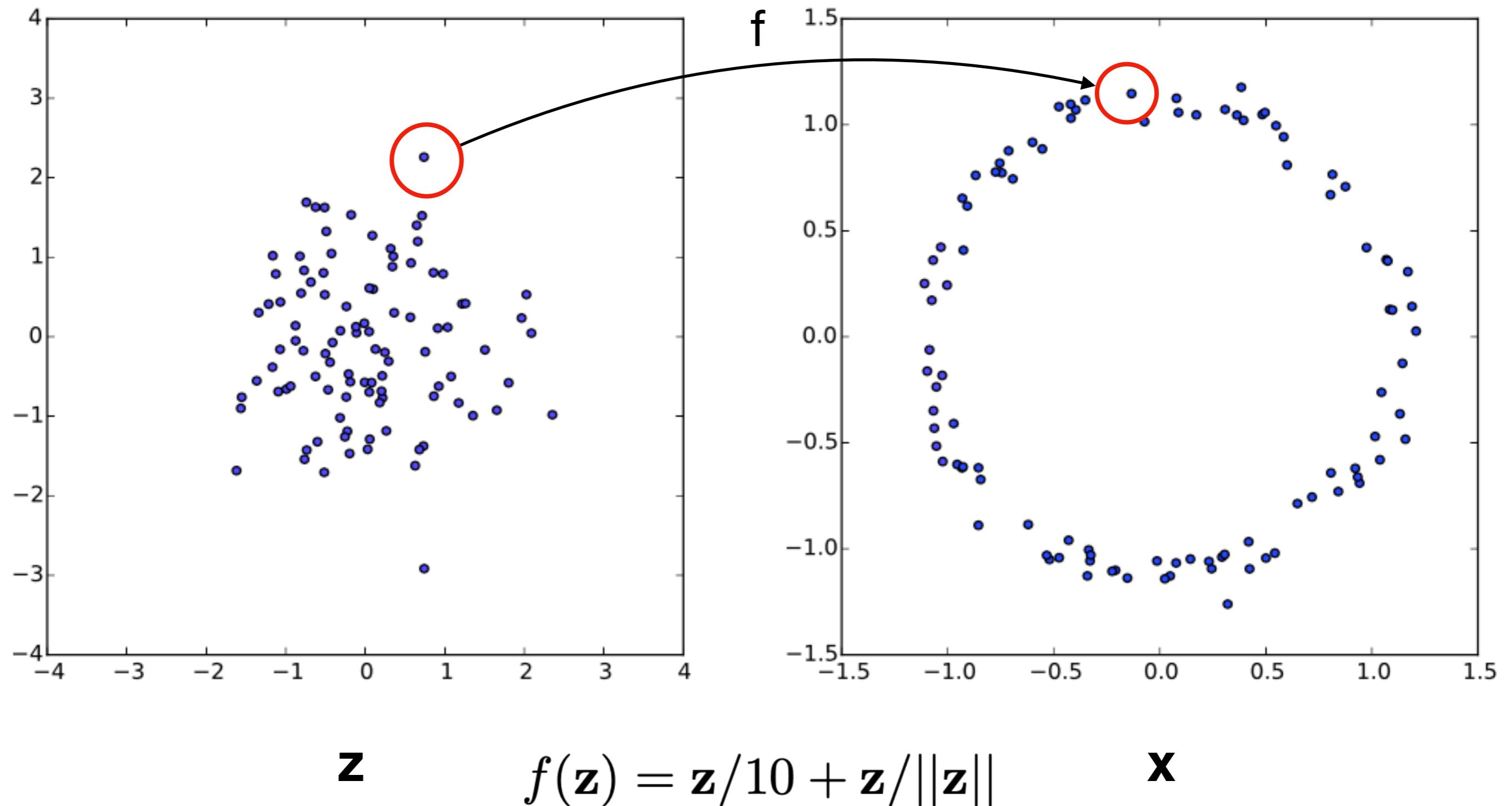
VAE as a Graphical Model

- We have an observed datapoint X in our dataset
- We have a latent variable \mathbf{z} sampled from a Gaussian
- We have a deterministic function $f(\mathbf{z}; \theta)$ that maps \mathbf{z} to the data space \mathcal{X} . If \mathbf{z} is a random variable (r.v.) in \mathcal{Z} , then $f(\mathbf{z}; \theta)$ is also a r.v. in \mathcal{X}
- If we repeat this sampling N times, we hope to approximate X by $f(\mathbf{z}; \theta)$

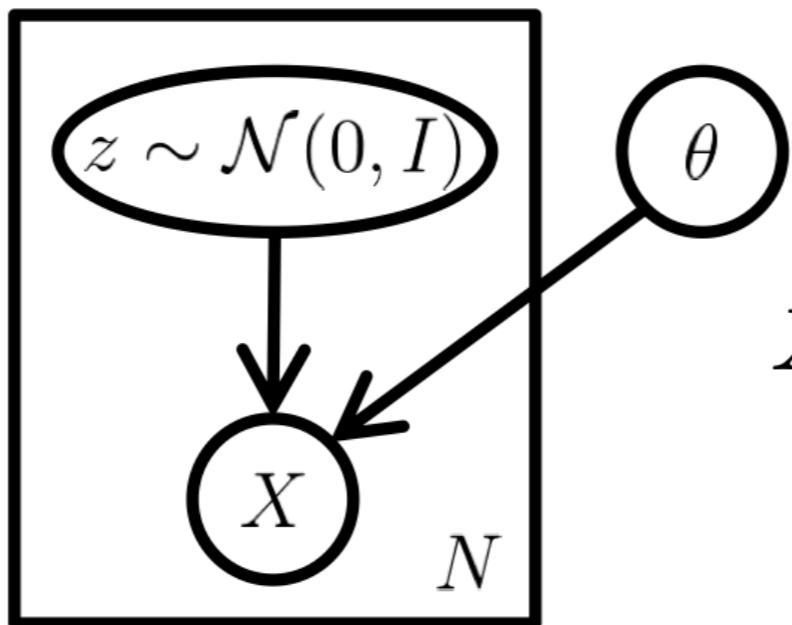


$$X \approx \frac{1}{N} \sum_{z \sim \mathcal{N}(0, I)} f(\mathbf{z}; \theta)$$

An Example (Goersch 2016)



A probabilistic perspective on Variational Auto-Encoder



$$X \approx \frac{1}{N} \sum_{z \sim \mathcal{N}(0, I)} f(\mathbf{z}; \theta)$$

- For each datapoint i :
 - Draw latent variables $z_i \sim p(z)$ (prior)
 - Draw data point $x_i \sim p_\theta(x|z)$
- Joint probability distribution over data and latent variables:
$$p(x, z) = p(z)p_\theta(x|z)$$
- Similar to Naive Bayes, HMM: only the prior differs

What is Our Loss Function?

- We would like to maximize the corpus log likelihood

$$\log P(\mathcal{X}) = \sum_{\mathbf{x} \in \mathcal{X}} \log P(\mathbf{x}; \theta)$$

- For a single example, the marginal likelihood is

$$P(\mathbf{x}; \theta) = \int P(\mathbf{x} | \mathbf{z}; \theta) P(\mathbf{z}) d\mathbf{z}$$

- We can approximate this by sampling **zs** then summing

$$P(\mathbf{x}; \theta) \approx \sum_{\mathbf{z} \in S(\mathbf{x})} P(\mathbf{x} | \mathbf{z}; \theta) \quad \text{where} \quad S(\mathbf{x}) := \{\mathbf{z}' ; \mathbf{z}' \sim P(\mathbf{z})\}$$

Variational Inference

Two tasks of interest:

- Learn the parameters θ of $p_\theta(x|z)$
- **Inference** over z with the *posterior* distribution: $p_\theta(z|x)$ given input x , what are its latent factors?

$$p_\theta(z|x) = \frac{p_\theta(x|z)p(z)}{p(x)}$$

$$p(x) = \int p(z)p_\theta(x|z)dz \text{ <- intractable}$$

- However, as $p(x)$ is intractable $p_\theta(z|x)$ is also intractable.
- Solution: **Variational inference** approximates the posterior $p_\theta(z|x)$ with a family of distributions $q_\phi(z|x)$

Variational Inference

- Variational inference approximates the true posterior $p_\theta(z|x)$ with a family of distributions $q_\phi(z|x)$
minimize : $\text{KL}(q_\phi(z|x) \parallel p_\theta(z|x))$
- One other target is to maximize the log-data likelihood which can be rewritten as:
$$\log p(x) = \text{ELBO} + \text{KL}(q_\phi(z|x) \parallel p_\theta(z|x))$$
- Combining the above two, this is equivalent to maximize the **Evidence Lower Bound (ELBO)**

$$\text{ELBO} = \mathbb{E}_{q_\phi(z|x)} [\log p_\theta(x|z)] - \text{KL}(q_\phi(z|x) \parallel p(z))$$

$$\text{KL}(q \parallel p) \geq 0 \Rightarrow \log p(x) \geq \text{ELBO}$$

Variational Inference

- Variational inference approximates the true posterior $p_\theta(z|x)$ with a family of distributions $q_\phi(z|x)$
minimize : $\text{KL}(q_\phi(z|x) || p_\theta(z|x))$
- One other target is to maximize the log-data likelihood which can be rewritten as:

$$\log p(x) = \text{ELBO} + \text{KL}(q_\phi(z|x) || p_\theta(z|x))$$

- **Evidence Lower Bound (ELBO)**

$$\text{ELBO} = \mathbb{E}_{q_\phi(z|x)} [\log p_\theta(x|z)] - \text{KL}(q_\phi(z|x) || p(z))$$

$$\text{KL}(q||p) \geq 0 \Rightarrow \log p(x) \geq \text{ELBO}$$

Variational Inference

- Variational inference approximates the true posterior $p_\theta(z|x)$ with a family of distributions $q_\phi(z|x)$

$$\text{minimize : } \text{KL}(q_\phi(z|x) || p_\theta(z|x))$$

$$\text{maximize: } \log p(x) = \text{ELBO} + \text{KL}(q_\phi(z|x) || p_\theta(z|x))$$

- Combining the above two objectives, it is equivalent to maximize ELBO

$$\text{maximize : ELBO}$$

Variational Auto-Encoders

$$\log p_{\theta}(\mathbf{x}) \geq \text{ELBO}$$

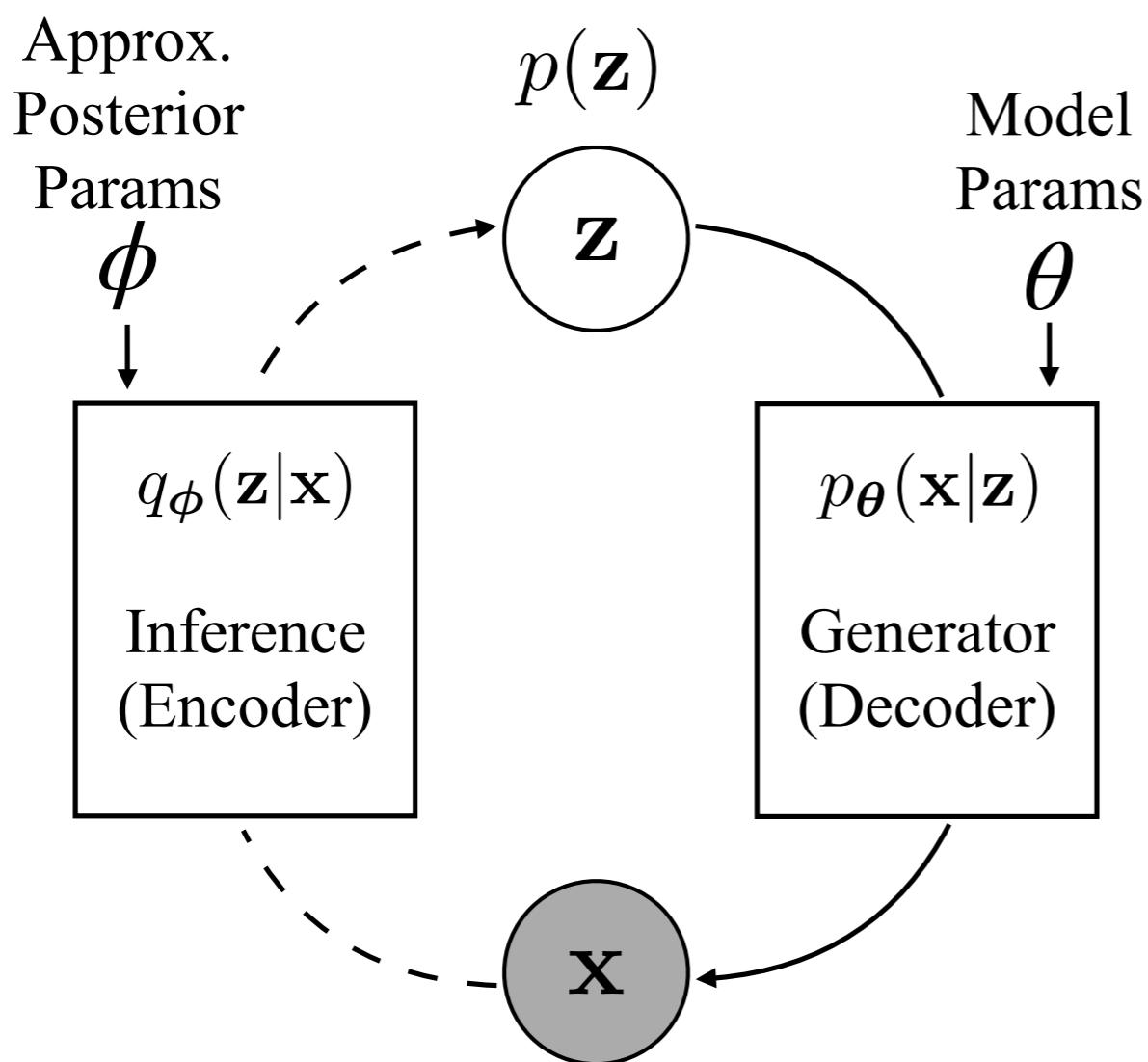
$$\underbrace{\mathbb{E}_{\mathbf{z} \sim q_{\phi}(\mathbf{z}|\mathbf{x})}[\log p_{\theta}(\mathbf{x}|\mathbf{z})]}_{\text{Reconstruction Loss}} - \underbrace{D_{\text{KL}}(q_{\phi}(\mathbf{z}|\mathbf{x}) \| p(\mathbf{z}))}_{\text{KL Regularizer}}$$

The inequality holds for any $q_{\phi}(z|x)$, but the lower bound is tight only if $p_{\theta}(z|x) = q_{\phi}(z|x)$

$p_{\theta}(z|x)$ is intractable

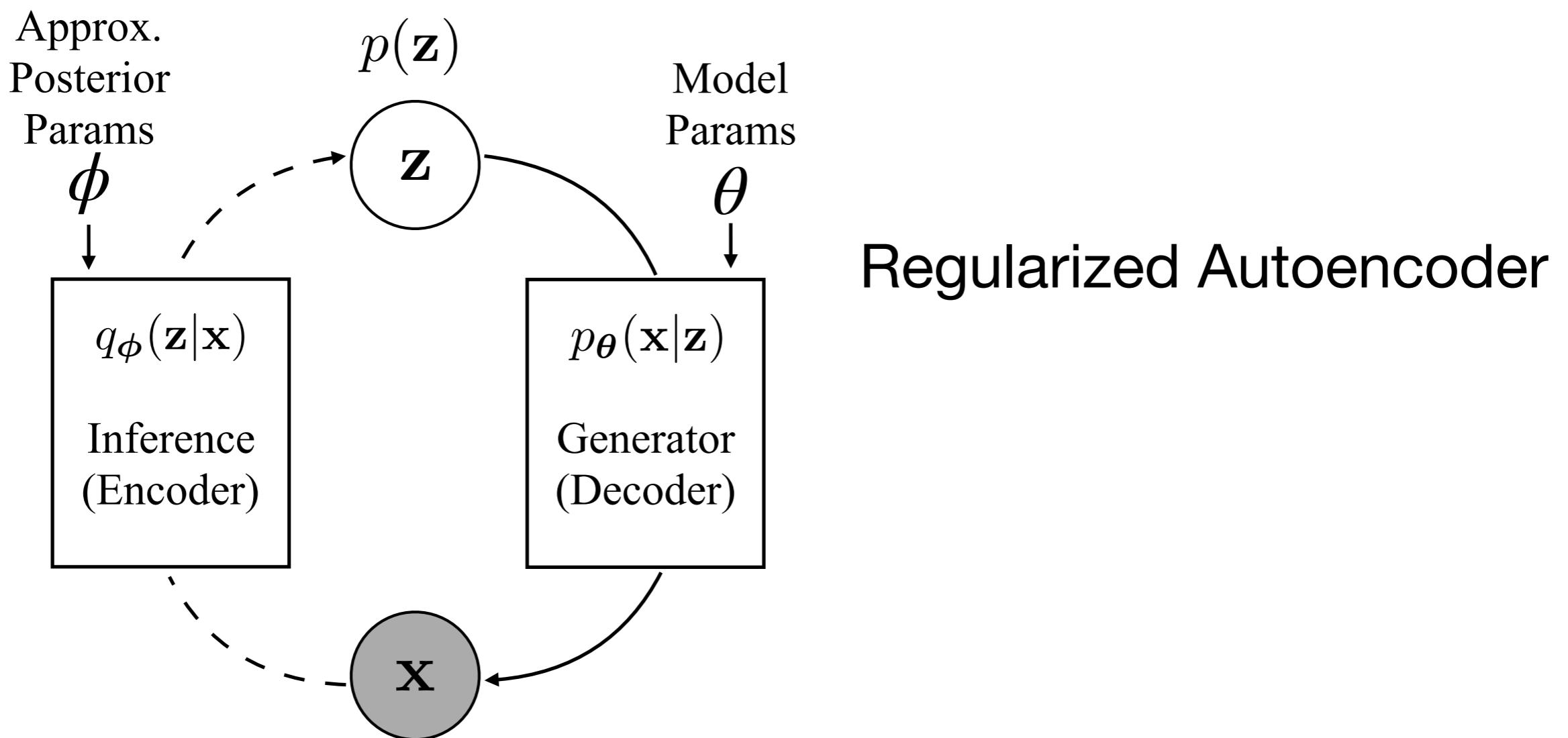
Variational Autoencoders

$$\log p_{\theta}(\mathbf{x}) \geq \underbrace{\mathbb{E}_{\mathbf{z} \sim q_{\phi}(\mathbf{z}|\mathbf{x})}[\log p_{\theta}(\mathbf{x}|\mathbf{z})]}_{\text{Reconstruction Loss}} - \underbrace{D_{\text{KL}}(q_{\phi}(\mathbf{z}|\mathbf{x})||p(\mathbf{z}))}_{\text{KL Regularizer}}$$



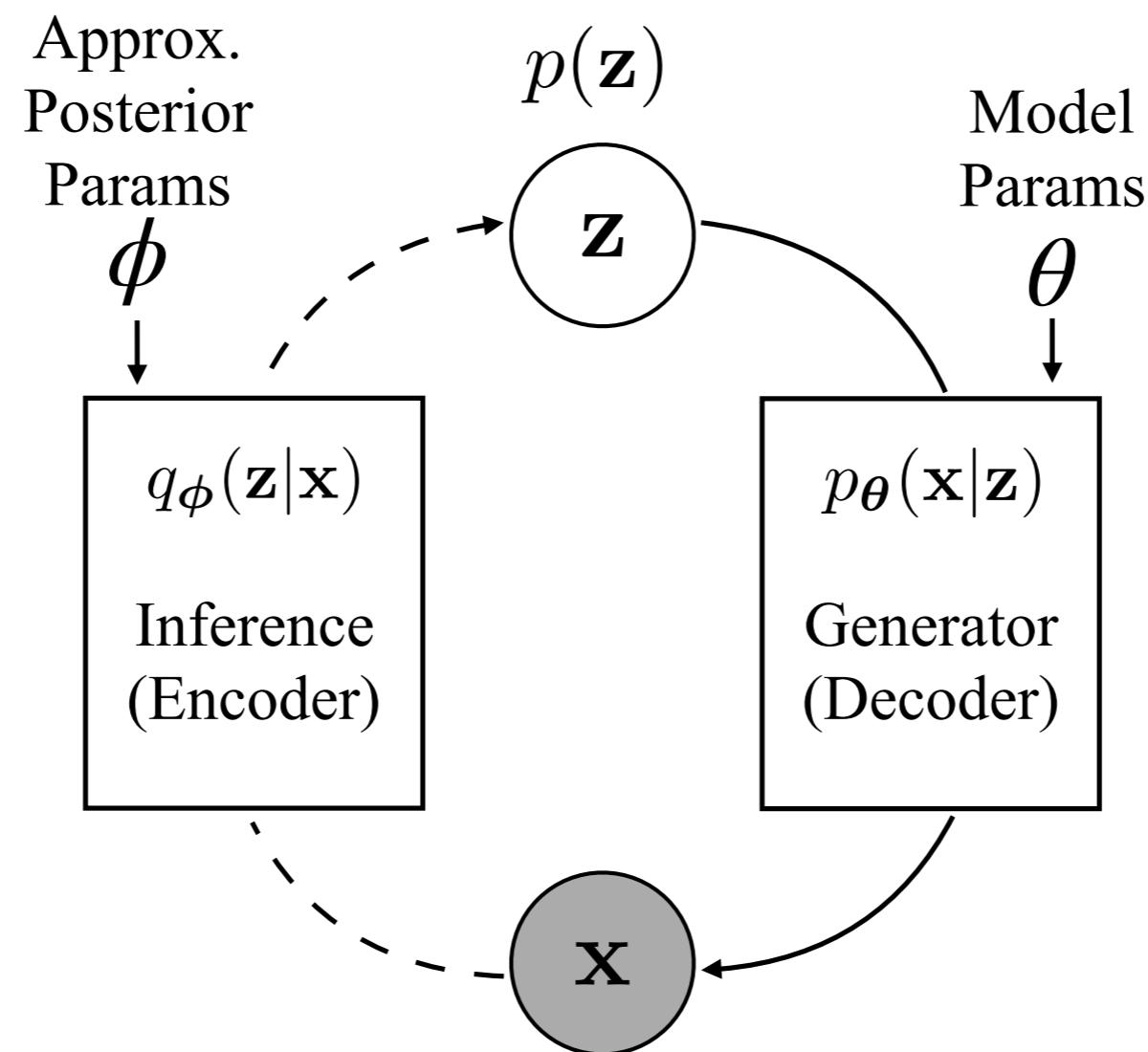
Variational Autoencoders

$$\log p_{\theta}(\mathbf{x}) \geq \underbrace{\mathbb{E}_{\mathbf{z} \sim q_{\phi}(\mathbf{z}|\mathbf{x})}[\log p_{\theta}(\mathbf{x}|\mathbf{z})]}_{\text{Reconstruction Loss}} - \underbrace{D_{\text{KL}}(q_{\phi}(\mathbf{z}|\mathbf{x})||p(\mathbf{z}))}_{\text{KL Regularizer}}$$



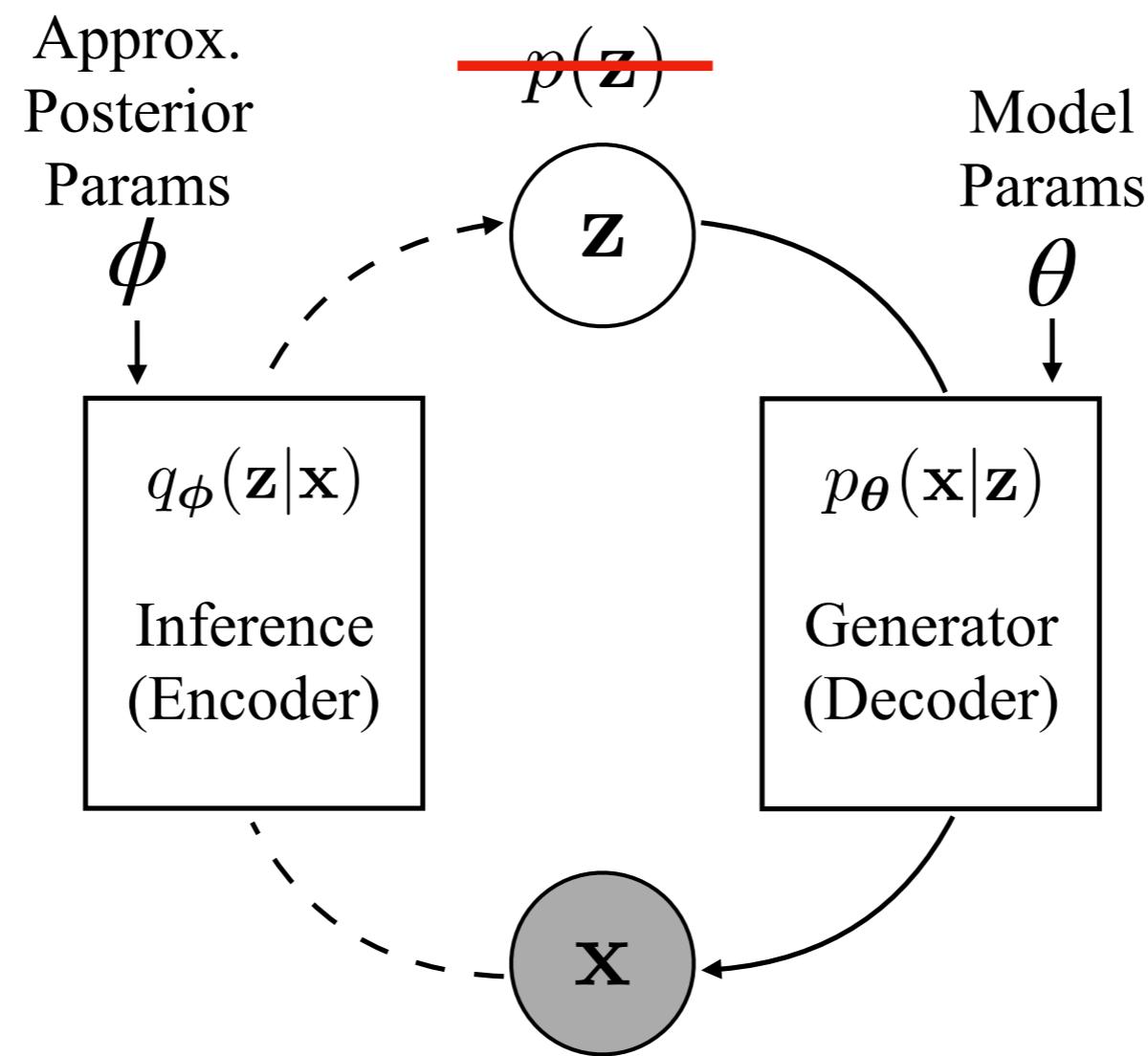
Why prior ?

$$\log p_{\theta}(\mathbf{x}) \geq \underbrace{\mathbb{E}_{\mathbf{z} \sim q_{\phi}(\mathbf{z}|\mathbf{x})}[\log p_{\theta}(\mathbf{x}|\mathbf{z})]}_{\text{Reconstruction Loss}} - \underbrace{D_{\text{KL}}(q_{\phi}(\mathbf{z}|\mathbf{x}) \| p(\mathbf{z}))}_{\text{KL Regularizer}}$$



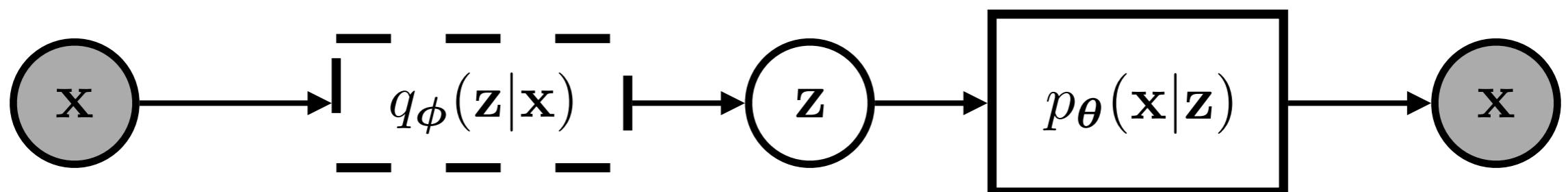
Why prior ?

$$\log p_{\theta}(\mathbf{x}) \geq \underbrace{\mathbb{E}_{\mathbf{z} \sim q_{\phi}(\mathbf{z}|\mathbf{x})} [\log p_{\theta}(\mathbf{x}|\mathbf{z})]}_{\text{Reconstruction Loss}} - \underbrace{D_{\text{KL}}(q_{\phi}(\mathbf{z}|\mathbf{x}) \| p(\mathbf{z}))}_{\text{KL Regularizer}}$$



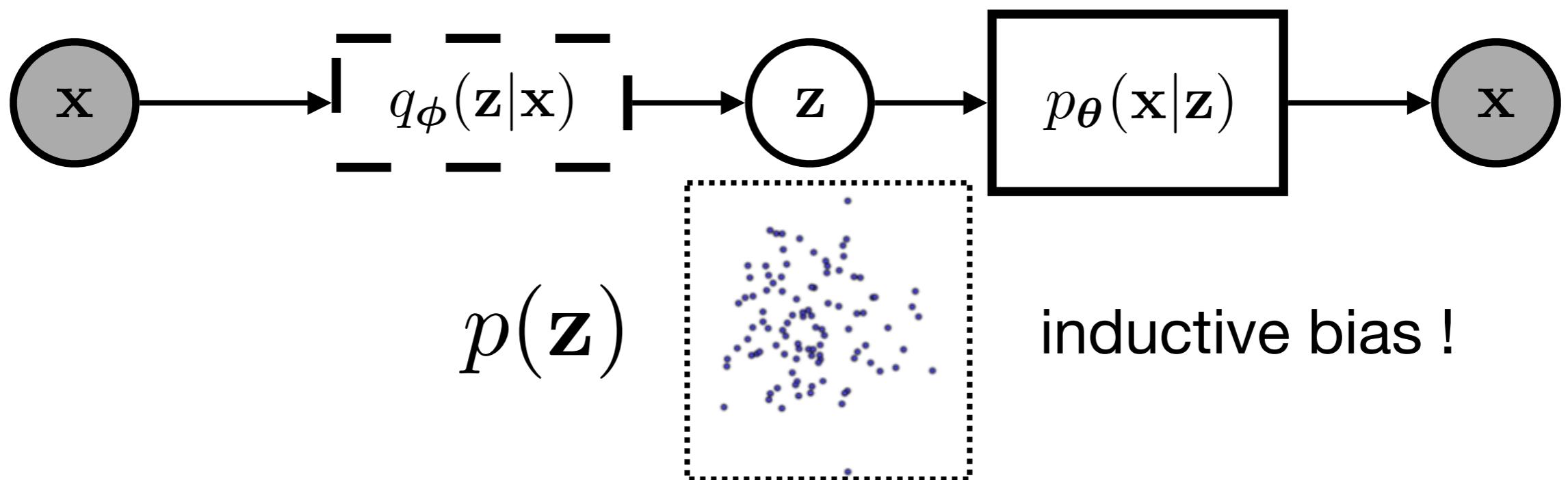
VAE vs. AE

VAE



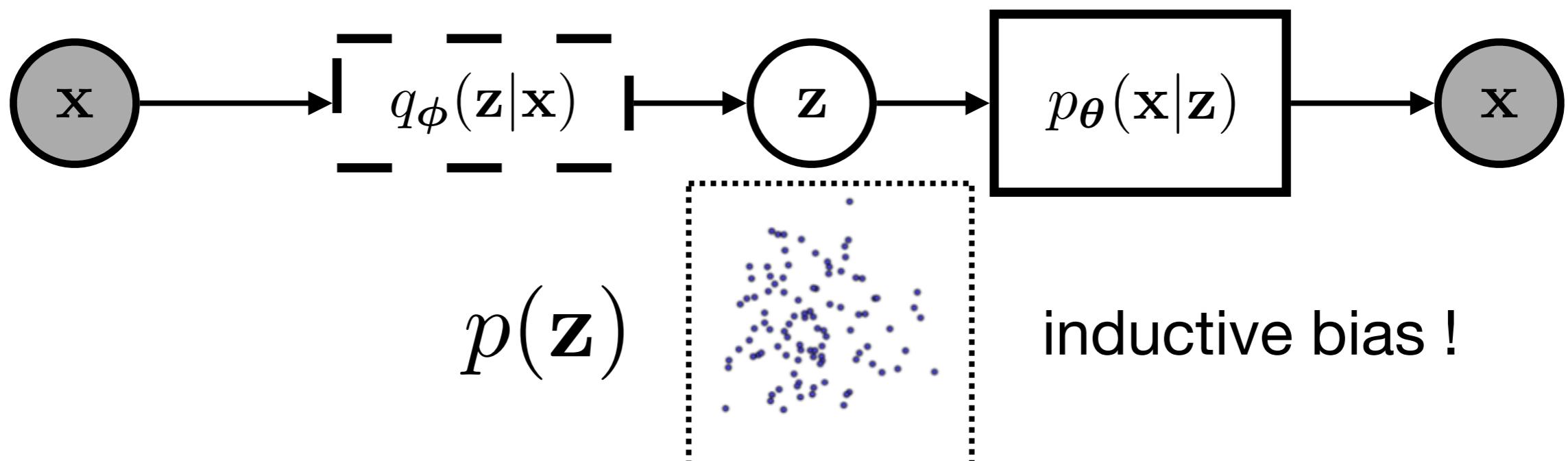
VAE vs. AE

VAE



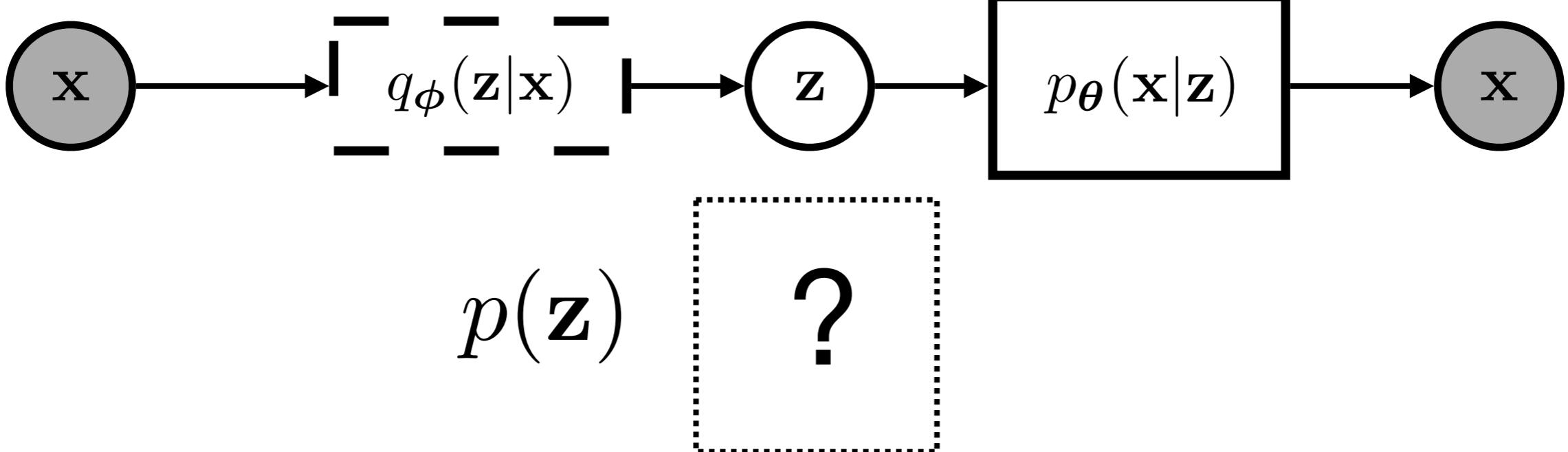
VAE vs. AE

VAE



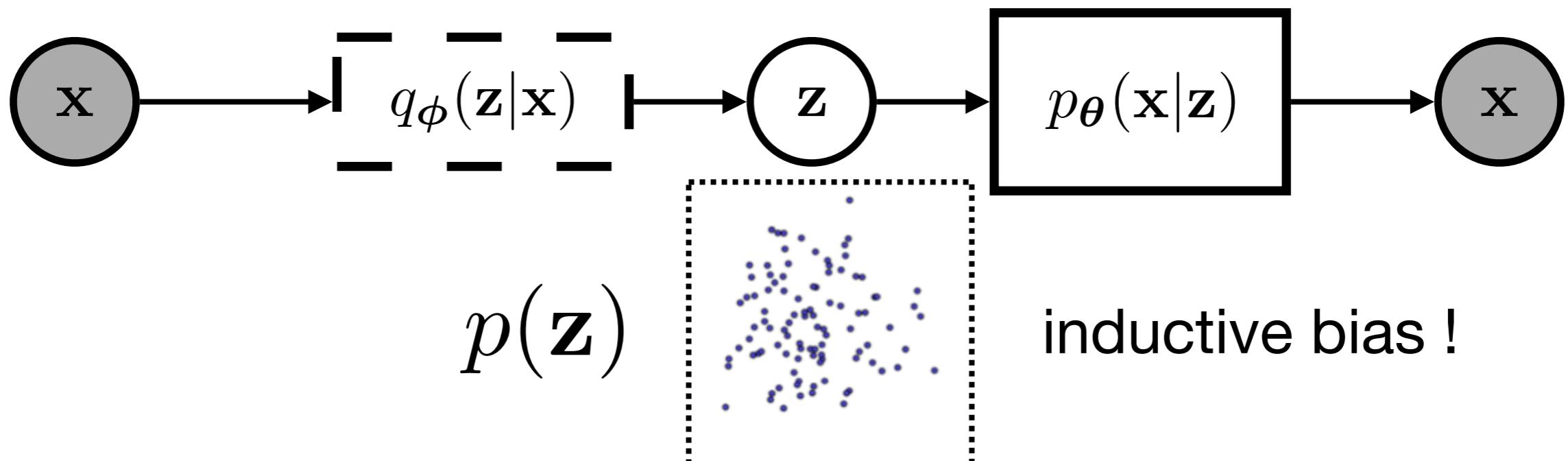
inductive bias !

AE



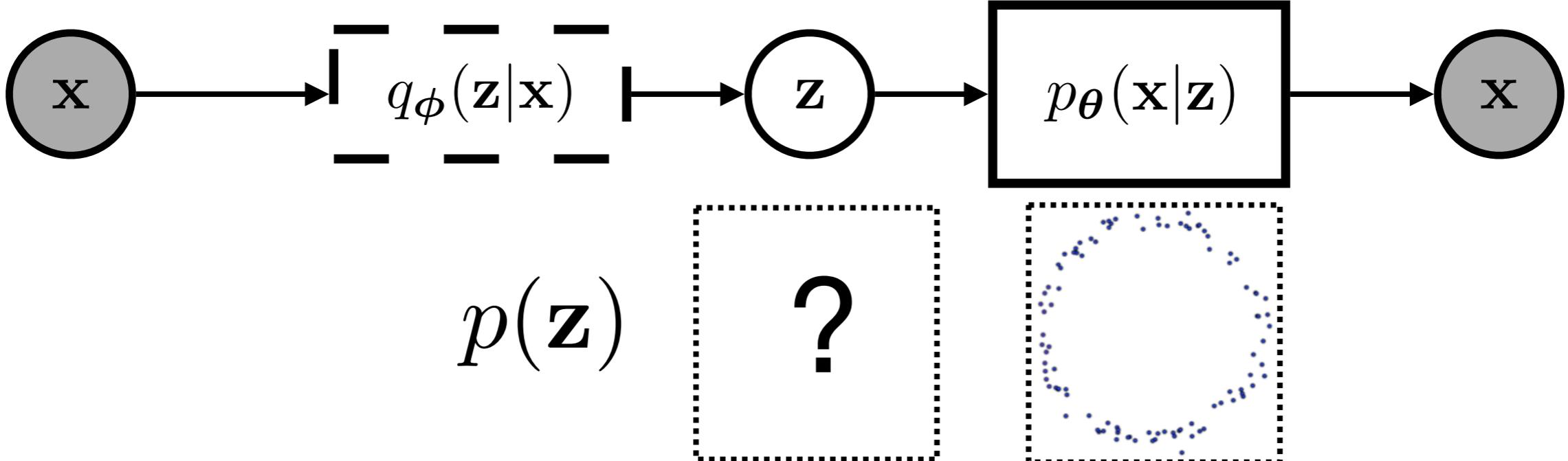
VAE vs. AE

VAE



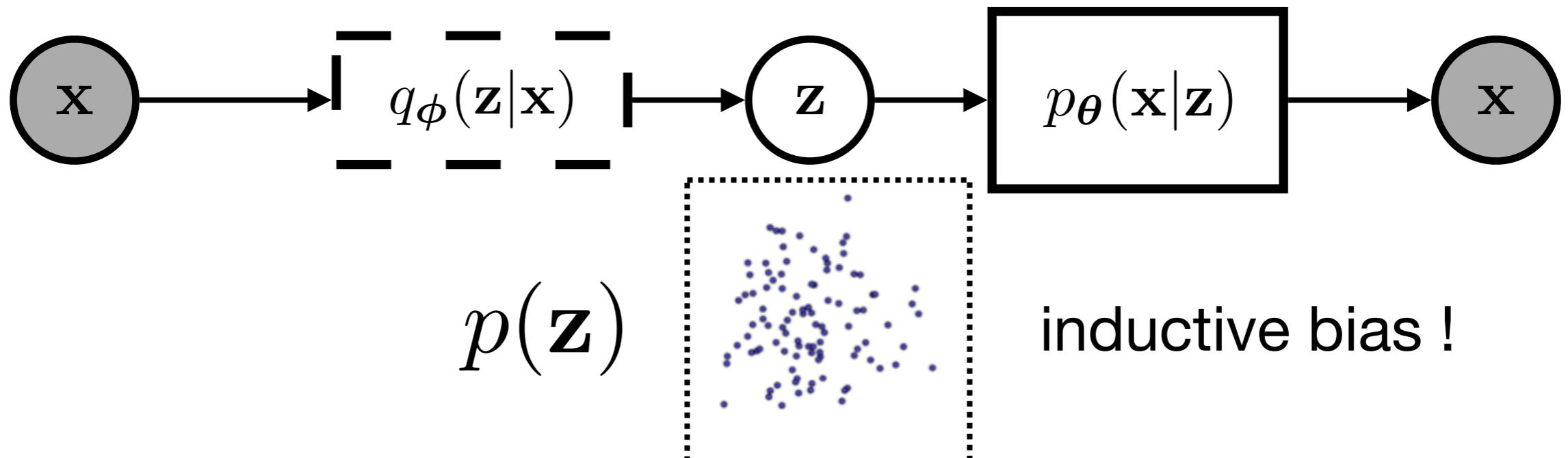
inductive bias !

AE



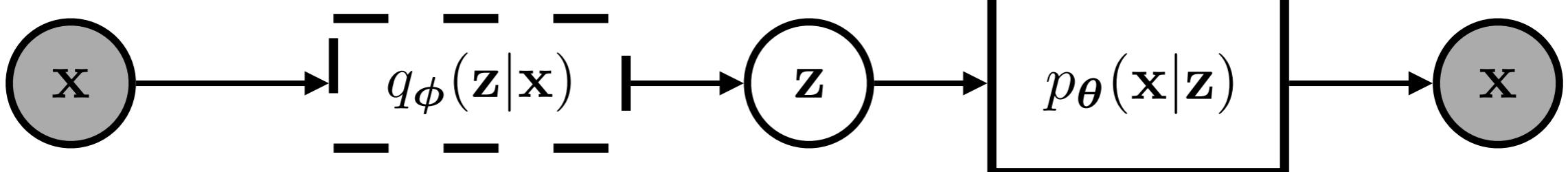
VAE vs. AE

VAE



inductive bias !

AE

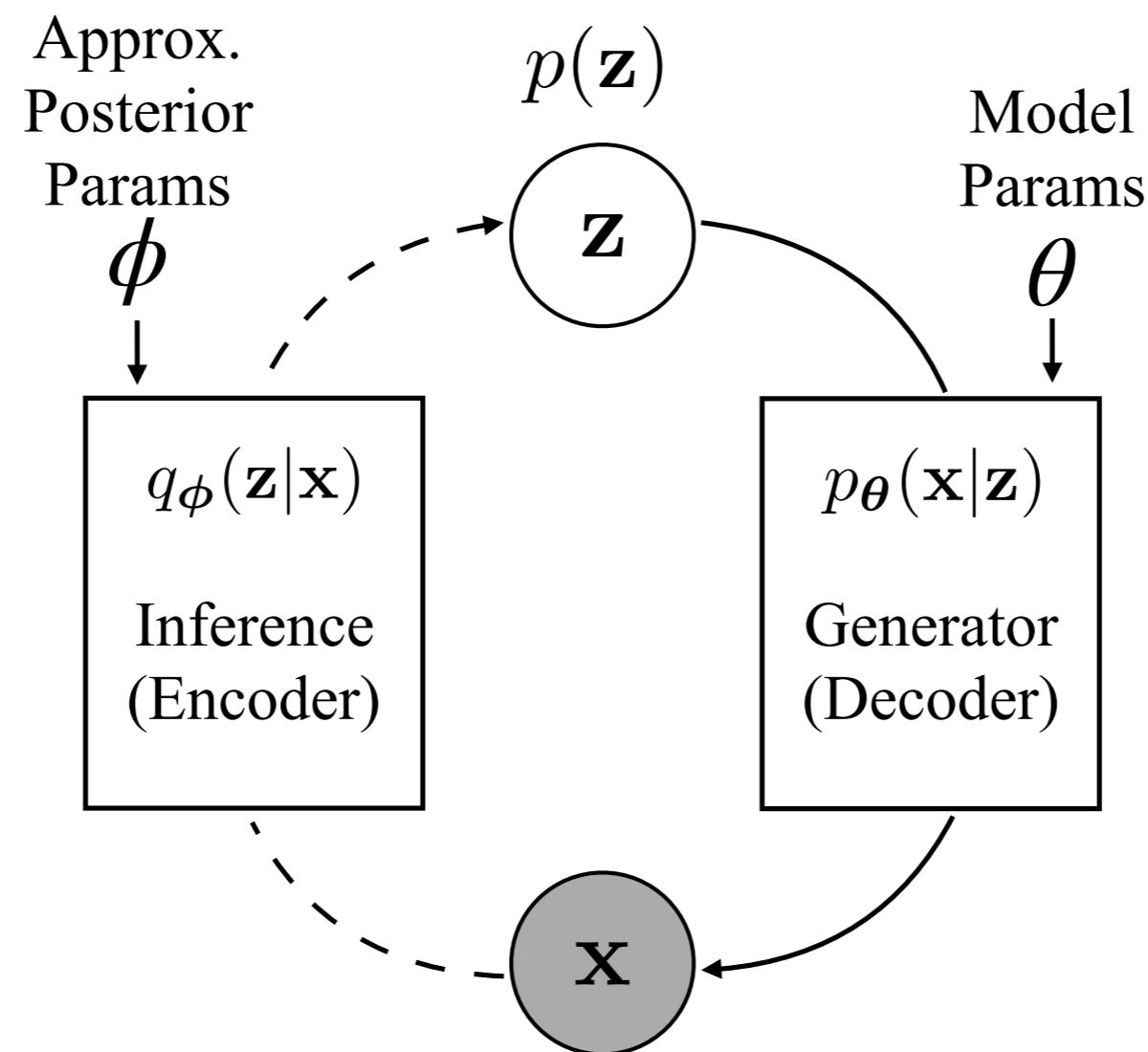


AE is not generative model:

- (1) Can't sample new data from AE
- (2) Can't compute the log likelihood of data x

Training of VAE

$$\log p_{\theta}(\mathbf{x}) \geq \underbrace{\mathbb{E}_{\mathbf{z} \sim q_{\phi}(\mathbf{z}|\mathbf{x})}[\log p_{\theta}(\mathbf{x}|\mathbf{z})]}_{\text{Reconstruction Loss}} - \underbrace{D_{\text{KL}}(q_{\phi}(\mathbf{z}|\mathbf{x}) \| p(\mathbf{z}))}_{\text{KL Regularizer}}$$



Problem!

Sampling Breaks Backprop

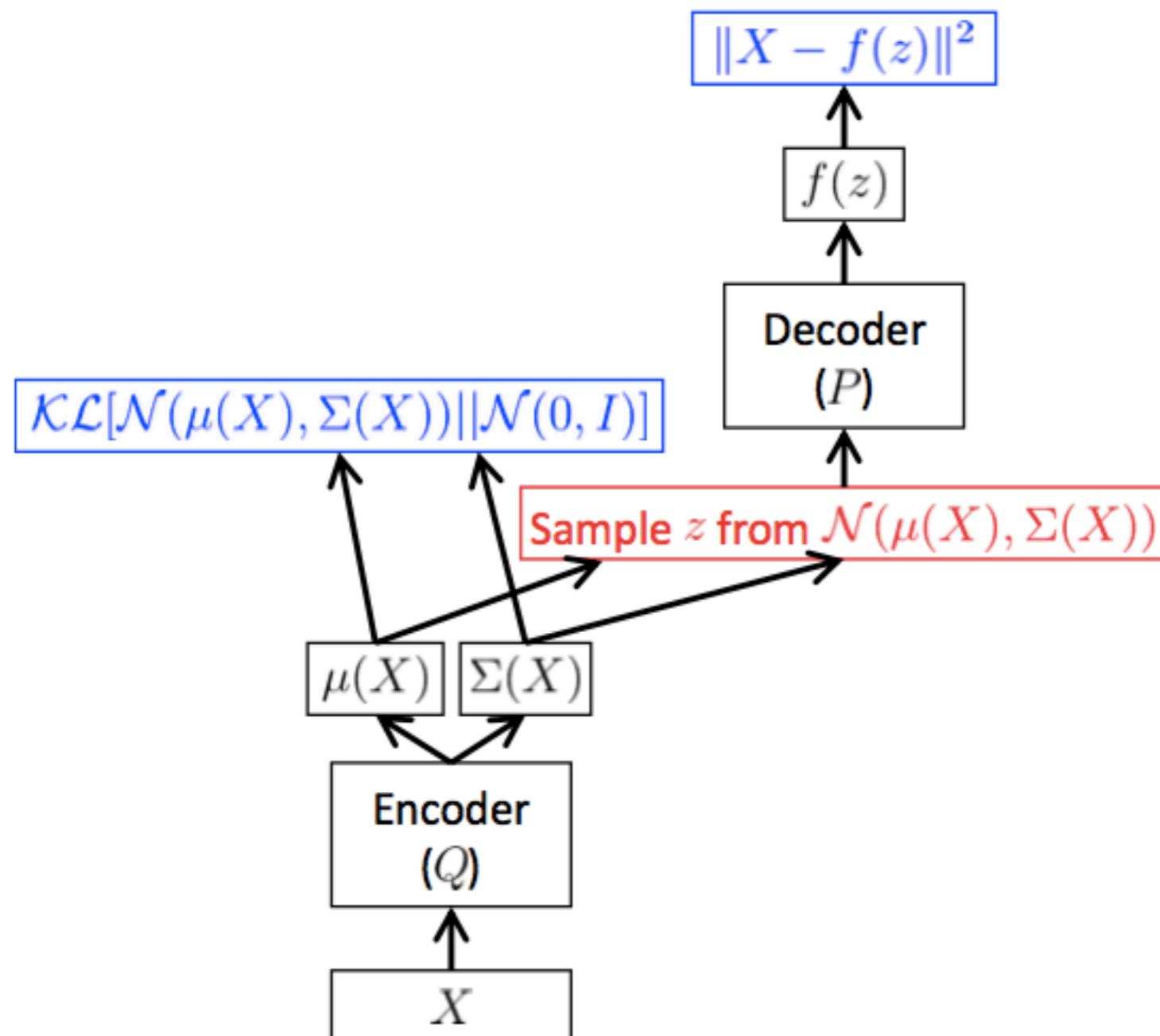


Figure Credit: Doersch (2016)

Solution: Re-parameterization Trick

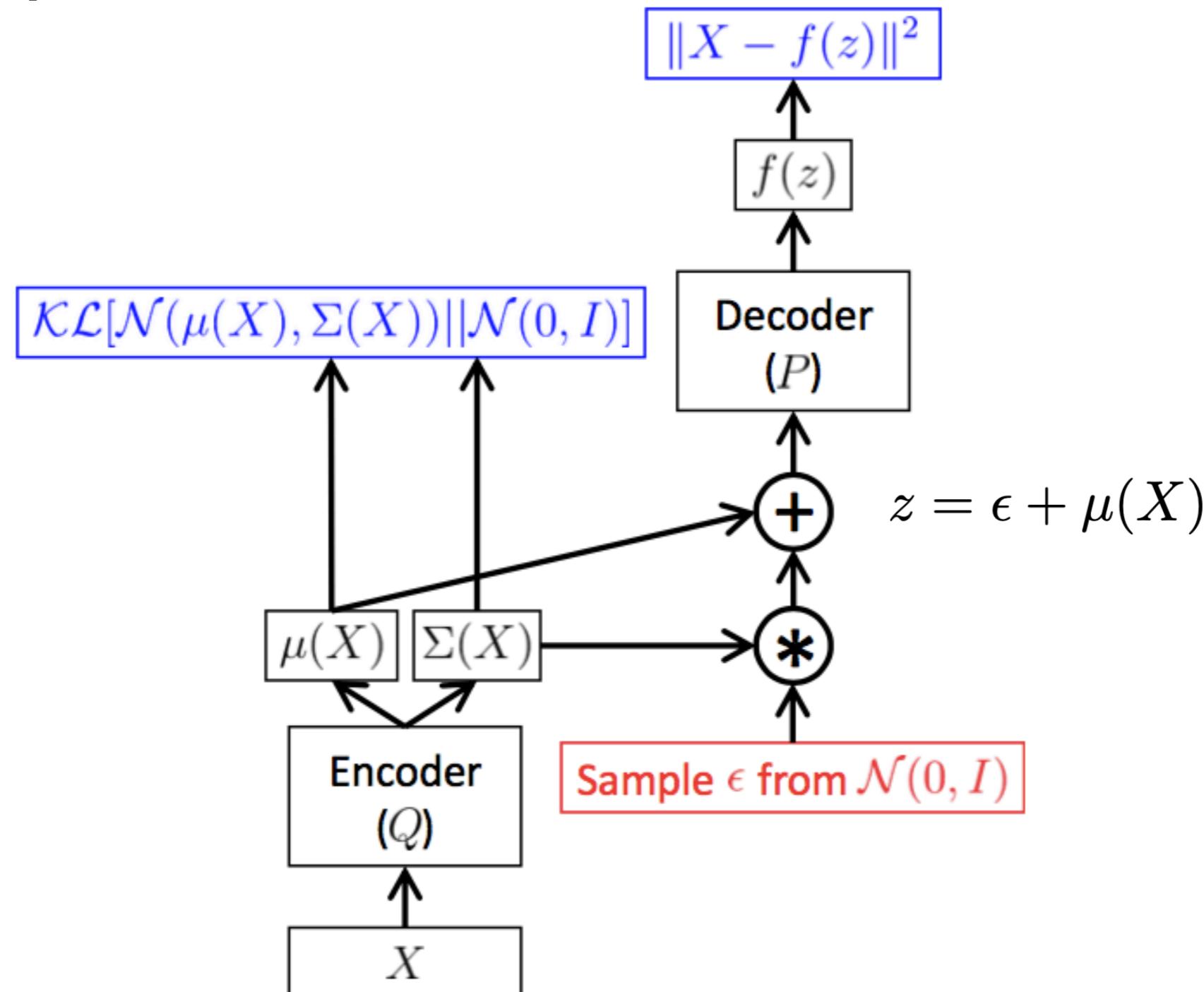


Figure Credit: Doersch (2016)

An Example: Generating Sentences w/ Variational Autoencoders

Generating from Language Models

- **Remember:** using ancestral sampling, we can generate from a normal language model

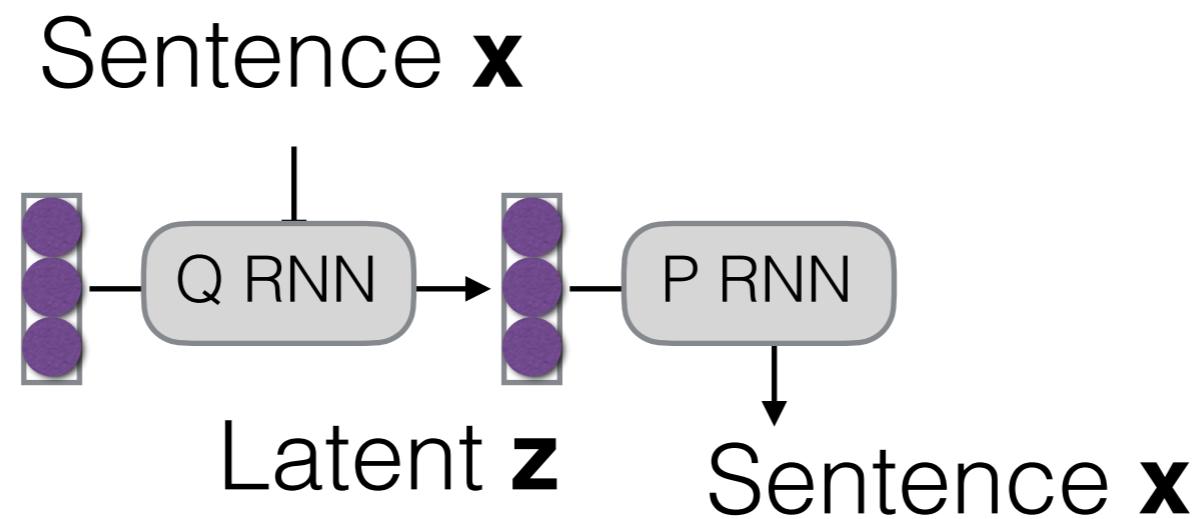
```
while  $x_{j-1} \neq "$ </s>" $":$   
 $x_j \sim P(x_j | x_1, \dots, x_{j-1})$ 
```

- We can also generate conditioned on something $P(\mathbf{y}|\mathbf{x})$ (e.g. translation, image captioning)

```
while  $y_{j-1} \neq "$ </s>" $":$   
 $y_j \sim P(y_j | X, y_1, \dots, y_{j-1})$ 
```

Generating Sentences from a Continuous Space (Bowman et al. 2015)

- The VAE-based approach is conditional language model that conditions on a latent variable \mathbf{z}
- Like an encoder-decoder, but latent representation is latent variable, input and output are identical



Motivation for Latent Variables

- Allows for a **consistent latent space** of sentences?
 - e.g. interpolation between two sentences

Standard encoder-decoder

i went to the store to buy some groceries .
i store to buy some groceries .
i were to buy any groceries .
horses are to buy any groceries .
horses are to buy any animal .
horses the favorite any animal .
horses the favorite favorite animal .
horses are my favorite animal .

VAE

“ i want to talk to you . ”
“i want to be with you . ”
“i do n’t want to be with you . ”
“i do n’t want to be with you . ”
she did n’t want to be with him .

he was silent for a long moment .
he was silent for a moment .
it was quiet for a moment .
it was dark and cold .
there was a pause .
it was my turn .

- **More robust to noise?** VAE can be viewed as standard model + regularization.

Difficulties in Training

- Of the two components in the VAE objective, the KL divergence term is much easier to learn!

$$\mathbb{E}_{z \sim Q(z|x)} [\log P(x | z)] - \mathcal{KL}[Q(z | x) || P(z)]$$

Requires good
generative model

Just need to
set the mean/variance
of Q to be same as P

- Results in the model learning to rely solely on decoder and ignore latent variable ($P(x|z) = P(x)$)
-> **Posterior Collapse**

Solution 1: KL Divergence Annealing

- Basic idea: Multiply KL term by a constant λ starting at zero, then gradually increase to 1
- Result: model can learn to use **z** before getting penalized

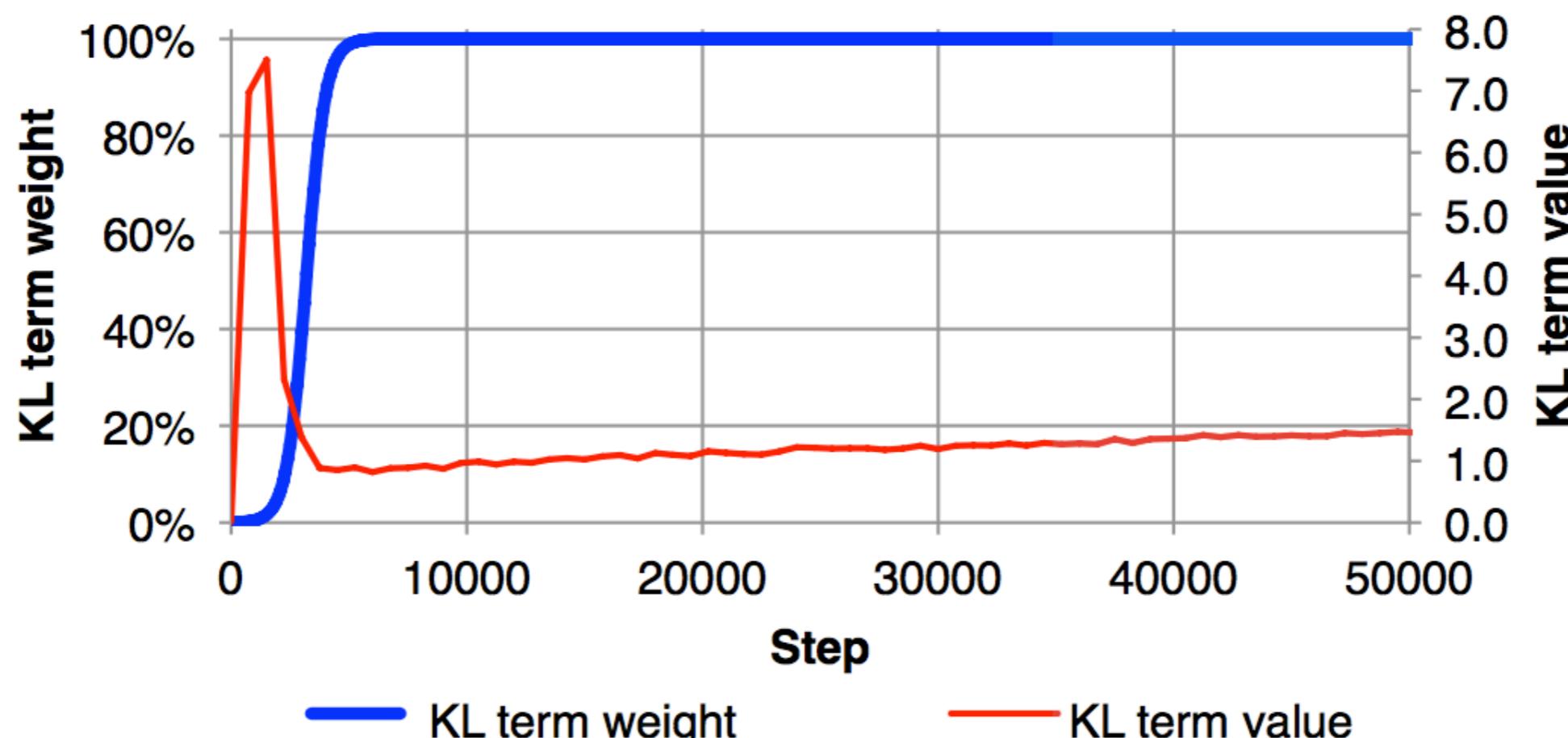


Figure Credit: Bowman et al. (2017)

Solution 2: Free bits / KL thresholding

- Free bits replaces the KL term in ELBO with a hinge loss that maximize each component of the original KL with a constant:

$$\sum_i \max[\lambda, D_{\text{KL}}(q_{\phi}(z_i|x) || p(z_i))]$$

- λ :Target rate

Solution 3: Weaken the Decoder

- But theoretically still problematic: it can be shown that the optimal strategy is to ignore \mathbf{z} when it is not necessary (Chen et al. 2017)
- Solution: weaken decoder $P(\mathbf{x}|\mathbf{z})$ so using \mathbf{z} is essential
 - Use word dropout to occasionally skip inputting previous word in \mathbf{x} (Bowman et al. 2015)
 - Use a convolutional decoder w/ limited context (Yang et al. 2017)

Solution 4:

Aggressive Inference Network Learning

$$\max_{\theta, \phi} \underbrace{\log p_{\theta}(\mathbf{x})}_{\text{marginal log data likelihood}} - \underbrace{D_{\text{KL}}(q_{\phi}(\mathbf{z}|\mathbf{x})|p_{\theta}(\mathbf{z}|\mathbf{x}))}_{\text{agreement between approximate and model posteriors}}$$



$$\max_{\theta} \max_{\phi} \underbrace{\log p_{\theta}(\mathbf{x})}_{\text{marginal log data likelihood}} - \underbrace{D_{\text{KL}}(q_{\phi}(\mathbf{z}|\mathbf{x})|p_{\theta}(\mathbf{z}|\mathbf{x}))}_{\text{agreement between approximate and model posteriors}}$$

(He et al. 2019)

Handling Discrete Latent Variables

Discrete Latent Variables?

- Many variables are better treated as discrete
 - Part-of-speech of a word
 - Class of a question
 - Writer traits (left-handed or right-handed, etc.)
- How do we handle these?

Method 1: Enumeration

- For discrete variables, our integral is a sum

$$P(\mathbf{x}; \theta) = \sum_{\mathbf{z}} P(\mathbf{x} \mid \mathbf{z}; \theta) P(\mathbf{z})$$

- If the number of possible configurations for \mathbf{z} is small, we can just sum over all of them

Method 2: Sampling

- Randomly sample a subset of configurations of \mathbf{z} and optimize with respect to this subset
- Various flavors:
 - Minimum risk training
 - Maximize ELBO loss
- Score function gradient estimator - Policy Gradient Method
 - Unbiased estimator but high variance - need to control variance

Method 3: Reparameterization

(Maddison et al. 2017, Jang et al. 2017)

- Reparameterization also possible for discrete variables!

Original Categorical Sampling Method:

$$\hat{z} = \text{cat-sample}(P(z | x))$$

Reparameterized Method

$$\hat{z} = \operatorname{argmax}(\log P(z | x) + \text{Gumbel}(0,1))$$

where the Gumbel distribution is

$$\text{Gumbel}(0, 1) = -\log(-\log(\text{Uniform}(0,1)))$$

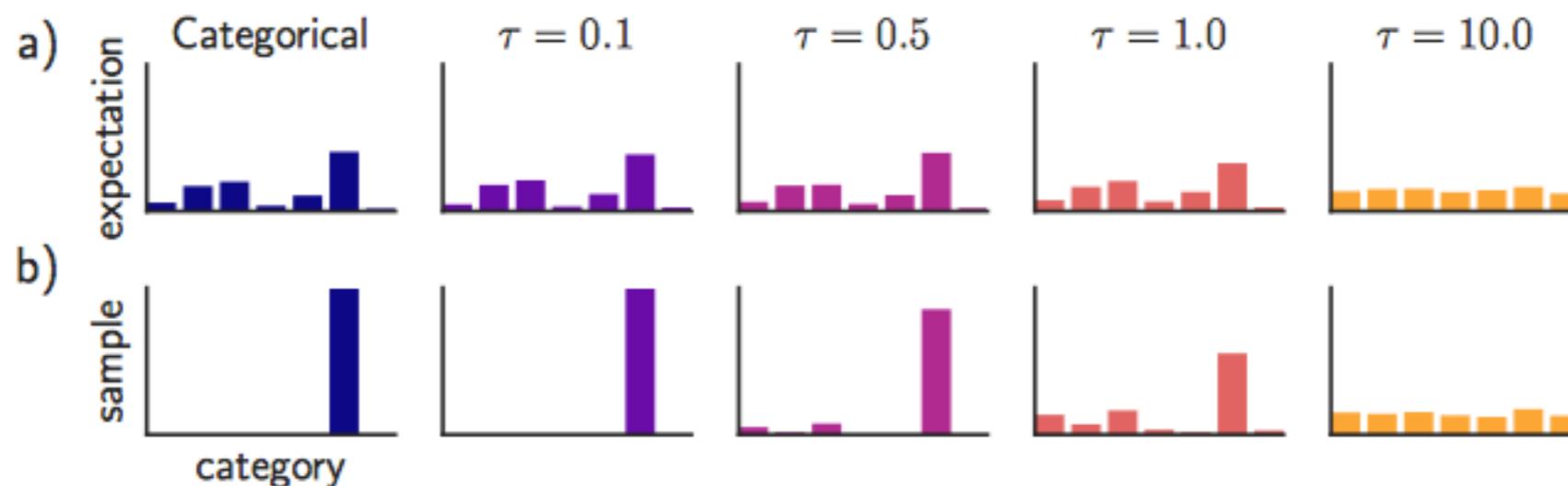
- Backprop is still not possible, due to argmax

Gumbel-Softmax

- A way to soften the decision and allow for continuous gradients
- Instead of argmax, take softmax with temperature τ

$$\hat{\mathbf{z}} = \text{softmax}((\log P(\mathbf{z} \mid \mathbf{x}) + \text{Gumbel}(0,1))^{1/\tau})$$

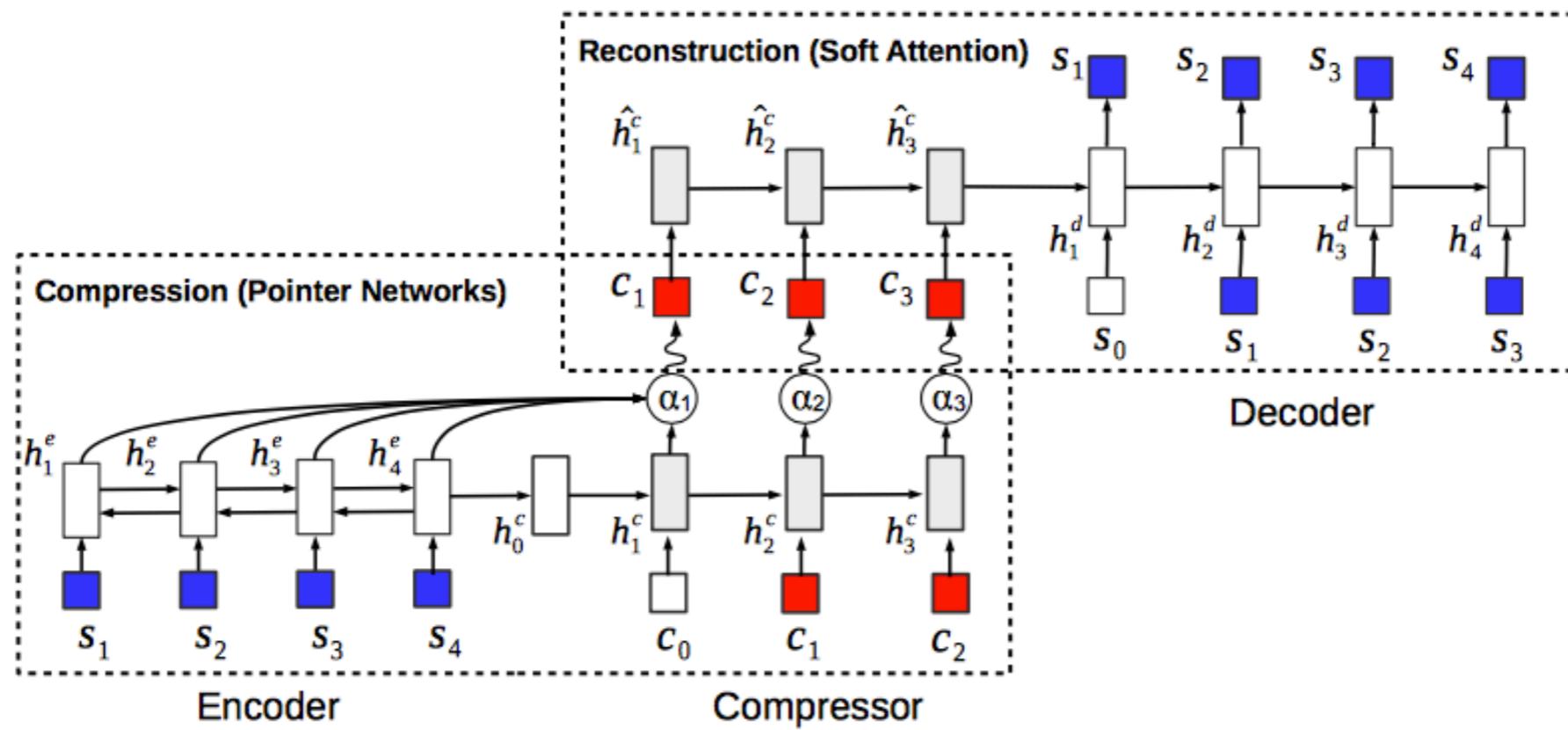
- As τ approaches 0, will approach max



Application Examples in NLP

Symbol Sequence Latent Variables (Miao and Blunsom 2016)

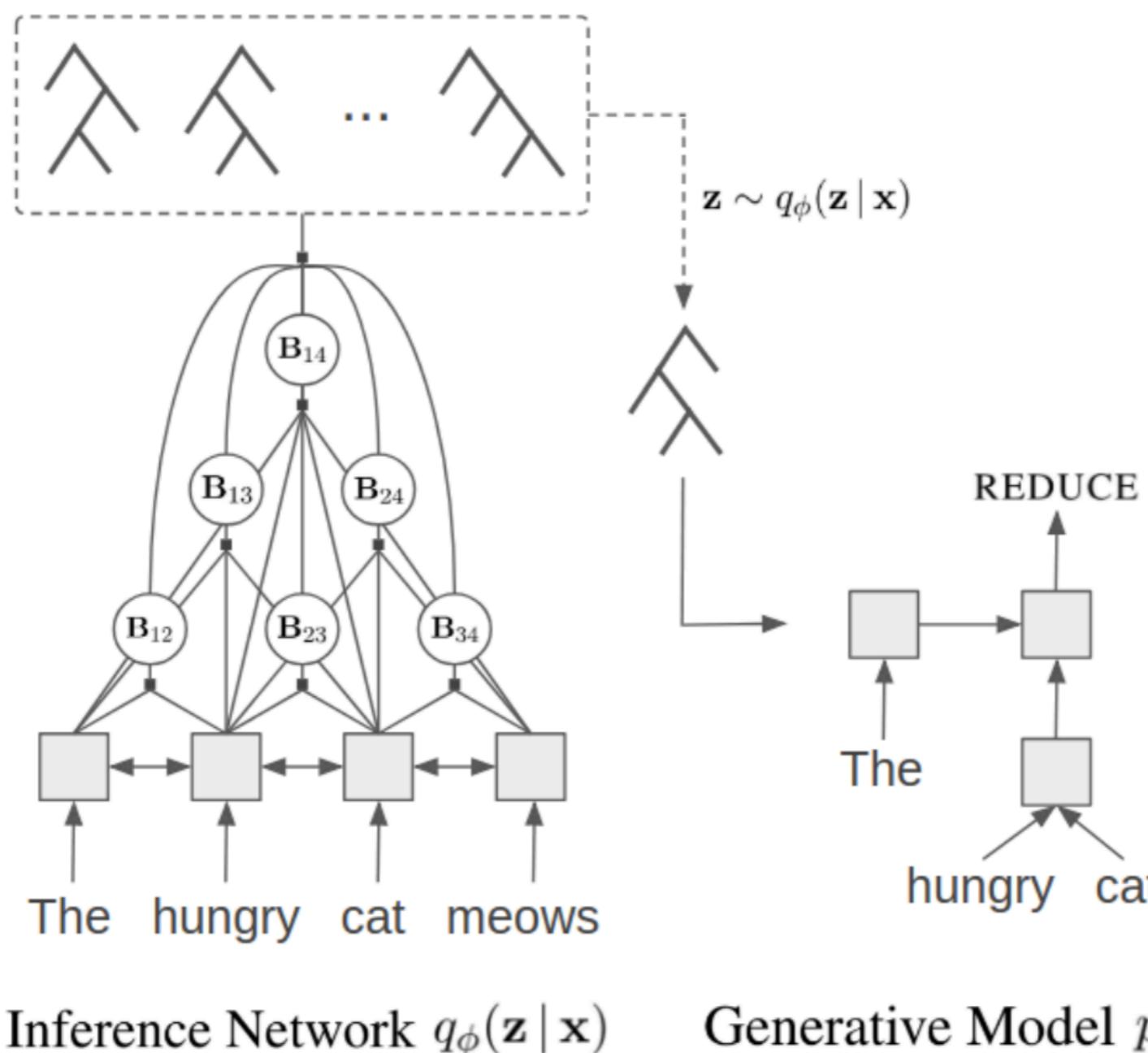
- Encoder-decoder with a sequence of latent symbols



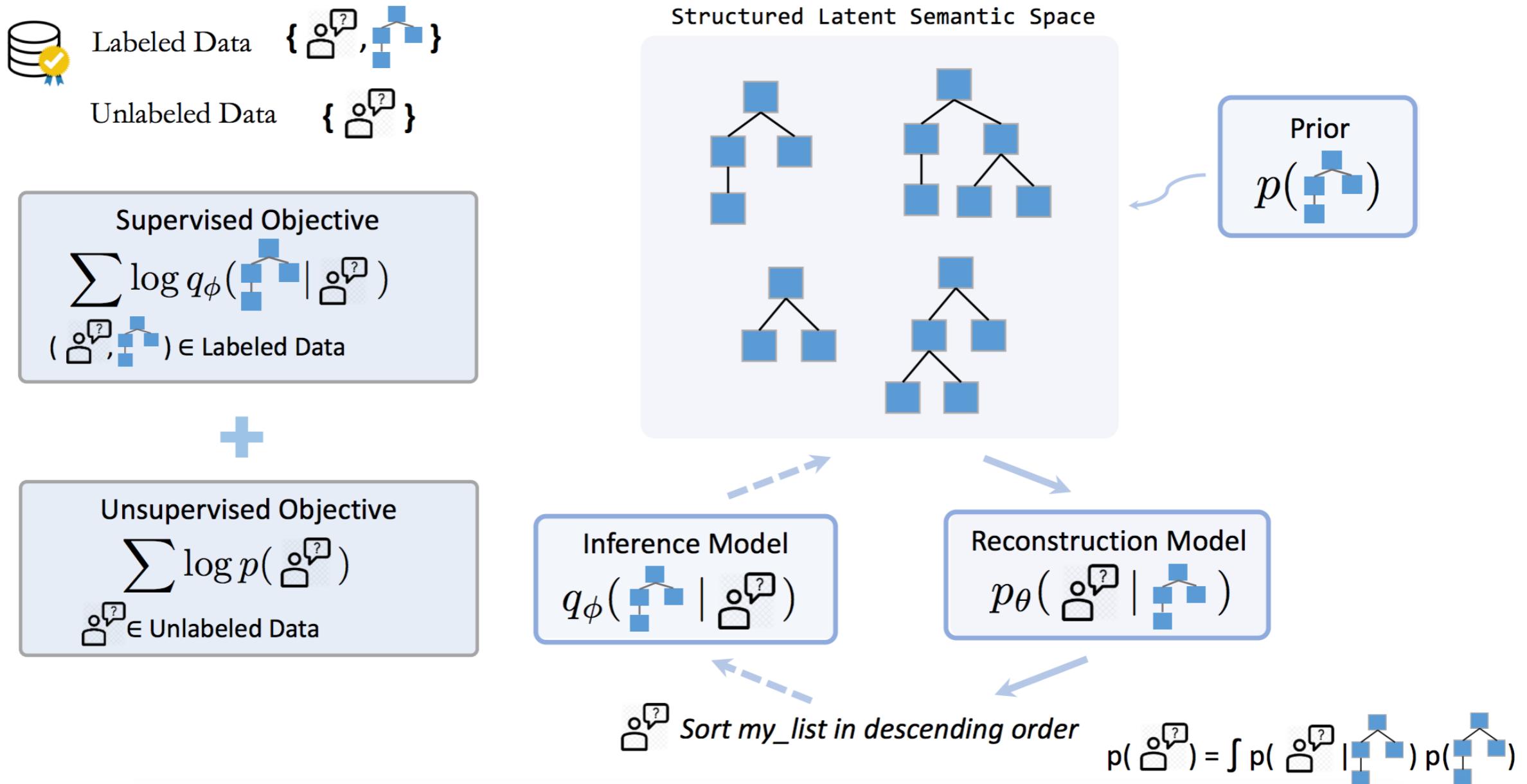
- Summarization in Miao and Blunsom (2016)
- Attempts to “discover” language (e.g. Havrylov and Titov 2017)
 - But things may not be so simple! (Kottur et al. 2017)

Unsupervised Recurrent Neural Network Grammars

(Kim et al., 2019)



STRUCTVAE: Tree-structured Latent Variable Models for Semi-supervised Semantic Parsing (Yin et al. 2018)



Questions?