Use the 4th-order Runge-Kutta integration method to find solutions for polytrope indecies using the Lane-Emden ODE.

Lane-Emden polytrope equation

$$\frac{1}{\xi^2}\frac{d}{d\xi}(\xi^2\frac{d\theta}{d\xi}) = -\theta^n$$

We can rewrite this as 2 first order equations:

$$\frac{dy}{d\xi} = z$$

$$\frac{dz}{d\xi} = -\frac{2}{\xi}z - y^n$$

For a given polytrope index n, the main class will integrate the system for us. When plotting the results, we can obtain the parameter ξ_1

Here, ξ_1 is our stellar radius. We will estimate the radius each step while ensuring that the next step does not take us past that estimate. This will prevent us from obtaining negative θ values. Given a point (ξ_0, y_0), and the derivative at that point, $z_0 = \frac{dy}{d\xi}\Big|_{\xi_0}$, we can write the following equation of a line:

$$y(\xi) - y_0 = z_0(\xi - \xi_0)$$

ξ_1 is defined to be our estimated stellar radius when y becomes zero. Thus:

$$\xi = -\frac{y_0}{z_0} + \xi_0$$

However, we must make sure that our stepsize $\Delta\xi$ is small enough that we do not go beyond this estimate.