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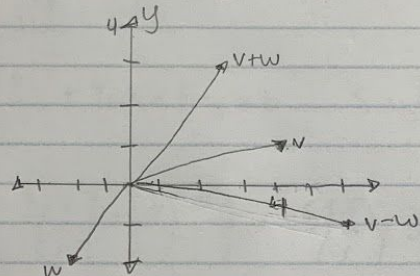
1/29
MA 232 HW 1
"I pledge my honor that I have abided by the Stevens Honor System" - Himanshu Rana

1.1

2) $v = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ $w = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$

$v+w = \begin{bmatrix} 4 \\ 1 \end{bmatrix} + \begin{bmatrix} -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$v-w = \begin{bmatrix} 4 \\ 1 \end{bmatrix} - \begin{bmatrix} -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ -1 \end{bmatrix}$



4) $v = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $w = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$3v + w = 3\begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 5 \end{bmatrix}$

$cv + dw = c\begin{bmatrix} 2 \\ 1 \end{bmatrix} + d\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2c \\ c \end{bmatrix} + \begin{bmatrix} d \\ 2d \end{bmatrix} = \begin{bmatrix} 2c+d \\ c+2d \end{bmatrix}$

5) $u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $v = \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix}$ $w = \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}$

$u+v+w = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix} + w = \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$2u+2v+w = 2\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 2\begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} + \begin{bmatrix} -6 \\ 2 \\ -4 \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$

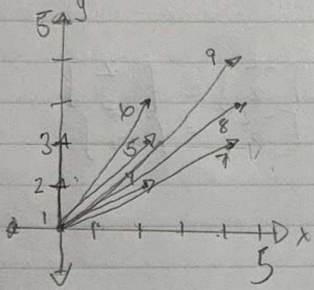
These lie in a plane because $u = -v - w$

7) $c\begin{bmatrix}2\\1\end{bmatrix} + d\begin{bmatrix}0\\1\end{bmatrix}$ with $c = 0, 1, 2$ and $d = 0, 1, 2$

1) $0\begin{bmatrix}2\\1\end{bmatrix}+0\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$ 4) $1\begin{bmatrix}2\\1\end{bmatrix}+0\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}2\\1\end{bmatrix}$ 7) $2\begin{bmatrix}2\\1\end{bmatrix}+0\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}4\\2\end{bmatrix}$

2) $0\begin{bmatrix}2\\1\end{bmatrix}+1\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}0\\1\end{bmatrix}$ 5) $1\begin{bmatrix}2\\1\end{bmatrix}+1\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}2\\2\end{bmatrix}$ 8) $2\begin{bmatrix}2\\1\end{bmatrix}+1\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}4\\3\end{bmatrix}$

3) $0\begin{bmatrix}2\\1\end{bmatrix}+2\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}0\\2\end{bmatrix}$ 6) $1\begin{bmatrix}2\\1\end{bmatrix}+2\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}2\\3\end{bmatrix}$ 9) $2\begin{bmatrix}2\\1\end{bmatrix}+2\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}4\\4\end{bmatrix}$



1.2

1) $u=\begin{bmatrix}-.6\\.8\end{bmatrix}$ $v=\begin{bmatrix}4\\3\end{bmatrix}$ $w=\begin{bmatrix}1\\2\end{bmatrix}$

$u \cdot v = \begin{bmatrix}-.6\\.8\end{bmatrix}\cdot\begin{bmatrix}4\\3\end{bmatrix} = (-.6)(4) + (.8)(3) = 0$

$u \cdot w = \begin{bmatrix}-.6\\.8\end{bmatrix}\cdot\begin{bmatrix}1\\2\end{bmatrix} = (-.6)(1) + (.8)(2) = 1$

$u \cdot (v+w)$ $\begin{bmatrix}-.6\\.8\end{bmatrix}\cdot\begin{bmatrix}4\\3\end{bmatrix} + \begin{bmatrix}-.6\\.8\end{bmatrix}\cdot\begin{bmatrix}1\\2\end{bmatrix}$ $([(-.6)(4)+(.8)(3)]+[(-.6)(1)+(.8)(2)] = 1$

$w \cdot v = \begin{bmatrix}1\\2\end{bmatrix}\cdot\begin{bmatrix}4\\3\end{bmatrix} = (1)(4) + 2(3) = 10$

2) $\|u\| = \sqrt{x^2+y^2} = \sqrt{(-.6)^2+(.8)^2} = \sqrt{1} = 1$

$\|v\| = \sqrt{x^2+y^2} = \sqrt{(4)^2+(3)^2} = \sqrt{25} = 5$

$\|w\| = \sqrt{x^2+y^2} = \sqrt{(1)^2+(2)^2} = \sqrt{5}$

$|u \cdot v| \le \|u\|\,\|v\|$

$0 \le 5$ ✓

$|v \cdot w| \le \|v\|\,\|w\|$

$10 \le 5\sqrt{5}$ ✓

6) a) every vector of form $w=\begin{bmatrix}c\\2c\end{bmatrix}$, where c is a constant

b) All vectors perpendicular to $v=(1,1,1)$ lie on a line in 3D

c) The vectors perpendicular to both $(1,1,1)$ and $(1,2,3)$ line on a plane

8) a) False, $\vec{v}$ and $\vec{w}$ are vectors perpendicular to $\vec{u}$
b) True, $\vec{u} \cdot (\vec{v} + 2\vec{w}) = \vec{u} \cdot \vec{v} + 2\vec{u} \cdot w = 0$
c) True, $\|u - v\|^2 = (u-v) \cdot (u-v) = u \cdot u + v \cdot v = 2,\ u \cdot v = 0$

13) any vector in the form $(c, d, -c)$ would be perpendicular to $(1,0,1)$, $v = (1,0,-1)$ and $w = (0,1,0)$
$\vec{v} \cdot \vec{w} = (1)(0) + (0)(1) + (-1)(0) = 0 + 0 + 0 = 0$

<u>1.3</u>

1) $3s_1 + 4s_2 + 5s_3 = b$ $\quad s_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\quad s_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ $\quad s_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$3\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + 4\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + 5\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ 12 \end{bmatrix}$$

4) $x_1 w_1 + x_2 w_2 + x_3 w_2$, $x_1 = 1$ $\quad w_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $\quad w_2 = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ $\quad w_3 = \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$
$x_1 = 1,\ x_2 = -2,\ x_3 = 1$

$$1\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + (-2)\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} + 1\begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$