HW_3

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Exercise 1

Write down Newton's formula for finding the minimum of

$$f(x) = (3x^4 - 4x^3)/12$$

in the range [-10, 10].Then, implement it in R.

Answer: Findning derivatives

$$f'(x) = \frac{12x^3 - 12x^2}{12} = x^2(x-1) = x^3 - x^2$$

$$f''(x) = 3x^2 - 2x$$

Then the Newton's formula to be:

$$x_{k+1} = x_k - \frac{x_k^3 - x_k^2}{3x_k^2 - 2x_k}$$

```
fun <- function(x) {
 x - (x^3 - x^2) / (3 * x^2 - 2 * x)
}
fun1 <- function(y) {
  values <- c()
  for(i in 1:10) {
    if(i == 1){
      values[[i]] <- fun(y)
    }
    else{
      values[[i]] <- fun(values[[i - 1]])
    }
  }
  print(values)
}
fun1(2)
```

```
## [[1]]
## [1] 1.5
##
## [[2]]
```

```
## [1] 1.2
##
## [[3]]
## [1] 1.05
##
## [[4]]
## [1] 1.004348
##
## [[5]]
## [1] 1.000037
##
## [[6]]
## [1] 1
##
## [[7]]
## [1] 1
##
## [[8]]
## [1] 1
##
## [[9]]
## [1] 1
##
## [[10]]
## [1] 1
```

The minimum is 1 when multiple starting values are used

Exercise 2

Explore optimize() in R and try to solve the previous problem.

```
fun2 <- function(x) {
  (3*x^4 - 4*x^3) / 12
}
x <- optimize(fun2, interval = c(-10,10), tol = 0.0001)
x
```

Answer:

```
## $minimum
## [1] 0.9999986
##
## $objective
## [1] -0.08333333
```

Exercise 3

Use any optimization algorithm to find the minimum of

$$f(x, y) = (x - 1)^2 + 100(y - x^2)^2$$

in the domain

$$-10 \leq x, y \leq 10$$

. Discuss any issues concerning the optimization process.

Answer:

$$x_{t+1} = x_t - H^{-1} \bigtriangledown f(x, y)$$

$$x_0 = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\bigtriangledown f(x, y) = \begin{bmatrix} (x-1)^2 \\ 100(y - x^2)^2 \end{bmatrix}$$

Next step to find the Hessian matrix by finding second derivative

$$f_{xx} = 1200x^2 + 2 - 400y$$

$$f_{xy} = -400x$$

$$f_{yy} = 200$$

$$H = \bigtriangledown^2 f(x, y) = \begin{bmatrix} 2 & -0 \\ -0 & 200 \end{bmatrix}$$

with

$$x_1 = x_0 - H^{-1} \bigtriangledown f(x, y)$$

$$x_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0.5 & 0 \\ 0 & 0.05 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix}$$

Moving forward the f(x,y) will move toward into a single point as it goes through more values of t. So the most confusing was the multivariate function to move forward.

Exercise 4 #### Answer: Explore the optimr package for R and try to solve the previous problem.

```
library(optimr)
```

```
fun4 <- function(x, y){
  (x - 1)^2 + 100 * (y - x^2)^2
}
optimr(c(-10,10), fun4, method = "L-BFGS-B")
```

```
## Error in fn(par, ...) : argument "y" is missing, with no default
```

```
## $convergence
## [1] 9999
##
## $par
## [1] NA NA
##
## $counts
## [1] NA NA
##
## $message
## [1] "optim method failure\n"
```