University of North Alabama

Programming Languages

JavaScript

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Contents

1 History

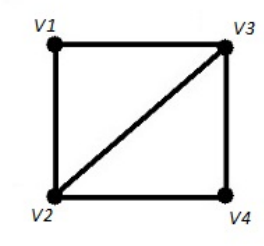


Figure 1

1.1 Adjacency matrix for A

$$A = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

1.2 A^2 and A^3

$$A^2 = \begin{pmatrix} 2 & 1 & 1 & 2 \\ 1 & 3 & 2 & 1 \\ 1 & 2 & 3 & 1 \\ 2 & 1 & 1 & 2 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 2 & 5 & 5 & 2 \\ 5 & 4 & 5 & 5 \\ 5 & 5 & 4 & 5 \\ 2 & 5 & 5 & 2 \end{pmatrix}$$

1.2.1 What values in A^n tell about the graph in Figure 1 proving your claim for A^2

The n in the expression A^n represents the amount of edges that must be used in order to travel between one node, to another. The adjacency matrix for A^1 represents 0 paths between V_1 and itself. In the adjacency matrix A^2, there are 2 possible paths from V_1 to itself utilizing 2 edges.

1.3 Compute the eigenvalues and eigenvectors for A

Eigenvalues

$$
\begin{array}{ccc}
\lambda_1 & \rightarrow & \frac{1}{2}(1+\sqrt{17}) \\
\lambda_2 & \rightarrow & \frac{1}{2}(1-\sqrt{17}) \\
\lambda_3 & \rightarrow & \text{-1} \\
\lambda_4 & \rightarrow & 0
\end{array}
$$

Eigenvectors

$$
\vec{x}_1 = \begin{pmatrix} 1 \\ \frac{1}{4}(1+\sqrt{17}) \\ \frac{1}{4}(1+\sqrt{17}) \\ 1 \end{pmatrix}
\vec{x}_2 = \begin{pmatrix} 1 \\ \frac{1}{4}(1-\sqrt{17}) \\ \frac{1}{4}(1-\sqrt{17}) \\ 1 \end{pmatrix}
\vec{x}_3 = \begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}
\vec{x}_4 = \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}
$$

1.3.1 Importance of eigenvalues and eigenvectors to graph

We observed that there is one zero, one positive, and two negative eigenvalues for the adjacency matrix A. The importance of these values related to the graph can be used to determine the structure of the graph. For this graph,

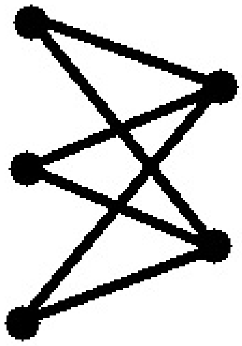


Figure 2

1.4 Draw the adjacency matrix A for Figure 2

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

1.5 Figure 2 Cycles

1.5.1 Amount of 2-cycles for the left-hand and right-hand vertices

Left-hand: 6
Right-Hand: 6

1.5.2 Amount of 4-cycles for the left-hand and right-hand vertices

Left-hand: 6
Right-Hand: 4

1.5.3 Amount of 6-cycles for the left-hand and right-hand vertices

Left-hand: 0
Right-Hand: 0

1.6 Eigenvalues and eigenvectors for graph in Figure 2

Eigenvalues

$$\begin{array}{ccc}
\lambda_1 & \rightarrow & -\sqrt{6} \\
\lambda_2 & \rightarrow & \sqrt{6} \\
\lambda_3 & \rightarrow & 0 \\
\lambda_4 & \rightarrow & 0 \\
\lambda_3 & \rightarrow & 0
\end{array}$$

Eigenvectors

$$\vec{x}_1 = \begin{pmatrix} 1 \\ \sqrt{\frac{3}{2}} \\ 1 \\ \sqrt{\frac{3}{2}} \\ 1 \end{pmatrix} \vec{x}_1 = \begin{pmatrix} 1 \\ -\sqrt{\frac{3}{2}} \\ 1 \\ -\sqrt{\frac{3}{2}} \\ 1 \end{pmatrix} \vec{x}_3 = \begin{pmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \vec{x}_4 = \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \vec{x}_5 = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

1.6.1 Importance of eigenvalues and eigenvectors

The eigenvalues and eigenvectors explain the structure of the graph. Observing that the greatest eigenvalue is positive form of the smallest eigenvalue for this graph, we can conclude that this graph is Bi-partite.

1.7 k-paths from A to B when $k = 1, 2, 3, 4, 5, 6$

$$\begin{array}{c}
k = 1 \rightarrow 0 \\
k = 2 \rightarrow 1 \\
k = 3 \rightarrow 0 \\
k = 4 \rightarrow 4 \\
k = 5 \rightarrow 0 \\
k = 6 \rightarrow 6
\end{array}$$

1.8 Properties Figure 3 has in common with Figure 2

These two graphs share the property of their respective adjacency matrices being symmetric.

1.9 Eigenvalues of the n-cube for $n = 1, 2, 3, 4$

$n = 1$

$$\lambda_1 \rightarrow 1$$

$n = 2$

$$\begin{array}{ccc} \lambda_1 & \rightarrow & \text{-2} \\ \lambda_2 & \rightarrow & 2 \\ \lambda_3 & \rightarrow & 0 \\ \lambda_4 & \rightarrow & 0 \end{array}$$

$n = 3$

$$\begin{array}{ccc} \lambda_1 & \rightarrow & \text{-3} \\ \lambda_2 & \rightarrow & 3 \\ \lambda_3 & \rightarrow & \text{-1} \\ \lambda_4 & \rightarrow & \text{-1} \\ \lambda_5 & \rightarrow & 1 \\ \lambda_6 & \rightarrow & 1 \\ \lambda_7 & \rightarrow & 0 \\ \lambda_8 & \rightarrow & 0 \end{array}$$

$n = 4$

$$\begin{array}{ccc} \lambda_1 & \rightarrow & 0 \\ \lambda_2 & \rightarrow & 0 \\ \lambda_3 & \rightarrow & 0 \\ \lambda_4 & \rightarrow & 0 \\ \lambda_5 & \rightarrow & 0 \\ \lambda_6 & \rightarrow & 0 \\ \lambda_7 & \rightarrow & \text{-2} \\ \lambda_8 & \rightarrow & \text{-2} \\ \lambda_1 & \rightarrow & \text{-2} \\ \lambda_2 & \rightarrow & \text{-2} \\ \lambda_3 & \rightarrow & 2 \\ \lambda_4 & \rightarrow & 2 \\ \lambda_5 & \rightarrow & 2 \\ \lambda_6 & \rightarrow & 2 \\ \lambda_7 & \rightarrow & \text{-4} \\ \lambda_8 & \rightarrow & 4 \end{array}$$

2 Digraphs and Ranking

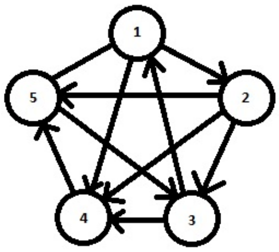


Figure 4

2.1 Draw the Transition Matrix A for Figure 4

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

2.2 Compute A^2 and A^3

$$A^2 = \begin{pmatrix} 0 & 0 & 2 & 1 & 2 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$A^3 = \begin{pmatrix} 2 & 0 & 2 & 2 & 1 \\ 1 & 1 & 1 & 2 & 2 \\ 0 & 0 & 3 & 1 & 2 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 & 2 \end{pmatrix}$$

2.2.1 What are the values in A^n telling you about the graph in Figure 1? Prove your claim for A^2

The n in A^n represents the number of edges that must be used in order to travel from one vertice to another. Referring to Figure 2, it is apparent that there doesn't exist a path $V1 \to V3$. However, the adjacency matrix A^2 calculates that there are 2 paths $V1 \to V3$. This is because the adjacency matrix has defined paths between vertices by utilizing 2 edges, such as $V1 \to V5 \to V3$ and $V1 \to V2 \to V3$.

2.3 Round Robin Tournament using method described in class

To illustrate the ideas introduced throughout this section, the digraph in Figure 2 will be viewed as a Street Fighter tournament where each vertice will be imagined as a player. The edge stemming from one vertice to another will represent a match played between players that will result in a win for one player, and a loss for the other. The player that wins will have its edge directed at the losing player. Taking the graph in Figure 2 and applying Person-Frobineus theorem to the tournament matrix, it can be calculated that the ranks of the players are:

1st: Player 1
2nd: Player 2
3rd: Player 3
4th: Player 5
5th: Player 4

2.3.1 Method

Since we are interested in the rankings of the players, we first organize the rankings of the players into a $rankingvector\vec{r}$.

$$r = \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \end{pmatrix}$$

Then, we associate the amount of wins with a player by setting $\vec{r} = A$. This yielded the following system

$$r_1 = \alpha(r_2 + r_4 + r_5)$$

$$r_2 = \alpha(r_3 + r_4 + r_5)$$

$$r_3 = \alpha(r_1 + r_4)$$

$$r_4 = \alpha(r_1)$$

$$r_5 = \alpha(r_3)$$

By manipulating this system, we transformed the equation

$$\vec{r} = \alpha A \vec{r}$$

to

$$A\vec{r} = \frac{1}{\alpha}\vec{r}$$

When the system is represented in this manner, along with the fact that A turns out to be a nonnegative matrix, then it is possible to apply the Perron-Frobenius Theorem to find a solution to this ranking problem. The Perron-Frobenius Theorem guarantees that there is a *unique* ranking vector $\vec{r}$. Using this fact, we solved for corresponding eigenvalues and eigenvectors for the tournament matrix A. This produced the equation:

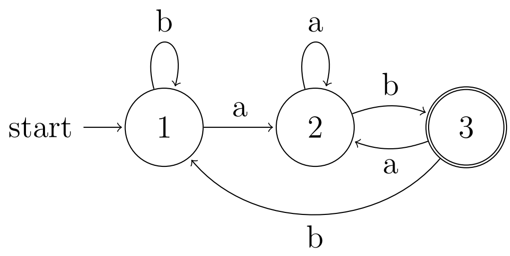
$$A \begin{pmatrix} 2.15062 \\ 1.96587 \\ 1.6593 \\ 0.602663 \\ 1 \end{pmatrix} = 1.6593 \begin{pmatrix} 2.15062 \\ 1.96587 \\ 1.6593 \\ 0.602663 \\ 1 \end{pmatrix}$$

Multiplying the left hand side of each equation by A^{-1}, we see that we have solved for the ranking vector $\vec{r}$

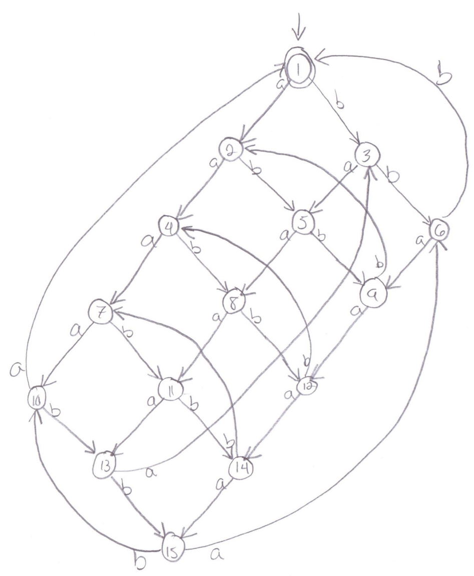
$$\begin{pmatrix} 2.1 \\ 1.9 \\ 1.6 \\ 0.6 \\ 1 \end{pmatrix} \rightarrow \vec{r_i} = \begin{pmatrix} 1st \\ 2nd \\ 3rd \\ 5th \\ 4th \end{pmatrix}$$

3 Automata and Matrices

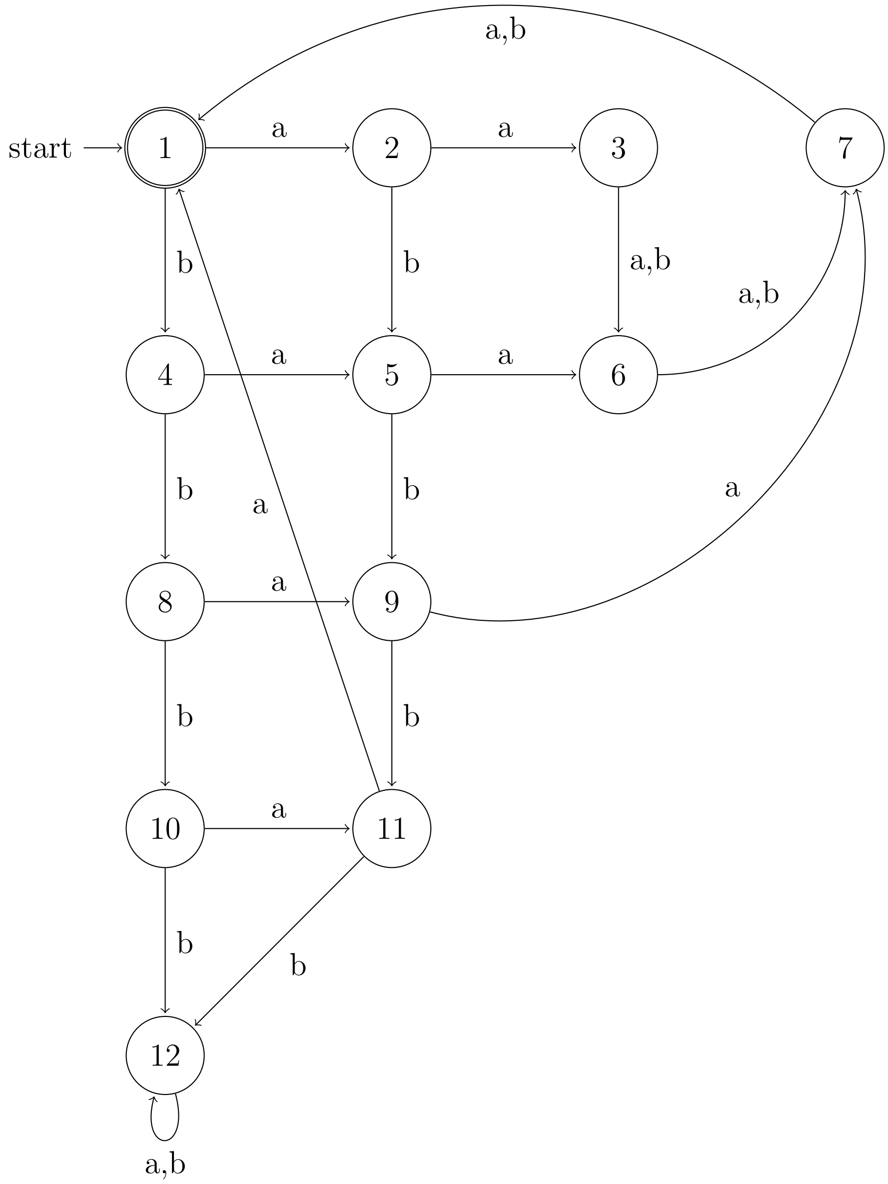
3.1 Finite automata that accepts all strings that end with ab



3.2 Finite automata that accepts all strings such that the number of $a's$ is divisible by 5, and the number of $b's$ is divisible by 3



3.3 Finite automata that accepts all strings such that every block of 5 symbols contains at least two $a's$



3.4 Strings that are accepted by automata in Figure 5

The automata in Figure 5 accepts strings that must include atleast one b and does not end with ba. For all strings that do not include b, it is not possible to exit state 1. All strings that end with ba will finish at state 3.

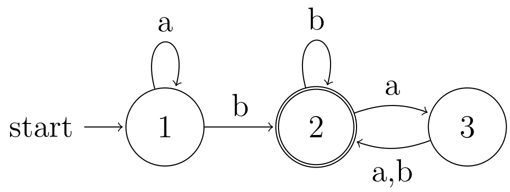


Figure 5

3.5 Transition matrix for automata in figure 5 labeled A, B

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$AB = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$BA = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$B^2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

3.5.1 Explanation of transition matrices

These matrices represent beginning and end states for a finite automata. For matrix index a_{ij}, we view row i as the beginning state, and j as the end state. For example, matrix A represents the act of feeding a single a character into the automata. Beginning at row 1, state 1 will have the end result of traveling to column 1, state 1 of the automata. This idea extends to the transition matrix A^2, which feeds two a characters into the automata. The same applies for B^2 and all other cases of transition matrices as well. This idea also extends to the case where we multiply transition matrix A by transition matrix B. This new transition matrix AB represents the event that two characters a

and b are put into the finite automata. Again, beginning at row 1, the end result will be that the end state will result at state 3.

3.6 Types of strings that are accepted by the automata in figure 6 labeled A,B

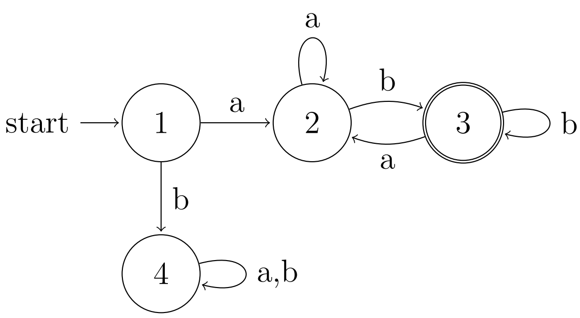


Figure 5

The string entered must begin with an a, and must end with a b. When a string begins with a, it is impossible to state 4. When a string does not conclude with b, it will finish at state 2.

3.7 Transition matrix for automata in figure 6 labeled A,B

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$AB = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$BA = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

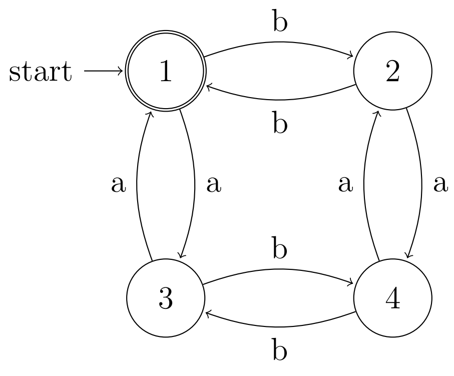
$$A^2 = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B^2 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3.7.1 Description of transition matrices for the automata in Figure 6

These matrices represent beginning and end states for a finite automata. For matrix index a_{ij}, we view row i as the beginning state, and j as the end state. For example, matrix A represents the act of feeding a single a character into the automata. Beginning at row 1, state 1 will have the end result of traveling to column 2, state 2 of the automata. This idea extends to the transition matrix A^2, which feeds two a characters into the automata. The same applies for B^2 and all other cases of transition matrices as well. This idea also extends to the case where we multiply transition matrix A by transition matrix B. This new transition matrix AB represents the event that two characters a and b are put into the finite automata. Again, beginning at row 1, the end result will be that the end state will result at state 3.

3.8 Finite automata that accepts all strings that have even numbers of $a's$ and $b's$



3.8.1 Transition matrices for previous automata

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$AB = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$BA = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B^2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3.8.2 Eigenvalues and eigenvectors

$$A$$

$$\begin{array}{ccc} \lambda_1 & \rightarrow & \text{-1} \\ \lambda_2 & \rightarrow & \text{-1} \\ \lambda_3 & \rightarrow & 1 \\ \lambda_4 & \rightarrow & 1 \end{array}$$

$$\vec{x}_1 = \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix} \; \vec{x}_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \; \vec{x}_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} \; \vec{x}_4 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$B$$

$$\begin{aligned}\lambda_1 &\rightarrow \text{-1}\\ \lambda_2 &\rightarrow \text{-1}\\ \lambda_3 &\rightarrow 1\\ \lambda_4 &\rightarrow 1\end{aligned}$$

$$\vec{x}_1 = \begin{pmatrix}0\\0\\-1\\1\end{pmatrix} \vec{x}_2 = \begin{pmatrix}-1\\1\\0\\0\end{pmatrix} \vec{x}_3 = \begin{pmatrix}0\\0\\1\\1\end{pmatrix} \vec{x}_4 = \begin{pmatrix}1\\1\\0\\0\end{pmatrix}$$

$$AB = BA$$

$$\begin{aligned}\lambda_1 &\rightarrow \text{-1}\\ \lambda_2 &\rightarrow \text{-1}\\ \lambda_3 &\rightarrow 1\\ \lambda_4 &\rightarrow 1\end{aligned}$$

$$\vec{x}_1 = \begin{pmatrix}-1\\0\\0\\1\end{pmatrix} \vec{x}_2 = \begin{pmatrix}0\\-1\\1\\0\end{pmatrix} \vec{x}_3 = \begin{pmatrix}1\\0\\0\\1\end{pmatrix} \vec{x}_4 = \begin{pmatrix}0\\1\\1\\0\end{pmatrix}$$

$$A^2 = B^2$$

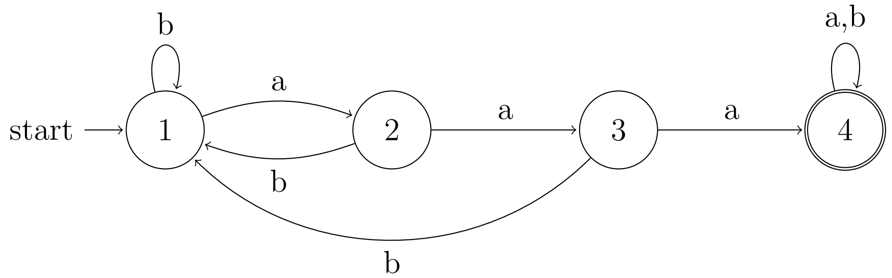
$$\begin{aligned}\lambda_1 &\rightarrow 1\\ \lambda_2 &\rightarrow 1\\ \lambda_3 &\rightarrow 1\\ \lambda_4 &\rightarrow 1\end{aligned}$$

$$\vec{x}_1 = \begin{pmatrix}0\\0\\0\\1\end{pmatrix} \vec{x}_2 = \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \vec{x}_3 = \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \vec{x}_4 = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$

3.8.3 Importance of eigenvalues and eigenvectors to automata

The absolute value of the eigenvalues for each calculated matrix is one. The eigenvalues for A, B, AB, and BA are symmetric. This represents that there are multiple nodes that have the same properties of adjacency.

3.9 Finite automata that accepts all strings with a substring aaa



$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$AB = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$BA = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$B^2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

3.9.1 Eigenvalues and eigenvectors

$$A$$

$$\begin{aligned}\lambda_1 &\to 1\\ \lambda_2 &\to 0\\ \lambda_3 &\to 0\\ \lambda_4 &\to 0\end{aligned}$$

$$\vec{x}_1 = \begin{pmatrix}1\\1\\1\\1\end{pmatrix} \ \vec{x}_2 = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \ \vec{x}_3 = \begin{pmatrix}0\\0\\0\\0\end{pmatrix} \ \vec{x}_4 = \begin{pmatrix}0\\0\\0\\0\end{pmatrix}$$

$$B$$

$$\begin{aligned}\lambda_1 &\to 1\\ \lambda_2 &\to 1\\ \lambda_3 &\to 0\\ \lambda_4 &\to 0\end{aligned}$$

$$\vec{x}_1 = \begin{pmatrix}0\\0\\0\\1\end{pmatrix} \ \vec{x}_2 = \begin{pmatrix}1\\1\\1\\0\end{pmatrix} \ \vec{x}_3 = \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \ \vec{x}_4 = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$

$$AB$$

$$\begin{aligned}\lambda_1 &\to 1\\ \lambda_2 &\to 1\\ \lambda_3 &\to 0\\ \lambda_4 &\to 0\end{aligned}$$

$$\vec{x}_1 = \begin{pmatrix}0\\0\\1\\1\end{pmatrix} \ \vec{x}_2 = \begin{pmatrix}1\\1\\0\\0\end{pmatrix} \ \vec{x}_3 = \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \ \vec{x}_4 = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$

$$BA$$

$$\begin{aligned}\lambda_1 &\to 1\\ \lambda_2 &\to 1\\ \lambda_3 &\to 0\\ \lambda_4 &\to 0\end{aligned}$$

$$\vec{x}_1 = \begin{pmatrix}0\\0\\0\\1\end{pmatrix} \; \vec{x}_2 = \begin{pmatrix}1\\1\\1\\0\end{pmatrix} \; \vec{x}_3 = \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \; \vec{x}_4 = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$$

A^2

$$\begin{aligned} \lambda_1 &\rightarrow 1 \\ \lambda_2 &\rightarrow 0 \\ \lambda_3 &\rightarrow 0 \\ \lambda_4 &\rightarrow 0 \end{aligned}$$

$$\vec{x}_1 = \begin{pmatrix}-1\\-1\\-1\\-1\end{pmatrix} \; \vec{x}_2 = \begin{pmatrix}0\\1\\0\\0\end{pmatrix} \; \vec{x}_3 = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \; \vec{x}_4 = \begin{pmatrix}0\\0\\0\\0\end{pmatrix}$$

B^2

$$\begin{aligned} \lambda_1 &\rightarrow 1 \\ \lambda_2 &\rightarrow 1 \\ \lambda_3 &\rightarrow 0 \\ \lambda_4 &\rightarrow 0 \end{aligned}$$

$$\vec{x}_1 = \begin{pmatrix}0\\0\\0\\1\end{pmatrix} \; \vec{x}_2 = \begin{pmatrix}1\\1\\1\\0\end{pmatrix} \; \vec{x}_3 = \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \; \vec{x}_4 = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$

3.9.2 Importance of eigenvalues and eigenvectors to automata