POWERFACTORY

PowerFactory 2021

Technical Reference

Cable System

ElmCabsys, TypCabsys

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POWER SYSTEM SOLUTIONS

MADE IN GERMANY

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1 Introduction

This document describes the definition of a cable system in terms of its geometry, the properties of the conducting, semi-conducting and insulating layers and installation characteristics (i.e. buried directly in ground, in a pipe). In addition, the calculation of frequency-dependent electrical parameters of the cable system are described.

A frequency-dependent cable system in *PowerFactory* can be defined using objects: a single core cable type *TypCab* which describes the construction characteristics of the cable and a cable system type *TypCabsys*, which defines the coupling between phases, i.e. the coupling between the single core cables in a multiphase/multi-circuit cable system.

A built-in cable constants function in the cable system type calculates then the frequency-dependent electrical parameters (impedance and admittance matrices). The function can handle coaxial cables consisting of a core, sheath and armour directly in ground or installed in a pipe (pipe-type cables). This function can be used in stand-alone mode by clicking on the *Calculate* button in the cable system dialog. The results are printed to the output window. Alternatively, the function may be automatically called by various simulation functions in *PowerFactory*, i.e. when running a frequency scan or when adjusting the model for an EMT simulation.

Finally, it should be noted that the cable system type (*TypCabsys*) supports the definition of the cable in terms of geometrical data; if the cable is to be defined in terms of electrical data, the reader is referred to [6], in which case the general line/cable element (*ElmLne*) in *PowerFactory* should be used instead.

2 Definition of the Cable System

2.1 The Single Core Cable Type (*TypCab*)

The single core cable type *TypCab* supports up to three tubular conducting layers in a coaxial arrangement, i.e. core, sheath and armour, separated by three insulating layers. Figure 2.1 shows the typical layout of a HV AC single core cable. The model also supports the definition of a core-outer and insulation-outer semiconducting layer.

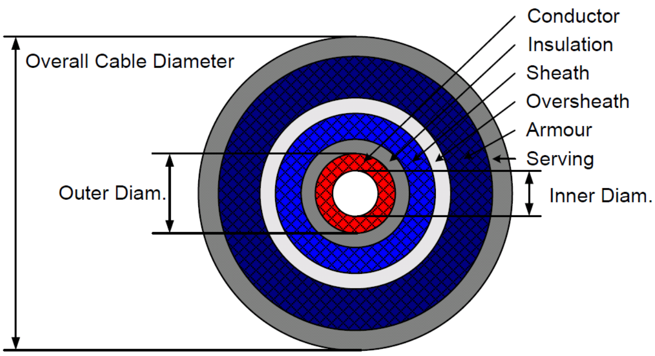


Figure 2.1: Cross-section of a single core cable including the core, sheath and armour

Section A shows the complete list of input parameters including units, range and the symbol used in this document. Hover the mouse pointer over the input parameters in the *TypCab* dialog to display the name of the input parameter. This is the name listed in the first column of the table.

The input data in the *TypCab* dialog is organised according to layers, i.e. the conducting, insulation and semiconducting layers, if available. Use *TypCab* to enter all the geometrical data defining the cross-section of the single core cable and the properties of all constituent materials.

2.1.1 Filling factor of conducting layers

To account for the compacting ratio of the cross-section of the conducting layers (stranded conductors, shaped compact, etc.), the user can enter a filling factor, *Cf*. This filling factor is related to the dc resistance of the cable by the following equation:

$$R_{DC}[\Omega/km] = \rho[\mu\Omega \cdot cm] \cdot \frac{1}{\pi \cdot (r^2 - q^2) \cdot C_f} \times 10$$

where *r* and *q* are the outer and inner radius of the conducting layer, respectively.

The user chooses the input parameter between the filling factor in % or the DC resistance in Ohm/km by clicking the selection arrow ➡. Note that one of them is always greyed out indicating its dependency on the other.

2.1.2 Temperature coefficient

If the temperature dependency of line/cables option is enabled in the Load Flow calculation, the resistivity of the conducting layers is adjusted by the following equation:

$$\rho_T = \rho_{20^\circ C} \cdot [1 + \alpha(T - 20)]$$

where α is the temperature coefficient of resistance. The resistivities and temperature coefficient of common metals are given in Table 2.1 for reference.

Table 2.1: Resistivities and temperature coefficient of resistance

Material	**Resistivity at** 20° **C [**$\mu\Omega \cdot cm$**]**	**Temperature coefficient at** $20^\circ C$ **[1/° C]**
Aluminium	DC Current Input	p.u.
Copper, hard-drawn	2.83	0,0039
Copper, annealed	1.77	0.00382
Brass	6.4 - 8.4	0.0020
Iron	10	0.0050
Silver	1.59	0.0038
Steel	12 - 88	0.001 - 0.005

2.2 The Cable System Type (*TypCabsys*)

The cable system type *TypCabsys* is used to complete the definition of a cable system. It defines the coupling between phases, i.e. the coupling between the single core cables in a multiphase/multi-circuit cable system. Cables are generally laid close together so this coupling should be taken into account.

Among other factors, this coupling depends on how the cables are laid. The *PowerFactory* model supports the following two types:

- **Parallel single-core cables (*TypCab*):** the cables are laid directly in the ground. This is the normal case for underground HV AC cables.
- **Multicore/Pipe-type cables (*TypCabmult*):** Used to represent submarine cables and cables that are drawn into a pipe, usually made of steel, and the pipe is laid in the ground. The pipe-type cable is widely used to model submarine cables and requires additional data for the pipe. The complete list of input parameters is given in Table A.2 in Section A.

2.3 Mutual Coupling

If the mutual coupling between cable systems is to be considered, then a cable system element (*ElmCabsys*) has to be defined. In this case, the line element *ElmLne* points to a cable system element (*ElmCabsys*) which in turns refers to the corresponding cable system type (*TypCabsys*). The latter contains the definition of the individual cables; e.g. *TypCab*.

2.3.1 AC/DC Couplings

AC and DC circuits with shared rights-of-way may result in electromagnetic coupling effects. AC/DC couplings can be modelled by using the cable system element (*ElmCabsys*) in conjunction with the cable system type (*TypCabsys*). The cable system type must have the *System Type* set to *AC/DC*. The impedances and admittances of the AC circuit/s will be solved using the user-defined *Nominal Frequency*, and of the DC circuit/s using the *Nominal Frequency (DC)*. AC/DC couplings are handled as follows in the various *PowerFactory* calculations:

- Load Flow: The transmission line equations are solved separately for the AC circuit/s and DC circuit/s. The DC circuit/s are always solved using the lumped parameter model, regardless of the selection of the *Line Model* in the cable system element (*ElmCabsys*).
- Short-Circuit: AC circuit/s are considered and any DC circuit/s are ignored.
- RMS Simulation: The DC circuit/s are always solved using the lumped parameter model, regardless of the selection of the *Line Model* in the cable system element (*ElmCabsys*).
- EMT Simulation: The frequency dependent phase-domain model is selected by default in the cable system element (*ElmCabsys*). The user can choose between the Universal Line Model (ULM) or the Frequency Dependent Cable Model (FDCM).

2.3.2 Cross-Bonding

Cross-bonding is a method used to suppress sheath voltages and reduce sheath currents. The cable system type offers an option to define the cable system as having *ideally* cross-bonded sheaths, i.e. corresponding to a perfect transposition of the sheaths. This is only available for 3-phase single core cables that have a sheath (not pipe-type or multicore cables). The use of the cross-bonding option causes modifications to sub-matrices of the cable system impedance and admittance matrices.

For a cable system comprising single core cables having a core and a sheath, the impedance matrix is given by:

$$[Z] = \begin{bmatrix} [Z_{cc}] & [Z_{cs}] \\ [Z_{sc}] & [Z_{ss}] \end{bmatrix} \tag{1}$$

Similarly, the admittance matrix is given by:

$$[Y] = \begin{bmatrix} [Y_{cc}] & [Y_{cs}] \\ [Y_{sc}] & [Y_{ss}] \end{bmatrix} \tag{2}$$

When the cable system is ideally cross-bonded, the core-sheath mutual impedances (sub-matrices $[Z_{cs}]$ and $[Z_{sc}]$) are averaged and the core self- and mutual impedances, contained in the $[Z_{cc}]$ sub-matrix, are left unchanged. The diagonal and mutual elements contained in the $[Z_{ss}]$ sub-matrix represent the mean of the self- and mutual impedances of the sheath [4].

Similarly, the Y_{cs} and Y_{sc} matrices are averaged row- and column-wise, respectively, and the diagonal and mutual elements contained in the $[Y_{ss}]$ sub-matrix represent the mean of the self- and mutual admittances of the sheath [4].

2.3.3 Conductor Elimination (*Reduced* Option)

The cable system type offers an option (*Reduced*) to mathematically eliminate conducting layers. This is applicable to the sheath, armour and pipe conducting layers (if present). This has the advantage of reducing the dimensions of the impedance and admittance matrices, thereby improving solution speed without a significant loss of accuracy. The Kron matrix reduction method is used to eliminate the grounded conducting layers, meaning that explicit modelling of these conductors is no longer possible. This is expressed mathematically (for ungrounded "u" and grounded "g" conductors) as follows [7]:

$$\frac{d}{dx}\begin{bmatrix}\underline{\boldsymbol{U}}_u\\ \underline{\boldsymbol{U}}_g\end{bmatrix} = -\begin{bmatrix}\underline{\boldsymbol{Z}}_{uu} & \underline{\boldsymbol{Z}}_{ug}\\ \underline{\boldsymbol{Z}}_{gu} & \underline{\boldsymbol{Z}}_{gg}\end{bmatrix}\cdot\begin{bmatrix}\underline{\boldsymbol{I}}_u\\ \underline{\boldsymbol{I}}_g\end{bmatrix} \tag{3}$$

$\underline{\boldsymbol{I}}_g$ can be eliminated since $\underline{\boldsymbol{U}}_g$ and $\frac{d}{dx}(\underline{\boldsymbol{U}}_g)$ are zero:

$$\frac{d}{dx}\begin{bmatrix}\underline{\boldsymbol{U}}_c\\ 0\end{bmatrix} = -\begin{bmatrix}\underline{\boldsymbol{Z}}_{uu} & \underline{\boldsymbol{Z}}_{ug}\\ \underline{\boldsymbol{Z}}_{gu} & \underline{\boldsymbol{Z}}_{gg}\end{bmatrix}\cdot\begin{bmatrix}\underline{\boldsymbol{I}}_c\\ 0\end{bmatrix} \tag{4}$$

which results in:

$$\frac{d}{dx}[\underline{\boldsymbol{U}}_u] = -[\underline{\boldsymbol{Z'}}_{reduced}]\cdot[\underline{\boldsymbol{I}}_u] \tag{5}$$

where

$$[\underline{\boldsymbol{Z'}}_{reduced}] = [\underline{\boldsymbol{Z'}}_{uu}] - [\underline{\boldsymbol{Z'}}_{ug}]\cdot[\underline{\boldsymbol{Z'}}_{gg}]\cdot[\underline{\boldsymbol{Z'}}_{gu}] \tag{6}$$

3 Calculation of Electrical Parameters

The calculation of the impedance and admittance of the cable is based on the cable constants equations formulated by [3], and makes the following assumptions:

- Coaxial arrangement of the conducting and insulating layers inside the single core cable;
- Single core cables inside the pipe are concentric with respect to the pipe;
- Each conducting layer of the cable has constant permeability. In addition, conducting layers are non-magnetic so that the cable model does not account for current-dependent saturation effects;
- Displacement currents and dielectric losses of the insulating layers are negligible.

A general formulation of the series impedance and shunt admittance of the cable is given by:

$$\frac{d}{dx}[\underline{\boldsymbol{U}}] = -[\underline{\boldsymbol{Z}}]\cdot[\underline{\boldsymbol{I}}] \tag{7}$$

$$\frac{d}{dx}[\underline{\boldsymbol{I}}] = -[\underline{\boldsymbol{Y}}]\cdot[\underline{\boldsymbol{U}}] \tag{8}$$

where $[\underline{\boldsymbol{U}}]$ and $[\underline{\boldsymbol{I}}]$ are the voltage and current vectors at a distance x along the cable.

The dimension of $[\underline{\boldsymbol{Z}}]$ and $[\underline{\boldsymbol{Y}}]$ depends on the total number of cables in the system and the total number of layers per single core cable. For instance, in a three-phase cable system with three conducting layers per single core cable (core, sheath and armour) the dimension of the $[\underline{\boldsymbol{Z}}]$ (i.e. $[\underline{\boldsymbol{Y}}]$) results in 9 (=3 phases x 1 single core cable/phase x 3 conducting layers/cable).

$[\underline{\boldsymbol{Z}}]$ and $[\underline{\boldsymbol{Y}}]$ are symmetric square matrices that can be expressed as follows:

$$[\underline{\boldsymbol{Z}}] = [\underline{\boldsymbol{Z}}_I] + [\underline{\boldsymbol{Z}}_P] + [\underline{\boldsymbol{Z}}_C] + [\underline{\boldsymbol{Z}}_0] \tag{9}$$

$$\begin{gathered}[\underline{\boldsymbol{Y}}] = s \cdot [\boldsymbol{P}]^{-1} \\ [\boldsymbol{P}] = [\boldsymbol{P}_I] + [\boldsymbol{P}_P] + [\boldsymbol{P}_C] + [\boldsymbol{P}_0]\end{gathered} \tag{10}$$

where $[\boldsymbol{P}]$ is a potential coefficient matrix and the Laplace's operator (complex frequency).

The matrices with subscript *I* account for the internal impedance and admittance, respectively, and matrices with subscript *O* account for the earth or air return path. In case of a pipe enclosure cable the matrices with subscript *C* and *P* define the impedance and admittance of the pipe; these matrices become zero if the cable is laid directly underground. In the next subsections we will discuss the physical meaning of these sub-matrices and the formulae used to calculate them.

The following naming convention is used throughout this document:

- Subscript *I* accounts for the internal impedance; subscript *O* for the earth or air return path, and subscripts *C* and *P* for the pipe enclosure (if available);
- Subscripts *c,s* and *a* (lower case) are used for core, sheath and armour in the cable layer equations;
- Subscripts *i,j* and *k* refer to the cables in the system (typically three cables in a three-phase cable system).

3.1 Internal Impedance

The internal impedance is associated with the longitudinal voltage drop due to the magnetic field inside the single core cable and it is given by the following equation:

$$\frac{d}{dx}\begin{bmatrix}\underline{\boldsymbol{U}}_c \\ \underline{\boldsymbol{U}}_s \\ \underline{\boldsymbol{U}}_a\end{bmatrix} = -\begin{bmatrix}\underline{\boldsymbol{Z}}_{cc} & \underline{\boldsymbol{Z}}_{cs} & \underline{\boldsymbol{Z}}_{ca} \\ \underline{\boldsymbol{Z}}_{sc} & \underline{\boldsymbol{Z}}_{ss} & \underline{\boldsymbol{Z}}_{sa} \\ \underline{\boldsymbol{Z}}_{ac} & \underline{\boldsymbol{Z}}_{as} & \underline{\boldsymbol{Z}}_{aa}\end{bmatrix} \cdot \begin{bmatrix}\underline{\boldsymbol{I}}_c \\ \underline{\boldsymbol{I}}_s \\ \underline{\boldsymbol{I}}_a\end{bmatrix} = -[\boldsymbol{Z}_I] \cdot \begin{bmatrix}\underline{\boldsymbol{I}}_c \\ \underline{\boldsymbol{I}}_s \\ \underline{\boldsymbol{I}}_a\end{bmatrix} \tag{11}$$

where the layer internal impedances in 11 are defined in terms of coaxial loop impedances as follows [7]:

$$
\begin{aligned}
\underline{\boldsymbol{Z}}_{cc} &= \underline{\boldsymbol{Z}}_{11} + 2 \cdot \underline{\boldsymbol{Z}}_{12} + \underline{\boldsymbol{Z}}_{22} + 2 \cdot \underline{\boldsymbol{Z}}_{23} + \underline{\boldsymbol{Z}}_{33} \\
\underline{\boldsymbol{Z}}_{cs} = \underline{\boldsymbol{Z}}_{sc} &= \underline{\boldsymbol{Z}}_{12} + \underline{\boldsymbol{Z}}_{22} + 2 \cdot \underline{\boldsymbol{Z}}_{23} + \underline{\boldsymbol{Z}}_{33} \\
\underline{\boldsymbol{Z}}_{ca} = \underline{\boldsymbol{Z}}_{ac} = \underline{\boldsymbol{Z}}_{sa} = \underline{\boldsymbol{Z}}_{as} &= \underline{\boldsymbol{Z}}_{23} + \underline{\boldsymbol{Z}}_{33} \\
\underline{\boldsymbol{Z}}_{ss} &= \underline{\boldsymbol{Z}}_{22} + 2 \cdot \underline{\boldsymbol{Z}}_{23} + \underline{\boldsymbol{Z}}_{33} \\
\underline{\boldsymbol{Z}}_{aa} &= \underline{\boldsymbol{Z}}_{33}
\end{aligned} \tag{12}
$$

The impedances with subscripts 1, 2 and 3 are referred to as loop impedances. For instance, $\underline{\boldsymbol{Z}}_{11}$ is the impedance of the innermost loop of the concentric tubular conductors and therefore that of the core-sheath loop.

$$
\begin{aligned}
\underline{\boldsymbol{Z}}_{11} &= \underline{\boldsymbol{Z}}_{c,OUT} + \underline{\boldsymbol{Z}}_{c/s,INS} + \underline{\boldsymbol{Z}}_{s,IN} \\
\underline{\boldsymbol{Z}}_{22} &= \underline{\boldsymbol{Z}}_{s,OUT} + \underline{\boldsymbol{Z}}_{s/a,INS} + \underline{\boldsymbol{Z}}_{a,IN} \\
\underline{\boldsymbol{Z}}_{33} &= \underline{\boldsymbol{Z}}_{a,OUT} + \underline{\boldsymbol{Z}}_{a/e,INS} + \underline{\boldsymbol{Z}}_{e} \\
\underline{\boldsymbol{Z}}_{12} = \underline{\boldsymbol{Z}}_{21} &= -\underline{\boldsymbol{Z}}_{s,MUTUAL} \\
\underline{\boldsymbol{Z}}_{23} = \underline{\boldsymbol{Z}}_{32} &= -\underline{\boldsymbol{Z}}_{a,MUTUAL}
\end{aligned} \tag{13}
$$

The impedances of the tubular conductors are found with the modified Bessel functions, with *tube = c, s* and *a* respectively:

$$
\begin{aligned}
\underline{Z}_{tube,IN} &= \frac{\rho \underline{m}}{2\pi q \underline{D}} \{ I_0(\underline{m}q) \cdot K_1(\underline{m}r) + K_0(\underline{m}q) \cdot I_1(\underline{m}r) \} \\
\underline{Z}_{tube,OUT} &= \frac{\rho \underline{m}}{2\pi r \underline{D}} \{ I_0(\underline{m}r) \cdot K_1(\underline{m}q) + K_0(\underline{m}r) \cdot I_1(\underline{m}q) \} \\
\underline{Z}_{tube,MUTUAL} &= \frac{\rho}{2\pi q r \underline{D}}
\end{aligned} \tag{14}
$$

where

$$
\underline{D} = I_1(\underline{m}r) \cdot K_1(\underline{m}q) - I_1(\underline{m}q) \cdot K_1(\underline{m}r) \tag{15}
$$

$$
\underline{m} = \sqrt{\frac{j\omega\mu}{\rho}} = \frac{1}{\underline{p}} \tag{16}
$$

The parameter $\underline{m}$ is the reciprocal of the depth of penetration, $\underline{p}$, and are both frequency-dependent, complex values.

To take into account different conductor shapes (compact, hollow and segmental) in *PowerFactory*, the correction factor, K_s, is introduced into Equation (16):

$$
\underline{m_s} = m \cdot \sqrt{K_s} \tag{17}
$$

where $K_s \leq 1$. The default value of K_s is 1 and it is set as follows:

Table 3.1: Skin effect correction factor values

Conductor shape	**Correction factor,** K_s	**Corresponding IEC conductor type**
Compact	$K_s = 1$	Round stranded, round compact
Hollow	$K_s = 1$	Hollow, helical stranded
Segmental	$K_s = 0.425$	Round segmental

It should be noted that this correction factor is exclusively applied to the core conducting layer (not for other conducting layers).

$\underline{\boldsymbol{Z}}_{INS}$ accounts for the longitudinal voltage drop due to the magnetic field in the insulating layers. For the general case of non-concentric tubular conductors in a pipe, it results in:

$$\underline{Z}_{INS} = j\omega \frac{\mu_0}{2\pi} \cdot \ln \left\{ \frac{q}{R_i} \left[1 - \left(\frac{d_i}{q} \right)^2 \right] \right\} \tag{18}$$

where d_i is the offset of single core cable i from the centre, R_i is the outside radius of cable i, and q is the inside radius of the pipe. Further details can be found in [7]. In the case of concentric tubular conductors, $d_i = 0$ and (18) can be expressed simply as:

$$\underline{\boldsymbol{Z}}_{INS} = j\omega \frac{\mu_0}{2\pi} \cdot \ln \left(\frac{r}{q} \right)$$

where r is the outside radius of the insulation and q is defined as the inside radius of the insulation (in identical units).

3.2 Internal Admittance

The internal admittance matrix is associated with the capacitive coupling and dielectric losses due to the insulating layers within the single core cable. The capacitance and dielectric losses of each insulating layer are given by:

$$\begin{aligned} C_i &= \frac{2\pi\varepsilon_0\varepsilon_r}{\ln(r/q)} = \frac{1}{P_i} \\ G_i &= \omega C_i \cdot tg(\delta) \end{aligned} \tag{19}$$

where P_i is the potential coefficient of the insulating layer.

Assuming that the single core cable consists of three layers, hence the insulation between core and sheath, sheath and armour and outermost insulating layer of the single core cable, it follows that:

$$[\boldsymbol{P}_I] = \begin{bmatrix} P_c + P_s + P_a & P_s + P_a & P_a \\ P_s + P_a & P_s + P_a & P_a \\ P_a & P_a & P_a \end{bmatrix} \tag{20}$$

$$[\boldsymbol{C}_I] = \frac{1}{[\boldsymbol{P}_I]} \tag{21}$$

$$[\underline{\boldsymbol{Y}}_I] = [\boldsymbol{G}_I] + j\omega[\boldsymbol{C}_I] \tag{22}$$

3.3 Semiconducting Layers

The model supports the definition of a semiconducting layer on the conductor's outer surface and the insulation's outer surface. These semiconducting layers mainly influence the admittance of the insulation. Their effect on the impedance of the conductor is minor and therefore not considered in the model.

The use of the *Advanced* option for the definition of semiconducting layers in either the *TypCab* or *TypCabmult* is only recommended in special cases and requires highly accurate data for *all* input fields in order to ensure correct capacitance. Semiconducting layers are usually defined by adjusting the radius and permittivity of the main insulation. For further details and calculation of the effective relative permittivity, the reader is referred to [5], [13].

Semiconducting layers can be considered in the following ways (the first two being the most common):

1. Implicitly via the adjustment of parameters related to the main insulation (using an *effective* relative permittivity [13]) and <u>no</u> utilisation of the semiconducting layers table. This should be used for projects that will be used in PowerFactory versions prior to PF2020;
2. Via use of the *Semiconducting Layers* table: Selection of the *Exists* flag and a user-defined thickness: In this case *PowerFactory* will internally recalculate the effective relative permittivity when calculating the shunt admittance; or
3. Via use of the *Semiconducting Layers* table: Selection of the *Exists* flag, a user-defined thickness and selection of the *Advanced* flag and user definition of parameters. This is only intended for specialised purposes and if accurate data is available.

The capacitance and conductance of the tubular semiconducting layer are given by the following equations:

$$C_{SC} = 2\pi\varepsilon_0 \cdot \varepsilon_{rSC} \frac{1}{\ln(r_{SC}/q_{SC})}$$

$$G_{SC} = \frac{2\pi}{\rho_{SC}} \cdot \frac{1}{\ln(r_{SC}/q_{SC})}$$

where r_{SC} and q_{SC} are the outer and inner radius of the tubular semiconducting layer respectively; ε_{rSC} is the relative permittivity and ρ_{SC} is the resistivity.

Hence the equivalent admittance of the insulation under consideration of the semiconducting layers is calculated as:

$$\frac{1}{G_{iIns}} = \frac{1}{G_{IL}} + \frac{1}{G_{SC}}$$

$$\frac{1}{C_{iIns}} = \frac{1}{C_{IL}} + \frac{1}{C_{SC}}$$

3.4 Parallel Single-Core Cables

Single-core cables are modelled using the single-core cable system type (*TypCab*). The impedance and admittance formulae are described below. Further details can be found in [7].

3.4.1 Impedance

If it assumed that *i, j, k* are three parallel single core cables, and each consists of core, sheath and armour. Equation (7) can then be expanded:

$$\frac{d}{dx}\begin{bmatrix}[\underline{\boldsymbol{U}}_i]\\ [\underline{\boldsymbol{U}}_j]\\ [\underline{\boldsymbol{U}}_k]\end{bmatrix} = \left\{\begin{bmatrix}[\underline{\boldsymbol{Z}}_{I,ii}] & [\mathbf{0}] & [\mathbf{0}]\\ \vdots & [\underline{\boldsymbol{Z}}_{I,jj}] & [\mathbf{0}]\\ \cdot^{\cdot^{\cdot}} & \cdots & [\underline{\boldsymbol{Z}}_{I,kk}]\end{bmatrix} + \begin{bmatrix}[\underline{\boldsymbol{Z}}_{0,ii}] & [\underline{\boldsymbol{Z}}_{0,ij}] & [\underline{\boldsymbol{Z}}_{0,ik}]\\ \vdots & [\underline{\boldsymbol{Z}}_{0,jj}] & [\underline{\boldsymbol{Z}}_{0,jk}]\\ \cdot^{\cdot^{\cdot}} & \cdots & [\underline{\boldsymbol{Z}}_{0,kk}]\end{bmatrix}\right\} \times \begin{bmatrix}[\underline{\boldsymbol{I}}_i]\\ [\underline{\boldsymbol{I}}_j]\\ [\underline{\boldsymbol{I}}_k]\end{bmatrix} \quad (23)$$

where $[\underline{\boldsymbol{Z}}_{0,s}]$ and $[\underline{\boldsymbol{Z}}_{0,m}]$ are the self- and mutual- earth-return impedance matrices of the cable system given as:

$$[\underline{\boldsymbol{Z}}_{0,s}] = \begin{bmatrix}\underline{Z}_{e,s} & \underline{Z}_{e,s} & \underline{Z}_{e,s}\\ \underline{Z}_{e,s} & \underline{Z}_{e,s} & \underline{Z}_{e,s}\\ \underline{Z}_{e,s} & \underline{Z}_{e,s} & \underline{Z}_{e,s}\end{bmatrix} ...s = jj,kk,ll \quad (24)$$

$$[\underline{\boldsymbol{Z}}_{0,m}] = \begin{bmatrix}\underline{Z}_{e,m} & \underline{Z}_{e,m} & \underline{Z}_{e,m}\\ \underline{Z}_{e,m} & \underline{Z}_{e,m} & \underline{Z}_{e,m}\\ \underline{Z}_{e,m} & \underline{Z}_{e,m} & \underline{Z}_{e,m}\end{bmatrix} ...m = jk,kl,lj \quad (25)$$

$\underline{Z}_{e,m}$ is the mutual earth-return impedance between two parallel cables *i, j* given by:

$$\underline{Z}_{e,jk} = j\frac{\omega\mu_0}{\pi}[K_0(\underline{m}\cdot d_{ik}) - K_0(\underline{m}\cdot D_{ik})] + (P_{ik} + jQ_{ik}) \quad (26)$$

and $P_{ik} + jQ_{ik}$ the terms of the Carson's series (see [1] for further information).

$\underline{Z}_{e,s}$ is the self earth-return impedance of the single core cable. Its value is obtained from (26) by replacing d with R; D with $2h$; and $h + y$ with $2h$.

3.4.2 Admittance

As the cable is directly laid underground and the earth surrounding the cable is assumed to be an equipotential surface, there is no capacitive coupling effect among the single core cables.

It then follows that $[\underline{\boldsymbol{P}}_0] = \Theta$ in equation 10 and therefore the admittance matrix of the cable results in:

$$[\boldsymbol{P}] = [\boldsymbol{P}_I] = \begin{bmatrix} [\boldsymbol{P}_{I,i}] & 0 & 0 \\ 0 & [\boldsymbol{P}_{I,j}] & 0 \\ 0 & 0 & [\boldsymbol{P}_{I,k}] \end{bmatrix} \tag{27}$$

$$\begin{gathered} [\underline{\boldsymbol{Y}}] = [\boldsymbol{P}]^{-1} \\ \frac{d}{dx}\begin{bmatrix} [\underline{\boldsymbol{I}}_i] \\ [\underline{\boldsymbol{I}}_j] \\ [\underline{\boldsymbol{I}}_k] \end{bmatrix} = -[\underline{\boldsymbol{Y}}] \cdot \begin{bmatrix} [\underline{\boldsymbol{U}}_i] \\ [\underline{\boldsymbol{U}}_j] \\ [\underline{\boldsymbol{U}}_k] \end{bmatrix} \end{gathered} \tag{28}$$

and the submatrices in the main diagonal represented according to equation (20).

3.5 Pipe Type Cable

The pipe-type cable is modelled using the multicore cable system type (*TypCabmult*). The impedance and admittance formulae are described below. Further details can be found in [7].

3.5.1 Impedance

Assuming again a system of three single core cables, *i, j, k*, each of them consisting of core, sheath and armour, equation (9) can be expanded as follows for the case of a pipe type cable:

$$\begin{aligned} \frac{d}{dx}\begin{bmatrix} [\underline{\boldsymbol{U}}_i] \\ [\underline{\boldsymbol{U}}_j] \\ [\underline{\boldsymbol{U}}_k] \\ [\underline{\boldsymbol{U}}_p] \end{bmatrix} = &\left\{ \begin{bmatrix} [\underline{\boldsymbol{Z}}_{I,ii}] & [0] & [0] & 0 \\ \vdots & [\underline{\boldsymbol{Z}}_{I,jj}] & [0] & 0 \\ \iddots & \cdots & [\underline{\boldsymbol{Z}}_{I,kk}] & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \right. \\ &+ \begin{bmatrix} [\underline{\boldsymbol{Z}}_{P,ii}] & [\underline{\boldsymbol{Z}}_{P,ij}] & [\underline{\boldsymbol{Z}}_{P,ik}] & 0 \\ \vdots & [\underline{\boldsymbol{Z}}_{P,jj}] & [\underline{\boldsymbol{Z}}_{P,jk}] & 0 \\ \iddots & \cdots & [\underline{\boldsymbol{Z}}_{P,kk}] & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ &+ \begin{bmatrix} [\underline{\boldsymbol{Z}}_{C1}] & [\underline{\boldsymbol{Z}}_{C1}] & [\underline{\boldsymbol{Z}}_{C1}] & \underline{\boldsymbol{Z}}_{C2} \\ \vdots & [\underline{\boldsymbol{Z}}_{C1}] & [\underline{\boldsymbol{Z}}_{C1}] & \underline{\boldsymbol{Z}}_{C2} \\ \iddots & \cdots & [\underline{\boldsymbol{Z}}_{C1}] & \underline{\boldsymbol{Z}}_{C2} \\ \underline{\boldsymbol{Z}}_{C2} & \underline{\boldsymbol{Z}}_{C2} & \underline{\boldsymbol{Z}}_{C2} & \underline{\boldsymbol{Z}}_{C3} \end{bmatrix} \\ &+ \left. \begin{bmatrix} [\underline{\boldsymbol{Z}}_0] & [\underline{\boldsymbol{Z}}_0] & [\underline{\boldsymbol{Z}}_0] & \underline{\boldsymbol{Z}}_0 \\ \vdots & [\underline{\boldsymbol{Z}}_0] & [\underline{\boldsymbol{Z}}_0] & \underline{\boldsymbol{Z}}_0 \\ \iddots & \cdots & [\underline{\boldsymbol{Z}}_0] & \underline{\boldsymbol{Z}}_0 \\ \underline{\boldsymbol{Z}}_0 & \underline{\boldsymbol{Z}}_0 & \underline{\boldsymbol{Z}}_0 & \underline{\boldsymbol{Z}}_0 \end{bmatrix} \right\} \times \begin{bmatrix} [\underline{\boldsymbol{I}}_i] \\ [\underline{\boldsymbol{I}}_j] \\ [\underline{\boldsymbol{I}}_k] \\ \underline{\boldsymbol{I}}_p \end{bmatrix} \end{aligned} \tag{29}$$

$[\underline{\boldsymbol{Z}}_P]$ defines the self- and mutual- impedances of the pipe-return path of the single core cables. A sub-matrix is given by:

$$[\underline{\boldsymbol{Z}}_{P,ij}] = \begin{bmatrix} \underline{\boldsymbol{Z}}_{P,ij} & \underline{\boldsymbol{Z}}_{P,ij} & \underline{\boldsymbol{Z}}_{P,ij} \\ \underline{\boldsymbol{Z}}_{P,ij} & \underline{\boldsymbol{Z}}_{P,ij} & \underline{\boldsymbol{Z}}_{P,ij} \\ \underline{\boldsymbol{Z}}_{P,ij} & \underline{\boldsymbol{Z}}_{P,ij} & \underline{\boldsymbol{Z}}_{P,ij} \end{bmatrix} \tag{30}$$

The self impedance with pipe-return path for the *i-th* cable *(i=j)* is:

$$\underline{Z}_{P,ii} = j\omega \frac{\mu_0}{2\pi} \left[\frac{\mu_r K_0(\underline{m}q)}{\underline{m}qK_1(\underline{m}q)} + \sum_{n=1}^{\infty} \left(\frac{d_i}{q} \right)^{2n} \frac{2\mu_r K_n(\underline{m}q)}{n\mu_r K_n(\underline{m}q) - \underline{m}qK_n'(\underline{m}q)} \right] \tag{31}$$

Mutual impedance between the *i-th* and the *j-th* cables with common pipe-return path (*1≠j*):

$$\begin{aligned} \underline{Z}_{P,ij} = j\omega \frac{\mu_0}{2\pi} \Bigg\{ & \ln \left(\frac{q}{\sqrt{d_i^2 + d_j^2 - 2d_i d_j \cos\vartheta_{ij}}} \right) + \mu_r \frac{K_0(\underline{m}q)}{\underline{m}qK_1(\underline{m}q)} \\ & + \sum_{n=1}^{\infty} \left(\frac{d_i d_j}{q^2} \right)^n \cos(n\vartheta_{ij}) \left[\frac{2\mu_r K_n(\underline{m}q)}{n\mu_r K_n(\underline{m}q) - \underline{m}qK_n'(\underline{m}q)} - \frac{1}{n} \right] \Bigg\} \end{aligned} \tag{32}$$

$[\underline{\boldsymbol{Z}}_C]$ is the connection impedance matrix between the pipe inner and outer surfaces. The sub-matrix $[\underline{\boldsymbol{Z}}_{C1}]$, $[\underline{\boldsymbol{Z}}_{C2}]$ and $[\underline{\boldsymbol{Z}}_{C3}]$ are given by:

$$[\underline{\boldsymbol{Z}}_{C1}] = \begin{bmatrix} \underline{Z}_{C1} & \underline{Z}_{C1} & \underline{Z}_{C1} \\ \underline{Z}_{C1} & \underline{Z}_{C1} & \underline{Z}_{C1} \\ \underline{Z}_{C1} & \underline{Z}_{C1} & \underline{Z}_{C1} \end{bmatrix} \tag{33}$$

where $\underline{Z}_{C1}$, $\underline{Z}_{C2}$ and $\underline{Z}_{C3}$ are calculated using equations (14) to (18) for the impedance of tubular conductors and *tube* being the pipe as follows:

$$\begin{aligned} \underline{Z}_{C1} &= \underline{Z}_{pipe,OUT} + \underline{Z}_{pipe,INS} - 2 \cdot \underline{Z}_{pipe,MUTUAL} \\ \underline{Z}_{C2} &= \underline{Z}_{pipe,OUT} + \underline{Z}_{pipe,INS} - \underline{Z}_{pipe,MUTUAL} \\ \underline{Z}_{C3} &= \underline{Z}_{pipe,OUT} + \underline{Z}_{pipe,INS} \end{aligned} \tag{34}$$

Finally, $[\underline{\boldsymbol{Z}}_0]$ represent the impedance of the earth return-path of the pipe. The diagonal sub-matrix $[\underline{\boldsymbol{Z}}_0]$ is given by:

$$[\underline{\boldsymbol{Z}}_0] = \begin{bmatrix} \underline{\boldsymbol{Z}}_0 & \underline{\boldsymbol{Z}}_0 & \underline{\boldsymbol{Z}}_0 \\ \underline{\boldsymbol{Z}}_0 & \underline{\boldsymbol{Z}}_0 & \underline{\boldsymbol{Z}}_0 \\ \underline{\boldsymbol{Z}}_0 & \underline{\boldsymbol{Z}}_0 & \underline{\boldsymbol{Z}}_0 \end{bmatrix} \tag{35}$$

where $\underline{Z}_0$ is the self earth return impedance of the pipe according to equation (26).

3.5.2 Admittance

The admittance follows the general definition in terms of the potential coefficient matrix as follows:

$$[\boldsymbol{P}] = \left\{ \begin{bmatrix} [\boldsymbol{P}_{I,ii}] & [0] & [0] & 0 \\ \vdots & [\boldsymbol{P}_{I,jj}] & [0] & 0 \\ \iddots & \cdots & [\boldsymbol{P}_{I,kk}] & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} [\boldsymbol{P}_{P,ii}] & [\boldsymbol{P}_{P,ij}] & [\boldsymbol{P}_{P,ik}] & 0 \\ \vdots & [\boldsymbol{P}_{P,jj}] & [\boldsymbol{P}_{P,jk}] & 0 \\ \iddots & \cdots & [\boldsymbol{P}_{P,kk}] & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} [\boldsymbol{P}_{C}] & [\boldsymbol{P}_{C}] & [\boldsymbol{P}_{C}] & \boldsymbol{P}_{C} \\ \vdots & [\boldsymbol{P}_{C}] & [\boldsymbol{P}_{C}] & \boldsymbol{P}_{C} \\ \iddots & \cdots & [\boldsymbol{P}_{C}] & \boldsymbol{P}_{C} \\ \boldsymbol{P}_{C} & \boldsymbol{P}_{C} & \boldsymbol{P}_{C} & \boldsymbol{P}_{C} \end{bmatrix} \right\} \tag{36}$$

where

$$[\underline{\boldsymbol{Y}}] = \begin{bmatrix} [\boldsymbol{G}_{I,i}] & 0 & 0 & 0 \\ 0 & [\boldsymbol{G}_{I,j}] & 0 & 0 \\ 0 & 0 & [\boldsymbol{G}_{I,k}] & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + j\omega\left([\underline{\boldsymbol{P}}]\right)^{-1} \tag{37}$$

$$\frac{d}{dx}\begin{bmatrix} [\underline{\boldsymbol{I}}_i] \\ [\underline{\boldsymbol{I}}_j] \\ [\underline{\boldsymbol{I}}_k] \\ \underline{I}_p \end{bmatrix} = -[\underline{\boldsymbol{Y}}] \cdot \begin{bmatrix} [\underline{\boldsymbol{U}}_i] \\ [\underline{\boldsymbol{U}}_j] \\ [\underline{\boldsymbol{U}}_k] \\ \underline{U}_p \end{bmatrix} \tag{38}$$

Note that equation (37) does not consider dielectric losses of the pipe.

Each of the $[\boldsymbol{P}_{I,ii}]$ submatrices of $[\boldsymbol{P}_I]$ is the internal potential coefficient matrix of the single core cable according to (27).

$[\boldsymbol{P}_P]$ is the pipe internal potential coefficient matrix and defines the capacitive coupling between the outermost layer of the single core cables and the pipe and hence the dielectric medium between the cables and the pipe. Each of the submatrices $[\boldsymbol{P}_{P,ij}]$ of $[\boldsymbol{P}_P]$ is a matrix with equal elements given in the following form:

$$[\boldsymbol{P}_{P,ij}] = \begin{bmatrix} P_{P,ij} & P_{P,ij} & P_{P,ij} \\ P_{P,ij} & P_{P,ij} & P_{P,ij} \\ P_{P,ij} & P_{P,ij} & P_{P,ij} \end{bmatrix} \tag{39}$$

with

$$P_{ii} = \frac{1}{2\pi\varepsilon_0\varepsilon_r} \ln \left\{ \frac{q}{R_i} \left[1 - \left(\frac{d_i}{q} \right)^2 \right] \right\} \tag{40}$$

$$P_{ij} = \frac{1}{2\pi\varepsilon_0\varepsilon_r} \left\{ \ln \left(\frac{q}{\sqrt{d_i^2 + d_j^2 - 2d_i d_j \cos\vartheta_{ij}}} \right) - \sum_{n=1}^{\infty} \frac{1}{n} \cdot \left(\frac{d_i d_j}{q^2} \right)^n \cdot \cos\vartheta_{ij} \right\} \tag{41}$$

$[\boldsymbol{P}_C]$ is the potential coefficient matrix between the pipe inner and outer surfaces and hence the capacitance due to the dielectric layer surrounding the pipe. A sub-matrix and the last column and row elements are given by:

$$[\boldsymbol{P}_C] = \begin{bmatrix} P_C & P_C & P_C \\ P_C & P_C & P_C \\ P_C & P_C & P_C \end{bmatrix} \tag{42}$$

$$P_C = \frac{1}{2\pi\varepsilon_0\varepsilon_r} \cdot \ln \left(\frac{r}{q} \right) \tag{43}$$

It is assumed in the model that the pipe is underground. Therefore the outer surface of the insulating layer surrounding the pipe is in direct contact with the earth (equipotential surface with $U = 0$). Hence no additional capacitive effect exists between the insulating layer of the pipe and ground.

3.6 Multicore Cable

The multicore cable was introduced for use primarily by the Cable Ampacity calculation and is modelled using the multicore/pipe-type cable system type (*TypCabmult*). As such, in terms of the calculation of the cable impedance and admittance, it is currently an approximate model which does not consider the common outer conducting layers (common sheath, etc). The impedance and admittance formulae are described below. Further details related to the calculation of the series impedance can be found in [11].

3.6.1 Impedance

The series impedance of cables is traditionally calculated using analytical formulae which are able to model the skin effect. Due to the underlying assumption of a uniform current distribution in the conductors, the proximity effect cannot be taken into account. For closely spaced cables, such as multicore cables, neglecting the proximity effect may result in an underestimation of losses at operating frequency [11]. The multicore/pipe-type cable system type (*TypCabmult*), when selected to be *Multicore*, uses the method of moments surface operator (MoM-SO) technique to calculate the frequency dependent series impedance [11]:

$$\frac{\partial \mathbf{V}}{\partial z} = -[\mathbf{R}(\omega) + j\omega\mathbf{L}(\omega)] \cdot \mathbf{I} \tag{44}$$

In summary, series impedance is calculated according to the following MoM-SO procedure:

- Each conductor (core and sheath) is replaced by the surrounding medium;
- An equivalent current density is introduced on the surface of each conductor;
- The electric field integral equation is then used to relate the current density and longitudinal electric field on all conductors;
- Finally, the p.u.l. parameters of the line are calculated using the combined discretised surface admittance operator and discretised electric field integral equation.

In the cable system type (*TypCabsys*), the option *Number of Fourier basis functions* available on the *Advanced* page determines the truncation order of the Fourier series (and in turn, the accuracy and computational expense). This input parameter can usually be left at the default value and is intended for advanced use only.

3.6.2 Admittance

Admittance formulae used for the multicore cable are identical to those given in Section 3.4.2.

4 EMT Simulation

Two distributed parameter models are provided in *PowerFactory* for the EMT simulation of cable systems. These are the constant parameter model and the frequency dependent (phase-domain) model. The latter is available formulated as either the Universal Line Model (ULM) [2] or the Frequency Dependent Cable Model (FDCM) [10].

4.1 Frequency Dependent Phase-Domain Models

In general, transmission lines can be characterised by two frequency-dependent matrix transfer functions:

- The propagation function, $\mathbf{A}(\omega)$
- The characteristic admittance, $\mathbf{Y}_c(\omega)$ (or characteristic impedance $\mathbf{Z}_c(\omega)$)

These are expressed mathematically as:

$$\mathbf{A} = e^{-\sqrt{\mathbf{YZ}} \cdot l} \tag{45}$$

and

$$\mathbf{Y}_c = \mathbf{Z}^{-1} \cdot \sqrt{\mathbf{ZY}} \tag{46}$$

where $\mathbf{Z}$ and $\mathbf{Y}$ are the impedance and admittance matrices, respectively, and l is the line length. The time-domain simulation could be formulated using the inverse Fourier transform of the above functions and then solving the associated equations in the time domain using numerical convolution. However, the preferred time-domain solution uses rational function approximations of low order to ensure a computationally-efficient solution. In *PowerFactory*, these rational function approximations are obtained using vector fitting [9].

4.1.1 Rational Function Approximation

The accuracy of the rational function approximations of $\mathbf{A}$ and $\mathbf{Y}_c$ (or $\mathbf{Z}_c$) strongly influences the quality of the time-domain solution. The fitting of $\mathbf{A}$ and $\mathbf{Y}_c$ (or $\mathbf{Z}_c$) in the phase domain means that the phase-domain transfer functions will be intrinsically stable [8].

Approximation of $\mathbf{Y}_c$ or $\mathbf{Z}_c$ The characteristic admittance, $\mathbf{Y}_c$, (or characteristic impedance, $\mathbf{Z}_c$), are smooth functions of frequency, hence the fitting is straightforward and can be carried out directly in the phase domain [2]. The trace of $\mathbf{Y}_c$ is fitted in order to calculate the poles, and the residues are subsequently calculated per element (k,l) of $\mathbf{Y}_c$ using the following approximation:

$$(\mathbf{Y}_c(s))_{k,l} = d + \sum_{m=1}^{N} \frac{r_{i,m}}{s - p_{i,m}} \tag{47}$$

where constant d is real, N is the number of poles, $r_{i,m}$ are the residues, and $p_{i,m}$ are the poles. The residues and poles may be real or in complex conjugate pairs. Fitting is performed in the phase domain using vector fitting, and all elements of $\mathbf{Y}_c$ get identical poles. This procedure is used for the fitting of $\mathbf{Y}_c$ (or $\mathbf{Z}_c$) for both the ULM and FDCM.

4.1.2 Universal Line Model

The Universal Line Model (ULM) proposed in [2] offers high accuracy and a phase-domain formulation.

Approximation of $\mathbf{A}$ The fitting of the propagation function, $\mathbf{A}$, is difficult because it contains modal components which have differing time delays [2]. In practicality, the differences in the time delays can be significant in the case of cables. This is due to the different permittivities in the insulation. Hence, the approximation of $\mathbf{A}$ is performed using a two-step approach; (i) fitting in the modal domain; followed by (ii) final fitting in the phase domain.

Modal-Domain Fitting of $\mathbf{A}$: The modes of $\mathbf{A}$ are calculated using a frequency-dependent transformation matrix, $\mathbf{T}$. The modal-domain fitting is comprised of three procedures:

Backwinding: Multiplication with a factor, $e^{(j \cdot \omega \cdot \tau)}$, in order to remove most of the oscillatory behaviour of the elements of $\mathbf{A}$. These elements are oscillating functions of frequency due to the time delay of the line [8]. The diagonal elements of the modal propagation matrix, $\mathbf{A}^m$, can be expressed as [12]:

$$a_i^m(\omega) = e^{-\alpha_i(\omega) + j \cdot \frac{\omega}{v_i(\omega)} \cdot l} \tag{48}$$

where α is the attenuation, v is the velocity, j is the imaginary unit and l is the line length. Each mode i is approximated by:

$$a_i^m(s) \approx \sum_{m=1}^{N} \frac{r_{i,m}}{s - p_{i,m}} \cdot e^{-s \cdot \tau_i} \tag{49}$$

where N is the number of poles for mode i, $r_{i,m}$ are the residues, and $p_{i,m}$ are the poles. The residues and poles may be real or in complex conjugate pairs.

Collapsing: The process of replacing modes having almost equal time delays with a single mode equal to the average of the modes. The criterion used for the formation of groups is provided in [2]. The concept of “groups” thereby replaces that of modes.

Fitting: Vector fitting is used to obtain the rational function approximation and all poles contribute to all elements of $\mathbf{A}$.

Phase-Domain Fitting of $\mathbf{A}$: The final fitting of $\mathbf{A}$ (i.e. the calculation of residues) is done in the phase domain using the poles and time delays found via fitting in the modal domain. Each element has the form:

$$\mathbf{A}(s) \approx \sum_{i=1}^{n}(\sum_{m=1}^{N} \frac{r_{i,m}}{s - p_{i,m}}) \cdot e^{-s \cdot \tau_i} \tag{50}$$

where n is now the number of groups, and the poles, $p_{i,m}$, and the time delays, τ_i are known.

4.1.3 Frequency Dependent Cable Model (FDCM)

The Frequency Dependent Cable Model (FDCM) introduced in [10] offers improved accuracy and stability via the simultaneous fitting of poles and residues in the phase domain. High residue-pole ratios are thereby avoided, making the FDCM less prone to divergence due to such high ratios in EMT simulations.

Approximation of $\mathbf{A}$ The difficulties associated with the fitting of the propagation function, $\mathbf{A}$, were mentioned in Section 4.1.2. The approximation of $\mathbf{A}$ for the FDCM is performed by fitting the modal contributions in the phase domain using a common set of poles for each modal contribution, i, below. The poles and residues of $\mathbf{A}$ are obtained simultaneously, as follows:

$$\mathbf{A}(s) \cong \sum_{i=1}^{n} \hat{\mathbf{A}}_i \cdot e^{-s \cdot \tau_i} \tag{51}$$

where n is the number of modal contribution groups. Fitting is then performed with the constant time delay, τ_i, removed:

$$\hat{\mathbf{A}}_i(s) \approx \sum_{j=1}^{M_i} \frac{\mathbf{R}_{i,j}}{s - p_{i,j}} \tag{52}$$

where M_i is the order of the approximation for the ith modal propagation function.

4.1.4 Fitting Procedure

The following steps are repeated, increasing the number of poles until either: (i) the user-defined RMS error threshold has been met; or (ii) the user-defined maximum number of poles has been reached:

1. Vector fitting is applied iteratively to reduce the RMS error. Iteration stops if either the user-defined maximum RMS error or the user-defined maximum number of iterations is reached.
2. If using option *Increase and flip unstable poles*, unstable poles are flipped to the left-half plane and the RMS error is recalculated based on the resulting set of poles. This generally worsens the RMS error, as expected.
3. If using option *Increase until rms error is met*, the RMS error is calculated based on the unadjusted set of poles. This generally results in a lower RMS error than the previous option, however sometimes at the cost of unstable poles which can result in time-domain divergence in some cases.

4. For both of these options, if the RMS error does not meet the RMS error threshold, the number of poles is increased, and the algorithm continues from the first step.

Following the fitting process, *PowerFactory* reports the RMS error and the poles obtained by vector fitting (if option *Output poles* is selected).

The stability of the time-domain solution is dependent not only upon the accuracy of the rational function approximation (i.e. the resulting RMS error), but also upon the proximity of the poles to each other, and the ratio between residue and pole.

For $\mathbf{Y}_c$ and $\mathbf{Z}_c$, a worse RMS error is usually obtained when fitting starting at very low frequencies, and a better RMS error is usually obtained using a higher-order approximation (i.e. more poles).

For $\mathbf{A}$, a low number of poles is usually sufficient for accurate time-domain simulation, which assists in avoiding high residue-to-pole ratios that tend to occur with higher-order approximations (in particular for the ULM).

A Parameter Definitions

Table A.1: Input parameter of the single core cable type (*TypCab*)

Parameter	Description	Unit	Range	Default	Symbol
loc_name	Name				
uline	Rated voltage	kV	$x \geq 0$	0	U
typCon	Shape of the core			Compact	
diaCon	Outer diameter of the core	mm	$x \geq 0$	5	r
diaTube	Inner diameter of the core	mm	$x \geq 0$	0	q
cHasEl	Exists: use this flag to enable/disable the conducting layers			1	
rho	Resistivity ($20°$C) of the conducting layers	$\mu\Omega \cdot cm$	$x > 0$	1.7241	ρ
my	Relative Permeability of the conducting layers			1	μ_r
cThEl	Thickness of the conducting layers	mm		2.5	
Cf	Filling factor of the conducting layers	%	$x > 0$ and $x \leq 100$	100	C_f
rpha	DC-Resistance ($20°$C) of the conducting layers	Ω/km		0.8780769	R_{dc}
alpha	Temperature coefficient of the conducting layers	1/K	$x \geq 0$	0.00393	α
cHasIns	Exists: use this flag to enable/disable the corresponding insulation layers			1	
tand (ctand)	Dielectric Loss Factor of the insulating layer, i.e. $tg\delta$ of the insulation. Set this value to zero to neglect insulation losses.			0.02	
epsr	Relative Permittivity of the insulating layer			3	ε_r
thIns	Thickness of the insulating layer	mm	$x \geq 0$	1	
cHasSc	Exists: use this flag to enable/disable the semiconducting layers			0	
rhoSc	Resistivity of the semiconducting layer	$\mu\Omega \cdot cm$	$x > 0$	1000000	
mySc	Relative permeability of the semiconducting layer			1	$\mu_{r,SC}$
epsrSc	Relative permittivity of the semiconducting layer			3	$\varepsilon_{r,SC}$
thSc	Thickness of the semiconducting layer	mm	$x \geq 0$	1	
tmax	Max. Operational Temperature	°C	$x \geq 0$	80	
rtemp	Max. End Temperature	°C	$x > 0$	80	
Ithr	Rated Short-Time (1s) Current	kA	$x \geq 0$	0	
diaCab	Overall Cable Diameter	mm		15	
materialCond	Conductor material			Unknown	
materialIns	Insulation material			Unknown	

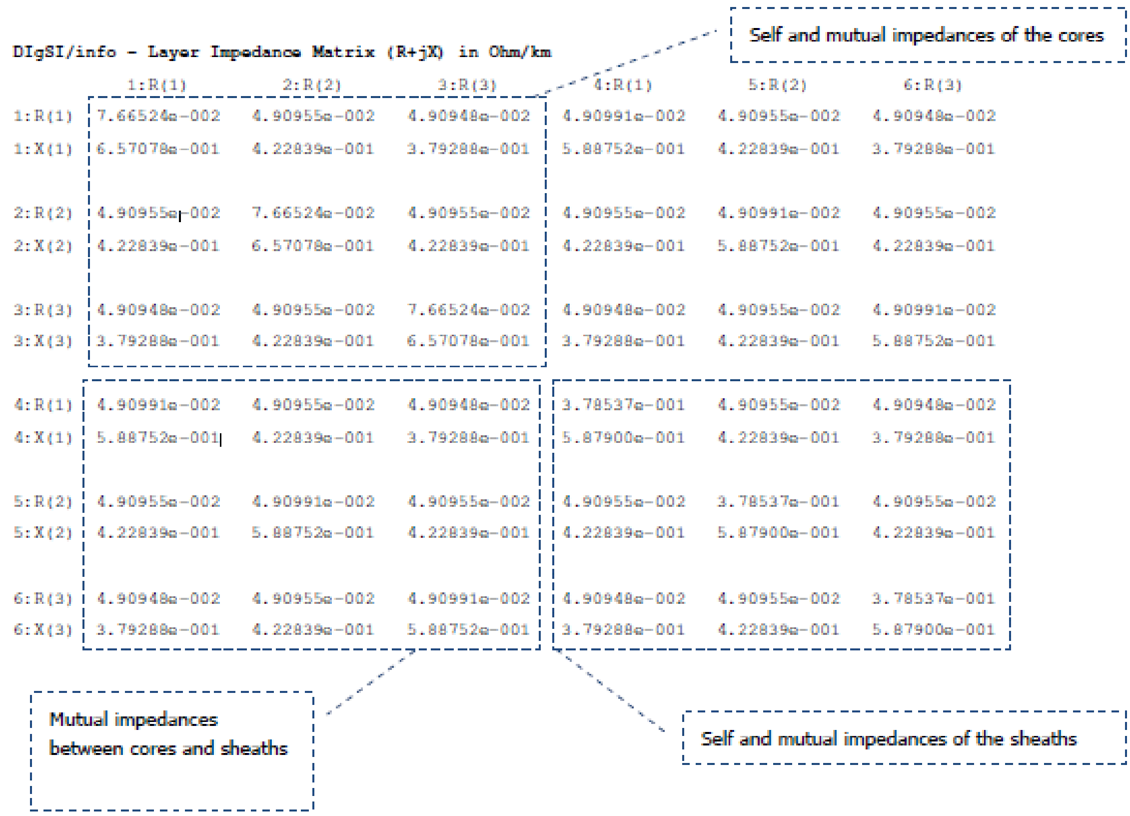
Table A.2: Input parameters of the cable system (*TypCabsys*)

Parameter	Description	Unit	Range	Default	Symbol
loc_name	Name				
frnom	Nom. Frequency	Hz		50	
rhoEarth	Resistivity of the earth return path	Ωm	$x > 0$	100	
cGearth	Conductivity of the earth return path = inverse of the earth resistivity.	$\mu S/cm$		100	
iopt_bur	To specify is the cable is laid direct in ground (parallel single core cables) or in a pipe (pipe-type cable)			gnd	
nlcir	Number of circuits defining the cable system		$x \geq 1$	1	
pcab_c	Single core cable type: select from the library the single core cable type (*TypCab*) of each circuit	*TypCab*			
nphas	Number of phases			3	
dInom	Rated current	kA		1	
red	Reduced: assert this option to automatically bond the sheaths and armours of the cable. This operation will reduce the Z/Y matrices of the cable to nphas x nphas.			0	
bond	Assert this option to cross bond the sheaths			0	
xy_c	Coordinate of Line Circuits: enter the coordinates of the single core cables in the cable systems. Cables buried direct underground have positive Y-distances with respect to the ground surface. For pipe type cables, X- and Y-coordinates are referred to the center of the pipe.	m		0	
dep_pipe	Depth of the center of the pipe (parameter only required for pipe-type cables).	m	$x \geq 0$	0	
rad_pipe	Outer Radius of the pipe	m	$x > 0$	0.1	
th_pipe	Thickness of the pipe	mm	$x \geq 0$	0	
th_ins	Thickness of the pipe outer insulation	mm	$x > 0$	1	
rho_pipe	Resistivity of the pipe	$\mu\Omega$cm	$x > 0$	20	
my_pipe	Relative permeability of the pipe		$x > 0$	1	
epsr_fil	Relative permittivity of the filling material (insulating material between the single core cables and the pipe)		$x > 0$	1	
epsr_ins	Relative permittivity of the pipe outer cover. Se		$x > 0$	1	

B Calculation Results

The cable constants function in stand-alone mode can be started via the *Calculate* button on the *EMT-Simulation* page of the dialog of the cable system type *TypCabsys*. Then *PowerFactory* prints the resulting impedance and admittance matrices to the output windows.

It follows an extract of the output window for a 132 kV, 3-phase cable system, 630 mm^2, directly underground. The first two matrices correspond to the unreduced layer impedances and admittances in phase components; cores first, followed by sheaths. Cables are in the same order as the input. Rows follow real and imaginary part.


DIgSI/info - Layer Impedance Matrix (R+jX) in Ohm/km

	1:R(1)	2:R(2)	3:R(3)	4:R(1)	5:R(2)	6:R(3)
1:R(1)	7.66524e-002	4.90955e-002	4.90948e-002	4.90991e-002	4.90955e-002	4.90948e-002
1:X(1)	6.57078e-001	4.22839e-001	3.79288e-001	5.88752e-001	4.22839e-001	3.79288e-001
2:R(2)	4.90955e-002	7.66524e-002	4.90955e-002	4.90955e-002	4.90991e-002	4.90955e-002
2:X(2)	4.22839e-001	6.57078e-001	4.22839e-001	4.22839e-001	5.88752e-001	4.22839e-001
3:R(3)	4.90948e-002	4.90955e-002	7.66524e-002	4.90948e-002	4.90955e-002	4.90991e-002
3:X(3)	3.79288e-001	4.22839e-001	6.57078e-001	3.79288e-001	4.22839e-001	5.88752e-001
4:R(1)	4.90991e-002	4.90955e-002	4.90948e-002	3.78537e-001	4.90955e-002	4.90948e-002
4:X(1)	5.88752e-001	4.22839e-001	3.79288e-001	5.87900e-001	4.22839e-001	3.79288e-001
5:R(2)	4.90955e-002	4.90991e-002	4.90955e-002	4.90955e-002	3.78537e-001	4.90955e-002
5:X(2)	4.22839e-001	5.88752e-001	4.22839e-001	4.22839e-001	5.87900e-001	4.22839e-001
6:R(3)	4.90948e-002	4.90955e-002	4.90991e-002	4.90948e-002	4.90955e-002	3.78537e-001
6:X(3)	3.79288e-001	4.22839e-001	5.88752e-001	3.79288e-001	4.22839e-001	5.87900e-001

DIgSI/info - Layer Admittance Matrix (G+jB) in uS/km

	1:G(1)	2:G(2)	3:G(3)	4:G(1)	5:G(2)	6:G(3)
1:G(1)	0.00000e+000	0.00000e+000	0.00000e+000	-0.00000e+000	0.00000e+000	0.00000e+000
1:B(1)	5.17257e+001	0.00000e+000	0.00000e+000	-5.17257e+001	0.00000e+000	0.00000e+000
2:G(2)	0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000	-0.00000e+000	0.00000e+000
2:B(2)	0.00000e+000	5.17257e+001	0.00000e+000	0.00000e+000	-5.17257e+001	0.00000e+000
3:G(3)	0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000	-0.00000e+000
3:B(3)	0.00000e+000	0.00000e+000	5.17257e+001	0.00000e+000	0.00000e+000	-5.17257e+001
4:G(1)	-0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000
4:B(1)	-5.17257e+001	0.00000e+000	0.00000e+000	5.85832e+002	0.00000e+000	0.00000e+000
5:G(2)	0.00000e+000	-0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000
5:B(2)	0.00000e+000	-5.17257e+001	0.00000e+000	0.00000e+000	5.85832e+002	0.00000e+000
6:G(3)	0.00000e+000	0.00000e+000	-0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000
6:B(3)	0.00000e+000	0.00000e+000	-5.17257e+001	0.00000e+000	0.00000e+000	5.85832e+002

DIgSI/info - 012 Impedance Matrix (R+jX) in Ohm/km

	1:R(0)	2:R(1)	3:R(2)	4:R(0)	5:R(1)	6:R(2)
1:R(0)	1.74843e-001	1.25721e-002	-1.25724e-002	1.47290e-001	1.25721e-002	-1.25724e-002
1:X(0)	1.47372e+000	-7.25883e-003	-7.25838e-003	1.40540e+000	-7.25883e-003	-7.25838e-003
2:R(1)	-1.25724e-002	2.75571e-002	-2.51443e-002	-1.25724e-002	3.81014e-006	-2.51443e-002
2:X(1)	-7.25838e-003	2.48756e-001	1.45177e-002	-7.25838e-003	1.80430e-001	1.45177e-002
3:R(2)	1.25721e-002	2.51448e-002	2.75571e-002	1.25721e-002	2.51448e-002	3.81014e-006
3:X(2)	-7.25883e-003	1.45168e-002	2.48756e-001	-7.25883e-003	1.45168e-002	1.80430e-001
4:R(0)	1.47290e-001	1.25721e-002	-1.25724e-002	4.76728e-001	1.25721e-002	-1.25724e-002
4:X(0)	1.40540e+000	-7.25883e-003	-7.25838e-003	1.40454e+000	-7.25883e-003	-7.25838e-003
5:R(1)	-1.25724e-002	3.81014e-006	-2.51443e-002	-1.25724e-002	3.29442e-001	-2.51443e-002
5:X(1)	-7.25838e-003	1.80430e-001	1.45177e-002	-7.25838e-003	1.79578e-001	1.45177e-002
6:R(2)	1.25721e-002	2.51448e-002	3.81014e-006	1.25721e-002	2.51448e-002	3.29442e-001
6:X(2)	-7.25883e-003	1.45168e-002	1.80430e-001	-7.25883e-003	1.45168e-002	1.79578e-001

The next two matrices are the impedances and admittances in symmetrical components in 0-1-2 sequence. Idem before, cores come first followed by the sheaths. Cables are in the same order as the input. Rows follow real and imaginary part.

DIgSI/info - 012 Admittance Matrix (G+jB) in uS/km

	1:G(0)	2:G(1)	3:G(2)	4:G(0)	5:G(1)	6:G(2)
1:G(0)	0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000
1:B(0)	5.17257e+001	0.00000e+000	0.00000e+000	-5.17257e+001	0.00000e+000	0.00000e+000
2:G(1)	0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000
2:B(1)	0.00000e+000	5.17257e+001	3.55271e-015	0.00000e+000	-5.17257e+001	-3.55271e-015
3:G(2)	0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000
3:B(2)	0.00000e+000	3.55271e-015	5.17257e+001	0.00000e+000	-3.55271e-015	-5.17257e+001
4:G(0)	0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000
4:B(0)	-5.17257e+001	0.00000e+000	0.00000e+000	5.85832e+002	0.00000e+000	0.00000e+000
5:G(1)	0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000
5:B(1)	0.00000e+000	-5.17257e+001	-3.55271e-015	0.00000e+000	5.85832e+002	5.68434e-014
6:G(2)	0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000	0.00000e+000
6:B(2)	0.00000e+000	-3.55271e-015	-5.17257e+001	0.00000e+000	5.68434e-014	5.85832e+002

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