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4/16
MA232 Hw 7
"I pledge my honor that I have abided by the Stevens Honor System" - Himanshu Rana

<u>4.3</u>

1) $A^TA\bar{x}=A^Tb$ $\quad A=\begin{bmatrix}1&0\\1&1\\1&3\\1&4\end{bmatrix}$ $\quad b=\begin{bmatrix}0\\8\\8\\20\end{bmatrix}$ $\quad A^TA=\begin{bmatrix}1&1&1&1\\0&1&3&4\end{bmatrix}\begin{bmatrix}1&0\\1&1\\1&3\\1&4\end{bmatrix}=\begin{bmatrix}4&8\\8&26\end{bmatrix}$

$A^Tb=\begin{bmatrix}1&1&1&1\\0&1&3&4\end{bmatrix}\begin{bmatrix}0\\8\\8\\20\end{bmatrix}=\begin{bmatrix}36\\112\end{bmatrix}$ $\quad \begin{bmatrix}4&8\\8&26\end{bmatrix}\begin{bmatrix}C\\D\end{bmatrix}=\begin{bmatrix}36\\112\end{bmatrix}$

$4C+8D=36$ $\quad 8C+26D=112$ $\quad \bar{x}=\begin{bmatrix}1\\4\end{bmatrix}$

$4C+32=36$ $\quad (8C+26D)-2(4C+8D)=40$

$C=1$ $\quad 10D=40 \Rightarrow D=4$

$p=A\bar{x}=\begin{bmatrix}1&0\\1&1\\1&3\\1&4\end{bmatrix}\begin{bmatrix}1\\4\end{bmatrix}=\begin{bmatrix}1\\5\\13\\17\end{bmatrix}$ $\quad e=b-p$ $\begin{bmatrix}0\\8\\8\\20\end{bmatrix}-\begin{bmatrix}1\\5\\13\\17\end{bmatrix}=\begin{bmatrix}-1\\3\\-5\\3\end{bmatrix}$

$E^2=(-1)^2+(3)^2+(-5)^2+3^2=44$

6) $b=(0,8,8,20)$ $\quad a=(1,1,1,1)$ $\quad \bar{x}=\dfrac{a^Tb}{a^Ta}=\dfrac{36}{4}=9$

$\begin{bmatrix}1\\1\\1\\1\end{bmatrix}\begin{bmatrix}0&8&8&20\end{bmatrix}=36$ $\quad \begin{bmatrix}1\\1\\1\\1\end{bmatrix}\begin{bmatrix}1&1&1&1\end{bmatrix}=4$

$p=9\begin{bmatrix}1&1&1&1\end{bmatrix}=\begin{bmatrix}9&9&9&9\end{bmatrix}$ $\quad e=b-p=\begin{bmatrix}-9\\-1\\-1\\11\end{bmatrix}$

$e^Ta=\begin{bmatrix}-9&-1&-1&11\end{bmatrix}\begin{bmatrix}1\\1\\1\\1\end{bmatrix}=0$ $\quad ,\sqrt{81+1+1+121}=\sqrt{204}$

9) $Ax=b$ $\quad \begin{bmatrix}1&0&0\\1&1&1\\1&3&9\\1&4&16\end{bmatrix}\begin{bmatrix}C\\D\\E\end{bmatrix}=\begin{bmatrix}0\\8\\8\\20\end{bmatrix}$ $\quad C=0,\ C+D+E=8,$ $C+3D+9E=8,\ C+4D+16E=20$

$A^TAx=A^Tb \Rightarrow \begin{bmatrix}1&1&1&1\\0&1&3&4\\0&1&9&16\end{bmatrix}\begin{bmatrix}1&0&0\\1&1&1\\1&3&9\\1&4&16\end{bmatrix}\begin{bmatrix}C\\D\\E\end{bmatrix}=\begin{bmatrix}1&1&1&1\\0&1&3&4\\0&1&9&16\end{bmatrix}\begin{bmatrix}0\\8\\8\\20\end{bmatrix}$

$\begin{bmatrix}4&8&26\\8&26&92\\26&92&338\end{bmatrix}\begin{bmatrix}C\\D\\E\end{bmatrix}=\begin{bmatrix}36\\112\\400\end{bmatrix}$ $\quad$ $4C+8D+26E=36$, $8C+26D+92E=112$, $26C+92D+338E=400$

12) a) $a^T a \bar{x} = a^T b$ $[1, \dots, 1]\begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}\bar{x} = [1, \dots, 1]\begin{bmatrix} b_1 \\ \vdots \\ b_m \end{bmatrix}$

$m\bar{x} = b_1 + b_2 + \cdots + b_m$

$\bar{x} = \dfrac{b_1 + b_2 + \cdots + b_m}{m}$

b) $e = b - a\bar{x}$ $\quad \|e\|^2 = \sum_{i=1}^{m} (b_i - \text{mean})^2 = \sigma^2$

c) $e = b - a\bar{x} \Rightarrow (1,2,6) - \frac{1+2+6}{3}(1,1,1)$

$(1,2,6) - 3(1,1,1) = (1,2,6) - (3,3,3)$

$(-2,-1,3)$

$P = \dfrac{aa^T}{a^Ta} = \dfrac{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}[1\ 1\ 1]}{[1\ 1\ 1]\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

22) $C + Dt$ to fit $b = 4, 2, -1, 0, 0$ at $t = -2, -1, 0, 1, 2$

$Ax = b$ $\quad \begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ -1 \\ 0 \\ 0 \end{bmatrix}$ $\quad x = (A^TA)^{-1}A^Tb$

$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -2 & -1 & 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 0 & 10 \end{bmatrix}^{-1} = \frac{1}{50}\begin{bmatrix} 10 & 0 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -2 & -1 & 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \\ -1 \\ 0 \\ 0 \end{bmatrix}$

$= \frac{1}{10}\begin{bmatrix} 2 & 2 & 2 & 2 & 2 \\ -2 & -1 & 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \\ -1 \\ 0 \\ 0 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 10 \\ -10 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ $\quad C = 1 \quad D = 1$

$\boxed{1 - t}$

4.4

2) $(2,2,-1)$ $(-1,2,2)$ $\quad \dfrac{(2,2,-1)}{\sqrt{4+4+1}} = \left(\frac{2}{3}, \frac{2}{3}, -\frac{1}{3}\right)$ $\quad Q = \begin{bmatrix} 2/3 & -1/3 \\ 2/3 & 2/3 \\ -1/3 & 2/3 \end{bmatrix}$

$\dfrac{(-1,2,2)}{\sqrt{1+4+4}} = \left(\frac{-1}{3}, \frac{2}{3}, \frac{2}{3}\right)$

$Q^TQ = \begin{bmatrix} 2/3 & 2/3 & -1/3 \\ -1/3 & 2/3 & 2/3 \end{bmatrix}\begin{bmatrix} 2/3 & -1/3 \\ 2/3 & 2/3 \\ -1/3 & 2/3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad QQ^T = \frac{1}{9}\begin{bmatrix} 5 & 2 & -4 \\ 2 & 8 & 2 \\ -4 & 2 & 5 \end{bmatrix}$

18) $a=(1,-1,0,0)$ $b=(0,1,-1,0)$ $c=(0,0,1,-1)$

$w_1 = \begin{bmatrix}1\\-1\\0\\0\end{bmatrix}$ $w_2 = \begin{bmatrix}0\\1\\-1\\0\end{bmatrix} - \dfrac{\begin{bmatrix}-1\\1\\0\\0\end{bmatrix}\begin{bmatrix}-1\\1\\0\\0\end{bmatrix}}{\begin{bmatrix}1\\-1\\0\\0\end{bmatrix}\begin{bmatrix}1\\-1\\0\\0\end{bmatrix}}\begin{bmatrix}1\\-1\\0\\0\end{bmatrix} = \begin{bmatrix}0\\1\\-1\\0\end{bmatrix} - \dfrac{(-1)}{2}\begin{bmatrix}1\\-1\\0\\0\end{bmatrix}$

$= \begin{bmatrix}1/2\\1/2\\-1\\0\end{bmatrix}$

$w_3 = \begin{bmatrix}0\\0\\1\\-1\end{bmatrix} - \dfrac{\begin{bmatrix}0\\0\\1\\-1\end{bmatrix}\begin{bmatrix}1\\-1\\0\\0\end{bmatrix}}{\begin{bmatrix}1\\-1\\0\\0\end{bmatrix}\begin{bmatrix}1\\-1\\0\\0\end{bmatrix}}\begin{bmatrix}1\\-1\\0\\0\end{bmatrix} - \dfrac{\begin{bmatrix}0\\0\\1\\-1\end{bmatrix}\begin{bmatrix}0\\1\\-1\\0\end{bmatrix}}{\begin{bmatrix}0\\1\\-1\\0\end{bmatrix}\begin{bmatrix}0\\1\\-1\\0\end{bmatrix}}\begin{bmatrix}0\\1\\-1\\0\end{bmatrix} = \begin{bmatrix}1/3\\1/3\\1/3\\-1\end{bmatrix}$

19) lower triangular times upper triangular

$A = \begin{bmatrix}-1 & 1\\2 & 1\\2 & 4\end{bmatrix} = \frac{1}{3}\begin{bmatrix}-1 & 1\\2 & 1\\2 & 4\end{bmatrix}\begin{bmatrix}3 & 3\\0 & 3\end{bmatrix}$ $A^TA = \begin{bmatrix}9 & 9\\9 & 18\end{bmatrix} = \begin{bmatrix}3 & 0\\3 & 3\end{bmatrix}\begin{bmatrix}3 & 3\\0 & 3\end{bmatrix} = R^TR$

20) a) True: $Q^TQ = I \Rightarrow (Q^{-1})(Q^{-1}) = I$

b) True: $\|Qx\|^2 = x_1 + x_2$ b/c $q_1 \cdot q_2 = 0$

$= x^TQ^TQx = x^Tx$

$\begin{bmatrix}3\\2\end{bmatrix}$

23) $A = \begin{bmatrix}1 & 2 & 4\\0 & 0 & 5\\0 & 3 & 6\end{bmatrix}$ $w_1 = \begin{bmatrix}1\\0\\0\end{bmatrix}$

$w_2 = \begin{bmatrix}2\\0\\3\end{bmatrix} - \dfrac{\begin{bmatrix}2\\0\\3\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix}}{\begin{bmatrix}1\\0\\0\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix}}\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}2\\0\\3\end{bmatrix} - \begin{bmatrix}2\\0\\0\end{bmatrix} = \begin{bmatrix}0\\0\\3\end{bmatrix}$

$w_3 = \begin{bmatrix}4\\5\\6\end{bmatrix} - \dfrac{\begin{bmatrix}4\\5\\6\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix}}{\begin{bmatrix}1\\0\\0\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix}}\begin{bmatrix}1\\0\\0\end{bmatrix} - \dfrac{\begin{bmatrix}4\\5\\6\end{bmatrix}\begin{bmatrix}2\\0\\3\end{bmatrix}}{\begin{bmatrix}2\\0\\3\end{bmatrix}\begin{bmatrix}2\\0\\3\end{bmatrix}}\begin{bmatrix}2\\0\\3\end{bmatrix} = \begin{bmatrix}4\\5\\6\end{bmatrix} - \begin{bmatrix}4\\0\\0\end{bmatrix} - \begin{bmatrix}0\\0\\6\end{bmatrix}$

$= \begin{bmatrix}0\\5\\0\end{bmatrix}$

$q_1 = \frac{1}{\sqrt{1}}\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}1\\0\\0\end{bmatrix}$

$q_2 = \frac{1}{\sqrt{9}}\begin{bmatrix}0\\0\\3\end{bmatrix} = \begin{bmatrix}0\\0\\1\end{bmatrix}$

$q_3 = \frac{1}{\sqrt{25}}\begin{bmatrix}0\\5\\0\end{bmatrix} = \begin{bmatrix}0\\1\\0\end{bmatrix}$

$\begin{bmatrix}1 & 0 & 0\\0 & 0 & 1\\0 & 1 & 0\end{bmatrix}\begin{bmatrix}1 & 2 & 4\\0 & 0 & 5\\0 & 3 & 6\end{bmatrix}$