# A handy-dandy introduction to the optical Thomson scattering diagnostic for pulsed-power experiments

Hannah Hasson

October 27, 2022

## Contents

## Preamble

Let's be honest, no matter what level you are at in research, Thomson scattering is confusing. This business between "collective" or "non-collective" scattering and all kinds of correlated behavior gets messy pretty darn fast. I hope this can be a simpler, more descriptive guide for folks who are new to the subject, since the literature out there can be pretty limited for the unfamiliar reader. Most of the information that I am conveying here was either from the textbooks by Sheffield et al. (2010); Hutchinson (2002) or from the PhD theses by Cornell graduates Rocco and Banasek. I hope you not only learn from but also enjoy this. Feel free to send any feedback or questions to hhasson@ur.rochester.edu. Happy scattering!

P.S. Please forgive me for my sins, as I have written all math in **CGS (cm, gram, sec) units** since the literature is all in those units. Temperatures, however, are typically in electron volts (eV).

## 1 What is scattering, and when is it of the Thomson variety?

Thomson scattering is one version of what happens when a photon scatters off of a free electron. There are two ways you can think about what it means to "scatter" here. Either:

1. Light as a particle: The photon and the electron both act like hard spheres that bounce off of each other, one gaining a little energy and another losing a little

   or

2. Light as a wave: The oscillating electric field of the photon wiggles the electron back and forth, then the electron emits radiation due to its being accelerated.

Because ions are so massive compared to electrons, in both of these pictures the ion is barely going to move at all and thus you are not going to get much scattered light out of the interaction. This is why we specifically talk about optical Thomson scattering done with *electrons.*

This interaction is distinguished from other types of photon-electron scattering by the energy (and thus wavelength) of the photon compared to the rest-mass energy of the electron. In the case of Thomson scattering **the photon energy is much smaller than the rest-mass energy of the electron ($\hbar\omega \ll m_e c^2$)**, which means the electron doesn't get to take a ton of energy from the photon and fly off into the sunset. Rather, it is barely affected by the interaction and the photon leaves traveling in a different direction with nearly the same energy it came in with. Think of it like throwing a Furby at a garbage truck (Fig. 1). The truck is so heavy it barely moves, and the Furby just goes bouncing off at some angle with almost the same speed. This is the case for photons in the visible light range or with longer wavelengths. If the photon is a much shorter wavelength with enough energy to accelerate the electron to relativistic speeds, we call this *Compton scattering.*

When we have particles scattering off of each other, we like to talk about something called a "**cross-section**." This is the probably of the interaction happening when any low-energy photon encounters a free electron. You can calculate the Thomson scattering cross-section by taking the ratio of total scattered power to incident power. The formula for the Thomson cross-section is

$$\sigma_T = \frac{8\pi r_e}{3} = 6.652 * 10^{-25} cm^2 \tag{1}$$

where $r_e$ is the classical electron radius, $r_e = e^2/(4\pi\epsilon_0 m_0 c^2)$. Note the name "cross-section" and the fact that it has units of area. That is because you can think of it as the

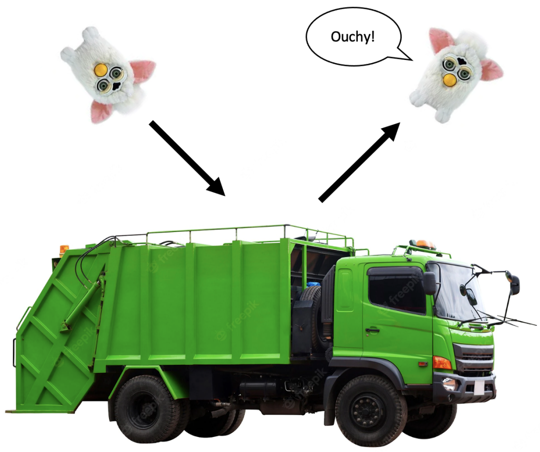


Figure 1: For your viewing pleasure, I illustrated what Thomson scattering would look like if photons were Furbys and electrons were garbage trucks. The Furby bounces off going the same speed with little change to the truck.

effective size of your electron in the view of the photon. When we want to estimate the amount of scattered light out for a certain amount of light encountering a plasma, we multiply the flux (energy/time/area/wavelength) of incident light by the cross section and number density of free electrons:

$$I_{scattered} = I_{incident} * \sigma_T * n_e \tag{2}$$

Since plasma has lots of free electrons flying around, it is a great candidate for using Thomson scattering on. There are two different *regimes* of Thomson scattering which allow you to measure different scales of physics in your plasma. One type of measurement will allow you to learn only of the electrons' conditions, and the other will allow you to measure behavior of the ions too. But first, let's get into how the electrons and ions can interact.

## 2 Debye shielding in plasma

In the frenzy of electrons and ions buzzing around in plasma, there is something that happens called **charge screening**. This means that, at some locations, a certain charge's electric field may not be detectable because other charges around it effectively cancel it out. The radius of a sphere around a given particle where an observer would see its field decrease by a factor of e is called the **Debye length** (pronounced "Duh-Buy"). This is naturally related to how dense the other charges are packed around the one we're trying to see (i.e. number density), and how fast the charges are moving around since we want to know how much time they spend close to one another (speed and thus temperature). The formula for the Debye length of an electron in a plasma in MKS units is:

$$\lambda_D = \sqrt{\frac{k_b T_e}{4\pi n_e e^2}}. \tag{3}$$

This is assuming that the electrons are moving around much faster than the ions, so we can basically treat the ions as stationary background positive charges as the electrons race around.

The reason the Debye length for electrons is so important for Thomson scattering is because if we probe at a scale smaller than $\lambda_D$, we must be looking at the physics on the scale of roughly an individual electron. On the other hand, if we make a measurement that probes on the scale larger than $\lambda_D$, we are statistically going to see overall behavior representative of groups of electrons following the ions around (Fig. 2). This is where the non-collective and collective measurements come from.

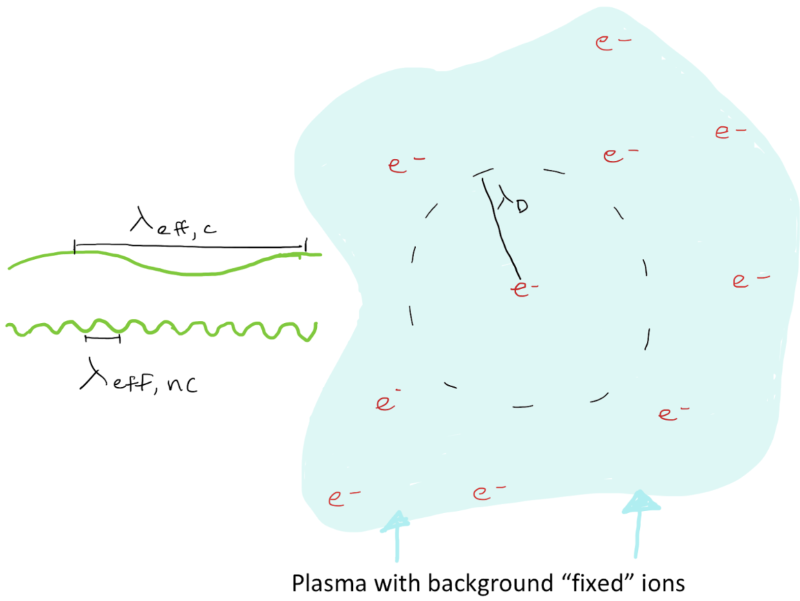


Figure 2: Sketch I made of two photons traveling toward a piece of plasma with background (roughly) stagnant ions and mobile electrons. The Debye sphere of one electron is shown as a dashed line, with the radius, $\lambda_D$, labeled. The effective wavelength of the top photon $\lambda_{eff,c}$ is greater than $\lambda_D$, so it will measure collective scattering. The effective wavelength of the bottom photon $\lambda_{eff,nc}$ is smaller than $\lambda_D$, so it will measure non-collective scattering.

## 3 Scattering vectors

Before we dig into which regime of scattering we have, we need to establish one of the quantities that determines the regime: the scattering vector. Every time we do a Thomson scattering measurement in an experiment, we have to draw a simple vector diagram with three wavevectors with respect to the scattering location: the input laser ($\vec{k_l}$), the scattering collection optics ($\vec{k_s}$), and the difference between the two ($\vec{k} = \vec{k_s} - \vec{k_l}$). The following is an example of one of these diagrams I made for a paper I'm writing:

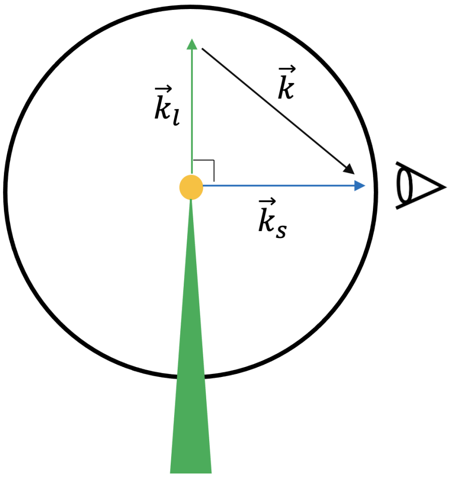


Figure 3: Thomson vectors diagram I made for a single scattering collection angle. The green line labeled $\vec{k_l}$ is the input laser wavevector, the blue line labeled $\vec{k_s}$ is the scattering vector we're observing. The black line labeled $\vec{k}$ is the scattering vector minus the laser vector, which is what we measure.

In this figure, the black hollow circle is the edge of the vacuum chamber, and the yellow filled circle is the scattering volume. The three vectors are drawn out for a scattering collection angle of 90° from the laser axis. This angle is important since you can observe slightly different spectra at each angle since this wavevector plays a key role in which regime (collective or non-collective) we are in.

How do we determine which regime we are in? As I said before, it is whether we are looking inside or outside the Debye length of the electron. This is determined by the product of the scattering wavevector and the electron Debye length. You will see this written in texts usually as the $\alpha$ parameter:

$$\alpha \equiv \frac{1}{k\lambda_D} = \frac{\lambda_{eff}}{\lambda_D} \tag{4}$$

You can think of this as the ratio of the effective wavelength of scattered light ($\lambda_{eff}$)

to the Debye length ($\lambda_D$). If $\alpha \ll 1$, then we are in the non-collective regime and are looking at uncorrelated electron behavior. If $\alpha \gtrsim 1$, then we are in the collective regime and are observing correlated electron behaviors. The relationships between these scales is shown in Fig. 2.

# 4 Non-collective scattering

## 4.1 Qualitative description of non-collective scattering

The first, simpler type of Thomson scattering that we have is **non-collective scattering**. You will also see this called "incoherent" scattering. This is the case where the Debye length of our plasma is much larger than the wavelength of the scattered wavevector. See my silly little drawing in Fig. 2 for a visual of what we're dealing with.

The electrons are shielded here by the background uniform(ish) distribution of ions. From outside of this sphere, anyone looking for this electron's field would see it reduced by e due to the positive background ion charge around it. Since we are in this case measuring at a scale smaller than one Debye length, we expect the motion we can measure from an electron to not be correlated with (affected by) other electrons around it. This is why we call it "non-collective;" it is a measurement reflecting individual electron motions, which are random.

## 4.2 Non-collective spectrum

There is an equation that you can use to model the theoretical spectrum of Thomson scattering at a certain angle in plasma of certain conditions. This theoretical equation has a few different names for whatever reason: the spectral density function, the **shape function**, or the form factor. I will mostly refer to it as the shape function here. The units of this equation are power per frequency. This means we can fit this model directly to our spectral data to get the plasma parameters.

If the individual electron motions are random in non-collective scattering, we would expect that with many scatterings we would get Doppler shifts in our scattered spectrum that follow some sort of Gaussian distribution... That's exactly what we get (assuming the electrons have a Maxwellian velocity distribution)! So our shape function will be a Gaussian. An example spectrum gracefully lifted from Hutchinson (2002) is shown in Fig. 4.

What does the equation for the shape function look like in the case of non-collective scattering? You will have something slightly ugly like this:

$$\frac{dP}{d\omega_s} = \frac{P_i r_e^2 L d\Omega}{2\pi}(1 + 2\omega/\omega_0) n_e f_{e0}(\omega/k) \tag{5}$$

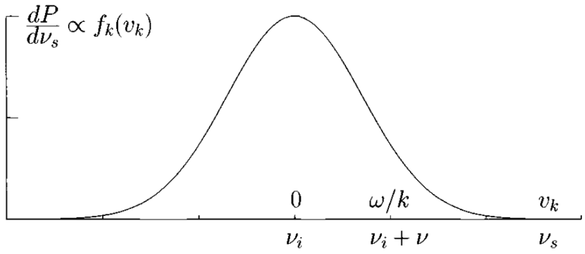


Figure 4: Theoretical spectrum from non-collective Thomson scattering off of electrons. The distribution of wavelengths of the scattered light is Gaussian because we are measuring individual electron motions, which will be random. The width of the Gaussian is determined by the temperature of the electrons in the plasma. Reproduced from Hutchinson (2002).

In reality, this is just some stuff multiplied by the electron velocity distribution function $f_{e0}$, which is what dictates the main features of the spectrum. So what quantities affect the shape of this spectrum? Primarily, the temperature of the electrons will dictate the width of the Gaussian. In general, we assume a Maxwell-Boltzmann velocity distribution in each spatial dimension:

$$f(v) = \left(\frac{m_e}{2\pi k_b T_e}\right)^{1/2} \exp\left(-\frac{m_e v^2}{2k_b T_e}\right) \tag{6}$$

If we take the ideal gas law for a monatomic gas, we can set its energy formula equal to the the kinetic energy formula

$$E = \frac{3}{2}k_b T_e = \frac{1}{2}m_e v^2 \tag{7}$$

and see that

$$v = \sqrt{\frac{3k_b T_e}{m_e}}. \tag{8}$$

So in this case the velocity is only a function of temperature and vice versa, making the Maxwell-Boltzmann distribution (Eq. 6) a function of only the electron's temperature. So the width can be fit to that equation to get the temperature of the plasma.

The spectrum has two more implicit variables not shown in the Maxwellian formula:

bulk velocity and number density. The bulk velocity of the plasma can be obtained from the Doppler shift of the entire spectrum using the basic Doppler formula and the input laser light frequency. The number density of the plasma is obtained from the absolute measurement of input and scattered photons in Equation 2. But in order to be able to get this, you must do an absolute calibration of how many photons are both injected into the volume and collected by your detector.

## 5 Collective scattering

### 5.1 Picture of the electron and ion plasma waves

Now we get to the juicier, but more confusing part of Thomson scattering. Collective scattering, also called "coherent" scattering, measures correlated electron behaviors, or how the electrons as a group move together. This means that we can get information about macroscopic features in the plasma- particularly plasma density waves in the electrons and ions. These are respectively called the **electron plasma wave (EPW)** and the **ion acoustic wave (IAW)**. The way I picture these is with two differently-zoomed views: the first is zooming out to the ion scale. We see the ions oscillating much, much slower than the electrons, in fact so much that the electron motions are sort of a blur of tiny movements that average out such that we just see them following the ions in their oscillations. The second view zooms way in on the speedy little electrons. We now see that at these tiny scales the electrons actually have their own natural oscillations too from moving amongst each other. So both the electron and ion plasma waves can exist at the same time, they just function at different scales.

### 5.2 Collective spectrum equation

Collective Thomson scattering will have a much more complicated spectrum due to the features of the electron and ion waves. However, we can still fit a shape function to our data to get all plasma parameters. Some qualitative things to know about the spectrum: There will be a double peak for both the IAW and the EPW. You can think of these as scattered light with smaller Doppler shifts applied to the wiggling motion of the waves, both as the particles oscillate away (redshift) and toward (blueshift) the observer. Since the ions are much heavier than the electrons, their resonance will be at a much lower frequency. This means that the IAW oscillations will cause a much smaller Doppler shift than the EPW in the scattered spectrum. However, the IAW pair will be much higher amplitude than the EPW because the electrons are much more affected by a process called Landau damping, which causes drag.

The collective scattering shape function for a plasma with one ion species is

$$S(\vec{k},\omega) = \frac{2\pi}{k}\left|1 - \frac{\chi_e}{\epsilon}\right| f_{0e}(\omega/k) + \frac{2\pi Z}{k}\left|\frac{\chi_e}{\epsilon}\right| f_{0i}(\omega/k), \tag{9}$$

where the lefthand term is the electron plasma wave and the righthand term is the ion plasma wave. Here, $\chi_x$ is the susceptibility (how polarizable it is in the presence of an E field) for species x. The two $f_0$ functions are the (typically Maxwellian) distributions of the electrons and ions, which are different due to their different masses and potentially different temperatures. Z here represents the effective charge, which is the amount of electrons ionized per atom.

Figure 5 is an example spectrum of both the electron and ion wave features on one spectrum (left), plus a zoomed-in view of the more detailed ion feature (right). This theoretical spectrum was produced using the Thomson scattering module of the PlasmaPy package. An extremely useful tutorial for this module can be found here.

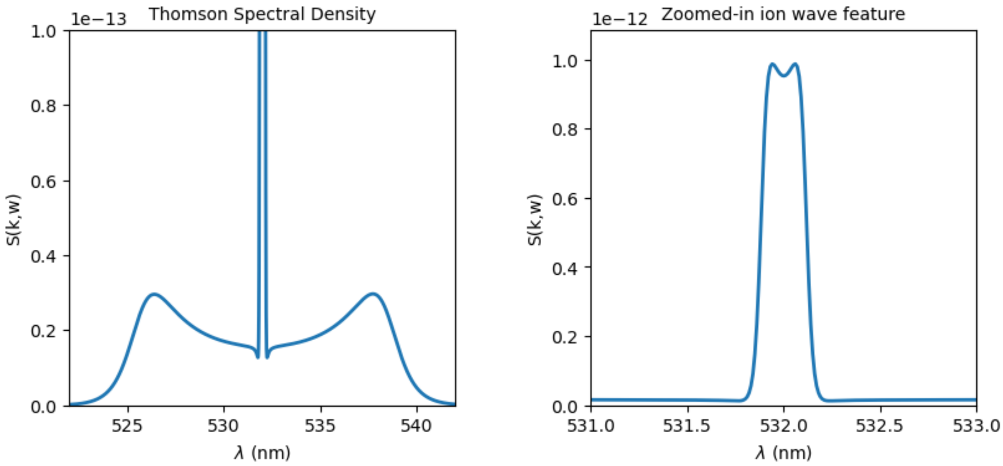


Figure 5: Example theoretical collective Thomson scattering spectrum. The left spectrum shows both the ion and electron wave features, while the right spectrum shows a zoom-in of the smaller-wavelength-shifted ion feature. Plots produced using PlasmaPy package (Murphy et al., 2022).

### 5.3 Ion Acoustic Wave (IAW) feature

We will start by breaking down the ion acoustic wave feature, which is the higher-signal feature in the spectrum. Fig. 6, reproduced from Rocco (2021), shows an example spectrum of the doublet IAW peaks. We see two peaks of slightly different heights with four of their properties highlighted. These peak properties are directly related to plasma parameters in the fit, which are shown next to letters A-D. Let's break these down one-by-one.

Just as with the non-collective spectrum, we can have a Doppler shift that applies to the entire plasma where the measurement is made. This is a measurement of **bulk velocity**, and shows up as the center of the spectrum shifting away from the input laser frequency. In Fig. 6, the center of the IAW feature is highlighted with the purple dashed line, and

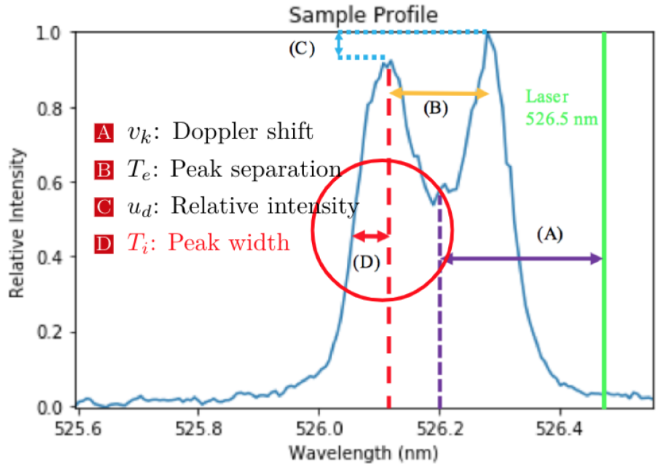


Figure 6: Example ion acoustic wave (IAW) spectrum showing the four important fit features and what plasma parameters they correspond to. Reproduced from Rocco (2021)

the Doppler shift from the original green laser line is marked as feature (A). Note that the Doppler velocity component measured will be along the vector $\vec{k}$ like that shown in Fig. 3.

The next important feature in the spectrum is the separation of the two ion peaks (yellow line marked (B) in Fig. 6). Remember that these correspond to the resonance in waves of ions wiggling back and forth. So you can imagine them as two different mini-Doppler-shifts in opposite directions. What affects how far apart they are? That would be the actual frequency of these waves. Larger frequency means they will oscillate faster, thus the mini-Doppler-shift will be larger. The formula for the IAW frequency is:

$$\omega_{IAW} \approx \pm k \left( \frac{\alpha^2 Z T_e}{(1+\alpha^2) m_i} + \frac{3T_i}{m_i} \right)^{1/2} \tag{10}$$

This looks kind of complicated, but stay with me for a moment. In order for the ion feature to show up at all, $ZT_e \gg T_i$. Otherwise, a process called Landau damping will squash out the wave altogether. With that in mind, we can say that the dominant term in Eq. 10 will be the lefthand one with $ZT_e$. Thus, if we have a good idea of the effective charge Z (number of free electrons per atom), we can get $T_e$ directly from fitting this feature.

The next feature that can be deduced from the spectral fit is the electron-ion drift velocity, $u_d$, shown in cyan and marked as (C) in Fig. 6. The drift velocity shows up in the spectrum as a favor of either the forward or backward ion wave peak. This amplitude difference occurs because the electrons moving slightly out of synch with the ions results in a slight damping of the wave in one direction. Although this is conceptually not too wild, the parameter $u_d$ is actually tangled up with $T_i$ and $T_e$, so there are usually multiple values of $u_d$ that will work for a given fit. Thus, we do not typically treat this as a reliable fit parameter and just let it float within reasonable bounds. It should not usually be included in results from the fit.

The final feature is the width of the ion peaks, marked in red as feature (D) in Fig. 6. As we might intuit from the non-collective spectrum, the ion feature's peak widths are related to the ion temperature $T_i$. But you didn't think it was going to be that easy, did you? It's actually also related to $T_e$ because we are still dealing with electron physics here! We are scattering off electrons, after all. The direct proportionality of peak width is actually to $ZT_e/T_i$. So if we can get $ZT_e$ from the peak separation, we can in theory get $T_i$ from the peak width measurement. BUT there are all kinds of extra caveats that can mess with the shape of the peaks and thus the width. So in general, we don't treat the ion temperature fit as reliable and thus do not take $T_i$ from the fit unless there is a compelling reason to believe the fit could only result from that ion temperature.

### 5.4 Electron plasma wave (EPW) feature

Finally, we will touch on the electron plasma wave (EPW) feature in the collective Thomson spectrum. This pair of peaks will be centered on the same wavelength as the IAW peaks, but will be much farther apart and much lower signal (Fig. 7). Therefore you would probably not want to use these to get the bulk Doppler shift of the spectrum's center if you can use the much stronger IAW peaks. As we will see, these features are great for getting electron properties though.

Remember, the EPW is NOT the same thing as the electron feature in the non-collective spectrum, which represents the random electron distribution. In this case, we are getting info specifically about oscillations within clumps of electrons bumping together. If you have heard of the plasma frequency ($\omega_{pe}$), this may sound similar- and you'd be correct! The frequency of the electron plasma wave is going to be very close to the plasma frequency. There is just going to be a small factor added on:

$$\omega_{EPW}^2 \approx \omega_{pe}^2 + \frac{3T_e}{m_e}k^2 \tag{11}$$

where the plasma frequency is

$$\omega_{pe}^2 = \frac{4\pi n_e e^2}{m_e}. \tag{12}$$

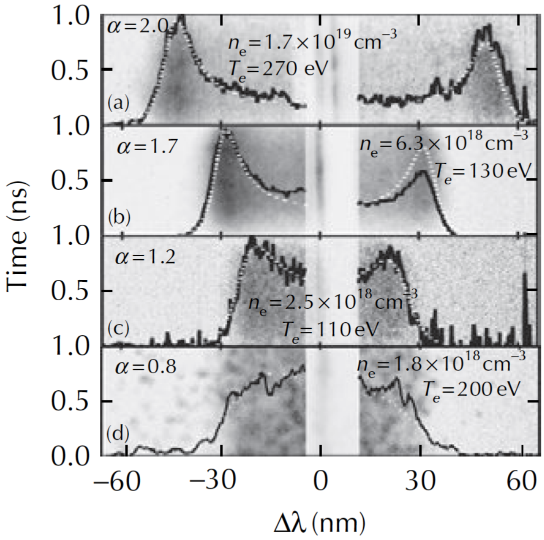


Figure 7: A few examples of the electron plasma wave double-peak for different $T_e$ and $n_e$. The middle of the plots with the IAW features are trimmed to focus on the EPW features. Reproduced from Sheffield et al. (2010).

In Eq. 11, the dominant term will be the number density $n_e$ from the plasma frequency term. So the separation of our peaks here will be primarily determined by $n_e$ with a small dependence on $T_e$ too.

The peak widths will be related to $n_e/T_e$, but since we can find $n_e$ from the peak separation, we can isolate $T_e$ after fitting the peak widths. Figure 7 shows the effects of varying $T_e$ and $n_e$.

When we want to take these measurements in practice, we may want to block the light from the part of the spectrum with the IAW feature, since it may be easier to see the EPW without the glare from that signal. It appears something like that was done in Fig. 7.

## 6 Things that can make this complicated mess even more complicated

Believe it or not, this can get even more complicated. In all of the above description, we are assuming that $\vec{B} = 0$ and that the electrons are non-relativistic. We also assume that the density in our scattering volumes is constant throughout. Density gradients, velocity gradients, and the presence of turbulence can all make the spectrum more nuanced and generally weird-looking. If you want to get into any of those details, check out the texts in the bibliography.

# 7 Setup of an optical Thomson scattering diagnostic for pulsed-power experiments

So now we get down to business: how do we actually collect these measurements in pulsed-power experiments? We have some idea from the scattering vector diagram in Figure 3 that the laser is injected from some angle and the scattered light is then collected at some other angle. This is where we will begin! One important detail that we glossed over, though, is that the laser needs to be focused to a small diameter so that we get enough photons per electron such that the intensity scattered (Eqtn. 2) and collected in some small solid angle is bright enough for our camera. This will be different depending on how sensitive your instruments are and how big/close to the plasma your collection optics are, but at Cornell the diameter of the focused Thomson scattering laser is $250\mu m$. But that is only the diameter of the scattering volume cylinder, so we need one more dimension to constrain how much light we get. That will be determined by the focal diameter of the collection optics! This can be understood much more clearly by looking at the diagram in Fig. 8.

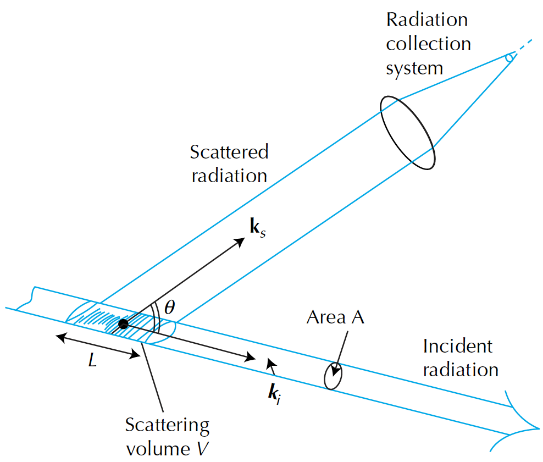


Figure 8: A diagram of the Thomson scattering volume in an experiment. The laser is focused to some small-cross-section circle (area A)and then the collection optics focus on some circle of a larger diameter (L) that is perpendicular to the laser path. Area A times diameter L gives roughly the collection volume. Reproduced from Sheffield et al. (2010).

The way this is set up in a lab is by marking where you want your scattering volume to be with a small reflective pin, focusing your laser there, and then tuning your collection optics to pick up the laser light reflected off the pin. This is of course not Thomson-scattered light (it's just a reflection), but it is a great way to make sure you are looking where you think you are, and that your spectrometer is tuned to the right wavelength.

Speaking of spectrometer, that is the next step in the diagnostic system after the collection optics. We will typically transport the collected scattered light to the spectrometer by focusing it with a lens into an optical fiber, and then feeding the other end of that fiber into the spectrometer we are using. This will then take your light and break it into a wavelength vs. intensity spectrum, which can be captured by a camera. You can even get multiple spectra from multiple points in the scattering volume by using what's called a fiber bundle. This is simply multiple fibers that are installed together in one mount side-by-side. The nice thing about this is that you can focus each fiber to a slightly different (vertical or horizontal) location in your plasma, then you can read each spectrum in a separate row of your image, since the wavelengths are only split along the horizontal. What the heck do I mean by all of this? Look at the example data in Figure 9. Each fiber has a separate little strip in the image, one of which is highlighted in red as an example. The wavelength of the input laser is highlighted by the dashed green line.

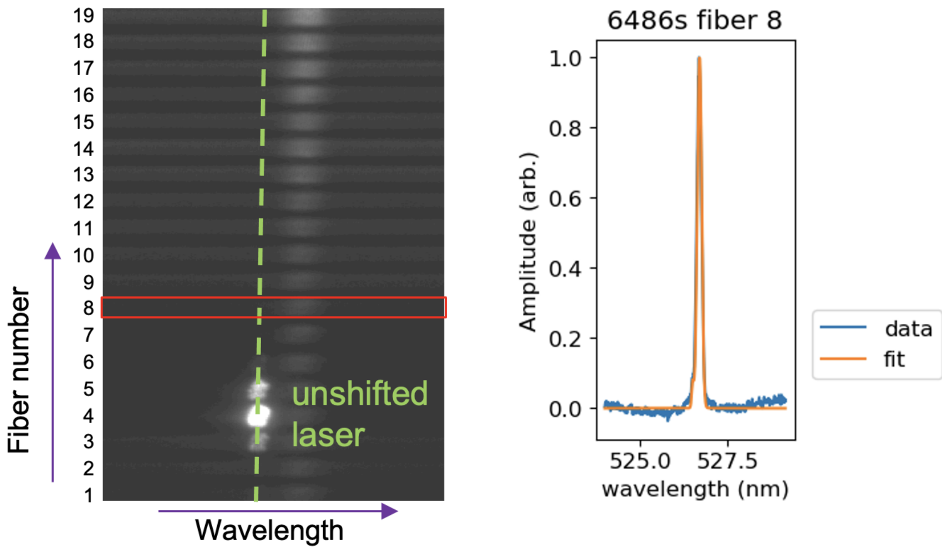


Figure 9: Some of my own raw Thomson scattering spectral data and a fitted spectrum of a single fiber. Each fiber in the bundle has a spectrum at a different vertical position (a single fiber is outlined in red), and the wavelength axis of each spectrum is on the horizontal. We see that there are several fibers picking up bright laser light that is likely reflected off of some hardware, while all the fibers collect a shifted, fainter signal corresponding to the Thomson-scattered light. The averaged, fitted spectrum of the spectrum in the red box is shown as well.

We see every fiber has a faint signal at a slightly longer wavelength than the input laser. THIS is the Thomson-scattered signal for each fiber! We can analyze these spectra individually by averaging the pixel values of all the rows of each fiber and then plotting that against the wavelength axis. Of course, calibration of the wavelength axis should be done prior to running any experiment with some light source that has a well-studied spectrum (at Cornell we use a Neon lamp). Once you have your wavelength calibrated, you can begin to fit your individual fibers' spectra to the shape functions that we mentioned above! Et viola, you've measured multiple properties with one instrument.

And that's the gist of it! Now you know the basics of how we use Thomson scattering in plasma experiments, and specifically in pulsed-power! Great job making it to the end, I am very proud of you :)

# Bibliography

J. T. Banasek. *Development of a Thomson scattering diagnostic on a pulsed power machine and its use in studying laboratory plasma jets focusing on the effect of current polarity.* PhD thesis, Cornell University, 2019.

I. H. Hutchinson. Principles of plasma diagnostics. *Plasma Physics and Controlled Fusion*, 44(12):2603, 2002.

N. A. Murphy, E. Everson, D. Stańczak, P. Heuer, P. Kozlowski, R. Malhotra, C. Arran, H. Bagherianlemraski, J. Beckers, J. Bergeron, and et al. Plasmapy. Jul 2022. doi: 10.5281/zenodo.6774350. Early development on PlasmaPy was partially supported by the U.S. Department of Energy through grant DE-SC0016363 that was funded through the NSF-DOE Partnership on Basic Plasma Science and Engineering; a Scholarly Studies grant awarded by the Smithsonian Institution; Google Summer of Code; and NASA Heliophysics Data Environment Enhancements (HDEE) grant 80NSSC20K0174. PlasmaPy is being developed with support from the U.S. National Science Foundation through grants 1931388, 1931393, 1931429, and 1931435 that were awarded through a collaborative proposal submitted to the Cyberinfrastructure for Sustained Scientific Innovation (CSSI) program.

S. V. R. Rocco. *Turbulence in gas-puff z-pinches: Applying Thomson scattering to diagnosing turbulent density and velocity fluctuations.* PhD thesis, Cornell University, 2021.

J. Sheffield, D. Froula, S. H. Glenzer, and N. C. Luhmann Jr. *Plasma scattering of electromagnetic radiation: theory and measurement techniques.* Academic press, 2010.