/Exercises Ch. 1/

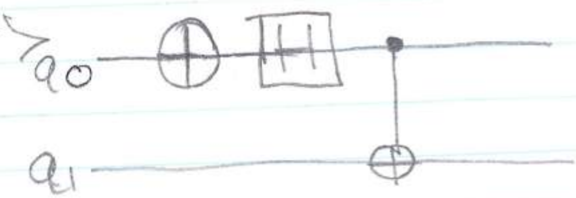
1.1 $|0\rangle = |000\rangle$

$|1\rangle = |001\rangle \ldots 7 = |111\rangle$

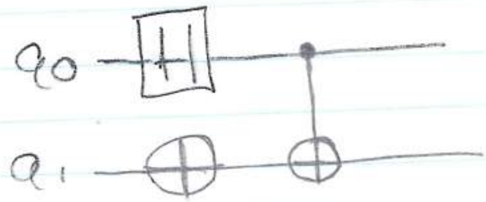
$|\psi\rangle = \sum_0^{2n-1} a_i |i\rangle$

basis states = 1

1.2. a) $\frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$

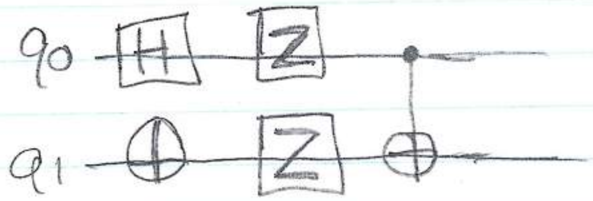
$\Rightarrow$ 

b) $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

$\Rightarrow$ 

c) $\frac{1}{\sqrt{2}}\left(|01\rangle - |10\rangle\right)$

$\Rightarrow$



1.3. b) $HXH = Z$

$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$HXH = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\underbrace{\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}}_{XH}$$

$$HXH = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 & 0 \\ 0 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = Z$$

c) $HYH = -Y$

$$HYH = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\underbrace{\begin{pmatrix} -i & i \\ i & i \end{pmatrix}}_{YH}$$

$$HYH = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} -i & i \\ i & i \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 & 2i \\ -2i & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} = -Y$$

a) $HZH = X$

$$HZH = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\underbrace{\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}}_{ZH}$$

$$HZH = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = X$$

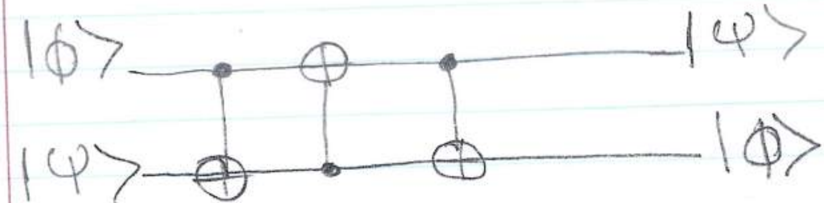
d) $CNOT_{1,0} = H^{\otimes 2}\, CNOT_{0,1}\, H^{\otimes 2}$

1.4. Classical XOR truth table

Bit 1	Bit 2	output
0	0	0
0	1	1
1	0	1
1	1	0

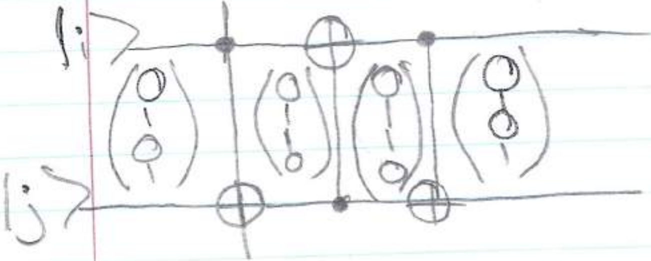
Boolean = A XOR B

Qubitswap gate:



truthtable Quantum XOR swap:

Qubit 1	Qubit 2	output 1	output 2
$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|1\rangle$
$\|1\rangle$	$\|1\rangle$	$\|1\rangle$	$\|0\rangle$



1.5. energy $\Delta = |0\rangle$ and $|1\rangle \approx 10\,GHz$

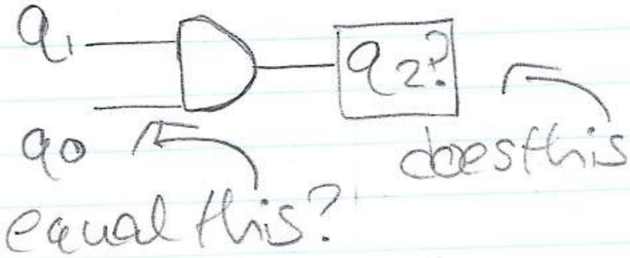
$kT \ll \hbar\omega$

$\omega/(2\pi) = 10\,GHz$

$T \ll 0.48\,K$

1.6.

$$|q_2 q_1 q_0\rangle = \frac{|010\rangle + |101\rangle + |001\rangle + |110\rangle}{2}$$



q_1	q_0	
1	0	$\Rightarrow 1$
0	1	$\Rightarrow 1$
0	1	$\Rightarrow 1$
1	0	$\Rightarrow 1$

q_2	q_1 Nand q_0
0	1
1	1
0	1
1	1

//PRINCIPLES OF SUPERCONDUCTING QUANTUM COMPUTERS//

CHAPTER 1:

1.11 Exercises:-

1.6. $|q_2 q_1 q_0\rangle = (|0\rangle \otimes |1\rangle \otimes |0\rangle + |1\rangle \otimes |0\rangle \otimes |1\rangle + |0\rangle \otimes |0\rangle \otimes |1\rangle + |1\rangle \otimes |1\rangle \otimes |0\rangle)/2$

which is linear combin. tensor product single qub. states
state is seperable and qubit q_2 isn't entangled w/ other 2

$|q_2 q_1 q_0\rangle = (|0\rangle \otimes |1\rangle)/2 + (|1\rangle \otimes |0\rangle)/2 + (|0\rangle \otimes |0\rangle)/2 + (|1\rangle \otimes |1\rangle)/2 \otimes |1\rangle$

first 4 seperable, last term $|1\rangle$ is state qubit q_2. q_2 isn't entangled w/ other 2 qubits.

1.7. (a) $|0\rangle = \dfrac{|+\rangle + |-\rangle}{\sqrt{2}}$

$$P(|+\rangle) = |\langle + | 0\rangle|^2 = \left| \frac{(\langle 0| + \langle 1|)}{\sqrt{2}} |0\rangle \right|^2$$

$$= |\langle 0|0\rangle + \langle 1|0\rangle|^2 / \sqrt{2}^2$$

$= (1+0)/2$

$= \underline{\underline{1/2}}$

1.7(b) $|1\rangle = (|+\rangle - |-\rangle)/\sqrt{2}$

$P(|+\rangle) = |\langle + | 1\rangle|^2 = \left| \frac{(\langle 0| + \langle 1|)}{\sqrt{2}} |1\rangle \right|$

$= (0+1)/2 = 1/2$

1.7(c) $|-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$

Use Born Rule:

$|\langle \varphi | \psi \rangle|^2$

$P|-\rangle = |\langle - | R|^2$

$= \left| \frac{\langle 0| - \langle 1|}{\sqrt{2}} * \frac{1}{\sqrt{2}|0\rangle + i\sqrt{2}|1\rangle} \right|^2$

$= |\langle 0|0\rangle/2 - \langle 0|1\rangle/2 - \langle 1|0\rangle/2 + \frac{\langle 1|1\rangle}{2}$

$+ \frac{i\langle 0|0\rangle}{2} -$

$\frac{\langle 0|1\rangle}{2} + \frac{\langle 1|0\rangle}{2}$

$+ \frac{\langle 1|1\rangle}{2} \Big|^2$

$= (1/2 - 0 - 0 + 0)^2 + (0 + 0 + 0 + \frac{1}{2})^2$

$= 1/2$

$|R\rangle = 1/\sqrt{2}\,|0\rangle + i\sqrt{2}$

1.8(a)

$$|\psi\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{(1+i)}{2}|1\rangle$$

$$P\,|0\rangle = |1/\sqrt{2}|^2 = \underline{\underline{1/2}}$$

1.8 (b) $|\psi\rangle = (1/2 + i/2)|+\rangle + (1/2 - i/2)|-\rangle$

$$P|+\rangle = |1/2 + i/2|^2 = \underline{\underline{1/2}}$$

1.8 (c) $|\psi\rangle = \frac{1+i}{2}|+\rangle + \frac{(1-i)}{2}|-\rangle$

$$P|-\rangle = |1 - i/2|^2 = \underline{\underline{1/2}}$$

/CHAPTER 2 Exercises/

2.1.(a)

$$E[Z^2] = (1^2)(Pr(Z=1)) + (-1^2)(Pr(Z=-1))$$
$$= p + (1-p) = 1$$

$$E[Z] = Pr(Z=1)(1) + Pr(Z=-1)(-1)$$
$$= p - (1-p) = 2p-1$$

2.1(b) $Var[Z] = E[Z^2] - E[Z]^2 = 1-(2p-1)^2$
$$= 4p(1-p)$$

2.1 (c) $\frac{d}{dp} Var[Z] = 4(1-2p) = 0$

$p = 1/2$ max. value $p(1-p)$

sub. $p = 1/2$

$$Var[Z] = 4(1/2)(1/2) = 1$$

upperbound.

$$\sigma/\sqrt{N} \leq 1/\sqrt{N}$$

upperbound on sigma

$$\sigma \leq 1$$

2.2 $\cos(\theta/2) = 0.8 - 0.2 = 0.6$

$\sin(\theta/2) * \cos(\phi) = -i(0.6-0.4)$
$$= -i(0.2) = -0.2i$$

$$\sin(\theta/2) \times \cos(\phi) = -i(0.6 - 0.4)$$
$$= -i(0.2) = -0.2i$$
$$\sin(\theta/2) \times \sin(\phi) = 2(0.2) - 1 = -0.6$$

$$\sin^2(\theta/2) \times (\cos^2(\phi) + \sin^2(\phi))$$
$$= |-0.2i|^2 + |-0.6|^2 = 0.4$$
$$\cos^2(\phi) + \sin^2(\phi) = 1$$
$$\sin(\theta/2) = \sqrt{0.4} = 0.632$$
$$\cos(\phi) = -0.2i/(-0.632) = 0.316i$$
$$\sin(\phi) = -0.6/(-0.632) = 0.949$$
$$\theta \approx 1.26\,\text{rad}$$
$$\phi \approx 1.22\,\text{rad.}$$

2.3. $XYX = X(YX) - (YX)X$
$$= X(-XY) - (-YX)X \quad (\text{bec. } YX = -XY)$$
$$= -X(YX) + (YX)X$$
$$= -XYX$$
$$\text{so } XYX = -Y$$
$$R_y(\theta) = e^{-i(\theta/2)\times Y}$$
$$XR_y(\theta)X$$
$$= Xe^{-i(\theta/2)\times Y}X \quad Y^2 = I$$
$$= X(\cos(\theta/2)I - i\sin(\theta/2)\times Y)X$$

$\cos(\emptyset/2)X^2 - i\sin(\emptyset/2)(XYX)$

$= \cos(\emptyset/2)X^2 + i\sin(\emptyset/2)xY$

$R_y(-\emptyset)$

$= e^{(ix(\emptyset/2)Y_i)}$

$= \cos(\emptyset/2)I + i\sin(\emptyset/2)Y$

$\cos(\emptyset/2)X^2 + i\sin(\emptyset/2)Y$

$= \cos(\emptyset/2)I + i\sin(\emptyset/2)Y$

$\cos(\emptyset/2)X^2 = \cos(\emptyset/2)I$

$X^2 = I$

therefore proved //

2.4 (a)

$ABC = (R_z(\phi)R_y(\emptyset/2)R_y(-\emptyset/2)R_z((-\lambda+\phi)/2)$
$R_z((\lambda-\phi)/2))$

$\cos(x)\cos(y) - \sin(x)\sin(y)$
$= \cos(x+y)$

$\sin(x)\cos(y) + \cos(x)\sin(y)$
$= \sin(x+y)$

$ABC = R_z(\phi)R_y(0)R_z((-\lambda+\phi)/2)R_z((\lambda-\phi)/2$

$= R_z(\phi)R_z((-\lambda+\phi)/2)R_z((\lambda-\phi)/2)$

$= R_z(\phi - (\lambda+\phi)/2 + (\lambda-\phi)/2)$

$= R_z(0) = I$

2.4(b) $AXBXC = R_z(\phi) R_z((\lambda-\phi)/2) R_z((-\lambda+\phi)/2) R_z(\phi)$

$= R_z(\phi) R_z(\lambda-\phi) R_z(-\lambda+\phi)) R_z(\phi)$

$= R_z(\lambda-\phi) R_z(-\lambda-\phi) = R_z(\lambda)$

so we proved this//
only differ by global phase

2.5

$\rho = 1/2 |a\rangle\langle a| + 1/2 |b\rangle\langle b|$

$= 1/2(\sqrt{0.90}\, |0\rangle + \sqrt{0.10}\, |1\rangle)(\sqrt{0.90}\, \langle 0| + \sqrt{0.10}\, \langle 1|)$

$+ 1/2(\sqrt{0.90}\, |0\rangle - \sqrt{0.10}\, |1\rangle)(\sqrt{0.90}\, \langle 0| - \sqrt{0.10}\, \langle 1|)$

$= 1/2\, (0.90\, |0\rangle\langle 0| + 0.19\, |0\rangle\langle 1| + 0.19\, |1\rangle\langle 0| + 0.10\, |1\rangle\langle 1|)$

$+ 1/2\, (0.90\, |0\rangle\langle 0| - 0.19\, |0\rangle\langle 1| - 0.19\, |1\rangle\langle 0| + 0.10\, |1\rangle\langle 1|)$

$= 0.90\, |0\rangle\langle 0| + 0.10\, |1\rangle\langle 1|$

dens. matrix corresponds to a mix of 2 pure states, equal prob. and orthogonal.

// CHAPTER 3 - Exercises //

3.1.

$CNOT = |00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 11|$
$\quad + |11\rangle\langle 10|$

$R_y(\theta) = exp(-i\,\theta/2 \times Y)$

$Y = \begin{bmatrix} [0, -1] \\ [i, 0] \end{bmatrix}$

$R_x(\theta) = exp(-i\,\theta/2 \times X)$

$X = \begin{bmatrix} [0, 1] \\ [1, 0] \end{bmatrix}$

$\sqrt{iSWAP} = exp(-i \times \pi/4) \times$
$SWAP \times exp(-i \times \pi/4) \times S$

$SWAP = |00\rangle\langle 00| + |01\rangle\langle 10| + |10\rangle\langle 01|$
$+ |11\rangle\langle 11|$

$S = \begin{bmatrix} [1, 0] \\ [0, i] \end{bmatrix}$

$CNOT = \left[I \otimes R_y(-\pi/2) \right] \left[R_x(-\pi/2) \otimes R_x(\pi/2) \right] \times$

$\rightarrow \sqrt{iSWAP} \times \left[I \otimes R_x(\pi) \right]$

$\rightarrow \sqrt{iSWAP} \left[I \otimes R_y(\pi/2) \right]$

$= \left[I \otimes exp(-i \times \pi/4 \times Y/2) \right] \left[exp(-i(-\pi/2)/2 * X) \otimes \right. \rightarrow$

$$\exp(-i*\pi/2*X/2)*(\exp(-i*\pi/4)*SWAP*$$
$$\exp(-i*\pi/4)*S)*[I\otimes\exp(-i*\pi*X/2)]*$$
$$(\exp(i*\pi/4)*SWAP*\exp(i*\pi/4)*S)$$

$$\exp(aX/2)\exp(bX/2) = \exp((a+b)X/2)$$

$$\to \cos(|a-b|/2) - i(X\,\mathrm{dot}(a\times b))$$

$$\to /2\sin(|a-b|/2)$$

$$= SWAP^2 = I$$

$$= S^2 = Z$$

$$= YZ = iZ$$

and we get the same CNOT //

3.2

$$I \times \cos^2(\beta \times \pi/2) - i * X * Z * \sin(\beta * \pi) + i * X * Z * \cos\left(\beta * \frac{\pi}{2}\right)$$

$$- X * Z \sin^2(\beta * \pi/2) = \left[e^{(-i * \pi * \beta/2)} * X * Z\right]^{\beta}$$

$$XZ = -ZX$$

$$I * \cos^2(\beta * \pi/2) + i * X * Z * \sin(\beta * \pi/2) - X * Z * \sin^2(\beta * \pi/2) = \left[e(-i * \pi * \beta/2) * X * Z\right]^{\beta}$$

$$\sqrt{I * \cos^2(\beta * \pi/2) + i * X * Z \sin(\beta\pi/2) - XZ \sin^2(\beta\pi/2)}$$

$$= \left[e^{(-i\pi\beta/4)} * XZ\right]^{(\beta/2)}$$

$$\cos(\emptyset/2) I + i \sin(\emptyset/2)(u \cdot X) i(u \cdot X)$$

$$\cos(\beta \times \pi/4) I - i \sin(\beta\pi/4) XZ$$

$$= \left[e^{(-i\pi\beta/4)} * XZ\right]^{\beta/2}$$

3.4 $I Z^{-1/2} X Z^{1/2} X I^{-1/2}$

$= 1/2(I-Z)\, 1/2(X-iY)\, 1/2(I-X)$

$= 1/4\,(I-Z)(X-iY)(I-X)$

$= 1/4\,(IX-ZX-iIY+ZY)(I-X)$

$= 1/4\,(I-X-iIY-Z+ZYX+iXY)$

$Y = iXZ$

$= 1/4\,(I-X-iIY-Z-iX+iYX)$

$= 1/4\,(I-X-iIY-Z-iY-X)$

$1/4\,(I-X-iIY-Z-iY-X)|00\rangle$

$= 1/4\,(I+Y)|00\rangle = |00\rangle$

$1/4\,(I-X-iIY-Z-iY-X)|01\rangle$

$= 1/4\,(I-Y)|01\rangle = |01\rangle$

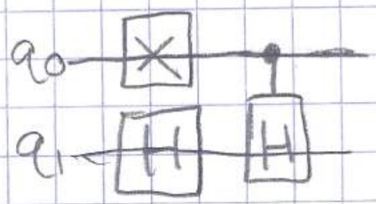
$1/4\,(I-X-iIY-Z-iY-X)|10\rangle$

$= 1/4\,(I-Z-YX)|11\rangle = |11\rangle$

$1/4\,(I-X-iIY-Z-iY-X)|11\rangle$

$= 1/4\,(I+Z-YX)|10\rangle = |10\rangle$ //

3.4



$|00\rangle \rightarrow |00\rangle$

$|01\rangle \rightarrow \frac{1}{\sqrt{2}}|0\rangle|1\rangle + \frac{1}{\sqrt{2}}|1\rangle|1\rangle$

$|10\rangle \rightarrow \frac{1}{\sqrt{2}}|1\rangle|0\rangle + \frac{1}{\sqrt{2}}|0\rangle|1\rangle$

$|11\rangle \rightarrow |11\rangle$

3.5

$|Q\rangle = (1/2) * |000\rangle + |001\rangle + |010\rangle + |111\rangle$

$= (1/2) * [1,0,0,0,1,0,0,0,1]^{+}$

$|q_2 q_1 q_0\rangle = |q_2\rangle \otimes |q_1\rangle \otimes |q_0\rangle$

$\rho_{q_2} = Tr(q_1 q_0 (|Q\rangle\langle Q|))$

$= (1/2) * Tr(q_1, q_0 (|000\rangle\langle 000| + |001\rangle\langle 001| + |010\rangle\langle 010| + |111\rangle\langle 111|))$

$= 1/2 * [1,0,0,1]$ //

//CHAPTER 4 - Exercises//

4.1

$c = 1/\sqrt{L*C}$

$$c = \frac{1}{\sqrt{L*C}} * Z_0$$

$$3*10^8 = \frac{1}{\sqrt{L*100*10^{-12}}} * 50$$

$L \approx 237.7\ nH/m$ //

4.2

$RL = 20\log_{10}(|\Gamma|)$

$\Gamma = (ZL - Z_0)/(ZL + Z_0)$

$ZL = Z_0 * (1+\Gamma)/(1-\Gamma)$

$15 = 20\log_{10}(|\Gamma|)$

$|\Gamma| = 0.1778$

$ZL = Z_0 * (1+0.1778)/(1-0.1778)$

$= 1.383\, Z_0$

perc. abs. $= 100*(1-|\Gamma|^2) = 100*(1-0.0316)$

$= 96.84\%$ //

4.3.

$Z_C = \sqrt{Z_0 * Z_L}$

$Z_0 = 300\,\Omega$, $Z_L = 75\,\Omega$

$Z_C = \sqrt{300\,\Omega * 75\,\Omega}$

$= \sqrt{22500\,\Omega^2} = 150\,\Omega$ //

4.4.

Aten. $= -10\log_{10}((R_2 + R_3)/(R_1 + R_2))^2$

$20 = -10\log_{10}((R_2 + R_3)/(R_1 + R_2))^2$

$(R_2 + R_3)/(R_1 + R_2)$

$= \sqrt{10^{(-20/10)}} = 0.1$

$R_1 = R_2 \| 50\,\Omega$

$R_3 = R_2 \| 50\,\Omega$

parallel combo of 2 resistors

$(R_2 + R_2 \| 50)/(R_2 \| 50 + 2R_2)$

$= 0.1$

$R_2 = 45.5\,\Omega$

$R_1 = R_3 = 30.7\,\Omega$

$\boxed{R_1 = R_3 = 30.7\,\Omega}$

$\boxed{R_2 = 45.5}$

4.5 $f_c = 1/2(\pi\sqrt{2LC})$

$f_c = 1/2(\pi\sqrt{3LC})$

for π network:

$f_c = 1/2(\pi\sqrt{2LC})$

$100e6 = 1/2\pi\sqrt{2LC}$

$LC = 1/(2\pi 100e6)^2$

$LC = 2.54e-17$, ass. $C = 1nF (1e-9F)$

$L = 2.54e-17/1e-9$

$LC = 25.4nH$ //

for T network:

$f_c = 1/2\pi\sqrt{3LC}$

$100e6 = 1/2\pi\sqrt{3LC}$

$3LC = (1/2\pi 100e6)^2$

$LC = ((1/2\pi 100e6)^2)/3$

$LC = 8.98e-18$

assume, $C = 1nF (1e-9F)$

$L = (8.98e-18)/1e-9$

$L = 8.98nH$ //

a

4.6

assume gain 10dB ($G=10$)
1Hz ($B=1$)

$$T_N \geq \frac{((10^{1/10}) + 6.626\times10^{-34}\times 6\times10^{9})}{2\times 1.38\times10^{-23}}$$

$T_N \geq 5.94K$ //

4.7

$NF = 10\log(F-1)$

$F = 1 + (T_{sub}N / T_{sub}0)$

$F = 1 + (4/290)$
$F = 1.0138$

$NF = 10\log(1.0138-1)$
$NF = 10\log(0.0138)$

$NF = -21.68\,dB$

so noise $\approx$ 21.68 dB //

//CHAPTER 5 – Exercises//

5.1.

$FWHM = f_0/Q$

$Q = f_0/FWHM$

$f_0 = 5GHz$, $FWHM = 50kHz$

$Q = 5\times10^9/50\times10^3$

$Q = 100$ //

5.2. $1/Q_{meas} = 1/Q_{int} + 1/Q_{ext}$

$1/5\times10^4 = 1/2\times10^5 + 1/Q_{ext}$

$1/Q_{ext} = 1/5\times10^4 - 1/2\times10^5$

$1/Q_{ext} = -0.0003$

$Q_{ext} = -1/0.0003$

$Q_{ext} = -3333.33$?//

losses in extern. circuit > losses in resonator

5.3 $f_0 = 1/2\pi\sqrt{LC}$

$Z_0 = 50\Omega$

$C = 1/4\pi^2 f_0^2 L$

$L = 1/4\pi^2 f_0^2 C$

$f_0 = 6GHz$, $Z_0 = 50\Omega$

$C = 1/4\pi^2(6\,\text{GHz})^2 L)$

$= 1.085\,\text{pF}$ //

$L = 1/4\pi^2(6\,\text{GHz})^2 C)$

$= 2.18\,\text{nH}$ //

5.4.

power trans. through input transmiss. = power output

half of the power is dissipated

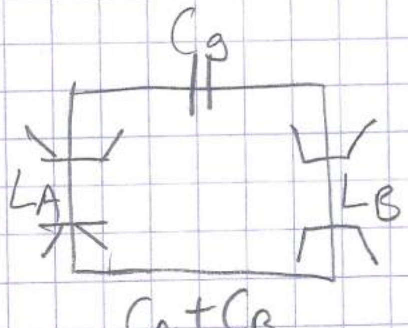
so insertion loss is 3 dB //

5.5.

$\Omega = 1/\sqrt{L_{eq} \times C_{eq}}$

$C_{eq} = C_A + C_B + 2C_g = 1\,\text{pF} + 1\,\text{pF} + 2 \times 50\,\text{fF}$

$= 2.1\,\text{pF}$



Based on Kirchhoff's Laws

$L_{eq} = L_A + + L_B = 2 \times 1\,\text{nH} = 2\,\text{nH}$

$$\Omega = 1/\sqrt{L_{eq} \times C_{eq}} = 1/\sqrt{2nH \times 2.1pF}$$
$$= 2.87 \times 10^{10} \text{ rad/s} \;//$$

5.6.

$$\Delta\Omega = x^2/(\Omega_0 - \Omega_{qub})$$
$$\Delta\Omega = x^2/(\Omega_0 - \Omega_{qub0})$$
$$= 7MHz^2/7GHz - 5GHz$$
$$= 98MHz \;//$$
$$\Delta\Omega_1 = x^2/\Omega_0 - \Omega_{qub1}$$
$$= 7MHz^2/7GHz - 5.007GHz$$
$$= 6.97MHz \;//$$

//CHAPTER 6 - Exercises//

6.1 (a)

$$\frac{dH}{dp} = p/m = \dot{x}$$

$$\frac{dH}{dx} = m\omega_0^2 x = \dot{p}$$

6.1 (b)

$$\ddot{p} = -m\omega_0^2 \dot{x}$$

$$= -m\omega_0^2 (p/m)$$

$$= -\omega_0^2 p$$

$$\ddot{p} = -\omega_0^2 (p/m)$$

$$m\ddot{x} + kx = 0$$

6.1 (c)

$$x(t) = A\exp(j\omega_0 t)$$

$$\dot{x}(t) = A(j\omega_0)\exp(j\omega_0 t) = j\omega_0 x(t)$$

$$\ddot{x}(t) = -A\omega_0^2 \exp(j\omega_0 t) = -\omega_0^2 x(t)$$

$$m(-\omega_0^2 x(t)) + kx(t) = 0$$

$$(-m\omega_0^2 + k)\,x(t) = 0$$

$$m\omega_0^2 = k$$

$$\omega_0 = \sqrt{k/m}$$

$$x(t) = A\exp(j\omega_0 t) = x_0\exp(j\omega_0 t)$$

6.3.

$x = 1/\sqrt{2m\omega_0\hbar}\ (a + a^\dagger)$

$p = -i(\sqrt{m\omega_0/2\hbar})(a - a^\dagger)$

$a = \sqrt{m\omega_0/2\hbar}\ (x + i(p/m\omega_0))$

$x = 1/\sqrt{2m\omega_0\hbar}\ (a - i(p/m\omega_0))$

$a^\dagger = \sqrt{m\omega_0/2\hbar}\ (x - i(p/m\omega_0))$

$p = -i(\sqrt{m\omega_0/2\hbar}\ (a - a^\dagger)$

$p = -i(\sqrt{\hbar m\omega_0/2})(a - a^\dagger)$

so, $x = \sqrt{\hbar/(2m\omega_0}\ (a + a^\dagger)$ //

$p = -i(\sqrt{\hbar m\omega_0/2}\ (a - a^\dagger)$ //

6.4

$\Phi^2 = \hbar/2\omega_0 C\,(a + a^\dagger)^2 = \hbar/2\omega_0 C\,(a^2 + 2a\cdot a^\dagger + a^{\dagger 2})$

$Q^2 = \hbar\cdot\omega_0\cdot C/2 - i(a - a^\dagger)^2 = \hbar\cdot\omega_0\cdot C/2\,(a^2 - 2a\cdot a^\dagger + a^{\dagger 2})$

$H = Q^2/2C + \Phi^2/2L$

$= \hbar/2(\omega_0 a^\dagger a + 1/2)$

$H = \hbar\cdot\omega_0(a^\dagger a + 1/2)$ //

6.5.

$$H = -(\hbar \cdot \omega_2/2) \cdot (\sigma_z \otimes I) \otimes (-\hbar\, \omega_1/2) \times I$$

$$\otimes \sigma_z + \hbar g \times \sigma_x \otimes \sigma_x$$

$$H = (\hbar^2 \cdot \omega_1 \cdot \omega_2/4)(\sigma_z \otimes I \otimes \sigma_z) - \hbar g \cdot \sigma_x \otimes \sigma_x$$

$$H = \begin{bmatrix} \hbar^2 \cdot \omega_1 \cdot \omega_2/4 - \hbar g, & 0, & 0, & 0 \\ 0, & -\hbar^2 \cdot \omega_1 \cdot \omega_2/4 - \hbar g, & 2 \cdot \hbar g, & 0 \\ 0, & 2\hbar g, & -\hbar^2 \cdot \omega_1 \cdot \omega_2/4 - \hbar g, & 0 \\ 0, & 0, & 0, & \hbar^2 \cdot \omega_1 \cdot \omega_2/4 - \hbar g \end{bmatrix}$$

6.5(a)

$$\det(H - \lambda I) = 0$$

$$\lambda 1 = -\hbar \cdot (\omega_1 + \omega_2)/2 - \hbar \cdot \sqrt{\omega_1^2 + \omega_2^2 + 4g^2}/2$$

$$\lambda 2 = -\hbar \cdot (\omega_1 - \omega_2)/2 - \hbar \cdot \sqrt{\omega_1^2 + \omega_2^2 + 4g^2}/2$$

$$\lambda 3 = \hbar \cdot (\omega_1 - \omega_2)/2 - \hbar \cdot \sqrt{\omega_1^2 + \omega_2^2 + 4g^2}/2$$

$$\lambda 4 = \hbar \cdot (\omega_1 + \omega_2)/2 - \hbar \cdot \sqrt{\omega_1^2 + \omega_2^2 + 4g^2}/2$$

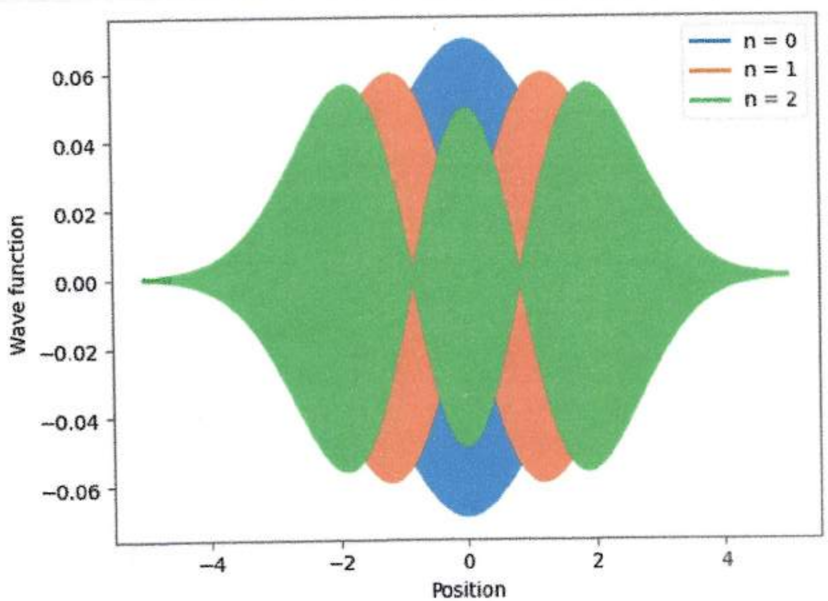
(b)

$$H = (\hbar^2 \cdot \omega_0^2/4) \cdot (\sigma_z \otimes I \otimes \sigma_z) - \hbar g \cdot \sigma_x \otimes \sigma_x$$

$$\omega_0 = \sqrt{\omega_1^2 + \omega_2^2}$$

$$|\psi\rangle = a|\uparrow\uparrow\rangle + b|\uparrow\downarrow\rangle + c|\downarrow\uparrow\rangle + d|\downarrow\downarrow\rangle //$$

[1]:
```
import numpy as np
import matplotlib.pyplot as plt

# Set up parameters
N = 1000  # number of points in the grid
L = 10.0  # Length of the grid
x = np.linspace(-L/2, L/2, N)  # position grid
dx = x[1] - x[0]  # grid spacing

# Define the potential function
V = 0.5 * x**2

# Define the kinetic energy operator
T = np.zeros((N, N))
for i in range(N):
    for j in range(N):
        if i == j:
            T[i, j] = -2.0
        elif i == j-1 or i == j+1:
            T[i, j] = 1.0
T /= dx**2

# Solve the eigenvalue problem
H = T + np.diag(V)
E, psi = np.linalg.eigh(H)

# Plot the wave functions
for i in range(3):
    plt.plot(x, psi[:, i], label=f"n = {i}", linewidth=2)

plt.xlabel("Position")
plt.ylabel("Wave function")
plt.legend()
plt.show()
```



[ ]:

//CHAPTER-7 : Exercises//

7.1. $E_f = (h^2/2m) \times (3\pi^2 \times n)^{2/3}$

$n = 8.47 \times 10^{28}\ m^{-3}$

$m = 9.11 \times 10^{-31}\ kg$

$h = 6.626 \times 10^{-34}\ Js$

$E_f = (6.626 \times 10^{-34}\ Js)^2 / (2 \times 9.11 \times 10^{-31}\ kg) \times$

$\rightarrow (3\pi^2 \times 8.47 \times 10^{28}\ m^{-3})^{2/3}$

$E_f = 7.00 \times 10^{-19}\ J$

$1eV = 1.602 \times 10^{-19}\ J$

$E_f = 7.00 \times 10^{-19}\ J / 1.602 \times 10^{-19}\ J/eV$

$E_f = 4.37 eV$ //

7.2. Debye freq. $= \omega_D = v/R$

$R = (3/(4\pi n))^{1/3}$

$n = 6.03 \times 10^{28}\ m^{-3}$

$v = 6420 m/s$

$R = (3/(4\pi \times 6.03 \times 10^{22}\ m^{-3})^{1/3}$

$R = 2.51 \times 10^{-10}\ m$

$\omega_D = 6420 m/s / 2.51 \times 10^{-10} m$

$\omega_D = 2.56 \times 10^{13}\ rad/s$

$\theta_D = \hbar\omega_D / k_B$

$\theta_D = (6.626 \times 10^{-34} Js/(2\pi) \times 2.56 \times 10^{13} rad/s) /$

$\rightarrow (1.381 \times 10^{-23} J/K)$

$\theta_D = 394K$ //

7.3. $\tau = m\mu/en$

$m^* = me = 9.11 \times 10^{-31} kg$

$\mu = \sigma / e^* n$

$n = 8.47 \times 10^{28} m^{-3}$

$\sigma = 5.96 \times 10^7 S/m$

$\mu = (5.96 \times 10^7 S/m)/(1.60 \times 10^{-19} C)/(8.47 \times 10^{28} m^{-3})$

$\mu = 4.14 \times 10^{-3} m^2/Vs$

$\tau = (9.11 \times 10^{-31} kg) \times (4.14 \times 10^{-3} m^2/Vs)/(1.60 \times 10^{-19} C) /$

$\rightarrow (8.47 \times 10^{28} m^{-3})$

$\tau = 2.33 \times 10^{-14} s$ //

7.4. $L_k = \mu_0 \times m^x / ne^2 A$

$m^* = me = 9.11 \times 10^{-31} kg$

$A = \pi \times (d/2)^2$

$n = 8.47 \times 10^{28} m^{-3}$

$d = 0.1 mm = 1 \times 10^{-4} m$

$A = \pi \times (1 \times 10^{-4} m / 2)^2$

$A = 7.85 \times 10^{-9} m^2$

$L_k = (4\pi \times 10^{-7}\,Tm/A) * (9.11 \times 10^{-31}\,kg)/(8.47 \times 10^{28}\,m^{-3} \times$

$(1.60 \times 10^{-19}\,C)^2 \times$

$7.85 \times 10^{-9}\,m^2)$

$L_k = 6.39 \times 10^{-15}\,H$ //

7.5.

$F_q = \int d^3r \exp(-\lambda r)/r * e^{(iqr)}$

$F_q = \int_0^{\infty} dr\, r^2 \int_0^{\pi} d\theta \sin(\theta) \int_0^{2\pi} d\phi \exp(-\lambda r)/r \times e^{iqr}$

$F_q = \int_0^{\infty} du (u/\lambda)^2 \int_0^{\pi} d\theta \sin(\theta) \int_0^{2\pi} d\phi \exp(-u)/u \times$

$e^{(iqr/\lambda \times u)}$

$F_q = 4\pi \int_0^{\infty} du (u/\lambda)^2 \exp(-u)/u \times J_1(qr/\lambda \times u)$

$J_1(x) \to$ 1st order Bessel func.

$F_q = 4\pi \int_0^{\infty} du (u/\lambda)^2 \exp(-u)/u \times \int_0^{\pi} d\varphi \cos(qr/\lambda \times u \times \sin(\varphi))$

$\int_0^{\pi} \cos(ax \times \sin(\varphi)) d\varphi = \pi J_0(ax)$

$J_0(x) \to$ Zeroth order Bessel func.

$F(q) = 4\pi \int_0^{\infty} du \exp(-u) \times \pi\, u/\lambda \times J_1(qr/\lambda \times u)$

$$F_q = 4\pi^2/\lambda^3 \int_0^\infty du \exp(-u) \times u \times (qr/\lambda \times u) \times J_1(qr/\lambda \times u)$$

$$J_1(x) \approx x/2$$

$$F_q = 4\pi^2/\lambda^3 \int_0^\infty du \exp(-u) \times (qr/\lambda \times u)^2/2$$

$$F_q = 2\pi/q^2$$

$$F_q = \int d^3r \, 1/r \times e^{iqr} = 1/q^2 \quad //$$

Another way.

7.5(a) $$F_q = \int d^3r (1/r) e^{iqr\cos\theta} dV$$

$$r = (r\sin\theta\cos\varphi, r\sin\theta\sin\varphi, r\cos\theta)$$

$$r \cdot q = qr\cos\theta$$

$$F_q = \int_0^\infty dr \int_0^\pi d\theta \sin\theta \int_0^{2\pi} d\varphi \exp(-\lambda r) e^{iqr\cos\theta}$$

$\hookrightarrow r\sin\theta$

7.5(b) $x = \cos\theta$

$$\sin\theta d\theta = -dx$$

$$F_q = \int_0^\infty dr \int_{-1}^{1} dx(-r) \exp(-\lambda r) e^{iqrx} r^2 dx$$

$$F_q = -2\int_0^\infty dr \, r \exp(-\lambda r) \sin(qr)$$

7.5(c) $$F_q = \left[-2r\exp(-\lambda r)\sin(qr)/\lambda\right]_0^\infty +$$

$$\hookrightarrow 2/\lambda \int_0^\infty \exp(-\lambda r) \cos(qr) dr - 2q/\lambda$$

$\rightarrow \int_0^\infty \exp(-\lambda r)\sin(qr)dr$

$$\int_0^\infty \exp(-\lambda r)\cos(qr)dr = \lambda/(\lambda^2+q^2)$$

$$\int_0^\infty \exp(-\lambda r)\sin(qr)dr = q/(\lambda^2+q^2)$$

$$F_q = 2/\lambda[(\lambda/(\lambda^2+q^2)) - q(\lambda^2+q^2)]$$

$$F_q = \lim(\lambda\to 0)2/\lambda[(\lambda/(\lambda^2+q^2)) - q/(\lambda^2+q^2)]$$

$$F_q = 2\lim(\lambda\to 0)[(1/(\lambda+q)) - (1/(\lambda-q))]$$

$$F_q = \lim(\lambda\to 0)2\,(q/(\lambda^2-q^2))$$

7.6.

$$\lambda_L = \sqrt{\frac{m^*}{\mu_0 n_s e^2}}$$

$$\lambda_L = \sqrt{\frac{1.4m_e}{\mu_0(18.1\times 10^{28}\,m^{-3})e^2}} \approx 79.8\,nm$$ //

//CHAPTER 8 – Exercises//

8.1. show $|R|^2+|T|^2=1$

$$|R|^2=\left|\frac{-i\eta\sinh\kappa a\, e^{-ika}}{\cosh\kappa a+i\epsilon\sinh\kappa a}\right|^2$$

$$=\frac{\eta^2\sinh^2\kappa a}{\cosh^2\kappa a+\epsilon^2\sinh^2\kappa a}$$

$$|T|^2=\left|\frac{e^{-ika}}{\cosh\kappa a+i\epsilon\sinh\kappa a}\right|^2=\frac{1}{\cosh^2\kappa a+\epsilon^2\sinh^2\kappa a}$$

$$|R|^2+|T|^2=\frac{\eta^2\sinh^2\kappa a}{\cosh^2\kappa a+\epsilon^2\sinh^2\kappa a}+\frac{1}{\cosh^2\kappa a+\epsilon^2\sinh^2\kappa a}$$

$$\cosh^2x-\sinh^2x=1$$

$$\frac{1}{\cosh^2x}-\frac{\sinh^2x}{\cosh^2x}=\frac{1}{\cosh^2x}-\tanh^2x$$

$$=\frac{1}{\cosh^2x+\sinh^2x}=\frac{1}{\cosh^2x+1}$$

$$|R|^2+|T|^2=\frac{\eta^2\sinh^2\kappa a}{\sinh^2\kappa a}\cdot\frac{1}{\cosh^2\kappa a+1}+\frac{1}{\cosh^2\kappa a+1}$$

$$=\frac{\eta^2}{\sinh^2\kappa a}\cdot\frac{1}{\cosh^2\kappa a+1}+\frac{1}{\cosh^2\kappa a+1}$$

$$\frac{\eta^2}{\sinh^2\kappa a}=\left(\frac{\kappa}{k}+\frac{k}{\kappa}\right)^2\cdot\frac{1}{\sinh^2\kappa a}$$

$$=\frac{k^2+\kappa^2}{k^2}=1+\frac{\kappa^2}{k^2}=1+\frac{n^2\sin^2\theta}{\cos^2\theta}$$

$$=\frac{1}{\cos^2\theta}$$

$= (1/\cosh^2(\varkappa a))[\cosh^2(\varkappa a) - \cosh^2(\varkappa a) + \cosh^2(\varkappa a)]$

$= 1$ //

8.2. (a)

$E_c = e^2/2C$

$E_c = (1.602 \times 10^{-19})^2/(2 \times 60 \times 10^{-15})$

$= 4.268 \times 10^{-23}$ J

8.2(b) $E_J = (h/2e) \times I_c$

$E_J = (6.626 \times 10^{-34})/(2 \times 1.602 \times 10^{-19}) \times 20 \times 10^{-9}$

$= 1.303 \times 10^{-20}$ J

8.2(c) $f_0 = 1/2\pi\sqrt{L_{eff}C}$

$L_{eff}C = h/2e^2/E_c$

$L_{eff} = 6.626 \times 10^{-34}/2 \times (1.602 \times 10^{-19})^2 / 4.268 \times 10^{-23}$

$= 5.828 \times 10^{-13}$ H

$f_0 = 1/2\pi\sqrt{5.828 \times 10^{-13} \times 60 \times 10^{-15}}$

$= 63.66$ Hz

8.2(d)

$f = \delta/2\pi h = \sqrt{8E_cE_J}/2\pi h$

$$\delta/2\pi h = \frac{\sqrt{8 \times 4.268 \times 10^{-23} \times 1.303 \times 10^{-20}}}{2\pi \times 6.626 \times 10^{-34}}$$

$= 30.9$ MHz //

8.3. $f = 1/2\pi\sqrt{L \times C}$

$E_J = h/2e \times I_c$

$L = (h/2e^2)/E_J C$

$E_J = (6.626 \times 10^{-34} Js)/(2 \times 1.602 \times 10^{-19} C) \times 15 \times 10^{-9} A$

$= 3.107 \times 10^{-24} J$

$L = ((6.626 \times 10^{-34} Js)/2 \times 1.602 \times 10^{-19} C)^2 / (3.107 \times 10^{-24} J \times (2\pi 6 \times 10^{9} Hz)^2)$

$= 1.704 \times 10^{-10} H$

$C = 1/4\pi^2 f^2 \times L$

$C = 1/(4\pi^2 \times (6 \times 10^{9} Hz)^2 \times 1.704 \times 10^{-10} H)$

$= 1.486 \times 10^{-15} F$

$= 1.486 pF$ //

8.4 (a)

$E_J = (\Phi_0/2\pi) \times I_{c1} \times I_{c2}/(I_{c1} + I_{c2})$

$\Phi_0 = (2.068 \times 10^{-15} Wb)$

$I_{c1} = I_{c2} = 20 nA$

$E_J = (\Phi_0/2\pi) \times 20nA \times 20nA / 20nA + 20nA$

$= (\Phi_0/2\pi) \times 200 nA^2 / 40 nA$

$= (\Phi_0/2\pi) \times 5nA$

$\approx 1.6 \times 10^{-23} J$ //

8.4 (b)

$E_J = E_{J1} + E_{J2} + 2E_J \cos(\pi\phi, \phi_0)$

$\phi_0 = h/2e$

$E_{J1} + E_{J2} = 2E_J \cos(\pi\phi/\phi_0)$

$\cos(\pi\phi/\phi_0) = (E_{J1} + E_{J2})/2E_J$

$E_{J1} = E_{J2} = E_J/2$

$\cos(\pi\phi/\phi_0) = 1/2$

sol. $\phi/\phi_0 = \pm 1/2, \pm 3/2, \pm 5/2 \ldots$

$\phi/\phi_0 = 0$ this is where we apply mag. flux //

8.5.

$I_T = I_c(\sin\phi_1 + \sin(\phi_1 + 2\pi\Phi_B/\Phi_0))$

$I_T = I_c(\sin\phi_1 + \sin\phi_1 \cos(2\pi\Phi_B/\Phi_0) + \cos\phi_1 \sin(2\pi\Phi_B/\Phi_0)$

$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$

$I_T = I_c(\sin\phi_1 + 1/2\sin(2\pi\Phi_B/\Phi_0)\sin(2\phi_1)$

$+ 1/2\cos(2\pi\Phi_B/\Phi_0)\cos(2\phi_1)$

$dI_T/d\phi_1 = I_c(\cos(2\phi_1) - \sin(2\pi\Phi_B/\Phi_0)\sin(2\phi_1) = 0$

$\phi_1 = \cos(2\phi_1) = \sin(2\pi\Phi_B/\Phi_0)\sin(2\phi_1)$

or

$\tan(2\phi_1) = 1/\tan(\pi\Phi_B/\Phi_0)$

$$2\phi_1 = \pi/2 - \pi\Phi_B/\Phi_0 + n\pi$$

$$I_T = I_c\left(\sin(\pi/4 - \pi\Phi_B/2\Phi_0 + n\pi/2) + \cos(\pi\Phi_B/2\Phi_0 + n\pi/2)\right)$$

$$I_T = I_c\left(\cos(\pi\Phi_B/2\Phi_0 + n\pi/2 - \pi/4) + \sin(\pi\Phi_B/2\Phi_0 + n\pi/2 - \pi/4)\right)$$

$$\pi\Phi_B/2\Phi_0 + n\pi/2 - \pi/4 = 0$$

$$\Phi_B = 2\Phi_0/n\pi - 1/2$$

$$I_T = 2I_c\left|\cos(\pi\Phi_B/\Phi_0)\right|$$

$$I_T = 2I_c\left|\cos(\pi\Phi_B/\Phi_0)\right| \quad //$$

//CHAPTER 9- Exercises//

9.1. $\delta(t) = Ae^{-t/T_2} + B$

$$\delta'(t) = Ae^{-t/T_2}\cos(\pi/16) + B$$

$$P(0) = \int [\delta'(t) = 0] \, p(\delta'(t))\,dt$$

$$\delta'(t) = 0$$

$$Ae^{-t/T_2}\cos(\pi/16) + B = 0$$

$$t = -T_2 \ln(\cos(\pi/16) - \Delta t$$

$$t \in [-T_2 \ln(\cos(\pi/16)) - \Delta t, \infty)$$

$$P(0) = \int [-T_2 \ln(\cos(\pi/16)) - \Delta t, \infty) \, p(\delta'(t))\,dt$$

$$\sigma^2 = (2\pi\delta)^{-1} \Delta\omega^2$$

$$\sigma^2 = (2\pi \times 1\,MHz)^{-1}(\pi/16)^2 \approx 6.25 \times 10^{-10}$$

$$p(\delta'(t)) = (2\pi\sigma^2)^{-1/2} \exp(-\delta'(t) - \mu)^2/2\sigma^2$$

$$\mu = \delta(t)$$

$$P(0) = \int [-T_2 \ln(\cos(\pi/16)) - \Delta t, \infty)(2\pi\sigma^2)^{-1/2} \exp(-(\delta'(t) - \mu)^2/2\sigma^2)\,dt$$

9.2. $P_{correct} = \left(1 - e^{(-100\Delta t/T_1)^n}\right)$

$$\Delta t = 100 \times 50\,nan.\,sec. = 5\,microsec.$$

$$0.85 = \left(1 - e^{(-100 \times 5\,microsec./T_1)^n}\right)$$

$T_1 = -100 \times 5\text{microsec.} / (\ln(1-0.85^{(1/n)})$

$T_1 = -100 \times 5\text{ microsec.} / (\ln(1-0.85^{1/5})$

$\approx 26.8\text{ microsec.} //$

9.3. $P_{success} = (1-2p)^m \quad m = n(d-1)$

$0.85 = (1-2p)^{d-1}$

$\ln(0.85) = (d-1)\ln(1-2p)$

$d = (\ln(0.85) / \ln(1-2p)) + 1$

$d = (\ln(0.85) / (\ln(1-2 \times 0.015)) + 1$

≈ 24

9.4. $f(x) = A_0 \times x^m + B_0$

$p_{error} = A_0 \times x^3 + B_0$

$f(x) = A_0 \times x^3 + B_0[(2^3-1)/2^3] \times (1-x)$

$f(x) = A_0 \times x^3 + B_0 \times (7/8) \times (1-x)$

$f(x) = A_0 \times x^3 + B_0 \times (7/8) \times (1-x) //$

//CHAPTER 10 - Exercises//

10.1

$|0_L\rangle = |0000000\rangle + |1111000\rangle + |1100110\rangle$
$+ |1010101\rangle + |0110011\rangle + |0101101\rangle$
$+ |0011110\rangle$

$|1_L\rangle = X|0_L\rangle$

$H = 1/\sqrt{2}\,(|0\rangle\langle 0| + |0\rangle\langle 1| + |1\rangle\langle 0| - |1\rangle\langle 1|)$

$H_{bar}|0_L\rangle = H \otimes H \otimes H \otimes H \otimes H \otimes H \otimes H\,(|0\rangle + |1\rangle) \otimes$
$(|0\rangle + |1\rangle) \otimes (|0\rangle + |1\rangle) \otimes (|0\rangle + |1\rangle) \otimes$
$(|0\rangle + |1\rangle) \otimes (|0\rangle + |1\rangle) \otimes (|0\rangle + |1\rangle)$

$= 1/2^7 \sum_{x,y,z,w,u,v,t} (-1)^{(x.y+z.w+u.v+t)} |x\rangle \otimes$
$|y\rangle \otimes |z\rangle \otimes |w\rangle \otimes |u\rangle \otimes |v\rangle \otimes |t\rangle$

$X_1 X_2 X_3 Z_4 Z_5 Z_6 Z_7 |0_L\rangle = |0_L\rangle$

$Z_1 Z_2 Z_3 X_4 X_5 X_6 X_7 |0_L\rangle = |0_L\rangle$

$= |0000000\rangle + |0001111\rangle + |0011001\rangle + |0010110\rangle$
$+ |0101010\rangle + |0100101\rangle + |0110011\rangle + |0111100\rangle$
$+ |1001100\rangle + |1000011\rangle + |1010101\rangle + |1011010\rangle$
$+ |1100110\rangle + |1101001\rangle + |1111111\rangle + |1110000\rangle$

10.2.

$$|0_L\rangle = |0000000\rangle + |1111000\rangle + |1100110\rangle$$
$$+ |1010101\rangle + |0110011\rangle + |0101101\rangle$$
$$+ |0011110\rangle$$

$$|1_L\rangle = X|0_L\rangle$$

$$S = [1\,0;\,0\,i]$$

$$\bar{S}|0_L\rangle = (S^{+} \otimes S^{+} \otimes S^{+} \otimes S^{+} \otimes S^{+} \otimes S^{+} \otimes S^{+})|0_L\rangle$$
$$= (S^{+} \otimes S^{+} \otimes S^{+} \otimes S^{+} \otimes S^{+} \otimes S^{+} \otimes S^{+})(|0\rangle + |1\rangle)$$
$$\otimes (|0\rangle + |1\rangle) \otimes (|0\rangle + |1\rangle) \otimes (|0\rangle + |1\rangle) \otimes$$
$$(|0\rangle + |1\rangle) \otimes (|0\rangle + |1\rangle) \otimes (|0\rangle + |1\rangle)$$

$$S^{+} = S$$

$$S^{+}|0\rangle = [1\,0;\,0\,-i](|0\rangle) = |0\rangle$$

$$S^{+}|1\rangle = [1\,0;\,0\,-i](|1\rangle) = i|1\rangle$$

$$\bar{S}|0_L\rangle = (|0\rangle + i|1\rangle) \otimes (|0\rangle + i|1\rangle) \otimes$$
$$(|0\rangle + i|1\rangle) \otimes (|0\rangle + i|1\rangle) \otimes (|0\rangle + i|1\rangle)$$
$$\otimes (|0\rangle + i|1\rangle) \otimes (|0\rangle + i|1\rangle)$$

$$\bar{S}|1_L\rangle = \bar{S}X|0_L\rangle$$
$$= X\bar{S}|0_L\rangle$$
$$= X(|1111111\rangle + i|1110111\rangle + i|1101111\rangle$$
$$- |1100111\rangle + i|1011111\rangle - |1010111\rangle -$$
$$|1001111\rangle - i|1000111\rangle - i|0111111\rangle$$
$$+ |0110111\rangle + |0101111\rangle - i|0100111\rangle + i|0011111\rangle$$
$$- i|0010111\rangle - |0001111\rangle + |0000111\rangle)$$

10.3.

$T = [1\ 0;\ 0\ \exp(i*\pi/4)]$

$|+_L\rangle = 1/2\,(|0_L\rangle + |1_L\rangle)$

$T|+_L\rangle = 1/2\,(T|0_L\rangle + T|1_L\rangle)$

$T|0_L\rangle = T(|0000000\rangle + |1111111\rangle)/\sqrt{2}$

$= (|0000000\rangle + \exp(i*\pi/4)|1111111\rangle)/\sqrt{2}$

$= (|0000000\rangle + (1+i)/\sqrt{2}\ |1111111\rangle)/\sqrt{2}$

$T|1_L\rangle = TX|0_L\rangle$

$= XT|0_L\rangle$

$= (|1111111\rangle + (1+i)/\sqrt{2}\ |0000000\rangle)/\sqrt{2}$

$T|+_L\rangle = 1/2\,[(|0000000\rangle + (1+i)/\sqrt{2}\ |1111111\rangle)$

$+ (|1111111\rangle + (1+i)/\sqrt{2}\ |0000000\rangle)]$

$= (|0000000\rangle + |1111111\rangle) + (1+i)/\sqrt{2}$

$(|0000000\rangle - |1111111\rangle)$

not valid //

10.5.

$\bar{X}|\lambda\rangle_L = |\rho\rangle_L$

$\bar{X}|\rho\rangle_L = |\lambda\rangle_L$

$\bar{Z}|\lambda\rangle_L = (-1)^{(\omega(|0\rangle_L))}|\lambda\rangle_L$

$\bar{Z}|\rho\rangle_L = (-1)^{(\omega(|1\rangle_L))}|\rho\rangle_L$

$\omega(|0\rangle_L), \omega(|1\rangle_L)$, weights

$$\bar{X}\bar{Z}|\lambda\rangle_L = \bar{X}(-1)^{(\omega|0\rangle_L)}|\lambda\rangle_L$$
$$= (-1)^{(\omega|0\rangle_L)}|\rho\rangle_L$$
$$\bar{Z}\bar{X}|\lambda\rangle_L = \bar{Z}|\rho\rangle_L$$
$$= (-1)^{(\omega(|1\rangle_L)}|\lambda\rangle_L$$
$$\bar{X}\bar{Z}|\lambda\rangle_L = -\bar{Z}\bar{X}|\lambda\rangle_L$$
$$\bar{X}\bar{Z} = -\bar{Z}\bar{X}$$
$$\to \bar{X}\bar{Z}|\lambda\rangle_L = (-1)^{(\omega|0\rangle_L)}|\rho\rangle_L$$
$$\bar{Z}\bar{X}|\lambda\rangle_L = (-1)^{(\omega(|1\rangle_L)}|\lambda\rangle_L$$
$|0\rangle_L$ and $|1\rangle_L$ orthog.
$$(-1)^{(\omega|0\rangle_L)} = -(-1)^{(\omega(|1\rangle_L))}$$

10.4.
If measure of Z-qubits changed

there must be error in stabilizer measurement

We need to perform operation to restore to correct state.

X operations correct Z errors and vice versa.

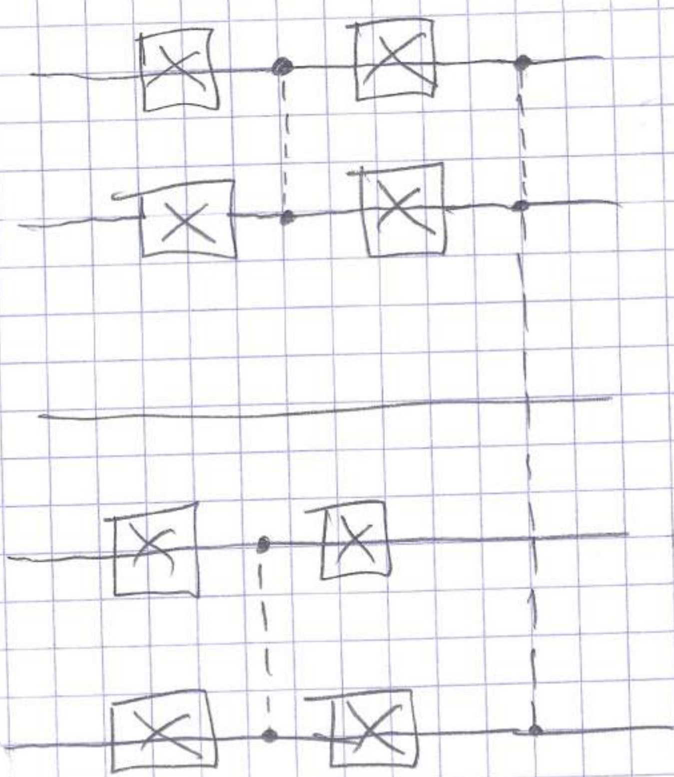
We apply these on the boundaries they are adjacent in the checkerboard pattern and adjacent to the Z-qubits.

We use transversal CNOTS also to do these operations simultaneously

where Z qubits are corrected by CNOTS (X, Z).

//CHAPTER 11 - Exercises//

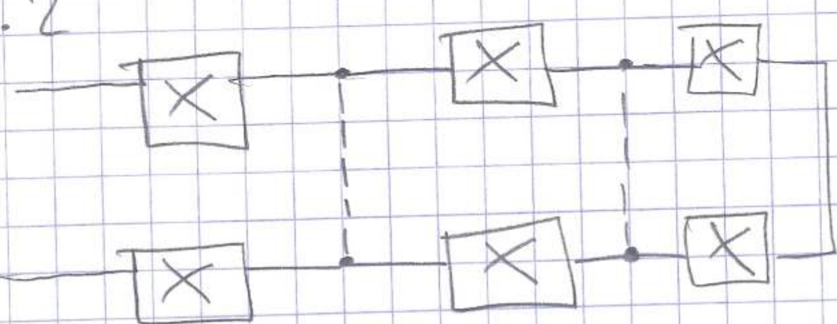
11.1 Reversible OR-gate:



C = A or B

this can be run inversely

11.2



inputs: q_0, q_1

outputs: q_0', q_1'

$SWAP|00\rangle = |00\rangle$

$SWAP|01\rangle = |10\rangle$

$SWAP|10\rangle = |01\rangle$

$SWAP|11\rangle = |11\rangle$

$$SWAP = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

$SWAP = CNOT(q_0, q_1)\,CNOT(q_1, q_0)\,CNOT(q_0 q_1$

11.3 ~~If we dont~~

dont uncompute carry bias: $O(n)$

each CNOT applied once.

~~~~ if we uncomputed carry bias it would be $O(\log(n))$, technique called 'carry lookahead'. reduces the # of CNOTS needed.

11.4. assume 2, 4 qubits A and B

Before swap:

$A = a_0 a_1 a_2 a_3$

$B = b_0 b_1 b_2 b_3$

After Swap1:

$A = a_0 b_1 a_2 a_3$

$B = b_0 a_1 b_2 b_3$

After swap 2:

$A = a_0 b_1 b_2 b_3$

$B = b_0 a_1 a_2 a_3$

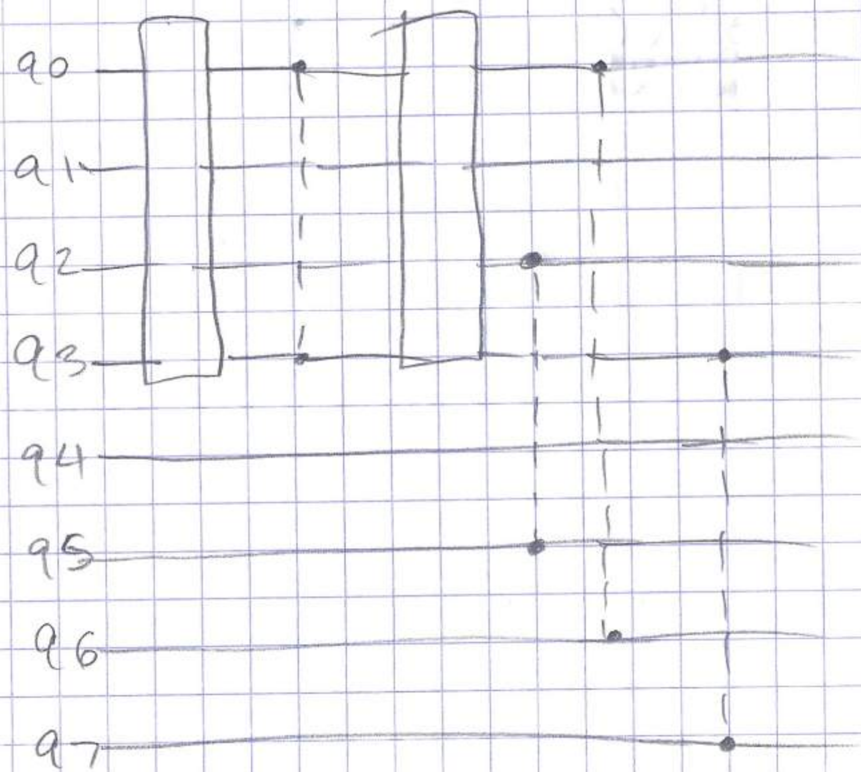
After swap 3:

$A = a_0 b_1 b_2 b_3$

$B = b_0 a_1 a_2 a_3$

$U = SWAP12 * SWAP23 * SWAP34$
~~~~

Sketch:



11.5. possible circuit:

11.6. $|a\rangle = |010\rangle$, $|b\rangle = |100\rangle$

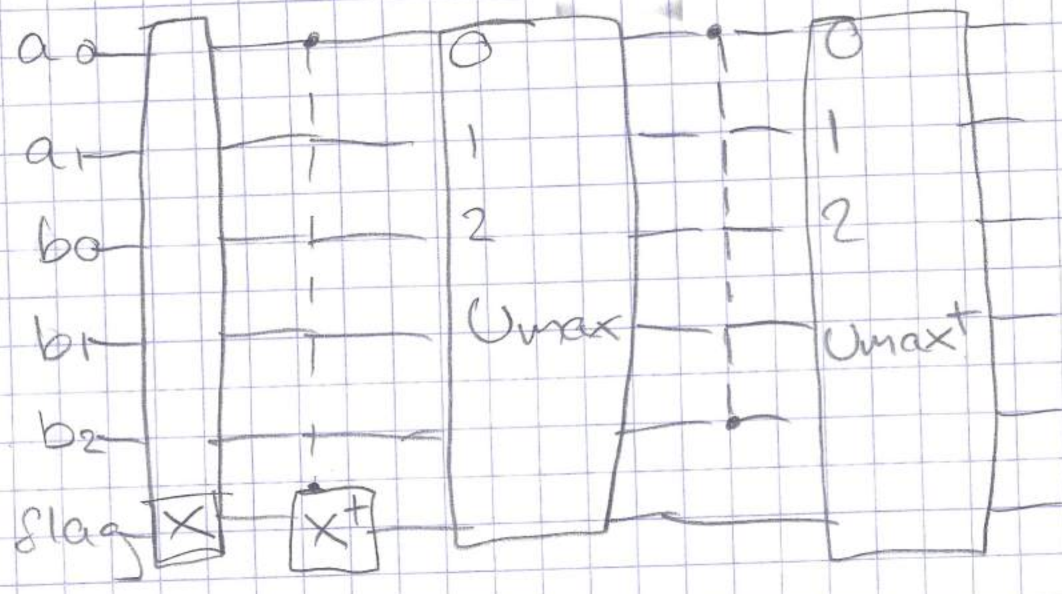
(a) output: $|\max(a,b)\rangle = |100\rangle$

$|\min(a,b)\rangle = |010\rangle$

$|b\rangle = |010\rangle$, $|a\rangle = |100\rangle$

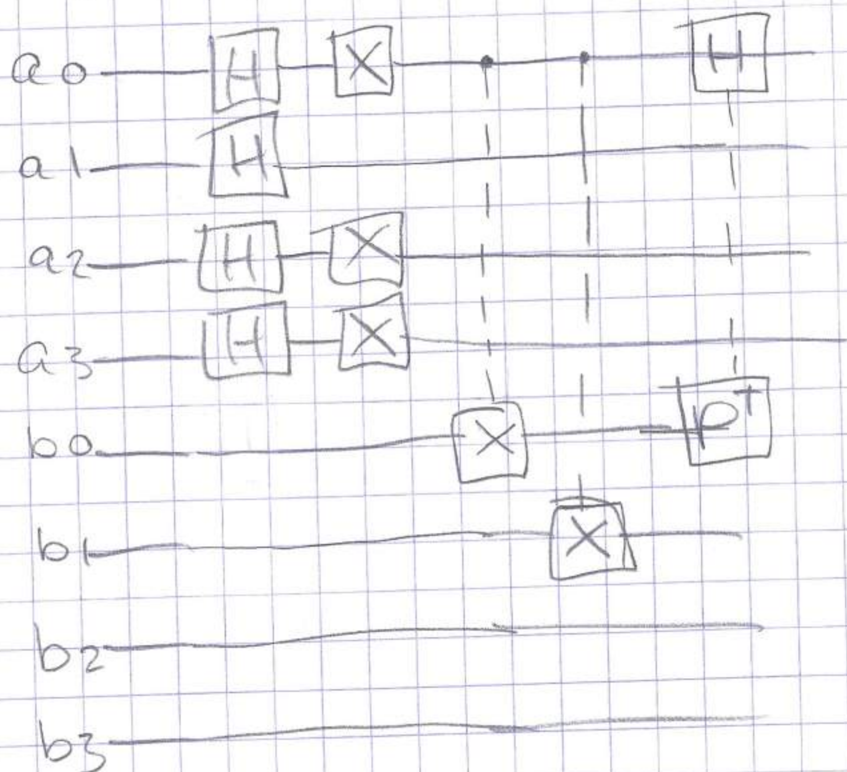
output is same.

b)



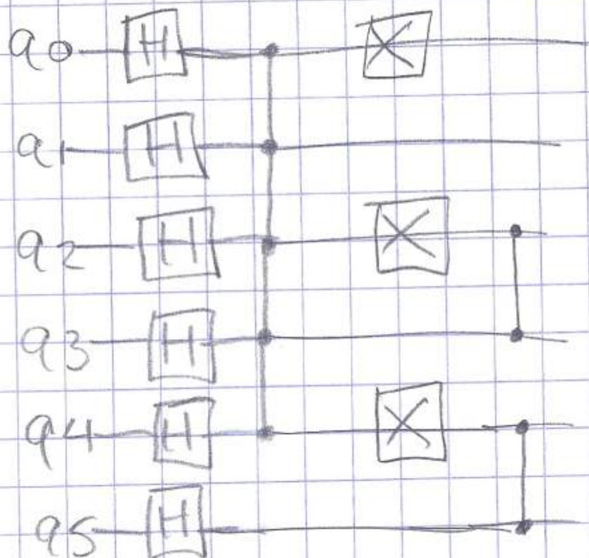
11.7.

possible circuit:

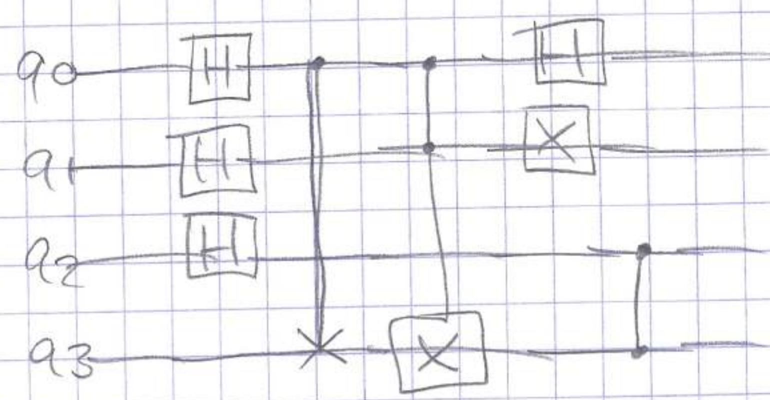


//CHAPTER 12 – Exercises//

12.1. (CPHASE)



12.2. $f'(x_2 x_1 x_0) = x_2 \oplus x_1 \oplus \mathrm{NOT}(x_0)$



12.3. $f(x) = 11^x \bmod 21$

$f(1) = 11^1 \bmod 21 = 11$

$f(2) = 11^2 \bmod 21 = 16$

$f(3) = 11^3 \bmod 21 = 5$

$f(4) = 11^4 \bmod 21 = 4$

$f(5) = 11^5 \bmod 21 = 17$

$f(6) = 11^6 \bmod 21 = 20$

$f(7) = 11^7 \bmod 21 = 3$

$f(8) = 11^8 \bmod 21 = 19$

$f(9) = 11^9 \bmod 21 = 18$

$f(10) = 11^{10} \bmod 21 = 2$

$f(11) = 11^{11} \bmod 21 = 15$

$f(12) = 11^{12} \bmod 21 = 13$

$f(13) = 11^{13} \bmod 21 = 8$

$f(14) = 11^{14} \bmod 21 = 14$

$f(15) = 11^{15} \bmod 21 = 10$

$f(16) = 11^{16} \bmod 21 = 7$

$f(17) = 11^{17} \bmod 21 = 12$

$f(18) = 11^{18} \bmod 21 = 1$

$r = 18$

(b) $U|y\rangle = |11y \bmod 21\rangle$

$U|1\rangle = |11 \bmod 21\rangle = |11\rangle$

$\vdots$ ....

$U^{18}|1\rangle = U(U^{17}|1\rangle) = U|12\rangle = |1\rangle$

$|u_{17}\rangle = (1/\sqrt{18})(|1\rangle + e^{i2\pi 17/18}|12\rangle + e^{i2\pi 34/18}|13\rangle$
$+ \dots + e^{i2\pi 289/18}|1\rangle)$

(c) $|u_0\rangle + |u_1\rangle + \dots + |u_{17}\rangle$

$e^{i2\pi 0/18} + e^{i2\pi 1/18} + \dots + e^{i2\pi \cancel{*} 17/18} = 0$

$|1\rangle + 0 = |1\rangle$

d) $a^2 \equiv 1 \pmod{n}$

$(a+1)(a-1) = a^2 - 1 \equiv 0 \pmod{n}$

$r = 18,\ r/2 = 9$

$11^9 + 1 = 2357947692$

$11^9 - 1 = 2357947690$