Consensus mechanism of HRD chain based on perpetual combinatorial auctions

Section one. Introduction

NFT (or heterogeneous token) is an encryption tool that can be used to token unique digital items. NFT is irreplaceable, which means that they can not be replaced by the same goods, and it can be proved that they are scarce, which makes them have intrinsic value.

As $NFTs$ become more and more financialized, they will also need new trading platforms, lending agreements and derivatives. Therefore, the price discovery of NFT will be the next problem to be solved. Efficient price discovery mechanism in the capital market will enable participants to trade more quickly, improve liquidity through token, allow NFT to become collateral without order book, and create rich derivatives with NFT as the target. In other words, price discovery will make the financialization of NFT asset classes possible. Nowadays, there are only several ways to find the price of NFT, such as historical transaction pricing, auction pricing, fragmented pricing and so on. At present, most sellers and buyers in the market prefer to buy NFT through auction and price it. Async.art uses perpetual auctions in their website galleries, and so does superrare. Beeple's auction, with a total turnover of 3.5 million US dollars, proved that although the auction is still in the development stage, it is strong enough. It is worth mentioning that the auction is very helpful for the sale of artworks, because the intrinsic value of NFT assets is often more subjective, and there will be more people in the wait-and-see state. Therefore, we propose the perpetual combinatorial auction model to solve the pricing problem of NFT, which allows the continuous addition of new NFTs in the ore pool. At the same time, we propose the POA (Proof of Aution) mechanism, which transforms the auction value of NFT into the mineral value generated by the block chain mining, and realizes the standardization of NFT assets, so as to solve the two basic problems of difficult circulation and pricing of NFT assets.

Section two. NFT pricing in perpetual combinatorial auctions

S_i	Mine price on the i-th day
N_{ij}	The maximum ore output of the j-th NFT single mining on the i-th day
Q_{ij}	The ore output of the j-th NFT combined mining on the i-th day
V_{ij}	The value of the j-th NFT on the i-th day
$Q_{i,j_1\dots j_n}$	The ore output of these n NFT $j_1,\dots,j_n$ combinations combined mining on the i-th day $(j_1<j_2<\dots<j_n)$
$V_{i,j_1\dots j_n}$	The value of these n NFT $j_1,\dots,j_n$ combinations on the i-th day $(j_1<j_2<\dots<j_n)$
k_i	Number of NFT on the i-th day
P_{ij}	The j-th NFT auction price on the i-th day
$P_{i,j_1\dots j_n}$	The price of these n NFT $j_1,\dots,j_n$ combinations on the i-th day $(j_1<j_2<\dots<j_n)$
Q	Total ore output per day
r	Fixed daily discount rate

Variation of NFT in ore pool:

$$
\underset{\text{Day } 1}{\begin{pmatrix} NFT_{11} \\ NFT_{12} \\ \vdots \\ NFT_{1k_1} \end{pmatrix}}
\quad
\underset{\text{Day } 2}{\begin{pmatrix} NFT_{21} \\ NFT_{22} \\ \vdots \\ NFT_{2k_1} \\ NFT_{2(k_1+1)} \\ \vdots \\ NFT_{2k_2} \end{pmatrix}}
\quad
\underset{\cdots\cdots}{\cdots\cdots}
\quad
\underset{\text{Day } i}{\begin{pmatrix} NFT_{i1} \\ NFT_{i2} \\ \vdots \\ NFT_{ik_1} \\ NFT_{i(k_1+1)} \\ \vdots \\ NFT_{ik_2} \\ NFT_{i(k_2+1)} \\ \vdots \\ NFT_{ik_i} \end{pmatrix}}
\quad
\underset{\cdots\cdots}{\cdots\cdots}
$$

The initial price of the j-th NFT digital asset on the i-th day is the auction price P_{ij}, which allows the NFT digital asset to mine in the public chain. According to the initial price P_{ij} of the j-th NFT digital asset, the calculation force is anchored, and according to the current mine price S_i, the output N_{ij} of the NFT digital asset can be determined. The daily output Q of the mine pool is constant. As time goes on, as shown in the figure above, there are new NFT digital assets in the mine pool, so the weight of each NFT digital asset is decreasing, and the output Q_{ij} of the mine is also decreasing. When the output of a NFT digital asset is less than a certain threshold, the NFT digital asset exits the mine pool. The price of the mine S_i is changing in real time, so the price V_{ij} of each NFT digital asset is not constant. In the following two sections, the pricing of NFT digital assets with auction portfolio and without auction portfolio under the perpetual combinatorial auction model is given in two cases: without discount and with known fixed discount rate r.

1. NFT pricing without discounting

(1) There is no auction portfolio

Theorem 1.1: $Q_{ij}=\begin{cases}\dfrac{N_{ij}}{\sum_{j=1}^{k_i} N_{ij}}Q & , \quad 1\le j\le k_i \\ 0 & , \quad k_i<j\end{cases}$.

Proof: When $1\le j\le k_i$, the proportion of the j-th in the total NFT on the i-th day is

$$\frac{P_{ij}}{\sum_{j=1}^{k_i} P_{ij}}=\frac{\dfrac{P_{ij}}{S_i}}{\dfrac{\sum_{j=1}^{k_i} P_{ij}}{S_i}}=\frac{N_{ij}}{\sum_{j=1}^{k_i} N_{ij}}.$$

When $k_i<j$, $Q_{ij}=0$,

so

$$Q_{ij}=\begin{cases}\dfrac{N_{ij}}{\sum_{j=1}^{k_i} N_{ij}}Q & , \quad 1\le j\le k_i \\ 0 & , \quad k_i<j\end{cases}.$$

Theorem 1.2: $V_{ij}=\begin{cases}\dfrac{N_{ij}}{\sum_{j=1}^{k_i} N_{ij}}QS_i & , \quad 1\le j\le k_i \\ 0 & , \quad k_i<j\end{cases}$

Proof: We know from the theorem 1.1

$$Q_{ij}=\begin{cases}\dfrac{N_{ij}}{\sum_{j=1}^{k_i}N_{ij}}Q & ,\quad 1\le j\le k_i\\ 0 & ,\quad k_i<j\end{cases},$$

so

$$V_{ij}=Q_{ij}S_i=\begin{cases}\dfrac{N_{ij}}{\sum_{j=1}^{k_i}N_{ij}}QS_i & ,\quad 1<j\le k_i\\ 0 & ,\quad k_i<j\end{cases}.$$

Theorem 1.3: $V_j=\sum_{n=1}^{\infty}\sum_{i=n}^{\infty}\dfrac{N_{ij}}{\sum_{j=1}^{k_i}N_{ij}}QS_i 1_{(k_{n-1},k_n]}(j)$, where $1_{(k_{n-1},k_n]}(j)=\begin{cases}1\ , & j\in(k_{n-1},k_n]\\ 0\ , & j\notin(k_{n-1},k_n]\end{cases}$, $k_0=0$

Proof: We know from the theorem 1.2

$$V_{ij}=\begin{cases}\dfrac{N_{ij}}{\sum_{j=1}^{k_i}N_{ij}}QS_i & ,\quad 1\le j\le k_i\\ 0 & ,\quad k_i<j\end{cases}$$

when $j\in(k_{n-1},k_n]$,

$$V_j=\sum_{i=1}^{\infty}V_{ij}=\sum_{i=n}^{\infty}V_{ij},$$

when $j\notin(k_{n-1},k_n]$,

$$V_j=0,$$

so

$$V_j=\sum_{n=1}^{\infty}\sum_{i=n}^{\infty}V_{ij}1_{(k_{n-1},k_n]}(j)=\sum_{n=1}^{\infty}\sum_{i=n}^{\infty}\frac{N_{ij}}{\sum_{j=1}^{k_i}N_{ij}}QS_i 1_{(k_{n-1},k_n]}(j),$$

where $1_{(k_{n-1},k_n]}(j)=\begin{cases}1\ , & j\in(k_{n-1},k_n]\\ 0\ , & j\notin(k_{n-1},k_n]\end{cases}$.

Theorem 1.4: $\lim_{i\to\infty}N_{ij}=0.$

Proof: Because $N_i=\dfrac{P_{ij}}{S_i}$, and because P_{ij} is a bounded quantity and $\lim_{i\to\infty}S_i=+\infty$,

so

$$\lim_{i\to\infty}N_{ij}=0.$$

Theorem 1.5: $\forall \varepsilon > 0, \{i \mid N_{ij} < \varepsilon\} \neq \varnothing$

Proof: We know from the theorem 1.4 and the definition of limit, $\forall \varepsilon > 0,\ \exists I = I(j,\varepsilon)$, when $i > I$, we have

$$N_{ij} < \varepsilon,$$

so

$$\{i \mid N_{ij} < \varepsilon\} \neq \varnothing.$$

Theorem 1.6: Set threshold $\varepsilon > 0$, when $N_{ij} < \varepsilon$, it is considered that the j −th NFT exits the ore pool at this time, let $\min\{i \mid N_{ij} < \varepsilon\} = T(j,\varepsilon)$, then

$$V_j = \sum_{n=1}^{\infty} \sum_{i=n}^{T(j,\varepsilon)} \frac{N_{ij}}{\sum_{j=1}^{k_i} N_{ij}} QS_i 1_{(k_{n-1},k_n]}(j)$$

where $k_0 = 0$.

Proof: When $N_{ij} < \varepsilon$, it is considered that the j −th NFT exits the ore pool at this time, here $N_{ij} = 0$.

when $i > T(j,\varepsilon)$, we have

$$\begin{aligned} V_j &= \sum_{i=n}^{\infty} V_{ij} = \sum_{n=1}^{\infty} \sum_{i=n}^{T(j,\varepsilon)} V_{ij} 1_{(k_{n-1},k_n]}(j) + \sum_{n=1}^{\infty} \sum_{i=T(j,\varepsilon)+1}^{\infty} V_{ij} 1_{(k_{n-1},k_n]}(j) \\ &= \sum_{n=1}^{\infty} \sum_{i=n}^{T(j,\varepsilon)} \frac{N_{ij}}{\sum_{j=1}^{k_i} N_{ij}} QS_i 1_{(k_{n-1},k_n]}(j) + \sum_{n=1}^{\infty} \sum_{i=T(j,\varepsilon)-1}^{\infty} \frac{N_{ij}}{\sum_{j=1}^{k_i} N_{ij}} QS_i 1_{(k_{n-1},k_n]}(j) \\ &= \sum_{n=1}^{\infty} \sum_{i=n}^{T(j,\varepsilon)} \frac{N_{ij}}{\sum_{j=1}^{k_i} N_{ij}} QS_i 1_{(k_{n-1},k_n]}(j) + \sum_{n=1}^{\infty} \sum_{i=T(j,\varepsilon)-1}^{\infty} \frac{0}{\sum_{j=1}^{k_i} N_{ij}} QS_i 1_{(k_{n-1},k_n]}(j) \\ &= \sum_{n=1}^{\infty} \sum_{i=n}^{T(j,\varepsilon)} \frac{N_{ij}}{\sum_{j=1}^{k_i} N_{ij}} QS_i 1_{(k_{n-1},k_n]}(j) \end{aligned}$$

(2) There is an auction portfolio

Theorem 2.1: $$Q_{i,j_1\ldots j_n} = \begin{cases} \dfrac{N_{i,j_1\ldots j_n}}{N_{i,j_1\ldots j_n} + \sum\limits_{\substack{j=1 \\ j\neq j_1\ldots j_n}}^{k_i} N_{ij}} Q & , \quad 1 \leq j_n \leq k_i \\ 0 & , \quad j_n > k_i \end{cases}$$

Proof: When $1 \leq j_n \leq k_i$, and $j_1 \ldots j_n$ Combinatorial auction, $P_{i,j_1\ldots j_n} < \sum_{k=1}^{n} P_{ij_k}$, so the proportion of corresponding $NFT_{j_1 \cdots j_n}$ in total ore production Q on day i is

$$\frac{P_{i,j_1\cdots j_n}}{p_{i,j_1\cdots j_n}+\sum\limits_{\substack{j=1\\ j\neq j_1\cdots j_n}}^{k_i} p_{ij}}=\frac{\dfrac{P_{i,j_1\cdots j_n}}{S_i}}{\dfrac{p_{i,j_1\cdots j_n}+\sum\limits_{\substack{j=1\\ j\neq j_1\cdots j_n}}^{k_i} p_{ij}}{S_i}}=\frac{N_{i,j_1\cdots j_n}}{N_{i,j_1\cdots j_n}+\sum\limits_{\substack{j=1\\ j\neq j_1\cdots j_n}}^{k_i} N_{ij}}$$

when $j_n > k_i$, $j_1 \cdots j_n$ does not exist at this time, so $\frac{P_{i,j_1\cdots j_n}}{p_{i,j_1\cdots j_n}+\sum\limits_{\substack{j=1\\ j\neq j_1\cdots j_n}}^{k_i} p_{ij}}=0$

Hence $$Q_{i,j_1\cdots j_n}=\begin{cases}\dfrac{N_{i,j_1\cdots j_n}}{N_{i,j_1\cdots j_n}+\sum\limits_{\substack{j=1\\ j\neq j_1\cdots j_n}}^{k_i} N_{ij}}Q & , \quad 1\le j_n\le k_i\\ 0 & , \quad j_n>k_i\end{cases}$$

Theorem 2.2: $$V_{i,j_1\cdots j_n}=\begin{cases}\dfrac{N_{i,j_1\cdots j_n}}{N_{i,j_1\cdots j_n}+\sum\limits_{\substack{j=1\\ j\neq j_1\cdots j_n}}^{k_i} N_{ij}}QS_i & , \quad 1\le j_n\le k_i\\ 0 & , \quad j_n>k_i\end{cases}$$

Proof: We know from the theorem 2.1

$$Q_{i,j_1\cdots j_n}=\begin{cases}\dfrac{N_{i,j_1\cdots j_n}}{N_{i,j_1\cdots j_n}+\sum\limits_{\substack{j=1\\ j\neq j_1\cdots j_n}}^{k_i} N_{ij}}Q & , \quad 1\le j_n\le k_i\\ 0 & , \quad j_n>k_i\end{cases}$$

Hence $$V_{i,j_1\cdots j_n}=Q_{i,j_1\cdots j_n}S_i=\begin{cases}\dfrac{N_{i,j_1\cdots j_n}}{N_{i,j_1\cdots j_n}+\sum\limits_{\substack{j=1\\ j\neq j_1\cdots j_n}}^{k_i} N_{ij}}QS_i & , \quad 1\le j_n\le k_i\\ 0 & , \quad j_n>k_i\end{cases}$$

Theorem 2.3:

$$V_{j_1\cdots j_n}=\sum_{m=1}^{\infty}\sum_{i=m}^{\infty}\frac{N_{i,j_1\cdots j_n}}{N_{i,j_1\cdots j_n}+\sum\limits_{\substack{j=1\\ j\neq j_1\cdots j_n}}^{k_i} N_{ij}}QS_i\,1_{(k_{m-1},k_m]}(j_n)$$

where $k_0=0$.

Proof: We know from the theorem 2.2

$$V_{i,j_1\cdots j_n}=\begin{cases}\dfrac{N_{i,j_1\cdots j_n}}{N_{i,j_1\cdots j_n}+\sum\limits_{\substack{j=1\\ j\neq j_1\cdots j_n}}^{k_i}N_{ij}}QS_i & , \quad 1\le j_n\le k_i\\ 0 & , \quad j_n>k_i\end{cases}$$

so

$$V_{j_1\cdots j_n}=\sum_{m=1}^{\infty}\sum_{i=m}^{\infty}\frac{N_{i,j_1\cdots j_n}}{N_{i,j_1\cdots j_n}+\sum\limits_{\substack{j=1\\ j\neq j_1\cdots j_n}}^{k_i}N_{ij}}QS_i\,1_{(k_{m-1},k_m]}(j_n)$$

Theorem 2.4: $\lim\limits_{i\to\infty}N_{i,j_1\cdots j_n}=0$

Proof: Because

$$N_{i,j_1\cdots j_n}=\frac{P_{i,j_1\cdots j_n}}{S_i}$$

And because $P_{i,j_1\cdots j_n}$ is a bounded quantity and

$$\lim_{i\to\infty}S_i=+\infty \quad\to\quad \lim_{i\to\infty}\frac{1}{S_i}=0,$$

so

$$\lim_{i\to\infty}N_{i,j_1\cdots j_n}=\lim_{i\to\infty}\frac{P_{i,j_1\cdots j_n}}{S_i}=0.$$

Theorem 2.5: $\forall\varepsilon>0,\{i\mid N_{i,j_1\cdots j_n}<\varepsilon\}\neq\varnothing.$

Proof: We know from the theorem 1.3 and the definition of limit, $\forall\varepsilon>0,\ \exists I=I(j_1\ldots j_n,\varepsilon)$, when $i>I$, we have

$$N_{ij}<\varepsilon,$$

so

$$\{i\mid N_{i,j_1\cdots j_n}<\varepsilon\}\neq\varnothing.$$

Theorem 2.6: Set threshold $\varepsilon>0$, let $\min\{i\mid N_{i,j_1\cdots j_n}<\varepsilon\}=T(j_1\ldots j_n,\varepsilon)$, when $N_{i,j_1\cdots j_n}<\varepsilon$, it is considered that the $j_1,\ldots,j_n$ NFT exits the ore pool at this time, then

$$V_{j_1\cdots j_n}=\sum_{m=1}^{\infty}\sum_{i=m}^{T(j_1\ldots j_n,\varepsilon)}\frac{N_{i,j_1\cdots j_n}}{N_{i,j_1\cdots j_n}+\sum\limits_{\substack{j=1\\ j\neq j_1\cdots j_n}}^{k_i}N_{ij}}QS_i\,1_{(k_{m-1},k_m]}(j_n)$$

where $k_0=0$.

Proof: when $N_{i,j_1\cdots j_n}<\varepsilon$, it is considered that the $j_1,\ldots,j_n$ NFT exits the ore pool at this time,

here $N_{i,j_1\cdots j_n}=0$, when $i>T(j_1\ldots j_n,\varepsilon)$, we have

$$
\begin{aligned}
V_{j_1\cdots j_n} &= \sum_{i=n}^{\infty} V_{i,j_1\cdots j_n} = \sum_{n=1}^{\infty}\sum_{i=n}^{T(j_1\cdots j_n,\varepsilon)} V_{i,j_1\cdots j_n} 1_{(k_{n-1},k_n]}(j_n) + \sum_{n=1}^{\infty}\sum_{i=T(j_1\cdots j_n,\varepsilon)+1}^{\infty} V_{i,j_1\cdots j_n} 1_{(k_{n-1},k_n]}(j_n) \\
&= \sum_{n=1}^{\infty}\sum_{i=n}^{T(j_1\cdots j_n,\varepsilon)} \frac{N_{i,j_1\cdots j_n}}{\sum_{j=1}^{k_i} N_{i,j_1\cdots j_n}} QS_i 1_{(k_{n-1},k_n]}(j_n) + \sum_{n=1}^{\infty}\sum_{i=T(j_1\cdots j_n,\varepsilon)+1}^{\infty} \frac{N_{ij}}{\sum_{j=1}^{k_i} N_{ij}} QS_i 1_{(k_{n-1},k_n]}(j_n) \\
&= \sum_{n=1}^{\infty}\sum_{i=n}^{T(j_1\cdots j_n,\varepsilon)} \frac{N_{i,j_1\cdots j_n}}{\sum_{j=1}^{k_i} N_{i,j_1\cdots j_n}} QS_i 1_{(k_{n-1},k_n]}(j_n) + \sum_{n=1}^{\infty}\sum_{i=T(j_1\cdots j_n,\varepsilon)+1}^{\infty} \frac{0}{\sum_{j=1}^{k_i} N_{ij}} QS_i 1_{(k_{n-1},k_n]}(j_n) \\
&= \sum_{n=1}^{\infty}\sum_{i=n}^{T(j_1\cdots j_n,\varepsilon)} \frac{N_{i,j_1\cdots j_n}}{\sum_{j=1}^{k_i} N_{i,j_1\cdots j_n}} QS_i 1_{(k_{n-1},k_n]}(j_n)
\end{aligned}
$$

2. NFT pricing with discount

(1) There is no auction portfolio

Theorem 3.1: $V_{ij} = \begin{cases} \dfrac{N_{ij}}{\sum_{j=1}^{k_i} N_{ij}} QS_i e^{-ir} & , \quad 1 \le j \le k_i \\ 0 & , \quad k_i < j \end{cases}$

Proof: When $1 \le j \le k_i$, the proportion of the j-th in the total NFT on the i-th day is

$$
\frac{P_{ij}}{\sum_{j=1}^{k_i} P_{ij}} = \frac{\frac{P_{ij}}{S_i}}{\frac{\sum_{j=1}^{k_i} P_{ij}}{S_i}} = \frac{N_{ij}}{\sum_{j=1}^{k_i} N_{ij}}.
$$

When $k_i < j$, $V_{ij} = 0$,

so

$$
V_{ij} = \begin{cases} \dfrac{N_{ij}}{\sum_{j=1}^{k_i} N_{ij}} QS_i e^{-ir} & , \quad 1 < j \le k_i \\ 0 & , \quad k_i < j \end{cases}
$$

Theorem 3.2: $V_j=\sum_{n=1}^{\infty}\sum_{i=n}^{\infty}\frac{N_{ij}}{\sum_{j=1}^{k_i}N_{ij}}QS_i 1_{(k_{n-1},k_n]}(j)e^{-ir}$, where $1_{(k_{n-1},k_n]}(j)=\begin{cases}1, & j\in(k_{n-1},k_n]\\ 0, & j\notin(k_{n-1},k_n]\end{cases}$, $k_0=0$

Proof: We know from the theorem 3.1

$$V_{ij}=\begin{cases}\frac{N_{ij}}{\sum_{j=1}^{k_i}N_{ij}}QS_i e^{-ir} & , \quad 1\le j\le k_i\\ 0 & , \quad k_i<j\end{cases}$$

when $j\in(k_{n-1},k_n]$,

$$V_j=\sum_{i=1}^{\infty}V_{ij}=\sum_{i=n}^{\infty}V_{ij},$$

when $j\notin(k_{n-1},k_n]$,

$$V_j=0,$$

so

$$V_j=\sum_{n=1}^{\infty}\sum_{i=n}^{\infty}V_{ij}1_{(k_{n-1},k_n]}(j)=\sum_{n=1}^{\infty}\sum_{i=n}^{\infty}\frac{N_{ij}}{\sum_{j=1}^{k_i}N_{ij}}QS_i 1_{(k_{n-1},k_n]}(j)e^{-ir},$$

where $1_{(k_{n-1},k_n]}(j)=\begin{cases}1, & j\in(k_{n-1},k_n]\\ 0, & j\notin(k_{n-1},k_n]\end{cases}$.

Theorem 3.3: $\lim_{i\to\infty}N_{ij}=0.$

Proof: Because $N_i=\frac{P_{ij}}{S_i}$, and because P_{ij} is a bounded quantity and $\lim_{i\to\infty}S_i=+\infty$,

so

$$\lim_{i\to\infty}N_{ij}=0.$$

Theorem 3.4: $\forall\varepsilon>0,\{i\mid N_{ij}<\varepsilon\}\ne\varnothing.$

Proof: We know from the theorem 3.3 and the definition of limit, $\forall\varepsilon>0,\ \exists I=I(j,\varepsilon)$, when $i>I$, we have

$$N_{ij}<\varepsilon,$$

so

$$\{i\mid N_{ij}<\varepsilon\}\ne\varnothing.$$

Theorem 3.5: Set threshold $\varepsilon>0$, when $N_{ij}<\varepsilon$, it is considered that the j-th NFT exits the ore pool at this time, let $\min\{i\mid N_{ij}<\varepsilon\}=T(j,\varepsilon)$, then

$$V_j = \sum_{n=1}^{\infty}\sum_{i=n}^{T(j,\varepsilon)} \frac{N_{ij}}{\sum_{j=1}^{k_i} N_{ij}} QS_i 1_{(k_{n-1},k_n]}(j) e^{-ir}$$

where $k_0 = 0$.

Proof：When $N_{ij} < \varepsilon$,it is considered that the j −th NFT exits the ore pool at this time，

here $N_{ij} = 0$.

when $i > T(j,\varepsilon)$， we have

$$\begin{aligned} V_j &= \sum_{i=n}^{\infty} V_{ij} = \sum_{n=1}^{\infty}\sum_{i=n}^{T(j,\varepsilon)} V_{ij} 1_{(k_{n-1},k_n]}(j) + \sum_{n=1}^{\infty}\sum_{i=T(j,\varepsilon)+1}^{\infty} V_{ij} 1_{(k_{n-1},k_n]}(j) \\ &= \sum_{n=1}^{\infty}\sum_{i=n}^{T(j,\varepsilon)} \frac{N_{ij}}{\sum_{j=1}^{k_i} N_{ij}} QS_i 1_{(k_{n-1},k_n]}(j) e^{-ir} + \sum_{n=1}^{\infty}\sum_{i=T(j,\varepsilon)+1}^{\infty} \frac{N_{ij}}{\sum_{j=1}^{k_i} N_{ij}} QS_i 1_{(k_{n-1},k_n]}(j) e^{-ir} \\ &= \sum_{n=1}^{\infty}\sum_{i=n}^{T(j,\varepsilon)} \frac{N_{ij}}{\sum_{j=1}^{k_i} N_{ij}} QS_i 1_{(k_{n-1},k_n]}(j) e^{-ir} + \sum_{n=1}^{\infty}\sum_{i=T(j,\varepsilon)+1}^{\infty} \frac{0}{\sum_{j=1}^{k_i} N_{ij}} QS_i 1_{(k_{n-1},k_n]}(j) e^{-ir} \\ &= \sum_{n=1}^{\infty}\sum_{i=n}^{T(j,\varepsilon)} \frac{N_{ij}}{\sum_{j=1}^{k_i} N_{ij}} QS_i 1_{(k_{n-1},k_n]}(j) e^{-ir}. \end{aligned}$$

(2) There is an auction portfolio

Theorem 4.1： $V_{i,j_1\cdots j_n} = \begin{cases} \dfrac{N_{i,j_1\cdots j_n}}{N_{i,j_1\cdots j_n} + \sum\limits_{\substack{j=1 \\ j\neq j_1\cdots j_n}}^{k_i} N_{ij}} QS_i e^{-ir} & , \quad 1 \le j_n \le k_i \\ 0 & , \quad j_n > k_i \end{cases}$

Proof：When $1 \le j_n \le k_i$,and $j_1 ... j_n$ Combinatorial auction，$P_{i,j_1\cdots j_n} < \sum_{k=1}^{n} P_{j_k}$,so the proportion of corresponding $NFT_{j_1\cdots j_n}$ in total ore production Q on day i is

$$\frac{P_{i,j_1\cdots j_n}}{p_{i,j_1\cdots j_n} + \sum\limits_{\substack{j=1 \\ j\neq j_1\cdots j_n}}^{k_i} p_{ij}} = \frac{\frac{P_{i,j_1\cdots j_n}}{S_i}}{\frac{p_{i,j_1\cdots j_n} + \sum\limits_{\substack{j=1 \\ j\neq j_1\cdots j_n}}^{k_i} p_{ij}}{S_i}} = \frac{N_{i,j_1\cdots j_n}}{N_{i,j_1\cdots j_n} + \sum\limits_{\substack{j=1 \\ j\neq j_1\cdots j_n}}^{k_i} N_{ij}}$$

when $j_n > k_i$ ，$j_1 \cdots j_n$ does not exist at this time， so $\dfrac{P_{i,j_1\cdots j_n}}{p_{i,j_1\cdots j_n} + \sum\limits_{\substack{j=1 \\ j\neq j_1\cdots j_n}}^{k_i} p_{ij}} = 0.$

$$\text{Hence } V_{i,j_1\cdots j_n}=\begin{cases}\dfrac{N_{i,j_1\cdots j_n}}{N_{i,j_1\cdots j_n}+\sum\limits_{\substack{j=1\\ j\neq j_1\cdots j_n}}^{k_i}N_{ij}}QS_ie^{-ir} & , \ 1\le j_n\le k_i\\ 0 & , \ j_n>k_i\end{cases}$$

Theorem 4.2：

$$V_{j_1\cdots j_n}=\sum_{m=1}^{\infty}\sum_{i=m}^{\infty}\frac{N_{i,j_1\cdots j_n}}{N_{i,j_1\cdots j_n}+\sum\limits_{\substack{j=1\\ j\neq j_1\cdots j_n}}^{k_i}N_{ij}}QS_i1_{(k_{m-1},k_m]}(j_n)e^{-ir}$$

where $k_0=0$.

Proof：We know from the theorem 4.1

$$V_{i,j_1\cdots j_n}=\begin{cases}\dfrac{N_{i,j_1\cdots j_n}}{N_{i,j_1\cdots j_n}+\sum\limits_{\substack{j=1\\ j\neq j_1\cdots j_n}}^{k_i}N_{ij}}QS_ie^{-ir} & , \ 1\le j_n\le k_i\\ 0 & , \ j_n>k_i\end{cases}$$

so

$$V_{j_1\cdots j_n}=\sum_{m=1}^{\infty}\sum_{i=m}^{\infty}\frac{N_{i,j_1\cdots j_n}}{N_{i,j_1\cdots j_n}+\sum\limits_{\substack{j=1\\ j\neq j_1\cdots j_n}}^{k_i}N_{ij}}QS_i1_{(k_{m-1},k_m]}(j_n)e^{-ir}$$

Theorem 4.3： $\lim\limits_{i\to\infty}N_{i,j_1\cdots j_n}=0$

Proof：Because

$$N_{i,j_1\cdots j_n}=\frac{P_{i,j_1\cdots j_n}}{S_i}$$

And because $P_{i,j_1\cdots j_n}$ P_{ij} is a bounded quantity and

$$\lim_{i\to\infty}S_i=+\infty\ \rightarrow\ \lim_{i\to\infty}\frac{1}{S_i}=0,$$

so

$$\lim_{i\to\infty}N_{i,j_1\cdots j_n}=\lim_{i\to\infty}\frac{P_{i,j_1\cdots j_n}}{S_i}=0.$$

Theorem 4.4： $\forall\varepsilon>0,\{i\mid N_{i,j_1\cdots j_n}<\varepsilon\}\neq\varnothing$

Proof：We know from the theorem 4.3 and the definition of limit ，$\forall\varepsilon>0,\ \exists I=I(j_1\ldots j_n,\varepsilon)$，when $i>I$， we have

$$N_{ij}<\varepsilon,$$

so

$$\{i\mid N_{i,j_1\cdots j_n}<\varepsilon\}\neq\varnothing.$$

Theorem 4.5：Set threshold $\varepsilon>0$，let $\min\{i\mid N_{i,j_1\cdots j_n}<\varepsilon\}=T(j_1\ldots j_n,\varepsilon)$，when $N_{i,j_1\cdots j_n}<\varepsilon$，it is considered that the $j_1,\ldots,j_n$ NFT exits the ore pool at this time, then

$$V_{j_1\cdots j_n}=\sum_{m=1}^{\infty}\sum_{i=m}^{T(j_1\ldots j_n,\varepsilon)}\frac{N_{i,j_1\cdots j_n}}{N_{i,j_1\cdots j_n}+\sum\limits_{\substack{j=1\\ j\neq j_1\cdots j_n}}^{k_i}N_{ij}}QS_i\,1_{(k_{m-1},k_m]}(j_n)e^{-ir}$$

where $k_0=0$.

Proof: when $N_{i,j_1\cdots j_n}<\varepsilon$, it is considered that the $j_1,\ldots,j_n$ NFT exits the ore pool at this time, here $N_{i,j_1\cdots j_n}=0$, when $i>T(j_1\ldots j_n,\varepsilon)$, we have

$$\begin{aligned}
V_{j_1\cdots j_n} &=\sum_{i=n}^{\infty}V_{i,j_1\cdots j_n}=\sum_{n=1}^{\infty}\sum_{i=n}^{T(j_1\ldots j_n,\varepsilon)}V_{i,j_1\cdots j_n}1_{(k_{n-1},k_n]}(j_n)+\sum_{n=1}^{\infty}\sum_{i=T(j_1\ldots j_n,\varepsilon)+1}^{\infty}V_{i,j_1\cdots j_n}1_{(k_{n-1},k_n]}(j_n)\\
&=\sum_{n=1}^{\infty}\sum_{i=n}^{T(j_1\ldots j_n,\varepsilon)}\frac{N_{i,j_1\cdots j_n}}{\sum\limits_{j=1}^{k_i}N_{i,j_1\cdots j_n}}QS_i1_{(k_{n-1},k_n]}(j_n)e^{-ir}+\sum_{n=1}^{\infty}\sum_{i=T(j_1\ldots j_n,\varepsilon)+1}^{\infty}\frac{N_{ij}}{\sum\limits_{j=1}^{k_i}N_{ij}}QS_i1_{(k_{n-1},k_n]}(j_n)e^{-ir}\\
&=\sum_{n=1}^{\infty}\sum_{i=n}^{T(j_1\ldots j_n,\varepsilon)}\frac{N_{i,j_1\cdots j_n}}{\sum\limits_{j=1}^{k_i}N_{i,j_1\cdots j_n}}QS_i1_{(k_{n-1},k_n]}(j_n)e^{-ir}+\sum_{n=1}^{\infty}\sum_{i=T(j_1\ldots j_n,\varepsilon)+1}^{\infty}\frac{0}{\sum\limits_{j=1}^{k_i}N_{ij}}QS_i1_{(k_{n-1},k_n]}(j_n)e^{-ir}\\
&=\sum_{n=1}^{\infty}\sum_{i=n}^{T(j_1\ldots j_n,\varepsilon)}\frac{N_{i,j_1\cdots j_n}}{\sum\limits_{j=1}^{k_i}N_{i,j_1\cdots j_n}}QS_i1_{(k_{n-1},k_n]}(j_n)e^{-ir}.
\end{aligned}$$

Section three. POA consensus mechanism(Proof of Aution)

In the HRD pool, the NFT asset value is used as the anchor of pool calculation force, the amount of HRD produced per day is constant, which is halved with the block height. Each NFT asset is priced in the perpetual combinatorial auction, and the weight $\frac{N_{ij}}{\sum_{j=1}^{k_i} N_{ij}}(1 \le j \le k_i)$ in the HRD pool is obtained from the price. As long as the mortgage is successful, HRD $Q_{ij}(=\frac{N_{ij}}{\sum_{j=1}^{k_i} N_{ij}}Q)$ can be produced according to the weight. In this way, NFT assets can be standardized through the output HDR, and users can replace NFT assets through HRD.

Section four. Conclusion

In this paper, under the model of perpetual combinatorial auction, we give the NFT pricing formula with and without auction portfolio without discount. Furthermore, we give the NFT pricing formula with and with auction portfolio with discount, it solves the problem of NFT assets pricing. We further abstract the consensus mechanism of POA to realize the standardized pricing of NFT assets with HDR. Thus, the two problems of NFT assets circulation and pricing are well solved .