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Stability of outer planetary orbits (P-types) in binaries

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Summary. The aim of the study is to derive stability limits of fictitious direct planetary orbits around both components in real double star systems. Since most of the double star orbits have high eccentricities we used as a model the elliptic restricted three body problem. We checked in detail the dependence of the stability limits for stable orbits on two parameters namely eccentricity and mass ratio of the primary.

The method of deriving such limits is purely numerical. Taking a certain well chosen subset in phase space close to the stability border we were able to determine two critical orbits: the Upper Critical Orbit (from this orbit on all others with greater semi-major axes were found to be stable) and Lower Critical Orbit (from this orbit on all others with initially smaller semi-major axes were found to be unstable). In between these two limits, in the "grey region", close to each other stable and unstable orbits are found, which is a well known phenomenon in dynamical systems near a stability limit.

As result of the study no dependence of the Critical Orbits on the mass ratio has been found for fictitious planets on P-type orbits surrounding both primaries in direct orbits. Then an already established formula (Dvorak, 1986) can be used to derive stability limits for such orbits. This has definitely to be considered for the S-type orbits, which stay "satellites" of one primary body.

Finally we applied this formula for 17 double stars where there exist well established elements (semimajor axes and eccentricities). From the celestial mechanics point of view we can conclude, that from the calculated limits on there could exist stable planetary orbits even in binaries. Their orbital periods around the two primary bodies would be in the order from ten to a few hundred years.

Key words: celestical mechanics – planets and satellites: general – stars: binaries: general

1. Introduction

In this paper we want to establish stability limits of direct planetary orbits surrounding both components of real double stars. Until now there exist on one hand analytical or semi-analytical studies in the circular restricted three body problem, where the different mass ratios of the primaries are fully taken into account (Hénon and Guyot, 1970; Szebehely, 1980; Szebehely and McKenzie, 1981). On the other hand numerical integrations in the framework of the elliptic restricted problem take into account the eccentricities of the orbits of the binaries (Harrington, 1975; Harrington, 1977; Dvorak, 1984; Dvorak, 1986 = Paper I; Rabl and Dvorak, 1988 = Paper II). But in all these numerical calculations the mass parameter $\mu = \text{mass}(1)/(\text{mass}(1)+\text{mass}(2))$ was fixed to special values (e.g. to $\mu=0.5$, or $\mu=$ Jupiter/Sun) or to a special double star (Benest, 1986, 1988). We know that binaries have eccentric orbits and that for most of them the mass parameter is different from 0.5, $(0.2<\mu<0.5)$. Table 1 shows the characteristics of 17 binary stars with well established elements (Heintz, 1978; Couteau, 1985). One can see from this table that both the eccentricity and the mass parameter μ may be important and cannot be neglected for the study of the motion of a planet in the gravitational field of a binary. Therefore the aim of this paper is to study for these selected stars stable fictitious planetary orbits from the purely dynamical point of view.

Finally we want to find stability limits for stable orbits surrounding the two components in direct orbits as functions of eccentricity and mass ratio of the primaries. This will be done with the help of critical orbits (= COs), separating stable planetary orbits from unstable orbits (see Paper I). In this connection a planetary orbit is defined as follows:

"A planetary orbit in a double star is either a near circular orbit around one component (TYPE-S) or around both components (TYPE-P); both orbits are direct ones."

All the details will be given in the next paragraphs; but for a better understanding we show in Fig. 1 a schematic picture of the COs for the P-types and the S-types. After reviewing some results we shall examine the dependence of the former mentioned COs as functions of eccentricity and mass-ratio of the binary.

Some examples of real double stars (following Table 1) will be discussed in the last section.

2. Recent studies in the circular restricted problem

There exist different studies of the problem in question in the framework of the planar circular restricted problem, where the two primary bodies (the components of the double star) have circular orbits. The influence of a third body (the planet) which is regarded to move in the same orbital plane, has no gravitational action on the two other bodies.

Szebehely (1980) and Szebehely and McKenzie, 1981 (= SK) used the well known integral of the circular restricted problem to determine stability limits for planetary orbits (for S- and P-types)

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and how they depend on the mass ratio. In the planar problem they calculated the value of the Jacobian constant C of orbits which are close to the estimated stability limit. Then they compared it to the critical value C corresponding to the zero velocity which passes through the Lagrange point L between the two primaries. The velocity of the planet was set to be circular with respect to the barycenter of the primaries. In this way they obtained an algebraic equation which can be solved with respect to distance r from the barycenter for fixed value of μ. This critical distance r can be regarded identical with the former defined COs which separates the stable region far away from the two primaries from the unstable one close to them (see Fig. 1 for the P-types).

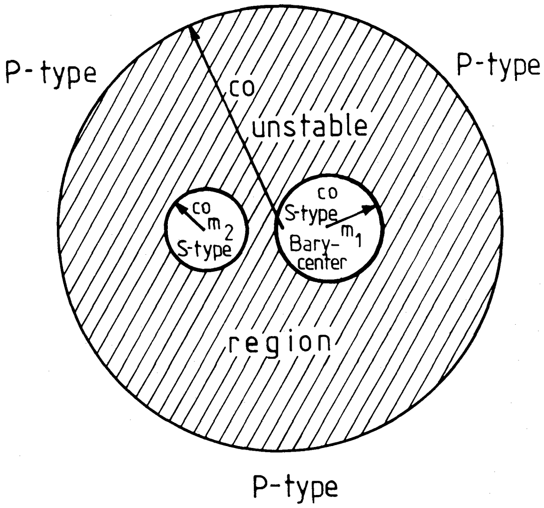

Fig. 1. P-and S-type orbits in binaries. This is a schematic picture of stable and unstable regions for the S-type orbits and the P-types in double stars. The radius CO is the stability limit. Around the two primaries there are stable regions for the S-types, farther outside there is the stable region for the P-types

One can see that there is a maximum value of the critical distances for values of the mass parameter $0.2 < \mu < 0.25$ (Table 2). Hénon and Guyot, 1970 (=HG) studied the linear stability of periodic orbits (=PO) of the Strømgren families *f, g, h, i, l* and *m* in the circular problem for all different mass ratios. The family *l* correspond in principle to the P-type and the family *g* to the S-type. HG found an unstable region close to the primaries, then from a distance of about 1.8 on (measured from the barycenter) the stable region begins. But at the distance of about 2.1 a small instability strip appears (corresponding to the 2:1 resonance). As defined in the introduction we want to find stability limits for planetary orbits which correspond to quasiperiodic orbits. According to the known structure of phase space it is expected to find in the vicinity of stable POs quasiperiodic orbits, while close to unstable PO one expects to find only escape orbits. In order to compare their results we have taken as the stability limit for the 3rd body the mean values of the two perpendicular crossings of the periodic orbit with the x-axes, which can be regarded as a "mean value" of their orbit. Both studies, SK and HG do coincide in their results quite well qualitatively as one can see from Table 2. Quantitatively there exists a 10% deviation, but this is not surprising due to the difference of the two methods used.

3. Numerical approach in the circular restricted problem

To confirm on the one hand the analytical and semi-analytical results, and to show the precision of our numerical method used on the other hand, we present now numerical work of the problem in question. We used a Lie-Integration program

Table. 1. Critical orbits of P-types in double stars with well established elements. In the columns 1 and 2 a current number and the name of the double star are listed. Columns 3, 4, 5 and 6 show the period in years, the parallax in arcseconds, the semi-major axis in AU and the eccentricity. Columns 7 and 8 list the mass of the more massive star in solar masses and the mass parameter for the binary. Columns 9 and 10 list values of the LCO and the UCO in AU. Column 11 shows the calculated period in years of a fictitious planet, which would have just the semi major axes of the UCO. In this sense they are minimum values for expected periods of planets of type P in this double star

No.	Name	Period	Parallax	A(AU)	Eccentricity	Mass 1	Ratio	LCO	UCO	Period
1	ADS 520	25.0	0.07	9.57	0.22	0.7	0.5	24.5	28.0	125.3
2	ε Cet	2.67	0.069	1.57	0.27	1.3	0.5	4.2	4.8	14.2
3	γ Vir	171.37	0.090	37.84	0.881	0.94	0.49	111	151	1355
4	α Com	25.87	0.038	12.49	0.5	1.43	0.49	37	43.6	170
5	ε CrB	41.56	0.059	13.98	0.276	0.79	0.497	37.8	42.7	221
6	ADS 9716	55.88	0.048	19.15	0.591	1.135	0.5	57.8	69.7	386
7	BD $-8°$ 4352	1.72	0.152	1.35	0.05	0.42	0.5	3.0	3.38	6.8
8	BD 45° 2505	12.98	0.160	4.58	0.73	0.285	0.5	13.9	17.5	97
9	ADS 11871	61.20	0.061	22.96	0.249	1.647	0.49	61.0	68.7	313
10	δ Equ	5.70	0.052	4.73	0.42	1.658	0.49	13.7	15.8	34.6
11	Kpr 37	21.85	0.074	9.67	0.15	1.20	0.427	13.7	28.1	103.0
12	99 Her	55.8	0.060	16.39	0.74	0.888	0.37	25.5	62.5	416
13	9 Pup	23.26	0.067	10.00	0.69	0.98	0.47	16.3	37.4	169
14	ADS 15972	44.60	0.248	9.53	0.41	0.2728	0.38	14.9	32.4	277
15	α CMa	50.09	0.375	19.89	0.592	2.11	0.33	30.5	74.7	364
16	α Cen	79.92	0.760	23.57	0.516	1.12	0.46	36.0	83.0	525
17	ξ Boo	151.51	0.147	33.14	0.512	0.858	0.46	50	117	1000

Table 2. Stability limits as functions of the mass. In the 1st column the mass parameter is listed. Column 2 shows M. Hénon and M. Guyot (1970) results for the Critical Orbit using a 1st order stability analysis of periodic orbits. Column 3 gives Szebehely and McKenzies (1981) results using the Jacobian constant in the restricted three body problem. The LCO and UCO are the values of the lower CO and upper CO as they were determined with our numerical method. The UCO can directly be compared to the other two results because from the UCO on all integrated orbits were found to be stable within the time of integration of 500 periods of the primaries. Dimensionless coordinates are used in the equations of motion

Mass parameter	HG	SK	LCO	UCO
0.1	2.12	2.27	1.7	2.15
0.15	2.13	2.39	1.7	2.2
0.2	2.14	2.44	1.75	2.4
0.25	2.14	2.45	1.8	2.3
0.3	2.14	2.43	1.9	2.25
0.35	2.14	2.40	1.9	2.25
0.4	2.14	2.33	1.9	2.3
0.45	2.14	2.26	1.9	2.3
0.5	2.13	2.17	1.9	2.3

developed by Hanslmeier and Dvorak (1984), Delva (1984, 1985), and by H. Lichtenegger (1985).

The full description of the method and the test calculation can be found in Paper I and Paper II. Nevertheless we want to give the main ideas of it briefly, as we use it throughout this paper. Since we wanted to find stable orbits we have to define first of all the meaning of stability in this study:

A stable orbit is defined as an orbit having elliptic orbital elements with an eccentricity smaller than 0.3 throughout the whole integration time of 500 periods of the primary bodies.

For the initial conditions of an orbit to be integrated we choose 4 (for $\mu=0.5$), respectively 8 (for $\mu \neq 0.5$) different positions of the planet:
Pos. 1 one the line of apse (for the elliptic problem), and Pos. 2 to Pos. 8 on lines 45, 90, 135, 180, 225, 270, 315 inclined to it. For the positions of the primaries in the elliptic case we fixed 2 different ones, namely the apoastron and the periastron as they have the largest respectively the smallest distance to each other. After fixing a position we integrated orbits with different initial distances from the barycenter, all of them with circular orbits with respect to one body at the barycenter having a mass of the sum of the two primaries. The chosen interval was 0.05 AU; the semi-major axes of the primaries was set to 1 AU.

As was shown in Paper I after integration times of 500 periods of the primaries about 90% of the escape orbits in the grey region were found. Therefore we fixed the integration time also to this value. It should be stressed again that stability is understood only in this numerical sense. Looking at the results it is evident that a sharp stability limit could not be found. As a consequence we were obliged to define two different COs which correspond to, two different limits: the lower critical orbit (= LCO) is the largest orbit unstable in all integrated positions, and the upper critical orbit (UCO) is the orbit with the smallest semi-major axis stable in all integrated positions.

Tables 3 show in detail the results of the calculations for different values of μ (0.1, 0.2, 0.3 and 0.4) fixing the eccentricity to $e=0$. In this diagram of initial conditions (= stability diagram) the escape time interval is marked by a number from 1 to 10 (0), which corresponds to the periods of the primaries in units of 50. "+" marks a stable orbit in the sense defined above. One can see the rather complicated structure of the grey region in between the LCO and UCO, where stable and unstable orbits are very close together. This is in principle a well known phenomenon of dynamical systems, but nevertheless a more detailed study of some features in the connection with the onset of chaos with the aid of the Liapunov exponents is still in progress.

The results of Table 3 concerning the LCOs and UCOs are summarized in Table 2 together with the corresponding values of HG and SK. We list there also values of the COs for the values of $\mu=0.15$, 0.25, 0.35 and 0.45 without showing the whole stability diagram. One can see, that the UCO is very close to the values found by the two authors mentioned above. It seems that there is also a maximum value of the UCO, but there is no clear mass dependence visible. We can conclude from the different studies, that for P-types the stability limit appears rather insensitive with respect to the mass parameter of the primaries in the circular restricted three body problem.

4. The critical orbit as function of eccentricity and mass-ratio

In the recent numerical study of Paper I such stability limits have been established for the first time as functions of the eccentricity

Table 3a. Stability diagram for $e=0$ and $\mu=0.1$. Column 1 lists the initial distance of the orbit to the barycenter of the primary bodies. Columns 2 to 9 correspond to the different starting positions 1 to 8 as they are explained in detail in the text. Stable orbits are marked by "+", the number marking an unstable orbit corresponds to the total number of revolutions of the primaries, in units of 50, within which the orbit became unstable. The lines with the asterisks mark the LCO respectively the UCO. The "grey region" between these two lines show that here stable and unstable orbits are found close to each other

Distance	Position							
	1	1	1	1	1	1	1	1
1.6	1	1	1	1	1	1	1	1
	1	1	1	1	1	1	1	1
1.7	1	2	1	1	6	1	1	1
*	*	*	*	*	*	*	*	*
	1	1	1	+	+	+	1	1
1.8	1	1	1	+	+	+	1	1
	+	1	+	+	+	+	+	1
1.9	+	1	1	2	+	1	1	1
	+	+	1	+	+	+	2	+
2.0	+	+	+	+	+	+	+	+
	+	+	+	+	+	+	+	+
2.1	+	+	2	+	+	+	1	+
*	*	*	*	*	*	*	*	*
	+	+	+	+	+	+	+	+
2.2	+	+	+	+	+	+	+	+
	+	+	+	+	+	+	+	+
2.3	+	+	+	+	+	+	+	+

Table 3b. Stability diagram for $e=0$ and $\mu=0.2$. For explanation see Table 3a

Distance	Position							
1.6	1	1	1	1	1	1	1	1
	1	1	1	1	1	1	1	1
1.7	1	1	1	1	1	1	1	1
	1	1	1	1	1	1	1	1
*	*	*	*	*	*	*	*	*
1.8	1	1	1	1	+	1	1	1
	1	1	1	+	+	+	1	1
1.9	+	1	1	1	1	1	1	1
	+	1	1	1	+	1	1	1
2.0	1	+	1	+	+	+	1	+
	+	+	1	+	+	+	1	+
2.1	+	1	1	1	+	1	1	2
	+	+	1	+	+	+	2	+
2.2	+	+	1	+	+	+	+	+
	+	+	2	+	+	+	+	+
2.3	+	+	1	+	+	+	+	+
	+	+	1	+	+	+	+	+
*	*	*	*	*	*	*	*	*
2.4	+	+	+	+	+	+	+	+
	+	+	+	+	+	+	+	+
2.5	+	+	+	+	+	+	+	+
	+	+	+	+	+	+	+	+

Table 3c. Stability diagram for $e=0$ and $\mu=0.3$. For explanation see Table 3a

Distance	Position							
	1	1	1	1	1	1	1	1
1.8	1	1	1	1	1	1	1	1
	1	1	1	1	1	1	1	1
1.9	1	1	1	1	1	1	1	1
*	*	*	*	*	*	*	*	*
	+	1	1	1	+	1	1	1
2.0	1	1	1	1	+	1	1	1
	4	+	1	+	+	+	1	+
2.1	+	2	1	+	4	+	1	1
	+	1	1	2	+	1	1	3
2.2	+	+	1	+	+	+	1	+
	+	+	+	+	+	+	+	+
2.3	+	+	+	+	+	+	+	+
	+	+	+	+	0	+	+	+
*	*	*	*	*	*	*	*	*
2.4	+	+	+	+	+	+	+	+
	+	+	+	+	+	+	+	+
2.5	+	+	+	+	+	+	+	+
	+	+	+	+	+	+	+	+

Table 3d. Stability diagram for $e=0$ and $\mu=0.4$. For explanation see Table 3a

Distance	Position							
	1	1	1	1	1	1	1	1
1.8	1	1	1	1	1	1	1	1
	1	1	1	1	1	1	1	1
1.9	1	1	1	1	1	1	1	1
*	*	*	*	*	*	*	*	*
	+	1	1	1	+	1	1	1
2.0	+	1	1	1	+	1	1	1
	1	+	1	+	+	+	1	+
2.1	+	+	1	+	+	+	1	+
	+	1	1	1	+	1	1	1
2.2	+	+	1	+	+	+	1	+
	+	+	8	+	+	+	+	+
*	*	*	*	*	*	*	*	*
2.3	+	+	+	+	+	+	+	+
	+	+	+	+	+	+	+	+
2.4	+	+	+	+	+	+	+	+
	+	+	+	+	+	+	+	+

Table 4. Critical orbits as functions of the eccentricity. Column 1 lists the eccentricities of the primaries, columns 2 and 3 show the LCO and UCO as they are defined in Table 2. The mass parameter is set to $\mu=0.5$

Eccentricity	LCO	UCO
0.00	2.09	2.37
0.05	2.22	2.51
0.10	2.35	2.64
0.15	2.46	2.76
0.20	2.56	2.88
0.25	2.66	2.99
0.30	2.74	3.10
0.35	2.81	3.21
0.40	2.87	3.31
0.45	2.92	3.40
0.50	2.97	3.49
0.55	3.00	3.57
0.60	3.02	3.65
0.65	3.02	3.72
0.70	3.02	3.79
0.75	3.01	3.86
0.80	2.99	3.91
0.85	2.96	3.96
0.90	2.92	4.01

of the primaries but irrespective of mass ratios. The mass parameter μ was fixed in this study to 0.5, which means that the primaries are assumed to be equally massive. The method used is formerly described in the preceding chapter. The parabolic fit established there for the LCO and the UCO is used for the Table 4. We can see immediately the increase of the COs with the eccentricity of the primaries' orbit, which can qualitatively be explained by the increase of the maximum distance between the primaries. But the aim of the paper is to determine the COs as function of eccentricity *and* mass ratio of the double star. So we had to choose the right model to describe the motion of the P-types around a binary system. As in other former studies the plane elliptic restricted 3-body problem was chosen. There we are able to study both parameters in detail namely mass ratio of the primaries and the eccentricity of their orbit. For this study we fixed the value of the eccentricity to $e=0.5$ and varied the mass parameter in the most interesting cases for binaries $0.1 \leq \mu \leq 0.5$. The method used has been described in some detail in the preceding section.

The results of the integrations are shown for $\mu=0.1$, 0.2, 0.3 and 0.4 in the stability diagrams in the Tables 5a to 5d. Unstable orbits are marked by numbers, which also indicate the onset of instability (in the upper defined meaning) in units of 50 periods of the primaries. We have a behaviour similar to the results from other numerical studies of the same kind: islands of stability are found in the "chaotic sea", or vice versa unstable lakes are found in the "main land" of stable orbits.

These are common features of nonlinear dynamical systems (Hénon and Heiles, 1964; Contopoulos, 1967). But we should stress that the tables presented here do not represent the whole phase space not even a surface of section. The "geographic" description is commonly used in the surface of section, an $n-1$ dimensional subspace of phase space. But in a certain sense the initial conditions are a special subset of phase space. Table 6 summarizes the results of Tables 5a to 5d and shows additionally the results of $\mu=0.5$, which was already studied in Paper I.

It is noteworthy that the UCOs seem to have the same characteristic feature of having a maximum (but not very significant) value around 0.25 as is visible for the circular restricted problem (Table 2). It is evident that the absolute values are significantly larger as the high eccentric orbit ($e=0.5$) causes closer approaches of the planet to the primaries. There is not such a feature for the LCOs visible.

To sum up all these results in the circular and in the elliptic restricted three body one can say that even for high eccentric orbits ($e=0.5$) there seem to exist no dependence of the COs on the masses. In general the COs for P-types are determined through the eccentricity of the primaries' orbit.

5. Stability limits for p-types in real binaries

As a consequence of the former results we could immediately use the two following formulae derived through a parabolic fit of numerical data in Paper I

$$\mathrm{LCO}=2.37+2.76e-1.04e^2 \tag{1}$$

$$\mathrm{UCO}=2.09+2.79e-2.07e^2 \tag{2}$$

With these two simple expressions we can calculate the COs for P-types for all real binaries of Table 1. But before using (1) and (2) we wanted to test their precision when the different mass-ratios of the primaries are neglected completely and the eccentricities are different from $e=0$ and $e=0.5$. In a numerical experiment we calculated again the LCOs and the UCOs for the 7 binaries from Table 1 with a mass parameter $\mu=0.5$. The numerical method is the same as it is explained in the preceding chapters. The derived results are shown only in 1 example of the stability diagram of α CMa in Table 7. Making use of the results for the stars 11–17 we can now compare the numerically determined COs with those calculated with formulas (1) and (2); we shall call them estimations. In this way we were able to determine the accuracy of the estimations of the COs which are represented in Table 8. In all the seven cases the estimated and calculated values are very close. Looking at the different mass parameters no shift to a less precise

Table 5a. Stability diagram for $e=0.5$ and $\mu=0.1$. Columns 1 to 8 correspond to positions 1 to 8 in the apoastron of the primaries, columns 9 to 19 to positions 1 to 8 in the periastron position. Column 9 shows the initial distance to the barycenter. For more explanations see Table 3a

Apoapsis								Distance	Periapsis·							
2	2	3	2	1	1	3	2	2.7	1	1	4	1	1	3	9	2
2	3	1	2	5	4	1	2		1	1	2	1	1	1	2	1
3	6	2	2	1	2	4	2	2.8	1	1	7	5	1	2	3	3
3	1	2	2	1	2	4	1		1	1	2	3	2	1	1	3
*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
7	2	3	2	10	1	5	1	2.9	2	+	4	1	3	3	9	2
+	3	9	7	2	3	5	2		5	+	+	3	3	6	+	+
9	2	7	+	4	+	3	5	3.0	+	+	4	2	3	3	+	+
+	3	5	+	5	5	3	3		2	8	+	10	6	10	5	7
+	4	4	5	8	6	7	5	3.1	4	+	+	10	5	5	9	5
5	+	6	+	+	6	10	5		6	+	7	4	6	4	+	+
+	5	9	+	+	+	5	7	3.2	8	+	+	7	8	+	10	+
9	6	3	+	+	+	3	7		3	10	+	3	9	3	5	+
7	+	+	+	9	+	+	+	3.3	7	+	8	+	+	+	+	+
9	+	+	+	+	8	+	+		+	+	+	+	7	+	+	8
*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
+	+	+	+	+	+	+	+	3.4	+	+	+	+	+	+	+	+
+	+	+	+	+	+	+	+		+	+	+	+	+	+	+	+
+	+	+	+	+	+	+	+	3.5	+	+	+	+	+	+	+	+
+	+	+	+	+	+	+	+		+	+	+	+	+	+	+	+

Table 5b. Stability diagram for $e=0.5$ and $\mu=0.2$. For explanation see Table 5a

Apoapsis								Distance	Periapsis							
2	2	1	1	1	1	1	1		1	1	1	1	1	1	2	1
1	2	1	1	1	1	2	1	2.8	1	1	2	2	1	1	1	1
1	2	1	2	2	2	1	1		1	1	3	1	1	2	2	1
2	3	4	2	1	2	1	3	2.9	2	1	3	1	1	1	2	1
*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
2	1	2	2	1	2	1	1		1	1	6	5	+	1	5	1
3	1	4	3	1	2	2	3	3.0	1	4	5	5	1	1	5	5
5	2	6	5	3	8	4	2		1	3	5	3	1	2	5	4
6	2	4	2	4	4	4	3	3.1	7	5	6	3	5	5	4	6
4	4	4	3	5	4	3	4		3	4	+	2	3	3	7	4
4	3	3	7	4	5	3	4	3.2	2	4	5	3	4	3	5	8
4	2	3	+	4	+	6	3		2	4	+	4	5	7	3	+
5	8	5	7	4	5	4	6	3.3	+	6	+	+	8	+	7	4
+	+	+	+	+	6	+	+		5	4	+	7	+	8	+	+
10	9	8	+	+	+	+	8	3.4	+	+	+	7	6	+	+	8
+	+	+	+	+	+	+	7		6	+	+	8	+	+	+	+
+	7	+	+	+	+	+	6	3.5	8	+	+	7	7	+	+	+
+	+	+	+	+	+	+	+		+	+	+	+	+	+	+	+
+	+	+	+	+	+	+	+	3.6	+	+	+	+	+	+	+	+
+	+	+	+	+	+	+	+		+	+	10	+	+	+	+	+
+	+	8	+	9	+	+	+	3.7	+	+	+	+	+	+	+	+
*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
+	+	+	+	+	+	+	+		+	+	+	+	+	+	+	+
+	+	+	+	+	+	+	+	3.8	+	+	+	+	+	+	+	+
+	+	+	+	+	+	+	+		+	+	+	+	+	+	+	+
+	+	+	+	+	+	+	+	3.9	+	+	+	+	+	+	+	+

Table 5c. Stability diagram for $e=0.5$ and $\mu=0.3$. For explanation see Table 5a

Apoapsis								Distance	Periapsis							
2	1	1	1	1	1	1	1	2.8	1	1	1	1	1	1	1	1
4	1	1	1	2	1	1	1		1	1	1	1	1	1	2	1
1	2	1	1	1	2	1	1	2.9	1	1	1	2	1	1	4	1
1	1	1	1	1	2	1	1		1	2	3	1	1	1	3	3
*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
1	2	3	1	1	1	1	2	3.0	1	3	+	2	1	1	+	2
5	2	1	2	3	3	2	3		2	2	5	4	1	6	4	1
+	2	5	1	3	2	6	4	3.1	2	9	5	3	2	+	4	4
6	4	8	3	6	4	+	3		1	5	+	2	2	5	6	0
4	3	6	4	4	5	6	4	3.2	2	4	5	3	3	4	5	7
6	3	4	6	5	+	7	3		1	6	5	3	3	5	5	6
5	3	4	+	5	+	3	7	3.3	1	6	8	3	3	+	4	5
6	8	9	6	+	+	+	8		1	4	+	+	+	+	+	5
+	8	+	+	+	6	5	+	3.4	4	7	+	4	+	+	+	8
8	+	+	+	+	+	+	7		+	+	+	8	+	0	+	9
+	8	+	+	+	+	+	9	3.5	7	+	+	+	4	+	+	+
+	+	+	+	+	+	+	+		7	+	+	+	+	+	+	+
+	+	+	+	+	+	+	+	3.6	+	+	+	+	+	+	+	+
+	+	+	+	+	+	+	+		+	+	+	+	+	+	+	+
8	+	+	+	+	+	+	+	3.7	+	+	+	+	+	0	+	0
*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
+	+	+	+	+	+	+	+		+	+	+	+	+	+	+	+
+	+	+	+	+	+	+	+	3.8	+	+	+	+	+	+	+	+
+	+	+	+	+	+	+	+		+	+	+	+	+	+	+	+
+	+	+	+	+	+	+	+	3.9	+	+	+	+	+	+	+	+

Table 5d. Stability diagram for $e=0.5$ and $\mu=0.4$. For explanation see Table 5a

Apoapsis								Distance	Periapsis							
7	1	1	1	1	1	1	2	2.8	1	1	1	1	1	1	1	1
1	1	2	1	2	1	1	1		1	1	2	1	1	1	3	1
1	1	1	1	1	1	1	2	2.9	1	1	1	1	1	1	2	1
1	1	2	2	1	3	3	1		1	1	3	1	1	2	5	3
1	2	1	3	7	1	3	2	3.0	1	2	+	1	1	1	+	4
2	2	3	2	+	2	1	2		1	3	+	2	1	5	+	2
+	4	4	2	+	7	3	4	3.1	1	5	8	0	1	9	8	6
+	+	+	+	0	+	+	+		2	+	+	+	2	+	+	+
8	+	8	4	9	5	+	+	3.2	2	6	8	7	2	+	+	+
+	+	6	8	+	9	8	9		1	6	9	5	3	+	+	9
+	+	+	+	+	+	+	7	3.3	1	+	8	5	4	+	+	+
6	+	+	+	6	+	+	+		2	+	+	7	+	+	+	+
+	+	3	9	+	+	+	+	3.4	3	7	+	+	+	5	+	7
+	+	+	+	+	+	+	+		9	+	+	+	+	+	+	+
+	+	+	+	+	+	+	+	3.5	+	+	+	+	3	+	+	+
*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
+	+	+	+	+	+	+	+		+	+	+	+	+	+	+	+
+	+	+	+	+	+	+	+	3.6	+	+	+	+	+	+	+	+
+	+	+	+	+	+	+	+		+	+	+	+	+	+	+	+
+	+	+	+	+	+	+	+	3.7	+	+	+	+	+	+	+	+

Table 6. Critical orbits for $e=0.5$ as functions of μ. Line 1 marks the mass parameter of the primaries, lines 2 and 3 show the LCO respectively the UCO from Table 5a–5d

Mass-ratio	0.1	0.2	0.3	0.4	0.5
LCO	2.85	2.9	2.95	2.95	2.95
UCO	3.4	3.75	3.75	3.55	3.5

estimation for binaries with a very different μ is visible. Star 15 with a $\mu=0.33$ has even a higher precision than star 11 with the largest "error" (which is in fact the difference expressed in percents of the mean value). Since 12 out of 14 estimations have errors smaller than 5% the method seems to work quite well, for all double stars, when one wants to derive stability limits for P-types.

The LCO is slightly better determined as the UCO (2.8% and 3.8%). The mean error of estimating a CO in such a system is

Table 7. Stability diagram for α CMA. For detailed explanations see Fig. 3a. The units for the distances are AU

Distance	Aphel								Perihel							
54	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1
	3	1	1	1	1	1	1	1	1	1	2	1	1	1	2	1
58	1	1	1	1	1	1	1	1	1	2	2	1	1	1	1	2
	2	1	1	1	1	1	1	1	1	1	9	1	1	1	4	2
*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
62	3	1	+	1	2	2	3	1	1	1	5	1	2	2	3	4
	4	2	3	4	4	4	4	4	2	4	6	2	1	4	7	6
66	4	3	7	+	5	+	6	3	1	+	+	3	3	7	+	7
	5	5	5	3	4	0	6	4	2	5	+	4	+	7	+	5
70	7	+	+	+	+	+	+	+	9	+	+	+	4	+	+	8
	+	+	+	+	+	+	+	+	8	+	8	+	+	+	+	+
74	7	+	+	+	+	+	+	+	+	+	+	8	+	8	+	+
	+	+	+	+	+	+	+	+	6	+	+	+	+	+	+	+
*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*	*
78	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
82	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

Table 8. Precision of "estimated" stability limits in critical orbits. Columns 1 and 2 list the star number from Table 1 and the mass parameter. In columns 3 and 4 the eccentricity and the semi-major axis in AU are given. Column 5 gives the estimated (after Eq. (1)) LCOs, column 6 the numerically determined values of the LCOs. Column 7 is the "error", that is here defined as the difference of the 2 values in percents of the mean value. Columns 8, 9 and 10 show the same for the UCOs; Eq. (2) is used to derive the values in column 8

Star	Mass	Ecc.	Axes	LCO(*e*)	LCO(*c*)	Error	UCO(*e*)	UCO(*c*)	Error
11	0.43	0.15	9.67	23.8	25.0	4.9	26.7	29.5	10
12	0.37	0.74	16.4	49.5	48.0	3.1	63.0	62.0	1.6
13	0.47	0.69	10.0	30.3	29.5	2.7	37.8	37.0	2.1
14	0.38	0.41	9.53	27.5	27.0	1.8	31.7	33.0	4.0
15	0.33	0.59	19.89	60.0	58.0	3.4	72.4	78.0	7.5
16	0.46	0.52	23.57	70.2	69.0	1.7	82.9	83.0	0.1
17	0.46	0.51	33.14	98.6	97.0	1.7	116.3	117.0	0.6

significantly smaller than 10% (3.3%), which is quite satisfying for any numerical method.

Finally we show the results for all the 17 binaries of Table 1. For the stars 11–17 the mean value of the calculated and estimated CO is taken in Table 9. The distances to the two primaries are in the most cases far away from any habitable zone (Benest, 1988). Only for ε Cet (2) and BD-8 4352 (7) the semi-major axes of the closest possible planet of type P is between 3 and 5 AU. In the last column we calculated the period of a fictitious planet which would be just on the UCO. Most of periods are of the order of some hundred years with the exception of (2) and (7). We want to emphasize that the case of the S-type planetary orbits is quite different. Doubtless there exists a strong dependence on the mass parameter which is physically clear from the fact, that the sphere of influence changes significantly with μ. The S-types are also more interesting than the P-types from the point of view of "habitability", since their orbits can be close enough to one component of a double star system (see Paper II; Benest, 1986, 1988). But unfortunately the numerical method is much more time consuming than for the P-types due to the very small step size necessary for the numerical integration; this work is still in progress.

As a conclusion of this study and the former ones concerning fictitious planetary orbits in binaries we believe that one should not all exclude all binaries when searching for planets outside the Solar System. Concerning the cosmogonical possibility of planet formation in double stars this question is not yet settled. When we think of the formation of planets in single stars, it looks like the existence of a large perturber of the orbits of small particles around the star in formation (like Jupiter in our Solar System) is triggering the growths of the particles (Lichtenegger, 1987). It could be that the existence from the early stages of such a second massive body would make the chance of formation of planets even significantly higher in double stars than in single stars.

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