chapter-2 homework

1. $A = \{\}$ (空集合)，求 $P(A)$。

$P(A) = \varnothing$

2. $A = \{a, b, c\}$，求 $P(A)$。

$P(A) = \{\varnothing, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

3. $A = \{a, b, c, d, e, f\}$，求 $P(A)$。

$P(A) = \{$
$\varnothing, \{a\}, \{b\}, \{c\}, \{d\}, \{e\}, \{f\},$
$\{a, b\}, \{a, c\}, \{a, d\}, \{a, e\}, \{a, f\},$
$\{b, c\}, \{b, d\}, \{b, e\}, \{b, f\},$
$\{c, d\}, \{c, e\}, \{c, f\},$
$\{d, e\}, \{d, f\},$
$\{e, f\},$
$\{a, b, c\}, \{a, b, d\}, \{a, b, e\}, \{a, b, f\}, \{a, c, d\}, \{a, c, e\}, \{a, c, f\}, \{a, d, e\}, \{a, d, f\}, \{a, e, f\},$
$\{b, c, d\}, \{b, c, e\}, \{b, c, f\}, \{b, d, e\}, \{b, d, f\}, \{b, e, f\},$
$\{c, d, e\}, \{c, d, f\}, \{c, e, f\},$
$\{d, e, f\},$
$\{a, b, c, d\}, \{a, b, c, e\}, \{a, b, c, f\}, \{a, b, d, e\}, \{a, b, d, f\}, \{a, b, e, f\}, \{a, c, d, e\}, \{a, c, d, f\}, \{a, c, e, f\}, \{a, d, e, f\},$
$\{b, c, d, e\}, \{b, c, d, f\}, \{b, c, e, f\}, \{b, d, e, f\},$
$\{c, d, e, f\},$
$\{a, b, c, d, e\}, \{a, b, c, d, f\}, \{a, b, c, e, f\}, \{a, b, d, e, f\}, \{a, c, d, e, f\},$
$\{b, c, d, e, f\},$
$\{a, b, c, d, e, f\}\}$

4. 令 $B = \{1, \{1\}, \{1, 2\}, 2\}$，以下何者為真，何者為假？

提示：B有四個元素：分別為1, $\{1\}$, $\{1, 2\}$, 2。

(1). $\{1\} \in B$：真
(2). $\{1\} \subseteq B$：真
(3). $\{1, 2\} \in B$：真
(4). $\{1, 2\} \subseteq B$：真
(5). $\{2\} \in B$：假
(6). $2 \in B$：真
(7). $\{2\} \subset B$：假
(8). $1 \in B$：真

(9). $\{2,\{1,2\}\} \subseteq B$：假
(10). $\{\{1\},\{2\},\{1,2\}\} \not\subset B$：真

5. 求第四題的 $P(B)$。

$P(B) = \{\varnothing, \{1\}, \{2\}, \{1,2\}, \{1,\{1,2\}\}, \{2,\{1,2\}\}, \{1,2,\{1,2\}\}\}$

6. 令 $U = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20\}$，$A = \{1,5,15,20\}$，$B = \{3,6,9,12,15,18\}$，$C = \{2,4,6,8,10,12,14,16,18,20\}$, $D = \{2,3,5,7,11,13,17\}$

整理

$U = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20\}$

$A = \{1,5,15,20\}$

$B = \{3,6,9,12,15,18\}$

$C = \{2,4,6,8,10,12,14,16,18,20\}$

$D = \{2,3,5,7,11,13,17\}$

(1). $\overline{B}$ 及 $\overline{C}$

$\overline{B} = \{1,2,4,5,7,8,10,11,13,14,16,17,19,20\}$

$\overline{C} = \{1,3,5,7,9,11,13,15,17,19\}$

(2). $(A \triangle B) \cup D$

$A \triangle B = \{1,3,5,6,9,12,18,20\}$

$(A \triangle B) \cup D = \{1,2,3,5,6,7,9,11,12,13,17,18,20\}$

(3). $\overline{(B \cup C) - A}$

$B \cup C = \{2,3,4,6,8,9,10,12,14,15,16,18,20\}$

$(B \cup C) - A = \{2,3,4,6,8,9,10,12,14,16,18\}$

$\overline{(B \cup C) - A} = \{1,5,7,11,13,15,17,19,20\}$

(4). $\overline{B} \cap D$

$\overline{B} = \{1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20\}$

$D = \{2, 3, 5, 7, 11, 13, 17\}$

$\overline{B} \cap D = \{2, 5, 7, 11, 13, 17\}$

(5). $C - ((A \cap B) \cup D)$

$A \cap B = \{15\}$

$(A \cap B) \cup D = \{2, 3, 5, 7, 11, 13, 15, 17\}$

$C - ((A \cap B) \cup D) = \{4, 6, 8, 10, 12, 14, 16, 18\}$

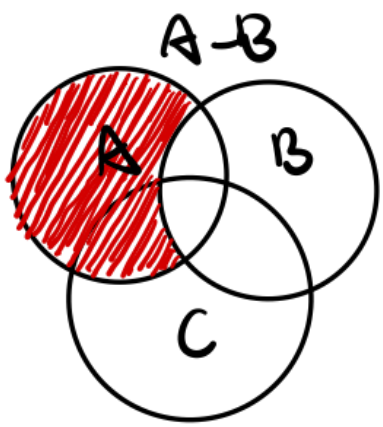
7. 畫出以下各小題之范氏圖

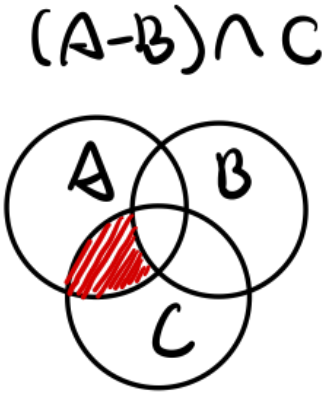
(1). $(A - B) \cap C$

(2). $B \cap (A \triangle B)$

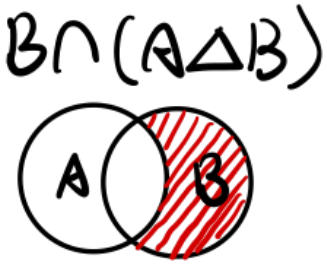
(3). $(A - (B \cup C)) \cup (B - (A \cup C)) \cup (C - (A \cup B))$

(1) $(A-B)\cap C$

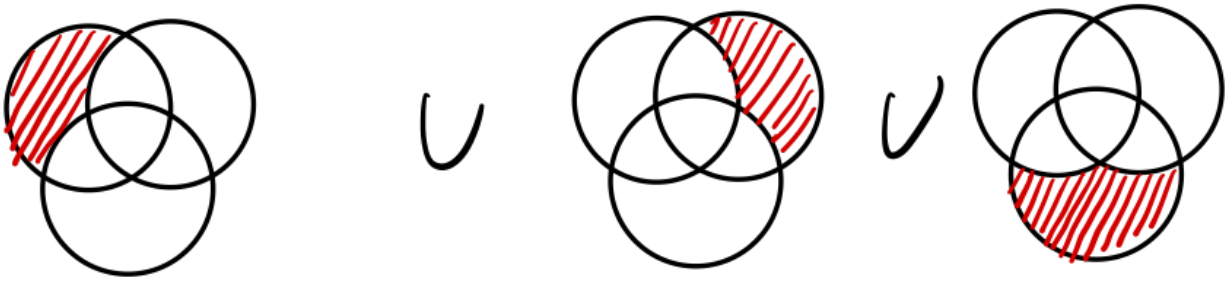




(2) $B\cap(A\triangle B)$



(3) $(A-(B\cup C))\cup(B-(A\cup C))\cup(C-(A\cup C))$



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(4). 用口語說明第 3 小題所代表的集合為何？

A B C 彼此沒有交集的部份的聯集。

8. 令 $U = \{x|x \in Z\}$且$x^2 \leq 1024$, $A = \{-11, -18, 25, 28, 11\}$, $B = \{3, 9, 16, 22, 25, 28\}$, $C = \{-18, 20, 22, -24, 26, 28, -30, -17\}$ 及 $D = \{17, 21, -6, 27\}$。

整理：

$U = \{x|x \in Z\}$且$x^2 \leq 1024$

$A = \{-11, -18, 11, 25, 28\}$

$B = \{3, 9, 16, 22, 25, 28\}$

$C = \{-30, -24, -18, -17, 20, 22, 26, 28\}$

$D = \{-6, 17, 21, 27\}$

(1). 若$E = \{-8, -1, 5, 7, 9, 11\}$。$A$ B C D 哪些集合與 E 互斥？

A C D 與 E 互斥。

(2). 任寫出兩個與 C 互斥的集合。

1. $\{1, 2, 3\}$
2. $\{4, 5, 6\}$

9. 分別用代數推導法、范氏圖及元屬隸屬表證明

$$A \triangle B = (A \cap \overline{B}) \cup (\overline{A} \cap B)$$

(1). 代數推導法

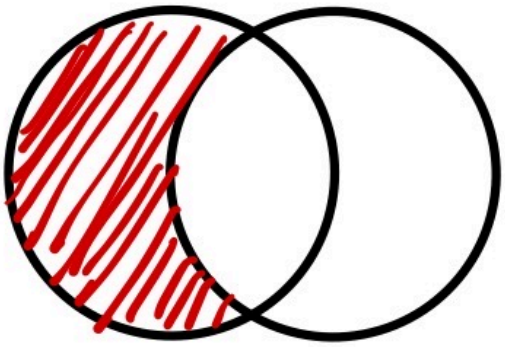
$A \triangle B = (A - B) \cup (B - A)$

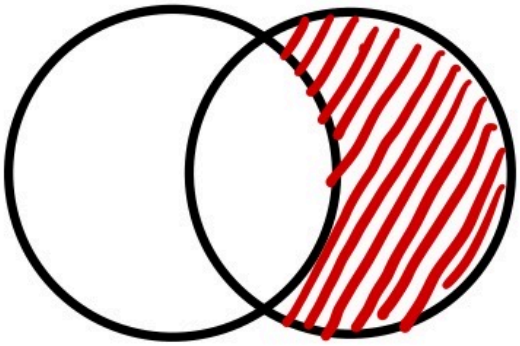
$= (A \cap \overline{B}) \cup (\overline{A} \cap B)$

$= (A \cap \overline{B}) \cup (\overline{A} \cap B)$

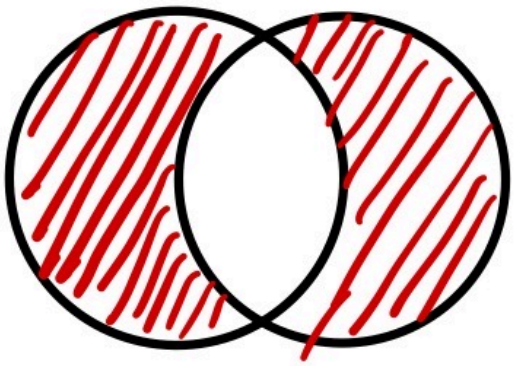
(2). 范氏圖

$A \cap \bar{B}$ $\bar{A} \cap B$





$(A \cap \bar{B}) \cup (\bar{A} \cap B)$



$= A \triangle B$

(3). 元屬隸屬表

A	B	$A\triangle B$	$A-B$	$B-A$	$(A\cap\overline{B})\cup(\overline{A}\cap B)$
0	0	0	0	0	0
0	1	1	0	1	1
1	0	1	1	0	1
1	1	0	0	0	0

10. 用元素隸屬表證明

$$\overline{\overline{(A\cup B)\cap C}\cup\overline{B}}=B\cap C$$

A	B	C	$A\cup B$	$(A\cup B)\cap C$	$\overline{(A\cup B)\cap C}$	$\overline{B}$	$\overline{\overline{(A\cup B)\cap C}\cup\overline{B}}$	$B\cap C$
0	0	0	0	0	1	1	0	0
0	0	1	0	0	1	1	0	0
0	1	0	1	0	1	0	0	0
0	1	1	1	1	0	0	1	1
1	0	0	1	0	1	1	0	0
1	0	1	1	1	0	1	0	0
1	1	0	1	0	1	0	0	0
1	1	1	1	1	0	0	1	1

11. 設 $A-B=\{1,5,7,8,16,20\}$,$B-A=\{2,12,18\}$且$A\cap B=\{3,4,6,9,10\}$。則 $|A|$ =? $|B|$=? $|A\cap B|$=? $|A\cup B|$=?

$|A|=11$

$|B|=8$

$|A\cap B|=5$

$|A\cup B|=14$

12. 設A與B互斥。在什麼條件下$|A\cup B|=|A|$?

$B=\varnothing$

13. 設 $|A| = 22, |B| = 30, |C| = 28, |A \cap B| = 10, |A \cap C| = 8, |B \cap C| = 9$**且** $|A \cup B \cup C| = 58$**。則** $|A \cap B \cap C| =?$ $|A - (B \cap C)| =?$ $|B - (A \cup C)| =?$

$|A \cap B \cap C| = 22$

$|A - (B \cap C)| = 17$

$|B - (A \cup C)| = 16$