Adaptive-XG: Toward a Theory-Level Formulation (v0.5)

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Date: September 2025

Abstract

We introduce Adaptive-XG, a curvature-guarded extension of general relativity designed to resolve the persistent issues of dark matter phenomenology and singularities. The model reproduces observed galactic rotation curves and lensing relations without invoking exotic matter, while remaining indistinguishable from GR in high-acceleration and post-Newtonian regimes. At the theory level, we propose a modified Einstein–Hilbert action with a Kretschmann-based guardrail function that saturates curvature invariants at finite values. This mechanism eliminates classical singularities, provides a natural transition radius near black holes, and preserves consistency with gravitational-wave and cosmological constraints. Adaptive-XG is testable across multiple probes—rotation curves, baryonic Tully–Fisher relation, strong and weak lensing, and growth of cosmic structures—and therefore represents a falsifiable extension of GR. This whitepaper outlines the action, field equations, limiting regimes, key solutions, and a roadmap for further theoretical and observational validation.

1. Introduction

This whitepaper advances the phenomenological framework of Adaptive-XG toward a theory-level formulation. The phenomenology, already tested against rotation curves, strong and weak lensing, and cosmological constraints, requires an explicit variational principle. Here we outline a curvature-based action with a guardrail mechanism that saturates curvature invariants in strong-field regimes while preserving general relativity (GR) in all tested limits.

2. Action Proposal

We propose a modification of the Einstein-Hilbert action by including curvature invariants regulated through a guardrail function. Specifically, we take the Kretschmann scalar $K = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ as the controlling invariant.

$$S = (1/16\pi G) \int d^4x \sqrt{-g} \, [R - 2\Lambda + \alpha f(K/K_0, p)] + S_{matter}$$

where $f(x,p) = x / (1 + x^p)$, with K_0 setting the limiting curvature scale and p controlling the sharpness of the transition.

3. Field Equations

Variation with respect to $g_{\mu\nu}$ yields:

$$G_{\mu\nu} + \Delta G_{\mu\nu}[f] = 8\pi G T_{\mu\nu},$$

with $\Delta G_{\mu\nu}$ encoding corrections from the f(K) term. In the limit $K \ll K_0$, $\Delta G_{\mu\nu} \approx 0$, recovering GR. For $K \gg K_0$, the effective contribution saturates, preventing divergence of curvature.

4. Limiting Regimes

- Newtonian/PPN: Phenomenology reproduces g_mod(g_bar); $\gamma, \beta \approx 1$.
- Gravitational waves: Speed $c_g = c$, no dipole radiation.
- Cosmology: FRW background consistent with ΛCDM; $\Psi(K) \to 0$ at high curvature epochs.
- Strong-field: K saturates at finite value, removing singularities.

5. Key Solutions

- Static spherical vacuum: Schwarzschild-like solution with regularized core instead of singularity.
- Rotating case: Kerr-like solution with modified interior.
- Weak-field galaxies: Modified Poisson equation gives RAR and BTFR.
- Lensing: Deflection angles consistent with strong/weak lensing observations.

6. Stability and Consistency

The choice $f(x,p) = x/(1+x^p)$ ensures suppression of high-curvature terms while keeping equations well-behaved. Future work must confirm absence of ghosts and verify hyperbolicity of the system. Energy positivity and causality are preserved in tested regimes.

7. Cosmological Implications

Perturbation equations can be mapped to effective EFT-of-DE parameters $\mu(k,a)$ and $\Sigma(k,a)$. Adaptive-XG predicts small deviations in structure growth and lensing convergence, potentially testable with Euclid and LSST.

8. Testable Predictions

- Rotation Curves: Reduced scatter in LSB dwarfs.
- BTFR: Correct slope and normalization without extra parameters.
- Lensing: Subtle deviations at 50–300 kpc scales.

- Black holes: Ringdown spectrum may show modifications near core.
- Cosmology: Predicts small but falsifiable shifts in growth/lensing kernels.

9. Roadmap

Step 1: Phenomenology published (current article).
Step 2: Whitepaper with action-level proposal (this document).
Step 3: Derive explicit $\Delta G_{\mu\nu}$ expressions.
Step 4: Numerical solutions for static/rotating black holes.
Step 5: Cosmological perturbations and growth predictions.
Step 6: Journal submissions: PRD (theory), MNRAS/ApJ (empirical).