Basic Cryptography

Overview

- Symmetric cryptography
  - Cæsar cipher, Vigènere cipher, and one-time pad
  - DES, AES
- Public key (asymmetric) cryptography
  - RSA
  - Digital signatures

Symmetric Cryptography

Symmetric Cryptography

- Sender, receiver share common key
  - Keys may be the same, or trivial to derive from one another
  - Sometimes called *secret key cryptography*
- Two basic types
  - Transposition ciphers
  - Substitution ciphers
  - Combinations are called *product ciphers*

Cæsar cipher

Cæsar cipher

- Earliest known substitution cipher
- By Julius Caesar
- First attested use in military affairs
- Replaces each letter by 3rd letter on
- example:
- meet me after the party
- PHHW PH DIWHU WKH SDUWB

Cæsar cipher

- Formal form
  - $\mathcal{M}$ = { sequences of letters }
  - $\mathcal{K}$ = { i | i is an integer and $0 \leq i \leq 25$ }
  - $\mathcal{E}$ = { E_k | $k \in \mathcal{K}$ and for all letters m, $E_k(m) = (m + k) \bmod 26$ }
  - $\mathcal{D}$ = { D_k | $k \in \mathcal{K}$ and for all letters c, $D_k(c) = (26 + c - k) \bmod 26$ }
  - $C = \mathcal{M}$

Caesar’s Problem

- Key is too short
  - Can be found by exhaustive search
  - Statistical frequencies not concealed well
    - They look too much like regular English letters
- So make it longer
  - Multiple letters in key
  - Idea is to smooth the statistical frequencies to make cryptanalysis harder

Attacks

Attacks

- Opponent whose goal is to break cryptosystem is the *adversary*
  - Assume adversary knows algorithm used, but not key
- Three types of attacks:
  - *ciphertext only*: adversary has only ciphertext; goal is to find plaintext, possibly key
  - *known plaintext*: adversary has ciphertext, corresponding plaintext; goal is to find key
  - *chosen plaintext*: adversary may supply plaintexts and obtain corresponding ciphertext; goal is to find key

Basis for Attacks

- Mathematical attacks
  - Based on analysis of underlying mathematics
- Statistical attacks
  - Make assumptions about the distribution of letters, pairs of letters (digrams), triplets of letters (trigrams), *etc.*
    - Called *models of the language*
  - Examine ciphertext, correlate properties with the assumptions.

Character Frequencies

a	0.07984	h	0.06384	n	0.06876	t	0.09058
b	0.01511	i	0.07000	o	0.07691	u	0.02844
c	0.02504	j	0.00131	p	0.01741	v	0.01056
d	0.04260	k	0.00741	q	0.00107	w	0.02304
e	0.12452	l	0.03961	r	0.05912	x	0.00159
f	0.02262	m	0.02629	s	0.06333	y	0.02028
g	0.02013					z	0.00057

Substitution Cipher

Substitution Ciphers

- Change characters in plaintext to produce ciphertext
- Example (Caesar cipher)
  - Plaintext is `HELLO WORLD`
  - Change each letter to the third letter following it (`X` goes to `A`, `Y` to `B`, `Z` to `C`)
    - Key is 3, usually written as letter ‘`D`’
  - Ciphertext is `KHOOR ZRUOG`

Vigènere Cipher

Vigènere Cipher

- Like Cæsar cipher, but use a phrase
- Example
  - Message THE BOY HAS THE BALL
  - Key VIG
  - Encipher using Cæsar cipher for each letter:

```
key    VIGVIGVIGVIGVIGV
plain  THEBOYHASTHEBALL
cipher OPKWWECIYOPKWIRG
```

Relevant Parts of Tableau

	G	*I*	*V*
A	G	I	V
B	H	J	W
E	L	M	Z
H	N	P	C
L	R	T	G
O	U	W	J
S	Y	A	N
T	Z	B	O
Y	E	H	T

- Tableau shown has relevant rows, columns only
- Example encipherments:
  - key V, letter T: follow V column down to T row (giving "O")
  - Key I, letter H: follow I column down to H row (giving "P")

Useful Terms

- *period*: length of key
  - In earlier example, period is 3
- *tableau*: table used to encipher and decipher
  - Vigènere cipher has key letters on top, plaintext letters on the left
- *polyalphabetic*: the key has several different letters
  - Cæsar cipher is monoalphabetic

Attacking the Cipher

- Approach
  - Establish period; call it *n*
  - Break message into *n* parts, each part being enciphered using the same key letter
  - Solve each part
    - You can leverage one part from another

One-Time Pad

One-Time Pad

- A Vigenère cipher with a random key at least as long as the message
  - Provably unbreakable
  - Why? Look at ciphertext `DXQR`. Equally likely to correspond to plaintext `DOIT` (key `AJIY`) and to plaintext `DONT` (key `AJDY`) and any other 4 letters
  - Warning: keys *must* be random, or you can attack the cipher by trying to regenerate the key
    - Approximations, such as using pseudorandom number generators to generate keys, are *not* random

Transposition Cipher

Transposition Cipher

- Rearrange letters in plaintext to produce ciphertext
- Example (Rail-Fence Cipher)
  - Plaintext is `HELLO WORLD`
  - Rearrange as

    ```
    HLOOL
    ELWRD
    ```

  - Ciphertext is `HLOOL ELWRD`

Example

- Arrange so the H and E are adjacent

```
HE
LL
OW
OR
LD
```

- Read across, then down, to get original plaintext

Attacking the Cipher

- Anagramming
  - If 1-gram frequencies match English frequencies, but other *n*-gram frequencies do not, probably transposition
  - Rearrange letters to form *n*-grams with highest frequencies

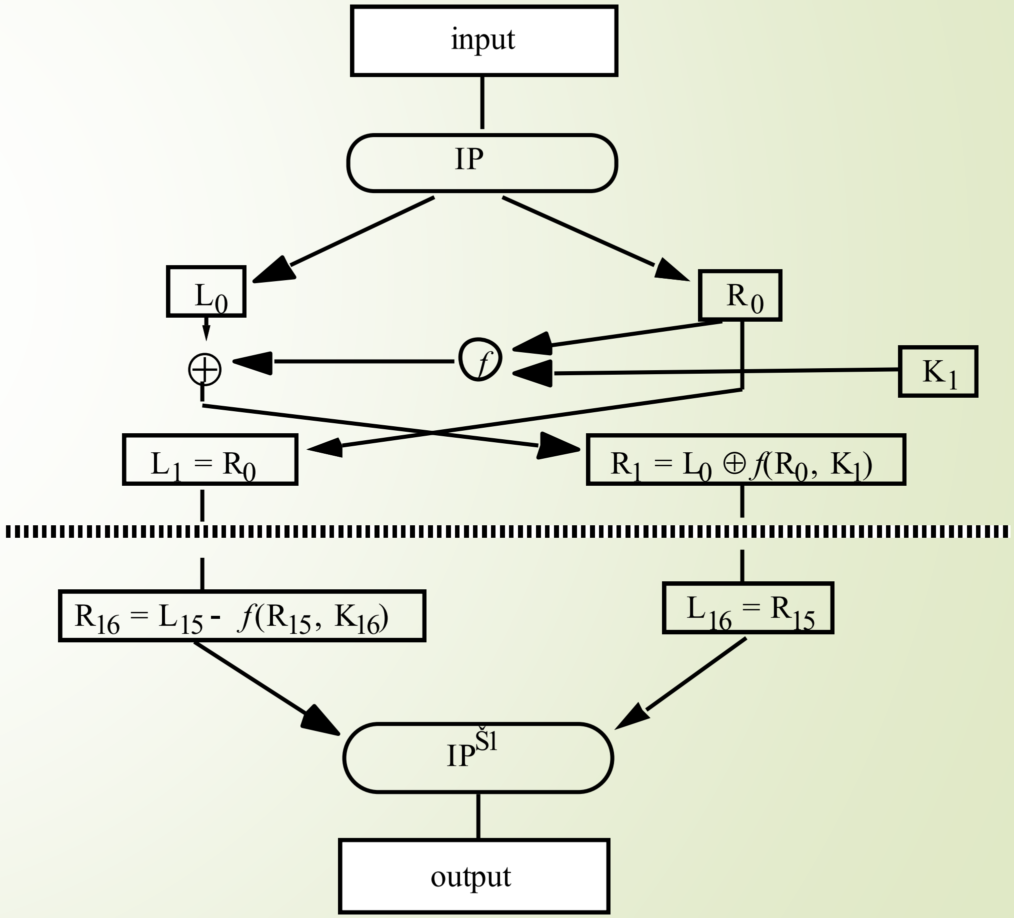
DES

Overview of the DES

- A block cipher:
  - Encrypts blocks of 64 bits using a 64 bit key
  - Outputs 64 bits of ciphertext
- A product cipher
  - Basic unit is the bit
  - Performs both substitution and transposition (permutation) on the bits
- Cipher consists of 16 rounds (iterations) each with a 48 bit round key generated from the user-supplied key

Structure of the DES

- Input is first permuted, then split into left half (L) and right half (R), each 32 bits
- R and round key run through function *f*
- R and L swapped
- After last round, L and R combined, permuted, forming DES output



Controversy

- Considered too weak
- Design decisions not public
  - S-boxes may have backdoors

AES

Advanced Encryption Standard

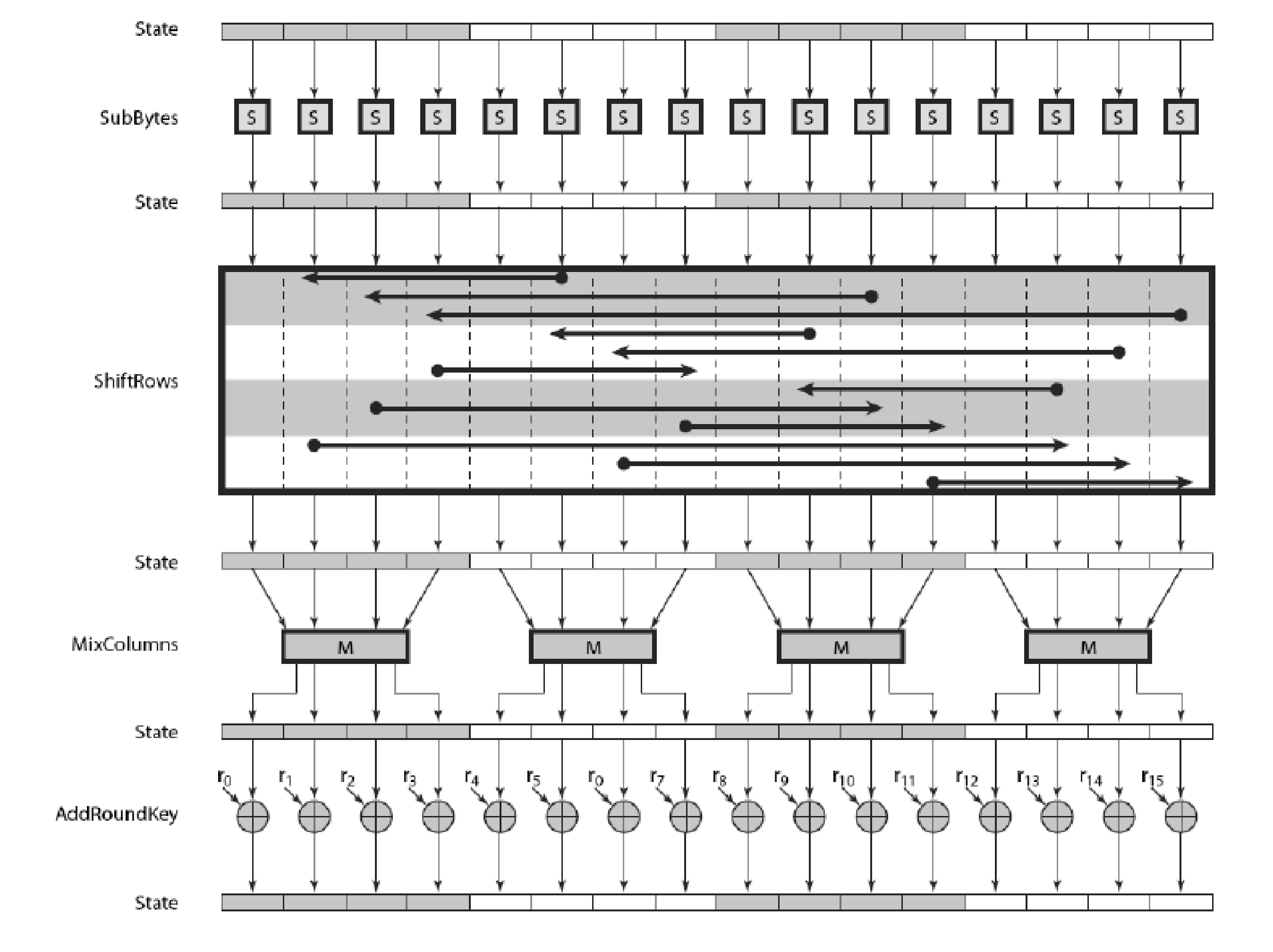
- Competition announces in 1997 to select successor to DES
  - Successor needed to be available for use without payment (no royalties, etc.)
  - Successor must encipher 128-bit blocks with keys of lengths 128, 192, and 256
- Rijndael selected as successor to DES, called the Advanced Encryption Standard (AES

Overview of the AES

- A block cipher:
  - encrypts blocks of 128 bits using a 128, 192, or 256 bit key
  - outputs 128 bits of ciphertext
- A product cipher
  - basic unit is the bit
  - performs both substitution and transposition (permutation) on the bits
- Cipher consists of rounds (iterations) each with a round key generated from the user-supplied key
  - If 128 bit key, then 10 rounds
  - If 192 bit key, then 12 rounds
  - If 256 bit key, then 14 rounds

Structure of the AES: Encryption

- byte substitution
- shift rows
- mix columns
- add round key



Public Key Cryptography

Public Key Cryptography

- Two keys
  - *Private key* known only to individual
  - *Public key* available to anyone
    - Public key, private key inverses
- Idea
  - Confidentiality: encipher using public key, decipher using private key
  - Integrity/authentication: encipher using private key, decipher using public one

Requirements

1. It must be computationally easy to encipher or decipher a message given the appropriate key
2. It must be computationally infeasible to derive the private key from the public key
3. It must be computationally infeasible to determine the private key from a chosen plaintext attack

RSA

RSA

- First described publicly in 1978
- RSA = “Rivest, Shamir, and Adleman”
- 2048 bit keys (at least)
- Exponentiation cipher
- Relies on the difficulty of determining the number of numbers relatively prime to a large integer *n*

Algorithm

- Choose two large prime numbers p, q
  - Let $n = pq$; then $\phi(n) = (p-1)(q-1)$
  - Choose $e < n$ such that e is relatively prime to $\phi(n)$.
  - Compute d such that $ed \bmod \phi(n) = 1$
- Public key: (e, n); private key: d
- Encipher: $c = m^e \bmod n$
- Decipher: $m = c^d \bmod n$

Example: Confidentiality

- Take p = 181, q = 1451, so n = 262631 and $\phi(n)$ = 261000
- Alice chooses e = 154993, making d = 95857
- Bob wants to send Alice secret message PUPPIESARESMALL (152015 150804 180017 041812 001111); encipher using public key
  - $152015^{154993} \bmod 262631 = 220160$
  - $150804^{154993} \bmod 262631 = 135824$
  - $180017^{154993} \bmod 262631 = 252355$
  - $041812^{154993} \bmod 262631 = 245799$
  - $001111_{154993} \bmod 262631 = 070707$
- Bob sends 220160 135824 252355 245799 070707
- Alice uses her private key to decipher it

Digital Signature

Digital Signature

- Construct that authenticates origin, contents of message in a manner provable to a disinterested third party (a “judge”)
- Sender cannot deny having sent message (service is “nonrepudiation”)

Public Key Digital Signatures

- Basically, Alice enciphers the message, or its cryptographic hash, with her private key
- In case of dispute or question of origin or whether changes have been made, a judge can use Alice's public key to verify the message came from Alice and has not been changed since being signed

Example

- Alice chooses e = 154993, making *d* = 95857
- Alice wants to send Bob the message PUPPIESARESMALL in such a way that Bob knows it comes from her and nothing was changed during the transmission
- Encipher using private key:
  - $152015^{95857} \bmod 262631 = 072798$
  - $150804^{95857} \bmod 262631 = 259757$
  - $180017^{95857} \bmod 262631 = 256449$
  - $041812^{95857} \bmod 262631 = 089234$
  - $001111^{95857} \bmod 262631 = 037974$
- Alice sends 072798 259757 256449 089234 037974
- Bob receives, uses Alice's public key to decipher it

Encryption and Digital Signature

Example: Both (Sending)

- Alice chooses *e* = 154993, making *d* = 95857, n = 262631
- Same *n* as for Alice; Bob chooses *e* = 45593, making *d* = 235457
- Alice wants to send PUPPIESARESMALL (152015 150804 180017 041812 001111) confidentially and authenticated
- Encipher:
  - $(152015^{95857} \bmod 262631)^{45593} \bmod 262631 = 249123$
  - $(150804^{95857} \bmod 262631)^{45593} \bmod 262631 = 166008$
  - $(180017^{95857} \bmod 262631)^{45593} \bmod 262631 = 146608$
  - $(041812^{95857} \bmod 262631)^{45593} \bmod 262631 = 092311$
  - $(001111^{95857} \bmod 262631)^{45593} \bmod 262631 = 096768$
- So Alice sends 249123 166008 146608 092311 096768

Example: Both (Receiving)

- Bob receives 249123 166008 146608 092311 096768
- Decipher:
  - $(249123^{235457} \bmod 262631)^{154993} \bmod 262631 = 152012$
  - $(166008^{235457} \bmod 262631)^{154993} \bmod 262631 = 150804$
  - $(146608^{235457} \bmod 262631)^{154993} \bmod 262631 = 180017$
  - $(092311^{235457} \bmod 262631)^{154993} \bmod 262631 = 041812$
  - $(096768^{235457} \bmod 262631)^{154993} \bmod 262631 = 001111$
- So Alice sent him 152015 150804 180017 041812 001111
  - Which translates to PUP PIE SAR ESM ALL or PUPPIESARESMALL

Key Points

Key Points

- Two main types of cryptosystems: symmetric and public key
- Symmetric key cryptosystems encipher and decipher using the same key
- Public key cryptosystems encipher and decipher using different keys
  - RSA, computationally infeasible to derive one from the other
- Digital signatures provide integrity of origin and content

  Much easier with public key cryptosystems than with classical cryptosystems