

Create a plot of the energy generation rate for both processes as a function of temperature (assuming solar composition).

Energy Generation Rates

The energy generation rate of the proton-proton chain is proportional to the rate of its slowest reaction, the $p + p \rightarrow d + e^+ + \nu_e$ reaction. The overall energy generation rate is:

$$q_{pp} = \frac{2.4 \times 10^4 \rho X^2}{T_9^{2/3}} e^{\frac{-3.380}{T_9^{1/3}}} \text{ erg} \cdot \text{g}^{-1} \cdot \text{s}^{-1}$$

The energy generation rate for the CNO cycle is proportional to the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ rate, and is:

$$q_{CNO} = \frac{4.4 \times 10^{25} \rho X Z}{T_9^{2/3}} e^{\frac{-15.228}{T_9^{1/3}}} \text{ erg} \cdot \text{g}^{-1} \cdot \text{s}^{-1}$$

Where $T_9 \equiv T/(10^9 \text{ K})$

Imagine expressing the reaction rate as a powerlaw,

$$q = q_0 \rho T^\nu$$

We can estimate ν around some temperature T_0 for both the pp-chain and the CNO cycle as:

$$\nu = \frac{d \log q}{d \log T}$$

We can compute the needed derivative by differencing:

$$\left. \frac{dq}{dT} \right|_{T_0} \approx \frac{q(T_0 + \delta T) - q(T_0)}{\delta T}$$

Where $\frac{\delta T}{T_0} \ll 1$