Data Science and Visualization (DSV, F23)

9. Association Rules

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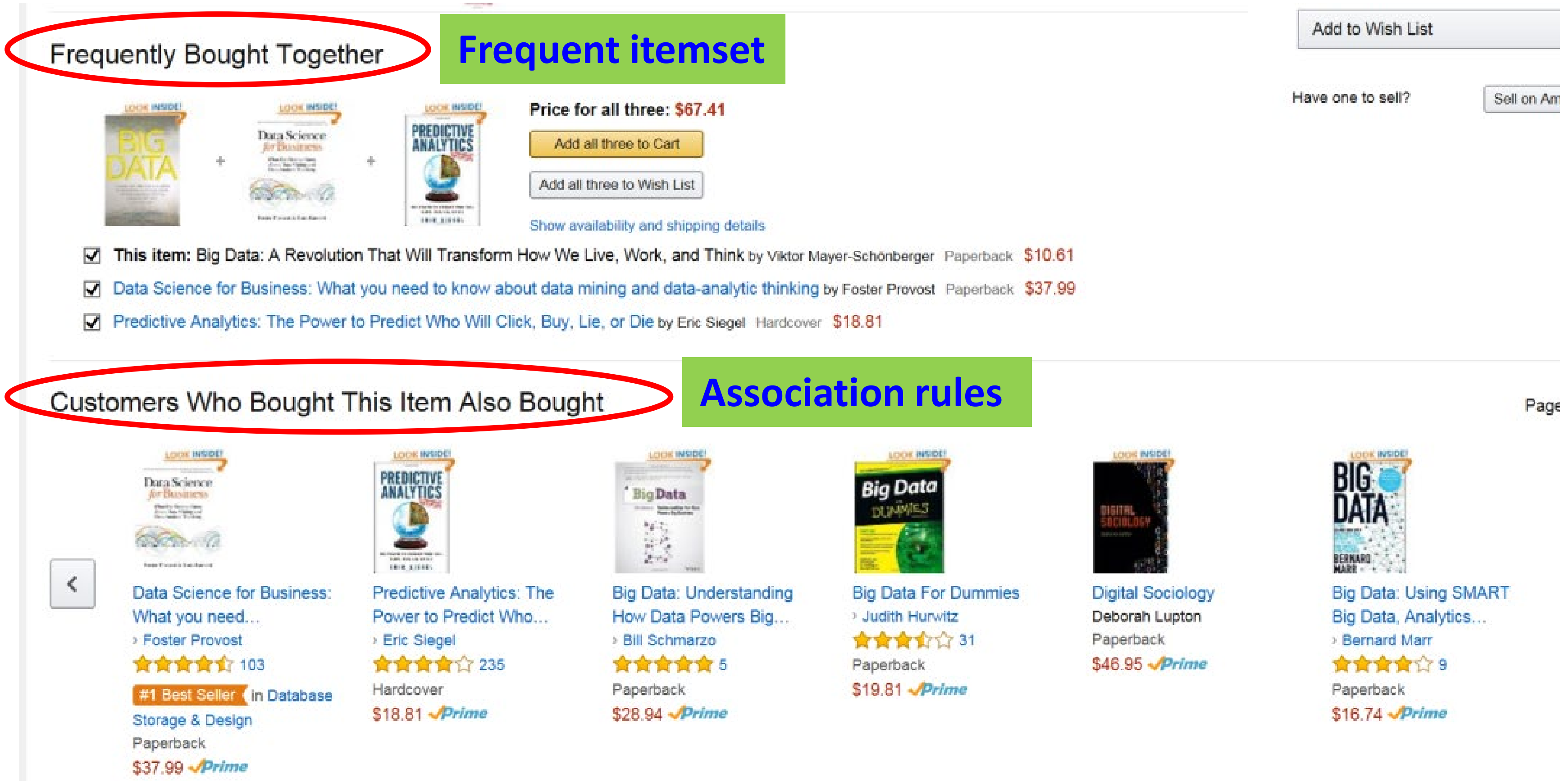
Recommendation in Amazon

- Two types of recommendation
  - 90% buyers who bought A also bought B.
  - Since you’ve bought A, you may also want B.



Agenda

- Problem definition
  - Support, confidence, lift, and association rule
  - Frequent itemsets
  - Steps for association rule mining
- Apriori principle, Apriori algorithm
- Deriving association rules from frequent itemsets

Market Basket Data

- Large set of *items*, i.e., things sold in a supermarket
- Large set of *baskets*, each a small subset of items, i.e., things that one customer buys in one **transaction**
- Transaction table **T**: market-basket data
  - Each record is a transaction, containing a set of items
  - Many-to-many mapping (association) between items and baskets
- What can we do with this type of data?
  - E.g., counting whether the combination {Milk, Bread} is *frequent* or not

TID	Items
1	{Milk, Bread, Beer, Diapers}
2	{Bread, Eggs}
3	{Bread, Diapers}
4	{Milk, Bread, Cola}
5	{Milk, Bread, Diapers}

Transaction table

What Is Association Rule Mining?

- Finding frequent patterns and associations (rules) among sets of items in a transaction table
- Motivation (market basket analysis):
  - How likely is that the customers buying *milk* are also buying *bread*?
  - Such rules help retailers making decisions
    - Plan the shelf space: placing milk close to bread, more convenient for the customers
    - Offer promotions/discounts for those products together

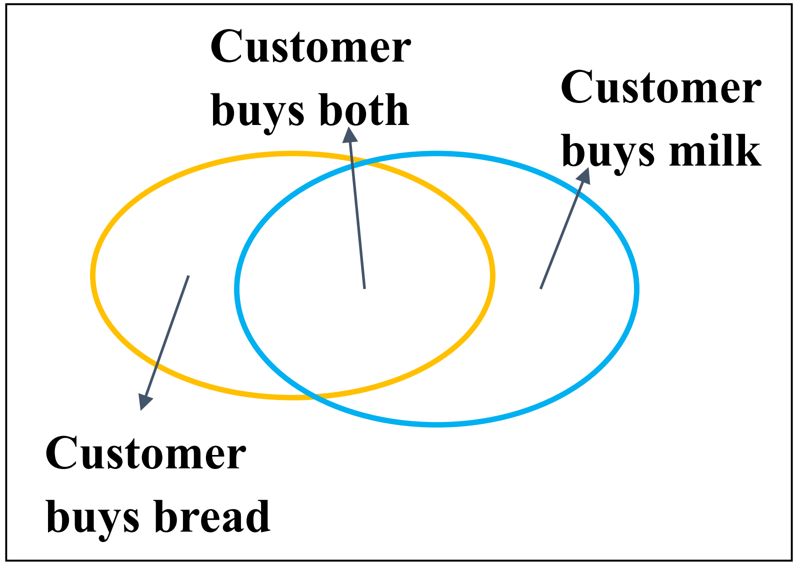
What Is an Association Rule?

- An **association rule** correlates (associates) the presence of one set of items with that of another set of items
- Examples
  - Rule form: Body $\Rightarrow$ Head [support, confidence]
  - milk $\Rightarrow$ bread [5%, 70%]
    - 5% of transactions buy both milk and bread
    - transactions that buy milk have 70% chance of buying also bread
- Applications: basket data analysis, catalog design
  - ** $\Rightarrow$ chocolate* (How to boost the sales of chocolate)
  - *Home Electronics $\Rightarrow$ ** (What other products should the store stock up?)

Interesting Rules

- A rule is said to be interesting (or valid) when:
  - Its items appear frequently in the transaction table (support)
  - It holds with a high probability (confidence)

  Example: $milk \Rightarrow bread$



NB: X and Y are itemsets.

Find all the rules $X \Rightarrow Y$ with confidence and support above given thresholds

- **support** *s*, probability that a transaction contains $X \cup Y$
- **confidence** *c*, conditional probability that a transaction having X also contains Y

Example (1)

- Find the support and confidence of the rule: {B,D} $\Rightarrow$ {A}
- Support value of *sup*(ABD):
  - percentage of tuples with {A,B,D}
    = (3/4)*100% = 75%
- Confidence value of *conf*(BD $\Rightarrow$ A)

$$\frac{\text{number of transactions that contain } \{A, B, D\}}{\text{number of transactions that contain } \{B, D\}} = \frac{3}{3} = 100\%$$

TID	items_bought
100	{F,A,D,B}
200	{D,A,C,E,B}
300	{C,A,B,E}
400	{B,A,D}

$$\text{prob}(Y \mid X) = \frac{\text{prob}(X \cup Y)}{\text{prob}(X)}$$

$$\text{conf}(X \Rightarrow Y) = \frac{\sup(X \cup Y)}{\sup(X)} = \frac{\text{frequency}(X \cup Y)}{\text{frequency}(X)}$$

Example (2)

- Find interesting rules

Transaction ID	Items Bought
2000	A,B,C
1000	A,C
4000	A,D
5000	B,E,F

Thresholds:

Min. support 50%
Min. confidence 50%

Frequent Itemset	Support
{A}	75%
{B}	50%
{C}	50%
{A,C}	50%

"$A \Rightarrow C$" is a valid rule because:

support = support($\{A \cup C\}$) = 2/4 = 50%

confidence = support($\{A \cup C\}$)/support($\{A\}$) = 50%/75% = 66.6%

Lift of A Rule

- **Lift**($X \Rightarrow Y$) = confidence($X \Rightarrow Y$)/support(Y)
  = support($X \cup Y$)/ (support(X) * support(Y))
  = (frequency($X \cup Y$)*$|T|$) / (frequency(X)*frequency(Y))

- Lift($X \Rightarrow Y$) refers to the increase in the ratio of sale of Y when X is sold
  - Lift = 1: No association between products X and Y.
  - Lift > 1: Products X and Y are more likely to be bought together.
  - Lift < 1: The two products are unlikely to be bought together.

Example of Lift

Transaction ID	Items Bought
2000	A,B,C
1000	A,C
4000	A,D
5000	B,E,F

Frequent Itemset	Support
{A}	75%
{B}	50%
{C}	50%
{A,C}	50%

Rule $A \Rightarrow C$:

support = support($\{A \cup C\}$) = 2/4 = 50%

confidence = support($\{A \cup C\}$)/support($\{A\}$) = 50%/75% = 66.6%

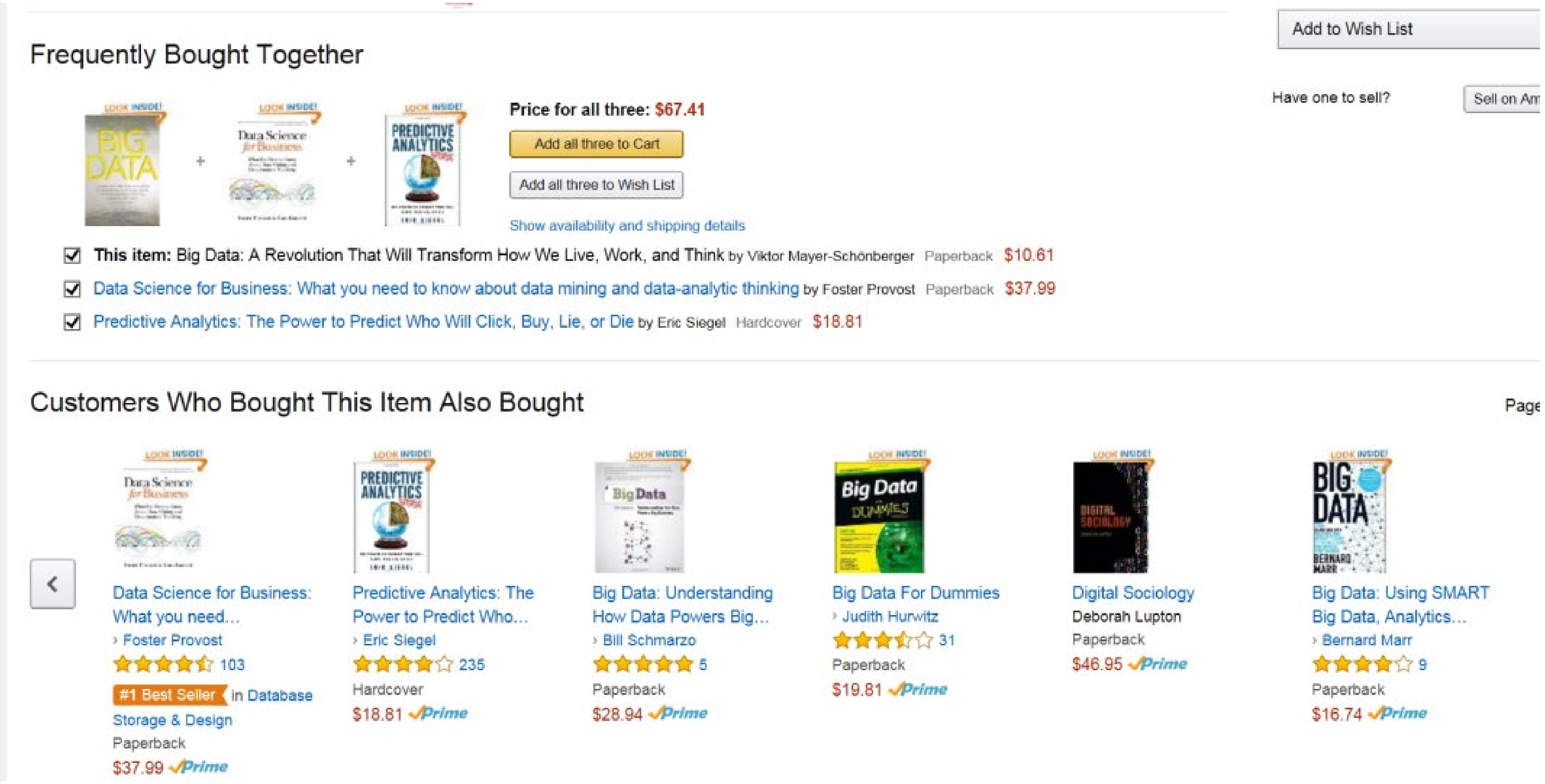
lift = confidence($A \Rightarrow C$)/support(C) = 66.6%/50% = 1.333

Lift's meaning: the likelihood of buying a A and C *together* is 1.33 times more than the likelihood of just buying the C.

A Real Application of Association Rules

- Amazon's recommendation
  - 90% buyers who bought A also bought B.
  - Since you've bought A, you may also want B.

Make recommendations based on rules of high support, confidence and lift.



Causality *vs.* Correlation

- Causality
    - From the very first day, humans are curious about *why*.
    - With big data, it may be very hard to see the exact reasons.
- Correlation
    - Instead, we can find interesting patterns or associations of different things from big data.
    - Probability instead of certainty (not totally random).
        - Association rule mining.

- **NB**: Association rules are empiricism! What they tell may not be the true cause and effect.

Steps of Association Rule Mining

1. Find the ***frequent itemsets***
    - The sets of items that have minimum support
    - How to do this efficiently?

2. Use the frequent itemsets to generate association rules

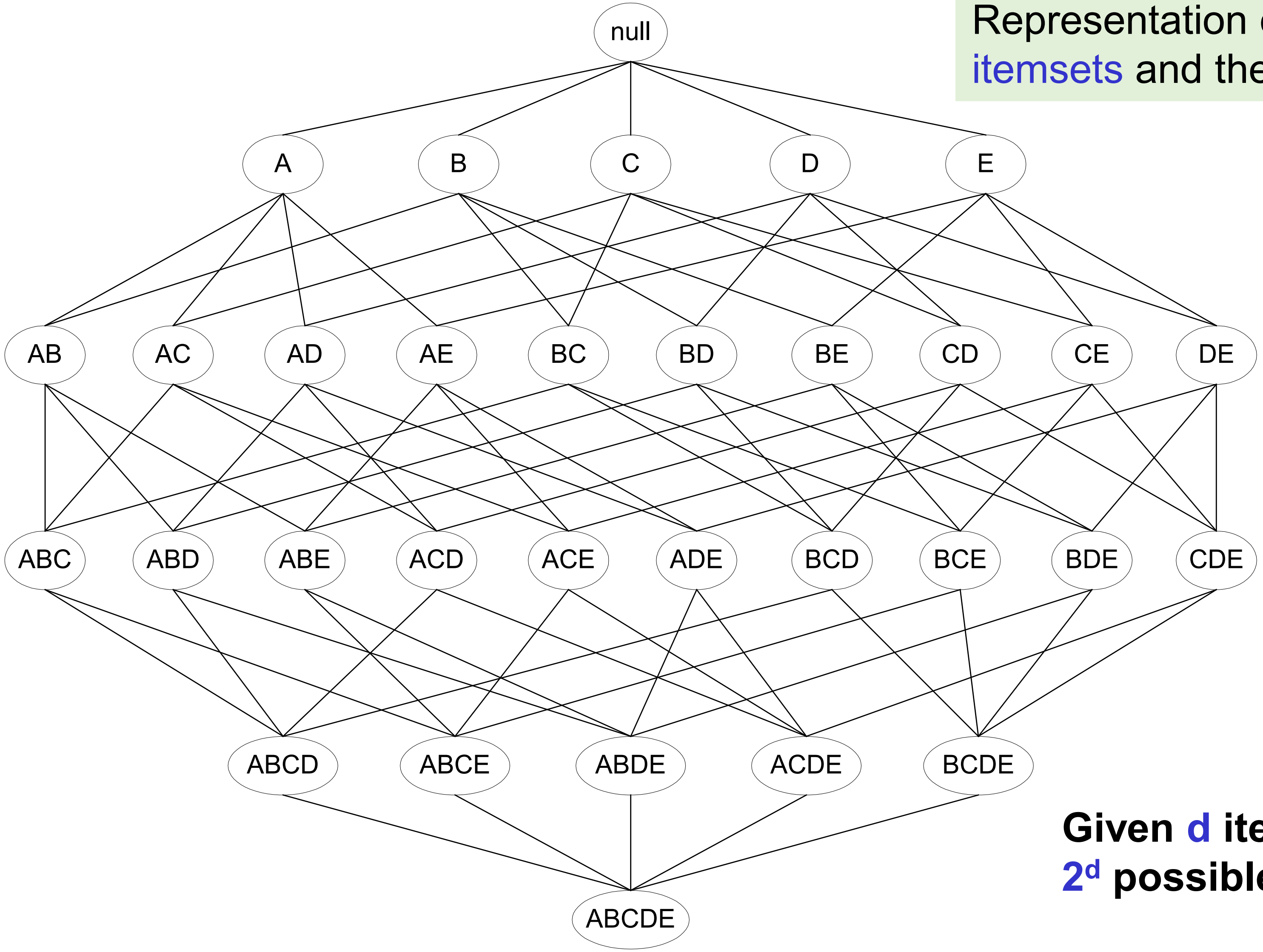
Mining Frequent Itemsets

- **Input**: A set of transactions **T**, over a set of items **I**
- **Output**: All itemsets with items in **I** having
  - support ≥ minsup (support threshold)

- Problem parameters:
  - N = |**T**|: number of transactions
  - d = |**I**|: number of (distinct) items
  - w: max width of a transaction
  - Number of possible itemsets: $M = 2^d$

- Scale of the problem:
  - WalMart sells 100,000 items and can store billions of baskets.
  - The Web has billions of words and many billions of pages.

The Itemset Lattice



Representation of all possible itemsets and their relationships

Given d items, there are 2^d possible itemsets

Agenda

- Problem definition
- Apriori principle, Apriori algorithm
- Deriving association rules from frequent itemsets

The Apriori Principle

- Main observations: $\forall \boldsymbol{X}, \boldsymbol{Y}: \boldsymbol{X} \subseteq \boldsymbol{Y} \Rightarrow \boldsymbol{s}(\boldsymbol{X}) \geq \boldsymbol{s}(\boldsymbol{Y})$
  - If an itemset is frequent, so are its subsets
  - If an itemset is infrequent, so are its supersets
- The Apriori principle: A *subset* of a frequent itemset must also be a frequent itemset
    - E.g., if $\{AB\}$ is a frequent itemset, both $\{A\}$ and $\{B\}$ must be a frequent itemset
  - Iteratively find frequent itemsets with cardinality from 1 to *m* (*m*-itemset): Use frequent k-itemsets to explore (k+1)-itemsets

Illustration of Apriori Principle

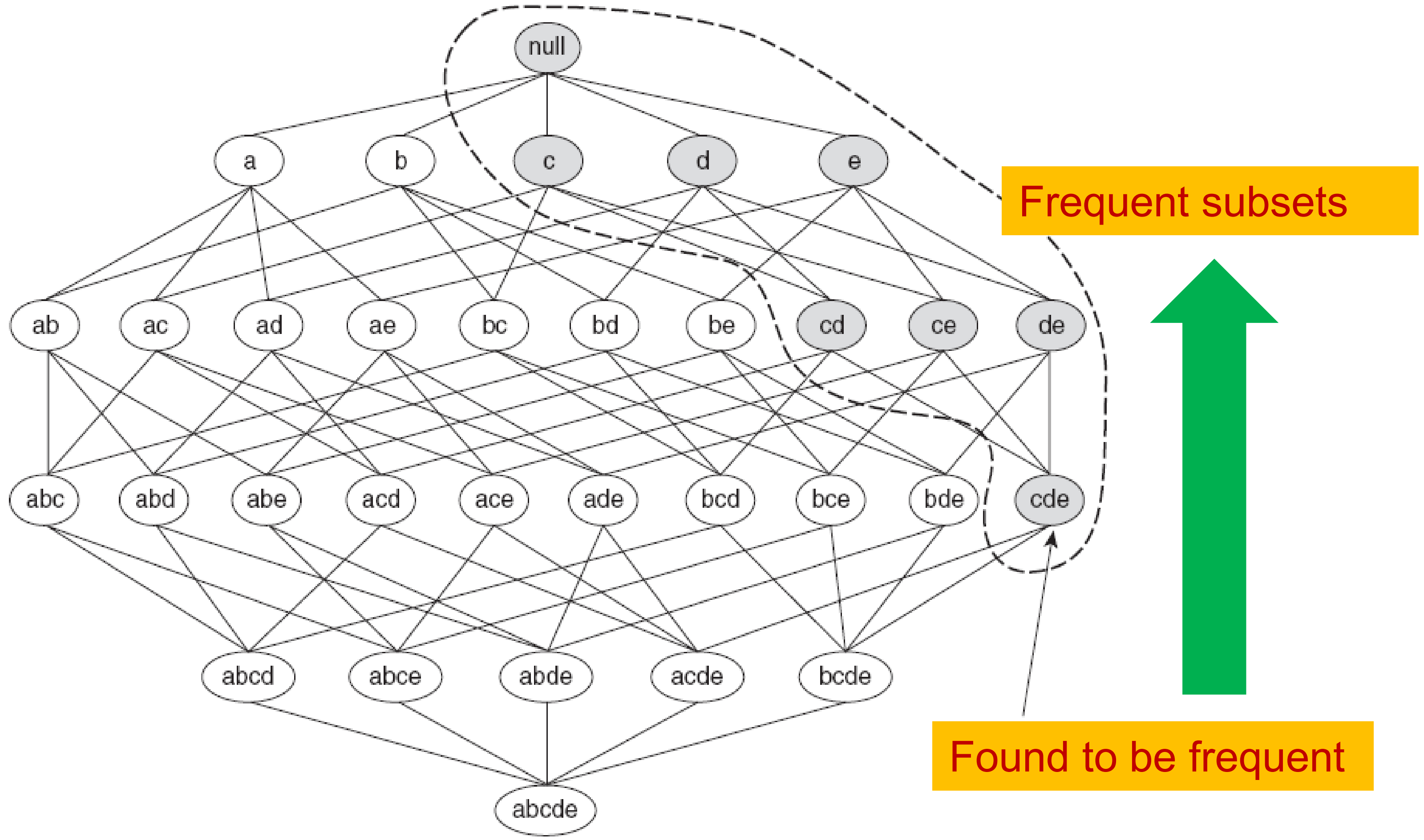
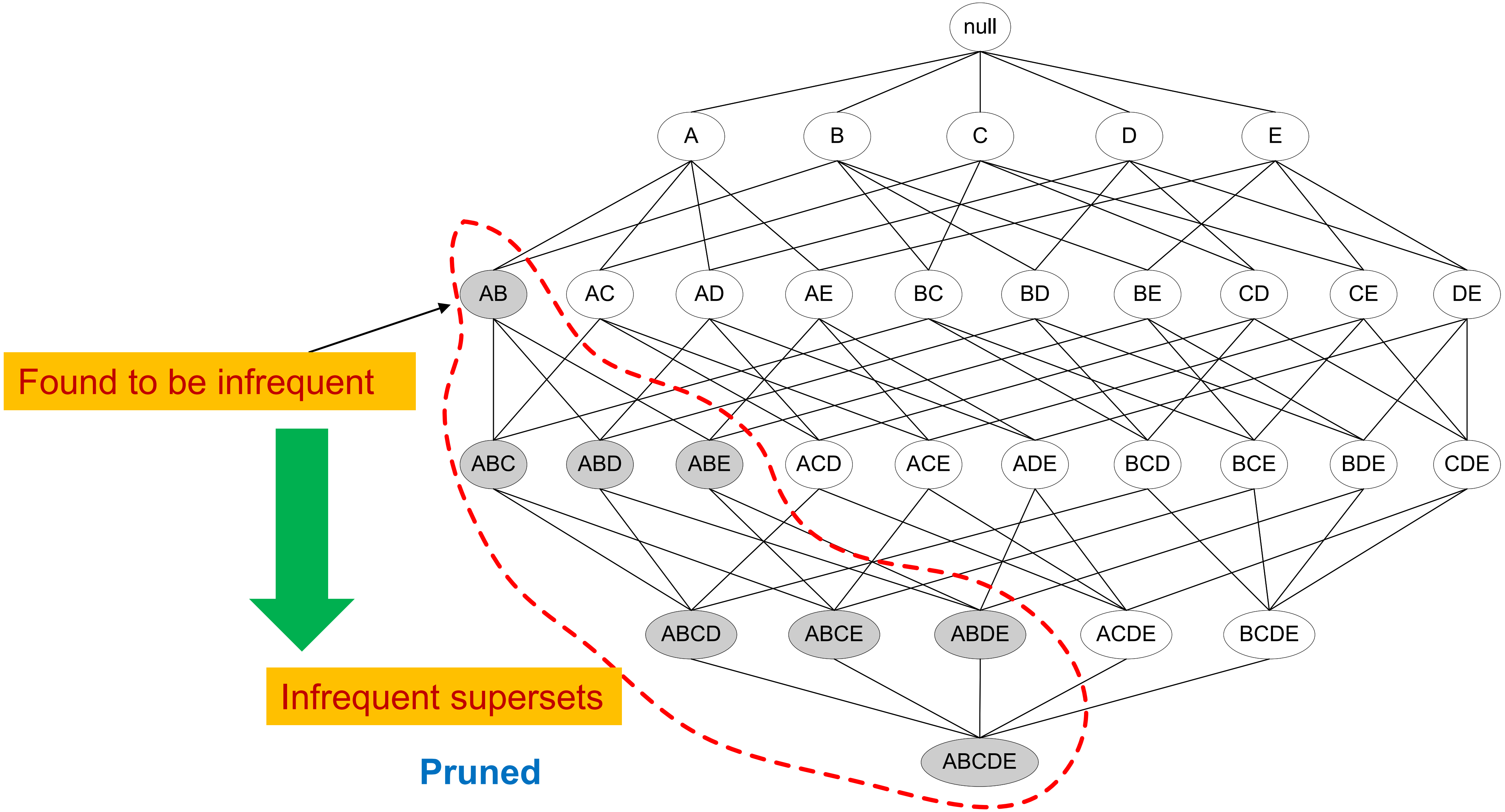


Figure 6.3. An illustration of the *Apriori* principle. If $\{c, d, e\}$ is frequent, then all subsets of this itemset are frequent.

Illustration of Apriori Principle (cont.)



Level-wise Process of Apriori Principle

Level 4 (frequent quadruples): {….}

Level 3 (frequent triplets): {ABD}, {BDF}

Level 2 (frequent pairs): {AB}, {AD}, {BD}, {BF}, {DF}

Level 1 (frequent items): {A}, {B}, {D}, {F}

> Remember:
>
> All subsets of a frequent itemset must be frequent as well

Question: Can ADF be frequent?

NO: because AF is not frequent

The Apriori Algorithm

Advanced

- Notations
  - C_k: Candidate itemset of size k
  - L_k: Frequent itemset of size k
- Important steps in candidate generation
  - Prune Step: Any k-itemset that is not frequent cannot be a subset of a frequent (k+1)-itemset
  - Join Step: C_{k+1} is generated by joining L_k with itself

$C_1 = \{\{\text{item}_1\}, \ldots, \{\text{item}_N\}\}$;
for ($k = 1$; L_k $!=\varnothing$; k++)
  for each transaction t in transaction table T
  increment the count of all candidates in C_k that are contained in t
  L_k = candidates in C_k with min_support (*frequent*)
  C_{k+1} = candidates generated from L_k;
return $\cup_k L_k$;

Special self-join!

The Apriori Algorithm Example (1)

Trans. Table T

TID	Items
100	1 3 4
200	2 3 5
300	1 2 3 5
400	2 5

min_sup=2 (or 50%)

Scan T

C_1

itemset	sup.
{1}	2
{2}	3
{3}	3
{4}	1
{5}	3

L_1

itemset	sup.
{1}	2
{2}	3
{3}	3
{5}	3

C_2

itemset
{1 2}
{1 3}
{1 5}
{2 3}
{2 5}
{3 5}

The Apriori Algorithm Example (2)

Trans. Table T

TID	Items
100	1 3 4
200	2 3 5
300	1 2 3 5
400	2 5

C_2

itemset
{1 2}
{1 3}
{1 5}
{2 3}
{2 5}
{3 5}

Scan T

C_2

itemset	sup
{1 2}	1
{1 3}	2
{1 5}	1
{2 3}	2
{2 5}	3
{3 5}	2

min_sup=2 (or 50%)

L_2

itemset	sup
{1 3}	2
{2 3}	2
{2 5}	3
{3 5}	2

C_3

itemset
{2 3 5}

The Apriori Algorithm Example (3)

Trans. Table T

TID	Items
100	1 3 4
200	2 3 5
300	1 2 3 5
400	2 5

min_sup=2 (or 50%)

C_3

itemset
{2 3 5}

$\xrightarrow{\text{Scan T}}$

L_3

itemset	sup
{2 3 5}	2

The Apriori Algorithm Example (4)

Trans. Table T

TID	Items
100	1 3 4
200	2 3 5
300	1 2 3 5
400	2 5

min_sup=2 (or 50%)

The result of frequent itemsets

itemset	sup.
{1}	2
{2}	3
{3}	3
{5}	3

itemset	sup
{1 3}	2
{2 3}	2
{2 5}	3
{3 5}	2

itemset	sup
{2 3 5}	2

$L_1 \cup L_2 \cup L_3$

Candidates Generation

Advanced

- Suppose the items in L_k are listed in an order
- Step 1: self-joining L_k to get C_{k+1} (In SQL)

  INSERT INTO C_{k+1}

  SELECT $p.item_1, p.item_2, \ldots, p.item_k, q.item_k$

  FROM $L_k\ p, L_k\ q$

  WHERE $p.item_1=q.item_1, \ldots, p.item_{k-1}=q.item_{k-1}, p.item_k < q.item_k$
- Step 2: pruning frequent itemsets in C_{k+1}

  forall ***itemsets c in*** C_{k+1} do

  forall ***k-subsets s of c*** do

  if *(s is not in* L_k*)* **then delete** *c* **from** C_{k+1}

The Previous Example

Advanced

Trans. Table T

TID	Items
100	1 3 4
200	2 3 5
300	1 2 3 5
400	2 5

min_sup=2 (or 50%)

We only need to match {2 3} with {2 5}

L_2

itemset	sup
{1 3}	2
{2 3}	2
{2 5}	3
{3 5}	2

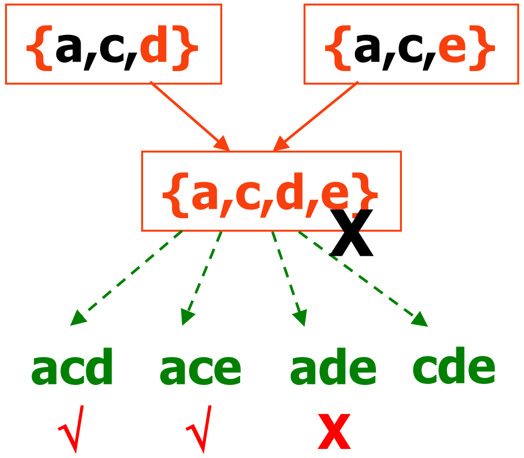
C_3

itemset
{2 3 5}

Example of Candidates Generation

Advanced

- L_3={*abc, abd, acd, ace, bcd*}
- Self-joining: $L_3 \bowtie L_3$
  - *abcd* from *abc* and *abd*
  - *acde* from *acd* and *ace*
  - No need to match other pairs
- Pruning:
  - *acde* is removed because *ade* is not in L_3
- C_4={*abcd*}
  - Scanning transaction table *T* is still needed to get the frequencies for items in C_4 *(to decide the correct L_4)*



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- Problem definition
- Apriori principle, Apriori algorithm
- Deriving association rules from frequent itemsets

Generating Association Rules from Frequent Itemsets

- Assume that we have discovered the frequent itemsets and their support
- How do we generate association rules?
- Frequent itemsets:

{1}	2
{2}	3
{3}	3
{5}	3
{1,3}	2
{2,3}	2
{2,5}	3
{3,5}	2
{2,3,5}	2

l →

Not a transaction table!

?

- For each frequent itemset *l*, find all nonempty subsets *s*.
- For each *s*, generate rule $s \Rightarrow l\text{-}s$, if $sup(l)/sup(s) \geq min\_conf$

Example: $l = \{2,3,5\}$, min_conf = 75%

$\{2,3\} \Rightarrow \{5\}$ 2/2=100% √

$\{2,5\} \Rightarrow \{3\}$ 2/3=66.6% X

$\{3,5\} \Rightarrow \{2\}$ 2/2=100% √

do the rest as an exercise

Example in Jupyter Notebook

- Library **mlxtend**
  - To install the library: pip install mlxtend in Anaconda Prompt
  - from mlxtend.frequent_patterns import apriori: frequent itemsets
  - from mlxtend.frequent_patterns import association_rules: rules
- Real data
  - store_data.csv (in Moodle)
  - (7501, 20)
    - 7501 transactions, each having at most 20 items
- Lecture9_Apriori_mlxtend_storedata.ipynb

Performance Bottlenecks of Apriori

Advanced

- Is Apriori fast enough?
- The core of the Apriori algorithm:
  - Use frequent *k*-itemsets to generate **candidate** frequent (*k+1*)-itemsets
  - Use full table scan and pattern matching to collect counts for the candidate itemsets
- The bottleneck of Apriori: **candidate generation**
  - Huge candidate sets:
    - A 10^4-sized frequent 1-itemset will generate 10^7 candidate 2-itemsets
    - To discover a frequent pattern of size 100, e.g., $\{a_1, a_2, \ldots, a_{100}\}$, one needs to generate $2^{100} \approx 10^{30}$ candidates.
  - Multiple scans of database table:
    - Needs $(n+1)$ scans, n is the length of the longest pattern

Methods to Improve Apriori's Efficiency

Advanced

- Transaction reduction
  - A transaction that does not contain any frequent k-itemset is useless in subsequent scans
- Partitioning
  - Any itemset that is potentially frequent in transaction table T must be frequent in at least one of the partitions of T.

Summary

- Association rule definition
  - Support, confidence, lift and association rule
  - Frequent itemsets
  - Steps for association rule mining
- Apriori algorithm
- Deriving association rules from frequent itemsets

Exercises

1. Refer to the transaection table to the right. Say sup(ab)=100
   - Determine the possible values of sup(a)
     - Conclusion: sup(a)___100
     - *Hint*: Is it possible that sup(a)=70? Why?
   - Determine the possible values of sup(abc)
     - Conclusion: sup(abc)___100
     - *Hint*: Is it possible that sup(abc)=120? Why?

Choose either "≤" or "≥"

Transaction table (1000 rows)

TID	Items
1	a,b,c
2	a,c
3	b,e,f
...	......

2. Write a Jupyter Notebook to find the association rules from the Bread Basket dataset (in Moodle)
   - Use the template provided in Moodle (Lecture9_Exercise_BreadBasket_template.ipynb)

Readings and References

- Mandatory readings
  - Association Rule: https://www.geeksforgeeks.org/association-rule/?ref=lbp
  - Frequent Itemsets: https://www.geeksforgeeks.org/frequent-item-set-in-data-set-association-rule-mining/?ref=lbp
  - Apriori Algorithm: https://www.geeksforgeeks.org/apriori-algorithm/?ref=lbp
- Further readings
  - Documentation of mlxtend's frequent
    - http://rasbt.github.io/mlxtend/user_guide/frequent_patterns/apriori/
    - http://rasbt.github.io/mlxtend/user_guide/frequent_patterns/association_rules/
  - Tutorials
    - https://www.geeksforgeeks.org/implementing-apriori-algorithm-in-python/
    - https://www.kaggle.com/code/annettecatherinepaul/apriori-algorithm-association-rule-mining
    - https://towardsdatascience.com/understand-and-build-fp-growth-algorithm-in-python-d8b989bab342 (FP-Growth, advanced)

Even Further Readings

Advanced

- Readings (optional. Only if you're interested in theory)
  - Jiawei Han, Micheline Kamber and Jian Pei. Data Mining: Concepts and Techniques (3rd edition), Elsevier Science Ltd, 2011.
    - Chapters 6 and 7 in the textbook
  - Rakesh Agrawal, Ramakrishnan Srikant: Fast Algorithms for Mining Association Rules in Large Databases. VLDB 1994: 487-499
  - Jiawei Han, Jian Pei, Yiwen Yin: Mining Frequent Patterns without Candidate Generation. SIGMOD 2000: 1-12
- Acknowledgment: Slides are from
  - Margaret H. Dunham (Data Mining: Introductory and Advanced Topics, Prentice Hall, 2002)
  - The HKP textbook
  - Man Lung Yiu and Panagiotis Karras