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2/7
MA 232 Hw #5
"I pledge my honer that I have abided by the stevens Honer System" - Himanshu Rana

3.3

1) $A = \begin{bmatrix} 2 & 4 & 6 & 4 \\ 2 & 5 & 7 & 6 \\ 2 & 3 & 5 & 2 \end{bmatrix}$ $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$

$$\left[\begin{array}{cccc|c} 2 & 4 & 6 & 4 & b_1 \\ 2 & 5 & 7 & 6 & b_2 \\ 2 & 3 & 5 & 2 & b_3 \end{array}\right] \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - R_1} \left[\begin{array}{cccc|c} 2 & 4 & 6 & 4 & b_1 \\ 0 & 1 & 1 & 2 & b_2 - b_1 \\ 0 & -1 & -1 & -2 & b_3 - b_1 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 + R_2} \left[\begin{array}{cccc|c} 2 & 4 & 6 & 4 & b_1 \\ 0 & 1 & 1 & 2 & b_2 - b_1 \\ 0 & 0 & 0 & 0 & b_3 + b_2 - 2b_1 \end{array}\right]$$

column space gives vector $\begin{bmatrix} 2 & 4 \\ 2 & 5 \\ 2 & 3 \end{bmatrix}$

$N(A) = \begin{bmatrix} -1 & 2 \\ -1 & -2 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ $A\bar{x} = \bar{b}$ $\begin{bmatrix} 2 & 4 & 6 & 4 & 4 \\ 0 & 1 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ $x = \begin{bmatrix} 4 \\ -1 \\ 0 \\ 0 \end{bmatrix}$

3) $x + 3y + 3z = 1$
$2x + 6y + 9z = 5$
$-x - 3y + 3z = 5$

$$\to \left[\begin{array}{ccc|c} 1 & 3 & 3 & 1 \\ 2 & 6 & 9 & 5 \\ -1 & -3 & 3 & 5 \end{array}\right] \xrightarrow[R_3 \to R_3 + R_1]{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 3 & 3 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 6 & 6 \end{array}\right]$$

$$\xrightarrow[R_3 \to R_3/6]{R_2 \to R_2/3} \left[\begin{array}{ccc|c} 1 & 3 & 3 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right] \xrightarrow{R_3 \to R_2 - R_3} \left[\begin{array}{ccc|c} 1 & 3 & 3 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 - 3x_2 - 3 \\ x_2 \\ 1 \end{bmatrix} \quad \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} + s\begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}$$

12) a) if $Ax_1 = b$ and $Ax_2 = b$ then $x_1 - x_2$ and $x = 0$ are solutions
b) $A(2x_1 - 2x_2) = b$ and $A(2x_1 - 2x_2) = 0$

3.4

1) $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $v_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $v_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $v_4 = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$

$A = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow[R_2 \to R_2 - R_3]{R_1 \to R_1 - R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \to \begin{array}{l} c_1 = 0 \\ c_2 = 0 \\ c_3 = 0 \end{array}$

$Ac = 0 \to v_1 \cdot 0 + v_2 \cdot 0 + v_3 \cdot 0 = 0 \to$ linearly independent

$A = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & 4 \end{bmatrix} \to \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 4 & 0 \end{array}\right]$ $c = (1, 1, -4, 1)$
$v_1 + v_2 - 4v_3 + v_4 = 0$

2) $v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$ $v_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$ $v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$ $v_4 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}$ $v_5 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}$ $v_6 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$

$v_4 = v_2 - v_1$, $v_5 = v_3 - v_1$, and $v_6 = v_3 - v_2$ which means v_4, v_5, v_6 are dependent on v_1, v_2, v_3

$A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \xrightarrow{R_2 \to R_2 + R_1} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \xrightarrow{R_3 \to R_3 + R_2} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} \xrightarrow{R_4 \to R_4 + R_3} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

# of pivots = 3, v_1, v_2, v_3 are independent, largest # $= \boxed{3}$

7) $av_1 + bv_2 + cv_3 = 0$
$a(w_2 - w_3) + b(w_1 - w_3) + c(w_1 - w_2) = 0$
$w_1(b + c) + w_2(a - c) + w_2(-a - b) = 0 \to a = 1, b = -1, c = 1$
$v_1 - v_2 + v_3 = 0$ $[v_1, v_2, v_3]$ is singular

10) $x + 2y - 3z - t = 0$

$$v_1 ; v_2 = \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{array}\right] \xrightarrow[R_1 \to R_1 + 2R_2]{R_1 \to R_1 + R_4} \begin{bmatrix} 0 & 0 \\ 0 & -1 \\ 0 & 0 \\ -1 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \\ 0 \end{bmatrix} \text{ are independent}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{array}\right] \xrightarrow[R_2 \to R_2 - R_3]{R_1 \to R_1 + R_4} \left[\begin{array}{ccc|c} 0 & 2 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{array}\right] \xrightarrow[R_1 \to R_1 + 2R_2]{R_1 \to R_1 - R_3} \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \end{array}\right]$$

$\begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$ are independent vectors on the plane

16) a) $\alpha(1,1,1,1)$ $\alpha \in \mathbb{R}$ $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

b) $\begin{bmatrix} x \\ y \\ z \\ -x-y-z \end{bmatrix} = x \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} + y \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} + z \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$ $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} \right\}$

c) $A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix} \Rightarrow Ax = \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{array}\right] \xrightarrow{R_1 \to R_1 - R_2} \left[\begin{array}{cccc|c} 0 & 1 & -1 & -1 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{array}\right]$

$x = \begin{bmatrix} -x_3 - x_4 \\ x_3 + x_4 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$ $\left\{ \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

d) $N(I) = \bar{0}$, columns of I is the basis for the column space

20) $x - 2y + 3z = 0$ in $\mathbb{R}^3$

normal vector $= \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ basis for vectors $\perp$ plane

$v_1 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$ $v_2 = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$ $\rightarrow$ orthogonal to normal vector

intersection of plane $x - y \rightarrow x = 2y$ free variable: y

$\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

38) a) two vectors cannot span $\mathbb{R}^3$

b) If you have four vectors, then they are dependent in $\mathbb{R}^3$

c)

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 2 & 2 & 8 & 0 \\ 2 & 1 & 0 & 0 \end{array}\right] \xrightarrow[R_3 \to R_3 - 2R_1]{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 4 & 8 & 0 \\ 0 & 3 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 \to R_2 - \frac{4}{3}R_3} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 3 & 0 & 0 \end{array}\right] \xrightarrow[R_3 \to \frac{1}{3}R_3]{R_2 \to \frac{1}{8}R_2} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{R_1 \to R_1 + R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right]$$ all vectors are independent

d) vectors are dependent