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The Impact of Covid-19 on Cryptocurrency

: Spanning Test of Bitcoin

1. Introduction

Bitcoin is a decentralized digitized means of transaction money based on blockchain technology. Bitcoin was created by Nakamoto in 2009 out of the need to maintain an objective monetary value regardless of government control. Since the 2007-2008 global financial crisis, each country has increased money supply through Quantitative Easing (QE), and a sharp increase in fiat money has caused strong distrust in the government's monetary control measure and raised concerns over inflation. Therefore, the need for a new currency that is independent of the government's supply could not be overemphasized. Unfortunately, since the Covid-19 pandemic in 2020, QE has led to currency depreciation and increased expectation of inflation. Recently, the value of Bitcoin has risen sharply incomparably with the previous period.

These expectations for Bitcoin have made comparisons with gold, a traditional safe-haven asset. Views on this notion have varied from negative to positive. The high volatility of returns has made it a risky asset. (Choi, Shin, 2021). On the other hand, some scholars have expressed a positive view of Bitcoin highlighting that it is not influenced by regulators, hence the payment system is more flexible and private and is suitable for complementing emerging market currencies. It is against this background that debate has arisen on whether Bitcoins can hedge against inflation from the government's QE policy and whether they can play a safe-haven role in the market crisis and uncertainties.

Since Markowitz (1959) expressed the foundations for the modern portfolio theory, investors have come to recognize that a correlation between assets is more important than the volatility of assets, and they realize that volatility can be reduced without compromising returns. Bitcoin can be a very risky investment due to its high volatility, but under Markowitz theory, the risk can be adjusted through its correlation with other assets. So far, stocks and real estate have a high correlation with each other, and

stocks and bonds have a low correlation of 0.2 ~ 0.3. However, Bitcoin has very high volatility even in its short history and has a correlation of 0.2 ~ 0.3 compared to the existing stock portfolio, and that correlation is getting lower now (Buttonwood, 2021). On the other hand, it shows profitability that exceeds that of bonds. Although past data cannot describe future expectations, these characteristics of Bitcoin can make it a very attractive alternative for portfolio managers.

Given the, another question also rises on whether these other characteristics will make Bitcoin a completely different asset from traditional assets such as stocks, bonds, commodities, gold, and real estate? In other words, if Bitcoin is added to an existing portfolio of assets, can it improve the portfolio output? How can this be verified? In this regard, this study examines whether new risky assets can expand investment opportunities for other sets of risky assets and examine that when investors choose a portfolio based on mean-variance analysis, they can expand minimum-variance boundaries in each set of risky assets (Huberman & Kandel, 1987).

The structure of this paper is as follows: Chapter2 reviews literature on the characteristics of Bitcoin by comparing cryptocurrency and gold, and previous studies on investment portfolio theory. Chapter 3 explains the methodology used by the Huberman and Kandel test (HK-test) to compare the possibility of investment expansion when Bitcoin is included in the portfolio. Chapter 4 describes the results of empirical analysis. Chapter 5 presents a summary and conclusion

2. literature review

It has been argued that cryptocurrency and gold share some similarities and differences. Bitcoin and gold are mined and supplied by non-central institutions, and unlike fiat currencies, they cannot be issued indefinitely.

Before Covid-19, Hwang (2019) discussed whether Bitcoin can be included in each country's legal system and laws as it has a pattern that is independent of other price indices or fluctuations in asset prices and has characteristics like gold as a means of payment or investment. He found risk hedging capabilities over a period through quantitative analysis using VGARCH, Multivector (VAR, VMA) and ASYMM models in Bitcoin.

After Covid-19 deepened inflation and uncertainty, Choi and Shin (2021) used the VAR (Vector Autoregression) to empirically analyze the function of Bitcoin as an inflation hedge like gold. But Bitcoin served as a weak haven unlike gold. However, it was analyzed that Bitcoin was not related to the policy uncertainty of the government authorities.

From the different characteristics in payment methods and hedging possibilities found by comparing Bitcoin and gold, in this paper, we study whether Bitcoin can expand the existing portfolio.

3. Methodology

This study adopts the analysis of the Markowitz portfolio optimization model, the mean-variance spanning test of Huberman and Kandel, and the step-down test of Kan and Zhou.

3-1. Markowitz portfolio optimization model

Markowitz (1959) systematized the Theory of Portfolio Selection theory: Diversifying assets and creating portfolios can reduce the risk of individual assets while preserving gross returns. Based on Markowitz's theory, Sharp et al. (1964) developed the Capital Asset Pricing Model (CAPM) including the assumptions of risk-free assets. CAPM is a model that derives the equilibrium rate of return for risky assets under the equilibrium of the capital market

3-2. Mean-variance spanning test

In a regression framework, this question can be answered using the mean variance spanning test described by Huberman and Kandel. This test examines whether the efficiency boundary improves with the inclusion of new assets. A new asset can be considered already included in an existing portfolio if its mean-variance boundary with the new asset matches the boundary already created by the reference asset. That is, no new improvements are created. Conversely, adding a new asset shifts the existing efficient frontier line to the left, allowing investors to improve their investment opportunities.

Huberman and Kandel describe that a set of K assets spans a set of $N + K$ assets if the minimum-variance frontier line of the K assets is equal to that of the K assets plus N assets.

$$rt = \alpha + \beta Rt + \varepsilon t \tag{1}$$

$$\alpha = 0N \quad and \quad \gamma = iN - \beta iK = 0 \tag{2}$$

Regression analysis is performed using the equation (1), As a regression method, the Wald test evaluates constraints on statistical parameters based on weighted distances between unconstrained estimates and hypothesized values. In the equation (1), the variable rt is the return vector (N-

dimensional vector) of N test assets in period t, and the variable *Rt* is the return vector (*K*-dimensional vector) of t *K* benchmark assets in period *t*. If the null hypothesis (2) is rejected, new assets can expand investment opportunities.

3-3. Step-down test

Kan and Zhou (2012) created two-step model to distinguish where HK-Test's null hypothesis was rejected: differences in tangents or shift in the global minimum

$$\alpha = 0N \tag{1}$$

$$\gamma = iN - \beta iK = 0 \text{ on condition } \alpha = 0N \tag{2}$$

The first step in the Step-Down test examines whether $\alpha = 0$. If null hypothesis is rejected at this step, ratios of tangent line are statistically different. The second step-down test examines whether $\delta = 0$ under the condition of $\alpha = 0$. When the null hypothesis is rejected, the global minimum variance of the two portfolios is statistically different

The figure on the left below shows the opportunity set expansion of the Maximum Sharpe Ratio (MSR) portfolio. The right side shows the shift of the Global Minimum Variance (GMV).

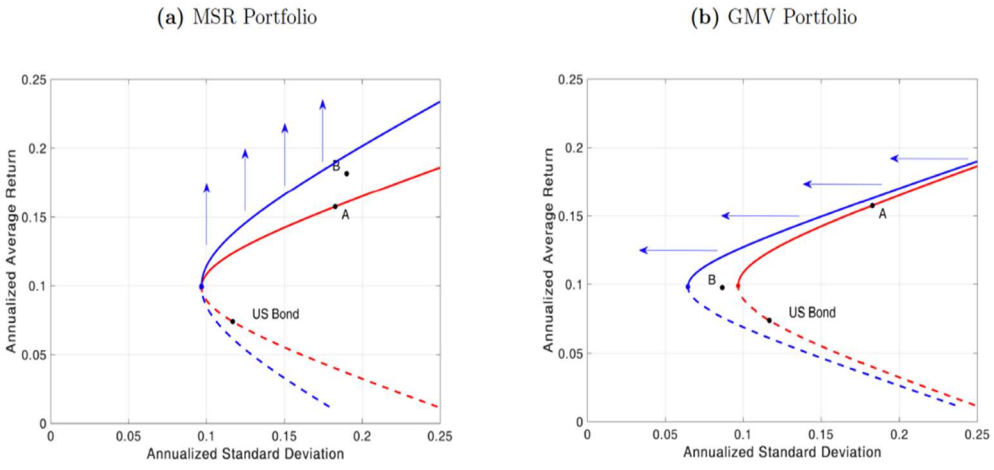


Fig1, expansion of the efficient frontier line (Boris et al., 2020)

4. Data

A set of K risky assets makes a benchmark portfolio using S&P 500 Index and long-term U.S. Treasuries bonds; a set of K risky assets uses Bitcoin and Ethereum

4.1 Benchmark Portfolio Set

This part applies a spanning test to examine the benefits of cryptocurrency for investors with traditional investment opportunities consisting of the S&P 500 index and U.S. long-term bonds. For convenience of analysis, the daily adjustment price of ^GSPC, a Yahoo Finance index, was selected as an alternative asset for S&P 500, and TLT, an ETF that tracks long-term treasury bonds made by iShares, was selected for long-term U.S. government bonds.

4.2 Test Assets

Test assets were Bitcoin and Ethereum, which account for the top 60% of market cap among cryptocurrencies. The value of Bitcoin is the adjusted daily price from April 28, 2013, recorded by finance.yahoo.com. The value of Ethereum is also adjusted daily from August 7, 2015, but they were sliced from Jan 1, 2017, to analyze the recent 5 years. The Binance coin is about 3% of the total assets, so we decided to exclude it

Market Rank	Ticker	Name	Start Date	Market Cap	Dominance
#1	BTC-USD	Bitcoin	1/3/2009	925.017B	40.57%
#2	ETH-USD	Ethereum	7/30/2015	476.975B	20.87%
#3	BNB-USD	Binance Coin	7/25/2017	88.777B	3.89%

Table 4.1 Summary of Cryptocurrency (Coinmarketcap, Dec 22,2021)

4.3 Impact of Covid-19

In the period analysis, based on January 2020, when the WHO pronounced an international public health emergency to analyze the impact of COVID-19, The period before (2017.11 ~ 2019.12.31) and the period thereafter (2020.1.1 ~ 2022.01.31) were separated to review the volatility of crypto currency investment due to the impact of COVID-19.

5. Results

5-1. Sharpe ratio and Global minimum variance

Portfolio Set	*Tangent line*		*Global minimum variance*		
	Sharpe ratio	*%*	*sigma*	*%*	*mean*
			Entire Period : 2017/11 ~ 2022/01		
SNP TLT	0.080768		0.625365		0.048239
SNP TLT BTC	0.086321		0.625290		0.047932
	0.005553	*6.87%*	*-0.000075*	*-0.01%*	*-0.000307*
SNP TLT ETH	0.082167		0.623821		0.047340
	0.001398	*1.73%*	*-0.001544*	*-0.25%*	*-0.000899*
SNP TLT BTC ETH	0.088174		0.622037		0.048574
	0.007405	*9.17%*	*-0.003328*	*-0.53%*	*0.000334*
			First Subperiod : 2017/11 ~ 2019/12 (Before Covid-19)		
SNP TLT	0.123123		0.445380		0.054242
SNP TLT BTC	0.124165		0.443802		0.053259
	0.001042	*0.85%*	*-0.001578*	*-0.35%*	*-0.000983*
SNP TLT ETH	0.133160		0.445191		0.053540
	0.010037	*8.15%*	*-0.000188*	*-0.04%*	*-0.000702*
SNP TLT BTC ETH	0.140777		0.442624		0.055135
	0.017654	*14.34%*	*-0.002756*	*-0.62%*	*0.000893*
			Second Subperiod : 2020/01 ~ 2022/01 (After Covid-19)		
SNP TLT	0.063153		0.755785		0.042659
SNP TLT BTC	0.104187		0.753847		0.037970
	0.041034	*64.98%*	*-0.001938*	*-0.26%*	*-0.004689*
SNP TLT ETH	0.104671		0.749474		0.033882
	0.041518	*65.74%*	*-0.006311*	*-0.83%*	*-0.008777*
SNP TLT BTC ETH	0.107601		0.748057		0.034901
	0.044448	*70.38%*	*-0.007728*	*-1.02%*	*-0.007758*

Table 5.1 Sharpe ratio and Global minimum variance

The period is divided into three sections. The entire period and separated before and after Covid-19. The first column shows portfolio assets: A portfolio consisting of S&P 500 (SNP) and long-term treasury bonds (TLT) was used as the benchmark portfolio, and as test assets, Bitcoin(BTC),

Ethereum(ETH), and a combination of Bitcoin and Ethereum The portfolio is divided into three areas.

In the second column, the Sharpe ratio was calculated. When calculating the Sharpe ratio, the risk-free rate is used by means of daily prices of SHY (1–3-year U.S. treasury Bond ETF made by iShares.co) during each period. It is 0.006418467% for the entire period, 0.01097434% for the first period (before Covid-19) and 0.002177914% for the second period (after Covid-19). Below each Sharpe ratio, the difference with the bench portfolio was calculated, and the ratio was expressed as a percentage next to it, showing the change in the ratio according to the change in assets.

In the third column, the global minimum variance value is indicated, and the difference from the basic portfolio is indicated below each, and the change amount is indicated in percent next to it.

5-2. Efficient frontier line

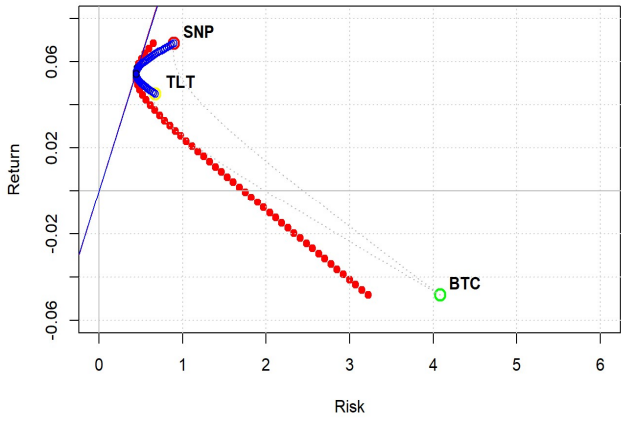
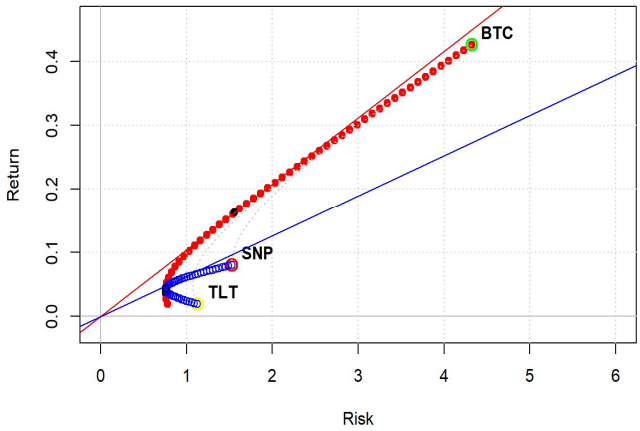
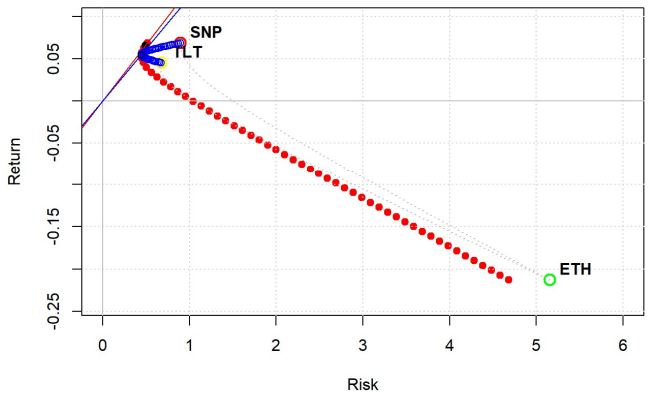
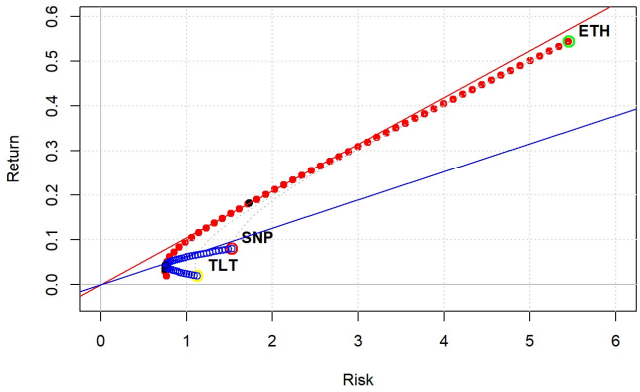
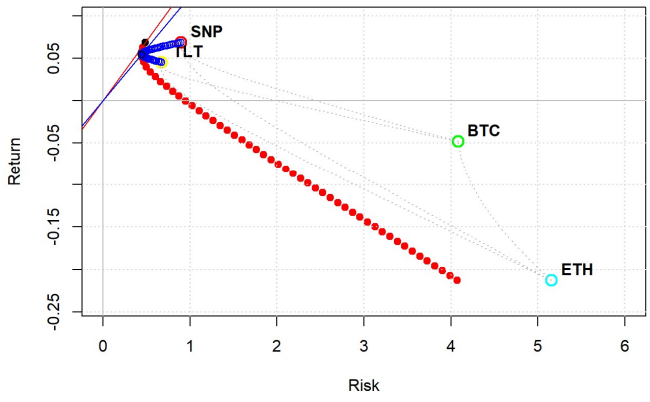
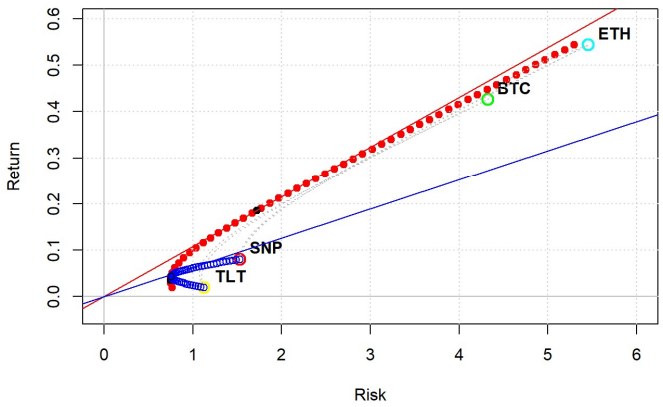
Before Covid-19 (2017.11 ~ 2019.12)	*After Covid-19 (2020.01 ~ 2022.01)*
	
Bitcoin	
	
Ethereum	
	
Bitcoin Ethereum	

Fig 5-1 efficient frontier line of new portfolio

Fig. 5-1 shows the change in portfolio adding Bitcoin and Ethereum before and after covid-19. All graphs show that there is an effect of portfolio improvement when Bitcoin and Ethereum are added.

1) Maximized Sharpe ratio (MSR)

In Table 5.1, Bitcoin's Sharpe ratio change compared to the benchmark portfolio increased by 0.85% before covid-19, and by 64.98% after covid-19. It shows that the effect of improving the portfolio of bitcoin has greatly increased since covid-19. Table 5.2 summarizes the improvement effect when each cryptocurrency is included in the portfolio. The improvement effect was high after COVID-19, and the effect of the portfolio combining Bitcoin and Ethereum was even higher.

	Before Covid-19		*after Covid-19*		*Entire Period*	
SNP TLT	*0.123123*		*0.063153*		*0.080768*	
SNP TLT BTC	*0.124165*	*0.85%*	*0.104187*	*64.98%*	*0.086321*	*6.87%*
SNP TLT ETH	*0.133160*	*8.15%*	*0.104671*	*65.74%*	*0.082167*	*1.73%*
SNP TLT BTC ETH	*0.140777*	*14.34%*	*0.107601*	*70.38%*	*0.088174*	*9.17%*

Table 5.2 Change in Sharp ratio

2) Global minimum variance (GMV)

It is difficult to know exactly how much the GMV point moved to the left in Fig.5.1, but the contents summarized through calculation are shown in Table 5.3 below. Compared to the basic portfolio, all moved to the left, and after covid-19, bitcoin moved to the left by 0.26%, Ethereum by 0.83%, and the joint portfolio by 1.02% to the left compared to the benchmark portfolio.

	Before Covid-19		*after Covid-19*		*Entire Period*	
SNP TLT	*0.445380*		*0.755785*		*0.625365*	
SNP TLT BTC	*0.443802*	*-0.35%*	*0.753847*	*-0.26%*	*0.625290*	*-0.01%*
SNP TLT ETH	*0.445191*	*-0.04%*	*0.749474*	*-0.83%*	*0.623821*	*-0.25%*
SNP TLT BTC ETH	*0.442624*	*-0.62%*	*0.748057*	*-1.02%*	*0.622037*	*-0.53%*

Table 5.3 Left Shift in Global minimum variance

So far, we compared the Sharpe ratio of Markowitz's efficient portfolio line and the left shift of the GMV point to see if there is any improvement when cryptocurrency is added to the existing portfolio.

However, there is a limit to the accurate analysis of whether it is statistically significant. Therefore, in the next chapter, we review using the statistical analysis framework suggested by HK-Test

5-3. Mean-variance spanning test

			HK test		*Step-Down test*			
	alpha	*beta*	*F-test*	*Pr(>F)*	F_1	*Pr(>F)*	F_2	*Pr(>F)*
Entire Period : 2017/11 ~ 2022/01								
BTC	0.001240	1.101088	0.501397	60.59%	0.743876	38.87%	0.259000	61.09%
ETH	0.000774	1.576825	2.166755	11.52%	0.182689	66.92%	4.155029	***4.18%***
BTC+ETH	0.002013	2.677913	6.153428	***0.22%***	0.420224	51.70%	11.895178	***0.06%***
First Subperiod : 2017/11 ~ 2019/12 (Before Covid-19)								
BTC	-0.000654	0.228036	1.512553	22.16%	0.098517	75.38%	2.933404	8.76%
ETH	-0.002596	0.665736	0.734719	48.03%	0.983085	32.21%	0.486373	48.60%
BTC+ETH	-0.003250	0.893772	0.279734	75.61%	0.521582	47.06%	0.037933	84.57%
Second Subperiod : 2020/01 ~ 2022/01 (After Covid-19)								
BTC	0.003329	1.381314	2.644391	7.22%	2.851954	9.20%	2.426054	12.01%
ETH	0.004246	1.875220	5.235524	***0.57%***	2.893921	8.97%	7.542870	***0.63%***
BTC+ETH	0.007575	3.256534	10.005349	***0.01%***	3.150328	7.66%	16.773872	***0.01%***

Table 5-1. Wald test

The table shows portfolio sets of mean-variance spanning tests for two cryptocurrencies using the S&P 500 index and the long-term U.S. Treasury bond as benchmark assets. The first test is an F-test of H_0 : $\alpha = 0$, $\beta = 1$ (β is the sum of coefficients excluding the intersection in regression) called the HK-test. The second test is a step-down test, where F_1 is *F-test* for $\alpha = 0$, and F_2 is *F-test* of $\beta = 1$ conditional on $\alpha = 0$. The two tests are conducted for each cryptocurrency as well as jointly for two cryptocurrencies. The results are presented as the entire sample period as well as for its two sub periods divided by Covid-19 outbreak.

The results of the regression test are like those of the previous frontier line analysis, but the statistical hypothesis test came up with a point that needs to be further reviewed. First, analyzing Bitcoin, no significant difference was found when a new portfolio was created in both the entire period, 1st and 2nd period in HK-test. In the period before Covid-19, the null hypothesis H_0 : $\alpha = 0$, $\beta = 1$ could not be rejected with a Wald-test result p-value of 22.16%, and in the period after Covid-19, the null hypothesis

was rejected with a p-value of 7.22% cannot do. In the entry-period, it was analyzed that Bitcoin could span the existing benchmark portfolio at 60.59%.

Ethereum has an improvement effect when adding new assets to 0.57% after covid-19. In the case of Wald-testing with the Bitcoin and Ethereum portfolio, the performance has increased after covid-19. As shown in the table below, there is a clear improvement effect with a p-value of 0.01%, and there is a significant improvement effect even in the entire period.

	Before Covid-19		*after Covid-19*		*Entire Period*	
	F-test	*p-value*	*F-test*	*p-value*	*F-test*	*p-value*
Bitcoin	*1.5126*	*22.16%*	*2.6444*	*7.22%*	*0.5014*	*60.59%*
Ethereum	*0.7347*	*48.03%*	*5.2355*	*0.57%*	*2.1668*	*11.52%*
Bitcoin Ethereum	*0.2797*	*75.61%*	*10.0053*	*0.01%*	*6.1534*	*0.22%*

Table 5.3 HK-test

The mean-variance spanning test described by HK-Test was used for each to more accurately analyze the effect of increasing the frontier line of the cryptocurrency portfolio to determine whether the Sharpe ratio improvement or the global minimum variance shifted to the left.

As a result of carrying out a step-down test for the period after Covid-19, Bitcoin has a p-value of 7.22% from the HK-test, which is greater than 5%, so the discussion of the step-down test is excluded. For Ethereum, a significant result with a p-value of 0.57% in HK-test was obtained, and 0.63% in step2 through step-test, indicating that the GMV effect is large. When examining the portfolio combining Bitcoin and Ethereum, it is 7.66% in the step-1 test and 0.01% in the step-2 test, showing a more significant improvement effect in the step-2 test, that is, movement of the global minimum variance point to the left is the result of bringing the portfolio improvement effect more than the Sharpe ratio increase effect.

As a result of analyzing the change in the frontier line only in %, the Sharpe ratio change is larger, but in the regression frame through the step-down test, the left shift effect of the global minimum variance is larger. This analysis is the same even when the entire period is analyzed, indicating that the left shift of GMV is important. Also, in the case of Ethereum, the HK-test result is 0.57% after covid-19 and 8.97% in the step 1 test, while 0.63% in the step-2 test, the effect of the GMV point part has a greater effect than the Sharpe-ratio.

As above, the difference between the efficient frontier line and the regression test is that the standard deviation of the denominator return is used as a substitute for the portfolio's total risk on the efficient frontier

line. This assumes that investment returns follow a normal distribution, but this is not always the case. Profits are usually biased in the opposite direction due to unexpected price spikes and falls in financial markets. Also, asset weights are limited from 0 to 1, but the efficient frontier line theoretically allows negative and infinite wealth weights.

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6. Conclusion

The above results indicate that adding cryptocurrency to the portfolio has the effect of improving the performance of investors (investors with the S&P 500 stock index and long-term treasury bonds as their basic assets). And it is analyzed with a regression analysis frame, the portfolio improvement effect is greater after covid-19 through the HK test. We found that the left shift of the global minimum variance had a more significant effect on portfolio improvement than the Sharpe ratio through the step test. In contras the effective line showed a high Sharpe ratio effect, but HK-test showed a high global minimum variance effect.

7. References

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Coinmarketcap *https://coinmarketcap.com, Dec 22,2021*

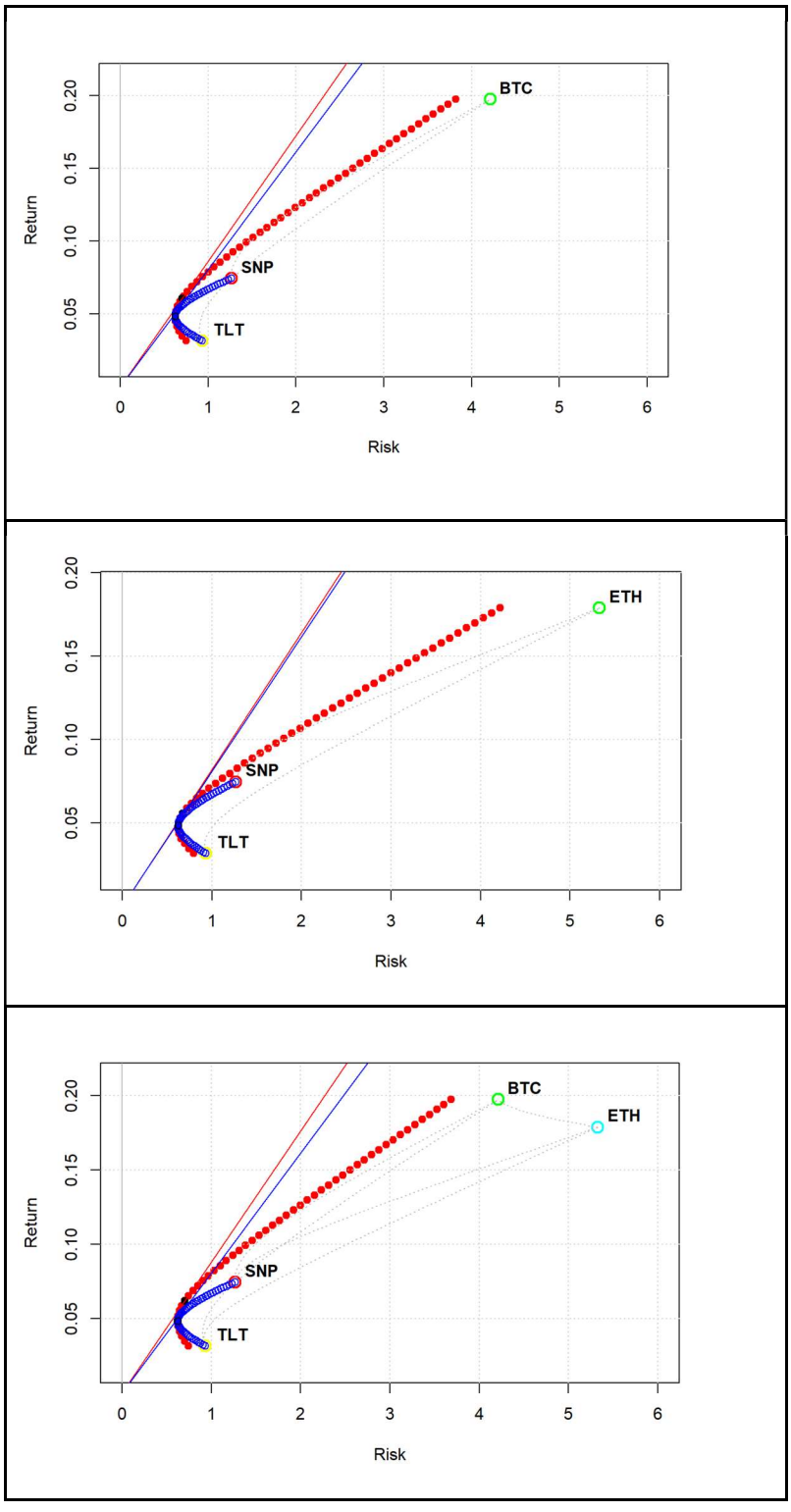
Appendix

1. Spanning test on Heteroskedasticity

			HK test		*Step-Down test*			
	alpha	*beta*	*F-test*	*Pr(>F)*	F_1	*Pr(>F)*	F_2	*Pr(>F)*
Entire Period : 2017/11 ~ 2022/01								
BTC	0.001240	1.101088	0.503565	0.604562	0.663035	0.415731	0.077998	0.780100
ETH	0.000774	1.576825	1.063837	0.345613	0.166130	0.683683	1.472315	0.225336
BTC+ETH	0.002013	2.677913	2.723794	0.066229	0.375397	0.540250	3.688094	0.055154
First Subperiod : 2017/11 ~ 2019/12								
BTC	-0.000654	0.228036	1.402610	0.247202	0.094740	0.758401	2.547930	0.111253
ETH	-0.002596	0.665736	0.771242	0.463148	0.938350	0.333308	0.403318	0.525755
BTC+ETH	-0.003250	0.893772	0.288698	0.749400	0.498744	0.480478	0.032032	0.858051
Second Subperiod : 2020/01 ~ 2022/01								
BTC	0.003329	1.381314	2.326376	0.098918	2.457334	0.117738	0.576694	0.448040
ETH	0.004246	1.875220	3.663534	***0.026473***	2.560977	0.110290	2.115951	0.146524
BTC+ETH	0.007575	3.256534	5.587576	***0.004031***	2.722466	0.099700	4.078634	***0.044068***

Under the heteroskedasticity condition, the null hypothesis is rejected only when cryptocurrencies are joined.

2. Entire Period : 2017/11 ~ 2022/01



3. Return of Bitcoin and Ethereum