

Assignment 1 - Part B : Forward & Backward PropagationData Point : $x = [20, 3, 4]$ $y = 18$

Initial weights and biases : $W_1 = \begin{bmatrix} 0.1 & 0.2 & 0.3 \\ 0.4 & 0.5 & 0.6 \\ 0.7 & 0.8 & 0.9 \end{bmatrix}$ $b_1 = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.3 \end{bmatrix}$

$$W_2 = [0.2 \quad 0.4 \quad 0.6] \quad b_2 = [0.5]$$

Forward Propagation- Step 1 : Compute $Z_1 = W_1 \times X + b_1$

$$Z_1 = (0.1 \times 20) + (0.2 \times 3) + (0.3 \times 4) + 0.1 = 3.9$$

$$(0.4 \times 20) + (0.5 \times 3) + (0.6 \times 4) + 0.2 = 12.1$$

$$(0.7 \times 20) + (0.8 \times 3) + (0.9 \times 4) + 0.3 = 20.3$$

$$\rightarrow Z_1 = \begin{bmatrix} 3.9 \\ 12.1 \\ 20.3 \end{bmatrix}$$

- Step 2 : $A_1 = \text{ReLU}(Z_1)$

$$\text{ReLU}(z) = \max(0, z)$$

All value $> 0 \rightarrow A_1 = Z_1$

$$\rightarrow A_1 = \begin{bmatrix} 3.9 \\ 12.1 \\ 20.3 \end{bmatrix}$$

- Step 3 : $Z_2 = W_2 \times A_1 + b_2$

$$Z_2 = (0.2 \times 3.9) + (0.4 \times 12.1) + (0.6 \times 20.3) + 0.5 = 18.3$$

$$\rightarrow Z_2 = 18.3$$

- Step 4 : $A_2 = \text{Sigmoid}(Z_2)$ where $\sigma(x) = 1 / (1 + e^{-x})$

$$\sigma(18.3) = 1 / (1 + e^{-18.3}) \rightarrow \text{it's almost } 0$$

$$\text{so } 1 / (1 + 0) \approx 1$$

$$\rightarrow A_2 \approx 1$$

- Step 5 : Calculate Loss = $(A_2 - Y)^2$ where $Y = 18$

$$(1 - 18)^2 = (-17)^2 = 289$$

Backward Propagation

1. $dL/dA_2 = 2(A_2 - y)$

$$= 2(1 - 18) = 2(-17) = -34$$

2. $dL/dz_2 = dL/dA_2 \times \text{Sigmoid}'(z_2)$ where $\text{Sigmoid}'(z) = \sigma(z)(1 - \sigma(z))$

$$\sigma(z_2) \approx 1 \rightarrow \text{Sigmoid}'(18 \cdot 3) \approx 1 \times (1 - 1) = 0$$

$$dL/dz_2 \approx -34 \times 0 = \approx 0$$

3. $dL/dw_2 = dL/dz_2 \times A_1^T$

$$\rightarrow dL/dz_2 \approx 0$$

$$dL/dw_2 \approx [0, 0, 0]$$

4. $dL/db_2 = dL/dz_2$

$$\rightarrow \approx 0$$

5. $dL/dA_1 = W_2^T \times dL/dz_2$

$$W_2^T = \begin{bmatrix} 0.2 \\ 0.4 \\ 0.6 \end{bmatrix} \text{ because } dL/dz_2 \approx 0$$

$$\rightarrow dL/dA_1 \approx \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

6. $dL/dz_1 = dL/dA_1 \times \text{ReLU}'(z_1)$ where $\text{ReLU}'(x) = 1 \quad if \ x > 0, \text{ else } 0$

$$\text{ReLU}'(z_1) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Since all $z_1 > 0$

$$dL/dA_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow$$

$$dL/dz_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

7. $dL/dw_1 = dL/dz_1 \times x^T$

$$\rightarrow dL/dz_1 \approx 0 \rightarrow dL/dw_1 \approx \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

8. $dL/db_1 = dL/dz_1$

$$\rightarrow dL/db_1 \approx \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$