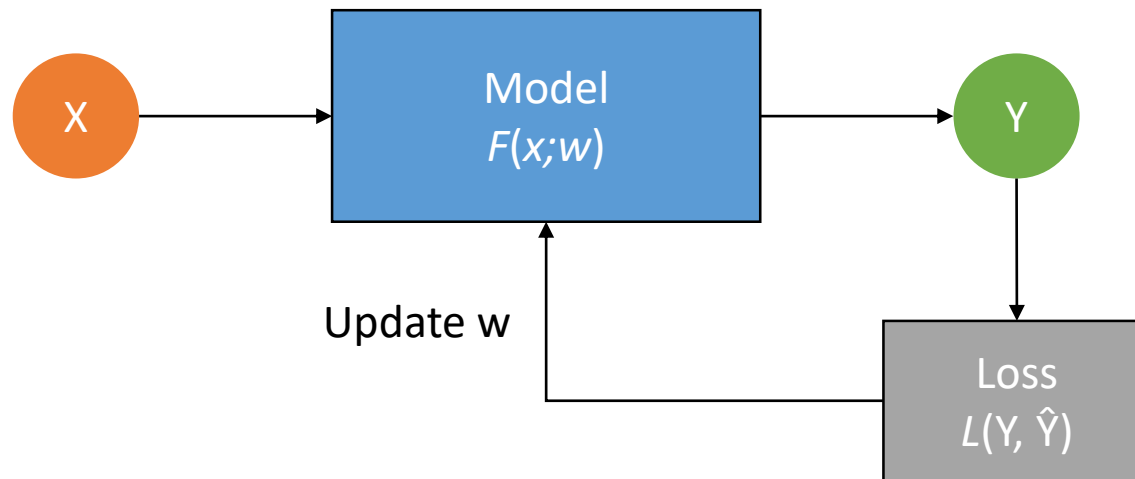


Loss function and cooperative learning

How ML works

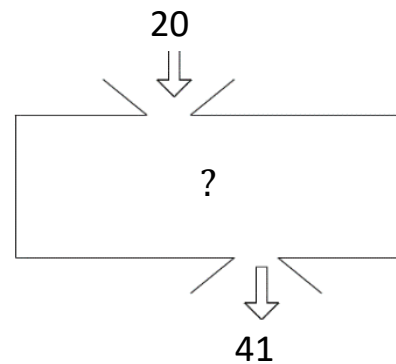
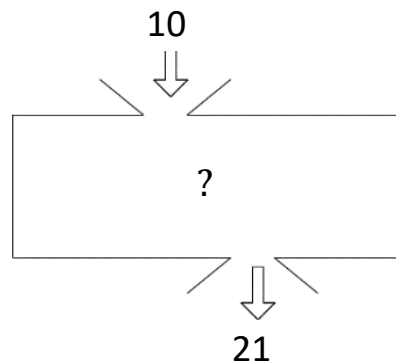
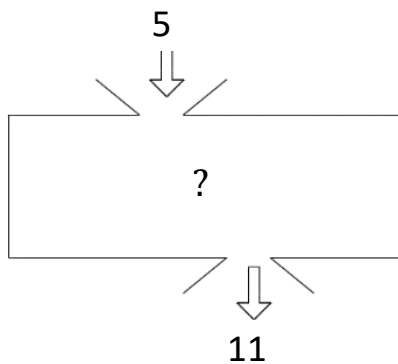
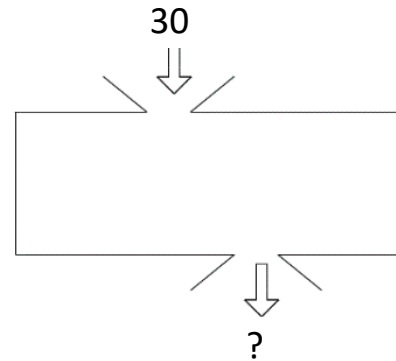
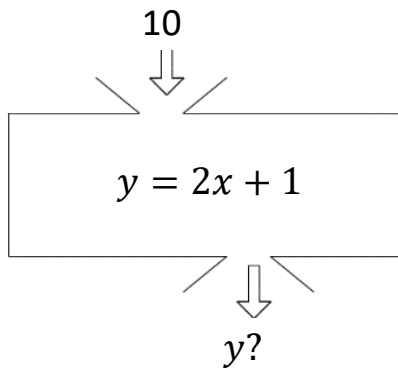


- Statistical view
 - 우도(likelihood)를 최대화하는 파라미터 탐색

$$\arg \max_w P(Y|X; w)$$

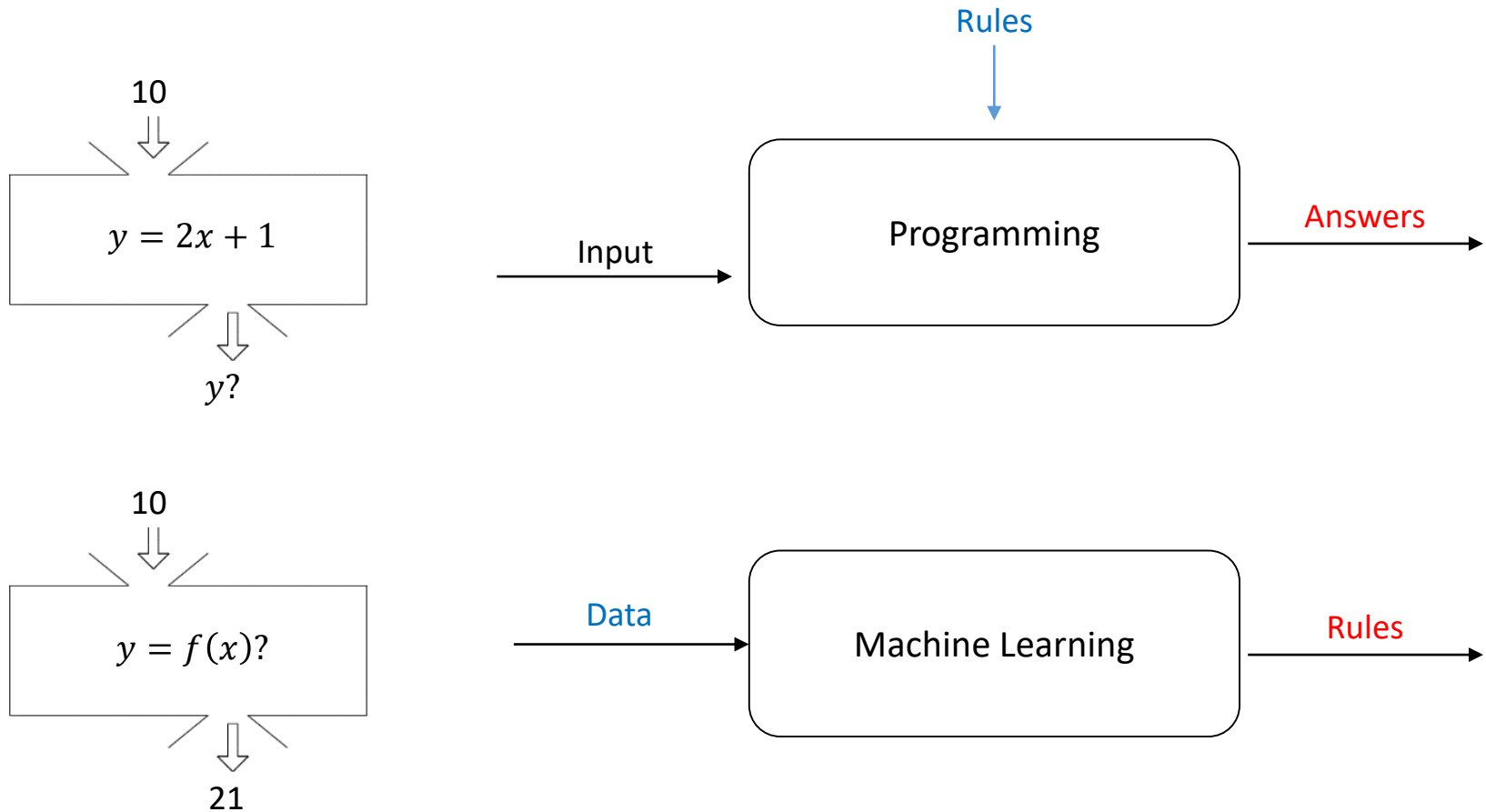
기계학습 좀 더 들여다보기...

- What is Machine Learning?



기계학습 좀 더 들여다보기...

- What is Machine Learning?



알고리즘 => 모델

- What is Machine Learning?

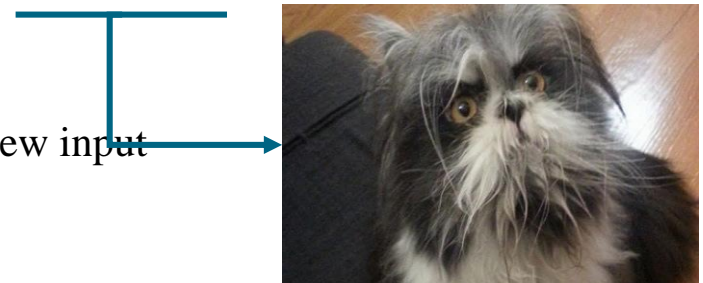


- Input :

- Output :  0, "dog"  

- Data : {  , 0}, {  , 0}, {  , 1} ...

- Finding a function which can predict the output from a new input



모델 찾기

- What is Machine Learning?

$$f(x) = ax + b$$

$$f(5) = 11$$

$$f(10) = 21$$



$$f(30) = ?$$

$f(x) = ax + b$ 에서 a, b 를 라고 한다.

머신러닝은 결국 를 찾아가는 것

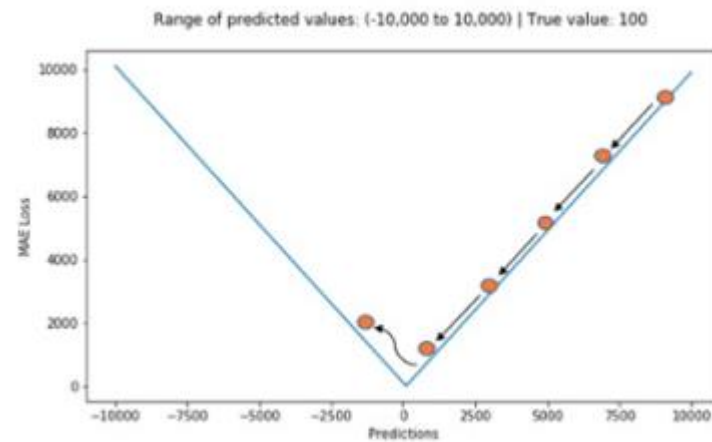
Loss function

- MAE (Mean Absolute Error)
- MSE (Mean Squared Error)
- RMSE (Rooted Mean Squared Error)
- Cross Entropy
- KLD

MAE

- Mean Absolute Error

$$L = |y - f(x)|$$



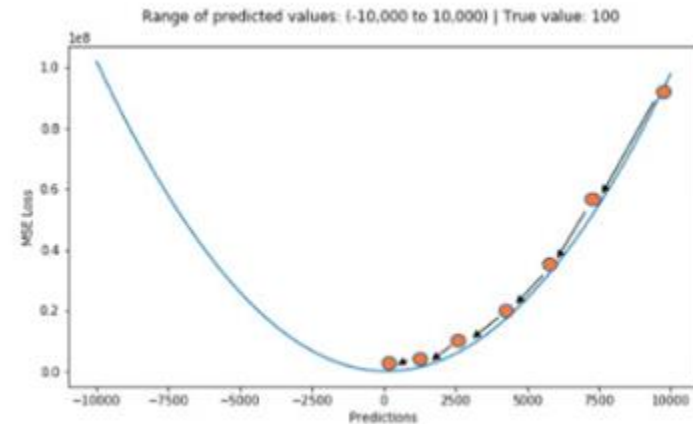
MSE and RMSE

- Mean Squared Error

$$L = (y - f(x))^2$$

- Rooted Mean Squared Error

$$L = \sqrt{(y - f(x))^2}$$



Huber Loss

- Combination of MSE and MAE

$$L = \begin{cases} \frac{1}{2} (y - f(x))^2, & \text{if } |y - f(x)| \leq \delta \\ \delta |y - f(x)| - \frac{1}{2} \delta^2, & \text{otherwise} \end{cases}$$

- Quadratic for smaller error
- Linear otherwise

Cross Entropy

■ Entropy: (최소)정보량의 평균

- $entropy = -\sum_{k=1}^m p_k \log_2 p_k$
- 정보량 $I(x) = -\log p(x)$

■ Cross Entropy:

- $H(P, Q) = \sum_i p_i \log_2 \frac{1}{q_i} = -\sum_i p_i \log_2 q_i$

■ Binary CE

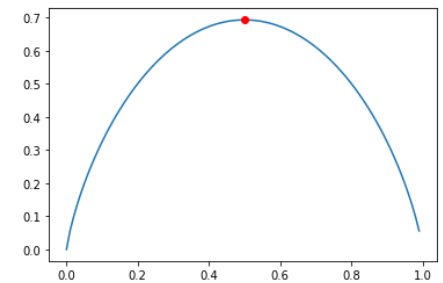
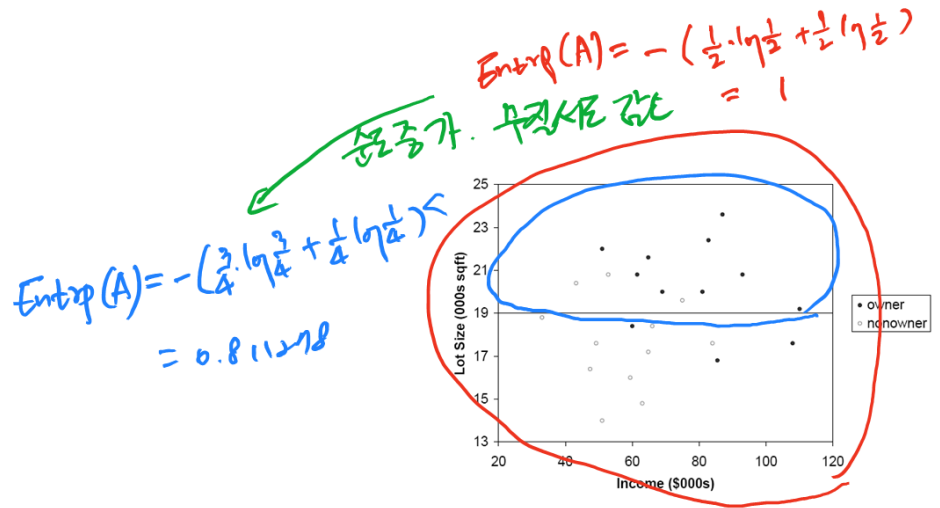
$$L = -y \cdot \log f(x) - (1 - y) \cdot \log(1 - f(x))$$



p: usually sigmoid function is used

■ Multi-class CE

$$L = -\sum_i y_i \cdot \log p_i$$



Cross entropy vs. MSE

Cross entropy	MSE
<ul style="list-style-type: none">-범주형 데이터 예측 시 주로 사용-각 클래스에 속할 확률을 의미하는 확률분포를 이용해 실제분포와의 엔트로피의 차이를 구한 것.	<ul style="list-style-type: none">-수치형 데이터 예측 시 주로 사용-정답값과 예측값의 차이를 제공하고 평균을 내준 것.
딥러닝에서 쓰이는 대표적인 손실 함수	

KL-Divergence

- The difference between two distributions
- 어떤 이상적인 분포에 대해, 그 분포를 근사하는 다른 분포를 사용해 샘플링을 한다면 발생할 수 있는 정보 엔트로피 차이를 계산

$$D_{KL}(P||Q) = - \sum_x (P(x) \cdot \log Q(x) - P(x) \cdot \log P(x)) = H(P, Q) - H(P, P)$$

H(P, P) is the entropy of P

H(P, Q) is the cross - entropy of P and Q

KLD(Kullback Leibler Divergence)

- In mathematical statistics, the Kullback–Leibler divergence, (also called relative entropy), is a measure of how one probability distribution is different from a second, reference probability distribution.

P: Distribution P represents the data, the observations, or a probability distribution precisely measured.

Q: Distribution Q represents instead a theory, a model, a description or an approximation of P

- The Kullback–Leibler divergence is then interpreted as the **average difference of the number of bits** required for encoding samples of P using a code optimized for Q rather than one optimized for P.

$$\begin{aligned} H(p, q) &= - \sum_i p_i \log q_i \\ &= - \sum_i p_i \log q_i \quad \begin{array}{l} \text{=0} \\ \underbrace{- \sum_i p_i \log p_i + \sum_i p_i \log p_i}_{=H(p)} \end{array} \\ &= H(p) + \sum_i p_i \log p_i - \sum_i p_i \log q_i \\ &= H(p) + \sum_i p_i \log \frac{p_i}{q_i} \end{aligned}$$

이만큼 더해지는 것이 무엇일까요?
⇒ 바로 분포 p와 분포 q의 **정보량 차이**입니다
⇒ 이것이 바로 KL-divergence입니다

p의 엔트로피에 **이만큼** 더해진 것이 cross entropy가 됩니다.

- 출처: https://en.wikipedia.org/wiki/Kullback%E2%80%93Leibler_divergence

KLD(Kullback Leibler Divergence)

- $KL(p||q)$

$$= H(p, q) - H(p)$$

$$= - \int p(x) \ln q(x) dx - (- \int p(x) \ln p(x) dx) = - \int p(x) \ln \left\{ \frac{q(x)}{p(x)} \right\} dx$$

- $H(p, q) = - \int p(x) \ln q(x) dx$

- $H(p) = - \int p(x) \ln p(x) dx$

- p : true distribution
- q : predicted distribution

- $KL(p||q) \neq KL(q||p)$

- p 와 q 가 같다면 $KL(p||q) = 0$

example

- 실제 사면체 주사위의 확률분포 $p = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$

추정한 사면체 주사위의 확률 분포 $q = (\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8})$ 라 가정.

- 추정한 확률분포 q 를 통해 코딩했을 경우
 - 코딩결과: (0, 10, 110, 111)
 - Entropy = Average code length = 2.25 => **Cross entropy!**
- 실제 분포 p 를 통한 최적의 코딩을 했을 경우
 - 코딩결과: (00,01,10,11)
 - Entropy = Average code length = 2

모델링한 p 와 q 가 달라서 엔트로피 차이가 발생 => **KL - divergence**

$$\begin{aligned} & (-\sum_x p(x) \log_2 q(x)) - (-\sum_x p(x) \log_2 p(x)) = -\sum_x p(x) \log_2 \frac{q(x)}{p(x)} \\ & = 2.25 - 2 = 0.25 \end{aligned}$$

■ VAE

- The structure is the same as autoencoder
 - We want encoder from autoencoder.
 - We want decoder from VAE
- Finding the posterior from simpler distribution

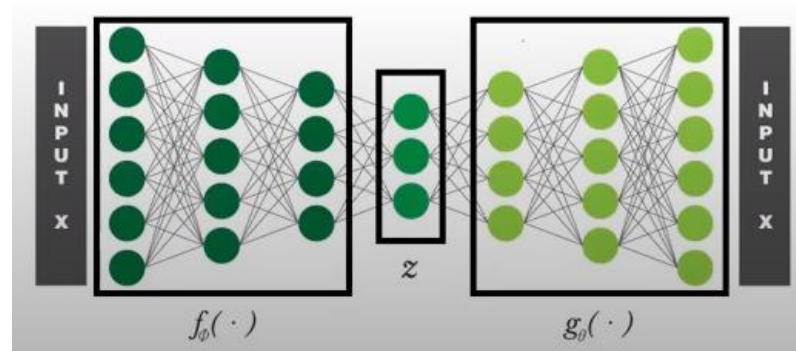
$$x = g_{\theta}(z)$$

$$p(x) = p(x|z) \quad \text{On the perspective of probability}$$

$$p(x) = E_{z \sim p_{\theta}(z)}[p(x|z)]$$

$$p(x) = E_{z \sim p_{\theta}(z|x)}[p(x|z)] \cong E_{z \sim q_{\phi}(z|x)}[p(x|z)]$$

Variational inference



$$\log(p(x)) = \log\left(\int p(x, z) dz\right) = \log\left(\int p(x|z)p(z) dz\right)$$

$$\log(p(x)) = \log\left(\int p(x|z) \frac{p(z)}{q_\phi(z|x)} q_\phi(z|x) dz\right)$$

$$\log(p(x)) \geq \int \log\left(p(x|z) \frac{p(z)}{q_\phi(z|x)}\right) q_\phi(z|x) dz \quad \text{By Jensen's Inequality}$$

$$\log(p(x)) \geq \int \log(p(x|z)) q_\phi(z|x) dz - \int \log\left(\frac{q_\phi(z|x)}{p(z)}\right) q_\phi(z|x) dz$$

$$ELBO(\phi) = \mathbb{E}_{q_\phi(z|x)}[\log(p(x|z))] - KL(q_\phi(z|x) || p(z))$$

Jensen's Inequality:

If $g(x)$ is a convex function on R_X , and $E[g(X)]$ and $g(E[X])$ are finite, then

$$E[g(X)] \geq g(E[X]).$$

$$\begin{aligned} \int \log(p(x|z)) q_\phi(z|x) dz &= \mathbb{E}_{q_\phi(z|x)}[\log(p(x|z))] \\ \int \log\left(\frac{q_\phi(z|x)}{p(z)}\right) q_\phi(z|x) dz &= KL(q_\phi(z|x) || p(z)) \end{aligned}$$

$$\log(p(x)) = \int \log(p(x)) q_\phi(z|x) dz$$

$$\begin{aligned} \log(p(x)) &= \int \log\left(\frac{p(x, z)}{p(z|x)}\right) q_\phi(z|x) dz \\ &= \int \log\left(\frac{p(x, z)}{q_\phi(z|x)} \cdot \frac{q_\phi(z|x)}{p(z|x)}\right) q_\phi(z|x) dz \\ &= \underbrace{\int \log\left(\frac{p(x, z)}{q_\phi(z|x)}\right) q_\phi(z|x) dz}_{\text{ELBO}} + \underbrace{\int \log\left(\frac{q_\phi(z|x)}{p(z|x)}\right) q_\phi(z|x) dz}_{D_{KL}} \end{aligned}$$

ELBO

D_{KL}

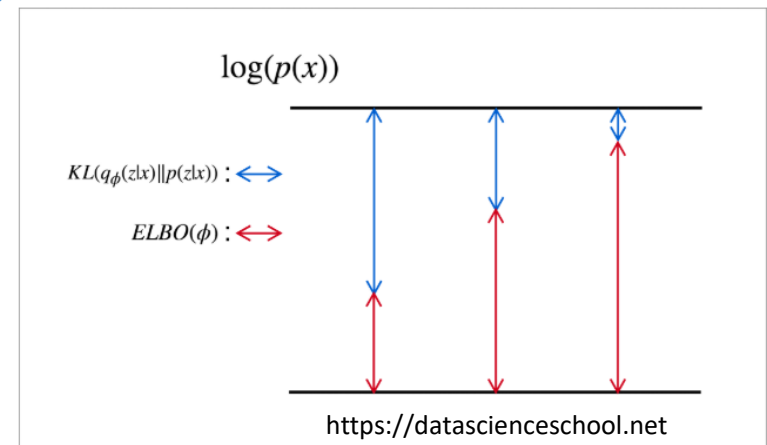
Minimize해야 하는 것

$$\begin{aligned} ELBO &= \int_z \log\left(\frac{p(z, x)}{q_\phi(z|x)}\right) q_\phi(z|x) dz = \int_z \log\left(\frac{p(z) p(x|z)}{q_\phi(z|x)}\right) q_\phi(z|x) dz \\ &= \int_z \log(p(x|z)) q_\phi(z|x) dz - \int_z \log\left(\frac{q_\phi(z|x)}{p(z)}\right) q_\phi(z|x) dz = E_{q_\phi(z|x)}[\log(p(x|z))] - D_{KL}(q_\phi(z|x) \| p(z)) \end{aligned}$$

ELBO; Derivation 1 의 것과 동일

$$\begin{aligned} \therefore \log p(x) &= E_{z \sim q(z|x)} [\log p(x|z)] - D_{KL}(q(z|x) \| p(z)) + D_{KL}(q(z|x) \| p(z|x)) \\ &= ELBO + D_{KL}(q(z|x) \| p(z|x)) \end{aligned}$$

➡ $L = -E_{z \sim q(z|x)} [\log p(x|z)] + D_{KL}(q(z|x) \| p(z))$



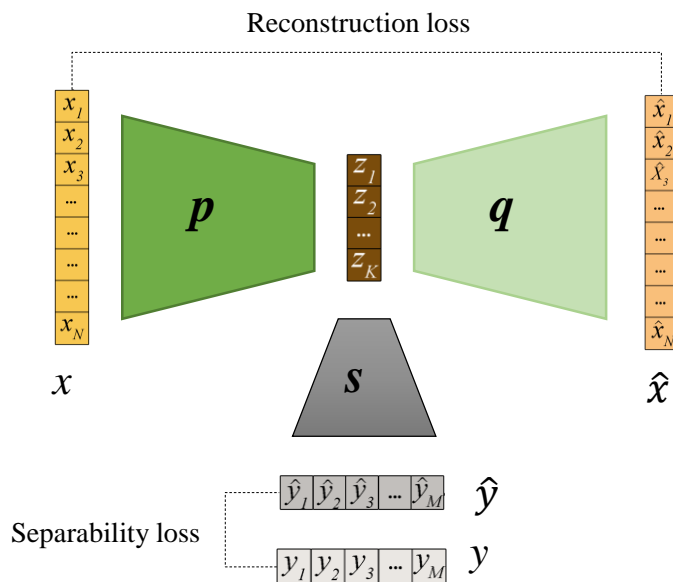


Cooperative and Non-cooperative Autonets: from Dimensionality Reduction to Classification

Imam Mustafa Kamal & Hyerim Bae

Super-encoder

- Super-encoder consists of **encoder** (p), **separator** (s), **decoder** (q) networks. The term super-encoder comes from a **super**vised encoder and **superior** **dimensionality reduction**.

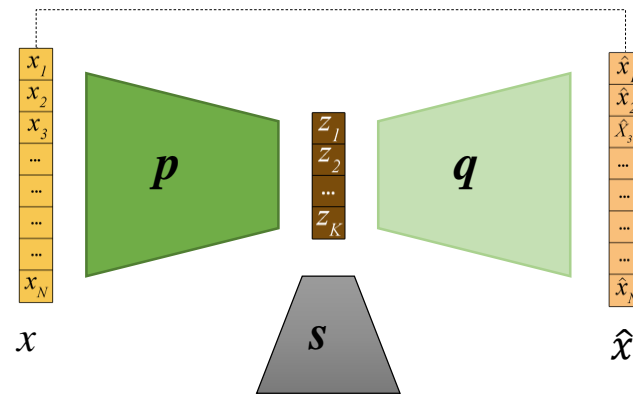


- p maps from x to latent-code z
 $z = p(x)$
- s assesses the z separability based on label y
 $\hat{y} = s(p(x))$
- q reconstructs latent-code z to original data x
 $\hat{x} = q(p(x|y))$

Cooperative Super-encoder

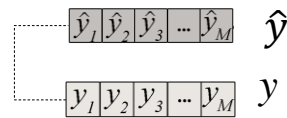
Reconstruction loss

$$\mathcal{L}_{dec}(\phi, \psi, \theta) = \mathbb{E}_{x,y}[\|x - p_{\theta}(q_{\phi}(x|y))\|_1]$$



Separability loss

$$\mathcal{L}_{enc}(\phi, \psi) = \mathbb{E}_{x,y}[\|y - s_{\psi}(q_{\phi}(x))\|_1]$$



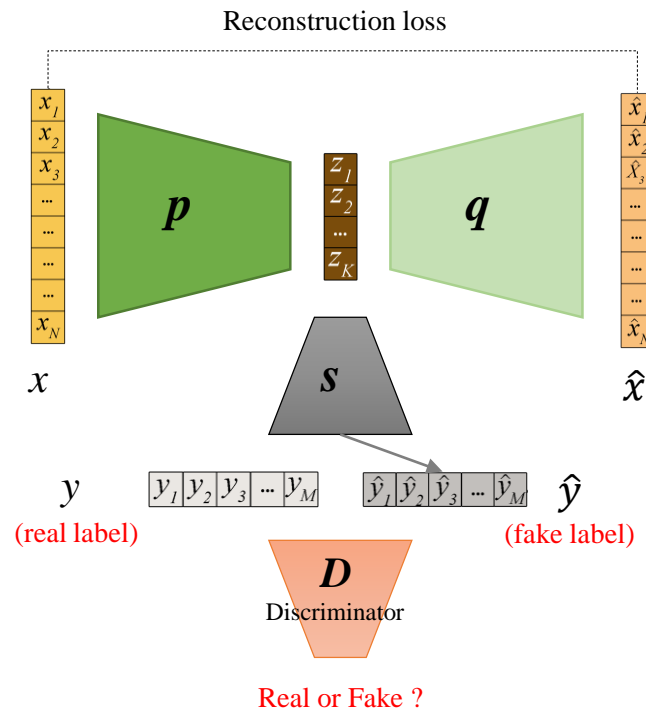
- θ Decoder parameter
- ψ Separator parameter
- ϕ Encoder parameter

Total / cooperative loss

$$\mathcal{L}(\phi, \psi, \theta) = \arg \min_{\phi, \psi, \theta} \alpha \mathcal{L}_{enc}(\phi, \psi) + \beta \mathcal{L}_{dec}(\phi, \psi, \theta)$$

Non-cooperative Super-encoder

$$\min_{p,q,s} \max_D V(\{p, q, s\}, D) = \mathbb{E}_{x,y} [\log D(y)] + \mathbb{E}_{x,y} [\log(1 - D(s(q(x))))] + \mathbb{E}_{x,y} [p(q(x|y))]$$



- p and s try to **minimize** the discrepancy between real and fake label
- While D wants to **maximize** the different between real and fake label

Super-encoder: result

Table 1. Performance of latent-code separability by encoder and reconstruction error by the decoder

Model	Performance evaluation							
	Encoder: latent-code separability (accuracy)				Decoder: reconstruction error			
	MNIST	Fashion-M	SVHN	CIFAR-10	MNIST	Fashion-M	SVHN	CIFAR-10
SE (co)	0.9893	0.9257	0.8813	0.7883	0.0123	0.0129	0.0336	0.0162
SE* (co)	0.9786	0.8735	0.7569	0.5553	0.0198	0.0183	0.0405	0.0163
SE (nc)	0.9898	0.9161	0.8489	0.8015	0.0132	0.0129	0.0356	0.0163
SE* (nc)	0.9742	0.8616	0.7435	0.5551	0.0262	0.0220	0.0413	0.0164
AE	0.9262	0.8493	0.4632	0.3933	0.0243	0.0586	0.0208	0.0148
DAE	0.9684	0.8411	0.3892	0.3093	0.0150	0.0610	0.0404	0.0155
VAE	0.9657	0.8267	0.2868	0.3011	0.0130	0.0145	0.0297	0.0161
DVAE	0.9475	0.7866	0.3064	0.2993	0.0172	0.0204	0.0500	0.0163
β -VAE	0.9493	0.7899	0.2256	0.2895	0.0158	0.0173	0.0386	0.0164
AAE	0.9630	0.8388	0.3877	0.3181	0.0644	0.0576	0.0903	0.0161
DAAE	0.9528	0.2740	0.2156	0.2976	0.0230	0.6060	0.0144	0.0153
iDAAE	0.9588	0.2737	0.2180	0.2991	0.0928	0.6035	0.0141	0.0156

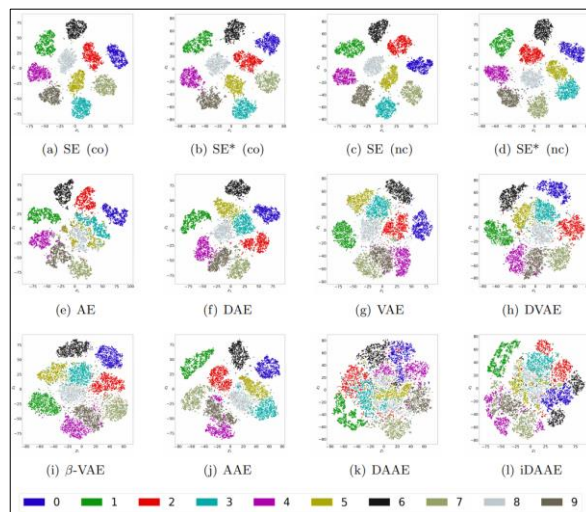
* Semi-supervised training with only 20% of label

- SE = Super-encoder
- SE* = Super-encoder with semi-supervised learning
- (co) = SE with Cooperation
- (nc) = SE with Non-cooperation

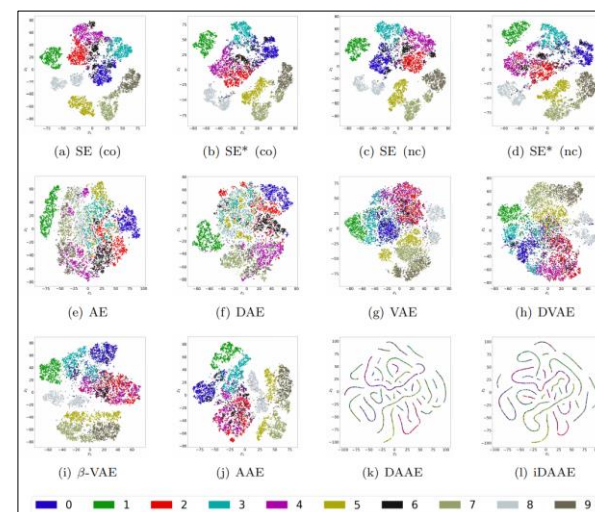
Latent-code Separability (1)

Compare
with
existing
autoencoder
models

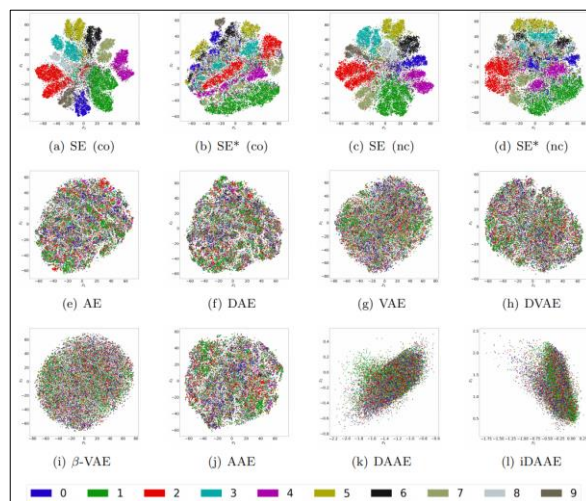
(a). MNIST



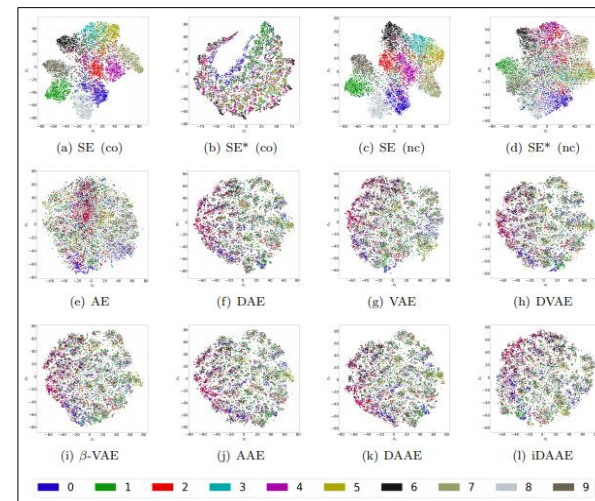
(b). Fashion-MNIST



(c). SVHN



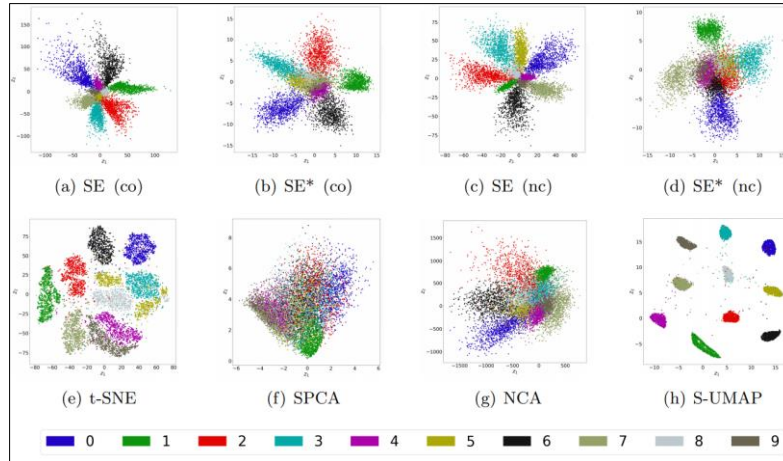
(d). CIFAR-10



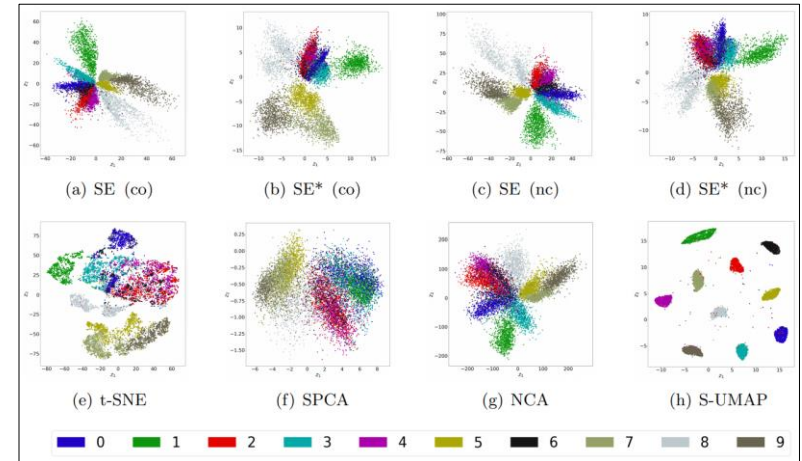
Latent-code Separability (2)

Compare with existing supervised and non-supervised dimensionality-reduction model

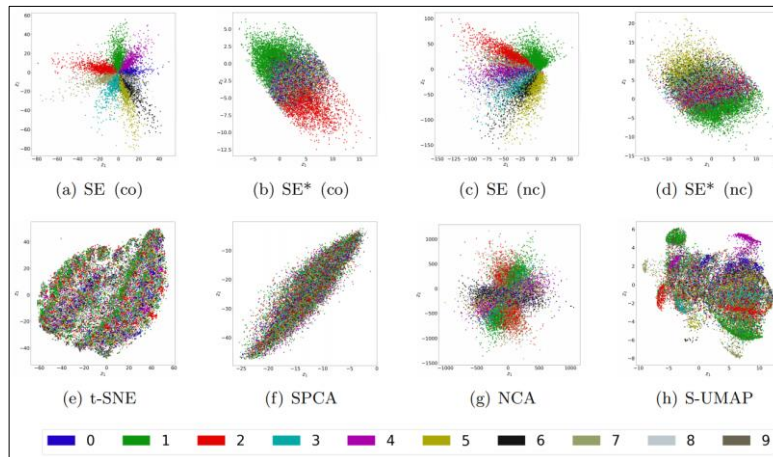
(a). MNIST



(b). Fashion-MNIST



(c). SVHN



(d). CIFAR-10

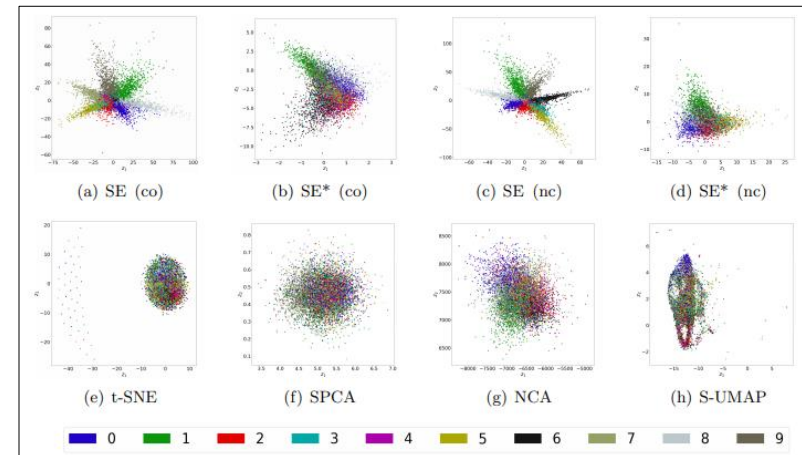
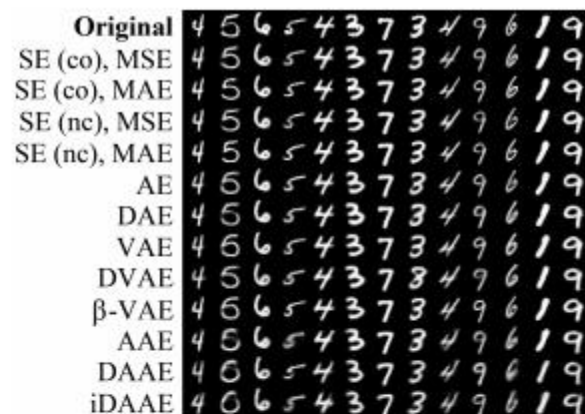
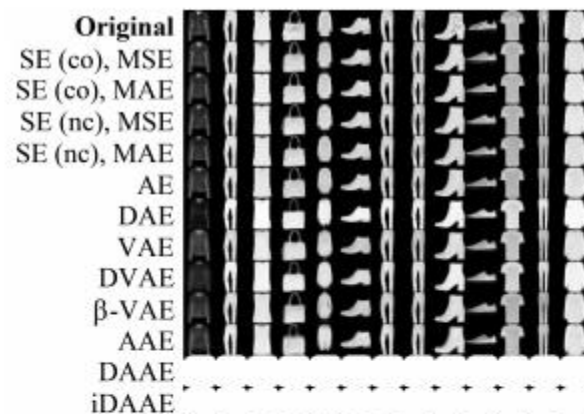


Image Reconstruction (Generation)

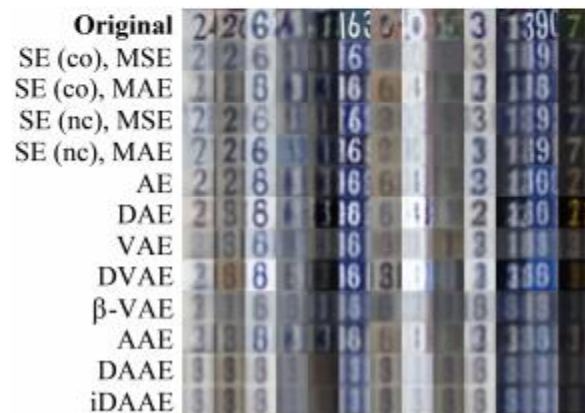
Compare
with
existing
autoencoder
models



(a) MNIST



(b) Fashion-M



(c) SVHN

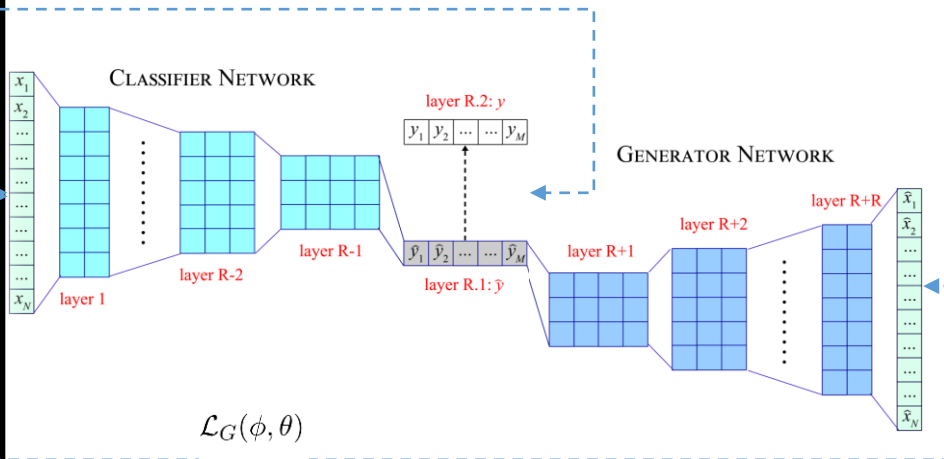


(d) CIFAR-10

Cooperative Auto-classifier

- Auto-classifier comprises **classifier (C)** and **generator (G)** networks
The term auto-classifier stands for an **automatic classifier**,
where the classifier is devised to **automatically deal with or exploit the dimensionality reduction**.

$$\mathcal{L}_C(\phi)$$



- C** estimates label (y) from input x ,
- G** must reconstruct label y to original data x ,
- Implicitly, label y also represents the latent representation of original data x .
- So, **C** is **two-fold model**: both as a **classifier** and **dimensionality-reduction model**.

- Strict cooperation (AC (co) v1) :

$$\mathcal{L}(\phi, \theta) = \arg \min_{\phi, \theta} \mathcal{L}_C(\phi) + \mathcal{L}_G(\phi, \theta) + |\mathcal{L}_C(\phi) - \mathcal{L}_G(\phi, \theta)|$$

- Adjusted cooperation (AC (co) v2):

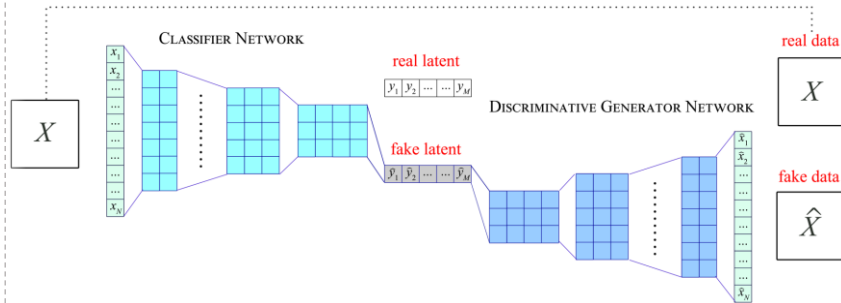
$$\mathcal{L}(\phi, \theta) = \arg \min_{\phi, \theta} \lambda(\mathcal{L}_C(\phi)) + \mathcal{L}_G(\phi, \theta)$$

Non-cooperative Auto-classifier

- Two types of games:

Classifier vs. Generator (AC (nc) v1)

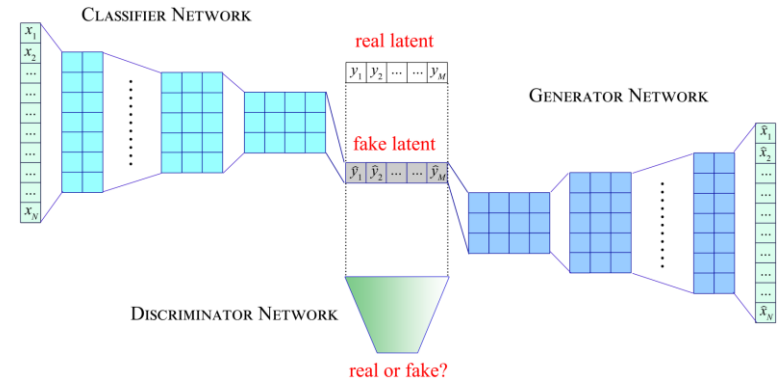
$$\min_C \max_{G^+} V(C, G^+) = \mathbb{E}_{x,y} [\log G^+(y)] + \mathbb{E}_{x,y} [\log(1 - G^+(C(x)))]$$



- Discriminative Generator:** distinguish the **real** and **fake image**
- The real latent must generate a real image
- The fake latent must generate a fake image

Classifier vs. Discriminator (AC (nc) v2)

$$\min_{C,G} \max_D V(\{C, G\}, D) = \mathbb{E}_{x,y} [\log D(y)] + \mathbb{E}_{x,y} [\log(1 - D(C(x)))] + \mathbb{E}_{x,y} [G(C(x))]$$



- Discriminator:** distinguish the **real** and **fake latent-code**

Auto-classifier: Result (1)

Table 2. Classification performance comparison of auto-classifier and some outstanding classifier models

Model	Classification accuracy (MSE)			
	MNIST	Chest X-Ray	Fashion-M	CIFAR-10
AC (co) v1, MSE	0.9962	0.8654	0.9408	0.8201
AC (co) v1, MAE	0.9944	0.8445	0.9218	0.8014
AC (co) v2, MSE	0.9959	0.8590	0.9392	0.8192
AC (co) v2, MAE	0.9939	0.8654	0.9235	0.8021
AC (nc) v1, MSE	0.9958	0.8718	0.9378	0.8208
AC (nc) v1, MAE	0.9955	0.8670	0.9181	0.8091
AC (nc) v2, MSE	0.9958	0.8718	0.9390	0.8238
AC (nc) v2, MAE	0.9933	0.9933	0.9241	0.8066
Independent CNN	0.9951	0.8429	0.9360	0.8017
Maxout Networks (Goodfellow et al. [109])	0.9400	N/A	N/A	0.8600
VAE (M1 + M2) (Makhzani et al. [40])	0.9904	N/A	N/A	N/A
VAT (Miyato et al. [110])	0.9936	N/A	N/A	N/A
CatGAN (Springenberg[111])	0.9909	N/A	N/A	0.8042
Task-Oriented GAN [90]	0.9312	N/A	N/A	N/A
Ladder Networks (Rasmus et al. [112])	0.9943	N/A	N/A	0.7960
SRC (Xie et al. [113])	0.8595	N/A	N/A	N/A
CRC (Xie et al. [113])	0.8325	N/A	N/A	N/A
KENRR (Linear) (Xie et al. [113])	0.8770	N/A	N/A	N/A
KENRR (POLY) (Xie et al. [113])	0.8795	N/A	N/A	N/A
KENRR (RBF) (Xie et al. [113])	0.8795	N/A	N/A	N/A
KENRR (Hellinger) (Xie et al. [113])	0.8845	N/A	N/A	N/A

- AC = auto-classifier
- (co) = cooperative learning
- (nc) = non-cooperative learning
- AC(co) v1 = strict losss
- AC (co) v2 = adjusted loss
- AC (nc) v1 = via generator
- AC (nc) v2 = via discriminator

Auto-classifier: Result (2)

Table 3. Performance measures of classification comparison among independent classifier, cooperative and non-cooperative classifier

Dataset	F1 score				
	Independent CNN	AC (co) v1	AC (co) v2	AC (nc) v1	AC (nc) v2
MNIST	0.9950	0.9962	0.9959	0.9957	0.9957
Chest X-Ray	0.8233	0.8506	0.8449	0.8615	0.8587
Fashion-M	0.9360	0.9409	0.9391	0.9376	0.9389
CIFAR-10	0.8012	0.8186	0.8185	0.8215	0.8222
Dataset	Precision				
	Independent CNN	AC (co) v1	AC (co) v2	AC (nc) v1	AC (nc) v2
MNIST	0.9950	0.9962	0.9958	0.9957	0.9957
Chest X-Ray	0.8102	0.8393	0.8359	0.8573	0.8487
Fashion-M	0.9360	0.9408	0.9392	0.9378	0.9390
CIFAR-10	0.8017	0.8201	0.8192	0.8208	0.8238
Dataset	Recall				
	Independent CNN	AC (co) v1	AC (co) v2	AC (nc) v1	AC (nc) v2
MNIST	0.9951	0.9962	0.9960	0.9958	0.9958
Chest X-Ray	0.8520	0.8712	0.8596	0.8668	0.8753
Fashion-M	0.9362	0.9412	0.9392	0.9378	0.9391
CIFAR-10	0.8100	0.8207	0.8208	0.8269	0.8222

Cooperative and non-cooperative networks are more robust than standalone CNN (classifier)

Reference

- <https://ratsgo.github.io/deep%20learning/2017/09/24/loss/>
- <https://www.analyticsvidhya.com/blog/2019/08/detailed-guide-7-loss-functions-machine-learning-python-code/>