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2/5
MA 232 Hw 2
"I pledge my honer that I have abided by the Stevens Honer System" - Himanshu Rana

2.1

9) a) $\begin{bmatrix} 1 & 2 & 4 \\ -2 & 3 & 1 \\ -4 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} (1)(2)+(2)(2)+(4)(3) \\ (-2)(2)+(3)(2)+(1)(2) \\ (-4)(2)+(1)(2)+(2)(3) \end{bmatrix} = \begin{bmatrix} 18 \\ 5 \\ 0 \end{bmatrix}$

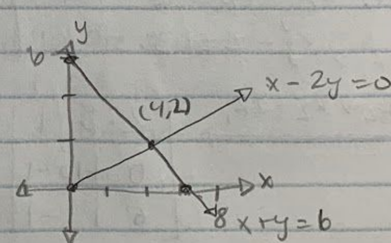
b) $\begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} (2)(1)+(1)(1)+(0)(1)+(0)(2) \\ (1)(1)+(2)(1)+(1)(1)+(0)(2) \\ (0)(1)+(1)(1)+(2)(1)+(1)(2) \\ (0)(1)+(0)(1)+(1)(1)+(2)(2) \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 5 \end{bmatrix}$

10) a) $Ax = 2\begin{bmatrix} 1 \\ -2 \\ -4 \end{bmatrix} + 2\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} + 3\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 18 \\ 5 \\ 0 \end{bmatrix}$ 9 seperate multiplications

b) $Ax = 1\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + 1\begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix} + 1\begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix} + 2\begin{bmatrix} 0 \\ 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \\ 5 \end{bmatrix}$

26) $x - 2y = 0$
$x + y = 6$

$\begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 6 \end{bmatrix}$



2.2

11) A system of linear equations can't have exactly two solutions because it can only have one, none, or infinitely many solutions, meaning when the lines intersect, are parallel, or when they lie on top of each other.

a) $\frac{1}{2}(x+X,\ y+Y,\ z+Z)$ would be another solution

b) They also meet along the line where the two points intersect.

12) $2x+3y+z=8$
$4x+7y+5z=20$
$-2y+2z=0$
$\rightarrow \left[\begin{array}{ccc|c} 2 & 3 & 1 & 8 \\ 4 & 7 & 5 & 20 \\ 0 & -2 & 2 & 0 \end{array}\right] \rightarrow$

$\xrightarrow{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc|c} 2 & 3 & 1 & 8 \\ 0 & 1 & 3 & 4 \\ 0 & -2 & 2 & 0 \end{array}\right] \xrightarrow{R_3 \to R_3 + 2R_2} \left[\begin{array}{ccc|c} 2 & 3 & 1 & 8 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 8 & 8 \end{array}\right]$

$2x+3y+z=8$   $x=2$
$y+3z=4$   $y=1$ $\rightarrow \boxed{(2,1,1)}$
$8z=8$   $z=1$

14) $2x+5y+z=0$
$4x+dy+z=2$
$y-z=3$
$\rightarrow \left[\begin{array}{ccc|c} 2 & 5 & 1 & 0 \\ 4 & d & 1 & 2 \\ 0 & 1 & -1 & 3 \end{array}\right] \rightarrow$

$d-10=0$
$d=10$

$\xrightarrow{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc|c} 2 & 5 & 1 & 0 \\ 0 & d-10 & -1 & 2 \\ 0 & 1 & -1 & 3 \end{array}\right] = \left[\begin{array}{ccc|c} 2 & 5 & 1 & 0 \\ 0 & 0 & -1 & 2 \\ 0 & 1 & 1 & 3 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 2 & 5 & 1 & 0 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & -1 & 2 \end{array}\right]$

$d=10$ forces row exchange

$$\left[\begin{array}{ccc|c} 2 & 5 & 1 & 0 \\ 0 & d-10 & -1 & 2 \\ 0 & 1 & -1 & 3 \end{array}\right] \xrightarrow{d=1} \left[\begin{array}{ccc|c} 2 & 5 & 1 & 0 \\ 0 & 1 & -1 & 2 \\ 0 & 1 & -1 & 3 \end{array}\right] \xrightarrow{R_3 \to R_3 - R_2} \left[\begin{array}{ccc|c} 2 & 5 & 1 & 0 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

2.4

6) $A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix}$

$(A+B)^2 \to A+B = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 3 & 0 \end{bmatrix} \to (A+B)^2$

$\begin{bmatrix} 2 & 2 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 10 & 4 \\ 6 & 6 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$ $B^2 = \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix}$

$2AB = 2\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix} = 2\begin{bmatrix} 7 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 14 & 0 \\ 0 & 0 \end{bmatrix}$

$A^2 + 2AB + B^2 = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 14 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} 16 & 2 \\ 3 & 0 \end{bmatrix}$

$(A+B)(A+B) = A^2 + AB + BA + B^2$

14) $(A-B)^2$: $(B-A)^2 = A(A-B) - B(A-B) = A^2 - AB - BA + B^2$

15) a) If $A^2$ is defined then $A$ is necessarily square – True

b) If $AB$ and $BA$ are defined then $A$ and $B$ are square – False

$A^{m \times n}$, $B^{n \times m}$ $\Rightarrow$ $AB$ will be $m \times m$ and $BA$ is $n \times n$

c) If $AB$ and $BA$ are defined then $AB$ and $BA$ are square – True

d) If $AB = B$, then $A = I$ – False

when $B = 0$ this statement fails

2.5

7) a) $Ax = (0, 0, -1)$ cannot have a solution because if you add equation 1 + equation 2 then subtract equation 3 you get $0 = 1$

b) For $(b_1, b_2, b_3)$ to be a solution then row 1 + row 2 = row 3 has to be true

c) In elimination, equation 3 becomes all zeroes, there is no pivot.

8) a) $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $x_1 + x_2 + 2x_3 = 0$, then $(x_1, x_2, x_3)$ has to be $(1, 1, -1)$

b) When we subtract col 1 and col 2 from col 3 we are left with all zeros. ∴ col 1 + col 2 = col 3. There is no third pivot.

25) $A = \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_3 \to 2R_3 - R_1]{R_2 \to 2R_2 - R_1} \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 3 & 1 & -1 & 2 & 0 \\ 0 & 1 & 3 & -1 & 0 & 2 \end{array}\right]$

$\xrightarrow{R_3 \to 3R_3 - R_2} \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 3 & 1 & -1 & 2 & 0 \\ 0 & 0 & 8 & -2 & -2 & 6 \end{array}\right] \xrightarrow{R_1 \to R_1 - R_2} \left[\begin{array}{ccc|ccc} 2 & -2 & 0 & 2 & -2 & 0 \\ 0 & 3 & 1 & -1 & 2 & 0 \\ 0 & 0 & 8 & -2 & -2 & 6 \end{array}\right]$

$\xrightarrow[R_2 \to \frac{R_2}{8}]{R_1 \to \frac{R_1}{2}} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & -1 & 0 \\ 0 & 3 & 1 & -1 & 2 & 0 \\ 0 & 0 & 1 & -1/4 & -1/4 & 3/4 \end{array}\right] \xrightarrow{R_2 \to R_2 - R_3} \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1/4 & 3/4 & -1/4 \\ 0 & 0 & 1 & -1/4 & -1/4 & 3/4 \end{array}\right]$

$\longrightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3/4 & -1/4 & -1/4 \\ 0 & 1 & 0 & -1/4 & 3/4 & -1/4 \\ 0 & 0 & 1 & -1/4 & -1/4 & 3/4 \end{array}\right]$ $A^{-1} = \begin{bmatrix} \frac{3}{4} & -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{4} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & 3/4 \end{bmatrix}$

$$B = \left[\begin{array}{ccc|ccc} 2 & -1 & -1 & 1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 1 & 0 \\ -1 & -1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_2 \to R_2 - R_3]{R_1 \to R_1 + R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 1 & 1 & 0 \\ 0 & 3 & -3 & 0 & 1 & -1 \\ -1 & -1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 + R_1} \left[\begin{array}{ccc|ccc} 1 & 1 & -2 & 1 & 1 & 0 \\ 0 & 3 & -2 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{array}\right]$$

Since there are 3 zeros in row 3 there is no inverse

29) a) A 4x4 matrix with a row of zeros is not invertible - True
- if $A$ has a row of zeros then so does $AB$ and $AB \neq I$

b) Every matrix with 1's down the main diagonal is invertible - True
- This is just an identity matrix so $I^{-1}$ is just $I$

c) If $A$ is invertible then $A^{-1}$ and $A^2$ are invertible - True
- inverse of $A^{-1} = A$ and $A^2 = (A^{-1})^2 = A^1$