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5/7
MA 232 HW #9
"I pledge my honor that I have abided by the Stevens Honor" - Himanshu Rana

6.2

1) $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1-\lambda & 2 \\ 0 & 3-\lambda \end{bmatrix}$ $(1-\lambda)(3-\lambda) - 0 = 0$ $\lambda = 1, 3$

$\lambda = 1$ $\begin{bmatrix} 0 & 2 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\begin{matrix} x = 1 \\ y = 0 \end{matrix}$ $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$\lambda = 3$ $\begin{bmatrix} -2 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\begin{matrix} x = 1 \\ y = 1 \end{matrix}$ $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$A = \begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1-\lambda & 1 \\ 3 & 3-\lambda \end{bmatrix}$ $(1-\lambda)(3-\lambda) - 3 = 0$ $3 - 4\lambda + \lambda^2 - 3$ $\lambda = 0, 4$

$\lambda = 0 = \begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\begin{matrix} x = 1 \\ y = -1 \end{matrix}$ $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} \frac{3}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{bmatrix}$

$\lambda = 4 = \begin{bmatrix} -3 & 1 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\begin{matrix} x = 1 \\ y = 3 \end{matrix}$ $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

4) a) A is invertible - False no eigenvalues
b) A is diagonalizable - True
c) X is invertible - True
d) X is diagonizable - False need eigenvectors

11) A has $\lambda = 2, 2, 5$
a) invertible - True $\lambda \neq 0$
b) diagonizable - False mult > 1
c) not diagonizable - False

12) eigenvectors are multiples of (1, 4)
a) no inverse - False, there could be $\lambda = 0$
b) repeated eigenvalue - True mult. ≥ 1
c) no diagonalization $X \Lambda X^{-1}$ - True only 1 line of eigenvectors

15) $A_1 = \begin{bmatrix} .6 & .9 \\ .4 & .1 \end{bmatrix}$ $A_2 = \begin{bmatrix} .6 & .9 \\ .1 & .6 \end{bmatrix} \rightarrow \begin{bmatrix} .6-\lambda & .9 \\ .1 & .6-\lambda \end{bmatrix} = (.6-\lambda)^2 - .9 = 0$

$\lambda_1 = .3,\ \lambda_2 = .9$

$|\lambda_i| < 1 \therefore A^k \rightarrow 0$ as $k \rightarrow \infty$

16) $A_1 = \begin{bmatrix} .6 & .9 \\ .4 & .1 \end{bmatrix}$ $\lambda_1 = 1$ $\lambda_2 = .2$ $\lambda_1 = 1$ $\begin{bmatrix} -.4 & .4 \\ .4 & -.4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$x(1,1)$

$X = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ $\Lambda = \begin{bmatrix} 1 & 0 \\ 0 & .2 \end{bmatrix}$ $\lambda_2 = .2 = \begin{bmatrix} .4 & .4 \\ .9 & .4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$x(1,-1)$

$X^{-1} = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{bmatrix}$

$\lim_{k\to\infty} \lambda^k = \lim_{k\to\infty} \begin{bmatrix} 1^k & 0 \\ 0 & (.2)^k \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \Lambda^k$

27) $A = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$ $\lambda_1 = 1,\ \lambda_2 = 9$ $\lambda = 1 \rightarrow \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $\lambda = 9 \rightarrow \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$R = \frac{1}{2}\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

matrix B has a negative λ so when you do $\sqrt{\lambda}$ you get $\pm i$ which are complex numbers

6.4

6) $S = \begin{bmatrix} -2 & 6 \\ 6 & 7 \end{bmatrix}$ $\Delta S = (S - \lambda I) = x^2 - 5x + 50 = (x+5)(x+10)$

$\lambda_1 = -5 \rightarrow \begin{bmatrix} -2 \\ 1 \end{bmatrix} \rightarrow \frac{1}{\sqrt{5}}\begin{bmatrix} -2 \\ 1 \end{bmatrix}$ $\lambda_1 = -5$ $\lambda_2 = 10$

$\lambda_2 = 10 \rightarrow \begin{bmatrix} 1 \\ 2 \end{bmatrix} \rightarrow \frac{1}{\sqrt{5}}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

eigenvalues are distinct so eigenvector is orthogonal

$Q = \begin{bmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & -\frac{1}{\sqrt{5}} \end{bmatrix}$ $Q^T A Q = \Lambda$ $\Lambda = \begin{bmatrix} -5 & 0 \\ 0 & 10 \end{bmatrix}$

7) $S = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & -2 \\ 2 & -2 & 0 \end{bmatrix} \to \begin{bmatrix} 1-\lambda & 0 & 2 \\ 0 & -1-\lambda & 2 \\ 2 & -2 & -\lambda \end{bmatrix} = -\lambda^3 + 9\lambda = -\lambda(\lambda+3)(\lambda-3)$

$\lambda_1 = 0 \quad \lambda_2 = -3 \quad \lambda_3 = 3$

$\lambda_1 = 0 \to \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & -1 & -2 & 0 \\ 2 & -2 & 0 & 0 \end{array}\right]$

23) a) False – $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

b) True – $A^T = Q\Lambda Q^T = A$

c) True – $S^{-1} = Q\Lambda^{-1}Q^T$

d) False – $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $S = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

6.5

12) $S = \begin{bmatrix} c & 1 & 1 \\ 1 & c & 1 \\ 1 & 1 & c \end{bmatrix}$ each leading principle submatrix should have strictly positive determinants

$\begin{bmatrix} c & 1 \\ 1 & c \end{bmatrix} > 0 \quad c^2 - 1 > 0, \quad c(c^2-1) - 1(c-1) + 1(1-c) > 0$

$c^3 - c - c + 1 + 1 - c > 0$

c has to be > 1

$(c-1)(c^2+c-2) > 0$

$(c-1)^2 (c+2) > 0$

$B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & d & 4 \\ 3 & 4 & 5 \end{bmatrix}$ $\quad 1 > 0 \quad d - 4 > 0 \quad 1(5d-16) - 2(-2) + 3(8-3d) > 0$

$d > 4$

$5d - 16 + 4 + 24 - 9d > 0$

$-4d + 12 > 0$

$4d < 12$

$d < 3$

$3 < d < 4$ is not possible so there is no solution, never spd