Name: **May 5, 2014**

Math 237: Final Exam (Part 1)

Please use separate paper for all your work and solutions. Make sure all your work/solutions are clearly marked, neat and organized. Staple your work to this exam. You must show all work on each question to get full points. This part of the exam is due Wednesday (5/7/14) at 10:15 am in class.

1. (5 points) Find all values of k for which the following vectors are orthogonal:
$$\vec{u} = [k, k, -2] \qquad \vec{v} = [-2, k-1, 5].$$
2. (10 points) Find the distance between the planes $2x - y + z = 1$ and $-6x + 3y - 3z = 1$.
3. (5 points) Given $\vec{u} = [2, 3, 0], \vec{v} = [0, -3, -2]$, find the orthogonal projection of $\overrightarrow{u+v}$ along $[0, 1, 1]$.
4. (10 points) Solve the following system, using Gaussian or Gauss-Jordan elimination:
$$\begin{aligned} x - 4y - z + w &= 3 \\ 2x - 8y + z - 4w &= 9 \\ -x + 4y - 2z + 5w &= -6. \end{aligned}$$
5. (5 points) Determine if the following set of vectors is linearly independent. If the set is linearly dependent, find a dependence relationship among the vectors.
$$\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \vec{y} = \begin{bmatrix} 2 \\ 5 \\ 7 \end{bmatrix}, \vec{z} = \begin{bmatrix} -1 \\ -4 \\ -5 \end{bmatrix}.$$
6. (5 points) Let $A = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$. Find A^{2014}.
7. (10 points) Use the Gauss-Jordan method to find the inverse of A in $\mathbb{Z}_4$
$$A = \begin{bmatrix} 2 & 3 & 0 \\ 0 & 3 & -3 \\ -2 & 3 & 3 \end{bmatrix}.$$
8. (5 points) Find the standard matrix representation of the linear transformation from $\mathbb{R}^2$ into $\mathbb{R}^2$ which corresponds to rotation counterclockwise around the origin by $45°$.
9. (20 points) Let $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$.
   (a) Find the eigenvalues of A.
   (b) Find a basis for the eigenspace of A.
   (c) Find an invertible matrix P and a diagonal matrix D such that $D = P^{-1}AP$.
10. (15 points) Apply the Gram-Schmidt process to the vectors to obtain an orthonormal basis for $\mathbb{R}^3$
$$\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

11. (10 points) Let $A = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$. Find an orthogonal matrix Q and a diagonal matrix D such that $Q^T AQ = D$.

12. (15 points) Find the QR factorization of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.

13. (10 points) Find a symmetric 2×2 matrix with eiginevalues $\lambda_1 = 8, \lambda_2 = 3$, and corresponding eigenvectors $\vec{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$.