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2/12
MA 232 HW 3
"I pledge my honor that I have abided by the Stevens Honor System" - Himanshu Rana

2.6

7) $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix}$ $\xrightarrow{R_2 \to R_2 - 2R_1,\ R_3 \to R_3 - 3R_1}$ $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 4 & 2 \end{bmatrix}$

$\xrightarrow{R_3 \to R_3 - 2R_2}$ $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = U$

$E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$ $E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

$L = E_{21}^{-1} E_{31}^{-1} E_{32}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$

13) $A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}$ $\xrightarrow{R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1,\ R_4 \to R_4 - R_1}$ $\begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & b-a & c-a & c-a \\ 0 & b-a & c-a & d-a \end{bmatrix}$

$\xrightarrow{R_3 \to R_3 - R_2,\ R_4 \to R_4 - R_2}$ $\begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & c-b & d-b \end{bmatrix}$ $\xrightarrow{R_4 \to R_4 - R_3}$ $\begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & 0 & d-c \end{bmatrix} = U$

$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

$a \neq 0$
$b \neq a$
$c \neq b$
$d \neq c$

16) $L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ $U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ $b = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$

$Lc = b \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ $c_1 = 4$, $c_2 = 1$, $c_3 = 1$

$Ux = c \rightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix}$ $x_1 = 3$, $x_2 = 0$, $x_3 = 1$

$LU = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix} = A$

2.7

21) $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$ $AB = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

$A^T = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ $B^T = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ $(AB)^T = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$

$B^T \cdot A^T = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix} = (AB)^T$

$A^T \cdot B^T = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 2 \\ 3 & 1 \end{bmatrix} \neq (AB)^T$

$AA^T = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$ $A^TA = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$

7) a) False - only if $A = A^T$
b) False - $(AB)^T = B^T A^T$
c) True - $AA^{-1} = I$
d) True - $(ABC)^T = C^T B^T A^T$

16) $A = A^T$ and $B = B^T$, then a) and c) are certainly symmetric because $(A^2 - B^2)^T = (AA)^T - (BB)^T = A^T A^T - B^T B^T = AA - BB$ and $(ABA)^T = (BA)^T A^T = A^T B^T A^T = ABA$

22) $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix} \xrightarrow{PA} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \xrightarrow{R_3 \to R_3 - 2R_1} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 3 & 2 \end{bmatrix}$

$\xrightarrow{R_3 \to R_3 - 3R_2} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix} = U \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix} \quad PA = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad PA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 4 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 1 \\ 2 & 4 & 1 \end{bmatrix} \xrightarrow{\substack{R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 2R_1}}$

$\to \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = U \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \quad PA = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

3.1

2) The rule that would be broken is (5) because $1 \cdot (2,3)$ should equal $(2,0)$ but rule (5) says $1 \cdot (2,3) = (2,3)$

10) (a) - let $\vec{v} = (a, a, b)$ and $\vec{w} = (c, c, d)$
$v+w = (a+c, a+c, b+d)$ and $cv = (ca, ca, cb)$
are both in the subspace

(d) - $\vec{v} = (1, 4, 0)$ and $\vec{w} = (2, 2, 2)$
$\vec{v}+\vec{w}$ and $c\vec{v}$ are all in the subset with all linear combinations

(e) - let $\vec{v} = (a, b, h)$ and $\vec{w} = (e, f, g)$
$\vec{v}+\vec{w} = (a+e, b+f, h+g) \rightarrow (a+b+h) + (e+f+g) = 0$
$c\vec{v} = (ca, cb, ch) = c(a+b+h) = 0$

14) a) all three types of $\mathbb{R}^2$
1) $Z = (0, 0)$
2) lines going through the origin
3) $\mathbb{R}^2$ itself

b) all subspaces of D
1) D itself
2) zero matrix
3) 1D matrices multiplied by a constant