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3/24
MA233 Hw #6
"I pledge my honor that I have abided by the Stevens Honor System" - Himanshu Rana

3.4

20) $x - 2y + 3z = 0$ in $\mathbb{R}^3$ $\quad A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

normal vector $= \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ basis for vectors $\perp$ plane

$v_1 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$ $v_2 = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$ $\rightarrow$ orthogonal to normal vector

intersection of plane $\rightarrow x = 2y$ $\quad$ free var $= y$ $\quad \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

38) a) two vectors cannot span $\mathbb{R}^3$

b) If you have four vectors, then they have to be dependent in $\mathbb{R}^3$

c) $\left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 2 & 2 & 8 & 0 \\ 2 & 1 & 0 & 0 \end{array}\right] \xrightarrow[R_3 \to R_3 - 2R_1]{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 4 & 8 & 0 \\ 0 & 3 & 0 & 0 \end{array}\right]$

$\xrightarrow{R_2 \to R_2 - \frac{4}{3}R_3} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 3 & 0 & 0 \end{array}\right] \begin{array}{l} R_2 \to \frac{1}{8}R_2 \\ R_3 \to \frac{1}{3}R_3 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right]$

$\xrightarrow{R_1 \to R_1 + R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right]$ all vectors are independent

d) vectors are dependent

3.5 13

2) $A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 5 & 8 \end{bmatrix}$

$A \xrightarrow{R_2 - R_1} \begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & 0 \end{bmatrix}$ row space basis = $(1, 2, 4)$

$N(A) = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix}$ Column space basis = $(1,2)$

$B \xrightarrow{R_2 - 2R_1} \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 0 \end{bmatrix}$ row space basis = $(1,2,4), (2,5,8)$
Column space basis $(1,2), (2,5)$

$N(B) = (-4, 0, 1)$

13) a) False: row space and column space don't have same dimension
b) True
c) False: if you have matrices A and B that are the same size and invertible, then they have same 4 subspaces

4.1

17) if $S = \vec{0}$ then $S^{\perp} = \mathbb{R}^3$

if $S = \text{span}\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right)$ then $S^{\perp} = (-1, 1, 0), (-1, 0, 1)$

$x_1 + x_2 + x_3 = 0$ $x_1 = -x_2 + -x_3$ $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$x_2 = x_2$ $\rightarrow$ $x_2 = x_2$

$x_3 = x_3$ $x_3 = x_3$

if $S = \text{span}\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}\right)$ then $S^{\perp} = (-1, 1, 0)$

$x_1 + x_2 = 0$ $\rightarrow$ $x_1 + x_2 = 0$ $\rightarrow$ $x_1 = -x_2$ $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

$x_3 = 0$ $x_2 = 0$ $x_2 = x_2$

$x_3 = 0$ $x_3 = 0$

21) $S = \text{span}\left(\begin{bmatrix}1\\2\\2\\3\end{bmatrix}, \begin{bmatrix}1\\3\\3\\2\end{bmatrix}\right)$ $A = \begin{bmatrix}1 & 2 & 2 & 3\\1 & 3 & 3 & 2\end{bmatrix}$

$A \xrightarrow{R_2 \to R_2 - R_1} \begin{bmatrix}1 & 2 & 2 & 3\\0 & 1 & 1 & -1\end{bmatrix} \xrightarrow{R_1 \to R_1 - 2R_2} \begin{bmatrix}1 & 0 & 0 & 5\\0 & 1 & 1 & -1\end{bmatrix}$

$S^\perp = (0,-1,1,0), (-5,1,0,1)$

22) P = plane in $\mathbb{R}^4$ where $x_1 + x_2 + x_3 + x_4 = 0$
normal vector for plane P is $n = \begin{bmatrix}1\\1\\1\\1\end{bmatrix}$
basis = $(1,1,1,1)$ for line $P^\perp$ orthogonal to P

26) a) there could be other vectors that both spaces share
b) dim = 5, needs more vectors to span orthogonal subspace
c) true for disjoint sets, lines could interesect at 0 w/o 90°

4.2

1) a) $A = \begin{bmatrix}1 & 1\\0 & 1\\0 & 0\end{bmatrix}$ $b = \begin{bmatrix}2\\3\\4\end{bmatrix}$ $A^TA\bar{x} = A^Tb$ $p = A\bar{x}$

$A^TA = \begin{bmatrix}1 & 0 & 0\\1 & 1 & 0\end{bmatrix}\begin{bmatrix}1 & 1\\0 & 1\\0 & 0\end{bmatrix} = \begin{bmatrix}1 & 1\\1 & 2\end{bmatrix}$ $= \begin{bmatrix}2 & -1\\-1 & 1\end{bmatrix}\begin{bmatrix}2\\5\end{bmatrix} = \begin{bmatrix}-1\\3\end{bmatrix}$

$A^Tb = \begin{bmatrix}1 & 0 & 0\\1 & 1 & 0\end{bmatrix}\begin{bmatrix}2\\3\\4\end{bmatrix} = \begin{bmatrix}2\\5\end{bmatrix}$ $p = \begin{bmatrix}1 & 1\\0 & 1\\0 & 0\end{bmatrix}\begin{bmatrix}-1\\3\end{bmatrix} = \begin{bmatrix}2\\3\\0\end{bmatrix}$

$\begin{bmatrix}2\\3\\4\end{bmatrix} - \begin{bmatrix}2\\3\\0\end{bmatrix} = \begin{bmatrix}0\\0\\4\end{bmatrix}$

b) $A = \begin{bmatrix}1 & 1\\1 & 1\\0 & 1\end{bmatrix}$ $b = \begin{bmatrix}4\\4\\6\end{bmatrix}$ $A^TA x = A^Tb$ $p = A\bar{x}$

$A^TA = \begin{bmatrix}1 & 1 & 0\\1 & 1 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\1 & 1\\0 & 1\end{bmatrix} = \begin{bmatrix}2 & 2\\2 & 3\end{bmatrix}$ $A^Tb = \begin{bmatrix}1 & 1 & 0\\1 & 1 & 1\end{bmatrix}\begin{bmatrix}4\\4\\6\end{bmatrix} = \begin{bmatrix}8\\14\end{bmatrix}$

$\frac{1}{2}\begin{bmatrix}3 & -2\\-2 & 2\end{bmatrix}\begin{bmatrix}8\\14\end{bmatrix} = \begin{bmatrix}-2\\6\end{bmatrix}$ $p = A\bar{x} = \begin{bmatrix}1 & 1\\1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}-2\\6\end{bmatrix} = \begin{bmatrix}4\\4\\6\end{bmatrix}$

$\begin{bmatrix}4\\4\\6\end{bmatrix} - \begin{bmatrix}4\\4\\6\end{bmatrix} = \bar{0}$

12) $P = A(A^TA)^{-1}A^T$

a) $(A^TA)^{-1} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$

$P = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

b) $(A^TA)^{-1} = \begin{bmatrix} 2 & 2 \\ 2 & 3 \end{bmatrix}^{-1} = \frac{1}{2}\begin{bmatrix} 3 & -2 \\ -2 & 2 \end{bmatrix}$

$\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

13) $P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $\quad p = Pb = P\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \end{bmatrix}$

16) $A = \begin{bmatrix} 1 & 1 \\ 2 & 0 \\ -1 & 1 \end{bmatrix}$ $\quad A^TA = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 2 & 0 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 2 \end{bmatrix}$

$x = (A^TA)^{-1}A^Tb =$

$\begin{bmatrix} \frac{1}{6} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 3 \\ 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{3}{2} \end{bmatrix}$

19) $N(A) = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ $\quad A = \begin{bmatrix} 1 & 2 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^TA = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 5 \end{bmatrix}$

$(A^TA)^{-1} = \frac{1}{6}\begin{bmatrix} 5 & -2 \\ -2 & 2 \end{bmatrix}$ $\quad P = A(A^TA)^{-1}A^T =$

$\frac{1}{6}\begin{bmatrix} 1 & 2 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

$= \frac{1}{6}\begin{bmatrix} 1 & 2 \\ 5 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 5 & 1 & 2 \\ 1 & 5 & -2 \\ 2 & -2 & 2 \end{bmatrix}$