Combined likelihood analysis of (dwarf galaxy) dark matter searches

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Preliminaries

- Poissonian probability mass function:

$$P(x|\mu) = \frac{\mu^x}{x!} e^{-\mu} \tag{1}$$

— probability to observe x events, when the expectation value is μ.

- **Likelihood function** $\mathcal{L}$ of a model, given some data:

$$\mathcal{L}(\mu|x) = P(x|\mu), \tag{2}$$

The *likelihood* of a model expectation μ, given the data point x, is equal to the *probability* of the data point x, given the expectation value μ.

Profile likelihood analyses

- Simple (1D) case: Assuming

$$\mathcal{L}\left(\vec{\pi} = (p, \vec{n}) | \vec{d}\right) \tag{3}$$

with the parameter of interest p and nuisance parameters $\vec{n}$, the *profile likelihood* is defined as:

$$\mathcal{PL}(p_0|\vec{d}) = \frac{\max(\mathcal{L}(p = p_0, \vec{n}))}{\max(\mathcal{L}(p, \vec{n}))}, \tag{4}$$

where the maximization is performed over the *complete* parameter range ($\forall \pi$) for the denominator, but only for the subrange with $p = p_0$ for the numerator.
($\longrightarrow$ "likelihood ratio test statistic")
- Wilks & Co.: $-2\ln(\mathcal{PL})$ approaches $\chi^2(1)$
$\longrightarrow$ possibility to infer parameter ranges / limits.

$\mathcal{L}$ for ACT observations

Two Poisson processes:

- signal
- background

Construct Likelihood as:

$$\mathcal{L}(s, b | N_{\text{on}}, N_{\text{off}}) = P(N_{\text{on}} | s + \alpha b) \times P(N_{\text{off}} | b) \tag{5}$$

Simplifying step: Assume $b = N_{\text{off}}$.

Dark matter signal expectation:

$$s = \frac{\langle \sigma \mathrm{v} \rangle}{8\pi m_\chi^2} \times T_{\text{obs}} \times \overline{J} \times \int dE \, A_{\text{eff}}(E) \frac{dN}{dE}(E) \tag{6}$$

Combined likelihood / stacking analysis

Combined likelihood of several data sets:

$$\mathcal{CL} = \prod_i \mathcal{L}_i \tag{7}$$

Include uncertainty of $\overline{J}$ for each data set:

$$\mathcal{L}_i \longrightarrow \mathcal{L}_i \left(\langle \sigma \mathrm{v} \rangle, m_\chi, \overline{J}_i | N_{\mathrm{on}}, N_{\mathrm{off}} \right) \times PDF(\overline{J}_i), \tag{8}$$

assuming a log-normal distribution of true $\overline{J}$ around the values determined by the people who do these things (Walker et al 2011, Charbonnier et al. 2011).

$$PDF(\overline{J}) = \frac{1}{\sqrt{2\pi}\,\ln(10)\overline{J}_i \sigma_{\overline{J},i}} \exp\left\{ -0.5 \frac{[\log(\overline{J}_i) - \log(\overline{J}_{\mathrm{mean},i})]^2}{\sigma^2_{\overline{J},i}} \right\} \tag{9}$$

Data: from selected publications of highest quality

Object (dSph)	Observation	Non	Noff	alpha	Nexcess	Reference
Sculptor	HESS	117	2283	0.04	25.7	...
Carina	HESS	86	1858	0.05	-6.9	
Sagittarius	HESS	846	14536	0.05	54.7	... unpublished
Segue1	VERITAS	1082	12479	0.08	33.8	
	MAGIC	52978	53301	1.0	-323.0	
Draco	VERITAS	305	3667	0.09	-28.4	
	MAGIC	10883	10996	1.0	-113.0	
Ursa Minor	VERITAS	250	3084	0.09	-30.4	
Fornax Cluster	HESS	160	122	1.0	38.0	
Sum / weighted ave.:		66707	102326	0.66	-349.5	—

Data: from selected publications of highest quality

Object (dSph)	Observations	$\log_{10}(\bar{J})$, $\sigma_{\bar{J}}$	(Possible) $\bar{J}$ reference
Sculptor	HESS	17.9, 0.2	Charbonnier
Carina	HESS	15.9, 0.4	Charbonnier
Sagittarius	HESS	17.3, 0.3	Viana
Segue1	VERITAS, MAGIC	19.1, 0.6	Essig
Draco	VERITAS, MAGIC	17.8, 0.2	Charbonnier
Ursa Minor	VERITAS	18.3, 0.3	Charbonnier
Fornax Cluster	HESS	16.5, 0.3	Abramowski

Combined likelihood

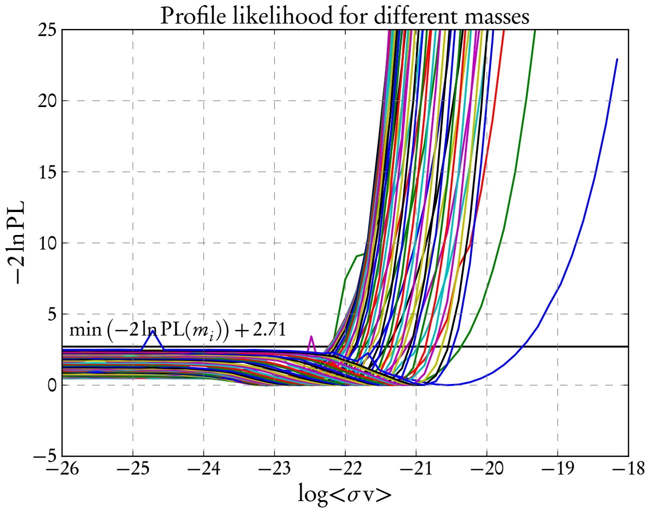
Hence, the full combined likelihood is a function of:

- $\langle \sigma \mathrm{v} \rangle$, m_χ and the ("true") $\bar{J}$ factors of all targets
- the experimental parameters of each observation ($T_{\mathrm{obs}}, A_{\mathrm{eff}}$)
- and, of course, the experimental results ($N_{\mathrm{on}}, N_{\mathrm{off}}$)

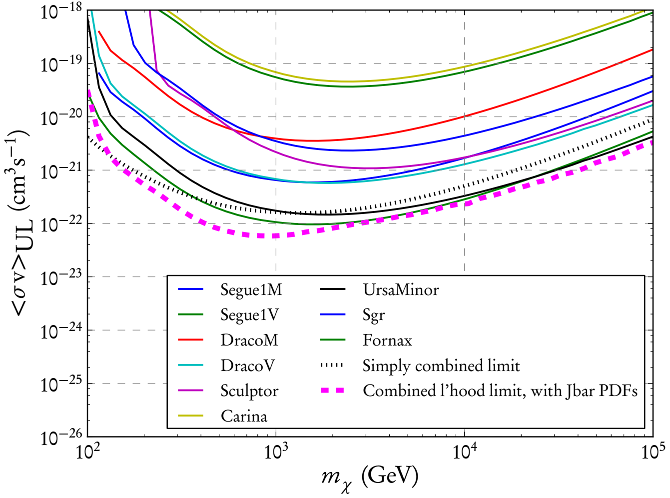
Next step: Calculate the combined profile likelihood $\mathcal{PL}$, as a function of fixed $\langle \sigma \mathrm{v} \rangle_f$, for each given DM mass $\longrightarrow$ maximize over the $\bar{J}$'s:

$$-2 \ln \mathcal{PL}(\langle \sigma \mathrm{v} \rangle_f, m_f) = -2 \ln \left(\frac{\max \left[\mathcal{CL}(\langle \sigma \mathrm{v} \rangle_f, m_f, \{\bar{J}\}) \right]}{\max \left[\mathcal{CL}(\langle \sigma \mathrm{v} \rangle, m_f, \{\bar{J}\}) \right]} \right) \qquad (10)$$

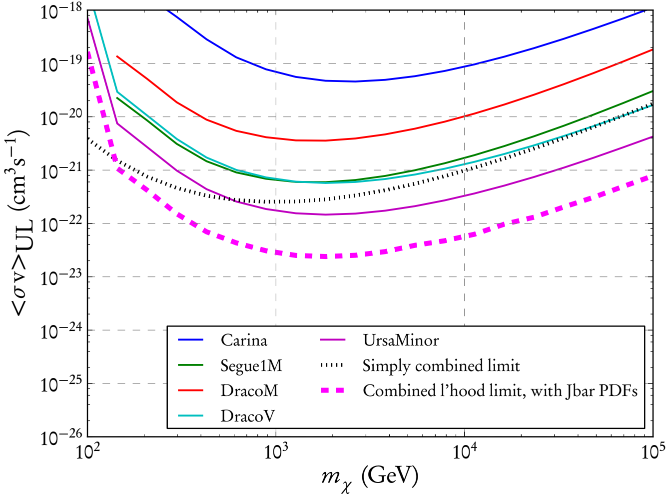
Profile likelihoods



Results: Combined upper limit on $\langle \sigma v \rangle$

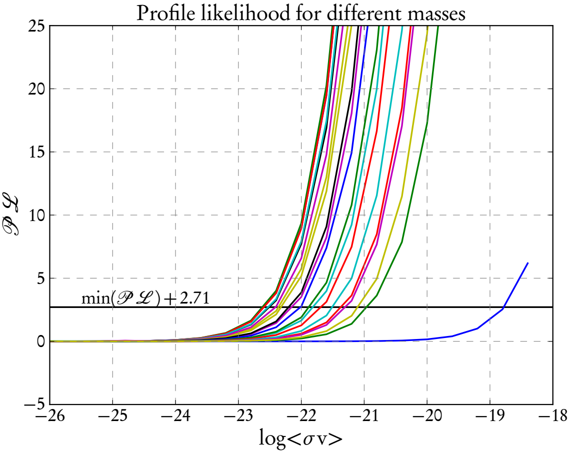


Backup: Results: negative excess obs. only



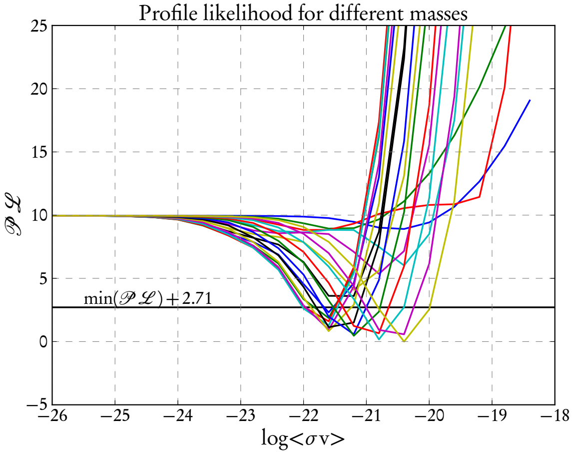
Profile likelihoods

Only observations with negative excess:



Profile likelihoods

Only observations with positive excess:



Issues

- How to deal with “negative excesses”, or their influence on the resulting UL?
- Implement profile likelihood for parameter *b*?
- Stability of minimizations ...
- Implement profile likelihood for single observation limits?
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