Lab2 Group11

Group 10

28/01/2020

Question 1: Optimizing a model parameter

```
## Warning in RNGkind("Mersenne-Twister", "Inversion", "Rounding"): non-uniform
## 'Rounding' sampler used
```

1.

2.

3.

4.

5.

6.

Question 2: Maximizing likelihood

(Started to write question 2 / Sofie)

1.

In this task we will use the file `data.RData` consists of a sample coming from normal distribution with parameters μ and σ. First we load the data set into R.

2.

The sample comes from a normal distribution with parameters μ and σ, where we set $\theta = (\mu, \sigma)$. Under the assumption that the sample $\boldsymbol{x} = (x_1, ..., x_{100})$ is iid, i.e. $\boldsymbol{X_i} \overset{iid}{\sim} N(\mu, \sigma^2)$, for $i = 1, ..., 100$, then the joint density function of all $n = 100$ observations can be written as

$$L(\theta; \boldsymbol{x}) = f(\boldsymbol{x}|\theta) = \prod_{i=1}^{100} f(x_i|\theta).$$

Now we let the number of observations be denoted by n in the following derivations. Using the density function of a normal distribution with parameter θ we obtain the likelihood function

$$L(\theta; \boldsymbol{x}) = \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left(\frac{x_i - \mu}{\sigma}\right)^2\right\}.$$

The log-likelihood function is given by

$$l(\theta; \boldsymbol{x}) = \log\ L(\theta; \boldsymbol{x}) = -\frac{n}{2}\log(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i - \mu)^2.$$

The maximum likelihood estimators (MLEs) $\hat{\mu}_{ML}$ and $\hat{\sigma}^2_{ML}$ of μ and σ^2 are obtained by maximizing the likelihood function. This is done by differentiating the log-likelihood functions and put them to zero. In more detail, we calculate the score functions $S(\theta; \boldsymbol{x})$ w.r.t. μ and σ seperately, and let them equal zero and solve for each parameter:

$$S(\mu) = \frac{\partial}{\partial\mu}\ l(\theta; \boldsymbol{x}) = -\frac{n(\overline{x} - \mu)}{\sigma^2} = 0,$$

where $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$. From this we obtain $\hat{\mu}_{ML} = \overline{x}$. Further,

$$S(\sigma) = \frac{\partial}{\partial\sigma}\ l(\theta; \boldsymbol{x}) = -\frac{n}{\sigma} + \frac{1}{\sigma^3}\sum_{i=1}^{n}(x_i - \mu)^2 = 0,$$

and $\hat{\sigma}^2_{ML} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2$.

Then we use the derived formulas in order to obtain the desired parameter estimates for the loaded data. So the data set with 100 observations gives the result $\hat{\mu}_{ML} = 1.275528$ and $\hat{\sigma}_{ML} = 2.005976$.

3.

4.

Appendix