POWERFACTORY

PowerFactory 2021

Technical Reference

Overhead Line Constants

TypGeo, TypTow, TypCon

PF2021

POWER SYSTEM SOLUTIONS

MADE IN GERMANY

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1 Introduction

This document describes the calculation of the electrical parameters of an overhead line system from its configuration characteristics such as tower geometry, conductor types, number, phasing and grounding condition of its circuits, etc. The calculation function is available for lines having a tower type (*TypTow*) or a tower geometry type (*TypGeo*).

The line parameters calculation function, or ‘line constants’, supports overhead line systems with any number of parallel circuits of the same or different nominal voltage, 3-ph, 2-ph and single-phase, with or without earth wires and neutral conductors and different types of transpositions. The calculation accounts for the skin effect in the conductors and for the frequency dependency of the earth return path.

The calculation function can be used in a stand-alone mode, in which case *PowerFactory* prints the calculation results (impedance and admittance matrices) to the output window, or it can be automatically called by the line (*ElmLne*) or line coupling (*ElmTow*) elements when associated with a tower type (*TypTow*) or a tower geometry type (*TypGeo*). In the latter case, the parameter calculation function will automatically return the resulting impedance and admittance matrices of the overhead line system to the simulation model.

Finally, the tower type (*TypTow*) also supports the definition of the tower in terms of its electrical parameters, so that the user has the option to enter the impedance and admittance matrices either in natural or in sequence components. This is useful when the user has to define an unbalanced system (eg. untransposed line) with multiple circuits not supported by the line type (*TypLne*).

2 Definition in Terms of Geometrical Data

The overhead line system is defined in terms of geometrical data, i.e. the physical dimensions of the towers (geometrical data) and the conductor data. The model consists in the conductor types (*TypCon*) and the tower geometries (*TypTow* or *TypGeo*).

The input parameters of the conductor type are listed in Table A.1 and discussed in 3.

The geometry of the tower is entered in either a tower type (*TypTow*) or a tower geometry type (*TypGeo*). In the tower type (*TypTow*) the user associates corresponding conductor types of each circuit to the geometry of the tower. The tower type (*TypTower*) therefore contains all required data of the overhead line system for the calculation of the electrical parameters. The tower geometry type (*TypGeo*) however, does not contain a reference to the conductor type, so that the definition is not complete. The conductor types are added later in the line (*ElmLne*) or coupling (*ElmTow*) elements. For that reason the tower geometry type (*TypGeo*) is more flexible, and should be therefore the preferred option when combining the same tower geometry with different conductor types. This is often the case in distribution systems.

The input parameters of the tower type (*TypTow*) are shown in Table A.3 and those of the tower geometry type (*TypGeo*) in Table A.2. The calculation of the electrical parameters is discussed in 4.

3 Conductor Type

The geometrical and electrical characteristics of the conductor are defined in a conductor type (*TypCon*). The user has the option between solid and tubular conductors, and between single conductors and a bundle of sub-conductors (number of sub-conductors >1) in which case the bundle spacing must be specified.

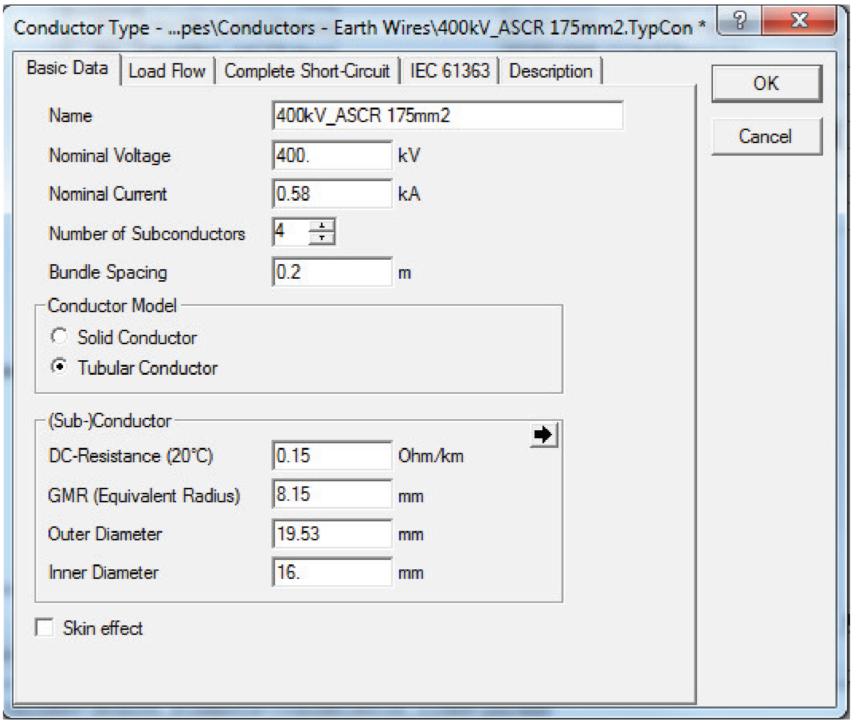

Figure 3.1: Conductor type dialog

The parameters of the sub-conductors are used to calculate the internal impedance. These parameters are the DC resistance, the diameter (or radius) and the internal inductance. The internal impedance can be also defined in terms of relative permeability or geometrical mean radius GMR as explained in 3.1.

If the *Skin effect* option is selected, the calculation of the internal impedance will account for the skin effect (using Bessel functions) as described in section 4.

3.1 Geometrical Mean Radius (GMR) of a Conductor

The Geometrical Mean Radius (GMR) is usually provided in manufacturers' data sheets. This is the data the user is encouraged to provide as input for the conductor type definition.

If not available, *PowerFactory* can calculate the GMR at power frequency using the conductor diameter (or radius) or the relative permeability (assuming a uniform current distribution over the conductor cross-section). The influence of elementary wires is not taken into account [1, 3, 4].

3.1.1 GMR of solid conductors

Consider a solid conductor of non-magnetic material and radius r as depicted in Figure 3.2.

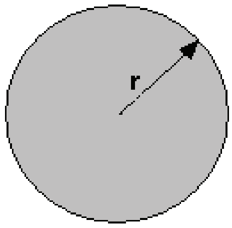

Figure 3.2: Solid conductor

From magnetic field theory, the self-inductance of the conductor can be calculated as in (1),

$$L_{self} = \frac{\mu_0}{8\pi} + \frac{\mu_0}{2\pi} \cdot \ln\left(\frac{1}{2 \cdot r}\right) \tag{1}$$

where the first term of the sum is the internal inductance associated with the magnetic flux inside the conductor (2) and the second term represents the external inductance associated with the external flux (3).

$$L_{internal} = \frac{\mu_0}{8\pi} \tag{2}$$

$$L_{external} = \frac{\mu_0}{2\pi} \cdot \ln\left(\frac{1}{2 \cdot r}\right) \tag{3}$$

In terms of the GMR of the conductor, (1) can be rewritten as (4)

$$L_{self} = \frac{\mu_0}{2\pi} \cdot \ln\left(\frac{1}{2 \cdot GMR}\right) \tag{4}$$

Therefore, between (1) and (4) the following expression can be deduced for the GMR of a solid conductor under the above-mentioned assumptions:

$$GMR = r \cdot e^{-\frac{1}{4}} \tag{5}$$

If a *Relative Permeability* has been defined in the conductor type, the GMR is calculated as:

$$GMR = r \cdot e^{-\frac{\mu_r}{4}} \tag{6}$$

3.1.2 GMR of tubular conductors

A similar procedure can be used to calculate the GMR of a tubular conductor. A uniform distribution of the current over the conductor cross-section is assumed as in the previous subsection.

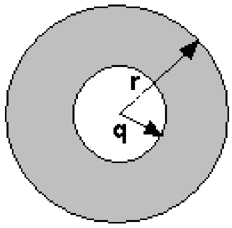

Figure 3.3: Tubular conductor

For the tubular conductor depicted in Figure 3.3, the self-inductance is calculated as in (7):

$$L_{self} = \frac{\mu_0}{2\pi} \cdot \left[\frac{q^4}{\left(r^2 - q^2\right)^2} \cdot \ln\frac{r}{q} - \frac{3q^2 - r^2}{4 \cdot (r^2 - q^2)} \right] + \frac{\mu_0}{2\pi} \cdot \ln\left(\frac{1}{2 \cdot r}\right) \tag{7}$$

Again, the first term represents the internal inductance, and thus for the tubular conductor:

$$L_{int} = \frac{\mu_0}{2\pi} \cdot \left[\frac{q^4}{\left(r^2 - q^2\right)^2} \cdot \ln\frac{r}{q} - \frac{3q^2 - r^2}{4 \cdot (r^2 - q^2)} \right] \tag{8}$$

Between (4) and (7), equation (9) results, and the GMR of a tubular conductor is expressed as:

$$GMR = r \cdot \exp\left[\frac{3q^2 - r^2}{4\left(r^2 - q^2\right)} - \frac{q^4}{\left(r^2 - q^2\right)^2} \cdot \ln\frac{r}{q} \right] \tag{9}$$

It should be re-emphasized that (5) and (9) assume a uniform distribution of the current over the cross-section of the conductor and therefore the elementary wires are not taken into account.

3.1.3 Numerical example

For an Al/St Conductor, 120/20 mm^2, Radius 7.75 mm and q/r= 0.226, the following values can be verified in *PowerFactory*:

- From Eq. (8), $L_{internal}$=0,045479 mH/km
- From Eq. (7), L_{self}= 0,878862 mH/km
- From Eq. (9), GMR = 6,17369 mm

3.2 Bundle Conductor

Bundle conductors are frequently used in high voltage transmission lines to reduce corona losses and fulfil electromagnetic radiation requirements. Two or more sub-conductors per phase are held together by spacers forming a symmetrical bundle conductor.

For the calculation of electrical parameters, *PowerFactory* replaces the bundle sub-conductors with a conductor of equivalent radius. Subsequent calculations of line parameters (internal impedance and geometrical impedance due to external flux) are then carried out considering this equivalent single conductor as if it were located in the middle of the bundle.

This approach is based on the following two assumptions:

- The bundle is symmetrical
- The current distribution among the sub-conductors within a bundle is uniform

Under these assumptions, the radius of the equivalent conductor will be:

$$r_B = \sqrt[n]{n \cdot r \cdot R^{n-1}} \tag{10}$$

where *r* is the radius of an individual sub-conductor, *n* is the number of sub-conductors and *R* the radius of the bundle (calculated from the bundle spacing *a* as depicted in Figure 3.4).

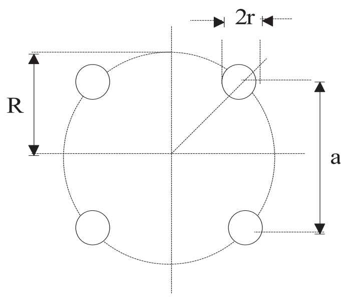

Figure 3.4: Symmetrical bundle of radius R with n sub-conductors. Bundle spacing a.

The relationship between the bundle spacing *a* and angle α is given by:

$$a = 2R \cdot \sin\left(\frac{\pi}{n}\right)$$

The equivalent GMR results:

$$GMR_B = \sqrt[n]{n \cdot GMR \cdot R^{n-1}} \tag{11}$$

where *GMR* is the geometrical mean radius of individual sub-conductor in bundle.

A different method for calculating line parameters of bundle conductors involves computing the parameters for each sub-conductor as if it were represented as an individual conductor. Since all sub-conductors within a bundle have the same voltage, the order of the geometrical matrices is then reduced to matrices of equivalent phase conductors. Even though a non-symmetrical bundle conductor can also be considered in this case, a uniform current distribution among sub-conductors within the bundle is still to be assumed. Differences between both methods seem to be very minor and therefore not relevant. *PowerFactory* supports the first approach described above.

For calculating the internal impedance of the bundle, the internal impedance of one sub-conductor must be divided by the number of sub-conductors *n*.

4 Calculation of Overhead Line Constants

4.1 Series Impedance

The line impedance consists of three components:

1. The internal impedance $Z'_{Int} = R'_{Int}(\omega) + j\omega \cdot L'_{Int}(\omega)$, which accounts for the voltage drop due to conductor resistance and the magnetic field inside the conductor itself. Because of the skin effect, both the internal reactance and resistance are frequency dependent.
2. Geometrical impedance Z'_G, being the impedance of an ideal conductor without any magnetic field inside and an ideally-conducting ground. The geometrical inductance is not frequency dependent.
3. Earth correction term $Z'_E = R'_t(\omega) + j\omega \cdot L'_t(\omega)$, a frequency-dependent term considering finite earth conductivity and proximity effects. It depends on the earth resistivity and the line geometry (different coefficients for self- and mutual-impedances).

4.1.1 Internal impedance

Without considering skin effect

If skin effect is not considered (*Skin effect* flag in the conductor type), the following formulae are used for the calculation of the internal impedance:

$$Z'_{Int} = R'_{Int} + j\omega L'_{Int} \tag{12}$$

$$R'_{Int} = R_{DC} \tag{13}$$

$$L'_{Int} = \begin{cases} \frac{\mu_0}{8\pi} & \text{solid conductor} \\ \frac{\mu_0}{2\pi} \cdot \left[\frac{q^4}{(r^2-q^2)^2} \cdot \ln\frac{r}{q} - \frac{3q^2-r^2}{4\cdot(r^2-q^2)} \right] & \text{tubular conductor} \end{cases} \tag{14}$$

where R_{DC} is the DC resistance of the conductor in Ω/km and *r* and *q* the outside and inside radius of the tubular conductor, respectively.

Considering skin effect

If the skin effect is considered, the internal impedance of the conductor becomes a function of the frequency and it is calculated using the complex Bessel functions:

$$Z_{Int} = \frac{1}{2} \cdot R_{DC} \cdot \left(\varsigma \cdot \sqrt{-j}\right) \cdot \frac{J_0\left(\varsigma \cdot \sqrt{-j}\right)}{J_1\left(\varsigma \cdot \sqrt{-j}\right)} \tag{15}$$

where

$$\varsigma = r \cdot \sqrt{\frac{\omega \cdot \mu_r \cdot \mu_0}{\rho}} = r \cdot m \tag{16}$$

$$J_n\left(x\right) = \sum_{k=0}^{\infty} \frac{\left(-1\right)^k}{k! \cdot \left(n+k\right)!} \cdot \left(\frac{x}{2}\right)^{n+2k} \tag{17}$$

The parameter

$$m = \sqrt{\frac{\omega \cdot \mu_r \cdot \mu_0}{\rho}}$$

in (16) is the reciprocal (absolute value) of the complex depth of penetration *p*. Eq. (16) can also be expressed in terms of the DC resistance of the conductor as follows:

$$\varsigma = \sqrt{\frac{\omega \cdot \mu_r \cdot \mu_0}{R_{DC} \cdot \pi}} \tag{18}$$

The relative permeability μ_r accounts for the conductor geometry ($\mu_r = 1$ for solid conductor and $\mu_r \leq 1$ for tubular conductors), with $\mu_r \neq 1$ if the conductor material is magnetic.

Temperature coefficient

If the *Temperature Dependency of lines/cables* option is enabled in the Load Flow command, the resistivity of the conducting layers is adjusted by the following equation:

$$\rho_T = \rho_{20^\circ C} \cdot \left[1 + \alpha\left(T - 20\right)\right]$$

where α is the temperature coefficient of resistance. The resistivities and temperature coefficient of common metals are given in Table 4.1 for reference.

Table 4.1: Resistivities and temperature coefficient of resistance

Material	**Resistivity at 20 °C [$\mu\Omega$.cm]**	**Temperature coefficient at 20°C [1/°C]**
Aluminum	2.83	0.0039
Copper, hard drawn	1.77	0.00382
Copper, annealed	1.72	0.00393
Brass	6.4-8.4	0.0020
Iron	10	0.0050
Silver	1.59	0.0038
Steel	12-88	0.001-0.005

4.1.2 Geometrical impedance

$$Z'_{Gii} = j\omega\frac{\mu_0}{2\pi}N_{ii} \tag{19}$$

$$Z'_{Gik} = j\omega\frac{\mu_0}{2\pi}N_{ik} \tag{20}$$

By default (i.e. when using Carson's formula), the frequency independent coefficients *N* are calculated as follows:

$$N_{ii} = \ln \frac{2h_i}{r_i} \tag{21}$$

$$N_{ik} = \ln \frac{d'_{ik}}{d_{ik}} \tag{22}$$

where h_i, d_{ij} and r_i are as defined in Figure 4.1.

If using the *Deri-Semlyen* option, these coefficients are instead [3]:

$$N_{ii} = \ln \frac{2(h_i + \bar{p})}{r_i} \tag{23}$$

$$N_{ik} = \ln \sqrt{\frac{(h_i + h_k + 2\bar{p})^2 + x_{ik}^2}{d_{ik}}} \tag{24}$$

where h_i, d_{ij} and r_i are as defined in Figure 4.1, and $\bar{p}$ represents a complex depth:

$$\bar{p} = \sqrt{\frac{\rho}{j\omega\mu_0}} \tag{25}$$

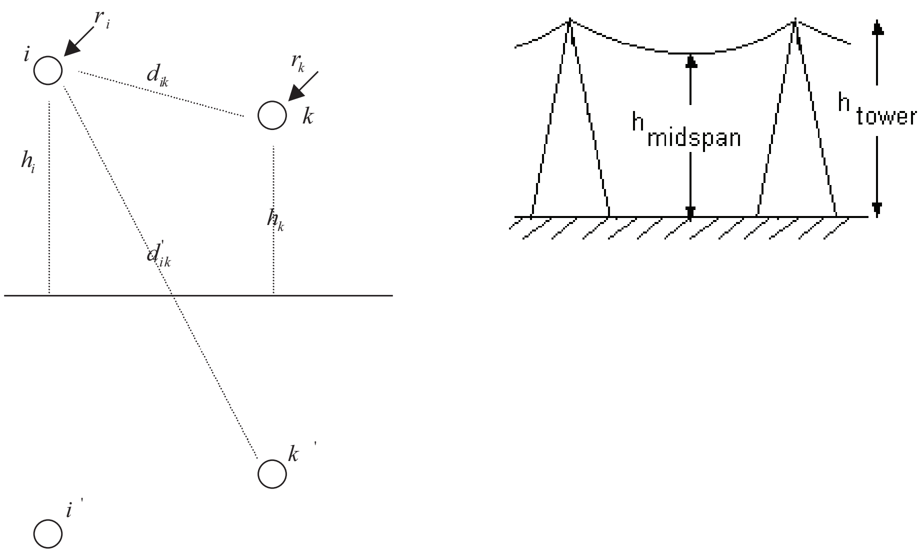

Figure 4.1: Calculation of the geometrical coefficients. Conductor profiles between towers.

For bundle conductors, the radius r_i in (16) is to be replaced by the equivalent radius as calculated in (10).

h_i is the average height above ground of conductor *i*. If the conductor profile can be described as a parabola (which is quite accurate for spans below 500 meters), then the average height above ground is:

$$h_i = h_{average} = \frac{2}{3} \cdot h_{midspan} + \frac{1}{3} \cdot h_{tower}$$

where $h_{midspan}$ is the conductor height at midspan and h_{tower} at tower as shown in (7).

4.1.3 Earth return impedance

Carson's Formula The impedance of the earth return path is calculated by default in *PowerFactory* according to the Carson's series which is given by:

$$Z_E' = \omega \frac{\mu_0}{\pi} \left(P + jQ\right) \tag{26}$$

The coefficients *P* and *Q* are highly frequency dependent and are calculated as follows:

$$\begin{aligned} P =& \frac{\pi}{8} - \\ &- b_1 x \cos\vartheta + b_2 \left[\left(c_2 - \ln x\right) x^2 \cos 2\vartheta + x^2 \vartheta \sin 2\vartheta\right] + b_3 x^3 \cos 3\vartheta - d_4 x^4 \cos 4\vartheta \\ &- b_5 x^5 \cos 5\vartheta + b_6 \left[\left(c_6 - \ln x\right) x^6 \cos 6\vartheta + x^6 \vartheta \sin 6\vartheta\right] + b_7 x^7 \cos 7\vartheta - d_8 x^8 \cos 8\vartheta \dots \end{aligned} \tag{27}$$

$$\begin{aligned} Q =& \frac{1}{2} \left(k - \ln x\right) x^6 + \\ &+ b_1 x \cos\vartheta - d_2 x^2 \cos 2\vartheta + b_3 x^3 \cos 3\vartheta - b_4 \left[\left(c_4 - \ln x\right) x^4 \cos 4\vartheta + x^4 \vartheta \sin 4\vartheta\right] \\ &+ b_5 x^5 \cos 5\vartheta - d_6 x^6 \cos 6\vartheta + b_7 x^7 \cos 7\vartheta - b_8 \left[\left(c_8 - \ln x\right) x^8 \cos 8\vartheta + x^8 \vartheta \sin 8\vartheta\right] \dots \end{aligned} \tag{28}$$

where:

$x = 2h_i\sqrt{\mu \cdot \kappa \cdot \omega}$ and $\vartheta = 0$ for the self-impedance

$x = d_{ij}\sqrt{\mu \cdot \kappa \cdot \omega}$ for the mutual impedance

$$b_i = \left|b_{i-2}\right| \frac{sign}{i \cdot (i+2)}$$

sign = +1 for *I = 1,2,3,4. . . .*

sign = +1 for *I = 5,6,7,8 . . .* and alternating after 4 terms

$$c_i = c_{i-2} + \frac{1}{i} + \frac{1}{i+2}$$

$$d_i = \frac{\pi}{4} \cdot b_i$$

$$k = \frac{1}{2} + \ln 2 - C = 0{,}61593$$

$$C = \lim_{n \to \infty} \left(\sum_{\nu=1}^{n} \frac{1}{\nu} - \ln n \right) = 0{,}577215$$

Following initial values are being used:

$$b_1 = \frac{\sqrt{2}}{6} b_2 = \frac{1}{16}$$

$$c_2 = \frac{5}{4} - C + \ln 2 = 1{,}3659315$$

Deri-Semlyen Approximation The impedance of the earth return path for overhead lines can be optionally calculated using the approximation proposed by Deri and Semlyen [2, 3] by selecting the option *Deri-Semlyen* in the associated line or tower dialog. This approximation is considered to be highly accurate [3] and uses relatively simple formulae for the series and mutual impedances, as shown below:

$$Z'_{ii} = R'_{i-internal} + j \cdot \left\{ \omega \frac{\mu_0}{2\pi} \ln \frac{2(h_i + \bar{p})}{r_i} + X'_{i-internal} \right\} \tag{29}$$

$$Z'_{ik} = j \cdot \omega \frac{\mu_0}{2\pi} \ln \sqrt{\frac{(h_i + h_k + 2\bar{p})^2 + x_{ik}^2}{d_{ik}}} \tag{30}$$

where h_i, h_k and r_i are as defined in Figure 4.1, and $R'_{i-internal}$ and $X'_{i-internal}$ are the per-unit length AC internal resistance and internal reactance of conductor i, respectively.

4.1.4 Series impedance matrix

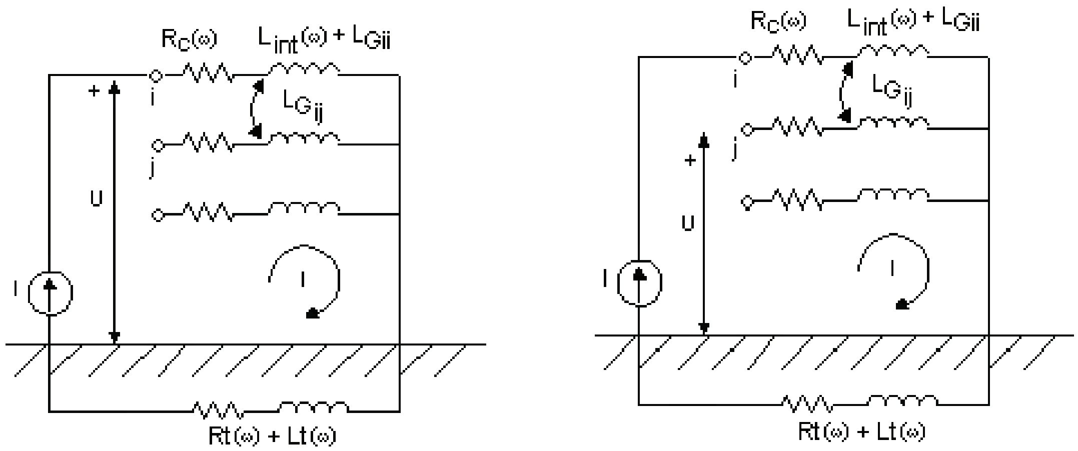

Figure 4.2: Definition of the self (left) and mutual (right) impedance of a line.

According to Figure 4.2 the self and mutual impedances can hence be expressed as follows:

$$Z'_{ii} = Z'_{Gii} + Z'_{Eii} + Z'_{Lii} \tag{31}$$

$$Z'_{ij} = Z'_{Gij} + Z'_{Eij} \tag{32}$$

It should be noted that even though Z'_{ij} defines a mutual impedance, it still has a real component due to the resistance of the earth return path.

For an overhead line with multiple conductors, the voltage drop along the line can be written in terms of an impedance matrix, which relates the voltage drop along every conductor to the currents across them:

$$\boldsymbol{\Delta U} = \boldsymbol{Z} \times \boldsymbol{I}$$

The dimension of the matrix depends on the total number of conductors in the line and therefore corresponds to the number of phase conductors plus the number of earth wires. Such a matrix is called the “natural” impedance matrix of the line.

$$\begin{bmatrix} \boldsymbol{\Delta U_E} \\ \boldsymbol{\Delta U_P} \end{bmatrix} = \begin{bmatrix} \boldsymbol{Z_{EE}} & \boldsymbol{Z_{EP}} \\ \boldsymbol{Z_{PE}} & \boldsymbol{Z_{PP}} \end{bmatrix} \times \begin{bmatrix} \boldsymbol{I_E} \\ \boldsymbol{I_P} \end{bmatrix}$$

When using the impedance matrix in a three-phase model, it must be reduced to the number of phase conductors. Therefore, an ideal grounding is considered leading to the assumption that there is no voltage drop across the earth conductors. The reduced matrix is hence:

$$\boldsymbol{\Delta U_P} = \left[\boldsymbol{Z_{PP}} - \boldsymbol{Z_{PE}} \boldsymbol{Z_{EE}^{-1}} \boldsymbol{Z_{EP}}\right] \boldsymbol{I_P} = \boldsymbol{Z_{red}} \boldsymbol{I_P}$$

If a detailed modelling of earth wires is required, earth wires must be entered as phase conductors.

The symmetrical impedance matrix is calculated as follows:

$$\boldsymbol{Z_{012}} = \boldsymbol{S} \cdot \boldsymbol{Z_{red}} \cdot \boldsymbol{T}$$

where $\boldsymbol{T} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$, $\boldsymbol{S} = \boldsymbol{T}^{-1}$ and $a = e^{j120^\circ}$

For perfectly transposed lines, the resulting symmetrical matrix is diagonal, with all off-diagonal elements equal to zero, and hence there is no coupling between sequence modes. In this case, the resulting matrix looks like:

$$\boldsymbol{Z_{012}} = \begin{bmatrix} Z_s + 2Z_m & 0 & 0 \\ 0 & Z_s - Z_m & 0 \\ 0 & 0 & Z_s - Z_m \end{bmatrix}$$

where Z_s is the self impedance and Z_m the mutual impedance of the perfectly transposed line.

4.2 Shunt Capacitance

The natural potential coefficient matrix **P** relates the voltage to the charge of each conductor. The dimension of this “natural” matrix corresponds to the number of phase conductors + the number of earth wires.

$$\boldsymbol{U} = \boldsymbol{P} \times \boldsymbol{Q}$$

Analogous to the impedance matrix, the matrix of potential coefficients is also reduced to the number of phase conductors, under the assumption that the voltage at the earth wires is zero. It follows:

$$\begin{bmatrix} \boldsymbol{U_E} \\ \boldsymbol{U_P} \end{bmatrix} = \begin{bmatrix} \boldsymbol{P_{EE}} & \boldsymbol{P_{EP}} \\ \boldsymbol{P_{PE}} & \boldsymbol{P_{PP}} \end{bmatrix} \times \begin{bmatrix} \boldsymbol{Q_E} \\ \boldsymbol{Q_P} \end{bmatrix}$$

$$\boldsymbol{U_E} = 0$$

$$\boldsymbol{U_P} = \left[\boldsymbol{P_{PP}} - \boldsymbol{P_{PE}} \boldsymbol{P_{EE}^{-1}} \boldsymbol{P_{EP}}\right] \cdot \boldsymbol{Q_P}$$

and hence the reduced coefficient matrix is

$$\boldsymbol{P_{red}} = \left[\boldsymbol{P_{PP}} - \boldsymbol{P_{PE}} \boldsymbol{P_{EE}^{-1}} \boldsymbol{P_{EP}}\right]$$

The matrix of capacitance coefficients can be obtained now by inverting the potential coefficient matrix:

$$\boldsymbol{C_{red}} = \boldsymbol{P_{red}^{-1}}$$

Using the same transformation matrix $\boldsymbol{T}$ and $\boldsymbol{S}$ as for the impedance, we can now calculate the symmetrical admittance matrix as follows:

$$\boldsymbol{C_{012}} = \boldsymbol{S} \cdot \boldsymbol{C_{RST}} \cdot \boldsymbol{T}$$

As in the impedance case, the resulting symmetrical matrix $\boldsymbol{Z_{012}}$ is diagonal only for perfectly transposed lines, with all off-diagonal elements equal to zero. Hence there is no coupling between sequence modes. In this case, the resulting admittance matrix looks like:

$$C_{012} = \begin{bmatrix} C_s - 2 \cdot C_m & 0 & 0 \\ 0 & C_s + C_m & 0 \\ 0 & 0 & C_s + C_m \end{bmatrix}$$

where C_s is the self capacitance and C_m the coupling capacitance of the perfectly transposed line.

4.3 Transposition

PowerFactory supports various transposition options for line and line coupling elements (*ElmLne* and *ElmTow*, respectively). The transposition can be selected either by a *Transpose* flag in the tower type (*TypTow*) and a selection of the type of *Transposition*; or by a *Transpose* flag in the line coupling element (*ElmTow*) (if a tower geometry type (*TypGeo*) instead of a tower type (*TypTow*) has been chosen) and a selection of the type of *Transposition*. The options available are described below.

4.3.1 None

The line is not transposed and each wire in the line maintains its position for the entire length of the line. There will be mutual coupling between the phases of each circuit and between the phases of one circuit and the phases of the other circuits.

4.3.2 Circuit-wise

Each 3-phase circuit can be separately transposed, depending on the user selection in the *Transposition* column of the table in the tower type. The phases of each individual circuit are therefore transposed. Circuit-wise transposition eliminates the positive and zero sequence off-diagonal elements of the mutual sub-matrices between transposed circuits. These mutual sub-matrices may differ from one another. The equivalent sequence impedance matrix is as follows (where X indicates a non-zero value):

$$Z_{012} = \begin{bmatrix} X & 0 & 0 & X & 0 & 0 \\ 0 & X & 0 & 0 & X & 0 \\ 0 & 0 & X & 0 & 0 & X \\ X & 0 & 0 & X & 0 & 0 \\ 0 & X & 0 & 0 & X & 0 \\ 0 & 0 & X & 0 & 0 & X \end{bmatrix}$$

4.3.3 Symmetrical

Each 3-phase circuit is transposed, followed by the transposition of the circuits at the same tower. Zero and positive sequence couplings between the two circuits exist but are equal. The equivalent sequence impedance matrix is as follows (where X indicates a non-zero value, and like colours indicate identical values):

$$Z_{012} = \begin{bmatrix} X & 0 & 0 & X & 0 & 0 \\ 0 & X & 0 & 0 & X & 0 \\ 0 & 0 & X & 0 & 0 & X \\ X & 0 & 0 & X & 0 & 0 \\ 0 & X & 0 & 0 & X & 0 \\ 0 & 0 & X & 0 & 0 & X \end{bmatrix}$$

4.3.4 Perfect

All phases of each circuit occupy all conductor positions, yielding perfectly symmetrical, balanced impedance and admittance matrices. All positive sequence couplings are eliminated. In practice, the perfectly-transposed impedance (or admittance) matrix can be calculated as the average of the main- and the off-diagonal element of the reduced impedance (or admittance) matrix. The equivalent sequence impedance matrix is as follows (where X indicates a non-zero value):

$$Z_{012} = \begin{bmatrix} X & 0 & 0 & 0 & 0 & 0 \\ 0 & X & 0 & 0 & 0 & 0 \\ 0 & 0 & X & 0 & 0 & 0 \\ 0 & 0 & 0 & X & 0 & 0 \\ 0 & 0 & 0 & 0 & X & 0 \\ 0 & 0 & 0 & 0 & 0 & X \end{bmatrix}$$

4.4 Output Results

The line constants calculation function in stand-alone mode can be started from the *Calculate* button on the edit dialog of the tower type *TypTow*. Then *PowerFactory* prints the resulting impedance and admittance matrices to the output windows.

The natural impedance matrix corresponds to the system of physical conductors including the earth wires. The size of the natural impedance matrix results:

$$Size\,(Zn) = N_{EW} + \sum_{j=1}^{Nc} (N_{ph})_j$$

where N_{EW} is the total number of earth wires, N_c the line circuits and N_{ph} the number of phases of the corresponding line circuit. Rows and columns of the natural impedance matrix proceed in the sequence ground wire 1, 2, . . . N_{EW} followed by phases A, B C for line circuits 1, 2 . . . .N_c.

The reduced impedance matrix represents the system of equivalent phase conductors after reduction of the earth wires. Rows and columns proceed in the same sequence as before but without the earth wires. The symmetrical components matrix proceed in the sequence 0, 1, 2 for line circuits 1, 2, . . . N_c. The size of the reduced impedance matrix Z_{ABC} is equal to the size of the symmetrical components matrix Z_{012}:

$$Size\left(Z_{ABC}\right) = Size\left(Z_{012}\right) = \sum_{j=1}^{Nc} \left(N_{ph}\right)_j$$

It follows for reference an extract of the output window for a 132 kV, 2 x 3-phase circuits, 1 x earth wire, circuit-wise transposed. First matrix corresponds to the natural impedance matrix (7 x 7), the second and third one to the reduced impedance matrix in natural components and symmetrical components respectively.

DIgSI/info - **Natural Impedance Matrix** (R+jX) [ohm/km]
DIgSI/info - Earth conductors first, followed by phase conductors in same order as the input.
DIgSI/info - Rows follow R,X, R,X... in [ohm/km]

Phase conductors, in same order as the input

1.93180e-001	4.48600e-002	4.52553e-002	4.55627e-002	4.48600e-002	4.52553e-002	4.55627e-002
6.87017e-001	2.64516e-001	2.32909e-001	2.19884e-001	2.64516e-001	2.32909e-001	2.19884e-001
4.48600e-002	1.13864e-001	4.58758e-002	4.61923e-002	4.54536e-002	4.58589e-002	4.61788e-002
2.64516e-001	6.63646e-001	2.91102e-001	2.60068e-001	2.70790e-001	2.47761e-001	2.40699e-001
4.52553e-002	4.58758e-002	1.14699e-001	4.66221e-002	4.58589e-002	4.62714e-002	4.66000e-002
2.32909e-001	2.91102e-001	6.62649e-001	3.08999e-001	2.47761e-001	2.42940e-001	2.46523e-001
4.55627e-002	4.61923e-002	4.66221e-002	1.15352e-001	4.61788e-002	4.66000e-002	4.69345e-002
2.19884e-001	2.60068e-001	3.08999e-001	6.61891e-001	2.40699e-001	2.46523e-001	2.59351e-001
4.48600e-002	4.54536e-002	4.58589e-002	4.61788e-002	1.13864e-001	4.58758e-002	4.61923e-002
2.64516e-001	2.70790e-001	2.47761e-001	2.40699e-001	6.63646e-001	2.91102e-001	2.60068e-001
4.52553e-002	4.58589e-002	4.62714e-002	4.66000e-002	4.58758e-002	1.14699e-001	4.66221e-002
2.32909e-001	2.47761e-001	2.42940e-001	2.46523e-001	2.91102e-001	6.62649e-001	3.08999e-001
4.55627e-002	4.61788e-002	4.66000e-002	4.69345e-002	4.61923e-002	4.66221e-002	1.15352e-001
2.19884e-001	2.40699e-001	2.46523e-001	2.59351e-001	2.60068e-001	3.08999e-001	6.61891e-001

Earth wire

Circuit 1

Circuit 2

```
DIgSI/info - Reduced Impedance Matrix (R+jX) [ohm/km]
DIgSI/info - Circuits (phases A,B,C...) follow in same order as the input.
DIgSI/info - Rows follow R,X, R,X... in [ohm/km]

  1.06521e-001   3.78915e-002   3.78915e-002   3.81026e-002   3.78741e-002   3.78741e-002
  5.79697e-001   2.04396e-001   2.04396e-001   1.74662e-001   1.62667e-001   1.62667e-001

  3.78915e-002   1.06521e-001   3.78915e-002   3.78741e-002   3.81026e-002   3.78741e-002
  2.04396e-001   5.79697e-001   2.04396e-001   1.62667e-001   1.74662e-001   1.62667e-001

  3.78915e-002   3.78915e-002   1.06521e-001   3.78741e-002   3.78741e-002   3.81026e-002
  2.04396e-001   2.04396e-001   5.79697e-001   1.62667e-001   1.62667e-001   1.74662e-001

  3.81026e-002   3.78741e-002   3.78741e-002   1.06521e-001   3.78915e-002   3.78915e-002
  1.74662e-001   1.62667e-001   1.62667e-001   5.79697e-001   2.04396e-001   2.04396e-001

  3.78741e-002   3.81026e-002   3.78741e-002   3.78915e-002   1.06521e-001   3.78915e-002
  1.62667e-001   1.74662e-001   1.62667e-001   2.04396e-001   5.79697e-001   2.04396e-001

  3.78741e-002   3.78741e-002   3.81026e-002   3.78915e-002   3.78915e-002   1.06521e-001
  1.62667e-001   1.62667e-001   1.74662e-001   2.04396e-001   2.04396e-001   5.79697e-001
```

```
DIgSI/info - Symmetrical Impedance Matrix (R+jX) [ohm/km]
DIgSI/info - Circuits (seq. 0,1,2...) follow in same order as the input.
DIgSI/info - Rows follow R,X, R,X... in [ohm/km]

  1.82304e-001   1.38778e-017   2.77556e-017   1.13851e-001   1.86483e-017   0.00000e+000
  9.88490e-001   0.00000e+000   0.00000e+000   4.99996e-001   4.33681e-018   9.54098e-018

  5.55112e-017   6.86296e-002   0.00000e+000   0.00000e+000   2.28545e-004  -1.82146e-017
  0.00000e+000   3.75301e-001   5.55112e-017   0.00000e+000   1.19951e-002   5.63785e-018

  5.55112e-017   0.00000e+000   6.86296e-002   0.00000e+000   1.60462e-017   2.28545e-004
  0.00000e+000   2.77556e-017   3.75301e-001   0.00000e+000   4.33681e-019   1.19951e-002

  1.13851e-001   1.86483e-017   0.00000e+000   1.82304e-001   1.38778e-017   2.77556e-017
  4.99996e-001   4.33681e-018   9.54098e-018   9.88490e-001   0.00000e+000   0.00000e+000

  0.00000e+000   2.28545e-004  -1.82146e-017   5.55112e-017   6.86296e-002   0.00000e+000
  0.00000e+000   1.19951e-002   5.63785e-018   0.00000e+000   3.75301e-001   5.55112e-017

  0.00000e+000   1.60462e-017   2.28545e-004   5.55112e-017   0.00000e+000   6.86296e-002
  0.00000e+000   4.33681e-019   1.19951e-002   0.00000e+000   2.77556e-017   3.75301e-001
```

The output admittance matrices follow the same structure.

5 Definition in Terms of Electrical Data

The overhead line system can be alternatively defined in terms of electrical data. In that case, only the tower type (*TypTow*) can be used.

In the tower type set the input parameter *i_mode* to 1 (electrical parameters, see Table A.3). On the load flow page you are able now to enter the impedance and admittance matrices in ohm/km. Note that you can choose between phase/symmetrical components, reactance/inductance and susceptance/capacitance by clicking . Use the  arrows swap between the impedance and admittance matrices.

On the basic data page, note that you still need to specify the number of circuits and the number of phases per circuit as they define the size of the Z/Y matrix. Besides you will be prompted to select a conductor type for each circuit as it defines the nominal voltage of the circuit (see parameter *uline* in *TypCon*, Table A.1).

It should also be noted that in case of electrical parameters the user just enters the reduced matrices, i.e. after elimination of the earth wires. In case that only the non-reduced natural matrices were available, the user shall define single-phase circuits for the earth wires and then ground these circuits externally in the network model.

A Parameter Definitions

Table A.1: Input parameters of the conductor type (*TypCon*)

Name	Description	Unit	Range	Default	Symbol
loc_name	Name				
uline	Nominal Voltage	kV	x>0	6	
sline	Nominal Current	kA	x>0	1	
ncsub	Number of Subconductors		x>0 and x<100	1	
dsubc	Bundle Spacing	m		0.1	
iModel	Conductor model: iModel = 0 : solid conductor iModel = 1 : tubular conductor; in this case inner diameter is compulsitory		x=0 or x=1	0	
rpha	(Sub-)Conductor: DC-Resistance (20°C)	Ohm/km	x>0	0.05	
erpha	(Sub-)Conductor: GMR (Equivalent Radius)	mm	x>0	11.682	
Lint	(Sub-)Conductor: Internal Inductance	mH/km	x>0	0.05	
my_r	(Sub-)Conductor: Relative Permeability		x>0	1	
diaco	(Sub-)Conductor: Outer Diameter	mm	x>0	30	
radco	(Sub-)Conductor: Outer Radius	mm	x>0	15	
diatub	(Sub-)Conductor: Inner Diameter	mm	x>0	16	
radtub	(Sub-)Conductor: Inner Radius	mm	x>0	8	
iskin	Consider skin effect (iskin=1) Neglect skin effect (iskin=0)		x=0 or x=1	1	
tmax	(Sub-)Conductor: Max. operational temperature	°C	x>=0	80	
rpha_tmax	(Sub-)Conductor: DC-Resistance at max. operational temperature	Ohm/km	x>=0	0.05	
alpha	(Sub-)Conductor: Temperature Coefficient	1/K	x>=0	0	
mlei	(Sub-)Conductor: Conductor Material			Al	
rtemp	(Sub-)Conductor: Max. End Temperature	°C	x>0	80	
Ithr	(Sub-)Conductor: Rated Short-Time (1s) Current	kA	x>=0	0	

Table A.2: Input parameters of the tower geometry type (*TypGeo*)

Name	Description	Unit	Range	Default	Symbol
loc_name	Name				
nlear	Number of Earth Wires		x>=0	1	
nlcir	Number of Line Circuits		x>0	1	
xy_e	Coordinates Earth Wires	m		0	
xy_c	Coordinates Phase Circuits	m		3	

Table A.3: Input parameters of the tower type (*TypTow*)

Name	Description	Unit	Range	Default	Symbol
loc_name	Name				
frnom	Nominal Frequency	Hz		50	
nlear	Number of Earth Wires		x>=0	1	
nlcir	Number of Line Circuits		x>=1	1	
iTransMode	Transposition		None Circuit-wise Symmetrical perfect	none	
i_mode	Input Mode: =0 : geometrical parameters. Coordinates of earth and phase conductors required (parameters “xy_e” and “xy_c”) = 1 : electrical parameters: R, X, G and B matrices required (parameters R_c, X_c, G_c and B_c or their equivalent matrices in sequence components)		x=0 or x=1	0	
gearth	Earth Conductivity	uS/cm	x>0	100	
rearth	Earth Resistivity	Ohmm	x>0	100	
pcond_e	Conductor types of earth wires	TypCon			
pcond_c	Conductor types of line circuits	TypCon			
nphas	Num. of Phases per line circuit			3	
cktrto	Assert this flag to enable transposition of the corresponding circuit in case if iTransMode=circuit-wise. For iTransMode other than circuit-wise this option is read only.			0	
xy_e	Coordinate of Earth Conductors	m		0	
xy_c	Coordinate of Line Circuits	m		0	
R_c	: Matrix of Resistances R_ij (natural components)	Ohm/km		0	
X_c	: Matrix of Reactances X_ij (natural components)	Ohm/km		0	
L_c	: Matrix of Inductances L_ij (natural components)	H/km		0	

R_c0	: Matrix of 0-Sequence-Resistances R_ij_0	Ohm/km		0	
R_c1	: Matrix of 1-Sequence-Resistances R_ij_1	Ohm/km		0	
X_c0	: Matrix of 0-Sequence-Reactances X_ij_0	Ohm/km		0	
X_c1	: Matrix of 1-Sequence-Reactances X_ij_1	Ohm/km		0	
L_c0	: Matrix of 0-Sequence-Inductances L_ij_0	H/km		0	
L_c1	: Matrix of 1-Sequence-Inductances L_ij_1	H/km		0	
G_c	: Matrix of Conductances G_ij (natural components)	uS/km		0	
B_c	: Matrix of Susceptances B_ij (natural components)	uS/km		0	
C_c	: Matrix of Capacitances C_ij (natural components)	uF/km		0	
G_c0	: Matrix of 0-Sequence-Conductances G_ij_0	uS/km		0	
G_c1	: Matrix of 1-Sequence-Conductances G_ij_1	uS/km		0	
B_c0	: Matrix of 0-Sequence-Susceptances B_ij_0	uS/km		0	
B_c1	: Matrix of 1-Sequence-Susceptances B_ij_1	uS/km		0	
C_c0	: Matrix of 0-Sequence-Capacitances C_ij_0	uF/km		0	
C_c1	: Matrix of 1-Sequence-Capacitances C_ij_1	uF/km		0	
pStoch	Stochastic model (StoTyplne)				

B References

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