main

Rick Holubec

2023-08-22

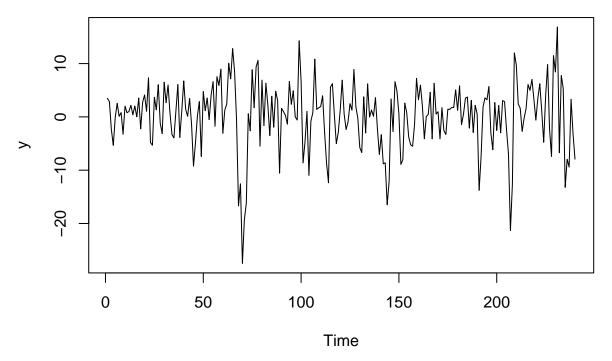
Set paths and get packages

```
rm(list = ls())
#path = '~/Documents/LSE/Dissertation/Code/ANOVA-kernel/'
\#path\_stan = '-\Documents/LSE/Dissertation/Code/Additive-GP-Kronecker-main/Code/Stan/'
library(ggplot2)
library(plyr)
library(cmdstanr)
## This is cmdstanr version 0.5.3
## - CmdStanR documentation and vignettes: mc-stan.org/cmdstanr
## - CmdStan path: /Users/rickholubec/.cmdstan/cmdstan-2.32.1
## - CmdStan version: 2.32.1
## A newer version of CmdStan is available. See ?install_cmdstan() to install it.
## To disable this check set option or environment variable CMDSTANR_NO_VER_CHECK=TRUE.
library(rstan)
## Loading required package: StanHeaders
## rstan (Version 2.21.8, GitRev: 2e1f913d3ca3)
## For execution on a local, multicore CPU with excess RAM we recommend calling
## options(mc.cores = parallel::detectCores()).
## To avoid recompilation of unchanged Stan programs, we recommend calling
## rstan_options(auto_write = TRUE)
```

Import data

```
source("Data-preprocessing.R")
```

```
## Registered S3 method overwritten by 'quantmod':
##
     method
                        from
##
     as.zoo.data.frame zoo
## New names:
## New names:
## New names:
## * '' -> '...2'
\#X_ts < -cbind(y_lag1, y_lag2, y_lag12, cpi, kilian, delta_stock, prod)
\#X < -as.data.frame(X_ts)
X_lag1_ts<-cbind(y_lag1, cpi_lag1, kilian_lag1, delta_stock_lag1, prod_lag1)</pre>
X_lag1<-as.data.frame(X_lag1_ts)</pre>
y<-as.vector(y_ts)
# Plots
ts.plot(y)
```

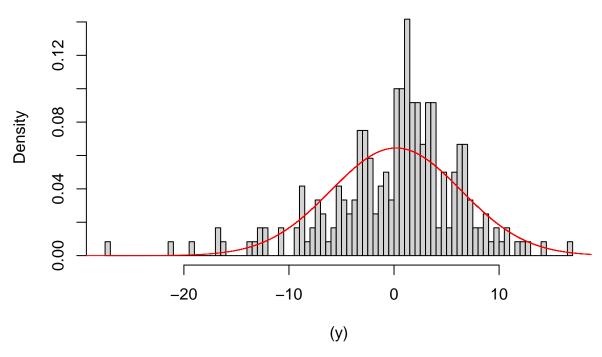


print(adf.test(y))

```
## Warning in adf.test(y): p-value smaller than printed p-value
##
## Augmented Dickey-Fuller Test
##
data: y
## Dickey-Fuller = -6.2293, Lag order = 6, p-value = 0.01
## alternative hypothesis: stationary
```

```
print(jarque.bera.test(y))
##
## Jarque Bera Test
## data: y
## X-squared = 77.649, df = 2, p-value < 2.2e-16
print(bds.test(y))
##
    BDS Test
##
##
## data: y
##
## Embedding dimension = 2 3
## Epsilon for close points = 3.0939 6.1878 9.2817 12.3756
##
## Standard Normal =
      [ 3.0939 ] [ 6.1878 ] [ 9.2817 ] [ 12.3756 ]
## [2]
           4.7721
                       6.1013
                                 7.1416
                                             8.0823
## [ 3 ]
            4.2824
                       6.1999
                                  7.0588
                                             7.9194
## p-value =
## [ 3.0939 ] [ 6.1878 ] [ 9.2817 ] [ 12.3756 ]
## [ 2 ]
                           0
                                0
                 0
                                                  0
## [ 3 ]
                 0
                            0
                                       0
                                                  0
hist((y), prob = TRUE,100)
xay < -seq(mean(y) - 10*sd(y), mean(y) + 10*sd(y), 0.0001)
lines(xay,dnorm(xay,mean = mean(y),sd = sd(y)), col="red")
```

Histogram of (y)



```
## model settings
#h-step
h<-1
# test
library(moments)
library(nonlinearTseries)</pre>
```

```
##
## Attaching package: 'nonlinearTseries'
##
## The following object is masked from 'package:grDevices':
##
## contourLines
```

abcd<-nonlinearityTest(y, verbose = TRUE)</pre>

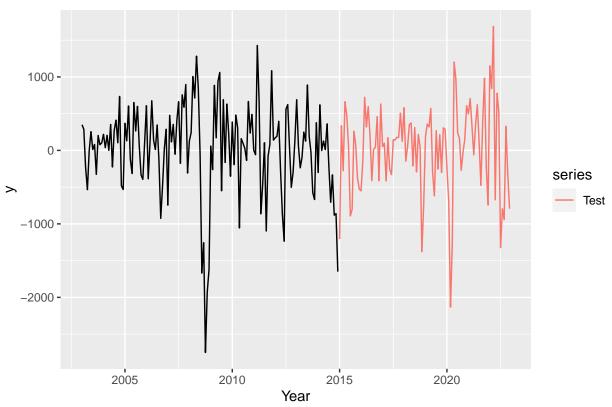
```
##
         ** Teraesvirta's neural network test **
##
         Null hypothesis: Linearity in "mean"
##
         X-squared = 9.027794 df = 2 p-value = 0.01095568
##
##
         ** White neural network test **
##
         Null hypothesis: Linearity in "mean"
##
         X-squared = 8.81693 df = 2 p-value = 0.01217385
##
##
         ** Keenan's one-degree test for nonlinearity **
##
        Null hypothesis: The time series follows some AR process
##
        F-stat = 8.418712 p-value = 0.00406494
##
```

```
##
         ** McLeod-Li test **
##
         Null hypothesis: The time series follows some ARIMA process
         Maximum p-value = 2.692434e-09
##
##
##
         ** Tsay's Test for nonlinearity **
##
         Null hypothesis: The time series follows some AR process
##
         F-stat = 8.418952 p-value = 0.004064421
##
##
         ** Likelihood ratio test for threshold nonlinearity **
##
         Null hypothesis: The time series follows some AR process
##
         Alternativce hypothesis: The time series follows some TAR process
##
         X-squared = 11.06862 p-value = 0.05291588
bds.test(y)
##
##
    BDS Test
##
## data: y
## Embedding dimension = 2 3
## Epsilon for close points = 3.0939 6.1878 9.2817 12.3756
##
## Standard Normal =
         [ 3.0939 ] [ 6.1878 ] [ 9.2817 ] [ 12.3756 ]
## [ 2 ]
                                   7.1416
             4.7721
                        6.1013
                                               8.0823
## [ 3 ]
             4.2824
                        6.1999
                                   7.0588
                                               7.9194
##
## p-value =
         [ 3.0939 ] [ 6.1878 ] [ 9.2817 ] [ 12.3756 ]
## [ 2 ]
                  0
                             0
                                        0
                                                    0
## [ 3 ]
                  0
                             0
                                        0
                                                    0
```

Train-Val-Test Split

```
set<-Train_Test_Split(X_lag1_ts,y_ts, c(0.6,0.4))
autoplot(100*set$y_train,ylab = "y",xlab = "Year",main = "Variable") +autolayer(100*set$y_test,series =</pre>
```





```
#Recession plot
#https://rpubs.com/rfrostbrewer/codethroughexample
library(dplyr)
```

```
##
## Attaching package: 'dplyr'
## The following objects are masked from 'package:plyr':
       arrange, count, desc, failwith, id, mutate, rename, summarise,
##
##
       summarize
## The following objects are masked from 'package:stats':
##
##
       filter, lag
## The following objects are masked from 'package:base':
##
##
       intersect, setdiff, setequal, union
library(ggpmisc)
```

```
## Loading required package: ggpp
```

Attaching package: 'ggpp'

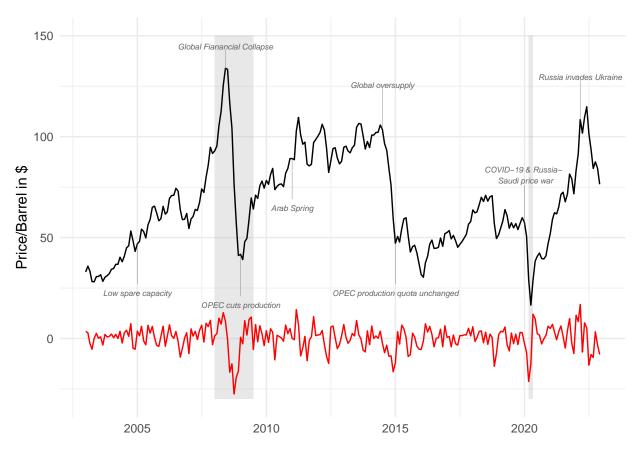
##

```
## The following object is masked from 'package:ggplot2':
##
##
       annotate
## Registered S3 method overwritten by 'ggpmisc':
##
     as.character.polynomial polynom
data_orig<-Data_Select_Period(data,transform = FALSE,end=c(2022,12)) #Data_Transform(data,transform = FA
recession_data_old<-read.csv("data/USREC-2.csv")[-(397:402),]
recession_data<-recession_data_old[(157:dim(recession_data_old)[1]),]</pre>
recession_data$DATA<-as.vector(data_orig)</pre>
recession_data$REC<-c(0,diff(recession_data$USREC))</pre>
recessions_start_end <- recession_data %>%
  mutate(recession_change = USREC - lag(USREC)) %>%
  filter(recession_change != 0)
recessions <- tibble(start = filter(recessions_start_end, recession_change == 1) $DATE,
                      end = filter(recessions_start_end, recession_change == -1)$DATE)
recession_data$RETURN<-y
ggplot(data = recession_data,aes(x = as.Date(DATE)))+
  geom rect(data = recessions,
            aes(xmin = as.Date(start), xmax = as.Date(end), ymin = -30, ymax = 150),
            inherit.aes = FALSE, fill = "grey70", alpha = 0.3) + #grey70, blue
  geom_line(aes(y = DATA,colour="WTI"),colour="black")+
  labs(x = NULL,
       y = "Price/Barrel in $",
       color = NULL
       #title = "WTI Crude Oil Prices from 2003-2023",
       \#subtitle = "From 1987-2021, comparing the top 1% and bottom 50% \n(National recessions shaded)"
       #caption = "Source: Federal Reserve Economic Data"
       ) +
  # annotate("segment", x = as.Date.factor("2001-06-01"),
             xend = as.Date.factor("2001-06-01"),
             y = 28, y = 48, colour = \#666666\%, size = 0.2, alpha = 0.6) +
  # annotate(geom = "text", x = as.Date.factor("2001-06-01"), y = 58,
             label = "Bush Tax Cut \n Jun 2001 & May 2003 ",
             fontface = "italic", vjust = 1, color = "#666666", size = 2) +
  annotate("segment", x = as.Date.factor("2008-06-01"),
           xend = as.Date.factor("2008-06-01"),
           y = 133, yend = 143, colour = "#666666", size=0.2, alpha=0.6) +
  annotate(geom = "text", x = as.Date.factor("2008-06-01"), y = 146,
           label = "Global Fianancial Collapse",
           fontface = "italic", vjust = 1, color = "#666666", size = 2) +
  annotate("segment", x = as.Date.factor("2020-01-01"),
           xend = as.Date.factor("2020-01-01"),
           y = 56, yend = 76, colour = "#666666", size=0.2, alpha=0.6) +
  annotate(geom = "text", x = as.Date.factor("2020-01-01"), y = 85,
           label = "COVID-19 & Russia- \n Saudi price war",
           fontface = "italic", vjust = 1, color = "#666666", size = 2) +
  annotate("segment", x = as.Date.factor("2009-01-01"),
           xend = as.Date.factor("2009-01-01"),
```

```
y = 41, yend = 21, colour = "#666666", size=0.2, alpha=0.6) +
annotate(geom = "text", x = as.Date.factor("2009-01-01"), y = 18,
        label = "OPEC cuts production",
        fontface = "italic", vjust = 1, color = "#666666", size = 2) +
# annotate("segment", x = as.Date.factor("2001-09-01"),
           xend = as.Date.factor("2001-09-01"),
           y = 26, yend = 46, colour = "#666666", size=0.2, alpha=0.6) +
# annotate(geom = "text", x = as.Date.factor("2001-09-01"), y = 49,
           label = "9/11 Attacks",
           fontface = "italic", vjust = 1, color = "#666666", size = 2) +
# annotate("segment", x = as.Date.factor("1999-03-01"),
           xend = as.Date.factor("1999-03-01"),
           y = 15, yend = 35, colour = "#666666", size=0.2, alpha=0.6) +
# annotate(geom = "text", x = as.Date.factor("1999-03-01"), y = 38,
           label = "OPEC cuts production",
           fontface = "italic", vjust = 1, color = "#666666", size = 2) +
annotate("segment", x = as.Date.factor("2022-03-01"),
        xend = as.Date.factor("2022-03-01"),
        y = 108, yend = 128, colour = "#666666", size=0.2, alpha=0.6) +
annotate(geom = "text", x = as.Date.factor("2022-03-01"), y = 131,
         label = "Russia invades Ukraine",
        fontface = "italic", vjust = 1, color = "#666666", size = 2) +
\# annotate("segment", x = as.Date.factor("2003-10-01"),
           xend = as.Date.factor("2003-10-01"),
           y = 36, yend = 56, colour = "#666666", size=0.2, alpha=0.6) +
# annotate(geom = "text", x = as.Date.factor("2003-10-01"), y = 59,
           label = "Iraq invades Kuwait",
           fontface = "italic", vjust = 1, color = "#666666", size = 2) +
annotate("segment", x = as.Date.factor("2014-07-01"),
        xend = as.Date.factor("2014-07-01"),
        y = 104, yend = 124, colour = "#666666", size=0.2, alpha=0.6) +
annotate(geom = "text", x = as.Date.factor("2014-07-01"), y = 127,
        label = "Global oversupply",
        fontface = "italic", vjust = 1, color = "#666666", size = 2) +
annotate("segment", x = as.Date.factor("2005-01-01"),
         xend = as.Date.factor("2005-01-01"),
        y = 47, yend = 27, colour = "#666666", size=0.2, alpha=0.6) +
annotate(geom = "text", x = as.Date.factor("2005-01-01"), y = 24,
        label = "Low spare capacity",
        fontface = "italic", vjust = 1, color = "#666666", size = 2) +
annotate("segment", x = as.Date.factor("2011-01-01"),
        xend = as.Date.factor("2011-01-01"),
        y = 89, yend = 69, colour = "#666666", size=0.2, alpha=0.6) +
annotate(geom = "text", x = as.Date.factor("2011-01-01"), y = 66,
        label = "Arab Spring",
        fontface = "italic", vjust = 1, color = "#666666", size = 2) +
annotate("segment", x = as.Date.factor("2015-01-01"),
        xend = as.Date.factor("2015-01-01"),
        y = 47, yend = 27, colour = "#666666", size=0.2, alpha=0.6) +
annotate(geom = "text", x = as.Date.factor("2015-01-01"), y = 24,
         label = "OPEC production quota unchanged",
         fontface = "italic", vjust = 1, color = "#666666", size = 2) +
geom_line(aes(x = as.Date(DATE), y = RETURN, colour="Return"), colour = "red")+
```

```
#scale_colour_manual("",
# values = c("black", "red"))
theme_minimal()
```

```
## Warning: Using 'size' aesthetic for lines was deprecated in ggplot2 3.4.0.
## i Please use 'linewidth' instead.
## This warning is displayed once every 8 hours.
## Call 'lifecycle::last_lifecycle_warnings()' to see where this warning was
## generated.
```



```
#theme_ipsum_rc()
#autoplot(data_orig,ylab = "y",xlab = "Year",main = "Crude Oil Prices from 2003-2023")
```

```
#save csv files for python
X_lag1_ts<-cbind(y_lag1, kilian_lag1, delta_stock_lag1, cpi_lag1,prod_lag1)#
set<-Train_Test_Split(X_lag1_ts,y_ts, c(0.6,0.4))
write.csv(set$X_train, file = "data/Pre-processed data/X_train.csv", row.names = FALSE)
write.csv(set$y_train, file = "data/Pre-processed data/y_train.csv", row.names = FALSE)
write.csv(set$X_val, file = "data/Pre-processed data/X_val.csv", row.names = FALSE)
write.csv(set$y_val, file = "data/Pre-processed data/y_val.csv", row.names = FALSE)
write.csv(set$X_test, file = "data/Pre-processed data/X_test.csv", row.names = FALSE)
write.csv(set$y_test, file = "data/Pre-processed data/y_test.csv", row.names = FALSE)
#write.csv(set$y_train, file = "X_train.csv", row.names = FALSE)</pre>
```

Random Walk

```
step <- 12
R <- as.integer(length(set$y_test)/step)</pre>
y_hat_RW <- c()</pre>
T1 <- dim(set$X_train)[1]+dim(set$X_test)[1]
T <- T1-R*step
for (i in 1:R){
    y1 \leftarrow window(y_ts, start = c(2003, (1+(i-1)*step)), end = c(2003, (T+step*(i-1))))
    model <- rwf(y1,step)</pre>
    #model<-mean(y1)</pre>
    y_pred <- predict(model,step)$mean</pre>
    #y_pred<-rep(model,step)</pre>
    y_hat_RW <- c(y_hat_RW,y_pred)</pre>
}
MSPE_RW <- sqrt(1/(dim(set$X_test)[1])*sum((y_hat_RW-as.vector(set$y_test))^2))
MAPE_RW <- 1/(dim(set$X_test)[1])*sum(abs(y_hat_RW-as.vector(set$y_test)))
print(paste("MSFE for RW =", MSPE_RW))
## [1] "MSFE for RW = 9.6173691070722"
print(paste("MAPE for RW =", MAPE_RW))
## [1] "MAPE for RW = 7.56552083333334"
```

\mathbf{ARMA}

```
library(forecast)
step <- 12
R <- as.integer(length(set$y_test)/step)</pre>
y_hat_ARMA <- c()</pre>
lower_ARMA<-c()</pre>
upper_ARMA<-c()
T1 <- dim(set$X_train)[1]+dim(set$X_test)[1]
T <- T1-R*step
for (i in 1:R){
    y1 \leftarrow window(y_ts, start = c(2003,(1+(i-1)*step)), end = c(2003,(T+step*(i-1))))
    model <- auto.arima(y1)</pre>
    y_pred <- as.vector(forecast(model,step)$mean)</pre>
    lower_ARMA <- c(lower_ARMA,as.vector(forecast(model,step)$lower[,2]))</pre>
    upper_ARMA <- c(upper_ARMA,as.vector(forecast(model,step)$upper[,2]))</pre>
    y_hat_ARMA <- c(y_hat_ARMA,y_pred)</pre>
MSPE_ARMA <- sqrt(1/(dim(set$X_test)[1])*sum((y_hat_ARMA-as.vector(set$y_test))^2))
MAPE_ARMA <- 1/(dim(set$X_test)[1])*sum(abs(y_hat_ARMA-as.vector(set$y_test)))
print(paste("MSFE for ARMA =", MSPE_ARMA))
```

[1] "MSFE for ARMA = 6.24144984110166"

```
print(paste("MAPE for ARMA =", MAPE_ARMA))
## [1] "MAPE for ARMA = 4.84811520135249"
ARDL
library(forecast)
library(ARDL)
## To cite the ARDL package in publications:
## Use this reference to refer to the validity of the ARDL package.
##
     Natsiopoulos, Kleanthis, and Tzeremes, Nickolaos G. (2022). ARDL
##
##
     bounds test for cointegration: Replicating the Pesaran et al. (2001)
##
     results for the UK earnings equation using R. Journal of Applied
##
     Econometrics, 37(5), 1079-1090. https://doi.org/10.1002/jae.2919
## Use this reference to cite this specific version of the ARDL package.
##
##
     Kleanthis Natsiopoulos and Nickolaos Tzeremes (2023). ARDL: ARDL, ECM
     and Bounds-Test for Cointegration. R package version 0.2.3.
##
     https://CRAN.R-project.org/package=ARDL
X_ARDL_ts<-cbind(delta_stock_lag12)</pre>
step <- 12
R <- as.integer(length(set$y_test)/step)</pre>
y_hat_ARDL <- c()</pre>
T1 <- dim(set$X train)[1]+dim(set$X test)[1]
T <- T1-R*step
for (i in 1:R){
    X1 \leftarrow window(X\_ARDL\_ts, start = c(2003, (1+(i-1)*step)), end = c(2003, (T+step*(i-1))))
    y1 \leftarrow window(y_ts, start = c(2003,(1+(i-1)*step)), end = c(2003,(T+step*(i-1))))
    X2 \leftarrow window(X_ARDL_ts, start = c(2003, (T+1+step*(i-1))), end = c(2003, (T+step*(i))))
    model <- auto.arima(y1,xreg=X1)</pre>
    y_pred <- predict(model,n.ahead = step,newxreg = X2)$pred</pre>
    y_hat_ARDL <- c(y_hat_ARDL,y_pred)</pre>
MSPE_ARDL <- sqrt(1/(dim(set$X_test)[1])*sum((y_hat_ARDL-as.vector(set$y_test))^2))
MAPE_ARDL <- 1/(dim(set$X_test)[1])*sum(abs(y_hat_ARDL-as.vector(set$y_test)))
print(paste("MSFE for ARDL =", MSPE_ARDL))
## [1] "MSFE for ARDL = 6.18532659248739"
print(paste("MAPE for ARDL =", MAPE_ARDL))
```

[1] "MAPE for ARDL = 4.8820786850432"

VAR.

```
library(vars)
## Loading required package: MASS
## Attaching package: 'MASS'
## The following object is masked from 'package:dplyr':
##
##
       select
## Loading required package: strucchange
## Loading required package: zoo
##
## Attaching package: 'zoo'
## The following objects are masked from 'package:base':
##
##
       as.Date, as.Date.numeric
## Loading required package: sandwich
##
## Attaching package: 'strucchange'
## The following object is masked from 'package:rstan':
##
##
       monitor
## Loading required package: urca
## Loading required package: lmtest
step <- 12
Y_ts<-cbind(y_ts,kilian,delta_stock)
R <- as.integer(length(set$y_test)/step)</pre>
y_hat_VAR <- c()</pre>
T1 <- dim(set\$X_train)[1]+dim(set\$X_test)[1]
T <- T1-R*step
for (i in 1:R){
    Y1 \leftarrow window(Y_{ts}, start = c(2003, (1+(i-1)*step)), end = c(2003, (T+step*(i-1))))
    order_select<-as.integer(VARselect(Y1)$selection[1]) #AIC
    model <- VAR(Y1,order_select)</pre>
    y_pred <- as.vector(predict(model,n.ahead =step)$fcst$y_ts[,1])</pre>
    y_hat_VAR <- c(y_hat_VAR,y_pred)</pre>
MSPE_VAR <- sqrt(1/(dim(set$X_test)[1])*sum((y_hat_VAR-as.vector(set$y_test))^2))
MAPE_VAR <- 1/(dim(set$X_test)[1])*sum(abs(y_hat_VAR-as.vector(set$y_test)))
print(paste("MSFE for VAR =", MSPE_VAR))
```

```
print(paste("MAPE for VAR =", MAPE_VAR))
## [1] "MAPE for VAR = 4.72446287269033"
tvAR.
library(tvReg)
## Loading required package: Matrix
## Funded by the Horizon 2020. Framework Programme of the European Union.
## Attaching package: 'tvReg'
## The following object is masked from 'package:forecast':
##
##
       forecast
library(Matrix)
step <- 12
R <- as.integer(length(set$y_test)/step)</pre>
y hat tvAR <- c()</pre>
T1 <- dim(set$X_train)[1]+dim(set$X_test)[1]
T <- T1-R*step
for (i in 1:R){
    y1 \leftarrow window(y_ts, start = c(2003,(1+(i-1)*step)), end = c(2003,(T+step*(i-1))))
    model <- tvAR(y1,1,type = "const",est = "ll",tkernel = "Epa")</pre>
    y_pred <- forecast(model,n.ahead=step)</pre>
    y_hat_tvAR <- c(y_hat_tvAR,y_pred)</pre>
}
## Calculating regression bandwidth...
MSPE_tvAR <- sqrt(1/(dim(set$X_test)[1])*sum((y_hat_tvAR-as.vector(set$y_test))^2))
MAPE_tvAR <- 1/(dim(set$X_test)[1])*sum(abs(y_hat_tvAR-as.vector(set$y_test)))
print(paste("MSFE for tvAR =", MSPE_tvAR))
## [1] "MSFE for tvAR = 6.23128082581619"
```

[1] "MSFE for VAR = 6.10912233191664"

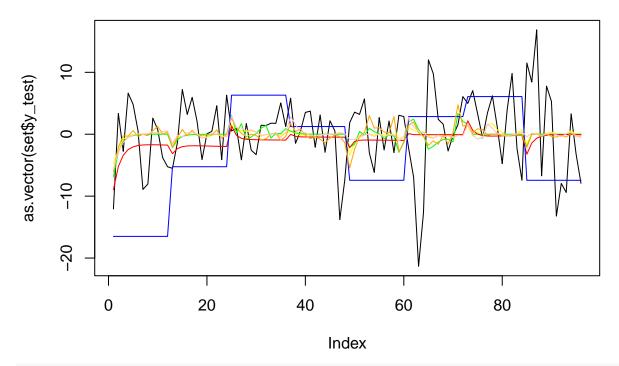
```
## [1] "MAPE for tvAR = 4.91363902702261"
tvVAR.
step <- 12
Y ts<-cbind(y ts,kilian,delta stock)
R <- as.integer(length(set$y_test)/step)</pre>
y hat tvVAR <- c()</pre>
T1 <- dim(set$X_train)[1]+dim(set$X_test)[1]
T <- T1-R*step
for (i in 1:R){
   Y1 \leftarrow window(Y_ts, start = c(2003, (1+(i-1)*step)), end = c(2003, (T+step*(i-1))))
   y1 < window(y_lag12, start = c(2003,(1+(i-1)*step)), end = c(2003,(T+step*(i-1))))
   y2 < -window(y_ts, start = c(2003, (1+(i-1)*step)), end = c(2003, (T+step*(i-1))))
   model <- tvVAR(Y1,1,type="const", est="ll", tkernel="Epa",z = as.vector(y1))</pre>
   y_pred <- forecast(model,n.ahead =step,newz = as.vector(tail(y2,step)))[,1]</pre>
   y_hat_tvVAR <- c(y_hat_tvVAR,y_pred)</pre>
}
## Calculating regression bandwidths... bandwidth(s) 208.9 24.59977 208.9
## Calculating regression bandwidths... bandwidth(s) 208.9 24.88 208.9
## Calculating regression bandwidths... bandwidth(s) 208.9 23.98 208.9
## Calculating regression bandwidths... bandwidth(s) 208.9 23.50991 208.9
## Calculating regression bandwidths... bandwidth(s) 208.9 23.37 208.9
## Calculating regression bandwidths... bandwidth(s) 208.9 15.43 208.9
## Calculating regression bandwidths... bandwidth(s) 40.32 34.73 208.9
## Calculating regression bandwidths... bandwidth(s) 178.05 27.96 34.96414
MSPE_tvVAR <- sqrt(1/(dim(set$X_test)[1])*sum((y_hat_tvVAR-as.vector(set$y_test))^2))
MAPE_tvVAR <- 1/(dim(set$X_test)[1])*sum(abs(y_hat_tvVAR-as.vector(set$y_test)))
print(paste("MSFE for tvVAR =", MSPE_tvVAR))
## [1] "MSFE for tvVAR = 6.0861540309294"
print(paste("MAPE for tvVAR =", MAPE_tvVAR))
## [1] "MAPE for tvVAR = 4.73132787002926"
FAR.
library(tvReg)
```

print(paste("MAPE for tvAR =", MAPE_tvAR))

step <- 12

R <- as.integer(length(set\$y_test)/step)</pre>

```
y_hat_FAR <- c()</pre>
T1 <- dim(set$X_train)[1]+dim(set$X_test)[1]
T <- T1-R*step
for (i in 1:R){
    y1 \leftarrow window(y_ts, start = c(2003, (1+(i-1)*step)), end = c(2003, (T+step*(i-1))))
    y1_{lag1}<-window(y_{lag12}, start = c(2003,(1+(i-1)*step)), end = c(2003,(T+step*(i-1))))
    model <- tvAR(y1,type=c("const"),1,z=y1_lag1, est="ll", tkernel="Epa")</pre>
    y pred <- forecast(model,n.ahead=step, newz = as.vector(tail(y1,step)))</pre>
    y_hat_FAR <- c(y_hat_FAR,y_pred)</pre>
## Calculating regression bandwidth...
MSPE_FAR <- sqrt(1/(dim(set$X_test)[1])*sum((y_hat_FAR-as.vector(set$y_test))^2))
MAPE_FAR <- 1/(dim(set$X_test)[1])*sum(abs(y_hat_FAR-as.vector(set$y_test)))
print(paste("MSFE for FAR =", MSPE_FAR))
## [1] "MSFE for FAR = 6.10457814879958"
print(paste("MAPE for FAR =", MAPE_FAR))
## [1] "MAPE for FAR = 4.7428440194226"
plot(as.vector(set$y_test),type="l")
lines(y hat FAR,col="yellow")
lines(y_hat_ARMA,col="green")
lines(y hat tvAR,col="red")
lines(y_hat_RW,col="blue")
lines(y_hat_ARDL,col="orange")
lines(y_hat_tvVAR,col="pink")
```



#lines(y_hat_GPR, col="purple")

```
#library(kableExtra)
library(xtable)
Methods_table <- c("RW", "ARMA", "ARDL", "VAR", "tvAR", "tvVAR", "FAR")</pre>
MSE_table <- round(c(MSPE_RW, MSPE_ARMA, MSPE_ARDL, MSPE_VAR, MSPE_tvAR, MSPE_tvVAR, MSPE_FAR),2)
MAPE_table <- round(c(MAPE_RW, MAPE_ARMA, MAPE_ARDL, MAPE_VAR, MAPE_tvAR, MAPE_tvVAR, MAPE_FAR),2)
# Create the dataframe
df_results <- data.frame(Method = Methods_table, MSPE = MSE_table, MAPE = MAPE_table)
df_results
     Method MSPE MAPE
##
## 1
         RW 9.62 7.57
## 2
       ARMA 6.24 4.85
       ARDL 6.19 4.88
## 3
        VAR 6.11 4.72
## 4
      tvAR 6.23 4.91
## 5
## 6 tvVAR 6.09 4.73
        FAR 6.10 4.74
## 7
#xtable(df_results)
```

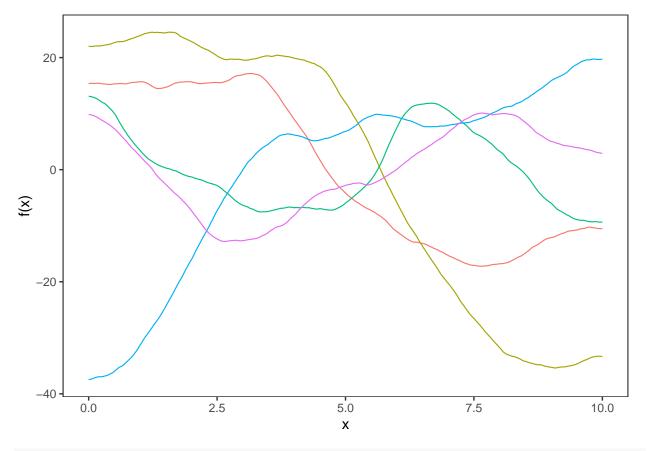
dm.test((y_hat_ARMA-as.vector(set\$y_test)),(y_hat_FAR-as.vector(set\$y_test)),h=12, alternative="greater"

```
##
##
  Diebold-Mariano Test
## data: (y_hat_ARMA - as.vector(set$y_test))(y_hat_FAR - as.vector(set$y_test))
## DM = 0.97981, Forecast horizon = 12, Loss function power = 1, p-value =
## 0.1648
## alternative hypothesis: greater
```

Gaussian Process Regression

BM kernel

```
library(ggplot2)
source("Kernels.R")
x <- seq(0, 10, length.out = 501)  # x-coordinates
N <- 5  # no. of draws
Y <- draw_samples(x, N, kernel_fn = bm_kernel ,centred=TRUE,squared=TRUE)
df <- data.frame(x = rep(x, N), y = as.vector(Y), group = rep(1:N, each = length(x)))
# Plot the data
plot1<-ggplot(df, aes(x = x, y = y, group = group, color = factor(group))) +
    geom_line(size = 0.4) +
    labs(x = "x", y = "f(x)")+ #, title = "Brownian motion kernel")+
    theme_bw()+
    theme(legend.position = "none",panel.grid.major = element_blank(),
        panel.grid.minor = element_blank())
ggsave("Brownian motion kernel paths (centred+squared).png", plot = plot1, width = 4, height = 4)
plot1</pre>
```

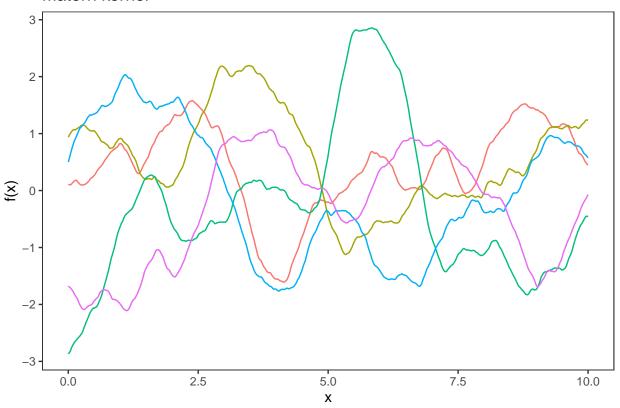


#scale_color_manual(values = col_list)

Matern Kernel

```
Y <- draw_samples(x, N, kernel_fn = matern_kernel,nu = 1.5, sigma = 1, l = 1)
df <- data.frame(x = rep(x, N), y = as.vector(Y), group = rep(1:N, each = length(x)))
# Plot the data
ggplot(df, aes(x = x, y = y, group = group, color = factor(group))) +
    geom_line(size = 0.5) +
    labs(x = "x", y = "f(x)", title = "Matern kernel")+
    theme_bw()+
    theme(legend.position = "none",panel.grid.major = element_blank(),
        panel.grid.minor = element_blank())</pre>
```

Matern kernel

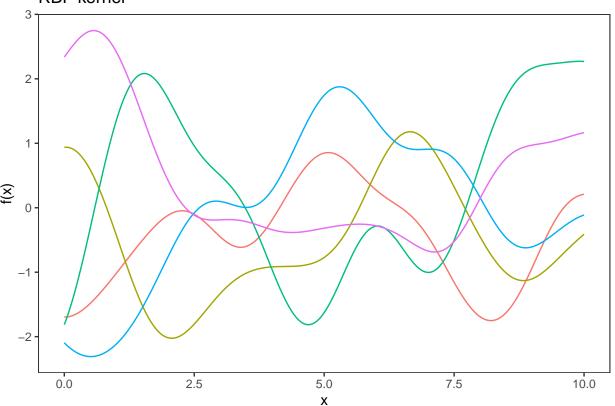


#scale_color_manual(values = col_list)

RBF Kernel

```
Y <- draw_samples(x, N, kernel_fn = se_kernel,sigma = 1, length=1)
df <- data.frame(x = rep(x, N), y = as.vector(Y), group = rep(1:N, each = length(x)))
# Plot the data
ggplot(df, aes(x = x, y = y, group = group, color = factor(group))) +
    geom_line(size = 0.5) +
    labs(x = "x", y = "f(x)", title = "RBF kernel")+
    theme_bw()+</pre>
```

RBF kernel



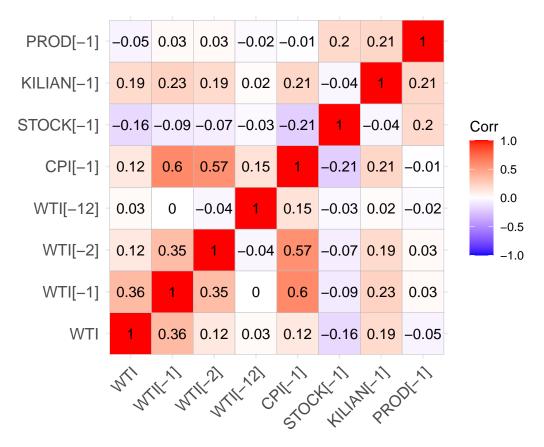
#scale_color_manual(values = col_list)

```
source("Kernels.R")
library(mvtnorm)
library(RColorBrewer)
library(plgp)
```

Loading required package: tgp

```
levelpersp <- function(x, y, z, colors=topo.colors, ...) {
    ## getting the value of the midpoint
    zz <- (z[-1,-1] + z[-1,-ncol(z)] + z[-nrow(z),-1] + z[-nrow(z),-ncol(z)])/4
    ## calculating the breaks
    breaks <- hist(zz, plot=FALSE)$breaks
    ## cutting up zz
    cols <- colors(length(breaks)-1)
    zzz <- cut(zz, breaks=breaks, labels=cols)
    ## plotting
    persp(x, y, z, col=as.character(zzz), ...)
    ## return breaks and colors for the legend
    list(breaks=breaks, colors=cols)
}</pre>
```

```
# Define the range and number of points for x1 and x2
n_points <- 20
x1 \leftarrow seq(0, 3, length.out = n_points)
x2 \leftarrow seq(0, 3, length.out = n_points)
# Sample from the bivariate normal distribution
N <- 1 # Number of samples
Y <- draw_samples_3D(x1, x2, N, seed = 125, kernel_fn = bm_kernel, centred = TRUE, squared = TRUE)
# Create the 3D plots with colors based on z-values using persp
\#par(mfrow = c(1, 3))
levelpersp(x1, x2, (matrix(Y, ncol = n_points)), theta = -30, phi = 30, colors = rainbow)
## $breaks
## [1] -20 -15 -10 -5 0
                             5 10 15
##
## $colors
## [1] "#FF0000" "#FFDB00" "#49FF00" "#00FF92" "#0092FF" "#4900FF" "#FF00DB"
\#levelpersp(x1, x2, (matrix(Y[2, ], ncol = n_points)), theta = -30, phi = 30, colors = topo.colors)
\#levelpersp(x1, x2, (matrix(Y[3, ], ncol = n_points)), theta = -30, phi = 30, colors = topo.colors)
library(corrplot)
## corrplot 0.92 loaded
source("Data-preprocessing.R")
## New names:
## * '' -> '...2'
## New names:
## New names:
## * '' -> '...1'
## * '' -> '...2'
```



```
# library(PerformanceAnalytics)
# chart.Correlation(correlation_matrix)
#ggpairs(X_corr)
# corrplot(correlation_matrix, method = "color", #type = "upper",
# tl.cex = 0.8, tl.col = "black", tl.srt = 45,
# #col = colorRampPalette(c("blue", "white", "red"))(100),
# addCoef.col = "black", number.cex = 0.7)
```

Gaussian Process Regression implementation

```
# print(fit$summary())
# library(shinystan)
```

```
# Fit the model
library(ggplot2)
library(plyr)
library(cmdstanr)
library(rstan)
N<-length(set$y_train)
mod = cmdstan_model("MCMC pred.stan",include_paths = "~/Documents/LSE/Dissertation/Code/ANOVA-kernel/")
y_stan<-as.vector(set$y_train)</pre>
\#data=list(N=N,K_gram1=K_gram(set\$X_train[,1],kernel_fn=bm_kernel_L2,centred=TRUE,squared=TRUE),
data_MCMC<-list(N=length(set$y_train),</pre>
          x1=as.vector(set$X_train[,1]),
          x2=as.vector(set$X_train[,2]),
          y=as.vector(set$y_train),
          x_new1=as.matrix(1),
          x new2=as.matrix(1)
          )
fit = mod$sample(
  data = data_MCMC,
  seed = 123,
  iter_warmup = 200,
  iter_sampling = 300,
  save_warmup = TRUE,
  chains = 2,
  parallel_chains = 2,
  refresh = 10
## Running MCMC with 2 parallel chains...
##
## Chain 1 Iteration:
                        1 / 500 [ 0%]
                                         (Warmup)
## Chain 1 Iteration: 10 / 500 [ 2%]
                                         (Warmup)
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 1.07451e+24, but A[2,1] = 1.0745
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 1.76399e+26, but A[2,1] = 1.7639
```

launch_shinystan(fit)

Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova

```
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 3.3645e+08, but A[2,1] = 3.3645e
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 2 Iteration:
                       1 / 500 [ 0%]
                                       (Warmup)
## Chain 2 Iteration: 10 / 500 [ 2%]
                                       (Warmup)
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = inf, but A[2,1] = inf (in '/var/
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = inf, but A[2,1] = inf (in '/var/
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 7.09232e+118, but A[2,1] = 7.092
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
```

Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th

Chain 2

```
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 1.81707e+32, but A[2,1] = 1.8170
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 1.41364e+11, but A[2,1] = 1.4136
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,19] = -6.83085e+06, but A[19,1] = -6.
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[10,71] = -5.24597e+06, but A[71,10] = -
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
```

Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = inf, but A[2,1] = inf (in '/var/a)

Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova:
Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m:
Chain 2

Chain 2

```
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 1.14789e+135, but A[2,1] = 1.147
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[19,137] = 7.57908e+06, but A[137,19] =
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 1 Iteration:
                      20 / 500 [ 4%]
                                        (Warmup)
## Chain 2 Iteration: 20 / 500 [ 4%]
                                        (Warmup)
## Chain 1 Iteration:
                      30 / 500 [ 6%]
                                        (Warmup)
## Chain 2 Iteration: 30 / 500 [ 6%]
                                        (Warmup)
## Chain 2 Iteration: 40 / 500 [ 8%]
                                        (Warmup)
## Chain 1 Iteration: 40 / 500 [ 8%]
                                        (Warmup)
## Chain 2 Iteration: 50 / 500 [ 10%]
                                        (Warmup)
## Chain 1 Iteration: 50 / 500 [ 10%]
                                        (Warmup)
## Chain 1 Iteration: 60 / 500 [ 12%]
                                        (Warmup)
## Chain 2 Iteration: 60 / 500 [ 12%]
                                        (Warmup)
## Chain 1 Iteration: 70 / 500 [ 14%]
                                        (Warmup)
## Chain 2 Iteration: 70 / 500 [ 14%]
                                        (Warmup)
## Chain 1 Iteration: 80 / 500 [ 16%]
                                        (Warmup)
## Chain 2 Iteration: 80 / 500 [ 16%]
                                        (Warmup)
## Chain 1 Iteration: 90 / 500 [ 18%]
                                        (Warmup)
## Chain 2 Iteration: 90 / 500 [ 18%]
                                        (Warmup)
## Chain 1 Iteration: 100 / 500 [ 20%]
                                        (Warmup)
## Chain 2 Iteration: 100 / 500 [ 20%]
                                        (Warmup)
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 2.533e+32, but A[2,1] = 2.533e+3
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 1 Iteration: 110 / 500 [ 22%]
```

```
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,11] = 2.24709e+07, but A[11,1] = 2.24
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 2 Iteration: 110 / 500 [ 22%]
                                        (Warmup)
## Chain 1 Iteration: 120 / 500 [ 24%]
                                        (Warmup)
## Chain 2 Iteration: 120 / 500 [ 24%]
                                        (Warmup)
## Chain 1 Iteration: 130 / 500 [ 26%]
                                        (Warmup)
## Chain 2 Iteration: 130 / 500 [ 26%]
                                        (Warmup)
## Chain 1 Iteration: 140 / 500 [ 28%]
                                        (Warmup)
## Chain 2 Iteration: 140 / 500 [ 28%]
                                        (Warmup)
## Chain 1 Iteration: 150 / 500 [ 30%]
                                         (Warmup)
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 3.15281e+08, but A[2,1] = 3.1528
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 2 Iteration: 150 / 500 [ 30%]
                                        (Warmup)
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 6.69044e+28, but A[2,1] = 6.6904
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 1 Iteration: 160 / 500 [ 32%]
                                        (Warmup)
## Chain 2 Iteration: 160 / 500 [ 32%]
                                        (Warmup)
## Chain 1 Iteration: 170 / 500 [ 34%]
                                        (Warmup)
## Chain 2 Iteration: 170 / 500 [ 34%]
                                        (Warmup)
## Chain 1 Iteration: 180 / 500 [ 36%]
                                        (Warmup)
## Chain 2 Iteration: 180 / 500 [ 36%]
                                        (Warmup)
```

(Warmup)

(Warmup)

(Warmup)

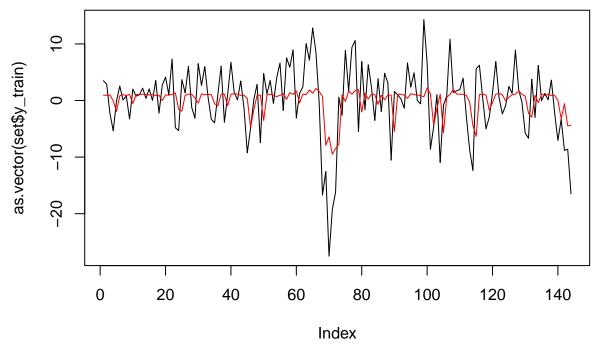
Chain 1 Iteration: 190 / 500 [38%]

Chain 2 Iteration: 190 / 500 [38%]

Chain 1 Iteration: 200 / 500 [40%]

```
## Chain 1 Iteration: 201 / 500 [ 40%]
                                          (Sampling)
## Chain 2 Iteration: 200 / 500 [ 40%]
                                          (Warmup)
## Chain 2 Iteration: 201 / 500 [ 40%]
                                          (Sampling)
## Chain 1 Iteration: 210 / 500 [ 42%]
                                         (Sampling)
## Chain 1 Iteration: 220 / 500 [ 44%]
                                          (Sampling)
## Chain 2 Iteration: 210 / 500 [ 42%]
                                          (Sampling)
## Chain 1 Iteration: 230 / 500 [ 46%]
                                          (Sampling)
## Chain 2 Iteration: 220 / 500 [ 44%]
                                          (Sampling)
## Chain 2 Iteration: 230 / 500 [ 46%]
                                          (Sampling)
## Chain 1 Iteration: 240 / 500 [ 48%]
                                          (Sampling)
## Chain 2 Iteration: 240 / 500 [ 48%]
                                          (Sampling)
## Chain 1 Iteration: 250 / 500 [ 50%]
                                          (Sampling)
## Chain 1 Iteration: 260 / 500 [ 52%]
                                          (Sampling)
## Chain 2 Iteration: 250 / 500 [ 50%]
                                          (Sampling)
## Chain 1 Iteration: 270 / 500 [ 54%]
                                          (Sampling)
## Chain 2 Iteration: 260 / 500 [ 52%]
                                          (Sampling)
## Chain 1 Iteration: 280 / 500 [ 56%]
                                          (Sampling)
## Chain 2 Iteration: 270 / 500 [ 54%]
                                          (Sampling)
## Chain 1 Iteration: 290 / 500 [ 58%]
                                          (Sampling)
## Chain 2 Iteration: 280 / 500 [ 56%]
                                          (Sampling)
## Chain 1 Iteration: 300 / 500 [ 60%]
                                          (Sampling)
## Chain 1 Iteration: 310 / 500 [ 62%]
                                          (Sampling)
## Chain 2 Iteration: 290 / 500 [ 58%]
                                          (Sampling)
## Chain 1 Iteration: 320 / 500 [ 64%]
                                          (Sampling)
## Chain 2 Iteration: 300 / 500 [ 60%]
                                          (Sampling)
## Chain 1 Iteration: 330 / 500 [ 66%]
                                          (Sampling)
## Chain 1 Iteration: 340 / 500 [ 68%]
                                          (Sampling)
## Chain 2 Iteration: 310 / 500 [ 62%]
                                          (Sampling)
## Chain 1 Iteration: 350 / 500 [ 70%]
                                          (Sampling)
## Chain 2 Iteration: 320 / 500 [ 64%]
                                          (Sampling)
## Chain 1 Iteration: 360 / 500 [ 72%]
                                          (Sampling)
## Chain 2 Iteration: 330 / 500 [ 66%]
                                          (Sampling)
## Chain 2 Iteration: 340 / 500 [ 68%]
                                          (Sampling)
## Chain 1 Iteration: 370 / 500 [ 74%]
                                          (Sampling)
## Chain 1 Iteration: 380 / 500 [ 76%]
                                          (Sampling)
## Chain 2 Iteration: 350 / 500 [ 70%]
                                          (Sampling)
## Chain 1 Iteration: 390 / 500 [ 78%]
                                          (Sampling)
## Chain 2 Iteration: 360 / 500 [ 72%]
                                          (Sampling)
## Chain 1 Iteration: 400 / 500 [ 80%]
                                          (Sampling)
## Chain 1 Iteration: 410 / 500 [ 82%]
                                          (Sampling)
## Chain 2 Iteration: 370 / 500 [ 74%]
                                          (Sampling)
## Chain 1 Iteration: 420 / 500 [ 84%]
                                          (Sampling)
## Chain 2 Iteration: 380 / 500 [ 76%]
                                          (Sampling)
## Chain 1 Iteration: 430 / 500 [ 86%]
                                          (Sampling)
## Chain 2 Iteration: 390 / 500 [ 78%]
                                          (Sampling)
## Chain 1 Iteration: 440 / 500 [ 88%]
                                          (Sampling)
## Chain 1 Iteration: 450 / 500 [ 90%]
                                          (Sampling)
## Chain 2 Iteration: 400 / 500 [ 80%]
                                          (Sampling)
## Chain 1 Iteration: 460 / 500 [ 92%]
                                          (Sampling)
## Chain 2 Iteration: 410 / 500
                                [ 82%]
                                          (Sampling)
## Chain 1 Iteration: 470 / 500 [ 94%]
                                          (Sampling)
                                          (Sampling)
## Chain 1 Iteration: 480 / 500 [ 96%]
## Chain 2 Iteration: 420 / 500 [ 84%]
                                          (Sampling)
## Chain 1 Iteration: 490 / 500 [ 98%]
                                         (Sampling)
```

```
## Chain 2 Iteration: 430 / 500 [ 86%]
                                         (Sampling)
## Chain 1 Iteration: 500 / 500 [100%]
                                         (Sampling)
## Chain 2 Iteration: 440 / 500 [ 88%]
                                         (Sampling)
## Chain 2 Iteration: 450 / 500 [ 90%]
                                         (Sampling)
## Chain 1 finished in 7.9 seconds.
## Chain 2 Iteration: 460 / 500 [ 92%]
                                         (Sampling)
## Chain 2 Iteration: 470 / 500 [ 94%]
                                         (Sampling)
## Chain 2 Iteration: 480 / 500 [ 96%]
                                         (Sampling)
## Chain 2 Iteration: 490 / 500 [ 98%]
                                         (Sampling)
## Chain 2 Iteration: 500 / 500 [100%]
                                         (Sampling)
## Chain 2 finished in 8.5 seconds.
##
## Both chains finished successfully.
## Mean chain execution time: 8.2 seconds.
## Total execution time: 8.6 seconds.
impact_1<-as.vector(fit$summary(variables = "f_one")$mean)</pre>
impact_2<-as.vector(fit$summary(variables = "f_two")$mean)</pre>
plot(as.vector(set$y_train),type="l")
lines(impact_1, col="red")
```



```
#lines(impact_2)
#lines(impact_1+impact_2,col="green")
```

```
\#geom\_line(aes(y = y\_hat\_GPR\_1\_step,colour="GPR"),alpha=0.7)+\#,colour="blue"
#
#
    qeom\_line(aes(y = y\_hat\_FAR, colour = "FAR"), alpha = 0.7) + \#, colour = "qreen"
#
    qeom\_line(aes(y = y\_hat\_ANN\_1\_step,colour="ANN"),alpha=0.7)+\#,colour="ANN"
   labs(x = NULL,
#
         y = "WTI"
#
         color = NULL
#
         #title = "WTI Crude Oil Prices from 2003-2023",
#
         #subtitle = "From 1987-2021, comparing the top 1% and bottom 50\% \setminus n(National\ recessions\ shaded
         #caption = "Source: Federal Reserve Economic Data"
#
#
#
    scale_color_manual(values = c( "ARMA" = "red", "FAR" = "green", "ANN" = "blue"),
                        breaks = c("ARMA", "FAR", "ANN"),
#
                        labels = c("ARMA", "FAR", "ANN")) +
#
#
   theme_minimal()+
   theme(legend.position = "bottom")
#mu_wti<-as.vector(fit$summary(variables = "mu_predicted")$mean)</pre>
#t_wti <-seq(-28,17,0.5)
#mu_kilian<-as.vector(fit$summary(variables = "mu_predicted")$mean)</pre>
\#t_kilian < -seq(-100,85,0.5)
#mu delta stock<-as.vector(fit$summary(variables = "mu predicted")$mean)</pre>
\#t_delta_stock < -seq(-90,84,0.5)
#plot(delta_stock_lag1,y)
\#lines(seq(-90,84,0.5),as.vector(fit\$summary(variables = "mu_predicted")\$mean), col="red")
#plot(kilian_lag1,y, col="red")
#points(kilian_lag1,y-impact_1-impact_3-impact_4-impact_5-impact_0)
\#lines(t_kilian, mu_kilian)
# library(ggplot2)
# # Create a data frame for your data
# data_123 <- data.frame(
  delta_stock_laq1 = delta_stock_laq1,
#
   y = y,
#
   impact_adjusted = y - impact_1 - impact_2 - impact_4 - impact_5 - impact_0
# )
# data_1234<-data.frame(
    t_delta_stock = t_delta_stock,
#
   mu delta stock = mu delta stock
# )
#
# # Create the ggplot with layers
# gg<-ggplot() +
    geom\_point(data = data\_123, aes(x = delta\_stock\_lag1, y = y), color = "chartreuse", size=1) +
#
   geom\_point(data = data\_123, aes(x = delta\_stock\_lag1, y = impact\_adjusted), color = "blue3", size=1
#
   geom\_line(data = data\_1234, aes(x = t\_delta\_stock, y = mu\_delta\_stock), color = "black") +
#
   labs(
#
     #title = "Kilian Lag Plot",
#
     x = "STOCK[-1]",
    y = "WTI"
#
   ) +
#
#
   theme_minimal() +
```

plot.title = element_text(size = 16, hjust = 0.5),

#

theme(

```
legend.position = "top",
#
                 aspect.ratio = 1
#
# #ggsave("Kilian.png",plot=gg, path="plots/",width = 10, height=10, units="cm")
# print(qq)
#impact_0<-as.vector(fit$summary(variables = "f_zero")$mean)</pre>
#impact_1<-as.vector(fit$summary(variables = "f_one")$mean)</pre>
#impact_2<-as.vector(fit$summary(variables = "f_two")$mean)</pre>
#impact_3<-as.vector(fit$summary(variables = "f_three")$mean)</pre>
\#impact\_4 < -as.vector(fit\$summary(variables = "f_four")\$mean)
#impact_5<-as.vector(fit$summary(variables = "f_five")$mean)</pre>
#total<-as.vector(fit$summary(variables = "f")$mean)</pre>
#time_idx1 < -as.Date(time(y_ts))
# ANOVA_data<-as.data.frame(cbind(time_idx1,impact_0,impact_1,impact_2,impact_3,impact_4,impact_5,y))[6
# qqplot(data = ANOVA \ data, aes(x = as.Date(time \ idx1))) +
        geom_line(aes(y = y),colour="black")+#,colour="black"
          geom_line(aes(y = impact_1,colour="WTI[-1]"),alpha=0.7)+#,colour="red"
        geom_line(aes(y = impact_2,colour="KILIAN[-1]"),alpha=0.7)+#,colour="green"
#
        geom\_line(aes(y = impact\_3, colour="STOCK[-1]"), alpha=0.7)+\#, colour="ANN"
          geom\_line(aes(y = (impact\_0 + impact\_4 + impact\_5), colour = "CONST + CPI[-1] + PROD[-1]"), alpha=0.7) + CPI[-1] +
#
        geom\_line(aes(y = (impact_1+impact_2+impact_3), colour="WTI[-1]+KILIAN[-1]+STOCK[-1]"))+
#
#
        labs(x = NULL,
#
                        y = "WTI",
#
                         color = NULL
#
                         #title = "WTI Crude Oil Prices from 2003-2023",
#
                         #subtitle = "From 1987-2021, comparing the top 1% and bottom 50\% \n(National recessions shaded
#
                         #caption = "Source: Federal Reserve Economic Data"
#
                         ) +
#
          scale\_color\_manual(values = c("WTI[-1]" = "darkblue", "KILIAN[-1]" = "royalblue", "STOCK[-1]" = "logarity = "logarity = "scale_color_manual(values = c("WTI[-1]" = "darkblue", "KILIAN[-1]" = "royalblue", "STOCK[-1]" = "logarity = "lo
#
                                                                breaks = c( "WTI[-1]", "KILIAN[-1]", "STOCK[-1]", "CONST+CPI[-1]+PROD[-1]", "WTI[-
#
                                                                labels = c( "WTI[-1]", "KILIAN[-1]", "STOCK[-1]", "REST", "TOTAL")) +
#
        theme minimal()+
        theme(legend.position = "bottom",
#
#
                plot.title = element_text(size = 16, hjust = 0.5),
#
                aspect.ratio = 1
#
 \begin{tabular}{ll} \# uncertain\_data1 <- as. data. frame (cbind(time\_idx, y\_hat\_ARMA, lower\_ARMA, upper\_ARMA, set \$y\_test)) [37:95,] \\ \end{tabular} 
# qqplot(data = uncertain data1, aes(x = as.Date(time idx))) +
          geom_ribbon(aes(ymin = lower_ARMA, ymax = upper_ARMA), fill = 'grey90') +
          geom\_line(aes(y = set\$y\_test[37:95]), colour="black")+
#
#
          geom_line(aes(y = y_hat_ARMA), colour="red") +#, colour="red"
#
         labs(x = NULL,
                         y = "WTI"
#
#
                         color = NULL
#
                        ) +
#
        theme_minimal()+
#
        theme(
#
                aspect.ratio = 1
#
```

```
# uncertain_data<-as.data.frame(cbind(time_idx,y_hat_GPR,lower_GPR,upper_GPR,set$y_test))[37:95,]
\# qqplot(data = uncertain_data, aes(x = as.Date(time_idx))) +
    qeom_ribbon(aes(ymin = lower_GPR, ymax = upper_GPR), fill = 'qrey90') +
   qeom\_line(aes(y = set\$y\_test[37:95]), colour="black")+
#
    qeom_line(aes(y = y_hat_GPR), colour="blue")+#, colour="red"
#
   labs(x = NULL,
#
         y = "WTI",
#
         color = NULL
#
         ) +
#
   theme minimal()+
#
   theme(
#
      aspect.ratio = 1
#
# Fit the model
library(ggplot2)
library(plyr)
library(cmdstanr)
library(rstan)
step <- 12
R <- as.integer(length(set$y_test)/step)</pre>
y hat GPR <- c()</pre>
lower GPR <- c()</pre>
upper GPR <- c()
T1 <- dim(set$X_train)[1]+dim(set$X_test)[1]
T <- T1-R*step
X_lag1_ts<-cbind(y_lag12,kilian_lag12,delta_stock_lag12,cpi_lag12, prod_lag12,y_lag1) #kilian_lag1
#X_ts<-cbind(y, cpi, kilian, delta_stock, prod)
for (i in 1:R){
    y_{train} \leftarrow window(y_{ts}, start = c(2003, (1+(i-1)*step)), end = c(2003, (T+step*(i-1))))
    X_{train} \leftarrow window(X_{lag1_ts}, start = c(2003,(1+(i-1)*step)), end = c(2003,(T+step*(i-1))))
    X_{\text{test}} \leftarrow \text{window}(X_{\text{lag1\_ts}}, \text{start} = c(2003, (T+1+\text{step*(i-1)})), end = c(2003, (T+\text{step*(i)})))
    data_MCMC<-list(N=length(y_train),</pre>
          x1=as.vector(X_train[,1]), #laq1
          x2=as.vector(X_train[,2]), #cpi
          x3=as.vector(X train[,3]), #kilian
          x4=as.vector(X_train[,4]), #delta stock
          x5=as.vector(X_train[,5]), #prod
          x6=as.vector(X_train[,6]), #y_lag2
          y=as.vector(y_train),
          M=step,
          x_new1=as.matrix(X_test[,1]),
          x_new2=as.matrix(X_test[,2]),
          x_new3=as.matrix(X_test[,3]),
          x_new4=as.matrix(X_test[,4]),
          x_new5=as.matrix(X_test[,5]),
          x_new6=as.matrix(X_test[,6])
    mod = cmdstan_model("MCMC.stan",include_paths = "~/Documents/LSE/Dissertation/Code/ANOVA-kernel/")
    fit <- mod$sample(</pre>
            data = data_MCMC,
             seed = 123,
             iter warmup = 200,
```

```
iter_sampling = 300,
            save_warmup = TRUE,
            chains = 2,
            parallel_chains = 2,
            refresh = 10
   y_pred <- as.vector(fit$summary(variables = c("mu_predicted"))$mean)</pre>
    sig<-fit$summary(variables = c("sigma"))$mean</pre>
   diag_cov <- diag(matrix(fit$summary(variables = c("var_predicted"))$mean,nrow=step,ncol=step))</pre>
   lower_GPR<-c(lower_GPR, y_pred-1.96*sqrt(diag_cov+sig^2))</pre>
   upper_GPR<-c(upper_GPR, y_pred+1.96*sqrt(diag_cov+sig^2))
   y_hat_GPR <- c(y_hat_GPR,y_pred)</pre>
   print(i)
}
## Running MCMC with 2 parallel chains...
##
## Chain 1 Iteration:
                        1 / 500 [ 0%]
                                         (Warmup)
## Chain 1 Iteration: 10 / 500 [ 2%]
                                         (Warmup)
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 1.73223e+35, but A[2,1] = 1.7322
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 5.62648e+37, but A[2,1] = 5.6264
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 1.78837e+11, but A[2,1] = 1.7883
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
```

```
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[82,86] = -5.93634e+06, but A[86,82] = -
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 2 Iteration: 1 / 500 [ 0%]
                                        (Warmup)
## Chain 2 Iteration: 10 / 500 [ 2%]
                                        (Warmup)
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = inf, but A[2,1] = inf (in '/var/
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = inf, but A[2,1] = inf (in '/var/
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like covariance
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 6.13117e+150, but A[2,1] = 6.131
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 3.17351e+40, but A[2,1] = 3.1735
```

Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova

- ## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m ## Chain 2 ## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th ## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 2.84851e+13, but A[2,1] = 2.8485 ## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova ## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m ## Chain 2 ## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th ## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,41] = -1.07897e+07, but A[41,1] = -1.07897e+07## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova ## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m ## Chain 2 ## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th ## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = inf, but A[2,1] = inf (in '/var/ ## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
- ## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m ## Chain 2
- ## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = inf, but A[2,1] = inf (in '/var/ ## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova

Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th

- ## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
- ## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th

Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 1.89677e+09, but A[2,1] = 1.8967

Chain 2

```
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
                                        (Warmup)
## Chain 2 Iteration:
                      20 / 500 [ 4%]
## Chain 1 Iteration:
                       20 / 500 [ 4%]
                                        (Warmup)
                       30 / 500 [ 6%]
                                        (Warmup)
## Chain 2 Iteration:
## Chain 1 Iteration:
                       30 / 500 [ 6%]
                                        (Warmup)
## Chain 2 Iteration: 40 / 500 [ 8%]
                                        (Warmup)
## Chain 1 Iteration: 40 / 500 [ 8%]
                                        (Warmup)
## Chain 2 Iteration: 50 / 500 [ 10%]
                                        (Warmup)
## Chain 1 Iteration: 50 / 500 [ 10%]
                                        (Warmup)
## Chain 2 Iteration: 60 / 500 [ 12%]
                                        (Warmup)
## Chain 2 Iteration: 70 / 500 [ 14%]
                                        (Warmup)
## Chain 1 Iteration: 60 / 500 [ 12%]
                                        (Warmup)
## Chain 2 Iteration: 80 / 500 [ 16%]
                                        (Warmup)
## Chain 1 Iteration: 70 / 500 [ 14%]
                                        (Warmup)
## Chain 2 Iteration: 90 / 500 [ 18%]
                                        (Warmup)
                      80 / 500 [ 16%]
## Chain 1 Iteration:
                                        (Warmup)
## Chain 2 Iteration: 100 / 500 [ 20%]
                                        (Warmup)
## Chain 1 Iteration: 90 / 500 [ 18%]
                                        (Warmup)
## Chain 2 Iteration: 110 / 500 [ 22%]
                                        (Warmup)
## Chain 2 Iteration: 120 / 500 [ 24%]
                                        (Warmup)
## Chain 2 Iteration: 130 / 500 [ 26%]
                                        (Warmup)
## Chain 2 Iteration: 140 / 500 [ 28%]
                                        (Warmup)
## Chain 1 Iteration: 100 / 500 [ 20%]
                                         (Warmup)
## Chain 2 Iteration: 150 / 500 [ 30%]
                                        (Warmup)
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 4.95589e+32, but A[2,1] = 4.9558
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 1 Iteration: 110 / 500 [ 22%]
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 7.96324e+21, but A[2,1] = 7.9632
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
```

Chain 1

```
## Chain 2 Iteration: 160 / 500 [ 32%]
                                         (Warmup)
## Chain 1 Iteration: 120 / 500 [ 24%]
                                         (Warmup)
## Chain 1 Iteration: 130 / 500 [ 26%]
                                         (Warmup)
## Chain 2 Iteration: 170 / 500 [ 34%]
                                         (Warmup)
## Chain 1 Iteration: 140 / 500 [ 28%]
                                         (Warmup)
## Chain 2 Iteration: 180 / 500 [ 36%]
                                         (Warmup)
## Chain 1 Iteration: 150 / 500 [ 30%]
                                         (Warmup)
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[2,67] = -97321.3, but A[67,2] = -97321.
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 2 Iteration: 190 / 500 [ 38%]
                                         (Warmup)
## Chain 1 Iteration: 160 / 500 [ 32%]
                                         (Warmup)
## Chain 2 Iteration: 200 / 500 [ 40%]
                                         (Warmup)
## Chain 2 Iteration: 201 / 500 [ 40%]
                                         (Sampling)
                                         (Warmup)
## Chain 1 Iteration: 170 / 500 [ 34%]
## Chain 2 Iteration: 210 / 500 [ 42%]
                                         (Sampling)
## Chain 2 Iteration: 220 / 500 [ 44%]
                                         (Sampling)
## Chain 1 Iteration: 180 / 500 [ 36%]
                                         (Warmup)
## Chain 2 Iteration: 230 / 500 [ 46%]
                                         (Sampling)
## Chain 1 Iteration: 190 / 500 [ 38%]
                                         (Warmup)
## Chain 2 Iteration: 240 / 500 [ 48%]
                                         (Sampling)
## Chain 1 Iteration: 200 / 500 [ 40%]
                                         (Warmup)
## Chain 1 Iteration: 201 / 500 [ 40%]
                                         (Sampling)
## Chain 1 Iteration: 210 / 500 [ 42%]
                                         (Sampling)
## Chain 2 Iteration: 250 / 500 [ 50%]
                                         (Sampling)
## Chain 2 Iteration: 260 / 500 [ 52%]
                                         (Sampling)
## Chain 2 Iteration: 270 / 500 [ 54%]
                                         (Sampling)
## Chain 1 Iteration: 220 / 500 [ 44%]
                                         (Sampling)
## Chain 1 Iteration: 230 / 500 [ 46%]
                                         (Sampling)
## Chain 2 Iteration: 280 / 500 [ 56%]
                                         (Sampling)
## Chain 1 Iteration: 240 / 500 [ 48%]
                                         (Sampling)
## Chain 2 Iteration: 290 / 500 [ 58%]
                                         (Sampling)
## Chain 1 Iteration: 250 / 500 [ 50%]
                                         (Sampling)
## Chain 2 Iteration: 300 / 500 [ 60%]
                                         (Sampling)
## Chain 2 Iteration: 310 / 500 [ 62%]
                                         (Sampling)
## Chain 1 Iteration: 260 / 500 [ 52%]
                                         (Sampling)
## Chain 1 Iteration: 270 / 500 [ 54%]
                                         (Sampling)
## Chain 2 Iteration: 320 / 500 [ 64%]
                                         (Sampling)
## Chain 2 Iteration: 330 / 500 [ 66%]
                                         (Sampling)
```

(Sampling)

(Sampling)

(Sampling)

(Sampling)

(Sampling)

Chain 1 Iteration: 280 / 500 [56%]

Chain 2 Iteration: 340 / 500 [68%]

Chain 1 Iteration: 290 / 500 [58%]

Chain 1 Iteration: 300 / 500 [60%]

Chain 2 Iteration: 350 / 500 [70%]

```
## Chain 1 Iteration: 310 / 500 [ 62%]
                                         (Sampling)
## Chain 2 Iteration: 360 / 500 [ 72%]
                                         (Sampling)
## Chain 1 Iteration: 320 / 500 [ 64%]
                                         (Sampling)
## Chain 2 Iteration: 370 / 500 [ 74%]
                                         (Sampling)
## Chain 2 Iteration: 380 / 500 [ 76%]
                                         (Sampling)
## Chain 1 Iteration: 330 / 500 [ 66%]
                                         (Sampling)
## Chain 2 Iteration: 390 / 500 [ 78%]
                                         (Sampling)
## Chain 1 Iteration: 340 / 500 [ 68%]
                                         (Sampling)
## Chain 1 Iteration: 350 / 500 [ 70%]
                                         (Sampling)
## Chain 2 Iteration: 400 / 500 [ 80%]
                                         (Sampling)
## Chain 1 Iteration: 360 / 500 [ 72%]
                                         (Sampling)
## Chain 2 Iteration: 410 / 500 [ 82%]
                                         (Sampling)
## Chain 1 Iteration: 370 / 500 [ 74%]
                                         (Sampling)
                                         (Sampling)
## Chain 2 Iteration: 420 / 500 [ 84%]
## Chain 1 Iteration: 380 / 500 [ 76%]
                                         (Sampling)
## Chain 2 Iteration: 430 / 500 [ 86%]
                                         (Sampling)
## Chain 1 Iteration: 390 / 500 [ 78%]
                                         (Sampling)
## Chain 2 Iteration: 440 / 500 [ 88%]
                                         (Sampling)
## Chain 2 Iteration: 450 / 500 [ 90%]
                                         (Sampling)
## Chain 1 Iteration: 400 / 500 [ 80%]
                                         (Sampling)
## Chain 2 Iteration: 460 / 500 [ 92%]
                                         (Sampling)
## Chain 1 Iteration: 410 / 500 [ 82%]
                                         (Sampling)
## Chain 1 Iteration: 420 / 500 [ 84%]
                                         (Sampling)
## Chain 2 Iteration: 470 / 500 [ 94%]
                                         (Sampling)
## Chain 1 Iteration: 430 / 500 [ 86%]
                                         (Sampling)
## Chain 2 Iteration: 480 / 500 [ 96%]
                                         (Sampling)
## Chain 1 Iteration: 440 / 500 [ 88%]
                                         (Sampling)
## Chain 2 Iteration: 490 / 500 [ 98%]
                                         (Sampling)
## Chain 2 Iteration: 500 / 500 [100%]
                                         (Sampling)
## Chain 2 finished in 8.6 seconds.
## Chain 1 Iteration: 450 / 500 [ 90%]
                                         (Sampling)
## Chain 1 Iteration: 460 / 500 [ 92%]
                                         (Sampling)
## Chain 1 Iteration: 470 / 500 [ 94%]
                                         (Sampling)
## Chain 1 Iteration: 480 / 500 [ 96%]
                                         (Sampling)
## Chain 1 Iteration: 490 / 500 [ 98%]
                                         (Sampling)
## Chain 1 Iteration: 500 / 500 [100%]
                                         (Sampling)
## Chain 1 finished in 9.1 seconds.
##
## Both chains finished successfully.
## Mean chain execution time: 8.9 seconds.
## Total execution time: 9.2 seconds.
##
## [1] 1
## Running MCMC with 2 parallel chains...
## Chain 1 Iteration:
                         1 / 500 [ 0%]
                                         (Warmup)
## Chain 1 Iteration: 10 / 500 [
                                   2%]
                                         (Warmup)
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,3] = -4.23895e+33, but A[3,1] = -4.23
```

Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova

```
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 7.26075e+36, but A[2,1] = 7.2607
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,3] = -3.07203e+10, but A[3,1] = -3.07203e+10
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 2 Iteration:
                       1 / 500 [ 0%]
                                        (Warmup)
## Chain 2 Iteration: 10 / 500 [ 2%]
                                        (Warmup)
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = inf, but A[2,1] = inf (in '/var/
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = inf, but A[2,1] = inf (in '/var/
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
```

Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th

```
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 7.01645e+145, but A[2,1] = 7.016
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 1.6904e+39, but A[2,1] = 1.6904e
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,3] = 1.0729e+13, but A[3,1] = 1.0729e
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[2,19] = -4.4983e+06, but A[19,2] = -4.4983e+06
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
```

Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th ## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = inf, but A[2,1] = inf (in '/var/

Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m

Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova

Chain 2

```
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 7.20949e+83, but A[2,1] = 7.2094
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Iteration: 20 / 500 [ 4%]
                                        (Warmup)
                      20 / 500 [ 4%]
                                        (Warmup)
## Chain 1 Iteration:
## Chain 2 Iteration: 30 / 500 [ 6%]
                                        (Warmup)
## Chain 1 Iteration: 30 / 500 [ 6%]
                                        (Warmup)
## Chain 2 Iteration: 40 / 500 [ 8%]
                                        (Warmup)
## Chain 2 Iteration: 50 / 500 [ 10%]
                                        (Warmup)
## Chain 1 Iteration: 40 / 500 [ 8%]
                                        (Warmup)
## Chain 2 Iteration: 60 / 500 [ 12%]
                                        (Warmup)
## Chain 1 Iteration: 50 / 500 [ 10%]
                                        (Warmup)
## Chain 1 Iteration: 60 / 500 [ 12%]
                                        (Warmup)
## Chain 2 Iteration: 70 / 500 [ 14%]
                                        (Warmup)
## Chain 2 Iteration: 80 / 500 [ 16%]
                                        (Warmup)
## Chain 1 Iteration: 70 / 500 [ 14%]
                                        (Warmup)
## Chain 2 Iteration: 90 / 500 [ 18%]
                                        (Warmup)
## Chain 1 Iteration: 80 / 500 [ 16%]
                                        (Warmup)
## Chain 2 Iteration: 100 / 500 [ 20%]
                                        (Warmup)
## Chain 2 Iteration: 110 / 500 [ 22%]
                                        (Warmup)
## Chain 2 Iteration: 120 / 500 [ 24%]
                                        (Warmup)
## Chain 2 Iteration: 130 / 500 [ 26%]
                                        (Warmup)
## Chain 2 Iteration: 140 / 500 [ 28%]
                                        (Warmup)
## Chain 2 Iteration: 150 / 500 [ 30%]
                                         (Warmup)
## Chain 1 Iteration: 90 / 500 [ 18%]
                                        (Warmup)
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 1.38397e+09, but A[2,1] = 1.3839
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Iteration: 160 / 500 [ 32%]
                                        (Warmup)
## Chain 2 Iteration: 170 / 500 [ 34%]
                                        (Warmup)
## Chain 2 Iteration: 180 / 500 [ 36%]
                                         (Warmup)
## Chain 1 Iteration: 100 / 500 [ 20%]
                                        (Warmup)
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
```

Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,3] = -9.24608e+46, but A[3,1] = -9.24

```
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 2 Iteration: 190 / 500 [ 38%]
                                         (Warmup)
## Chain 1 Iteration: 110 / 500 [ 22%]
                                         (Warmup)
## Chain 2 Iteration: 200 / 500 [ 40%]
                                         (Warmup)
## Chain 2 Iteration: 201 / 500 [ 40%]
                                         (Sampling)
## Chain 1 Iteration: 120 / 500 [ 24%]
                                         (Warmup)
## Chain 2 Iteration: 210 / 500 [ 42%]
                                         (Sampling)
## Chain 1 Iteration: 130 / 500 [ 26%]
                                         (Warmup)
## Chain 2 Iteration: 220 / 500 [ 44%]
                                         (Sampling)
## Chain 2 Iteration: 230 / 500 [ 46%]
                                         (Sampling)
## Chain 1 Iteration: 140 / 500 [ 28%]
                                         (Warmup)
## Chain 2 Iteration: 240 / 500 [ 48%]
                                         (Sampling)
## Chain 2 Iteration: 250 / 500 [ 50%]
                                         (Sampling)
## Chain 1 Iteration: 150 / 500 [ 30%]
                                         (Warmup)
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[2,19] = -4.25692e+06, but A[19,2] = -4.
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 2 Iteration: 260 / 500 [ 52%]
                                         (Sampling)
## Chain 2 Iteration: 270 / 500 [ 54%]
                                         (Sampling)
## Chain 1 Iteration: 160 / 500 [ 32%]
                                         (Warmup)
                                         (Sampling)
## Chain 2 Iteration: 280 / 500 [ 56%]
## Chain 1 Iteration: 170 / 500 [ 34%]
                                         (Warmup)
## Chain 2 Iteration: 290 / 500 [ 58%]
                                         (Sampling)
## Chain 2 Iteration: 300 / 500 [ 60%]
                                         (Sampling)
## Chain 1 Iteration: 180 / 500 [ 36%]
                                         (Warmup)
## Chain 2 Iteration: 310 / 500 [ 62%]
                                         (Sampling)
## Chain 2 Iteration: 320 / 500 [ 64%]
                                         (Sampling)
## Chain 1 Iteration: 190 / 500 [ 38%]
                                         (Warmup)
## Chain 2 Iteration: 330 / 500 [ 66%]
                                         (Sampling)
## Chain 1 Iteration: 200 / 500 [ 40%]
                                         (Warmup)
## Chain 1 Iteration: 201 / 500 [ 40%]
                                         (Sampling)
## Chain 2 Iteration: 340 / 500 [ 68%]
                                         (Sampling)
## Chain 2 Iteration: 350 / 500 [ 70%]
                                         (Sampling)
## Chain 1 Iteration: 210 / 500 [ 42%]
                                         (Sampling)
## Chain 2 Iteration: 360 / 500 [ 72%]
                                         (Sampling)
## Chain 1 Iteration: 220 / 500 [ 44%]
                                         (Sampling)
## Chain 1 Iteration: 230 / 500 [ 46%]
                                         (Sampling)
```

(Sampling)

(Sampling)

Chain 2 Iteration: 370 / 500 [74%]

Chain 2 Iteration: 380 / 500 [76%]

```
## Chain 1 Iteration: 240 / 500 [ 48%]
                                          (Sampling)
## Chain 2 Iteration: 390 / 500 [ 78%]
                                          (Sampling)
## Chain 1 Iteration: 250 / 500 [ 50%]
                                          (Sampling)
## Chain 2 Iteration: 400 / 500 [ 80%]
                                          (Sampling)
## Chain 2 Iteration: 410 / 500 [ 82%]
                                          (Sampling)
## Chain 1 Iteration: 260 / 500 [ 52%]
                                          (Sampling)
## Chain 2 Iteration: 420 / 500 [ 84%]
                                          (Sampling)
## Chain 1 Iteration: 270 / 500 [ 54%]
                                          (Sampling)
## Chain 2 Iteration: 430 / 500 [ 86%]
                                          (Sampling)
## Chain 1 Iteration: 280 / 500 [ 56%]
                                          (Sampling)
## Chain 2 Iteration: 440 / 500 [ 88%]
                                          (Sampling)
## Chain 1 Iteration: 290 / 500 [ 58%]
                                          (Sampling)
## Chain 2 Iteration: 450 / 500 [ 90%]
                                          (Sampling)
## Chain 2 Iteration: 460 / 500 [ 92%]
                                          (Sampling)
## Chain 2 Iteration: 470 / 500 [ 94%]
                                          (Sampling)
## Chain 1 Iteration: 300 / 500 [ 60%]
                                          (Sampling)
## Chain 2 Iteration: 480 / 500 [ 96%]
                                          (Sampling)
## Chain 1 Iteration: 310 / 500 [ 62%]
                                          (Sampling)
## Chain 2 Iteration: 490 / 500 [ 98%]
                                          (Sampling)
## Chain 1 Iteration: 320 / 500 [ 64%]
                                          (Sampling)
## Chain 2 Iteration: 500 / 500 [100%]
                                          (Sampling)
## Chain 2 finished in 8.6 seconds.
## Chain 1 Iteration: 330 / 500 [ 66%]
                                          (Sampling)
## Chain 1 Iteration: 340 / 500 [ 68%]
                                         (Sampling)
## Chain 1 Iteration: 350 / 500 [ 70%]
                                          (Sampling)
## Chain 1 Iteration: 360 / 500 [ 72%]
                                          (Sampling)
## Chain 1 Iteration: 370 / 500 [ 74%]
                                          (Sampling)
## Chain 1 Iteration: 380 / 500 [ 76%]
                                          (Sampling)
## Chain 1 Iteration: 390 / 500 [ 78%]
                                          (Sampling)
## Chain 1 Iteration: 400 / 500 [ 80%]
                                          (Sampling)
## Chain 1 Iteration: 410 / 500 [ 82%]
                                          (Sampling)
## Chain 1 Iteration: 420 / 500 [ 84%]
                                          (Sampling)
## Chain 1 Iteration: 430 / 500 [ 86%]
                                          (Sampling)
## Chain 1 Iteration: 440 / 500 [ 88%]
                                          (Sampling)
## Chain 1 Iteration: 450 / 500 [ 90%]
                                          (Sampling)
                                          (Sampling)
## Chain 1 Iteration: 460 / 500 [ 92%]
## Chain 1 Iteration: 470 / 500 [ 94%]
                                          (Sampling)
## Chain 1 Iteration: 480 / 500 [ 96%]
                                          (Sampling)
## Chain 1 Iteration: 490 / 500 [ 98%]
                                          (Sampling)
## Chain 1 Iteration: 500 / 500 [100%]
                                          (Sampling)
## Chain 1 finished in 10.7 seconds.
##
## Both chains finished successfully.
## Mean chain execution time: 9.6 seconds.
## Total execution time: 10.8 seconds.
##
## [1] 2
## Running MCMC with 2 parallel chains...
## Chain 1 Iteration:
                         1 / 500 [
                                          (Warmup)
## Chain 1 Iteration:
                       10 / 500 [
                                    2%]
                                          (Warmup)
```

Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th

```
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 3.48441e+36, but A[2,1] = 3.4844
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 4.12994e+38, but A[2,1] = 4.1299
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,3] = -7.20176e+10, but A[3,1] = -7.20
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[58,74] = 6.40993e+06, but A[74,58] = 6.40993e+06
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 2 Iteration:
                        1 / 500 [ 0%]
                                        (Warmup)
## Chain 2 Iteration: 10 / 500 [ 2%]
                                        (Warmup)
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
```

Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = inf, but A[2,1] = inf (in '/var/:
Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova:
Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or may

- ## Chain 2 ## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th ## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = inf, but A[2,1] = inf (in '/var/ ## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova ## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m ## Chain 2 ## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th ## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 1.16448e+156, but A[2,1] = 1.164 ## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova ## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m ## Chain 2 ## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th ## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 1.34414e+42, but A[2,1] = 1.3441 ## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova ## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m ## Chain 2 ## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th ## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 1.46646e+14, but A[2,1] = 1.4664 ## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova ## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m ## Chain 2
- ## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of the ## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,4] = -1.84947e+07, but A[4,1] = -1.84947e+07

Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova

```
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[64,97] = 4.96968e+06, but A[97,64] = 4.
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = inf, but A[2,1] = inf (in '/var/
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 1.32163e+25, but A[2,1] = 1.3216
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 1 Iteration: 20 / 500 [ 4%]
                                        (Warmup)
## Chain 1 Iteration: 30 / 500 [ 6%]
                                        (Warmup)
## Chain 2 Iteration: 20 / 500 [ 4%]
                                        (Warmup)
## Chain 1 Iteration: 40 / 500 [ 8%]
                                        (Warmup)
## Chain 2 Iteration: 30 / 500 [ 6%]
                                        (Warmup)
## Chain 1 Iteration: 50 / 500 [ 10%]
                                        (Warmup)
## Chain 2 Iteration: 40 / 500 [ 8%]
                                        (Warmup)
## Chain 1 Iteration:
                      60 / 500 [ 12%]
                                        (Warmup)
## Chain 1 Iteration: 70 / 500 [ 14%]
                                        (Warmup)
## Chain 2 Iteration: 50 / 500 [ 10%]
                                        (Warmup)
## Chain 1 Iteration: 80 / 500 [ 16%]
                                        (Warmup)
## Chain 2 Iteration: 60 / 500 [ 12%]
                                        (Warmup)
```

(Warmup)

(Warmup)

(Warmup)

Chain 1 Iteration: 90 / 500 [18%]

Chain 2 Iteration: 70 / 500 [14%]

Chain 1 Iteration: 100 / 500 [20%]

```
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 1.49581e+08, but A[2,1] = 1.4958
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 2 Iteration: 80 / 500 [ 16%]
                                         (Warmup)
## Chain 1 Iteration: 110 / 500 [ 22%]
                                         (Warmup)
## Chain 1 Iteration: 120 / 500 [ 24%]
                                         (Warmup)
## Chain 1 Iteration: 130 / 500 [ 26%]
                                         (Warmup)
## Chain 2 Iteration: 90 / 500 [ 18%]
                                         (Warmup)
## Chain 1 Iteration: 140 / 500 [ 28%]
                                         (Warmup)
## Chain 1 Iteration: 150 / 500 [ 30%]
                                         (Warmup)
## Chain 1 Iteration: 160 / 500 [ 32%]
                                         (Warmup)
## Chain 2 Iteration: 100 / 500 [ 20%]
                                         (Warmup)
## Chain 1 Iteration: 170 / 500 [ 34%]
                                         (Warmup)
## Chain 1 Iteration: 180 / 500 [ 36%]
                                         (Warmup)
## Chain 2 Iteration: 110 / 500 [ 22%]
                                         (Warmup)
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,4] = -1.92002e+10, but A[4,1] = -1.92002e+10
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 1 Iteration: 190 / 500 [ 38%]
                                         (Warmup)
## Chain 1 Iteration: 200 / 500 [ 40%]
                                         (Warmup)
## Chain 1 Iteration: 201 / 500 [ 40%]
                                         (Sampling)
## Chain 1 Iteration: 210 / 500 [ 42%]
                                         (Sampling)
## Chain 1 Iteration: 220 / 500 [ 44%]
                                         (Sampling)
## Chain 2 Iteration: 120 / 500 [ 24%]
                                         (Warmup)
## Chain 1 Iteration: 230 / 500 [ 46%]
                                         (Sampling)
## Chain 1 Iteration: 240 / 500 [ 48%]
                                         (Sampling)
## Chain 2 Iteration: 130 / 500 [ 26%]
                                         (Warmup)
## Chain 1 Iteration: 250 / 500 [ 50%]
                                         (Sampling)
## Chain 1 Iteration: 260 / 500 [ 52%]
                                         (Sampling)
## Chain 2 Iteration: 140 / 500 [ 28%]
                                         (Warmup)
                                         (Sampling)
## Chain 1 Iteration: 270 / 500 [ 54%]
## Chain 2 Iteration: 150 / 500 [ 30%]
                                         (Warmup)
## Chain 1 Iteration: 280 / 500 [ 56%]
                                         (Sampling)
## Chain 2 Iteration: 160 / 500 [ 32%]
                                         (Warmup)
```

(Sampling)

(Warmup)

Chain 1 Iteration: 290 / 500 [58%]

Chain 2 Iteration: 170 / 500 [34%]

```
## Chain 1 Iteration: 300 / 500 [ 60%]
                                          (Sampling)
## Chain 1 Iteration: 310 / 500 [ 62%]
                                          (Sampling)
## Chain 2 Iteration: 180 / 500 [ 36%]
                                          (Warmup)
## Chain 1 Iteration: 320 / 500 [ 64%]
                                         (Sampling)
## Chain 2 Iteration: 190 / 500 [ 38%]
                                          (Warmup)
## Chain 1 Iteration: 330 / 500 [ 66%]
                                         (Sampling)
## Chain 2 Iteration: 200 / 500 [ 40%]
                                          (Warmup)
                                         (Sampling)
## Chain 2 Iteration: 201 / 500 [ 40%]
## Chain 1 Iteration: 340 / 500 [ 68%]
                                          (Sampling)
## Chain 2 Iteration: 210 / 500 [ 42%]
                                          (Sampling)
## Chain 1 Iteration: 350 / 500 [ 70%]
                                          (Sampling)
## Chain 1 Iteration: 360 / 500 [ 72%]
                                          (Sampling)
## Chain 2 Iteration: 220 / 500 [ 44%]
                                          (Sampling)
## Chain 1 Iteration: 370 / 500 [ 74%]
                                          (Sampling)
## Chain 2 Iteration: 230 / 500 [ 46%]
                                          (Sampling)
## Chain 1 Iteration: 380 / 500 [ 76%]
                                          (Sampling)
## Chain 2 Iteration: 240 / 500 [ 48%]
                                          (Sampling)
## Chain 1 Iteration: 390 / 500 [ 78%]
                                          (Sampling)
## Chain 2 Iteration: 250 / 500 [ 50%]
                                          (Sampling)
## Chain 1 Iteration: 400 / 500 [ 80%]
                                          (Sampling)
## Chain 2 Iteration: 260 / 500 [ 52%]
                                          (Sampling)
## Chain 2 Iteration: 270 / 500 [ 54%]
                                          (Sampling)
## Chain 1 Iteration: 410 / 500 [ 82%]
                                          (Sampling)
## Chain 1 Iteration: 420 / 500 [ 84%]
                                          (Sampling)
## Chain 2 Iteration: 280 / 500 [ 56%]
                                          (Sampling)
## Chain 1 Iteration: 430 / 500 [ 86%]
                                          (Sampling)
## Chain 2 Iteration: 290 / 500 [ 58%]
                                          (Sampling)
## Chain 1 Iteration: 440 / 500 [ 88%]
                                          (Sampling)
## Chain 2 Iteration: 300 / 500 [ 60%]
                                          (Sampling)
## Chain 1 Iteration: 450 / 500 [ 90%]
                                          (Sampling)
## Chain 2 Iteration: 310 / 500 [ 62%]
                                          (Sampling)
## Chain 1 Iteration: 460 / 500 [ 92%]
                                          (Sampling)
## Chain 2 Iteration: 320 / 500 [ 64%]
                                          (Sampling)
## Chain 1 Iteration: 470 / 500 [ 94%]
                                          (Sampling)
## Chain 1 Iteration: 480 / 500 [ 96%]
                                          (Sampling)
## Chain 2 Iteration: 330 / 500 [ 66%]
                                          (Sampling)
## Chain 1 Iteration: 490 / 500 [ 98%]
                                          (Sampling)
## Chain 2 Iteration: 340 / 500 [ 68%]
                                          (Sampling)
## Chain 1 Iteration: 500 / 500 [100%]
                                          (Sampling)
## Chain 2 Iteration: 350 / 500 [ 70%]
                                          (Sampling)
## Chain 1 finished in 8.8 seconds.
## Chain 2 Iteration: 360 / 500 [ 72%]
                                          (Sampling)
## Chain 2 Iteration: 370 / 500 [ 74%]
                                          (Sampling)
## Chain 2 Iteration: 380 / 500 [ 76%]
                                          (Sampling)
## Chain 2 Iteration: 390 / 500 [ 78%]
                                          (Sampling)
## Chain 2 Iteration: 400 / 500 [ 80%]
                                          (Sampling)
## Chain 2 Iteration: 410 / 500 [ 82%]
                                          (Sampling)
## Chain 2 Iteration: 420 / 500 [ 84%]
                                          (Sampling)
## Chain 2 Iteration: 430 / 500 [ 86%]
                                          (Sampling)
## Chain 2 Iteration: 440 / 500
                                [ 88%]
                                          (Sampling)
                                          (Sampling)
## Chain 2 Iteration: 450 / 500 [ 90%]
                                          (Sampling)
## Chain 2 Iteration: 460 / 500 [ 92%]
## Chain 2 Iteration: 470 / 500 [ 94%]
                                          (Sampling)
## Chain 2 Iteration: 480 / 500 [ 96%]
                                         (Sampling)
```

```
## Chain 2 Iteration: 490 / 500 [ 98%]
                                        (Sampling)
## Chain 2 Iteration: 500 / 500 [100%]
                                       (Sampling)
## Chain 2 finished in 10.2 seconds.
##
## Both chains finished successfully.
## Mean chain execution time: 9.5 seconds.
## Total execution time: 10.2 seconds.
## [1] 3
## Running MCMC with 2 parallel chains...
## Chain 1 Iteration:
                        1 / 500 [ 0%]
                                        (Warmup)
## Chain 1 Iteration: 10 / 500 [ 2%]
                                        (Warmup)
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,3] = -6.63521e+44, but A[3,1] = -6.63
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,2] = -1.51479e+47, but A[2,1] = -1.51479e+47
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,2] = -1.60488e+13, but A[2,1] = -1.60488e+13
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,5] = 1.67898e+07, but A[5,1] = 1.6789
```

Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova

- ## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m ## Chain 1 ## Chain 2 Iteration: 1 / 500 [0%] (Warmup) ## Chain 2 Iteration: 10 / 500 [2%] (Warmup) ## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th ## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = -inf, but A[2,1] = -inf (in '/va ## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova ## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m ## Chain 2 ## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th ## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = -inf, but A[2,1] = -inf (in '/va ## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova ## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m ## Chain 2 ## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th ## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = -6.21968e+153, but A[2,1] = -6.2 ## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova ## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m ## Chain 2 ## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th ## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = -1.75349e+41, but A[2,1] = -1.75## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova ## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
- ## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th

```
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = -4.11051e+13, but A[2,1] = -4.11
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,5] = 3.31713e+07, but A[5,1] = 3.3171
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[5,110] = 6.47991e+06, but A[110,5] = 6.
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = nan, but A[2,1] = nan (in '/var/
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
```

Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th ## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = -1.17852e+79, but A[2,1] = -1.17

Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
Chain 2

Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova

```
## Chain 1 Iteration: 20 / 500 [ 4%]
                                         (Warmup)
## Chain 2 Iteration: 20 / 500 [ 4%]
                                         (Warmup)
## Chain 1 Iteration: 30 / 500 [ 6%]
                                         (Warmup)
## Chain 2 Iteration: 30 / 500 [ 6%]
                                         (Warmup)
## Chain 2 Iteration: 40 / 500 [ 8%]
                                         (Warmup)
## Chain 1 Iteration: 40 / 500 [ 8%]
                                         (Warmup)
## Chain 1 Iteration: 50 / 500 [ 10%]
                                         (Warmup)
## Chain 2 Iteration: 50 / 500 [ 10%]
                                         (Warmup)
## Chain 1 Iteration: 60 / 500 [ 12%]
                                         (Warmup)
## Chain 2 Iteration: 60 / 500 [ 12%]
                                         (Warmup)
## Chain 1 Iteration: 70 / 500 [ 14%]
                                         (Warmup)
                       70 / 500 [ 14%]
## Chain 2 Iteration:
                                         (Warmup)
## Chain 1 Iteration: 80 / 500 [ 16%]
                                         (Warmup)
## Chain 2 Iteration:
                                         (Warmup)
                       80 / 500 [ 16%]
                       90 / 500 [ 18%]
## Chain 1 Iteration:
                                         (Warmup)
## Chain 2 Iteration: 90 / 500 [ 18%]
                                         (Warmup)
## Chain 1 Iteration: 100 / 500 [ 20%]
                                         (Warmup)
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 8.76106e+12, but A[2,1] = 8.7610
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 2 Iteration: 100 / 500 [ 20%]
                                         (Warmup)
## Chain 1 Iteration: 110 / 500 [ 22%]
                                         (Warmup)
## Chain 1 Iteration: 120 / 500 [ 24%]
                                         (Warmup)
## Chain 2 Iteration: 110 / 500 [ 22%]
                                         (Warmup)
## Chain 1 Iteration: 130 / 500 [ 26%]
                                         (Warmup)
## Chain 2 Iteration: 120 / 500 [ 24%]
                                         (Warmup)
## Chain 1 Iteration: 140 / 500 [ 28%]
                                         (Warmup)
## Chain 2 Iteration: 130 / 500 [ 26%]
                                         (Warmup)
## Chain 2 Iteration: 140 / 500 [ 28%]
                                         (Warmup)
## Chain 1 Iteration: 150 / 500 [ 30%]
                                         (Warmup)
## Chain 1 Iteration: 160 / 500 [ 32%]
                                         (Warmup)
## Chain 2 Iteration: 150 / 500 [ 30%]
                                         (Warmup)
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[9,46] = -6.48615e+06, but A[46,9] = -6.48615e+06
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
```

```
## Chain 2 Iteration: 160 / 500 [ 32%]
                                          (Warmup)
## Chain 1 Iteration: 170 / 500 [ 34%]
                                         (Warmup)
## Chain 1 Iteration: 180 / 500 [ 36%]
                                          (Warmup)
## Chain 2 Iteration: 170 / 500 [ 34%]
                                         (Warmup)
## Chain 1 Iteration: 190 / 500 [ 38%]
                                          (Warmup)
                                         (Warmup)
## Chain 2 Iteration: 180 / 500 [ 36%]
                                          (Warmup)
  Chain 1 Iteration: 200 / 500 [ 40%]
## Chain 1 Iteration: 201 / 500 [ 40%]
                                         (Sampling)
## Chain 2 Iteration: 190 / 500 [ 38%]
                                          (Warmup)
## Chain 1 Iteration: 210 / 500 [ 42%]
                                          (Sampling)
## Chain 2 Iteration: 200 / 500 [ 40%]
                                          (Warmup)
## Chain 2 Iteration: 201 / 500 [ 40%]
                                          (Sampling)
## Chain 1 Iteration: 220 / 500 [ 44%]
                                          (Sampling)
## Chain 2 Iteration: 210 / 500 [ 42%]
                                          (Sampling)
## Chain 1 Iteration: 230 / 500 [ 46%]
                                          (Sampling)
## Chain 2 Iteration: 220 / 500 [ 44%]
                                          (Sampling)
## Chain 1 Iteration: 240 / 500 [ 48%]
                                          (Sampling)
## Chain 2 Iteration: 230 / 500 [ 46%]
                                          (Sampling)
## Chain 1 Iteration: 250 / 500 [ 50%]
                                          (Sampling)
## Chain 1 Iteration: 260 / 500 [ 52%]
                                          (Sampling)
## Chain 2 Iteration: 240 / 500 [ 48%]
                                          (Sampling)
## Chain 1 Iteration: 270 / 500 [ 54%]
                                          (Sampling)
## Chain 2 Iteration: 250 / 500 [ 50%]
                                          (Sampling)
## Chain 2 Iteration: 260 / 500 [ 52%]
                                          (Sampling)
## Chain 1 Iteration: 280 / 500 [ 56%]
                                          (Sampling)
## Chain 2 Iteration: 270 / 500 [ 54%]
                                          (Sampling)
## Chain 1 Iteration: 290 / 500 [ 58%]
                                          (Sampling)
## Chain 1 Iteration: 300 / 500 [ 60%]
                                          (Sampling)
## Chain 2 Iteration: 280 / 500 [ 56%]
                                          (Sampling)
## Chain 1 Iteration: 310 / 500 [ 62%]
                                          (Sampling)
## Chain 2 Iteration: 290 / 500 [ 58%]
                                          (Sampling)
## Chain 1 Iteration: 320 / 500 [ 64%]
                                          (Sampling)
## Chain 1 Iteration: 330 / 500 [ 66%]
                                          (Sampling)
## Chain 2 Iteration: 300 / 500 [ 60%]
                                          (Sampling)
## Chain 1 Iteration: 340 / 500 [ 68%]
                                          (Sampling)
## Chain 2 Iteration: 310 / 500 [ 62%]
                                          (Sampling)
## Chain 1 Iteration: 350 / 500 [ 70%]
                                          (Sampling)
## Chain 2 Iteration: 320 / 500 [ 64%]
                                          (Sampling)
## Chain 1 Iteration: 360 / 500 [ 72%]
                                          (Sampling)
## Chain 2 Iteration: 330 / 500 [ 66%]
                                          (Sampling)
  Chain 1 Iteration: 370 / 500 [ 74%]
                                          (Sampling)
## Chain 2 Iteration: 340 / 500 [ 68%]
                                          (Sampling)
## Chain 1 Iteration: 380 / 500 [ 76%]
                                          (Sampling)
## Chain 2 Iteration: 350 / 500 [ 70%]
                                          (Sampling)
## Chain 1 Iteration: 390 / 500 [ 78%]
                                          (Sampling)
## Chain 1 Iteration: 400 / 500 [ 80%]
                                          (Sampling)
## Chain 2 Iteration: 360 / 500 [ 72%]
                                          (Sampling)
## Chain 1 Iteration: 410 / 500 [ 82%]
                                          (Sampling)
## Chain 2 Iteration: 370 / 500 [ 74%]
                                          (Sampling)
## Chain 1 Iteration: 420 / 500
                                [ 84%]
                                          (Sampling)
## Chain 2 Iteration: 380 / 500 [ 76%]
                                          (Sampling)
## Chain 1 Iteration: 430 / 500 [ 86%]
                                          (Sampling)
## Chain 2 Iteration: 390 / 500 [ 78%]
                                          (Sampling)
## Chain 1 Iteration: 440 / 500 [ 88%]
                                         (Sampling)
```

```
## Chain 2 Iteration: 400 / 500 [ 80%]
                                         (Sampling)
## Chain 1 Iteration: 450 / 500 [ 90%]
                                        (Sampling)
## Chain 1 Iteration: 460 / 500 [ 92%]
                                         (Sampling)
## Chain 2 Iteration: 410 / 500 [ 82%]
                                         (Sampling)
## Chain 1 Iteration: 470 / 500 [ 94%]
                                         (Sampling)
## Chain 2 Iteration: 420 / 500 [ 84%]
                                         (Sampling)
## Chain 1 Iteration: 480 / 500 [ 96%]
                                         (Sampling)
## Chain 2 Iteration: 430 / 500 [ 86%]
                                         (Sampling)
## Chain 1 Iteration: 490 / 500 [ 98%]
                                         (Sampling)
## Chain 2 Iteration: 440 / 500 [ 88%]
                                         (Sampling)
## Chain 1 Iteration: 500 / 500 [100%]
                                         (Sampling)
## Chain 2 Iteration: 450 / 500 [ 90%]
                                         (Sampling)
## Chain 1 finished in 9.1 seconds.
## Chain 2 Iteration: 460 / 500 [ 92%]
                                         (Sampling)
## Chain 2 Iteration: 470 / 500 [ 94%]
                                         (Sampling)
## Chain 2 Iteration: 480 / 500 [ 96%]
                                         (Sampling)
## Chain 2 Iteration: 490 / 500 [ 98%]
                                         (Sampling)
## Chain 2 Iteration: 500 / 500 [100%]
                                         (Sampling)
## Chain 2 finished in 9.6 seconds.
## Both chains finished successfully.
## Mean chain execution time: 9.4 seconds.
## Total execution time: 9.7 seconds.
## [1] 4
## Running MCMC with 2 parallel chains...
## Chain 1 Iteration:
                        1 / 500 [ 0%]
                                         (Warmup)
## Chain 1 Iteration: 10 / 500 [
                                   2%]
                                         (Warmup)
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,2] = -1.98599e+48, but A[2,1] = -1.98
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,2] = -4.76292e+50, but A[2,1] = -4.76
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
```

```
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,2] = -1.14133e+15, but A[2,1] = -1.14
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,3] = -7.23754e+06, but A[3,1] = -7.23
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: Matrix m is not positive definite (in '/var/folders/v7/0072jt
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 2 Iteration:
                       1 / 500 [ 0%]
                                        (Warmup)
## Chain 2 Iteration: 10 / 500 [ 2%]
                                       (Warmup)
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = -inf, but A[2,1] = -inf (in '/va
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
```

Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = -inf, but A[2,1] = -inf (in '/va

Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova

Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m

- ## Chain 2 ## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th ## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = -1.63006e+173, but A[2,1] = -1.6## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova ## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m ## Chain 2 ## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th ## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = -3.61757e+46, but A[2,1] = -3.61## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova ## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m ## Chain 2 ## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th ## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = -2.68179e+15, but A[2,1] = -2.68## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova ## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
- ## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th

 ## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = -2.55499e+07, but A[2,1] = -2.55

 ## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova

 ## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m

Chain 2

Chain 2

Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th ## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[34,40] = 7.23672e+06, but A[40,34] = 7.

```
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = -inf, but A[2,1] = -inf (in '/va
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,3] = -6.91829e+32, but A[3,1] = -6.91
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 1 Iteration: 20 / 500 [ 4%]
                                        (Warmup)
## Chain 2 Iteration: 20 / 500 [ 4%]
                                        (Warmup)
## Chain 1 Iteration: 30 / 500 [ 6%]
                                        (Warmup)
## Chain 2 Iteration: 30 / 500 [ 6%]
                                        (Warmup)
## Chain 2 Iteration: 40 / 500 [ 8%]
                                        (Warmup)
## Chain 1 Iteration: 40 / 500 [ 8%]
                                        (Warmup)
## Chain 2 Iteration: 50 / 500 [ 10%]
                                        (Warmup)
## Chain 1 Iteration: 50 / 500 [ 10%]
                                        (Warmup)
                                        (Warmup)
## Chain 2 Iteration: 60 / 500 [ 12%]
## Chain 1 Iteration: 60 / 500 [ 12%]
                                        (Warmup)
## Chain 2 Iteration: 70 / 500 [ 14%]
                                        (Warmup)
## Chain 2 Iteration: 80 / 500 [ 16%]
                                        (Warmup)
## Chain 1 Iteration: 70 / 500 [ 14%]
                                        (Warmup)
## Chain 2 Iteration: 90 / 500 [ 18%]
                                        (Warmup)
## Chain 1 Iteration: 80 / 500 [ 16%]
                                        (Warmup)
## Chain 2 Iteration: 100 / 500 [ 20%]
                                        (Warmup)
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = -5.10504e+13, but A[2,1] = -5.10
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
```

```
## Chain 2 Iteration: 110 / 500 [ 22%]
                                         (Warmup)
## Chain 1 Iteration: 90 / 500 [ 18%]
                                         (Warmup)
## Chain 2 Iteration: 120 / 500 [ 24%]
                                         (Warmup)
## Chain 2 Iteration: 130 / 500 [ 26%]
                                         (Warmup)
## Chain 2 Iteration: 140 / 500 [ 28%]
                                         (Warmup)
## Chain 1 Iteration: 100 / 500 [ 20%]
                                         (Warmup)
## Chain 2 Iteration: 150 / 500 [ 30%]
                                         (Warmup)
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = -2.81213e+10, but A[2,1] = -2.81
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Iteration: 160 / 500 [ 32%]
                                         (Warmup)
## Chain 1 Iteration: 110 / 500 [ 22%]
                                         (Warmup)
## Chain 1 Iteration: 120 / 500 [ 24%]
                                         (Warmup)
                                         (Warmup)
## Chain 1 Iteration: 130 / 500 [ 26%]
## Chain 2 Iteration: 170 / 500 [ 34%]
                                         (Warmup)
## Chain 1 Iteration: 140 / 500 [ 28%]
                                         (Warmup)
## Chain 2 Iteration: 180 / 500 [ 36%]
                                         (Warmup)
## Chain 1 Iteration: 150 / 500 [ 30%]
                                         (Warmup)
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,2] = -8.37264e+09, but A[2,1] = -8.37264e+09
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 2 Iteration: 190 / 500 [ 38%]
                                         (Warmup)
## Chain 1 Iteration: 160 / 500 [ 32%]
                                         (Warmup)
## Chain 2 Iteration: 200 / 500 [ 40%]
                                         (Warmup)
## Chain 2 Iteration: 201 / 500 [ 40%]
                                         (Sampling)
## Chain 1 Iteration: 170 / 500 [ 34%]
                                         (Warmup)
## Chain 2 Iteration: 210 / 500 [ 42%]
                                         (Sampling)
## Chain 1 Iteration: 180 / 500 [ 36%]
                                         (Warmup)
## Chain 1 Iteration: 190 / 500 [ 38%]
                                         (Warmup)
## Chain 2 Iteration: 220 / 500 [ 44%]
                                         (Sampling)
## Chain 1 Iteration: 200 / 500 [ 40%]
                                         (Warmup)
## Chain 1 Iteration: 201 / 500 [ 40%]
                                         (Sampling)
## Chain 2 Iteration: 230 / 500 [ 46%]
                                         (Sampling)
## Chain 1 Iteration: 210 / 500 [ 42%]
                                         (Sampling)
```

(Sampling)

Chain 2 Iteration: 240 / 500 [48%]

```
## Chain 1 Iteration: 220 / 500 [ 44%]
                                          (Sampling)
## Chain 1 Iteration: 230 / 500 [ 46%]
                                          (Sampling)
                                          (Sampling)
## Chain 2 Iteration: 250 / 500 [ 50%]
## Chain 1 Iteration: 240 / 500 [ 48%]
                                          (Sampling)
## Chain 2 Iteration: 260 / 500 [ 52%]
                                          (Sampling)
## Chain 2 Iteration: 270 / 500 [ 54%]
                                          (Sampling)
## Chain 1 Iteration: 250 / 500 [ 50%]
                                          (Sampling)
## Chain 1 Iteration: 260 / 500 [ 52%]
                                          (Sampling)
## Chain 1 Iteration: 270 / 500 [ 54%]
                                          (Sampling)
## Chain 2 Iteration: 280 / 500 [ 56%]
                                          (Sampling)
## Chain 1 Iteration: 280 / 500 [ 56%]
                                          (Sampling)
## Chain 2 Iteration: 290 / 500 [ 58%]
                                          (Sampling)
## Chain 1 Iteration: 290 / 500 [ 58%]
                                          (Sampling)
## Chain 1 Iteration: 300 / 500 [ 60%]
                                          (Sampling)
## Chain 2 Iteration: 300 / 500 [ 60%]
                                          (Sampling)
## Chain 1 Iteration: 310 / 500 [ 62%]
                                          (Sampling)
## Chain 2 Iteration: 310 / 500 [ 62%]
                                          (Sampling)
## Chain 1 Iteration: 320 / 500 [ 64%]
                                          (Sampling)
## Chain 2 Iteration: 320 / 500 [ 64%]
                                          (Sampling)
## Chain 1 Iteration: 330 / 500 [ 66%]
                                          (Sampling)
## Chain 1 Iteration: 340 / 500 [ 68%]
                                          (Sampling)
## Chain 2 Iteration: 330 / 500 [ 66%]
                                          (Sampling)
## Chain 1 Iteration: 350 / 500 [ 70%]
                                          (Sampling)
## Chain 2 Iteration: 340 / 500 [ 68%]
                                          (Sampling)
## Chain 2 Iteration: 350 / 500 [ 70%]
                                          (Sampling)
## Chain 1 Iteration: 360 / 500 [ 72%]
                                          (Sampling)
## Chain 1 Iteration: 370 / 500 [ 74%]
                                          (Sampling)
## Chain 1 Iteration: 380 / 500 [ 76%]
                                          (Sampling)
## Chain 2 Iteration: 360 / 500 [ 72%]
                                          (Sampling)
## Chain 2 Iteration: 370 / 500 [ 74%]
                                          (Sampling)
## Chain 1 Iteration: 390 / 500 [ 78%]
                                          (Sampling)
## Chain 2 Iteration: 380 / 500 [ 76%]
                                          (Sampling)
## Chain 1 Iteration: 400 / 500 [ 80%]
                                          (Sampling)
## Chain 1 Iteration: 410 / 500 [ 82%]
                                          (Sampling)
## Chain 2 Iteration: 390 / 500
                                [ 78%]
                                          (Sampling)
## Chain 1 Iteration: 420 / 500 [ 84%]
                                          (Sampling)
## Chain 2 Iteration: 400 / 500 [ 80%]
                                          (Sampling)
## Chain 1 Iteration: 430 / 500 [ 86%]
                                          (Sampling)
## Chain 2 Iteration: 410 / 500 [ 82%]
                                          (Sampling)
## Chain 1 Iteration: 440 / 500 [ 88%]
                                          (Sampling)
## Chain 1 Iteration: 450 / 500 [ 90%]
                                          (Sampling)
## Chain 2 Iteration: 420 / 500 [ 84%]
                                          (Sampling)
## Chain 1 Iteration: 460 / 500 [ 92%]
                                          (Sampling)
## Chain 2 Iteration: 430 / 500 [ 86%]
                                          (Sampling)
## Chain 2 Iteration: 440 / 500 [ 88%]
                                          (Sampling)
## Chain 1 Iteration: 470 / 500 [ 94%]
                                          (Sampling)
## Chain 1 Iteration: 480 / 500 [ 96%]
                                          (Sampling)
## Chain 2 Iteration: 450 / 500 [ 90%]
                                          (Sampling)
## Chain 1 Iteration: 490 / 500 [ 98%]
                                          (Sampling)
## Chain 2 Iteration: 460 / 500 [ 92%]
                                          (Sampling)
## Chain 1 Iteration: 500 / 500 [100%]
                                          (Sampling)
## Chain 1 finished in 9.0 seconds.
## Chain 2 Iteration: 470 / 500 [ 94%]
                                          (Sampling)
## Chain 2 Iteration: 480 / 500 [ 96%]
                                         (Sampling)
```

```
## Chain 2 Iteration: 490 / 500 [ 98%]
                                        (Sampling)
## Chain 2 Iteration: 500 / 500 [100%]
                                       (Sampling)
## Chain 2 finished in 9.4 seconds.
##
## Both chains finished successfully.
## Mean chain execution time: 9.2 seconds.
## Total execution time: 9.5 seconds.
## [1] 5
## Running MCMC with 2 parallel chains...
## Chain 1 Iteration:
                        1 / 500 [ 0%]
                                        (Warmup)
## Chain 1 Iteration: 10 / 500 [ 2%]
                                        (Warmup)
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,2] = -1.97021e+46, but A[2,1] = -1.97021e+46
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 1.30678e+48, but A[2,1] = 1.3067
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,2] = -1.93153e+14, but A[2,1] = -1.93164
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,22] = 185347, but A[22,1] = 185347 (i.
```

Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova

- ## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m ## Chain 1 ## Chain 2 Iteration: 1 / 500 [0%] (Warmup) ## Chain 2 Iteration: 10 / 500 [2%] (Warmup) ## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th ## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = inf, but A[2,1] = inf (in '/var/ ## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova ## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m ## Chain 2 ## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th ## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = inf, but A[2,1] = inf (in '/var/ ## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova ## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m ## Chain 2 ## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th ## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 7.25323e+169, but A[2,1] = 7.253 ## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova ## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m ## Chain 2 ## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th ## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 1.6896e+45, but A[2,1] = 1.6896e ## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova ## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m ## Chain 2
- ## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th

```
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 3.30122e+14, but A[2,1] = 3.3012
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,9] = 6.94525e+06, but A[9,1] = 6.9452
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[3,47] = -4.36536e+06, but A[47,3] = -4.
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = nan, but A[2,1] = nan (in '/var/
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
```

Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 1.26991e+61, but A[2,1] = 1.2699

Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova

Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m

Chain 2

```
## Chain 2 Iteration: 20 / 500 [ 4%]
                                         (Warmup)
## Chain 1 Iteration: 30 / 500 [ 6%]
                                         (Warmup)
## Chain 2 Iteration: 30 / 500 [ 6%]
                                         (Warmup)
## Chain 1 Iteration: 40 / 500 [ 8%]
                                         (Warmup)
## Chain 2 Iteration: 40 / 500 [ 8%]
                                         (Warmup)
## Chain 1 Iteration: 50 / 500 [ 10%]
                                         (Warmup)
## Chain 2 Iteration: 50 / 500 [ 10%]
                                         (Warmup)
## Chain 1 Iteration: 60 / 500 [ 12%]
                                         (Warmup)
## Chain 2 Iteration: 60 / 500 [ 12%]
                                         (Warmup)
## Chain 2 Iteration: 70 / 500 [ 14%]
                                         (Warmup)
## Chain 1 Iteration: 70 / 500 [ 14%]
                                         (Warmup)
## Chain 2 Iteration: 80 / 500 [ 16%]
                                         (Warmup)
## Chain 1 Iteration: 80 / 500 [ 16%]
                                         (Warmup)
## Chain 2 Iteration: 90 / 500 [ 18%]
                                         (Warmup)
## Chain 2 Iteration: 100 / 500 [ 20%]
                                         (Warmup)
## Chain 2 Iteration: 110 / 500 [ 22%]
                                         (Warmup)
## Chain 1 Iteration: 90 / 500 [ 18%]
                                         (Warmup)
## Chain 2 Iteration: 120 / 500 [ 24%]
                                         (Warmup)
## Chain 2 Iteration: 130 / 500 [ 26%]
                                         (Warmup)
## Chain 2 Iteration: 140 / 500 [ 28%]
                                         (Warmup)
## Chain 2 Iteration: 150 / 500 [ 30%]
                                         (Warmup)
## Chain 1 Iteration: 100 / 500 [ 20%]
                                         (Warmup)
## Chain 1 Iteration: 110 / 500 [ 22%]
                                         (Warmup)
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,3] = 1.0178e+23, but A[3,1] = 1.0178e
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 2 Iteration: 160 / 500 [ 32%]
                                         (Warmup)
## Chain 2 Iteration: 170 / 500 [ 34%]
                                         (Warmup)
## Chain 1 Iteration: 120 / 500 [ 24%]
                                         (Warmup)
## Chain 1 Iteration: 130 / 500 [ 26%]
                                         (Warmup)
## Chain 2 Iteration: 180 / 500 [ 36%]
                                         (Warmup)
## Chain 1 Iteration: 140 / 500 [ 28%]
                                         (Warmup)
## Chain 2 Iteration: 190 / 500 [ 38%]
                                         (Warmup)
## Chain 2 Iteration: 200 / 500 [ 40%]
                                         (Warmup)
## Chain 2 Iteration: 201 / 500 [ 40%]
                                         (Sampling)
## Chain 2 Iteration: 210 / 500 [ 42%]
                                         (Sampling)
## Chain 1 Iteration: 150 / 500 [ 30%]
                                         (Warmup)
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 2.62241e+11, but A[2,1] = 2.6224
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
```

(Warmup)

Chain 1 Iteration: 20 / 500 [4%]

Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m

```
## Chain 1
```

```
## Chain 2 Iteration: 220 / 500 [ 44%]
                                          (Sampling)
## Chain 1 Iteration: 160 / 500 [ 32%]
                                          (Warmup)
                                          (Sampling)
## Chain 2 Iteration: 230 / 500 [ 46%]
## Chain 1 Iteration: 170 / 500 [ 34%]
                                          (Warmup)
## Chain 2 Iteration: 240 / 500 [ 48%]
                                          (Sampling)
## Chain 1 Iteration: 180 / 500 [ 36%]
                                          (Warmup)
## Chain 2 Iteration: 250 / 500 [ 50%]
                                          (Sampling)
## Chain 2 Iteration: 260 / 500 [ 52%]
                                          (Sampling)
## Chain 1 Iteration: 190 / 500 [ 38%]
                                          (Warmup)
## Chain 2 Iteration: 270 / 500 [ 54%]
                                          (Sampling)
## Chain 2 Iteration: 280 / 500 [ 56%]
                                          (Sampling)
## Chain 1 Iteration: 200 / 500 [ 40%]
                                          (Warmup)
## Chain 1 Iteration: 201 / 500 [ 40%]
                                          (Sampling)
## Chain 2 Iteration: 290 / 500 [ 58%]
                                          (Sampling)
## Chain 1 Iteration: 210 / 500 [ 42%]
                                          (Sampling)
## Chain 2 Iteration: 300 / 500 [ 60%]
                                          (Sampling)
## Chain 1 Iteration: 220 / 500 [ 44%]
                                          (Sampling)
## Chain 2 Iteration: 310 / 500 [ 62%]
                                          (Sampling)
## Chain 1 Iteration: 230 / 500 [ 46%]
                                          (Sampling)
## Chain 2 Iteration: 320 / 500 [ 64%]
                                          (Sampling)
## Chain 2 Iteration: 330 / 500 [ 66%]
                                          (Sampling)
## Chain 1 Iteration: 240 / 500 [ 48%]
                                          (Sampling)
## Chain 2 Iteration: 340 / 500 [ 68%]
                                          (Sampling)
## Chain 2 Iteration: 350 / 500 [ 70%]
                                          (Sampling)
## Chain 1 Iteration: 250 / 500 [ 50%]
                                          (Sampling)
## Chain 2 Iteration: 360 / 500 [ 72%]
                                          (Sampling)
## Chain 1 Iteration: 260 / 500 [ 52%]
                                          (Sampling)
## Chain 2 Iteration: 370 / 500 [ 74%]
                                          (Sampling)
## Chain 1 Iteration: 270 / 500 [ 54%]
                                          (Sampling)
## Chain 2 Iteration: 380 / 500 [ 76%]
                                          (Sampling)
## Chain 2 Iteration: 390 / 500 [ 78%]
                                          (Sampling)
## Chain 1 Iteration: 280 / 500 [ 56%]
                                          (Sampling)
## Chain 2 Iteration: 400 / 500 [ 80%]
                                          (Sampling)
## Chain 2 Iteration: 410 / 500 [ 82%]
                                          (Sampling)
## Chain 1 Iteration: 290 / 500 [ 58%]
                                          (Sampling)
## Chain 2 Iteration: 420 / 500 [ 84%]
                                          (Sampling)
## Chain 1 Iteration: 300 / 500 [ 60%]
                                          (Sampling)
## Chain 2 Iteration: 430 / 500 [ 86%]
                                          (Sampling)
## Chain 1 Iteration: 310 / 500 [ 62%]
                                          (Sampling)
## Chain 2 Iteration: 440 / 500 [ 88%]
                                          (Sampling)
## Chain 2 Iteration: 450 / 500 [ 90%]
                                          (Sampling)
## Chain 1 Iteration: 320 / 500 [ 64%]
                                          (Sampling)
## Chain 2 Iteration: 460 / 500 [ 92%]
                                          (Sampling)
## Chain 1 Iteration: 330 / 500 [ 66%]
                                          (Sampling)
## Chain 2 Iteration: 470 / 500 [ 94%]
                                          (Sampling)
## Chain 1 Iteration: 340 / 500 [ 68%]
                                          (Sampling)
## Chain 2 Iteration: 480 / 500 [ 96%]
                                          (Sampling)
## Chain 1 Iteration: 350 / 500 [ 70%]
                                          (Sampling)
## Chain 2 Iteration: 490 / 500 [ 98%]
                                          (Sampling)
## Chain 1 Iteration: 360 / 500 [ 72%]
                                         (Sampling)
```

```
## Chain 2 Iteration: 500 / 500 [100%]
                                        (Sampling)
## Chain 2 finished in 9.2 seconds.
## Chain 1 Iteration: 370 / 500 [ 74%]
                                        (Sampling)
## Chain 1 Iteration: 380 / 500 [ 76%]
                                        (Sampling)
## Chain 1 Iteration: 390 / 500 [ 78%]
                                        (Sampling)
## Chain 1 Iteration: 400 / 500 [ 80%]
                                        (Sampling)
## Chain 1 Iteration: 410 / 500 [ 82%]
                                        (Sampling)
## Chain 1 Iteration: 420 / 500 [ 84%]
                                        (Sampling)
## Chain 1 Iteration: 430 / 500 [ 86%]
                                        (Sampling)
## Chain 1 Iteration: 440 / 500 [ 88%]
                                        (Sampling)
## Chain 1 Iteration: 450 / 500 [ 90%]
                                        (Sampling)
## Chain 1 Iteration: 460 / 500 [ 92%]
                                        (Sampling)
## Chain 1 Iteration: 470 / 500 [ 94%]
                                        (Sampling)
## Chain 1 Iteration: 480 / 500 [ 96%]
                                        (Sampling)
## Chain 1 Iteration: 490 / 500 [ 98%]
                                        (Sampling)
## Chain 1 Iteration: 500 / 500 [100%]
                                        (Sampling)
## Chain 1 finished in 10.7 seconds.
##
## Both chains finished successfully.
## Mean chain execution time: 9.9 seconds.
## Total execution time: 10.9 seconds.
## [1] 6
## Running MCMC with 2 parallel chains...
##
## Chain 1 Iteration:
                        1 / 500 [ 0%]
                                        (Warmup)
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 4.64649e+21, but A[2,1] = 4.6464
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 1.65977e+24, but A[2,1] = 1.6597
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 1.16464e+08, but A[2,1] = 1.1646
```

```
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 2 Iteration:
                        1 / 500 [ 0%]
## Chain 2 Iteration: 10 / 500 [ 2%]
                                        (Warmup)
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = inf, but A[2,1] = inf (in '/var/
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = inf, but A[2,1] = inf (in '/var/
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 2.45755e+142, but A[2,1] = 2.457
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 3.82265e+38, but A[2,1] = 3.8226
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
```

```
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 1.4378e+13, but A[2,1] = 1.4378e
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,6] = 7.59987e+06, but A[6,1] = 7.5998
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[9,103] = -4.5829e+06, but A[103,9] = -4
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = inf, but A[2,1] = inf (in '/var/
```

Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = inf, but A[2,1] = inf (in '/var/
Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
Chain 2

Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 6.50894e+36, but A[2,1] = 6.5089
Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova

Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m

```
## Chain 2
## Chain 1 Iteration:
                       10 / 500 [
                                    2%]
                                         (Warmup)
## Chain 2 Iteration:
                       20 / 500 [
                                    4%]
                                         (Warmup)
## Chain 1 Iteration:
                       20 / 500 [
                                    4%]
                                         (Warmup)
## Chain 2 Iteration:
                       30 / 500 [
                                    6%]
                                         (Warmup)
## Chain 1 Iteration:
                       30 / 500 [ 6%]
                                         (Warmup)
## Chain 2 Iteration:
                       40 / 500 [
                                   8%]
                                         (Warmup)
## Chain 1 Iteration:
                       40 / 500 [ 8%]
                                         (Warmup)
## Chain 2 Iteration:
                       50 / 500 [ 10%]
                                         (Warmup)
## Chain 2 Iteration:
                       60 / 500 [ 12%]
                                         (Warmup)
## Chain 1 Iteration:
                       50 / 500 [ 10%]
                                         (Warmup)
                       70 / 500 [ 14%]
## Chain 2 Iteration:
                                         (Warmup)
## Chain 2 Iteration: 80 / 500 [ 16%]
                                         (Warmup)
## Chain 1 Iteration:
                       60 / 500 [ 12%]
                                         (Warmup)
## Chain 2 Iteration:
                       90 / 500 [ 18%]
                                         (Warmup)
## Chain 1 Iteration:
                       70 / 500 [ 14%]
                                         (Warmup)
## Chain 1 Iteration: 80 / 500 [ 16%]
                                         (Warmup)
## Chain 2 Iteration: 100 / 500 [ 20%]
                                         (Warmup)
## Chain 2 Iteration: 110 / 500 [ 22%]
                                         (Warmup)
## Chain 2 Iteration: 120 / 500 [ 24%]
                                         (Warmup)
## Chain 2 Iteration: 130 / 500 [ 26%]
                                         (Warmup)
## Chain 2 Iteration: 140 / 500 [ 28%]
                                         (Warmup)
## Chain 2 Iteration: 150 / 500 [ 30%]
                                         (Warmup)
## Chain 1 Iteration: 90 / 500 [ 18%]
                                         (Warmup)
## Chain 2 Iteration: 160 / 500 [ 32%]
                                         (Warmup)
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 9.58466e+14, but A[2,1] = 9.5846
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Iteration: 170 / 500 [ 34%]
                                         (Warmup)
                                         (Warmup)
## Chain 2 Iteration: 180 / 500 [ 36%]
## Chain 2 Iteration: 190 / 500 [ 38%]
                                         (Warmup)
## Chain 1 Iteration: 100 / 500 [ 20%]
                                         (Warmup)
## Chain 2 Iteration: 200 / 500 [ 40%]
                                         (Warmup)
## Chain 2 Iteration: 201 / 500 [ 40%]
                                         (Sampling)
## Chain 2 Iteration: 210 / 500 [ 42%]
                                         (Sampling)
## Chain 1 Iteration: 110 / 500 [ 22%]
                                         (Warmup)
## Chain 2 Iteration: 220 / 500 [ 44%]
                                         (Sampling)
## Chain 2 Iteration: 230 / 500 [ 46%]
                                         (Sampling)
                                         (Warmup)
## Chain 1 Iteration: 120 / 500 [ 24%]
## Chain 2 Iteration: 240 / 500 [ 48%]
                                         (Sampling)
```

(Sampling)

(Sampling)

(Warmup)

Chain 2 Iteration: 250 / 500 [50%]

Chain 1 Iteration: 130 / 500 [26%]

Chain 2 Iteration: 260 / 500 [52%]

```
## Chain 1 Iteration: 140 / 500 [ 28%]
                                          (Warmup)
## Chain 2 Iteration: 270 / 500 [ 54%]
                                          (Sampling)
## Chain 1 Iteration: 150 / 500 [ 30%]
                                          (Warmup)
## Chain 2 Iteration: 280 / 500 [ 56%]
                                         (Sampling)
## Chain 1 Iteration: 160 / 500 [ 32%]
                                          (Warmup)
                                         (Sampling)
## Chain 2 Iteration: 290 / 500 [ 58%]
                                          (Warmup)
  Chain 1 Iteration: 170 / 500 [ 34%]
## Chain 2 Iteration: 300 / 500 [ 60%]
                                         (Sampling)
## Chain 1 Iteration: 180 / 500 [ 36%]
                                          (Warmup)
## Chain 2 Iteration: 310 / 500 [ 62%]
                                          (Sampling)
## Chain 1 Iteration: 190 / 500 [ 38%]
                                          (Warmup)
## Chain 2 Iteration: 320 / 500 [ 64%]
                                          (Sampling)
## Chain 1 Iteration: 200 / 500 [ 40%]
                                          (Warmup)
## Chain 2 Iteration: 330 / 500 [ 66%]
                                          (Sampling)
## Chain 1 Iteration: 201 / 500 [ 40%]
                                          (Sampling)
## Chain 1 Iteration: 210 / 500 [ 42%]
                                          (Sampling)
## Chain 2 Iteration: 340 / 500 [ 68%]
                                          (Sampling)
## Chain 1 Iteration: 220 / 500 [ 44%]
                                          (Sampling)
## Chain 2 Iteration: 350 / 500 [ 70%]
                                          (Sampling)
## Chain 1 Iteration: 230 / 500 [ 46%]
                                          (Sampling)
## Chain 2 Iteration: 360 / 500 [ 72%]
                                          (Sampling)
## Chain 1 Iteration: 240 / 500 [ 48%]
                                          (Sampling)
## Chain 2 Iteration: 370 / 500 [ 74%]
                                          (Sampling)
## Chain 1 Iteration: 250 / 500 [ 50%]
                                          (Sampling)
## Chain 2 Iteration: 380 / 500 [ 76%]
                                          (Sampling)
## Chain 1 Iteration: 260 / 500 [ 52%]
                                          (Sampling)
## Chain 2 Iteration: 390 / 500 [ 78%]
                                          (Sampling)
## Chain 1 Iteration: 270 / 500 [ 54%]
                                          (Sampling)
## Chain 2 Iteration: 400 / 500 [ 80%]
                                          (Sampling)
## Chain 1 Iteration: 280 / 500 [ 56%]
                                          (Sampling)
## Chain 2 Iteration: 410 / 500 [ 82%]
                                          (Sampling)
## Chain 1 Iteration: 290 / 500 [ 58%]
                                          (Sampling)
## Chain 2 Iteration: 420 / 500 [ 84%]
                                          (Sampling)
## Chain 1 Iteration: 300 / 500 [ 60%]
                                          (Sampling)
## Chain 2 Iteration: 430 / 500
                                [ 86%]
                                          (Sampling)
## Chain 2 Iteration: 440 / 500 [ 88%]
                                          (Sampling)
## Chain 1 Iteration: 310 / 500 [ 62%]
                                          (Sampling)
## Chain 1 Iteration: 320 / 500 [ 64%]
                                          (Sampling)
## Chain 2 Iteration: 450 / 500 [ 90%]
                                          (Sampling)
## Chain 1 Iteration: 330 / 500 [ 66%]
                                          (Sampling)
  Chain 2 Iteration: 460 / 500 [ 92%]
                                          (Sampling)
## Chain 1 Iteration: 340 / 500 [ 68%]
                                          (Sampling)
## Chain 2 Iteration: 470 / 500 [ 94%]
                                          (Sampling)
## Chain 2 Iteration: 480 / 500 [ 96%]
                                          (Sampling)
## Chain 1 Iteration: 350 / 500 [ 70%]
                                          (Sampling)
## Chain 2 Iteration: 490 / 500 [ 98%]
                                          (Sampling)
## Chain 2 Iteration: 500 / 500 [100%]
                                          (Sampling)
## Chain 2 finished in 9.9 seconds.
## Chain 1 Iteration: 360 / 500 [ 72%]
                                          (Sampling)
## Chain 1 Iteration: 370 / 500 [ 74%]
                                          (Sampling)
                                          (Sampling)
## Chain 1 Iteration: 380 / 500 [ 76%]
## Chain 1 Iteration: 390 / 500 [ 78%]
                                          (Sampling)
## Chain 1 Iteration: 400 / 500 [ 80%]
                                          (Sampling)
## Chain 1 Iteration: 410 / 500 [ 82%]
                                         (Sampling)
```

```
## Chain 1 Iteration: 430 / 500 [ 86%]
                                        (Sampling)
## Chain 1 Iteration: 440 / 500 [ 88%]
                                        (Sampling)
## Chain 1 Iteration: 450 / 500 [ 90%]
                                        (Sampling)
## Chain 1 Iteration: 460 / 500 [ 92%]
                                        (Sampling)
## Chain 1 Iteration: 470 / 500 [ 94%]
                                        (Sampling)
## Chain 1 Iteration: 480 / 500 [ 96%]
                                        (Sampling)
## Chain 1 Iteration: 490 / 500 [ 98%]
                                        (Sampling)
## Chain 1 Iteration: 500 / 500 [100%]
                                        (Sampling)
## Chain 1 finished in 11.3 seconds.
## Both chains finished successfully.
## Mean chain execution time: 10.6 seconds.
## Total execution time: 11.4 seconds.
##
## [1] 7
## Running MCMC with 2 parallel chains...
                        1 / 500 [ 0%]
## Chain 1 Iteration:
                                        (Warmup)
## Chain 1 Iteration: 10 / 500 [ 2%]
                                        (Warmup)
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 2.05378e+22, but A[2,1] = 2.0537
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 9.62182e+24, but A[2,1] = 9.6218
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 6.80316e+08, but A[2,1] = 6.8031e+08
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
```

(Sampling)

Chain 1 Iteration: 420 / 500 [84%]

```
1 / 500 [ 0%]
## Chain 2 Iteration:
## Chain 2 Iteration: 10 / 500 [ 2%]
                                       (Warmup)
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = inf, but A[2,1] = inf (in '/var/
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = inf, but A[2,1] = inf (in '/var/
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 1.26522e+131, but A[2,1] = 1.265
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 3.05636e+36, but A[2,1] = 3.0563
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
```

Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = 2.24197e+13, but A[2,1] = 2.2419

Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova

```
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,3] = 6.32163e+06, but A[3,1] = 6.3216
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = inf, but A[2,1] = inf (in '/var/
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,3] = 2.79449e+65, but A[3,1] = 2.7944
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Iteration: 20 / 500 [ 4%]
                                        (Warmup)
## Chain 2 Iteration: 30 / 500 [ 6%]
                                        (Warmup)
## Chain 1 Iteration: 20 / 500 [ 4%]
                                        (Warmup)
## Chain 2 Iteration: 40 / 500 [ 8%]
                                        (Warmup)
## Chain 1 Iteration: 30 / 500 [ 6%]
                                        (Warmup)
                      50 / 500 [ 10%]
## Chain 2 Iteration:
                                        (Warmup)
## Chain 1 Iteration: 40 / 500 [ 8%]
                                        (Warmup)
## Chain 1 Iteration: 50 / 500 [ 10%]
                                        (Warmup)
## Chain 2 Iteration:
                      60 / 500 [ 12%]
                                        (Warmup)
## Chain 2 Iteration:
                      70 / 500 [ 14%]
                                        (Warmup)
## Chain 1 Iteration: 60 / 500 [ 12%]
                                        (Warmup)
## Chain 2 Iteration: 80 / 500 [ 16%]
                                        (Warmup)
## Chain 1 Iteration: 70 / 500 [ 14%]
                                        (Warmup)
## Chain 2 Iteration: 90 / 500 [ 18%]
                                        (Warmup)
## Chain 2 Iteration: 100 / 500 [ 20%]
                                        (Warmup)
```

(Warmup)

(Warmup)

Chain 1 Iteration: 80 / 500 [16%]
Chain 2 Iteration: 110 / 500 [22%]

```
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = -5.9717e+16, but A[2,1] = -5.971
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = -1.47526e+08, but A[2,1] = -1.47526e+08
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = -6.53684e+64, but A[2,1] = -6.53
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 2
## Chain 2 Iteration: 120 / 500 [ 24%]
                                        (Warmup)
## Chain 2 Iteration: 130 / 500 [ 26%]
                                        (Warmup)
## Chain 2 Iteration: 140 / 500 [ 28%]
                                        (Warmup)
## Chain 2 Iteration: 150 / 500 [ 30%]
                                        (Warmup)
## Chain 2 Iteration: 160 / 500 [ 32%]
                                        (Warmup)
## Chain 2 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 2 Exception: cholesky_decompose: A is not symmetric. A[1,2] = -2.67249e+06, but A[2,1] = -2.67
## Chain 2 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 2 but if this warning occurs often then your model may be either severely ill-conditioned or m
```

```
## Chain 2 Iteration: 170 / 500 [ 34%]
                                         (Warmup)
## Chain 1 Iteration: 90 / 500 [ 18%]
                                         (Warmup)
## Chain 2 Iteration: 180 / 500 [ 36%]
                                         (Warmup)
## Chain 2 Iteration: 190 / 500 [ 38%]
                                         (Warmup)
## Chain 2 Iteration: 200 / 500 [ 40%]
                                         (Warmup)
## Chain 2 Iteration: 201 / 500 [ 40%]
                                         (Sampling)
## Chain 2 Iteration: 210 / 500 [ 42%]
                                         (Sampling)
## Chain 1 Iteration: 100 / 500 [ 20%]
                                         (Warmup)
## Chain 2 Iteration: 220 / 500 [ 44%]
                                         (Sampling)
## Chain 2 Iteration: 230 / 500 [ 46%]
                                         (Sampling)
## Chain 2 Iteration: 240 / 500 [ 48%]
                                         (Sampling)
## Chain 2 Iteration: 250 / 500 [ 50%]
                                         (Sampling)
## Chain 2 Iteration: 260 / 500 [ 52%]
                                         (Sampling)
## Chain 1 Iteration: 110 / 500 [ 22%]
                                         (Warmup)
## Chain 2 Iteration: 270 / 500 [ 54%]
                                         (Sampling)
## Chain 2 Iteration: 280 / 500 [ 56%]
                                         (Sampling)
## Chain 2 Iteration: 290 / 500 [ 58%]
                                         (Sampling)
## Chain 1 Iteration: 120 / 500 [ 24%]
                                         (Warmup)
## Chain 2 Iteration: 300 / 500 [ 60%]
                                         (Sampling)
## Chain 2 Iteration: 310 / 500 [ 62%]
                                         (Sampling)
## Chain 2 Iteration: 320 / 500 [ 64%]
                                         (Sampling)
## Chain 1 Iteration: 130 / 500 [ 26%]
                                         (Warmup)
## Chain 2 Iteration: 330 / 500 [ 66%]
                                         (Sampling)
## Chain 1 Iteration: 140 / 500 [ 28%]
                                         (Warmup)
## Chain 2 Iteration: 340 / 500 [ 68%]
                                         (Sampling)
## Chain 1 Iteration: 150 / 500 [ 30%]
                                         (Warmup)
## Chain 1 Informational Message: The current Metropolis proposal is about to be rejected because of th
## Chain 1 Exception: cholesky_decompose: A is not symmetric. A[1,2] = -2.12377e+19, but A[2,1] = -2.12
## Chain 1 If this warning occurs sporadically, such as for highly constrained variable types like cova
## Chain 1 but if this warning occurs often then your model may be either severely ill-conditioned or m
## Chain 1
## Chain 2 Iteration: 350 / 500 [ 70%]
                                         (Sampling)
## Chain 2 Iteration: 360 / 500 [ 72%]
                                         (Sampling)
## Chain 2 Iteration: 370 / 500 [ 74%]
                                         (Sampling)
## Chain 1 Iteration: 160 / 500 [ 32%]
                                         (Warmup)
## Chain 2 Iteration: 380 / 500 [ 76%]
                                         (Sampling)
## Chain 1 Iteration: 170 / 500 [ 34%]
                                         (Warmup)
## Chain 2 Iteration: 390 / 500 [ 78%]
                                         (Sampling)
## Chain 1 Iteration: 180 / 500 [ 36%]
                                         (Warmup)
## Chain 2 Iteration: 400 / 500 [ 80%]
                                         (Sampling)
## Chain 2 Iteration: 410 / 500 [ 82%]
                                         (Sampling)
```

(Warmup)

(Warmup)

(Sampling)

(Sampling)

(Sampling)

Chain 1 Iteration: 190 / 500 [38%]

Chain 2 Iteration: 420 / 500 [84%]

Chain 2 Iteration: 430 / 500 [86%]

Chain 1 Iteration: 200 / 500 [40%]

Chain 1 Iteration: 201 / 500 [40%]

```
## Chain 2 Iteration: 440 / 500 [ 88%]
                                         (Sampling)
## Chain 1 Iteration: 210 / 500 [ 42%]
                                         (Sampling)
                                         (Sampling)
## Chain 2 Iteration: 450 / 500 [ 90%]
## Chain 1 Iteration: 220 / 500 [ 44%]
                                         (Sampling)
## Chain 2 Iteration: 460 / 500 [ 92%]
                                         (Sampling)
## Chain 1 Iteration: 230 / 500 [ 46%]
                                         (Sampling)
## Chain 2 Iteration: 470 / 500 [ 94%]
                                         (Sampling)
## Chain 1 Iteration: 240 / 500 [ 48%]
                                         (Sampling)
## Chain 2 Iteration: 480 / 500 [ 96%]
                                         (Sampling)
## Chain 2 Iteration: 490 / 500 [ 98%]
                                         (Sampling)
## Chain 1 Iteration: 250 / 500 [ 50%]
                                         (Sampling)
## Chain 2 Iteration: 500 / 500 [100%]
                                         (Sampling)
## Chain 2 finished in 8.0 seconds.
## Chain 1 Iteration: 260 / 500 [ 52%]
                                         (Sampling)
## Chain 1 Iteration: 270 / 500 [ 54%]
                                         (Sampling)
## Chain 1 Iteration: 280 / 500 [ 56%]
                                         (Sampling)
## Chain 1 Iteration: 290 / 500 [ 58%]
                                         (Sampling)
## Chain 1 Iteration: 300 / 500 [ 60%]
                                         (Sampling)
## Chain 1 Iteration: 310 / 500 [ 62%]
                                         (Sampling)
## Chain 1 Iteration: 320 / 500 [ 64%]
                                         (Sampling)
## Chain 1 Iteration: 330 / 500 [ 66%]
                                         (Sampling)
## Chain 1 Iteration: 340 / 500 [ 68%]
                                         (Sampling)
## Chain 1 Iteration: 350 / 500 [ 70%]
                                         (Sampling)
## Chain 1 Iteration: 360 / 500 [ 72%]
                                         (Sampling)
## Chain 1 Iteration: 370 / 500 [ 74%]
                                         (Sampling)
## Chain 1 Iteration: 380 / 500 [ 76%]
                                         (Sampling)
## Chain 1 Iteration: 390 / 500 [ 78%]
                                         (Sampling)
## Chain 1 Iteration: 400 / 500 [ 80%]
                                         (Sampling)
## Chain 1 Iteration: 410 / 500 [ 82%]
                                         (Sampling)
## Chain 1 Iteration: 420 / 500 [ 84%]
                                         (Sampling)
## Chain 1 Iteration: 430 / 500 [ 86%]
                                         (Sampling)
## Chain 1 Iteration: 440 / 500 [ 88%]
                                         (Sampling)
## Chain 1 Iteration: 450 / 500 [ 90%]
                                         (Sampling)
## Chain 1 Iteration: 460 / 500 [ 92%]
                                         (Sampling)
## Chain 1 Iteration: 470 / 500 [ 94%]
                                         (Sampling)
## Chain 1 Iteration: 480 / 500 [ 96%]
                                         (Sampling)
## Chain 1 Iteration: 490 / 500 [ 98%]
                                         (Sampling)
## Chain 1 Iteration: 500 / 500 [100%]
                                         (Sampling)
## Chain 1 finished in 10.8 seconds.
##
## Both chains finished successfully.
## Mean chain execution time: 9.4 seconds.
## Total execution time: 10.9 seconds.
##
## [1] 8
MSPE GPR <- sqrt(1/(dim(set$X test)[1])*sum((y hat GPR-as.vector(set$y test))^2))
MAPE_GPR <- 1/(dim(set$X_test)[1])*sum(abs(y_hat_GPR-as.vector(set$y_test)))
print(paste("MSFE for GPR =", MSPE_GPR))
```

```
print(paste("MAPE for GPR =", MAPE_GPR))
```

[1] "MAPE for GPR = 4.79852656478769"

```
library(bridgesampling)
library(rstan)
# cores <- 4
# options(mc.cores = cores)
# mod_bridge<-stan_model("MCMC test.stan")</pre>
# stanfit <- bridge <- vector("list", 3)</pre>
# for (k in 1:3) {
  stanfit[[k]] <- sampling(mod bridge,</pre>
#
     data = list(N=N)
#
            K_gram1=K_gram1,
#
            K_gram2=K_gram2,
#
             y=y\_stan
#
            ),
#
     iter = 11000, warmup = 1000, chains = 4,
   #init = init_fun(nchains = 4, k = k, m = 1),
#
#
     cores = cores, seed = 1)
#
   bridge[[k]] <- bridge_sampler(stanfit[[k]], method = "warp3",</pre>
     repetitions = 10, cores = cores)
# }
mod_bridge<-stan("MCMC.stan", data=data_MCMC,</pre>
          seed = 123.
  iter= 30.
  save_warmup = TRUE,
  chains = 2
```

```
##
## SAMPLING FOR MODEL 'MCMC' NOW (CHAIN 1).
## Chain 1:
## Chain 1: Gradient evaluation took 0.001867 seconds
## Chain 1: 1000 transitions using 10 leapfrog steps per transition would take 18.67 seconds.
## Chain 1: Adjust your expectations accordingly!
## Chain 1:
## Chain 1:
## Chain 1: WARNING: No variance estimation is
## Chain 1:
                    performed for num_warmup < 20
## Chain 1:
## Chain 1: Iteration: 1 / 30 [ 3%]
                                        (Warmup)
## Chain 1: Iteration: 3 / 30 [ 10%]
                                        (Warmup)
## Chain 1: Iteration: 6 / 30 [ 20%]
                                        (Warmup)
## Chain 1: Iteration: 9 / 30 [ 30%]
                                        (Warmup)
## Chain 1: Iteration: 12 / 30 [ 40%]
                                        (Warmup)
## Chain 1: Iteration: 15 / 30 [ 50%]
                                        (Warmup)
## Chain 1: Iteration: 16 / 30 [ 53%]
                                        (Sampling)
## Chain 1: Iteration: 18 / 30 [ 60%]
                                        (Sampling)
## Chain 1: Iteration: 21 / 30 [ 70%]
                                        (Sampling)
## Chain 1: Iteration: 24 / 30 [ 80%]
                                        (Sampling)
## Chain 1: Iteration: 27 / 30 [ 90%]
                                        (Sampling)
## Chain 1: Iteration: 30 / 30 [100%]
                                       (Sampling)
```

```
## Chain 1:
## Chain 1: Elapsed Time: 0.722791 seconds (Warm-up)
                           1.01519 seconds (Sampling)
## Chain 1:
## Chain 1:
                           1.73798 seconds (Total)
## Chain 1:
##
## SAMPLING FOR MODEL 'MCMC' NOW (CHAIN 2).
## Chain 2:
## Chain 2: Gradient evaluation took 0.001106 seconds
## Chain 2: 1000 transitions using 10 leapfrog steps per transition would take 11.06 seconds.
## Chain 2: Adjust your expectations accordingly!
## Chain 2:
## Chain 2:
## Chain 2: WARNING: No variance estimation is
## Chain 2:
                    performed for num_warmup < 20
## Chain 2:
## Chain 2: Iteration: 1 / 30 [ 3%]
                                       (Warmup)
## Chain 2: Iteration: 3 / 30 [ 10%]
                                       (Warmup)
## Chain 2: Iteration: 6 / 30 [ 20%]
                                       (Warmup)
## Chain 2: Iteration: 9 / 30 [ 30%]
                                       (Warmup)
## Chain 2: Iteration: 12 / 30 [ 40%]
                                       (Warmup)
## Chain 2: Iteration: 15 / 30 [ 50%]
                                       (Warmup)
## Chain 2: Iteration: 16 / 30 [ 53%]
                                       (Sampling)
## Chain 2: Iteration: 18 / 30 [ 60%]
                                       (Sampling)
## Chain 2: Iteration: 21 / 30 [ 70%]
                                       (Sampling)
## Chain 2: Iteration: 24 / 30 [ 80%]
                                       (Sampling)
## Chain 2: Iteration: 27 / 30 [ 90%]
                                       (Sampling)
## Chain 2: Iteration: 30 / 30 [100%]
                                       (Sampling)
## Chain 2:
## Chain 2: Elapsed Time: 0.144438 seconds (Warm-up)
## Chain 2:
                           0.550138 seconds (Sampling)
## Chain 2:
                           0.694576 seconds (Total)
## Chain 2:
## Warning: There were 1 chains where the estimated Bayesian Fraction of Missing Information was low. S
## https://mc-stan.org/misc/warnings.html#bfmi-low
## Warning: Examine the pairs() plot to diagnose sampling problems
## Warning: The largest R-hat is 1.72, indicating chains have not mixed.
## Running the chains for more iterations may help. See
## https://mc-stan.org/misc/warnings.html#r-hat
## Warning: Bulk Effective Samples Size (ESS) is too low, indicating posterior means and medians may be
## Running the chains for more iterations may help. See
## https://mc-stan.org/misc/warnings.html#bulk-ess
## Warning: Tail Effective Samples Size (ESS) is too low, indicating posterior variances and tail quant
## Running the chains for more iterations may help. See
## https://mc-stan.org/misc/warnings.html#tail-ess
```

```
bridge_sampler(mod_bridge)
## Iteration: 1
## Iteration: 2
## Iteration: 3
## Iteration: 4
## Iteration: 5
## Iteration: 6
## Iteration: 7
## Iteration: 8
## Iteration: 9
## Iteration: 10
## Iteration: 11
## Iteration: 12
## Iteration: 13
## Iteration: 14
## Iteration: 15
## Iteration: 16
## Iteration: 17
## Iteration: 18
## Iteration: 19
## Bridge sampling estimate of the log marginal likelihood: -475.2413
## Estimate obtained in 19 iteration(s) via method "normal".
library(shinystan)
## Loading required package: shiny
## This is shinystan version 2.6.0
launch_shinystan(fit)
## Launching ShinyStan interface... for large models this may take some time.
## Listening on http://127.0.0.1:3584
fit$summary(variables = c("lp__"))$mean
## [1] -476.9455
```