Himanshu Rana
4/25
MA 232 Hw #8
"I pledge my honor that I have abided by the Stevens Honor System" - Himanshu

5.1

1) $\det(A) = \frac{1}{2}$ $\quad \det(2A) = 2^4\left(\frac{1}{2}\right) = 8 \quad \det(A^2) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$\det(-A) = (-1)^4\left(\frac{1}{2}\right) = \frac{1}{2} \quad \det(A^{-1}) = \frac{1}{1/2} = 2$

2) $\det(A) = -1$ $\quad \det\left(\frac{1}{2}A\right) = \left(\frac{1}{2}\right)^3(-1) = -1/8 \quad \det(A^2) = (-1)^2 = 1$

$\det(-A) = (-1)^3(-1) = 1 \quad \det(A^{-1}) = -1/1 = -1$

13) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix} \xrightarrow{R_3 \to R_3 - R_2} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_2 \to R_2 - R_1} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ $\det(A) = 1$

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 3 \\ 3 & 3 & 3 \end{bmatrix} \xrightarrow{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -3 \\ 3 & 3 & 3 \end{bmatrix} \xrightarrow[R_3 \to R_3 - \left(\frac{3}{2}\right)R_2]{R_3 \to R_3 - 3R_1} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -3 \\ 0 & 0 & -3/2 \end{bmatrix}$ $\det(A) = 1(-2)\left(-\frac{3}{2}\right) = 3$

28) a) True - $\det(A) = 0 \quad \det(A)\det(B) = 0$

b) False - if you do a row exchange then $\det(A) = -\det(A)$

c)

d) True - $\det(AB) = \det(A)\det(B) = \det(BA)$

6.1

3) $A = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix} \to \begin{bmatrix} -\lambda & 2 \\ 1 & 1-\lambda \end{bmatrix} = 0$ $\quad (-\lambda)(1-\lambda) - 2 = 0$

$-\lambda + \lambda^2 - 2 = 0$

$\lambda^2 - \lambda - 2 = 0$

$\lambda_1 = -1 \quad \lambda_2 = 2$

$\lambda_1 = -1 = \begin{bmatrix} -1 & 2 \\ 1 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \vec{x} = 0 \quad x_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

$\lambda_2 = 2 = \begin{bmatrix} -2 & 2 \\ 1 & -1 \end{bmatrix} \to \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \vec{x} = 0 \quad x_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -1/2 & 1 \\ 1/2 & 0 \end{bmatrix} \to \begin{bmatrix} -1/2-\lambda & 1 \\ 1/2 & -\lambda \end{bmatrix} \quad -\lambda(-\frac{1}{2}-\lambda) - \frac{1}{2} = 0$

$\frac{1}{2}\lambda + \lambda^2 - \frac{1}{2} = 0$

$\lambda^2 + \frac{1}{2}\lambda - \frac{1}{2} = 0 \quad \lambda_1 = \frac{1}{2} \quad \lambda_2 = -1$

$\lambda_1 = \frac{1}{2} = \begin{bmatrix} -1 & 1 \\ 1/2 & -1/2 \end{bmatrix} \to$

$\lambda_2 = -1 = \begin{bmatrix} \frac{1}{2} & 1 \\ 1/2 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \vec{x} = 0 \quad x + 2y = 0 \quad x_2 = \begin{bmatrix} 1 \\ -1/2 \end{bmatrix}$

$x = 1 \quad y = -1/2$

A^{-1} has the same eigenvectors, with eigenvalues $1/\lambda = \frac{1}{2}$ and -1

10) $A = \begin{bmatrix} .6 & .2 \\ .4 & .8 \end{bmatrix} \quad \det(A - \lambda I) = \begin{bmatrix} .6-\lambda & .2 \\ .4 & .8-\lambda \end{bmatrix}$

$\lambda^2 - 1.4\lambda + .4 \Rightarrow \lambda_1 = .4 \quad \lambda_2 = 1$

$A^\infty = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ 2/3 & 2/3 \end{bmatrix} \to \begin{bmatrix} 1/3-\lambda & 1/3 \\ 2/3 & 2/3-\lambda \end{bmatrix} \quad \lambda^2 - \lambda = 0$

$\lambda_1 = 0 \quad \lambda_2 = 1$

$\lambda = .4 \begin{bmatrix} .2 & .2 \\ .4 & .4 \end{bmatrix} \quad x = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad \lambda = 1 \begin{bmatrix} -.4 & .2 \\ .4 & -.2 \end{bmatrix} \quad x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\lambda = 0 \begin{bmatrix} 1/3 & 1/3 \\ 2/3 & 2/3 \end{bmatrix} \quad x = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \quad \lambda = 1 \begin{bmatrix} -2/3 & 1/3 \\ 2/3 & -1/3 \end{bmatrix} \quad x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

24) $A = \begin{bmatrix} 2 & 1 & 2 \\ 4 & 2 & 4 \\ 2 & 1 & 2 \end{bmatrix} \quad \det(A - \lambda I) = \begin{bmatrix} 2-\lambda & 1 & 2 \\ 4 & 2-\lambda & 4 \\ 2 & 1 & 2-\lambda \end{bmatrix} = (2-\lambda)^3 + 12\lambda - 8 = 0$

$\lambda_1 = 0$ mult. 2

$\lambda_2 = 6$ mult. 1

$\lambda_1 = 0 \to \begin{bmatrix} 2 & 1 & 2 \\ 4 & 2 & 4 \\ 2 & 1 & 2 \end{bmatrix} \quad x = \begin{bmatrix} -1/2 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$\lambda_2 = 6 \to \begin{bmatrix} -4 & 1 & 2 \\ 4 & -4 & 4 \\ 2 & 1 & -4 \end{bmatrix} \quad x = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$